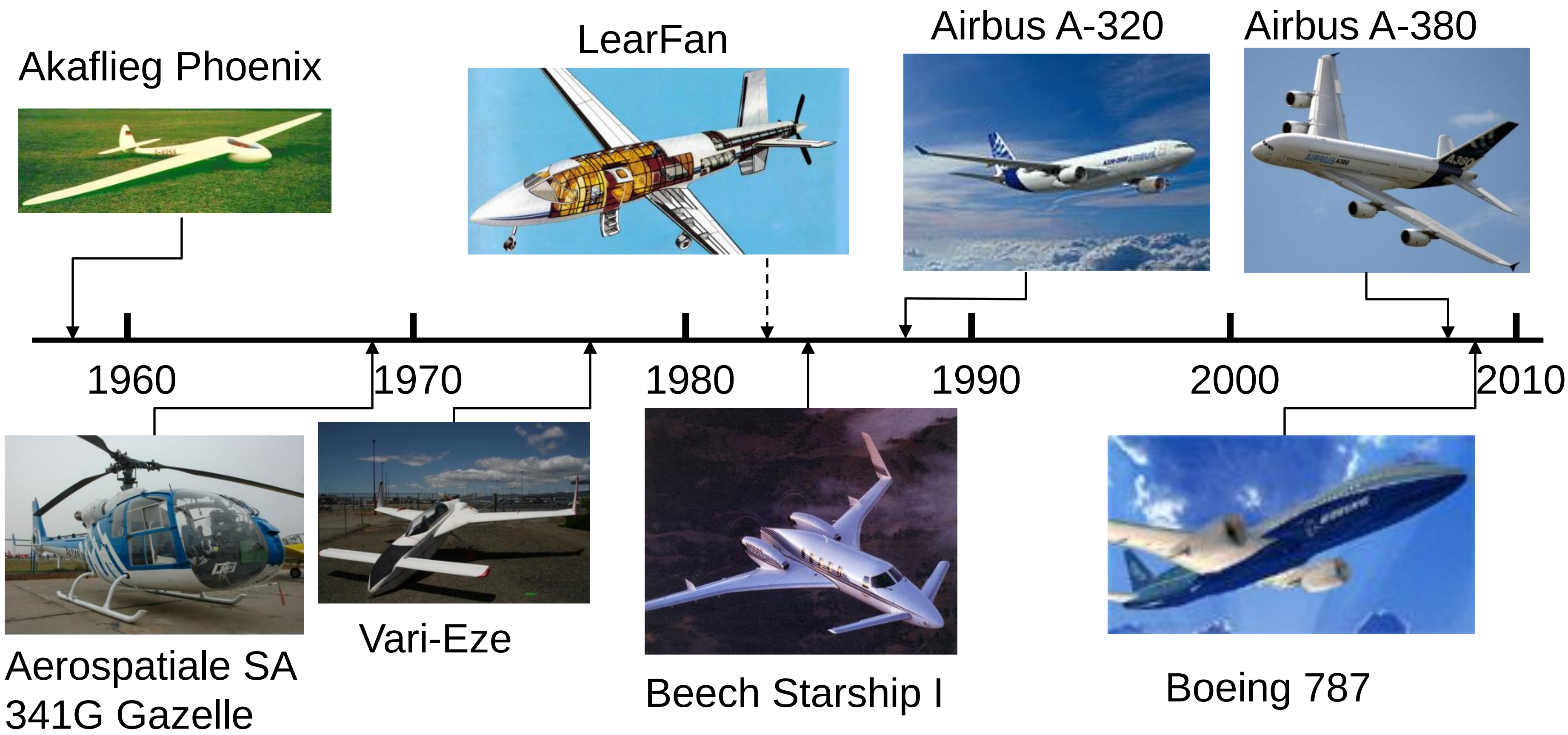
Significant Composites Usage in Aircraft (commercial A/C)

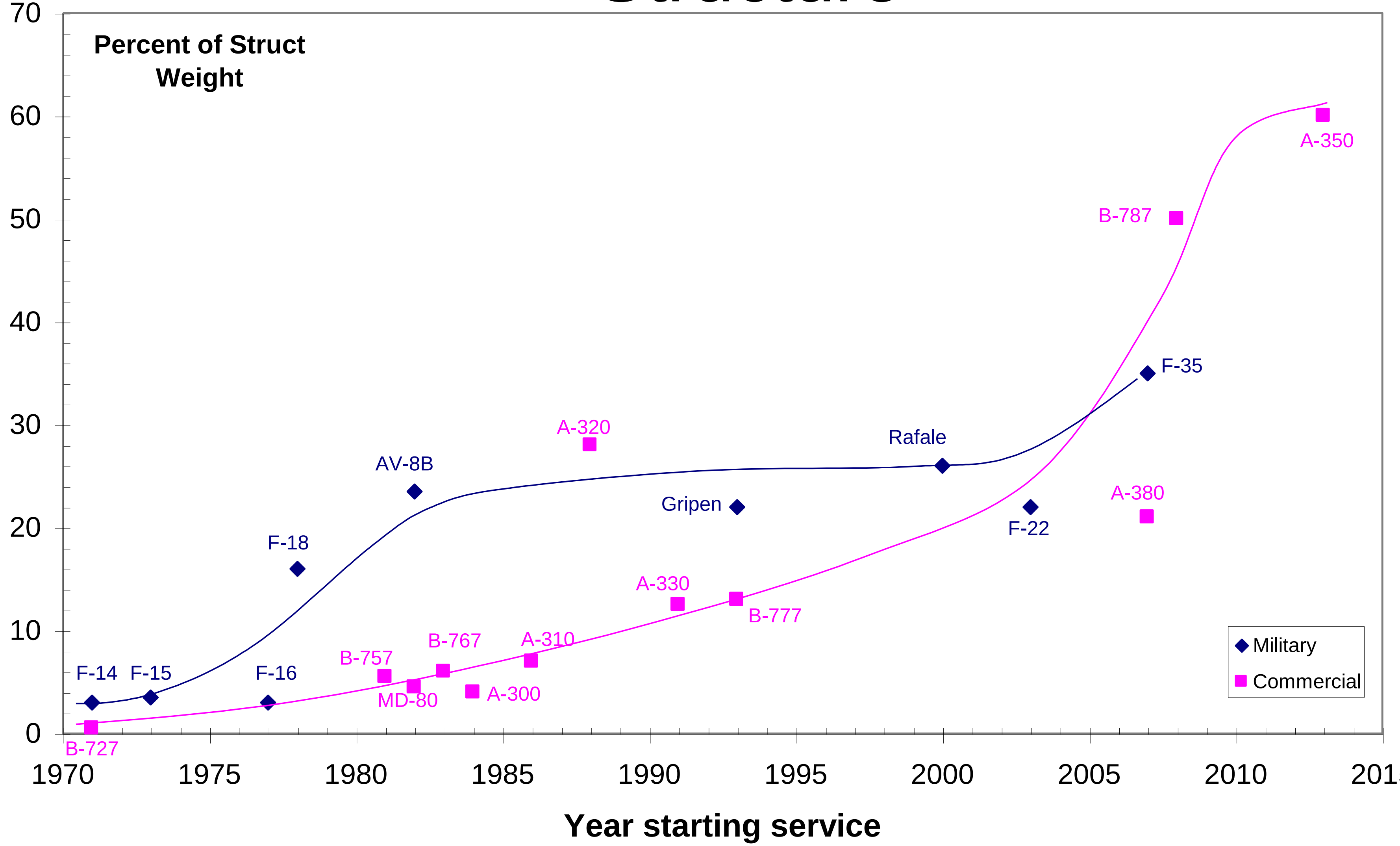


Sailplanes & Helicopters

Ultralights / Civil Aviation

Commercial Transport

Implementation of Composites in A/C Structure



Review of Classical Laminated-Plate Theory

- In metals, we have plane stress, plane strain, generalized three-dimensional **isotropic** elasticity…
- In composites we still have plane stress, plane strain, generalized three-dimensional **anisotropic** elasticity…

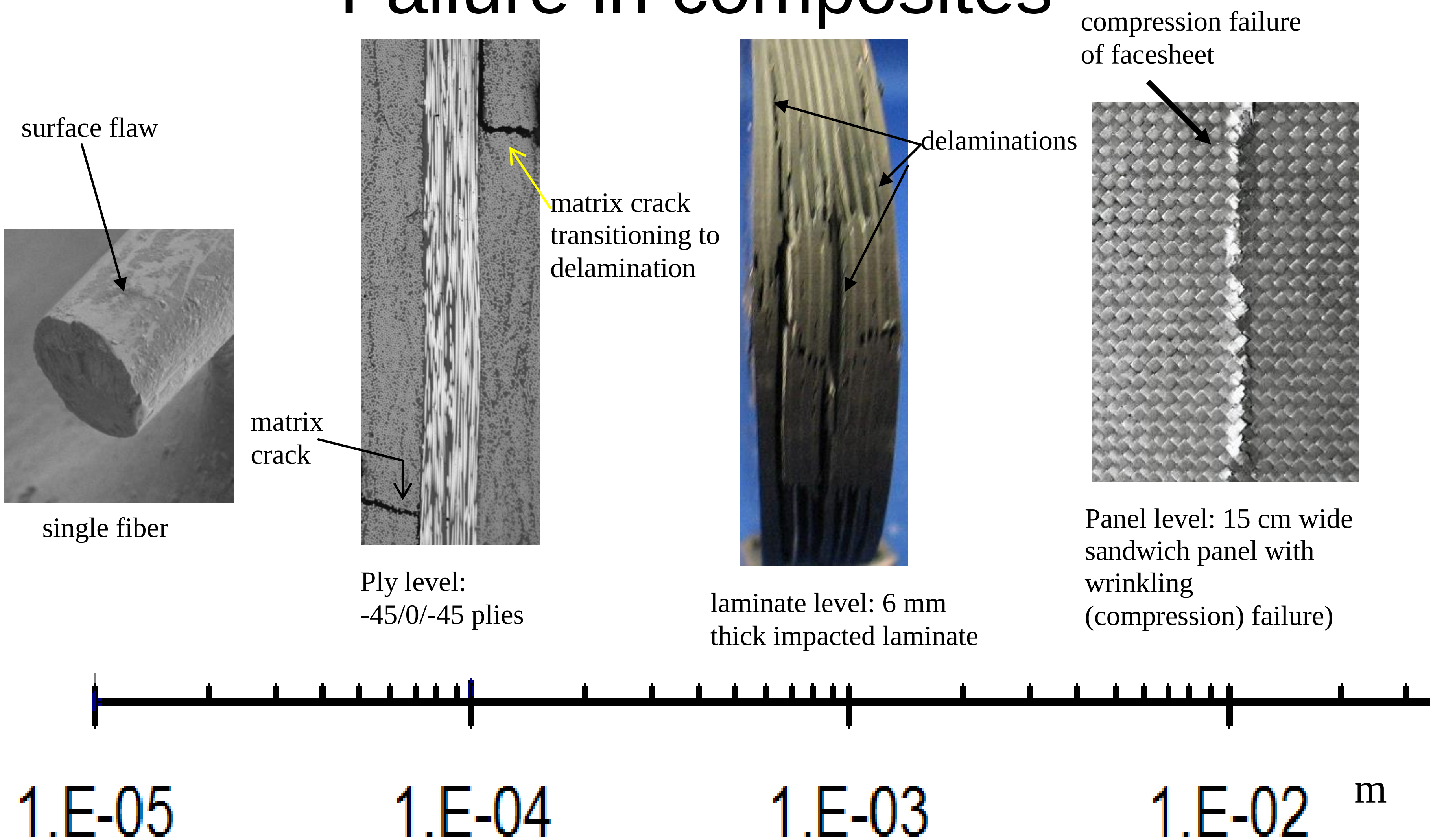
Composites vs Metals (theoretical modeling)

- The main differences are:
  - Elastic response:
    - Isotropic materials need two elastic constants, any two of E, G, v.
    - Anisotropic materials need many more, as many as 21
    - We will concentrate in a class of anisotropic materials called **orthotropic** for which four elastic constants are sufficient: E_x, E_y, G_{xy}, v_{xy}

Composites vs Metals (theoretical modeling)

- The main differences are (cont’d):
  - Failure:
    - Unlike yielding in metals, which is well defined and relates to one phenomenon (plastic deformation) composites exhibit multiple failure modes which interact:
      - matrix yielding
      - matrix cracking
      - delamination (separation of layers in a laminate)
      - fiber cracking
      - failure of fiber/matrix interface

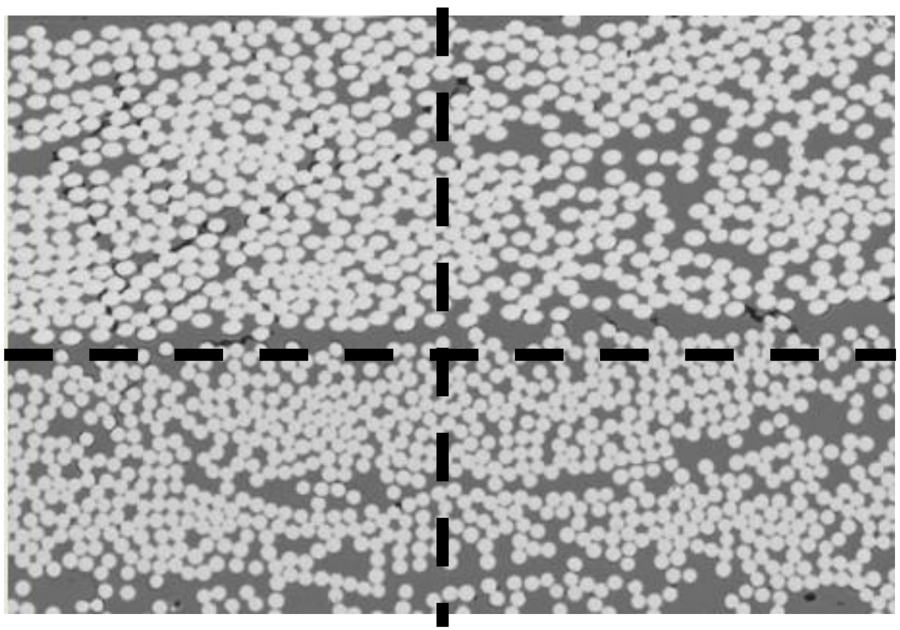
Failure in composites



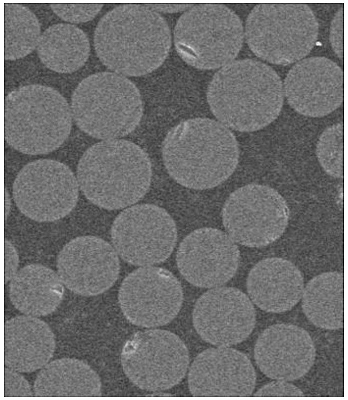
- different failures at different scales

Orthotropic composite materials

- Composite: consisting of more than one constituents, e.g. fibers and matrix
- Orthotropic materials: There are two planes of symmetry perpendicular to each other

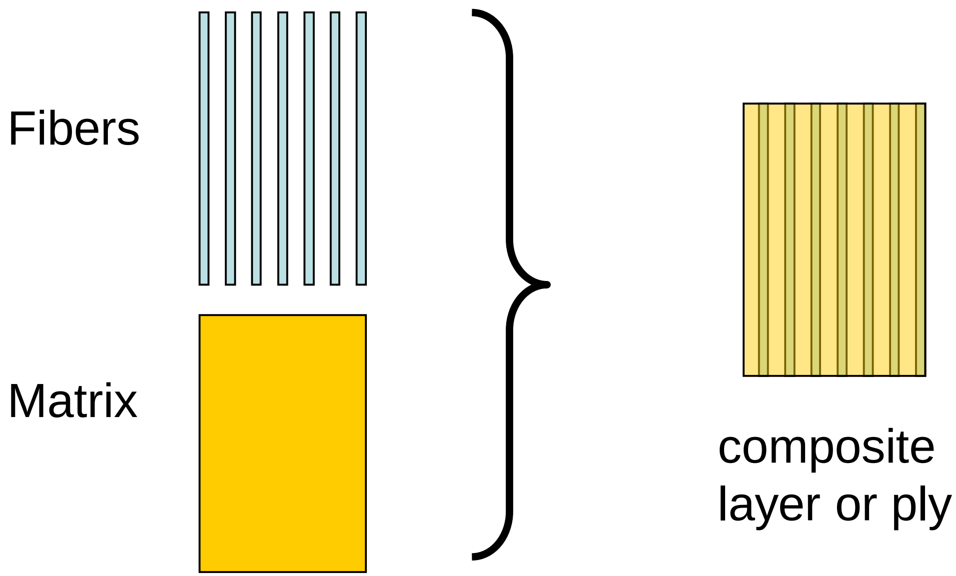


Note: In a "real" composite, the symmetry may not be perfect. It will also depend on how closely we zoom in



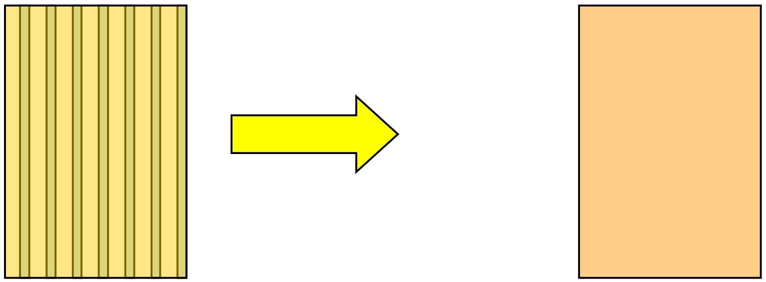
Modeling of composites

- Micromechanics: Model fiber and matrix separately



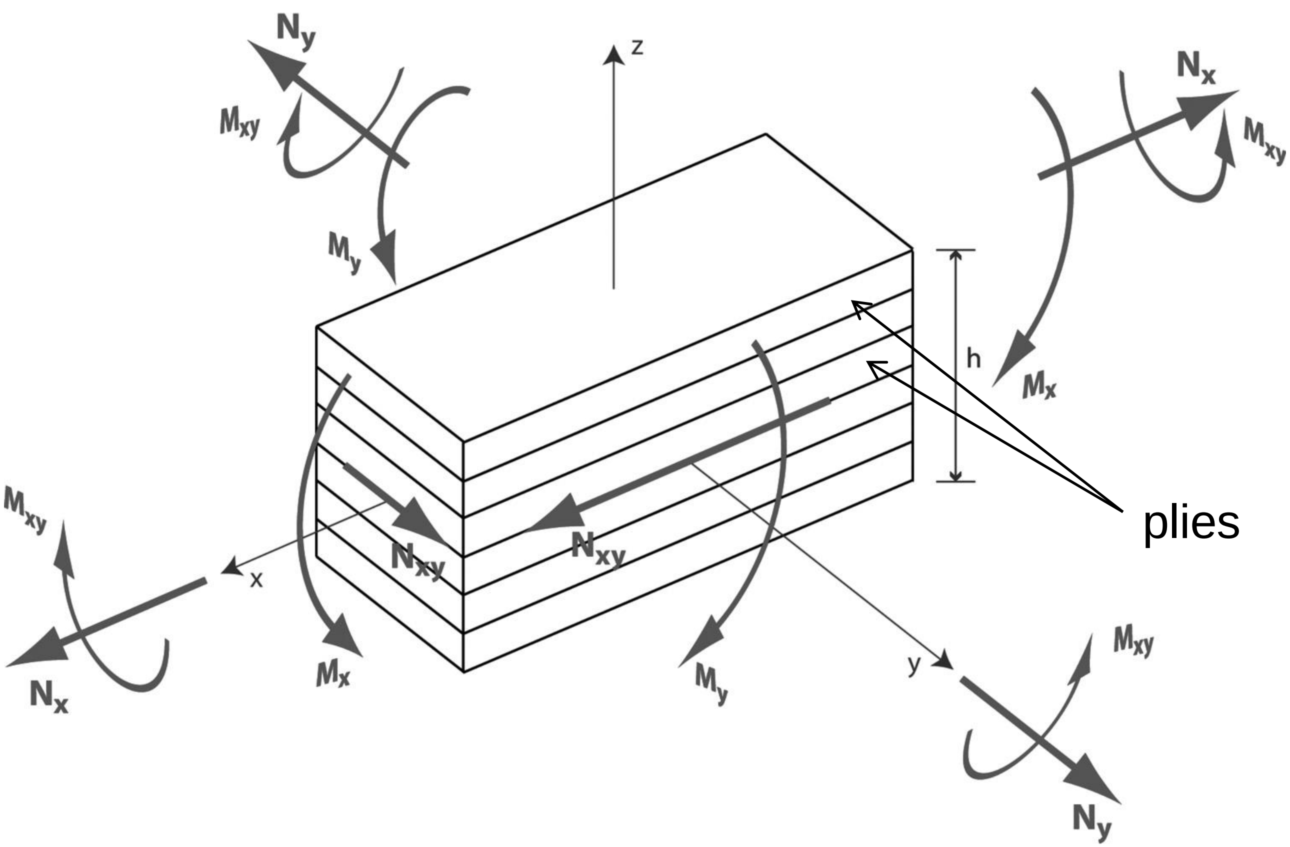
Modeling of composites

- "Meso-mechanics" (ply level)
  - smear fiber and matrix properties to an equivalent orthotropic material



The building block

- Stack plies (or laminae) of different orientations together to get a laminate



General equations

- Constitutive relations for a single ply (stress-strain equations)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 \\ E_{12} & E_{22} & E_{23} & 0 & 0 & 0 \\ E_{13} & E_{23} & E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$

x and 1 parallel to the fibers in this case

Compare to…

$$\sigma_x = \frac{1-\nu}{(1+\nu)(1-2\nu)} E\varepsilon_x + \frac{\nu}{(1+\nu)(1-2\nu)} E\varepsilon_y + \frac{\nu}{(1+\nu)(1-2\nu)} E\varepsilon_z$$

$$\sigma_y = \frac{\nu}{(1+\nu)(1-2\nu)} E\varepsilon_x + \frac{1-\nu}{(1+\nu)(1-2\nu)} E\varepsilon_y + \frac{\nu}{(1+\nu)(1-2\nu)} E\varepsilon_z$$

$$\sigma_z = \frac{\nu}{(1+\nu)(1-2\nu)} E\varepsilon_x + \frac{\nu}{(1+\nu)(1-2\nu)} E\varepsilon_y + \frac{1-\nu}{(1+\nu)(1-2\nu)} E\varepsilon_z$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{xz} = G\gamma_{xz}$$

$$\tau_{xy} = G\gamma_{xy}$$

… for metals

Simplification: “Thin” laminates

- If laminate is thin enough,

$$\sigma_z \approx \tau_{yz} \approx \tau_{xz} \approx 0$$

- Then,

$$\left.\begin{aligned}
\sigma_x &= E_{11}\varepsilon_x + E_{12}\varepsilon_y + E_{13}\varepsilon_z \\
\sigma_y &= E_{12}\varepsilon_x + E_{22}\varepsilon_y + E_{23}\varepsilon_z \\
0 &= E_{13}\varepsilon_x + E_{23}\varepsilon_y + E_{33}\varepsilon_z \\
0 &= E_{44}\gamma_{yz} \\
0 &= E_{55}\gamma_{xz} \\
\tau_{xy} &= E_{66}\gamma_{yz}
\end{aligned}\right\}$$

eliminate all strains that have one “z” subscript to obtain the 2-D stress-strain equations

2-D stresses for single ply

- For a coordinate system with one axis aligned with the fibers

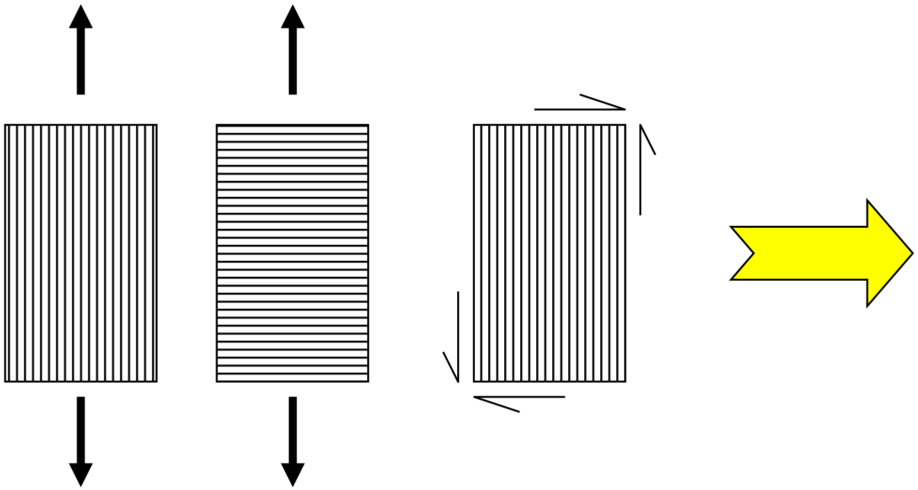
$$
\begin{aligned}
\sigma_x &= \underbrace{\left(E_{11} - \frac{E_{13}^2}{E_{33}}\right)}_{\mathbf{Q_{xx}}} \varepsilon_x + \underbrace{\left(E_{12} - \frac{E_{13}E_{23}}{E_{33}}\right)}_{\mathbf{Q_{xy}}} \varepsilon_y \\
\sigma_y &= \left(E_{12} - \frac{E_{13}E_{23}}{E_{33}}\right) \varepsilon_x + \underbrace{\left(E_{22} - \frac{E_{23}^2}{E_{33}}\right)}_{\mathbf{Q_{yy}}} \varepsilon_y \\
\tau_{xy} &= \underbrace{E_{66}}_{\mathbf{Q_{ss}}} \gamma_{xy}
\end{aligned}
$$

- or, using matrix notation:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{SS} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}
$$

Relation of elastic constants to engineering constants

- Standard engineering tests,



E_L, E_T, G_{LT}, ν_{LT}

engineering constants

- It can be shown that:

$$Q_{xx} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{yy} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{xy} = \frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}} = \frac{\nu_{TL}E_L}{1 - \nu_{LT}\nu_{TL}}$$

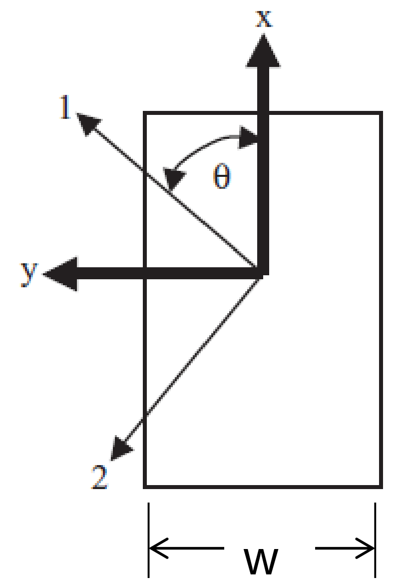
$$Q_{ss} = G_{LT}$$

Stress transformation

- What happens if the coordinate system does not have one axis aligned with the fibers?
- Recall that stress transforms according to:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & (\cos^2\theta - \sin^2\theta) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

(2nd order tensor)



- In an analogous fashion, stiffness transforms according to

$$Q_{11}^{(\theta)} = m^4 Q_{xx} + n^4 Q_{yy} + 2m^2n^2 Q_{xy} + 4m^2n^2 Q_{ss}$$

$$Q_{22}^{(\theta)} = n^4 Q_{xx} + m^4 Q_{yy} + 2m^2n^2 Q_{xy} + 4m^2n^2 Q_{ss}$$

$$Q_{12}^{(\theta)} = m^2n^2 Q_{xx} + m^2n^2 Q_{yy} + (m^4 + n^4) Q_{xy} - 4m^2n^2 Q_{ss}$$

$$Q_{66}^{(\theta)} = m^2n^2 Q_{xx} + m^2n^2 Q_{yy} - 2m^2n^2 Q_{xy} + (m^2 - n^2)^2 Q_{ss}$$

$$Q_{16}^{(\theta)} = m^3n Q_{xx} - mn^3 Q_{yy} + (mn^3 - m^3n) Q_{xy} + 2(mn^3 - m^3n) Q_{ss}$$

$$Q_{26}^{(\theta)} = mn^3 Q_{xx} - m^3n Q_{yy} + (m^3n - mn^3) Q_{xy} + 2(m^3n - mn^3) Q_{ss}$$

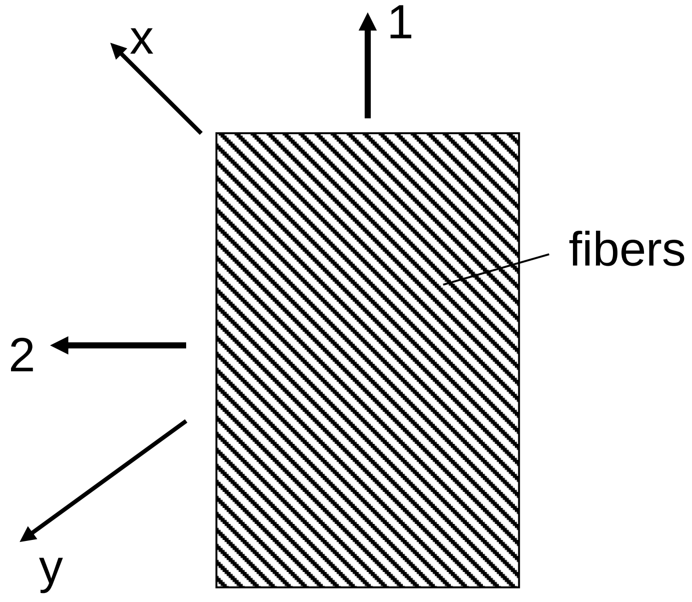
(4th order tensor)

where $m = \cos\theta$ and $n = \sin\theta$

Resulting in…

- For a ply of any orientation:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \qquad (1.1)$$



Note that if the fibers in a ply are not aligned with the coordinate system of interest, 12 in this case, Q_{16} and Q_{26} are different than zero!

From ply to laminate

- All this describes a single ply. How do we go from ply stiffness to laminate stiffness?
- Recall cross-sections of multiple materials
- The equivalent stiffness $(EA)_{eq}$ is given as the sum of the individual (EA) values of the components
- This means that we can use the transformation equations to get the stiffness of each ply in the axis system of interest and multiply by the ply cross-sectional area:

$$(EA)_{ply\ \ i} = \left(Q_{ij}\right)_{ply\ \ i} A_{ply\ \ i} = \left(Q_{ij}\right)_{ply\ \ i} t^{(i)} w$$

$t^{(i)}$ is thickness of ply i and w is the width

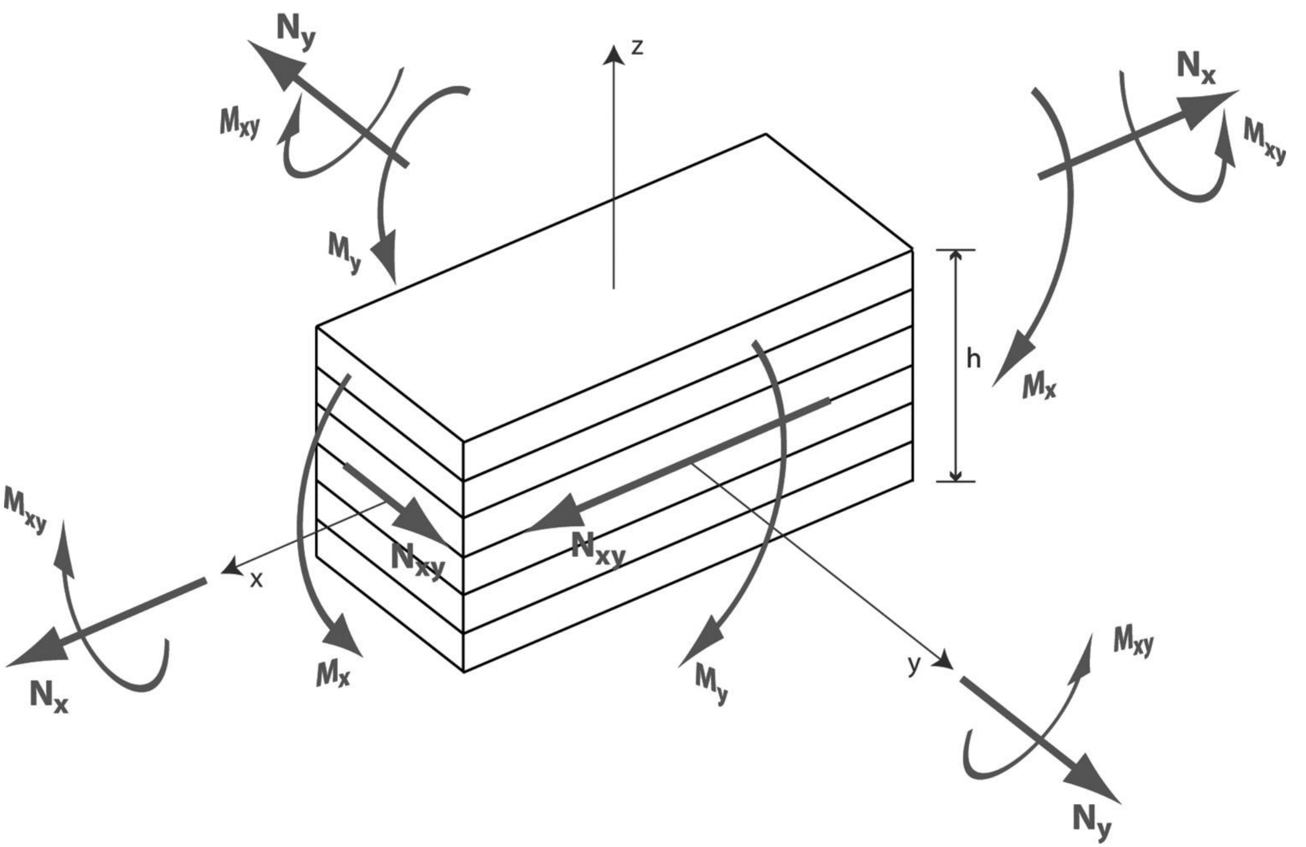
$$(EA)_{lam} = \sum_{i=1}^{n} (EA)_{ply\ \ i}$$

Stress resultants

- Before proceeding, it is convenient to invoke the fact that most laminates are very thin compared to their in-plane dimensions
- As a (good) approximation, one can average stresses over the laminate thickness
- Then, instead of stresses, which change from ply to ply, we work with stress resultants: N_x, N_y, Nxy,Mx, M_y, M_{xy}

Stress resultants

- Define force and moment resultants: N_x, N_y, Nxy,Mx, M_y, M_{xy}



$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \mathrm{d}z \qquad (1.2)$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y \mathrm{d}z$$

$$N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} \mathrm{d}z$$

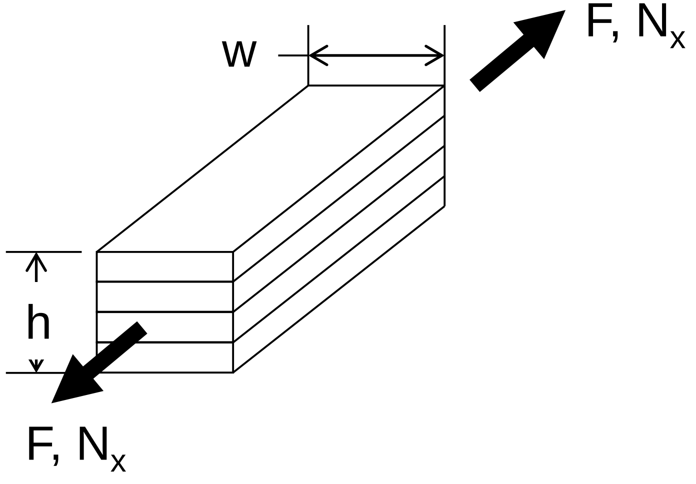
$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z \mathrm{d}z$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z \mathrm{d}z$$

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z \mathrm{d}z$$

Stress resultants

- This means that the force and moment resultants are forces and moments **per unit width**
- For example, for the case of uniaxial tension



$$\sigma_o = \frac{F}{wh}$$

with σ_o the average applied stress

- But the average applied stress is, by definition:

$$\sigma_o = \frac{1}{h}\int_{-h/2}^{h/2} \sigma_x dz$$

- Using our definition of N_x:

$$\sigma_o = \frac{N_x}{h}$$

Membrane (in-plane) behavior

- Assume in-plane loads are applied, and laminate has no bending (=> symmetric layup)
- This means that the strains ε_1, ε_2, and γ_{12} are constant through the thickness and equal to the mid-plane strains, ε_{1o}, ε_{2o}, and γ_{12o}
- Take eq. (1.1) and integrate with respect to z. For example the first equation will be:

$$\int_{-h/2}^{h/2} \sigma_{11} dz = \int_{-h/2}^{h/2} Q_{11} \varepsilon_{xo} dz + \int_{-h/2}^{h/2} Q_{12} \varepsilon_{yo} dz + \int_{-h/2}^{h/2} Q_{16} \gamma_{xyo} dz \Rightarrow$$

$$N_x = \underbrace{\left[\int_{-h/2}^{h/2} Q_{11} dz \right]}_{A_{11}} \varepsilon_{xo} + \underbrace{\left[\int_{-h/2}^{h/2} Q_{12} dz \right]}_{A_{12}} \varepsilon_{yo} + \underbrace{\left[\int_{-h/2}^{h/2} Q_{16} dz \right]}_{A_{16}} \gamma_{xyo}$$

Membrane (in-plane) behavior

- Similarly for the other two equations in (1.1). Using matrix notation:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{1.3}$$

- where A_{ij} is given by:

$$A_{ij} = \int_{-h/2}^{h/2} Q_{ij} dz$$

Note we are mixing 1,2 and x,y here indiscriminantly; (not a good idea but convenient)

- and because Q_{ij} is constant within each ply, the integral can be substituted by a summation:

$$A_{ij} = \sum_{k=1}^{N} \left(Q_{ij}\right)^{(k)} \underbrace{\left(z_k - z_{k-1}\right)}_{\text{ply thickness}}$$

where k denotes the kth ply, z_o is at the bottom of the laminate and N is the total number of plies in the laminate