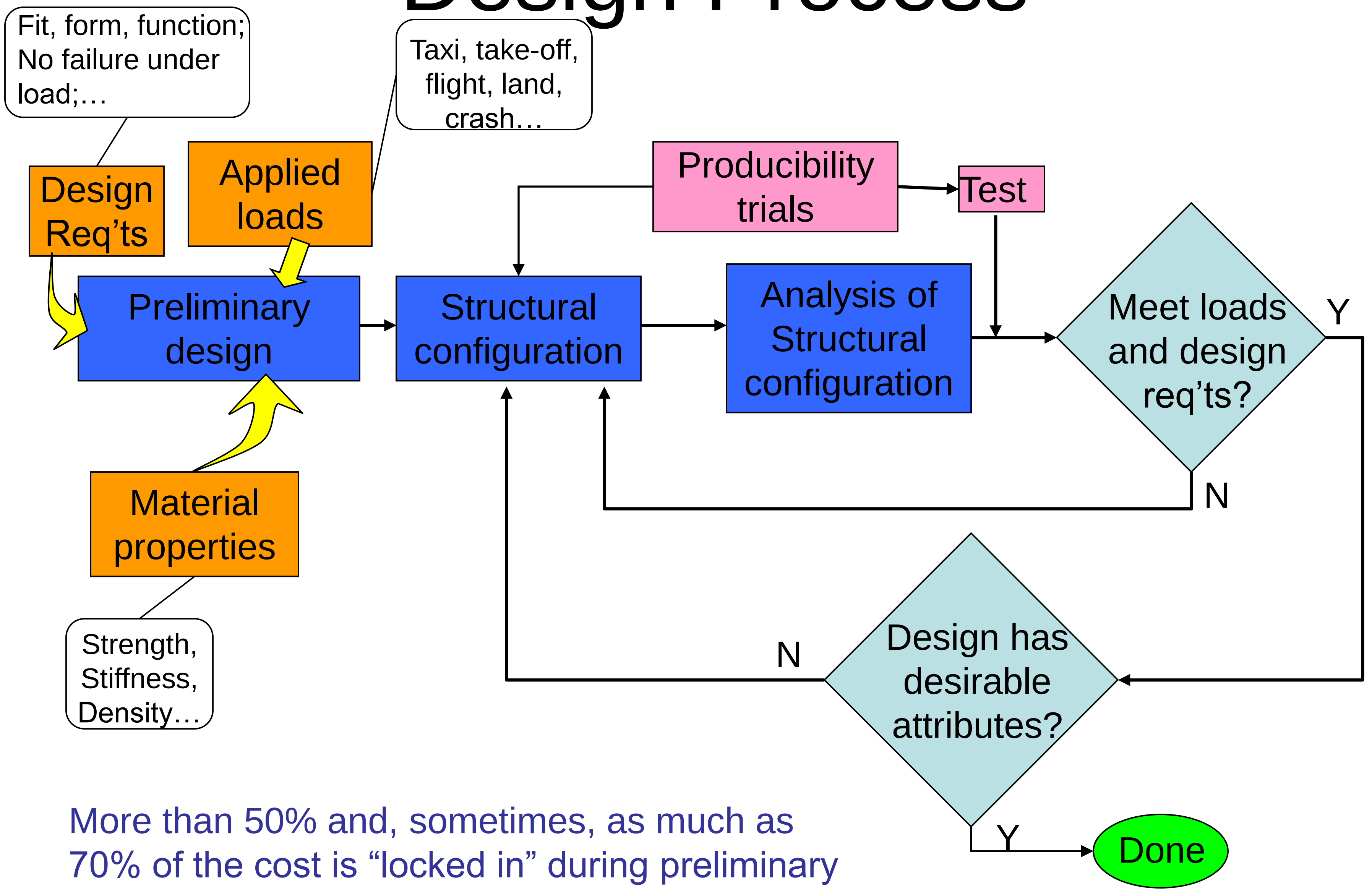
Design Process

- Obtain applied loads
- Obtain material properties
- Come up with a structural configuration
- Analyze structural configuration to
  - meet loads without failure (static and fatigue)
  - minimize weight
  - minimize cost
  - optimize other quantities (frequency, radar signature, etc.)
- Iterate

check if
concept is
manufacturable
verify analysis
by tests

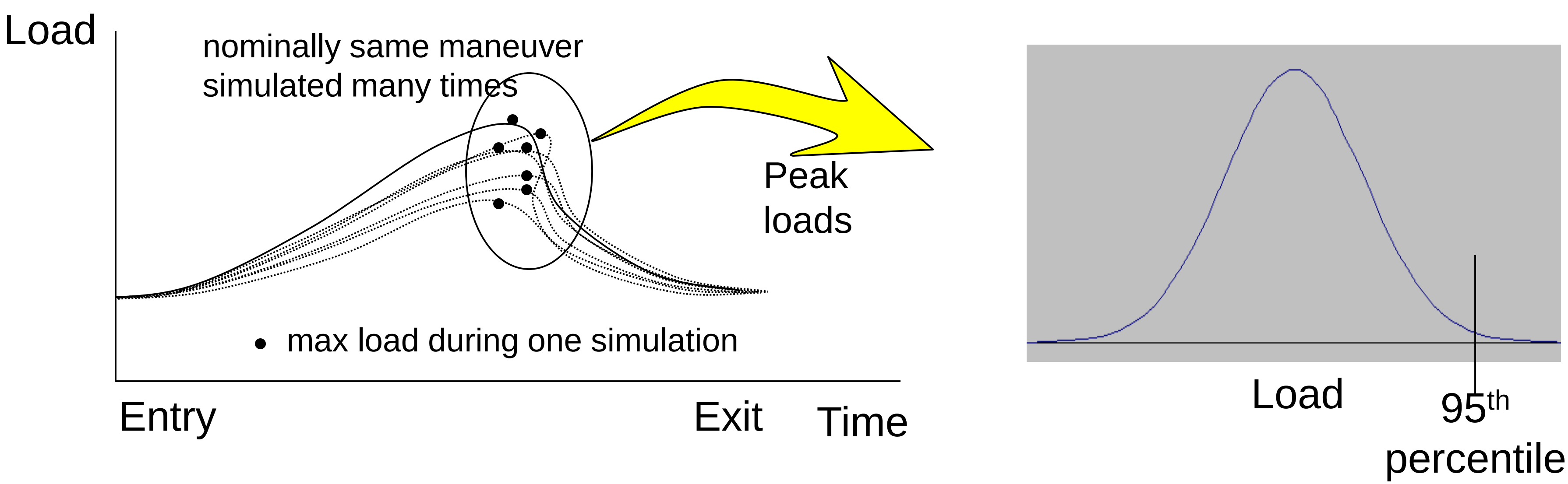
Design Process



More than 50% and, sometimes, as much as 70% of the cost is "locked in" during preliminary design!!

Applied Loads and Usage

- Different users perform the same maneuver differently resulting in different loads
- 95th percentile (or some other high percentile) of max load occurring in maneuver simulation) to cover most cases

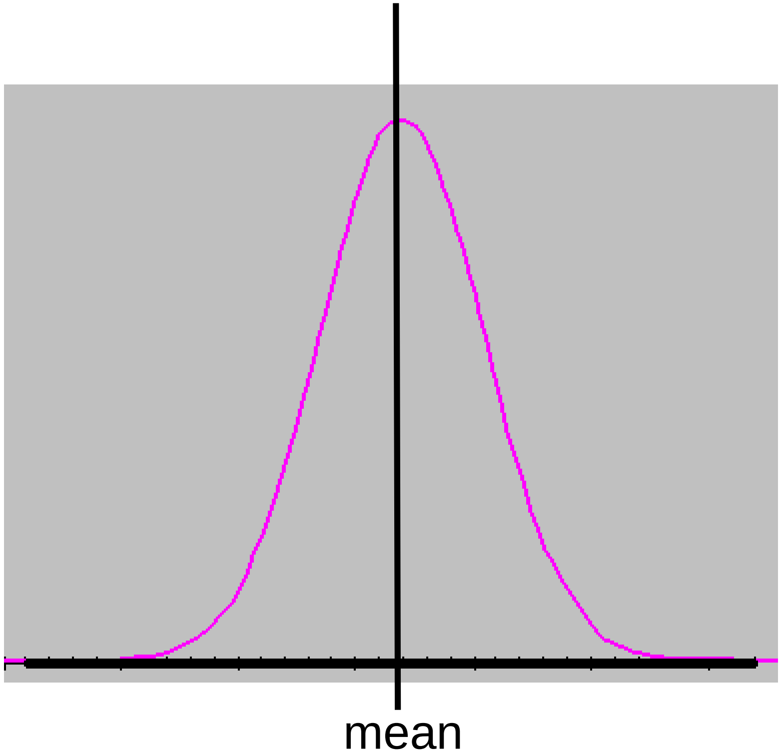


Material

- Variability (scatter)
  - raw material
  - manufacturing
  - etc.

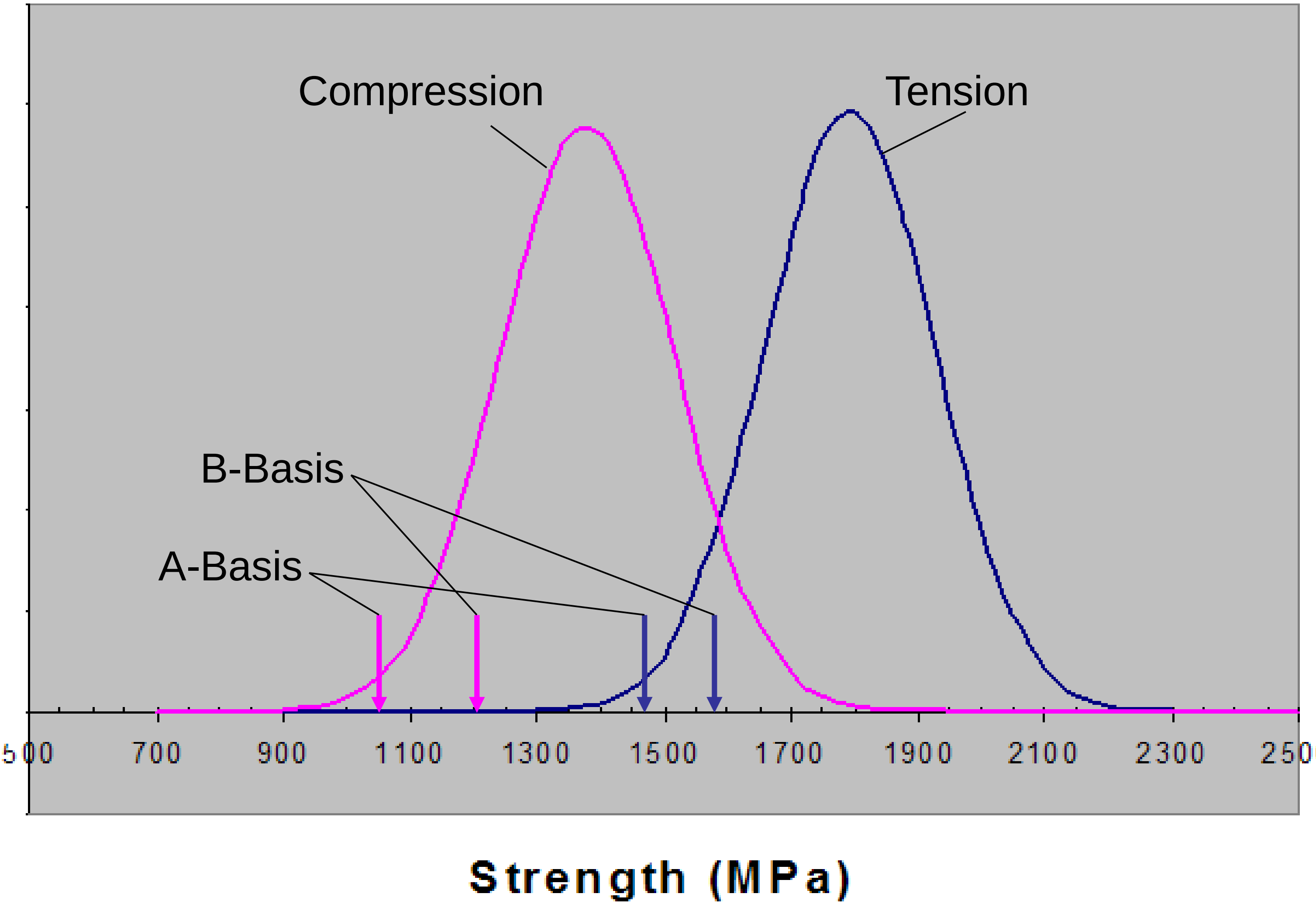
A,B-Basis values

- Environmental effects
- Effect of damage



Material scatter

Typical Uni-directional Gr/E (0 deg)



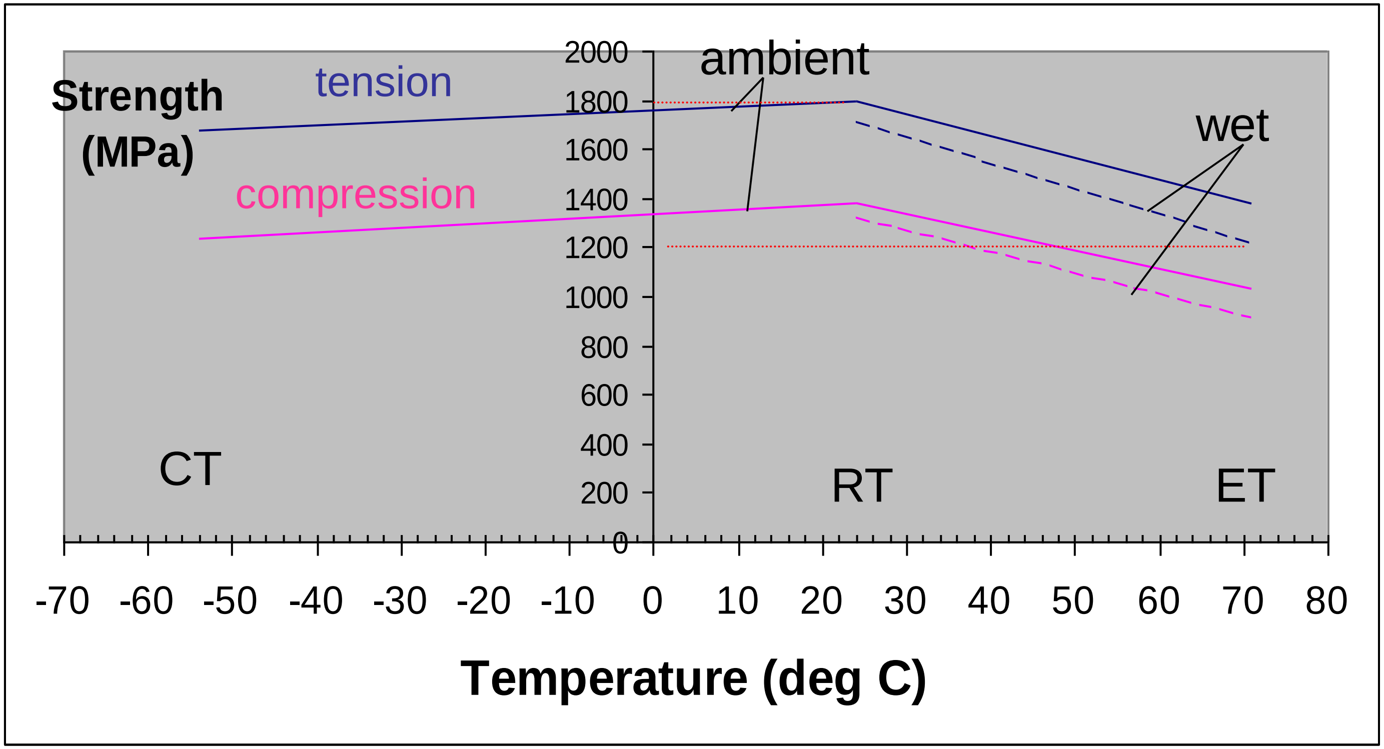
Material scatter

- B-Basis (10^{th} percentile): 90% of the strength tests will have higher failure load
- A-Basis (1 percentile): 99% of the strength tests will have higher failure load

• typically, A-Basis is used for single-load path primary structure and B-Basis is used for multiple-load path primary or secondary structure (failure does not lead to loss of vehicle)

Effect of environment

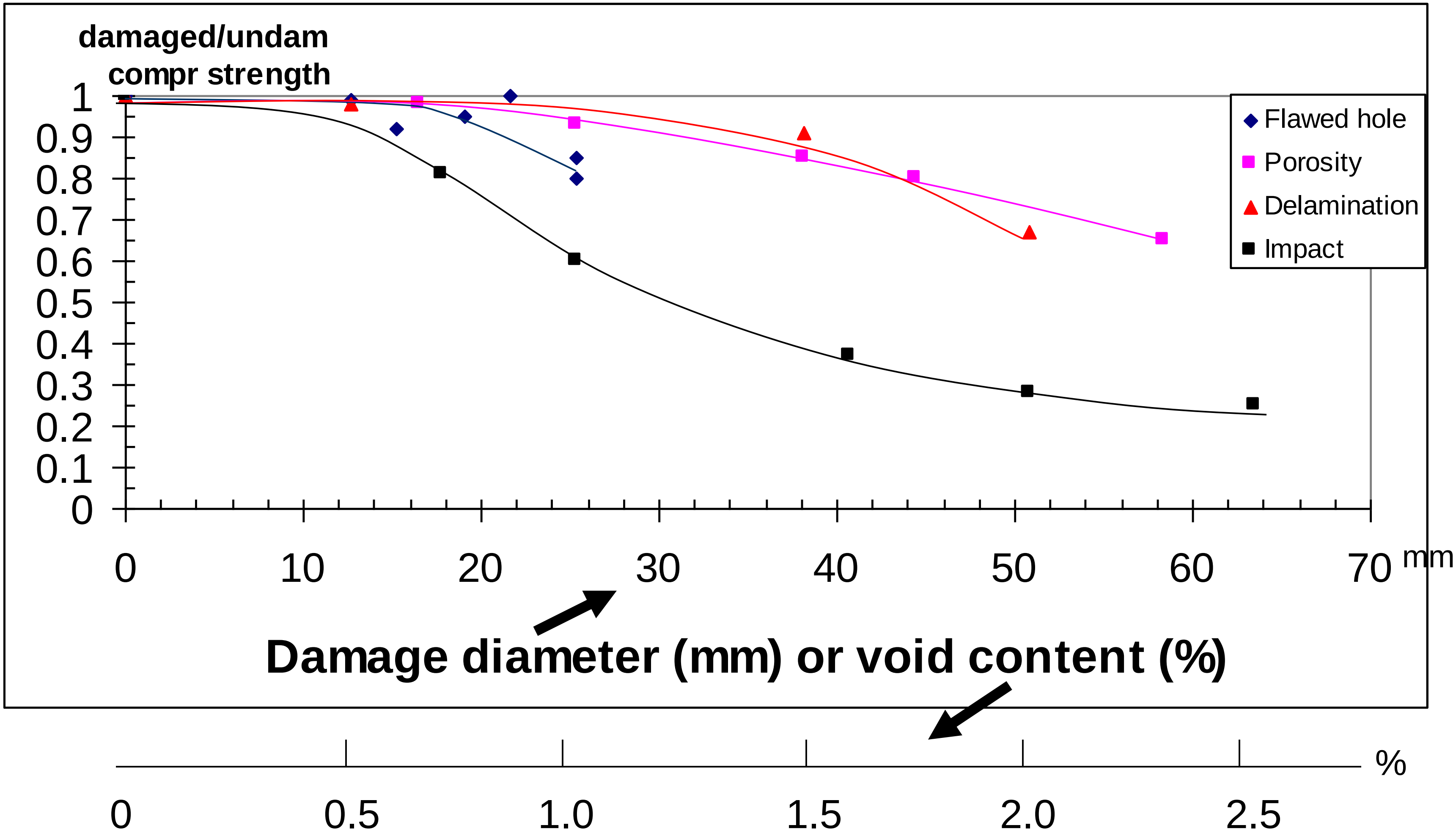
Typical Uni-directional Gr/E



- "knockdown due to environment (=separation between red horizontal lines): 5-30% depending on property

Effect of damage

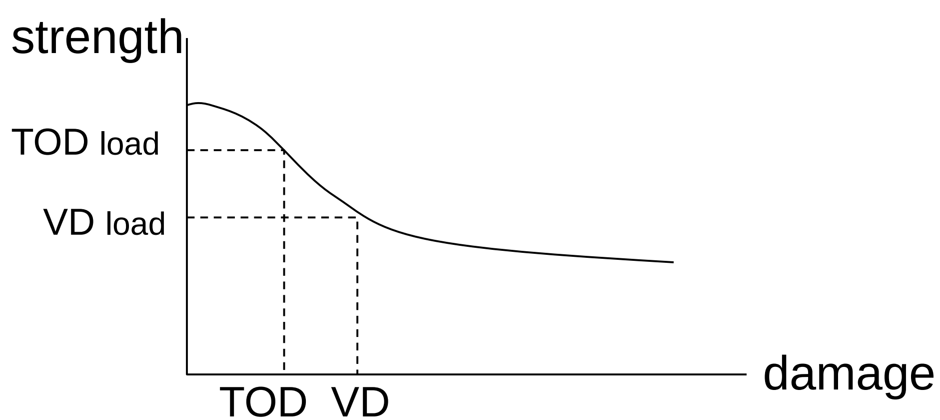
Compression loading[1]



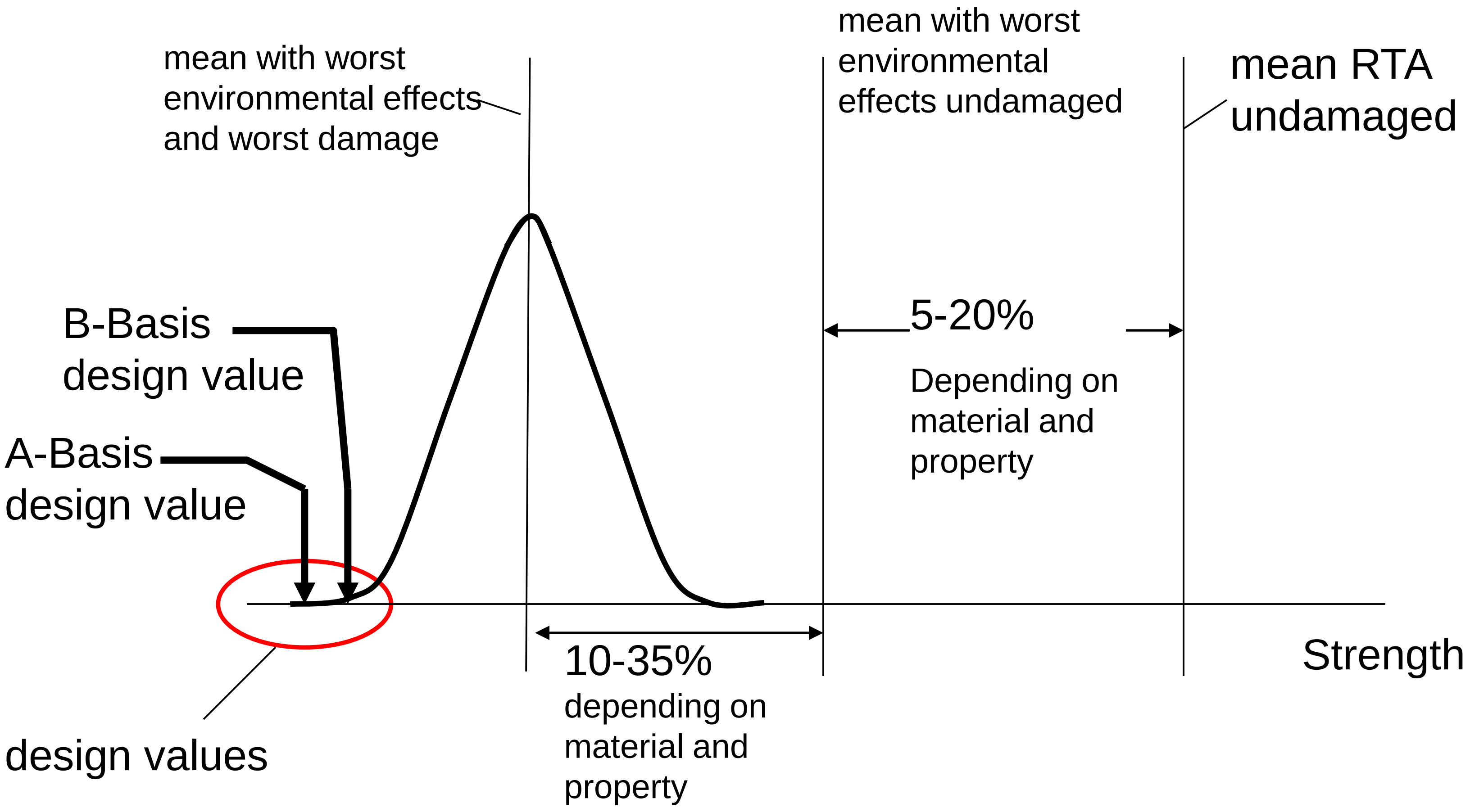
(1) Whitehead, R.S., "Lessons Learned for Composite Structures", Proc First NASA Advanced Composites Technology Conference, Seattle WA, 1990, pp 399-415

Effect of Damage (cont'd)

- Design structure to take ultimate load in presence of Threshold Of Detectability or Barely Visible Impact Damage
- Design structure to take limit load in presence of (some) Visible Damage (VD) (e.g. 6 mm dia hole)
- If $\text{TOD}_{\text{load capability}}$ / $\text{VD}_{\text{load capability}}$ <1.5, TOD is critical; otherwise, VD is critical



Design value



Cutoff strains (or stresses)

- Combine "worst" effects for material scatter, environment, and damage

Mean undamaged failure strain (compression)
~ **11000 microstrain (0.011)**

Worst Knockdown source	Knockdown fraction
Environment (ETW compr or shear)	0.8
Damage (BVID)	0.65
Material scatter (CV~11%)	0.8

Cutoff strain value= 11000 x 0.8 x 0.65 x 0.8 =**4576** microstrain (=0.0045 mm/mm)

Independent of loading case, environment, layup, etc.=> conservative

(CV=std. dev/mean x 100)

Weight comparison: Al versus composites

	Aluminum (7075-T6)	Quasi-Isotropic Gr/Epoxy	Gr/E layup used in compr*
Density (kg/m^3)	2777	1611	1611
Young's modulus (GPa)	68.9	48.2	71.7
Compr. (yield) failure strain (μs)	5700	4576	~4500
Compr. failure stress (MPa)	392.7	220.8	~322.6

* $[45/-45/0_2/\overline{90}]s$

including knockdowns for material scatter, environment and damage

• Aluminum is, typically, stronger than Gr/E but also has higher density

Weight comparison: Al versus composites

$$Weight, \quad W = \rho t (Area)$$

$$at \quad failure,$$

$$\sigma_{fail} = \frac{F_a}{wt} \Rightarrow t = \frac{F_a}{w\sigma_{fail}}$$

$$W = \rho \frac{F_a}{w\sigma_{fail}} (Area)$$

$$\frac{W_{Gr}}{W_{Al}} = \frac{\left(\dfrac{\rho}{\sigma_{fail}}\right)_{Gr}}{\left(\dfrac{\rho}{\sigma_{fail}}\right)_{Al}}$$

	QI/Al	[45/-45/0$_2$/$\overline{90}$]s/Al
$\frac{W_{Gr}}{W_{Al}}$	**1.03**	**0.706**

black Aluminum!

29.4% savings!

Some added considerations of the design process

Material capability (strength, stiffness)

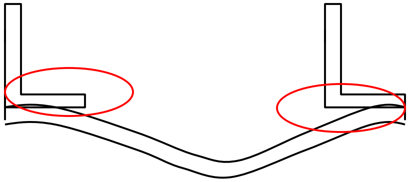
Configuration performance (different failure modes)

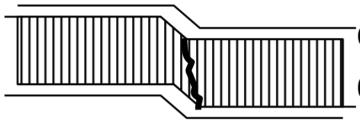
Conservative(1) analysis to determine stresses and strains

Failure criteria

Cutoff values

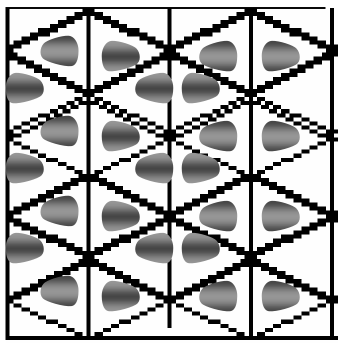
out-of-plane failure





eccentricity driven

local (bay) buckling



(1) Reasonably conservative, reasonably accurate and fast tends to be preferable to very accurate but computationally very expensive methods

Some added considerations of the design process

Material capability (strength, stiffness)

Configuration performance (different failure modes)

Conservative[1] analysis to determine stresses and strains

Failure criteria

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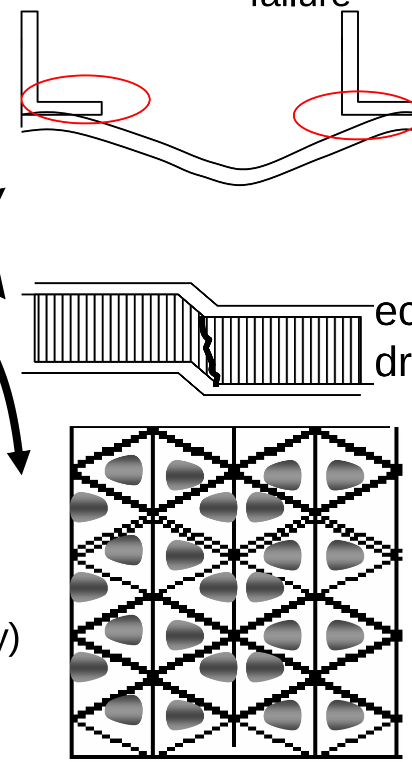
out-of-plane failure



eccentricity driven

local (bay) buckling

Discuss failure modes briefly and how some of them are difficult to pick-up in analysis

Discuss problems with failure criteria

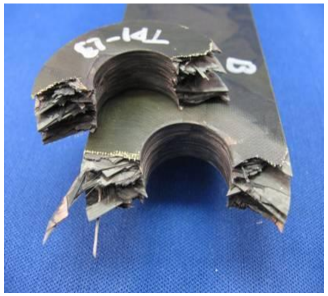
Segway into cutoff values

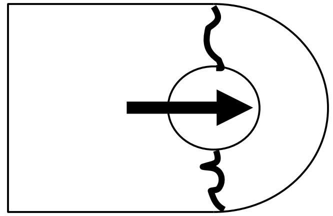
(1) Reasonably conservative, reasonably accurate and fast tends to be preferable to very accurate but computationally very expensive methods

Related issues/considerations

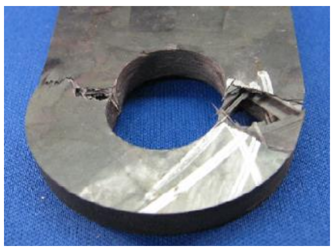
- Being able to obtain accurate stresses and/or strains is not enough to quantify failure correctly and thus not enough to generate a good design
- Need to know the failure mode in advance
- Design to specific failure mode(s) and not on the basis of highest stress in a model
  - e.g. buckling vs crippling analysis
  - Buckling of bays vs buckling of plate (isogrid)
  - Interlaminar stresses require much higher mesh density in FE model so a model could be good from every other respect but if you did not know the possibility of delamination you would not capture the critical failure mode (e.g. skin-stiffener separation, stiffener termination)
- Modelling issues (e.g. fasteners, BC’s between ss and clamped, etc)

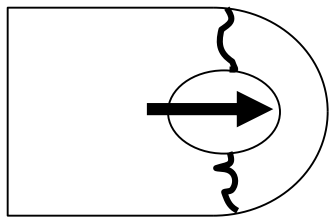
Multiplicity and interaction of failure modes (lugs)



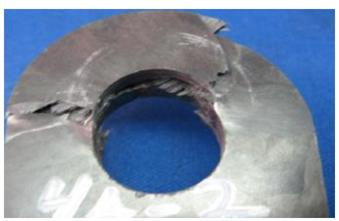


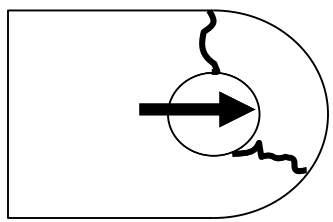
Net section failure



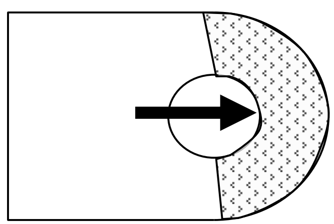


Bearing, (hole elongates and material ahead of pin fails) and net section failure combined





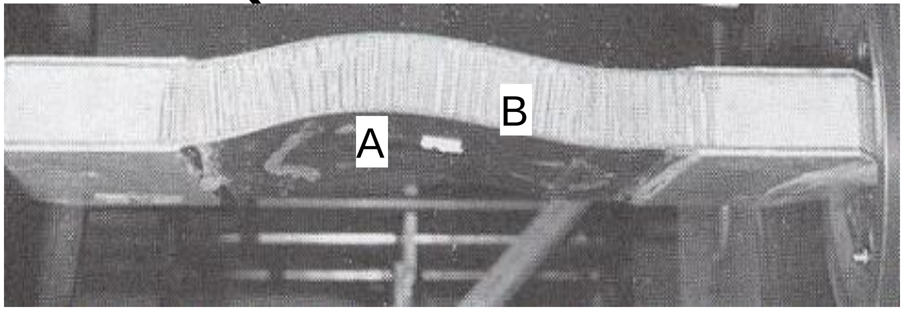
Shearout, (shear failure ahead of pin hole along loading plane) and net section failure combined



Delamination

Even for a “simple” detail like a lug, the multiplicity of failure modes can make failure prediction extremely difficult. FE cannot help much.

Multiplicity and interaction of failure modes (sandwich structure)



sandwich under compression at failure

- Wavy shape of facesheet can lead to:
  - material failure of the facesheet (bending combined with compression)-A
  - material failure of the adhesive (tension, compression, shear)- A, B
  - material failure of the core (tension, compression, shear)
- Stability driven/related failure of the facesheet
  - facesheet buckling (plate on elastic foundation)
  - wrinkling
  - intra-cellular buckling
- etc.

Governing Equations - Linear

(Cartesian coordinates)

- Equilibrium (no body forces)

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

- or in terms of force and moment resultants:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = 0$$

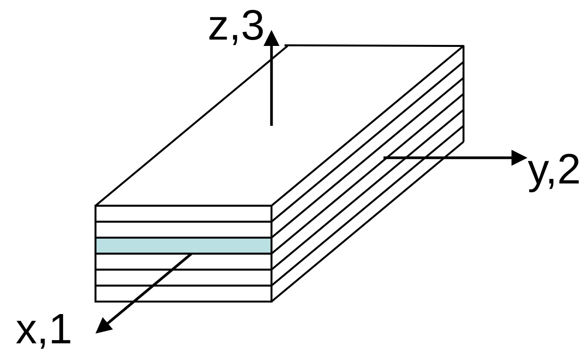
$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}$$

$$Q_y = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y}$$

Governing Equations (cont'd)

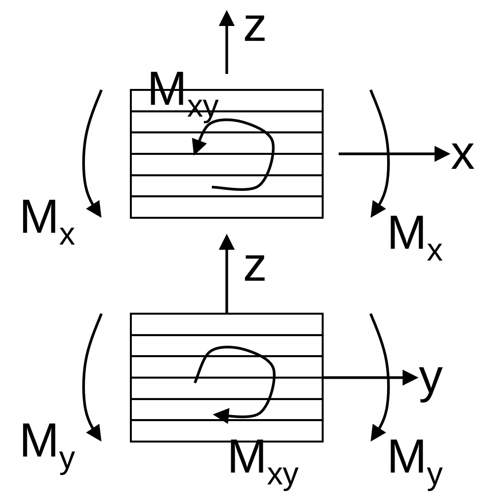
- Stress-strain equations (e.g. per ply)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & E_{16} \\ E_{12} & E_{22} & E_{23} & 0 & 0 & E_{26} \\ E_{13} & E_{23} & E_{33} & 0 & 0 & E_{36} \\ 0 & 0 & 0 & E_{44} & E_{45} & 0 \\ 0 & 0 & 0 & E_{45} & E_{55} & 0 \\ E_{16} & E_{26} & E_{36} & 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$



- or, in terms of force and moment resultants

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{\,o} \\ \varepsilon_y^{\,o} \\ \gamma_{xy}^{\,o} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$



(e.g. per laminate)

Governing Equations (cont'd)

- Strain-displacement equations

$$\varepsilon_x{}^o = \frac{\partial u}{\partial x} \qquad \kappa_x = -\frac{\partial^2 w}{\partial x^2}$$

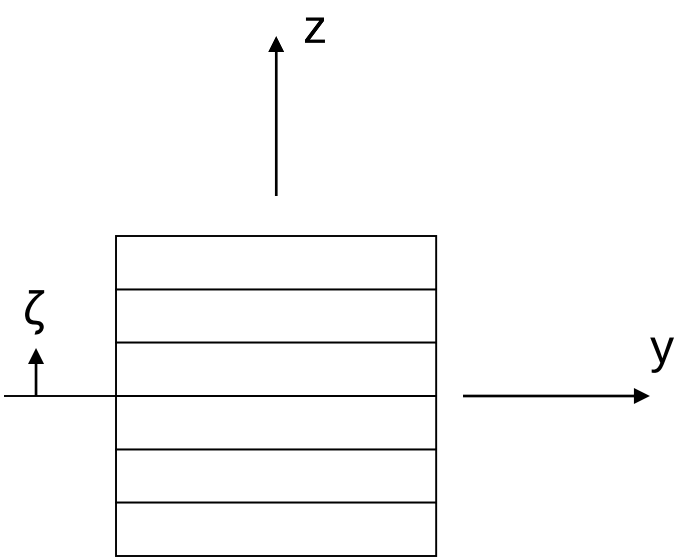
$$\varepsilon_y{}^o = \frac{\partial v}{\partial y} \qquad \kappa_y = -\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy}{}^o = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

$$\varepsilon_x = \varepsilon_x{}^o + \zeta\kappa_x$$

$$\varepsilon_y = \varepsilon_y{}^o + \zeta\kappa_y$$

$$\gamma_{xy} = \gamma_{xy}{}^o + \zeta\kappa_{xy}$$



- 17 eqns in the 17 unknowns: u, v, w, N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y, ε_x, ε_y, γ_{xy}, κ_x, κ_y, κ_{xy}

Governing Equations (cont'd)

- eliminating strains and forces and moments (e.g. Jones section 5.2.2):

$$A_{11}\frac{\partial^2 u}{\partial x^2}+2A_{16}\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 u}{\partial y^2}+A_{16}\frac{\partial^2 v}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}+A_{26}\frac{\partial^2 v}{\partial y^2}$$

$$-B_{11}\frac{\partial^3 w}{\partial x^3}-3B_{16}\frac{\partial^3 w}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial y^2}-B_{26}\frac{\partial^3 w}{\partial y^3}=0$$

$$A_{16}\frac{\partial^2 u}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{26}\frac{\partial^2 u}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}+2A_{26}\frac{\partial^2 v}{\partial x\partial y}+A_{22}\frac{\partial^2 v}{\partial y^2}$$

$$-B_{16}\frac{\partial^3 w}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w}{\partial x\partial y^2}-B_{22}\frac{\partial^3 w}{\partial y^3}=0$$

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}$$

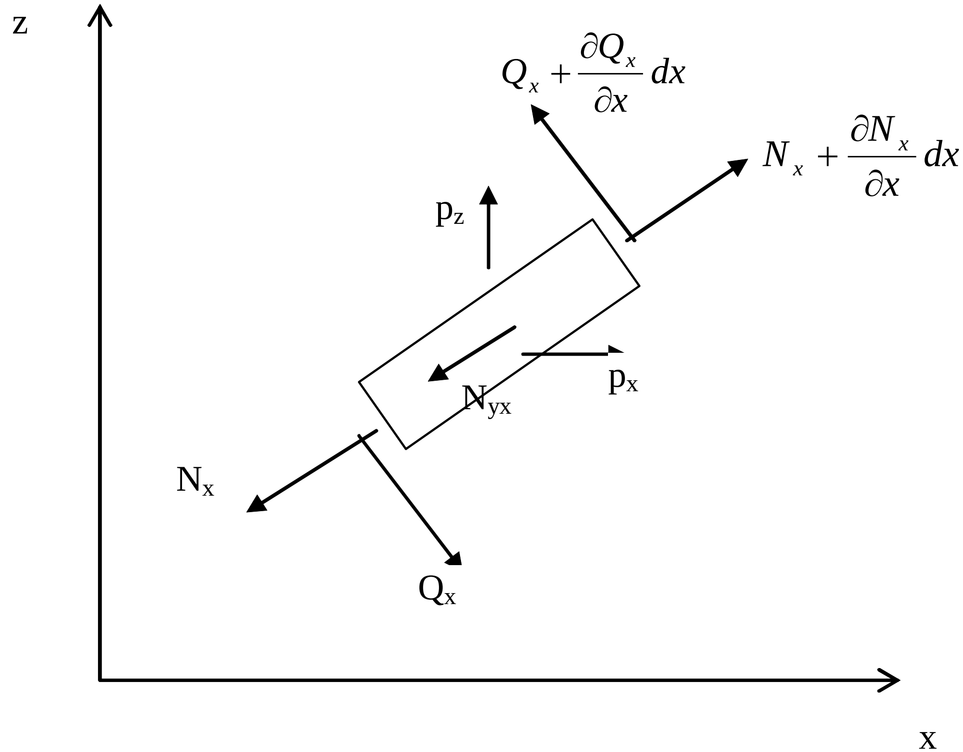
$$+D_{22}\frac{\partial^4 w}{\partial y^4}-B_{11}\frac{\partial^3 u}{\partial x^3}-3B_{16}\frac{\partial^3 u}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 u}{\partial x\partial y^2}-B_{26}\frac{\partial^3 u}{\partial y^3}$$

$$-B_{16}\frac{\partial^3 v}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 v}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 v}{\partial x\partial y^2}-B_{22}\frac{\partial^3 v}{\partial y^3}=0$$

(no body force)

Governing Equations (cont'd)

- if in-plane forces Nx, Ny, Nxy ≠ 0, additional terms from $\Sigma F_z=0$



$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} +$$

$$2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} -$$

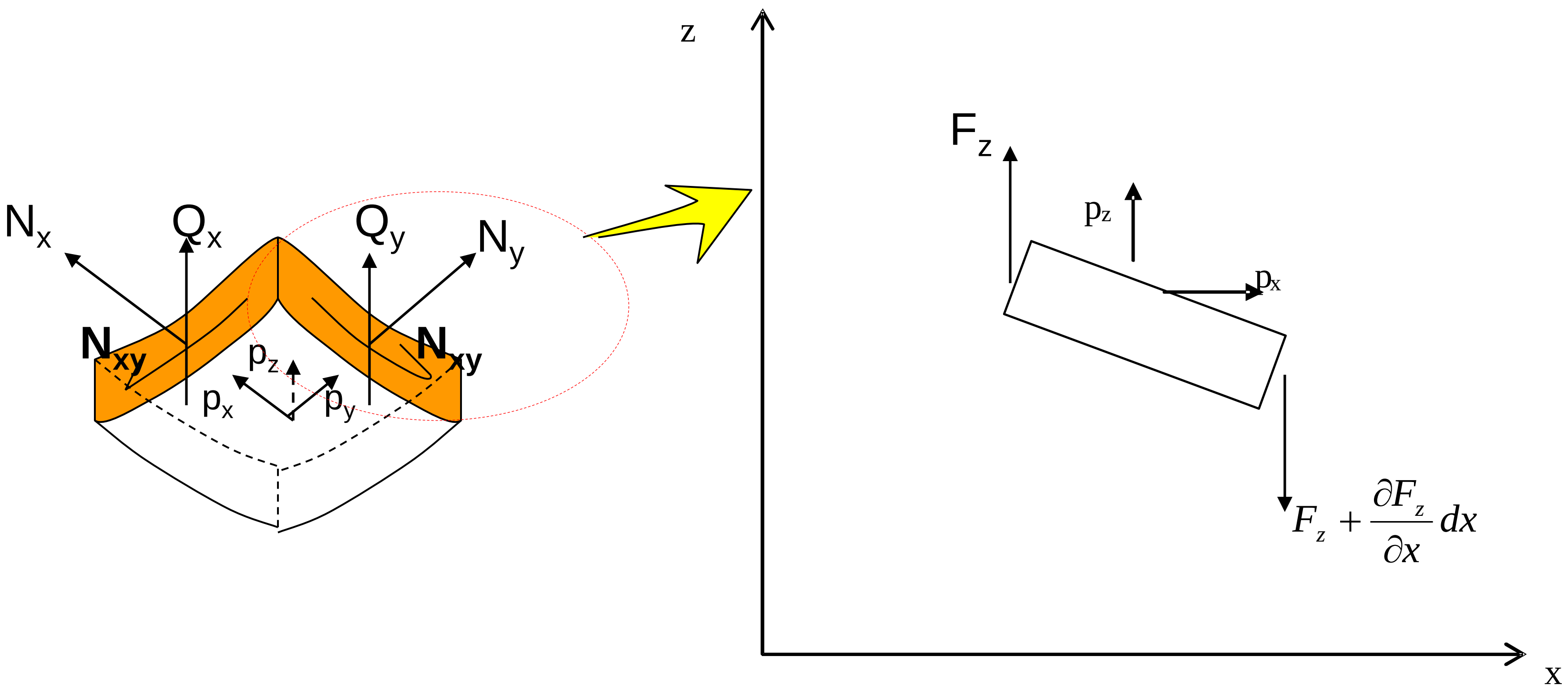
$$p_x \frac{\partial w}{\partial x} - p_y \frac{\partial w}{\partial y} + p_z = LHS$$

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = p_z + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} -$$

$$p_x\frac{\partial w}{\partial x} - p_y\frac{\partial w}{\partial y}$$

Governing Equations (cont'd)

- to see how additional terms are derived, consider F_z force at two ends

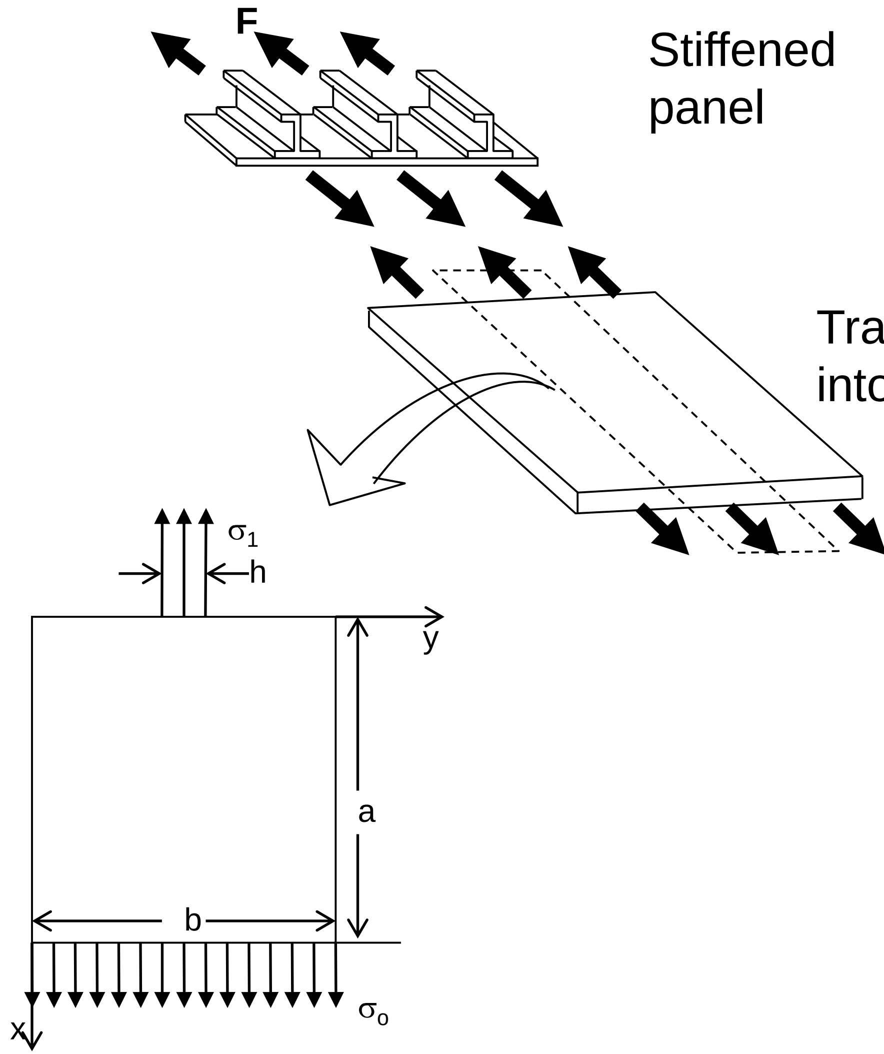


$$F_z = N_x \frac{\partial w}{\partial x} dy + Q_x dy + Q_y dx + N_y \frac{\partial w}{\partial y} dx + N_{xy} \frac{\partial w}{\partial x} dx + N_{xy} \frac{\partial w}{\partial y} dy$$

Example:

- Composite plate under localized in-plane load

Motivation: Stiffener termination



Stiffened panel

Transitioning into flat panel

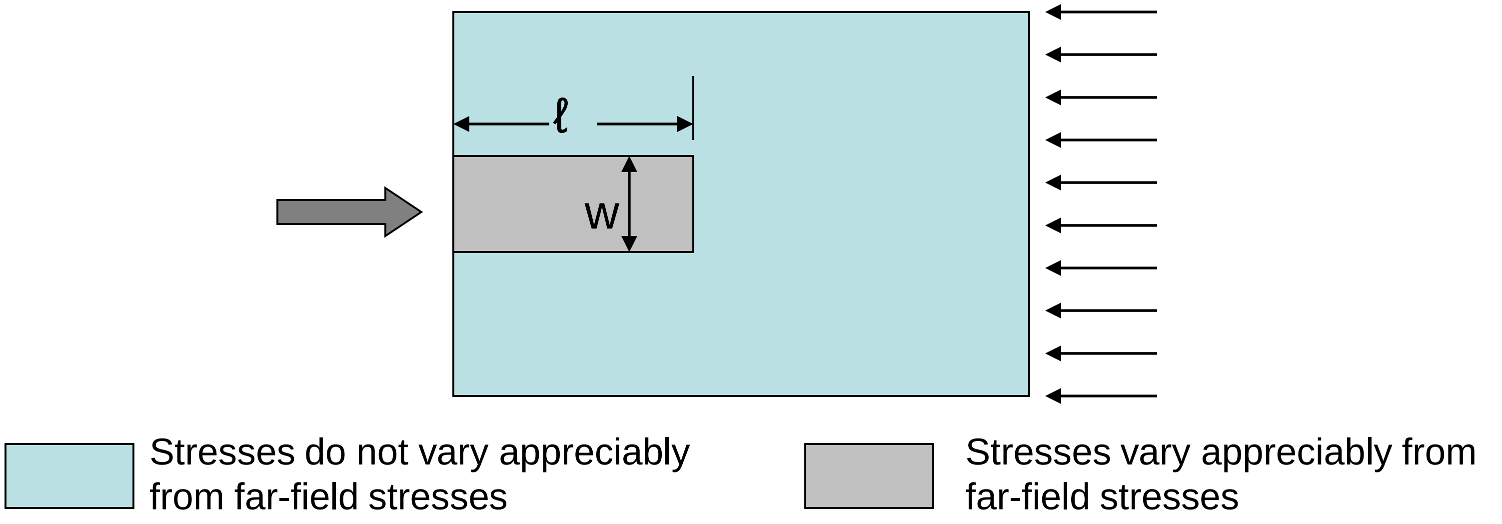
Simplified problem to be solved

Objectives

- determine the stresses in the plate so they can be used in some form of failure criterion to predict failure

- determine the length ℓ and width w of the region where stresses exceed significantly their far-field values (near the point of load application) to get an idea of the geometry of the region that needs reinforcement (doubler)

- design transition region for load introduction into the plate to be used in further analysis



Concentrated load acting on composite plate – solution[1]

- Assumptions
  - Homogeneous orthotropic plate
  - Layup is symmetric (B matrix=0)
  - Layup is balanced (no stretching/shearing coupling=> $A_{16}=A_{26}=0$)
  - There is no twisting/bending coupling ($D_{16}=D_{26}=0$)
  - Plate is sufficiently long and wide so solution is not affected by boundary proximity

(1) Kassapoglou, C., and Bauer, G., "Composite Plates Under Concentrated Load on One Edge and Uniform Load on the Opposite Edge", Mechanics of Advanced Materials and Structures, 17, 2010 pp 196-203

Derive governing PDE

- stress-strain eqns

$$Nx = A_{11}\varepsilon_x + A_{12}\varepsilon_y$$

$$Ny = A_{12}\varepsilon_x + A_{22}\varepsilon_y$$

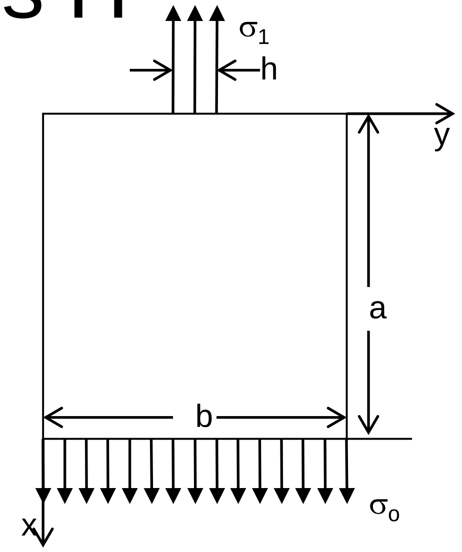
$$Nxy = A_{66}\gamma_{xy}$$

- averaged over plate thickness H

$$\sigma_x = \frac{A_{11}}{H}\varepsilon_x + \frac{A_{12}}{H}\varepsilon_y$$

$$\sigma_y = \frac{A_{12}}{H}\varepsilon_x + \frac{A_{22}}{H}\varepsilon_y$$

$$\tau_{xy} = \frac{A_{66}}{H}\gamma_{xy}$$



Governing PDE (cont'd)

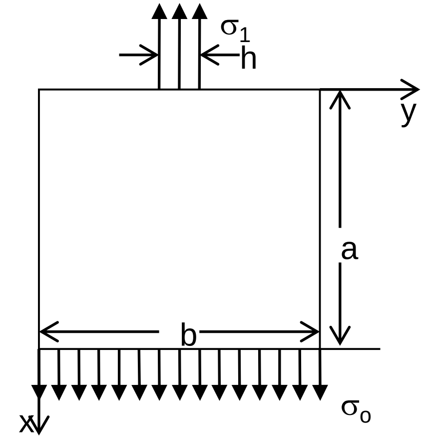
- No dependence on out-of-plane coordinate z:
  - $\frac{\partial}{\partial z} = 0$
- out-of-plane stresses $\tau_{xz} = \tau_{yz} = \sigma_z = 0$
- equilibrium eqns have the form:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$

Governing PDE (cont'd)

- Solving for the strains



$$\varepsilon_x = \frac{HA_{22}\sigma_x - HA_{12}\sigma_y}{A_{11}A_{22} - A_{12}{}^2} \qquad \gamma_{xy} = H\frac{\tau_{xy}}{A_{66}}$$

$$\varepsilon_y = \frac{HA_{11}\sigma_y - HA_{12}\sigma_x}{A_{11}A_{22} - A_{12}{}^2}$$

- Eliminating the displacements from the strain-displacement equations gives the strain compatibility:

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2}$$

- Substituting for the strains in the strain compatibility eqn:

$$\frac{A_{11}A_{22} - A_{12}{}^2}{A_{66}}\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = A_{22}\frac{\partial^2 \sigma_x}{\partial y^2} - A_{12}\frac{\partial^2 \sigma_y}{\partial y^2} + A_{11}\frac{\partial^2 \sigma_y}{\partial x^2} - A_{12}\frac{\partial^2 \sigma_x}{\partial x^2}$$

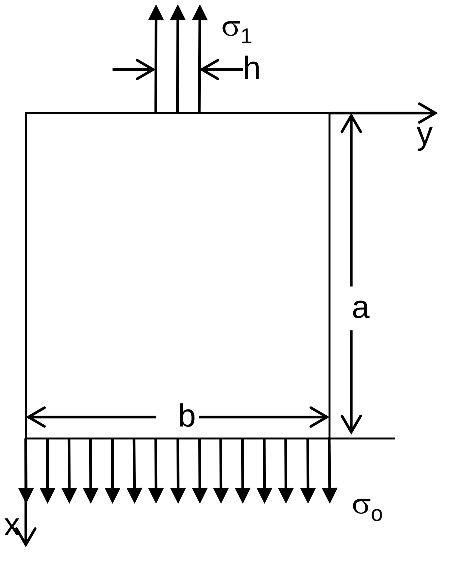
Governing PDE (cont'd)

- Use stress equilibrium equations and successive differentiations to substitute in the above equation

$$\frac{\partial^4 \sigma_x}{\partial x^4} + \underbrace{\left[\frac{A_{11}A_{22} - A_{12}{}^2}{A_{11}A_{66}} - 2\frac{A_{12}}{A_{11}} \right]}_{\beta} \frac{\partial^4 \sigma_x}{\partial x^2 \partial y^2} + \underbrace{\frac{A_{22}}{A_{11}}}_{\gamma} \frac{\partial^4 \sigma_x}{\partial y^4} = 0$$

or:

$$\frac{\partial^4 \sigma_x}{\partial x^4} + \beta \frac{\partial^4 \sigma_x}{\partial x^2 \partial y^2} + \gamma \frac{\partial^4 \sigma_x}{\partial y^4} = 0$$



Boundary Conditions

$$\sigma_x(x=0)=0 \quad 0 \le y \le \frac{b-h}{2} \quad and \quad \frac{b+h}{2} \le y \le b$$

$$\sigma_x(x=0)=\sigma_1=\frac{F}{Hh} \quad for \; \frac{b-h}{2} \le y \le \frac{b+h}{2}$$

applied load

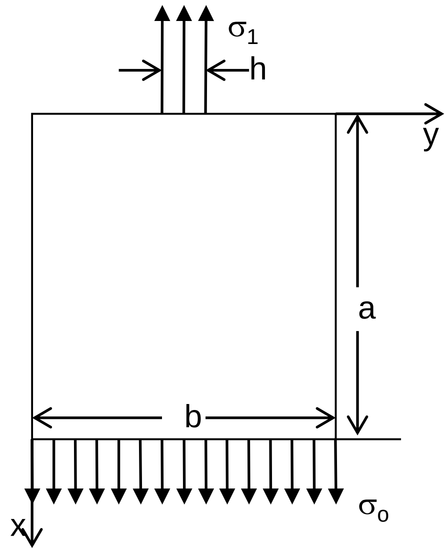
$$\sigma_x(x=a)=\sigma_o=\frac{F}{bH}$$

reaction

$$\sigma_y(y=0)=\sigma_y(y=b)=0$$

$$\tau_{xy}(x=0)=\tau_{xy}(x=a)=\tau_{xy}(y=0)=\tau_{xy}(y=b)=0$$

Stress-free condition



Solution of PDE

- Assume solution of the form (f_n unknown)

$$\sigma_x \approx f_n(x)\cos\frac{n\pi y}{b}$$

- Substituting, f_n is found to satisfy the eqn:

$$\frac{d^4 f_n}{dx^4} - \beta\left(\frac{n\pi}{b}\right)^2 \frac{d^2 f_n}{dx^2} + \gamma\left(\frac{n\pi}{b}\right)^4 f_n = 0$$

- from which, $f_n = Ce^{\varphi x}$ with

$$\phi = \pm\frac{1}{\sqrt{2}}\left(\frac{n\pi}{b}\right)\sqrt{\beta \pm \sqrt{\beta^2 - 4\gamma}}$$

- combining, the final form of the solution is

$$\sigma_x = Ko + \sum_{n=1}^{\infty} A_n\left[e^{\phi_1 x} + C_n e^{\phi_2 x}\right]\cos\frac{n\pi y}{b}$$

Fourier cosine series at any given x!

Solution of PDE (cont'd)

- only the two φ solutions with negative real parts are used (decaying exponentials) provided the plate is "long enough"; otherwise, all four solutions must be used
- a constant K_o is introduced to get the most general form of the solution

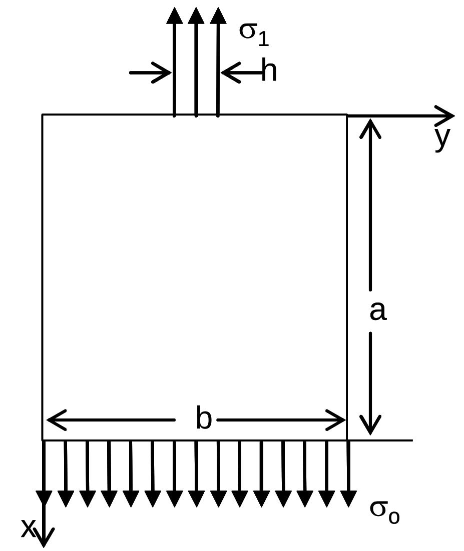
Determination of all stresses

- using equilibrium equations and boundary conditions (except at x=0) the stresses are found to be:

$$\sigma_x = Ko + \sum_{n=1}^{\infty} A_n \left(e^{\phi_1 x} - \frac{\phi_1}{\phi_2} e^{\phi_2 x} \right) \cos \frac{2n\pi y}{b}$$

$$\sigma_y = \sum_{n=1}^{\infty} \left(\frac{b}{2n\pi} \right)^2 \phi_1 A_n \left(\phi_1 e^{\phi_1 x} - \phi_2 e^{\phi_2 x} \right) \left(1 - \cos \frac{2n\pi y}{b} \right)$$

$$\tau_{xy} = -\sum_{n=1}^{\infty} \frac{b}{2n\pi} \phi_1 A_n \left(e^{\phi_1 x} - e^{\phi_2 x} \right) \sin \frac{2n\pi y}{b}$$



- only even terms contribute to the solution
- K_o and A_n are still unknown

Boundary condition at x=0

- K_o and A_n are determined as Fourier cosine series coefficients:

$$Ko = \frac{F}{bH}$$

average of σ_x at any x location

$$\int_0^b \sigma_x(x=0)\cos\frac{2q\pi y}{b}dy = \int_0^b \left(K_o + \sum A_n \left(e^{\phi_1 x} - \frac{\phi_1}{\phi_2} e^{\phi_2 x} \right)_{x=0} \cos\frac{2n\pi y}{b} \right) \cos\frac{2q\pi y}{b}\, dy \Rightarrow$$

$$A_n = \frac{F}{hH}\frac{\phi_2}{\phi_2 - \phi_1}\frac{2}{n\pi}\cos n\pi \sin\frac{n\pi h}{b}$$