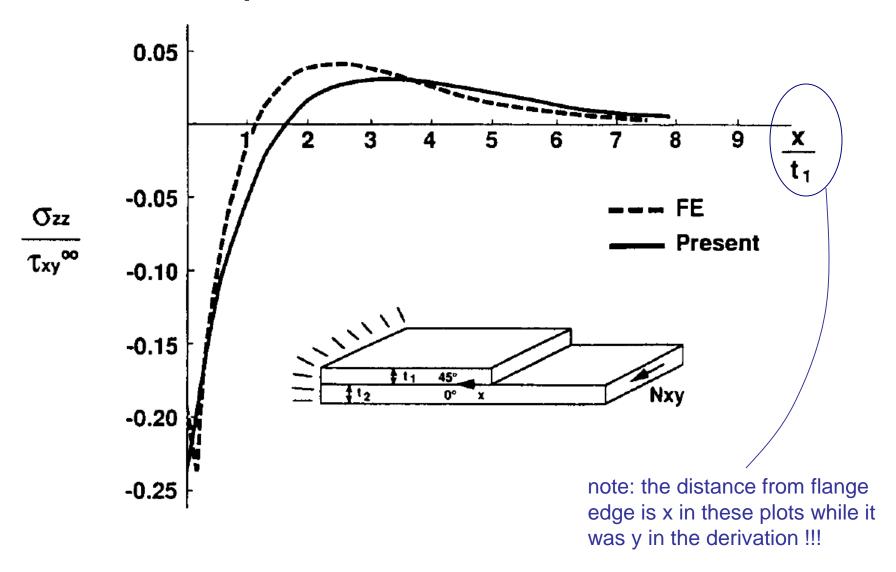
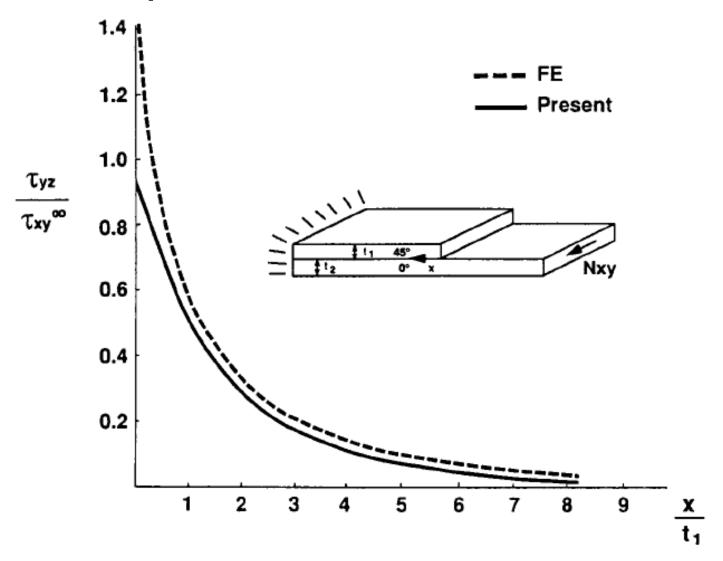
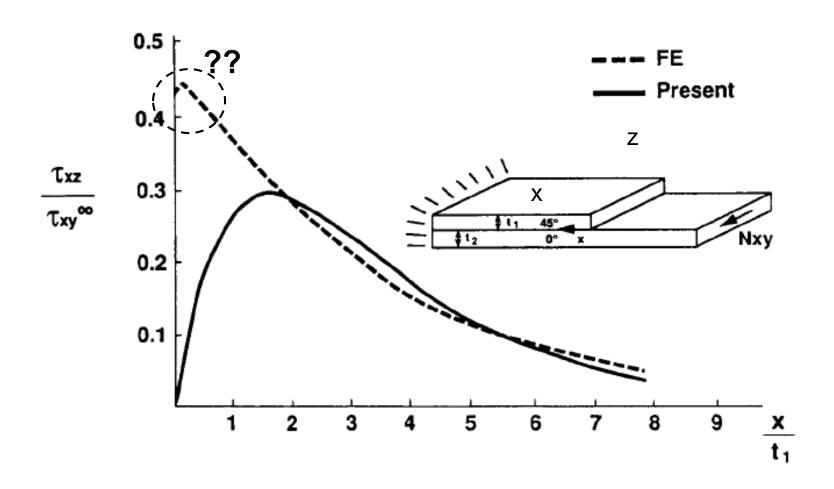
Comparison of solution to FE



Comparison of solution to FE



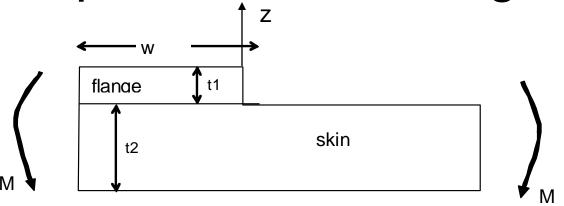
Comparison of solution to FE



More elaborate solutions to the skin-stiffener separation problem

 Cohen, D. and Hyer, M.W., "Calculation of skin-stiffener interface stresses in stiffened composite panels" NASA CR 184682, 1987

Skin-stiffener separation: Implications for design



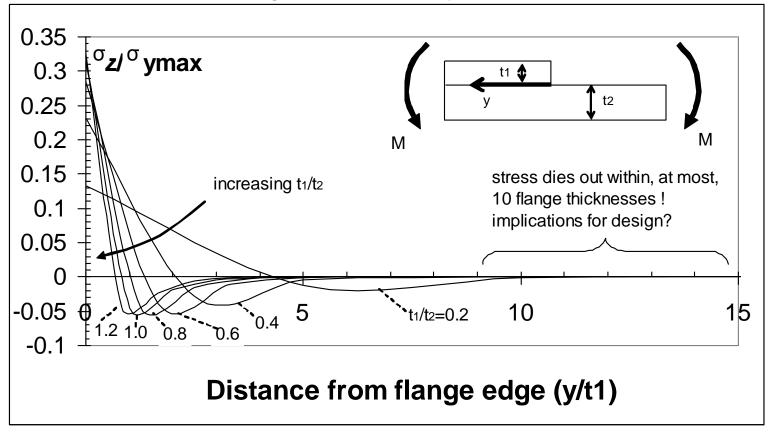
Material properties:

 $v_{xy} = 0.29$ vxz = 0.29

tply=0.152 mm vyz=0.4

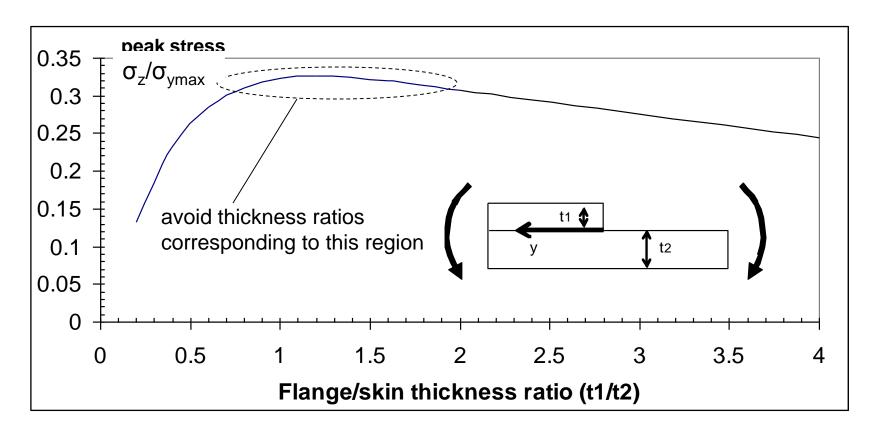
results apply to other load cases as well!

Effect of flange/skin thickness ratio skin and flange have same layup: [45/-45/-45/45]n



- stresses die out within 10 (or less) flange thicknesses
- as flange thickness increases, peak stress increases (BUT see next Figure)

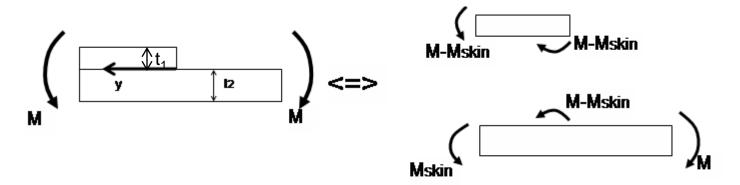
Effect of flange/skin thickness ratio on peak interlaminar stress



• Flange thickness should not be close to skin thickness!!

Why should we have $t_1 \neq t_2$?

free-body diagram of structure



- the interlaminar normal stress σ_z gives rise to M-M_{skin}
- the higher M-M_{skin} the higher the force couple created by σ_z and the higher the peak value of σ_z

Why should we have t₁≠t₂?

• from beam theory,

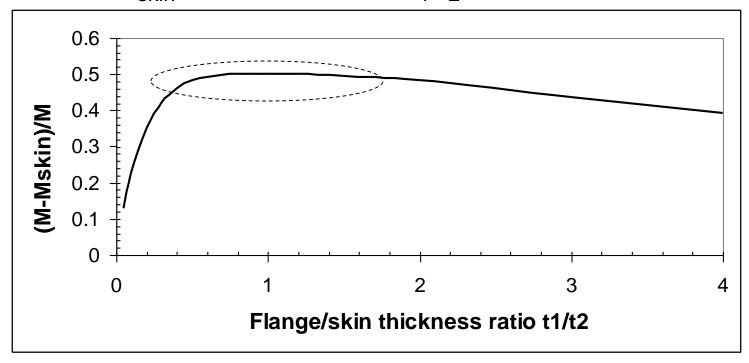
$$M - M_{skin} = M \left(1 - \frac{\frac{t_1}{t_2} \left(\left(\frac{t_1}{t_2} \right)^2 + 3 \right)}{\left(1 + \frac{t_1}{t_2} \right)^3} \right) \quad for \quad t_2 < t_1$$

$$= M \left(1 - \frac{\left(3 \left(\frac{t_1}{t_2} \right)^2 + 1 \right)}{\left(1 + \frac{t_1}{t_2} \right)^3} \right) \quad for \quad t_2 > t_1$$

$$\left(1 + \frac{t_1}{t_2} \right)^3 \quad for \quad t_2 > t_1$$

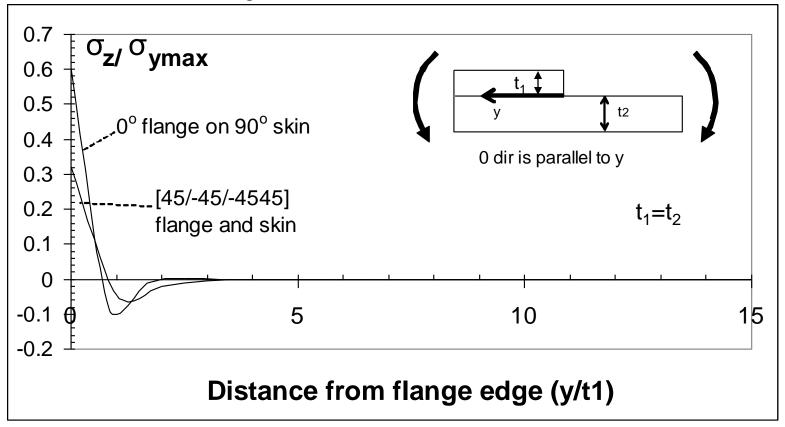
Why should we have t₁≠t₂?

plot M-M_{skin} as a function of t₁/t₂



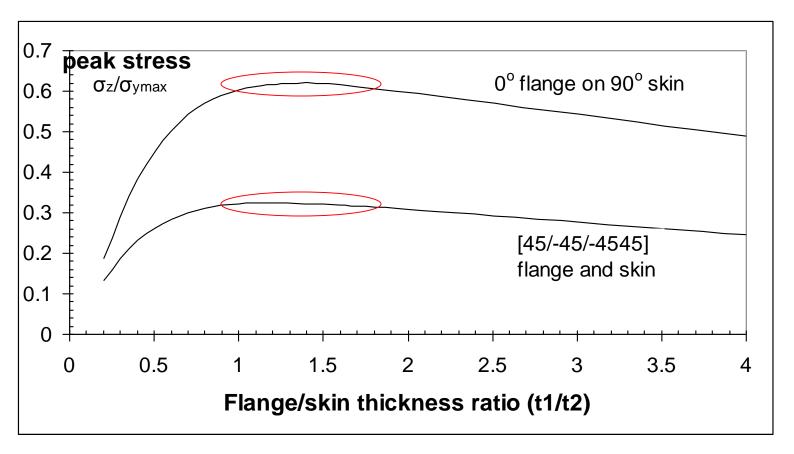
M-Mskin is maximized when t₁≈t₂!!

Effect of layup on skin-stiffener separation stresses



- 0° flange on 90° skin is worst mismatch: peak stress is twice the value of [45/-45/-45/45] flange and skin
- the higher the peak stress the faster the rate of decay (why?)

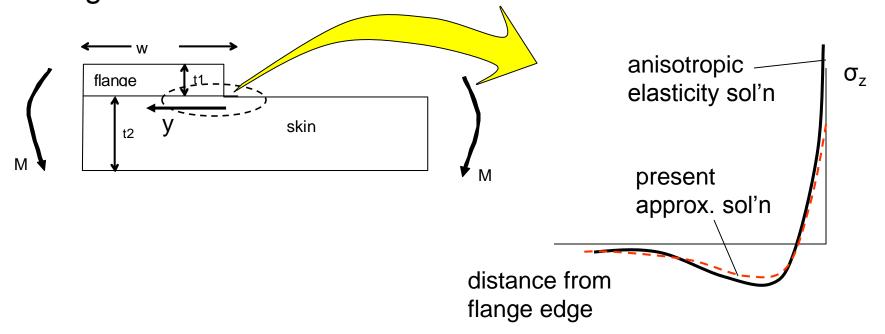
Effect of stiffness mismatch



- the stiffness mismatch almost doubles the interlaminar stresses
- roughly equal flange and skin thicknesses must still be avoided

Brief discussion on stress singularity

- comparisons based on the peak stress can be misleading
- the exact value at the flange edge is unknown; a full 3-D anisotropic elasticity solution would predict the stress is singular there

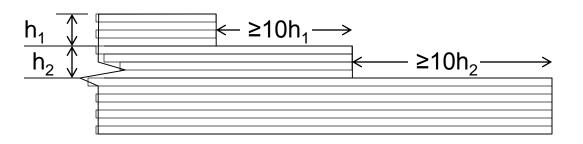


Brief discussion on stress singularity

- however, the elasticity solution is based on assuming homogeneous plies (matrix and fiber are not explicitly accounted for)
- the strength of the singularity is weak (logarithmic or less) implying that the singularity is significant over distances from the flange edge that are of the order of a few fiber diameters where the homogeneity assumption breaks down anyway; so singularity is of no consequence in design (but still need to decide what to do with peak stresses calculated)
- can combine the peak stress or some average stress or stress at a distance with an onset of delamination criterion

Skin/stiffener separation – Summary of findings

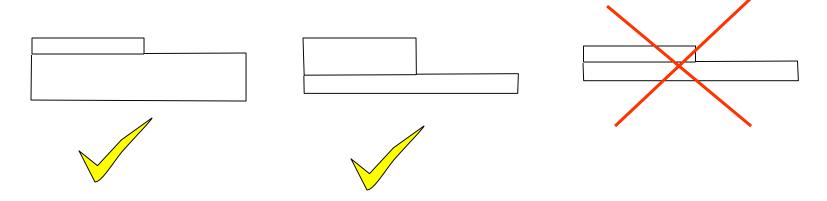
- two factors tend to make the interlaminar stresses worse:
 - flange thickness; in general, the higher it is the higher the interlaminar stresses
 - stiffness mismatch; the higher it is the higher the interlaminar stresses
- interlaminar stresses die out within ~10 flange thicknesses (important for successive or staggered plydrops):



(similar conditions are used for internal plydrops)

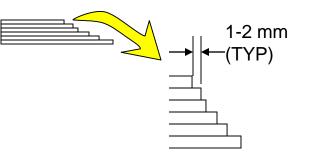
Skin/stiffener separation – Summary of findings

• in general, the flange and skin thickness should not be close to one another

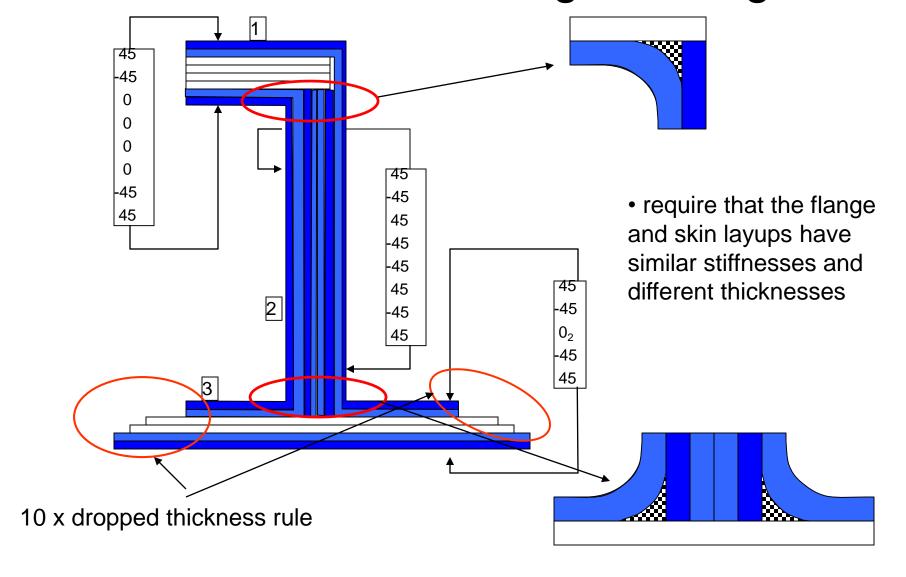


one final note: if one ply is dropped,
10(tply) is only 1-2 mm.
Placing/dropping with such accuracy in

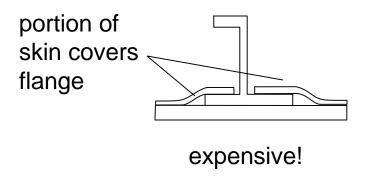
production is difficult (and expensive)

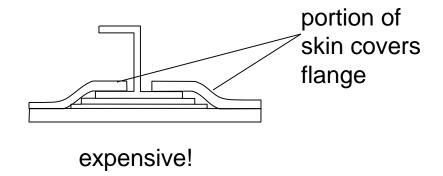


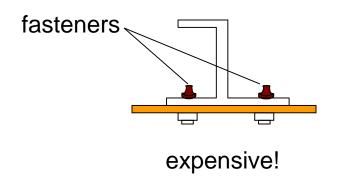
Revisiting the stiffener cross-section we have been considering all along

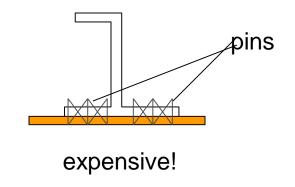


Other options for delaying skin/stiffener separation

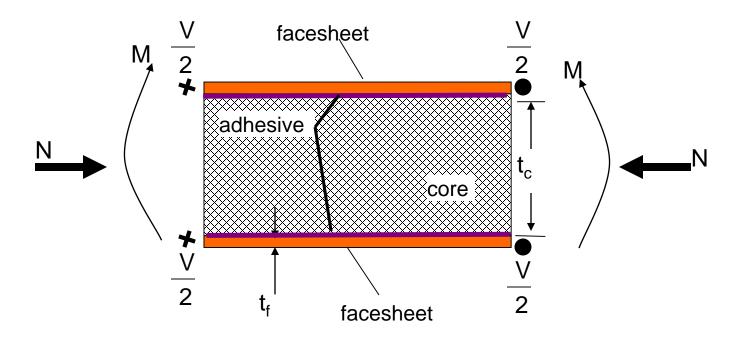








Sandwich Structure

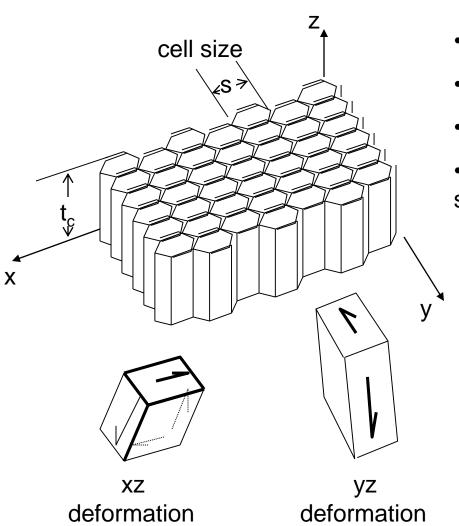


See: Plantema, F.J., Sandwich Construction, John Wiley & Sons, NY, 1966

Sandwich structure components

- facesheet: any load-carrying composite material
 - preferred to have the outer-most ply as fabric to minimize damage extent due to impact
 - layup of each facesheet does not have to be symmetric (even though it would be preferred) as long as the entire sandwich layup is symmetric
- core: honeycomb, foam, pins (X-cor[™], K-cor[™])...; required to have "sufficient" flatwise strength and stiffness and two transverse shear strengths and stiffnesses
- adhesive: film adhesive, at least
 0.08 mm thick; in some cases (e.g.
 X-cor™) it can be omitted

Honeycomb Core properties of importance



- transverse shear moduli, G_{xz} , G_{vz}
- transverse shear strengths F_{xz} , F_{yz}
- out-of-plane stiffness E_z
- out-of-plane tension and compression strengths F_z^t , F_z^c

Sandwich Structure

- by moving material away from the neutral axes, core increases drastically the bending stiffness of the structure=> increase in buckling load
- at the same time, for cores thicker than 6-7 mm, transverse shear effects become significant
- sandwich bending stiffness:

$$D_{ij} = 2(D_{ij})_f + 2(A_{ij})_f \left(\frac{t_c + t_f}{2}\right)^2$$
(5.5.1)

subscript "f" denotes single facesheet

of course, one can also get Dij by defining entire laminate with core stiffness values=0

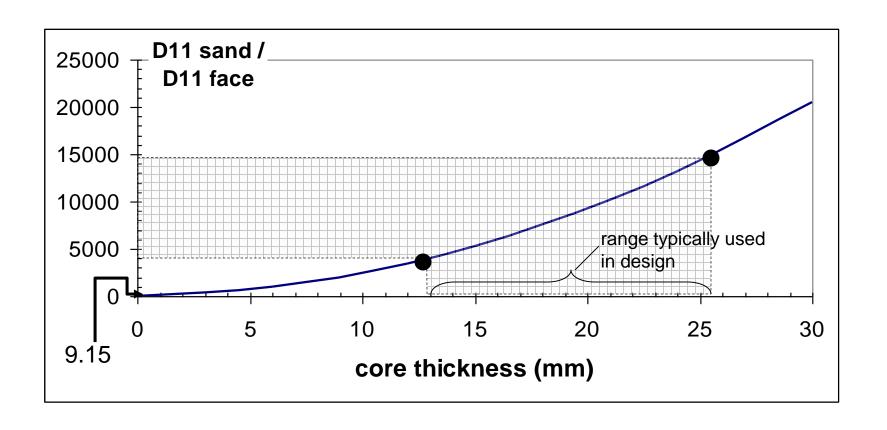
Effect of core thickness on bending stiffness of sandwich

- example we saw before; facesheet: (±45)/(0/90)/(±45)
- facesheet thickness: 0.5715 mm

A11	28912.44	N/mm
A12	12491.43	N/mm
A22	28912.44	N/mm
A66	13468.58	N/mm

D11	659.7	Nmm
D12	466.9	Nmm
D22	659.7	Nmm
D66	494.0	Nmm

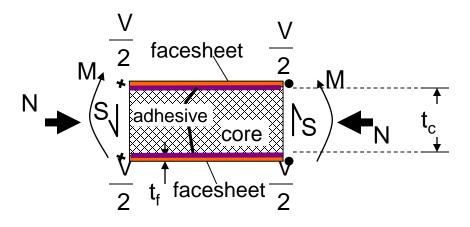
Effect of core thickness on bending stiffness of sandwich



Everything comes at a price...

- the huge increase in bending stiffness and buckling load would make the sandwich the ideal structural element, BUT
 - more failure modes must be accounted for; every component (facesheet, adhesive, core) can fail and in more than one ways
 - transitioning to adjacent structure (rampdowns) is not easy
 - attachments usually require inserts which can be expensive
 - susceptibility to moisture absorption and freezethaw cycles

Standard practice for analysis



- all loads shown are taken by the facesheet except transverse shear S which is taken by the core
- moments are resolved as force couples:

$$F_{face} = \frac{M}{t_c + t_f} \tag{5.5.2.1}$$

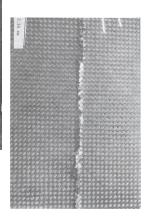
Exceptions to the rule

• the assumption that all loads (except for transverse shear) are carried by the facesheet is valid as long as the core is very soft, which is typical of most applications; if the core used has stiffness comparable to that of the facesheet (e.g. solid aluminum or carbon core) the assumption is not valid

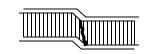
Failure analysis

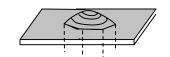
- panel buckling (buckling of sandwich panel as a whole)
- facesheet strength failure (tension, compression, shear)
- facesheet wrinkling (local buckling of facesheet on elastic foundation)
- shear crimping (precipitated by core shear failure usually after facesheet antisymmetric wrinkling)
- facesheet dimpling or intra-cellular buckling (facesheet buckling between cell boundaries)
- adhesive strength failure (tension, shear)
- core strength failure (tension, compression, shear)





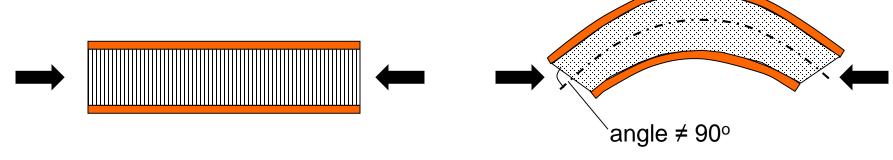






5.5.3.1

Sandwich panel buckling - compression



- unless the core is very thin, transverse shear effects are important! Plane sections remain plane but are no longer perpendicular to the mid-plane
- treat the sandwich as a wide column; then from [1], the buckling load (per unit width) for an isotropic beam is given by

$$N_{crit} = \frac{N_{Ecrit}}{1 + \frac{kN_{Ecrit}}{t_c G_c}}$$

N_{Ecrit}=Buckling load without transv shear effects

k=shear correction factor (5.5.3.1.1)

G_c=(core) shear modulus (in dir of loading)

t_c=(core) thickness

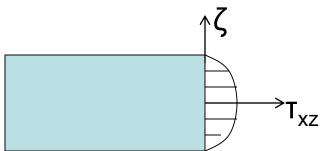
1. Timoshenko, SP, and Gere, JM, *Theory of Elastic Stability*, McGraw-Hill, NY, 1961, p 351...

 inconsistency between derived and assumed throughthe thickness strain distributions

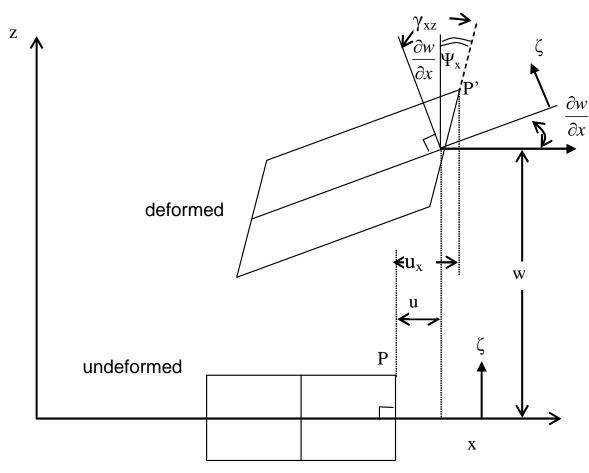
derived:

$$\tau_{xz} = \frac{3}{2} \frac{Q_x}{h} \left[1 - \left(\frac{\zeta}{h/2} \right)^2 \right]$$

 $\tau_{xz} = \frac{3}{2} \frac{Q_x}{h} \left[1 - \left(\frac{\zeta}{h/2} \right)^2 \right]$ quadratic in $\zeta =>$ strain γ_{xz} is also quadratic in ζ



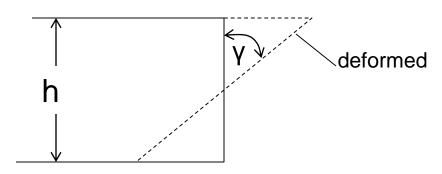
assumed in first order shear deformation theory:



$$\gamma_{xz} = \Psi_x + \frac{\partial w}{\partial x}$$

independent of ζ!

 reconcile inconsistency by making sure the work done is the same for both approaches



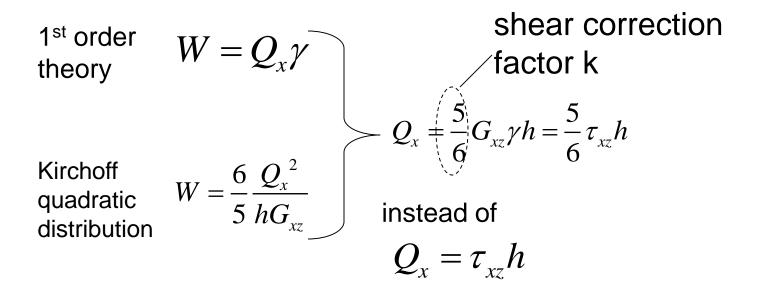
In general, work done:

$$W = \int_{-h/2}^{h/2} \tau_{xz} \gamma_{xz} dz$$

1st order theory
$$\gamma_{xz} = const = \gamma \Rightarrow W = \gamma \int_{\frac{-h/2}{2}}^{h/2} \tau_{xz} dz$$

shear force Q_x

Kirchoff quadratic
$$\tau_{xz} = \frac{3}{2} \frac{Q_x}{h} \left[1 - \left(\frac{\zeta}{h/2} \right)^2 \right] \Rightarrow W = \int_{-h/2}^{h/2} \frac{\left[\frac{3}{2} \frac{Q_x}{h} \left(1 - \left(\frac{\zeta}{h/2} \right)^2 \right) \right]^2}{G_{xz}} d\zeta = \frac{6}{5} \frac{Q_x^2}{hG_{xz}}$$



Sandwich panel buckling - compression

• for a sandwich, the through-the-thickness shear distribution is (very nearly) uniform and k≈1; then, rearranging eq. (5.5.3.1.1):

$$N_{crit} = \frac{t_c G_c}{\frac{t_c G_c}{N_{Ecrit}} + 1}$$
 (5.5.3.1.2)

• N_{Ecrit} for uni-axial compression was found before in eq. (5.2.3.1):

$$N_{Ecrit} = \frac{\pi^2 \left[D_{11} m^4 + 2(D_{12} + 2D_{66}) m^2 (AR)^2 + D_{22} (AR)^4 \right]}{a^2 m^2}$$
 (5.5.3.1.3)

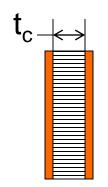
with a the panel length (load // a) and Dij given by (5.5.1)

Sandwich panel buckling - Example

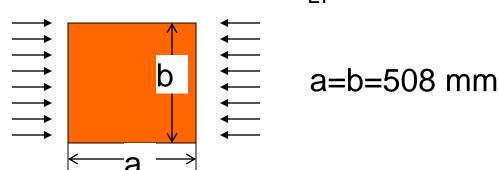
(±45)/(0/90)/ (±45) facesheet with Nomex HRH-10 1/8-3.0 core

D11	659.7			Nmm	
D12	466.9			Nmm	
D22	659.7			Nmm	
D66	494.0		Nmm		
A11	28912		N	N/mm	
A12		12491	N	N/mm	
A22	28912		N	N/mm	
A66		13469	N	N/mm	

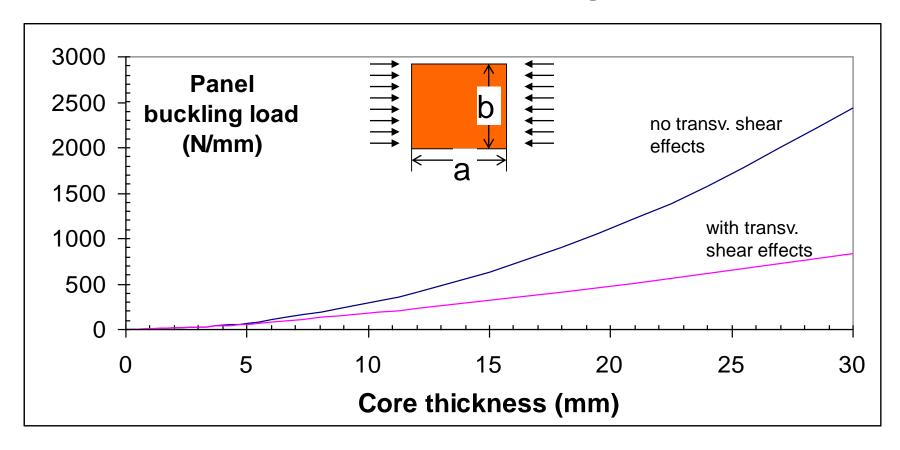
facesheet properties



Core shear stiffness G_{LT}=42.1 N/mm²

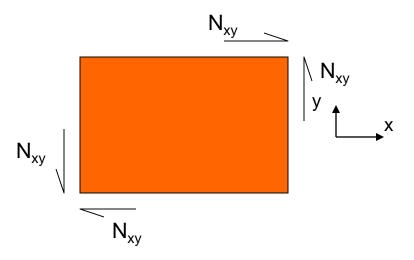


Transverse shear effect on sandwich buckling load



at t_c=3 mm the difference in buckling loads with and without shear effects is already 21%

Sandwich panel buckling under shear



$$N_{xycrit} = \frac{\left(G_{xz} + G_{yz}\right)t_c}{\frac{\left(G_{xz} + G_{yz}\right)t_c}{N_{xyEcr}} + 2}$$

(5.5.3.1.4)

• where N_{xyEcr} is the buckling load under shear for simply supported plate without shear correction

Sandwich panel under shear

• for N_{xyEcr} can use the expression derived at the very beginning of the course:

$$N_{xyEcr} = \frac{9\pi^4b}{32a^3} \left(D_{11} + 2(D_{12} + 2D_{66}) \frac{a^2}{b^2} + D_{22} \frac{a^4}{b^4} \right) \times 0.79$$
 factor introduced to correct the expression derived by 2x2 eigenvalue problem

Of (see Advanced Composites Design Guide, DoD/NASA, 1983)

$$N_{xyEcr} = \frac{\frac{\pi^4 b}{a^3}}{\sqrt{\frac{14.28}{D1^2} + \frac{40.96}{D1D2} + \frac{40.96}{D1D3}}}$$

$$D1 = D_{11} + D_{22} \left(\frac{a}{b}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{a}{b}\right)^2$$

$$D2 = D_{11} + 81D_{22} \left(\frac{a}{b}\right)^4 + 18(D_{12} + 2D_{66}) \left(\frac{a}{b}\right)^2$$

$$D3 = 81D_{11} + D_{22} \left(\frac{a}{b}\right)^4 + 18(D_{12} + 2D_{66}) \left(\frac{a}{b}\right)^2$$

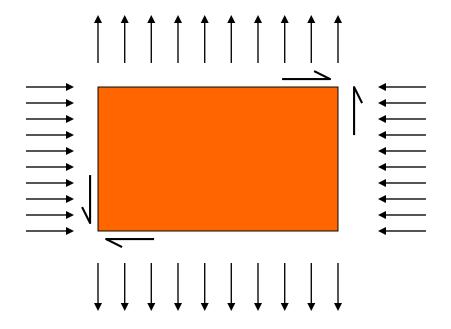
Sandwich panel under shear

• for 0≤a/b<0.5, interpolate between the value for a/b=0 given by

$$\begin{split} N_{xyEcr} &= \left(\frac{2}{a}\right)^2 \left[D_{11}^{\ 3}D_{22}\right]^{1/4} + \left(8.125 + \frac{5.05\left(D_{12} + 2D_{66}\right)}{\sqrt{D_{11}D_{22}}}\right) \\ if & \frac{\sqrt{D_{11}D_{22}}}{\left(D_{12} + 2D_{66}\right)} > 1 \\ N_{xyEcr} &= \left(\frac{2}{a}\right)^2 \sqrt{D_{11}\left(D_{12} + 2D_{66}\right)} \left[11.7 + 0.532\frac{\sqrt{D_{11}D_{22}}}{\left(D_{12} + 2D_{66}\right)} + 0.938\frac{D_{11}D_{22}}{\left(D_{12} + 2D_{66}\right)^2}\right] \\ if & \frac{\sqrt{D_{11}D_{22}}}{\left(D_{12} + 2D_{66}\right)} < 1 \end{split}$$

and the value for a/b=0.5 given in the previous page

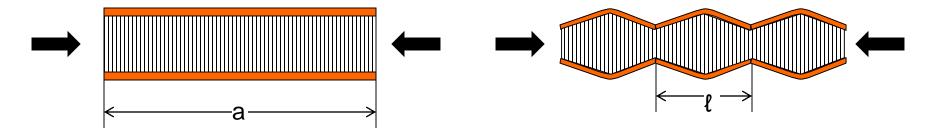
Sandwich panel buckling under combined loads



 use the interaction curves we had before but correct the individual buckling loads for for transverse shear effects

Wrinkling⁽¹⁾

 wrinkling is a local buckling phenomenon where the facesheet buckles over a characteristic half-wave length \ell unrelated to the panel length or width



(1) Hoff, N.J., Mautner, S.E., "The Buckling of Sandwich-Type Panels", J Aeronautical Sciences, July 1945, pp 285-297

Wrinkling

- two different failure modes:
 - symmetric⁽¹⁾





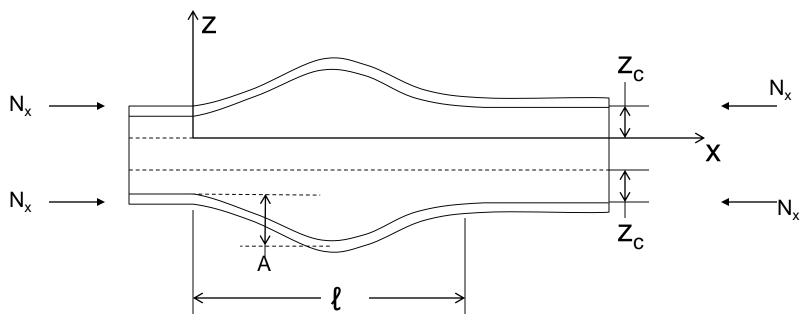
antisymmetric



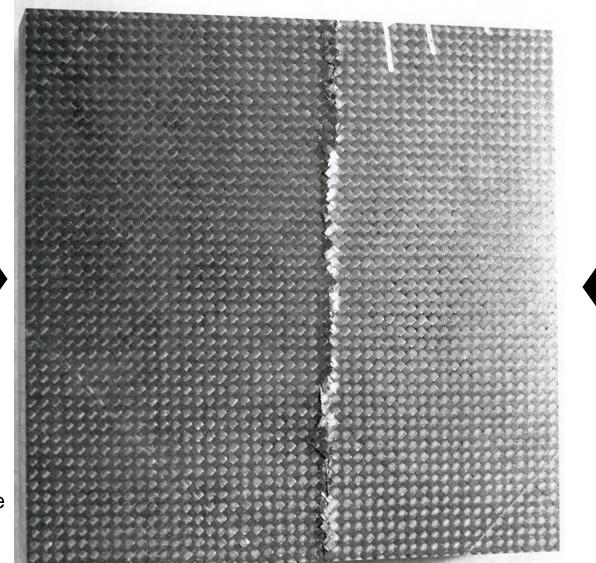
mixed modes are also possible



(1) symmetry refers to the local half-wave and is about an axis perpendicular to the plane of the panel

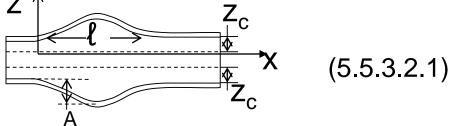


- sandwich is infinite in y direction
- facesheet buckles locally over a half-wavelength \(\ell \) with simply supported BC's
- deflections vary linearly with z over a portion z_c of the core



failure localized over a short distance; broken fibers (brooming), delaminations, adhesive and core failure

$$w = A \frac{z}{z_c} \sin \frac{\pi x}{\ell}$$



- determine the wrinkling load by energy minimization
- total energy (=potential-work done) per unit width:

$$\Pi_c = 2U_f + U_c - 2W \tag{5.5.3.2.2}$$

where U_f is energy stored in each facesheet, U_c is energy stored in the core, and W is work done by applied force N_x per facesheet

- assume that deformations in the plane of the facesheet (u,v) are negligible (=> ϵ_{xo} = ϵ_{yo} = γ_{xyo} =0)
- strains and stresses in the facesheet are given by

$$\left.\begin{array}{c}
\varepsilon_{x} = -z \frac{\partial^{2} w}{\partial x^{2}} \\
\sigma_{x} = E_{f} \varepsilon_{x}
\end{array}\right\} \qquad \left.\begin{array}{c}
\sigma_{x} \varepsilon_{x} = E_{f} z^{2} \left(\frac{\partial^{2} w}{\partial x^{2}}\right)^{2} \\
\end{array}\right. \qquad (5.5.3.2.3)$$

where E_f is the facesheet (membrane) Young's modulus appropriately calculated

then, per unit width,

$$U_{f} = \frac{1}{2} \iint E_{f} z^{2} \left[\left(\frac{\partial^{2} w}{\partial x^{2}} \right)^{2} \right]_{z=z_{c}} dz dx = \frac{1}{2} E_{f} I \int_{0}^{\ell} \left[\left(\frac{\partial^{2} w}{\partial x^{2}} \right)^{2} \right]_{z=z_{c}} dx = \frac{(E\overline{I})_{f}}{2} \int_{0}^{\ell} \left[\left(\frac{\partial^{2} w}{\partial x^{2}} \right)^{2} \right]_{z=z_{c}} dx \quad (5.5.3.2.4)$$
where $\overline{I} = t_{f}^{3}/12$ and $(\overline{EI})_{f} \approx (D_{11})_{f}$

strains and stresses in the core are given by

$$\varepsilon_{z} = \frac{\partial w}{\partial z} \qquad \text{u\approx0 by previous}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \qquad \text{assumption}$$

$$\sigma_{z} = E_{c} \varepsilon_{z}$$

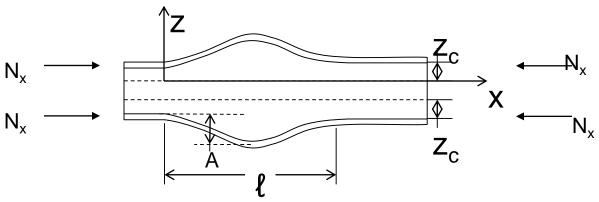
$$\tau_{xz} = G_{xz} \gamma_{xz} \qquad \qquad \tau_{xz} = G_{xz} \gamma_{xz}$$

$$(5.5.3.2.5)$$

where E_c and G_{xz} are/core axial and shear moduli respectively

then, per unit width,

$$U_{c} = \frac{1}{2} \int_{0}^{\ell} \int_{-z_{c}}^{z_{c}} \left(E_{c} \left(\frac{\partial w}{\partial z} \right)^{2} + G_{xz} \left(\frac{\partial w}{\partial x} \right)^{2} \right) dz dx$$
 (5.5.3.2.6)



work done per facesheet (per unit width):

$$W = N_{x}\delta$$

$$\delta = \ell - \int_{0}^{\ell} dx$$

$$(dx)^{2} + (dw)^{2} = (ds)^{2} \Rightarrow dx = ds\sqrt{1 - \left(\frac{dw}{ds}\right)^{2}}$$

$$\sqrt{1 - \left(\frac{dw}{ds}\right)^{2}} \approx 1 - \frac{1}{2}\left(\frac{dw}{ds}\right)^{2} \quad for \quad small \quad \left(\frac{dw}{ds}\right)^{2}$$

$$\frac{dw}{ds} \approx \frac{dw}{dx} \quad for \quad small \quad w$$

$$(5.5.3.2.7)$$

• use (5.5.3.2.1) to calculate necessary derivatives:

$$\left(\frac{\partial w}{\partial x}\right)^2 = \frac{A^2 z^2}{z_c^2} \frac{\pi^2}{2\ell^2} \left(1 + \cos\frac{2\pi x}{\ell}\right)$$

$$\left(\frac{\partial w}{\partial z}\right)^2 = \frac{A^2}{z_c^2} \frac{1}{2} \left(1 - \cos\frac{2\pi x}{\ell}\right)$$

$$\left(\frac{\partial^2 w}{\partial x^2}\right)^2 = \frac{A^2 z^2}{z_c^2} \frac{\pi^4}{2\ell^4} \left(1 - \cos\frac{2\pi x}{\ell}\right)$$

$$(5.5.3.2.8)$$

• substitute in (5.5.3.2.2) and carry out the integrations

$$\Pi_{c} = \frac{\pi^{4}}{2\ell^{3}} (E\overline{I})_{f} A^{2} + \frac{1}{2} \left[\frac{E_{c}\ell}{z_{c}} + \frac{1}{3} G_{xz} z_{c} \frac{\pi^{2}}{\ell} \right] A^{2} - N_{x} \frac{A^{2} \pi^{2}}{2\ell}$$
(5.5.3.2.9)

• to determine the wrinkling load N_{xwr} , minimize the energy (differentiate Π_c with respect to A and set the result equal to zero):

$$\frac{\partial \Pi_{c}}{\partial A} = 0 \Rightarrow 2A \left[\frac{\pi^{4}}{2\ell^{3}} (E\overline{I})_{f} + \frac{1}{2} \left[\frac{E_{c}\ell}{z_{c}} + \frac{1}{3} G_{xz} z_{c} \frac{\pi^{2}}{\ell} \right] - N_{x} \frac{\pi^{2}}{2\ell} \right] = 0 \Rightarrow$$

$$N_{xwr} = \frac{\pi^{2} (E\overline{I})_{f}}{\ell^{2}} \left(\frac{E_{c}\ell^{2}}{\pi^{2}z_{c}} + G_{xz} \frac{z_{c}}{3} \right) \qquad (5.5.3.2.10)$$

$$\text{contribution from beam on elastic foundation contribution when it consists of torsional springs}$$

$$\text{with result from with result from section 5.3.2} \qquad K_{mm} = \frac{\pi^{2} EI}{I_{c}^{2}} \left(m^{2} + \frac{kL^{4}}{\pi^{4} (EI) m^{2}} \right)$$

same when m=1 and k= E_c/z_c

- the expression for the wrinkling load, eq. (5.5.3.2.10) is still in terms of two unknown constants, ℓ and z_c
- since we are looking for the lowest buckling load, determine the value of ℓ that minimizes N_{xwr}

$$\frac{\partial N_{xwr}}{\partial \ell} = 0 \Rightarrow \ell = \pi \left(\frac{(E\overline{I})_f}{E_c} z_c\right)^{1/4} \tag{5.5.3.2.11}$$

• substituting in (5.3.2.10):

$$N_{xwr} = \frac{2\sqrt{E_c(\overline{EI})_f}}{\sqrt{z_c}} + \frac{G_{xz}z_c}{3}$$
 (5.5.3.2.12)

in a similar manner, the "active" portion of the core,
 z_c can be determined,

$$\frac{\partial N_{xwr}}{\partial z_c} = 0 \Rightarrow z_c = 3^{2/3} \left(\frac{E_c(\overline{EI})_f}{G_{xz}^2} \right)^{1/3}$$

• and substituting for $\overline{I}_f = t_f^3/12$

$$z_c = \frac{3^{2/3}}{12} t_f \left(\frac{E_c E_f}{G_{xz}} \right)^{1/3} = 0.91 t_f \left(\frac{E_c E_f}{G_{xz}} \right)^{1/3}$$

(5.5.3.2.13)

• use this expression to substitute in (5.5.3.2.11) to get \(\ell\)

$$\ell = \frac{\pi 3^{1/6}}{12^{1/3}} t_f \left(\frac{E_f}{\sqrt{E_c G_{xz}}} \right)^{1/3} = 1.648 t_f \left(\frac{E_f}{\sqrt{E_c G_{xz}}} \right)^{1/3}$$
 (5.5.3.2.14)

• use (5.5.3.2.13) and (5.5.3.2.14) to substitute in (5.5.3.2.12) to get

$$N_{xwr} = 0.91t_f (E_f E_c G_{xz})^{1/3}$$
 (5.5.3.2.15)

• this expression has been derived by many people making different assumptions and using different methods; typically, the only thing that changes as the approach and assumptions change is the coefficient; for a good review of most of the methods, see: Ley, R.P., Lin, W., and Mbanefo, U., "Facesheet Wrinkling in Sandwich Structures", NASA/CR-1999-208994, January 1999