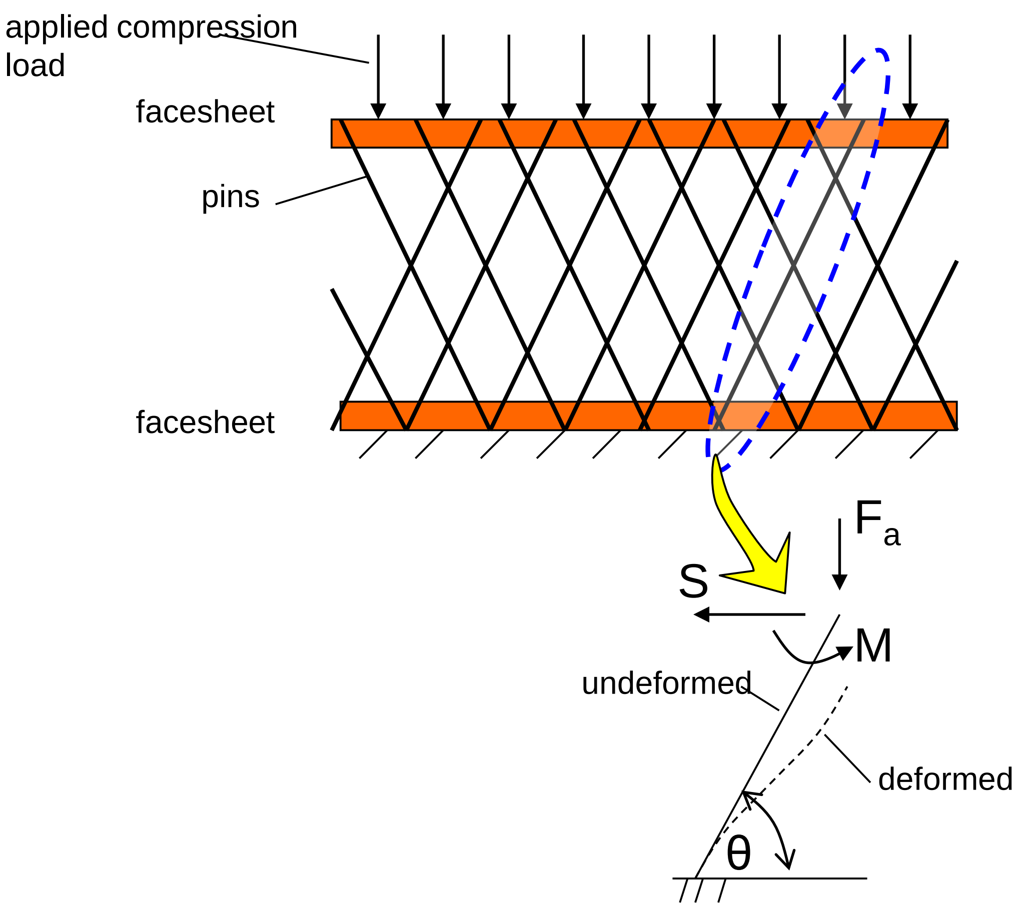
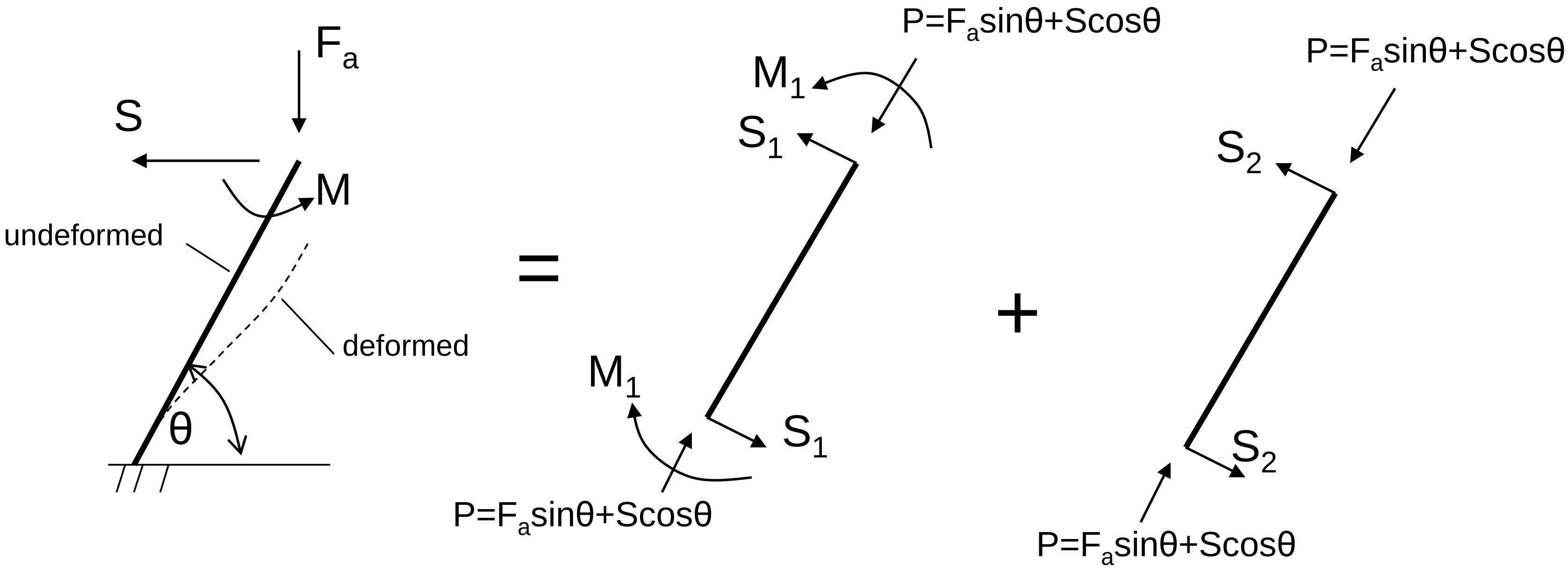
# Example: Flatwise compression of sandwich with Gr pins for core



# Individual pin under flatwise load




After some algebra:

$$M = \frac{\ell}{2}(F_a \cos\theta - S\sin\theta)$$

$$\beta = \sqrt{\frac{P}{E_p I_p}}$$

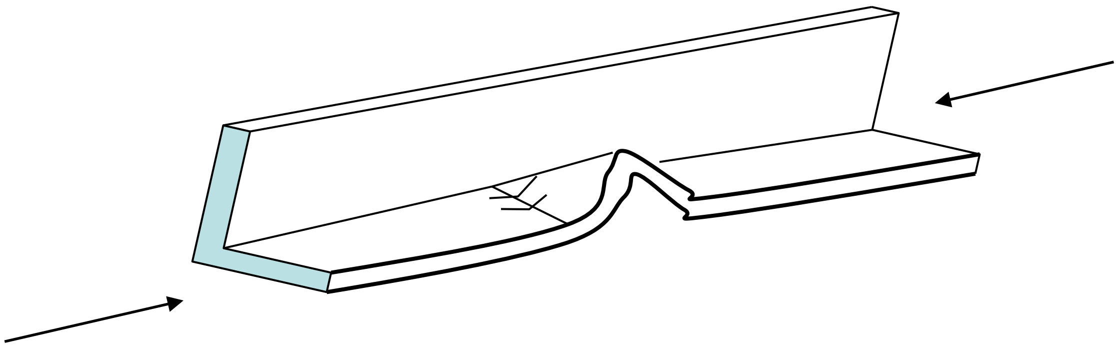
$$M_1 = -\frac{\ell}{2}(F_a \cos\theta - S\sin\theta)\frac{\sin\beta\ell}{\sin\beta\ell - \beta\ell(1+\cos\beta\ell)}$$

$$S_1 = \beta\frac{\ell}{2}(F_a \cos\theta - S\sin\theta)\frac{1+\cos\beta\ell}{\sin\beta\ell - \beta\ell(1+\cos\beta\ell)}$$

$$S_2 = \frac{2\sin\beta\ell - \beta\ell(1+\cos\beta\ell)}{2\left(\sin\beta\ell - \beta\ell(1+\cos\beta\ell)\right)}(-F_a \cos\theta + S\sin\theta)$$

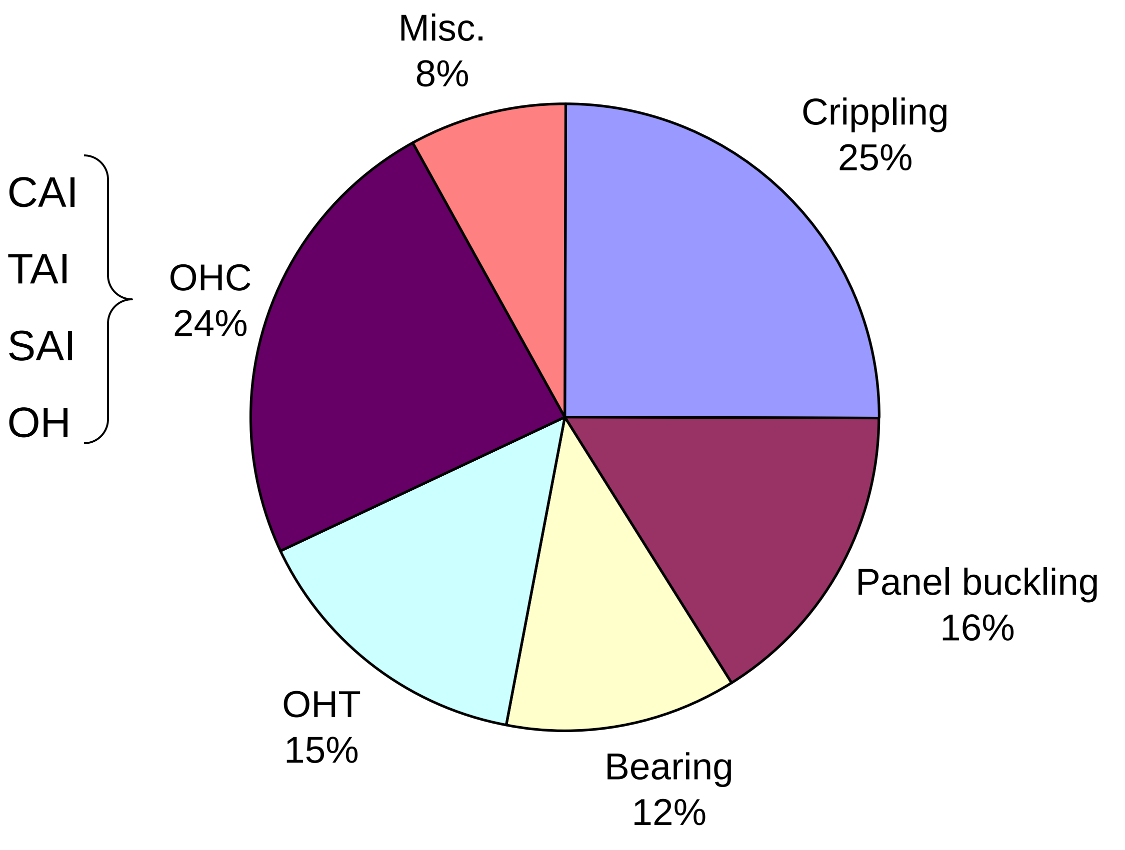
etc…

# Stiffener crippling

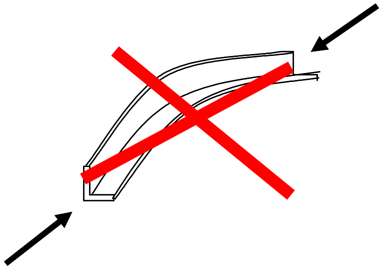


- Perhaps the most common stiffener failure and one of the most common failure modes in a fuselage

# Fuselage failure modes by %



# Stiffener crippling



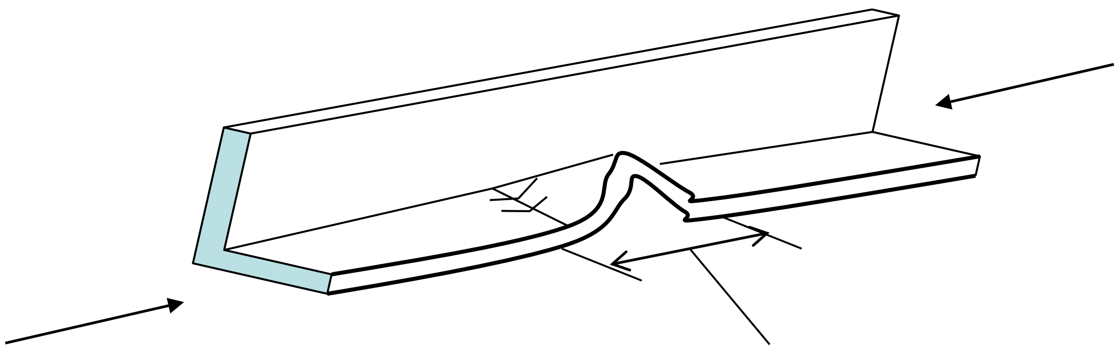
• in a good design, stiffeners do not fail by column buckling but by flange crippling

• if column buckling is an issue, the effective beam length is decreased and/or the bending stiffness of the stiffener is increased until flange crippling becomes the primary failure mode

**Reason**: in column buckling the entire stiffener is “gone”. In stiffener crippling, local flange failure occurs which may be confined in one flange, without immediate failure of entire stiffener, to absorb enough of a crash load to protect passengers

# Stiffener crippling

• stiffener crippling occurs when one or more flanges buckle in a **local** buckling mode with wavelength unrelated to the length of the beam



half-wave length=f(b,t, Dij)

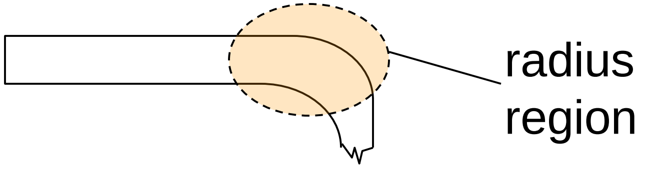
b=flange width

t=flange thickness

Dij= bending stiffnesses of flange

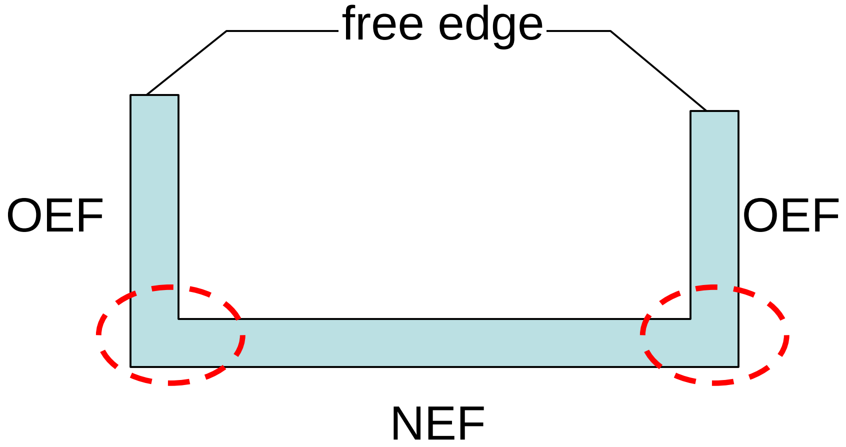
# Stiffener crippling

- ideally, one first obtains the individual loads on each flange
- then computes short-wavelength buckling load of the flange of interest,
- then goes through a post-buckling solution similar to the one for plate under compression
- determine load for final failure
- very cumbersome (plus modelling issues with radius regions)



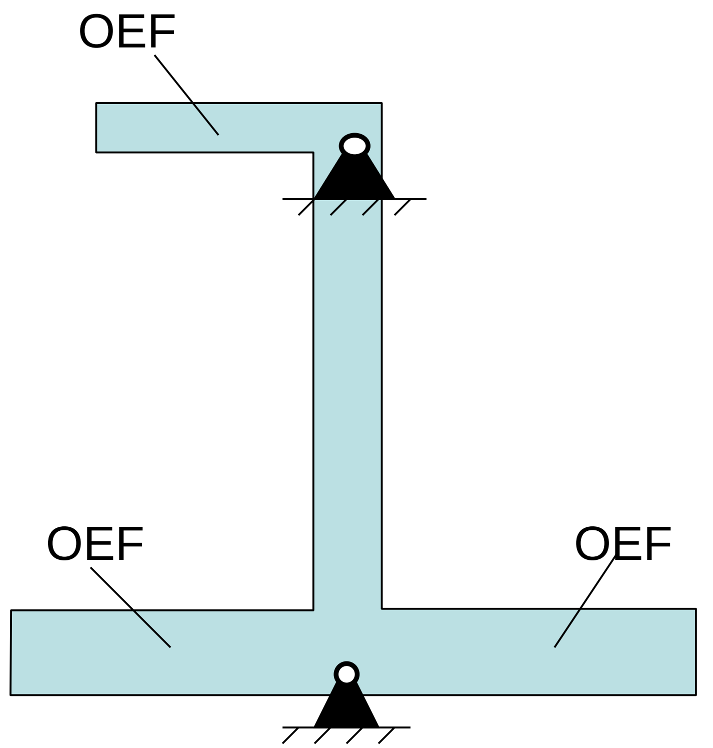
# Stiffener crippling

- Distinguish two cases:
  - One-Edge-Free (OEF)
  - No-Edge-Free (NEF)



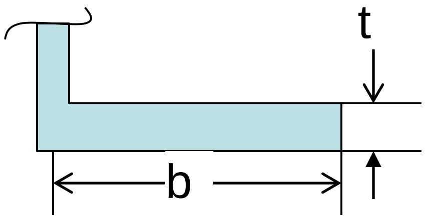

if an edge is not supported or stabilized by another member of the cross-section (or via other means) it is free

# OEF Crippling




For OEF, the buckling problem has been solved before as the case of a very long plate under compression with <u>three sides simply supported and one side free</u>

# OEF crippling - predictions




- buckling load was found to be

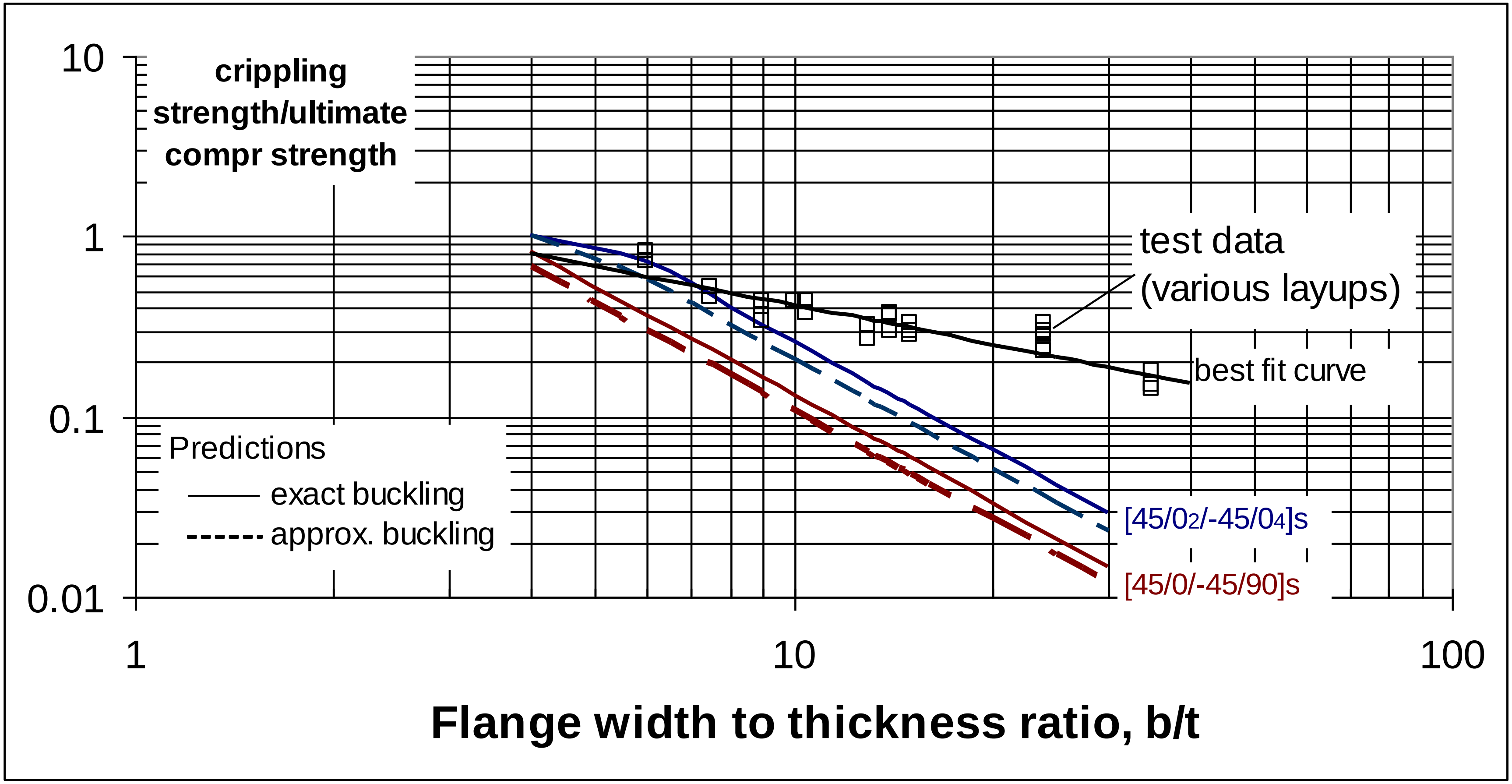
$$N_{xcrit} = \frac{12D_{66}}{b^2}$$ exact expression

$$N_{xcrit} = \frac{4\pi^2}{b^2}\lambda^2 D_{66} + \frac{2\pi^2}{b^2} D_{12}$$ , λ=5/12 our approx. expression

- **therefore, to maximize the crippling load of a flange of given width b, one must maximize $D_{66}$ for the flange.**

what does that imply for the flange layup? and how does this implication match our requirement for high I for the entire cross-section?

# OEF Crippling: Buckling versus Final Failure; Comparisons with test results




Test results from Mil-Hdbk 17-3F, vol 3, ch 5, Fig 5.7.2.h, Jun 17, 2002

# OEF crippling: test vs theory

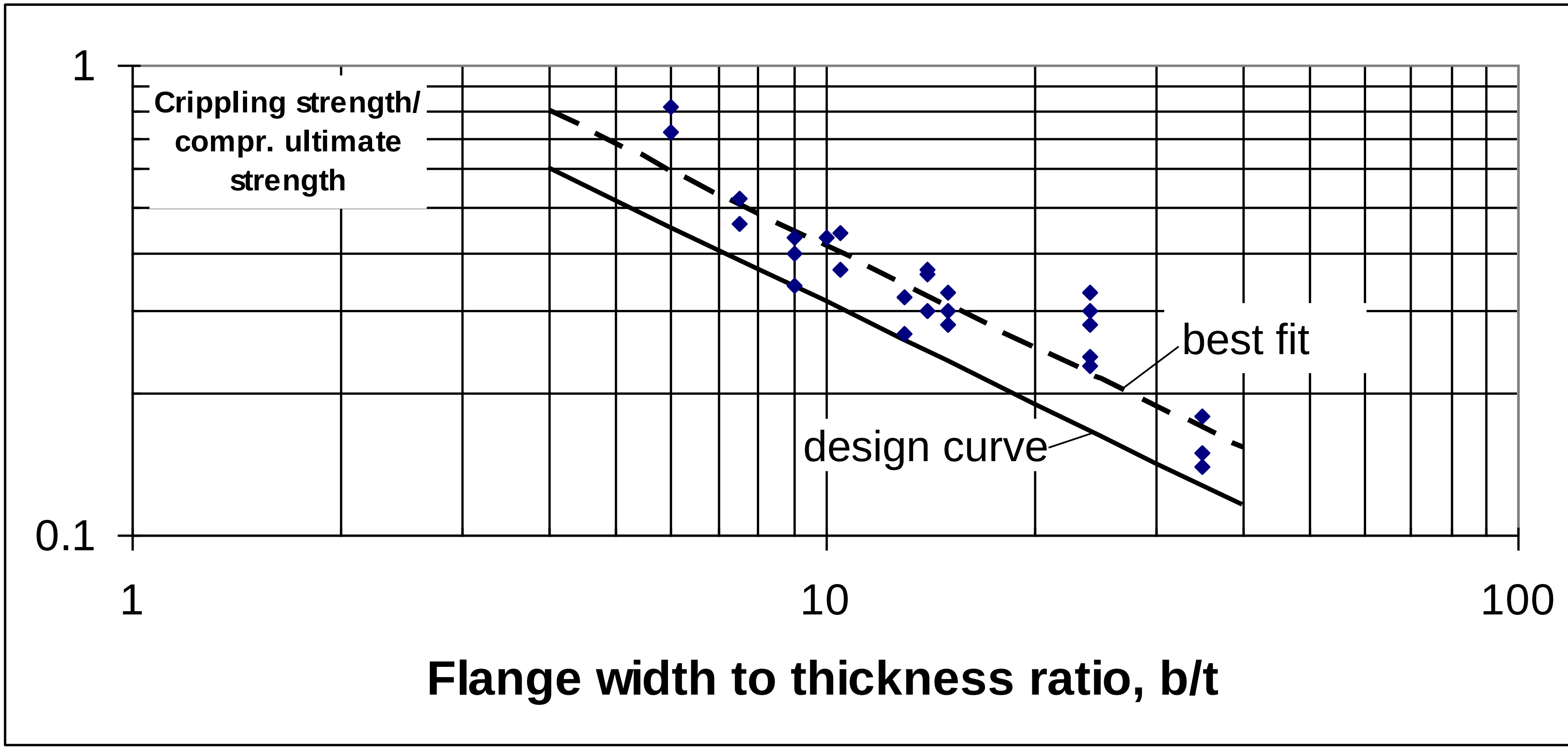
- irrespective of layup, test results follow a single curve:

$$\frac{\sigma_{crip}}{\sigma_c^{\ u}} = \frac{2.151}{\left(\dfrac{b}{t}\right)^{0.717}} \quad \Rightarrow \quad \boxed{\frac{\sigma_{crip}}{\sigma_c^{\ u}} = \frac{1.63}{\left(\dfrac{b}{t}\right)^{0.717}}}$$

preliminary design curve (valid for at least 25%0, 25%45) b/t>2.9

- predictions are a function of layup
- approx. predictions are close to exact theoretical predictions

# OEF crippling design curve

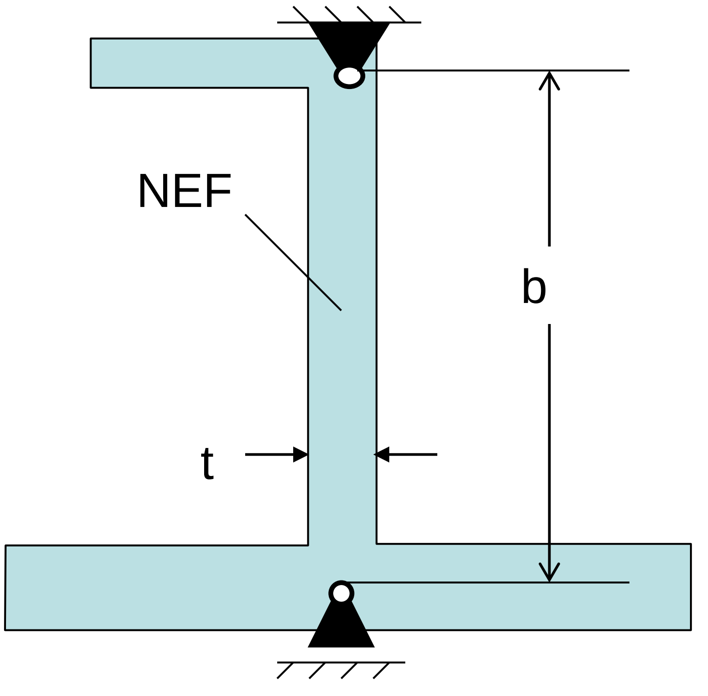


# OEF crippling – Test vs Theory (cont'd)

- predictions are very conservative especially for high b/t values (recall plates have high post-buckling ability)

- for low b/t values, effect of radii and transverse shear effects (not included in the predictions) may be important

# NEF Crippling




This is the case of a long plate under compression, simply-supported all around which was solved before as part of the biaxially loaded plate

# NEF crippling - predictions

- buckling load was found to be

$$N_o = \frac{\pi^2\left[D_{11}m^4 + 2(D_{12}+2D_{66})m^2n^2(AR)^2 + D_{22}n^4(AR)^4\right]}{a^2(m^2+kn^2(AR)^2)}$$

- which for k=0 and n=1 can be rearranged to:

$$N_{xcrit} = \frac{\pi^2}{b^2}\sqrt{D_{11}D_{22}}\left[\frac{m^2b^2}{a^2\sqrt{\frac{D_{22}}{D_{11}}}} + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + \sqrt{\frac{D_{22}}{D_{11}}}\frac{a^2}{b^2m^2}\right]$$

≈1 for long plate

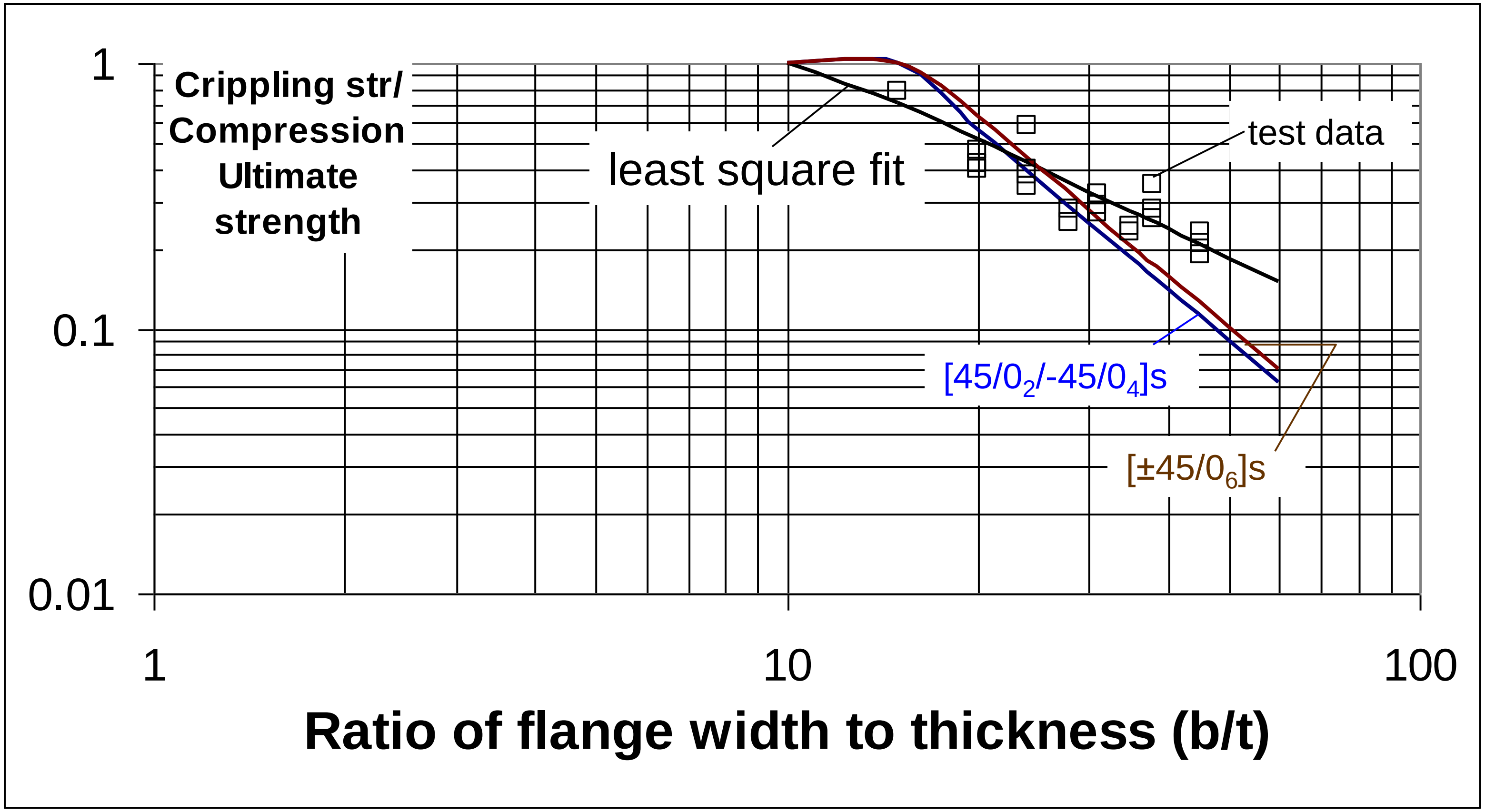
(hint: find m that minimizes Nxcrit)

- and for a very long plate,

$$N_{xcrit} = \frac{2\pi^2}{b^2}\left[\sqrt{D_{11}D_{22}} + (D_{12}+2D_{66})\right]$$

it still "pays" to have ±45 plies away from the mid-plane as in the OEF condition

# NEF Crippling: Buckling versus Final Failure; Comparisons with test results




Test results from Mil-Hdbk 17-3F, vol 3, ch 5, Fig 5.7.2.f, Jun 17, 2002

# NEF crippling: test vs theory

- irrespective of layup, test results follow a single curve:

$$\frac{\sigma_{crip}}{\sigma_c{}^u} = \frac{14.92}{\left(\frac{b}{t}\right)^{1.124}}$$

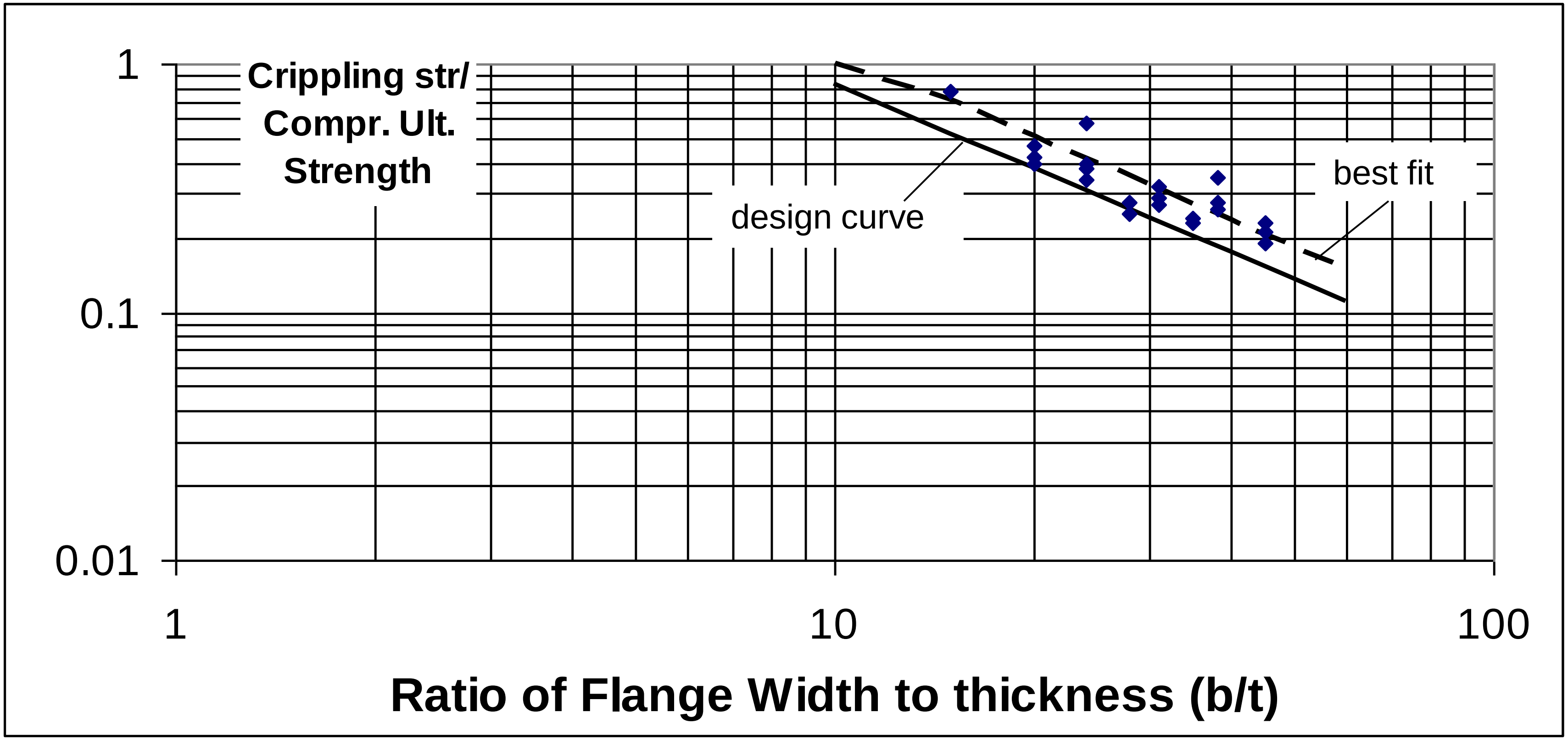
$$\frac{\sigma_{crip}}{\sigma_c{}^u} = \frac{11.0}{\left(\frac{b}{t}\right)^{1.124}}$$

preliminary design curve

$b/t \geq 8.5$

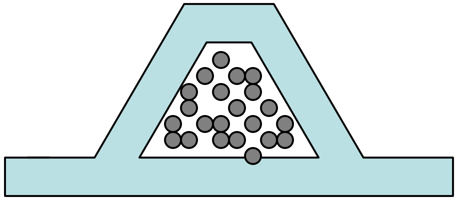
- predictions are a function of layup
- predictions are very conservative especially for high b/t values (recall plates have high post-buckling ability)
- for low b/t values, effect of radii and transv shear effects (not included in the predictions) may be important

# NEF crippling design curve



# Stiffener crippling – Other considerations

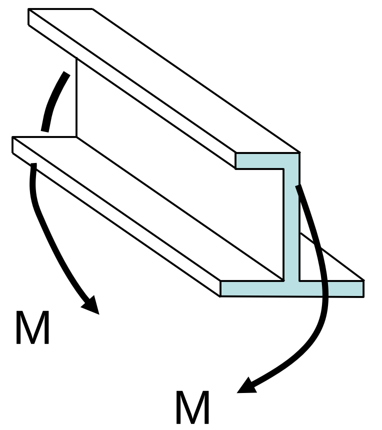
- closed section stiffeners:



- hollow: analysis as before
- filled (with foam for example): see section on sandwich

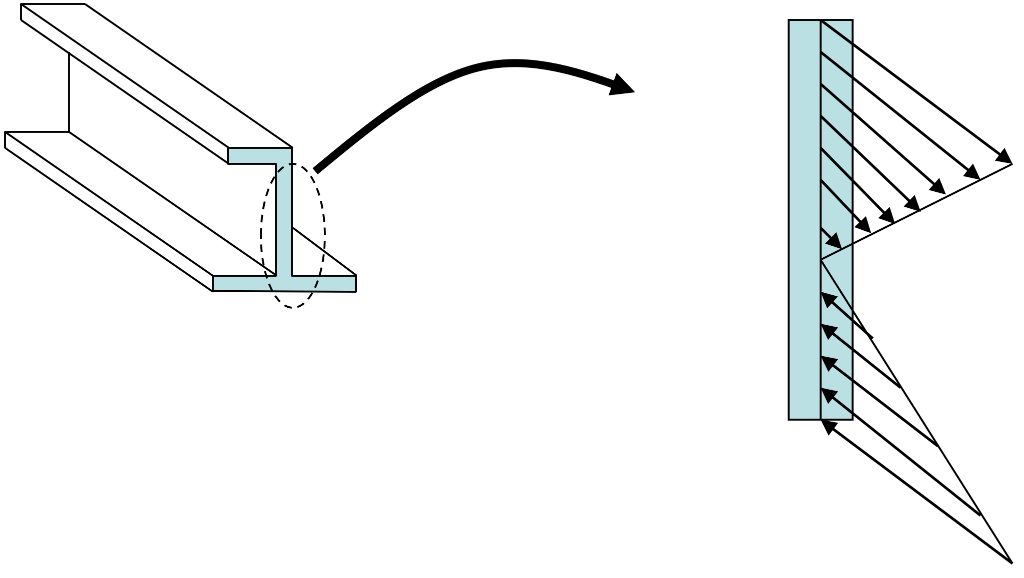
# Stiffener crippling – Other considerations

- stiffeners under bending moment loads => flanges in bending



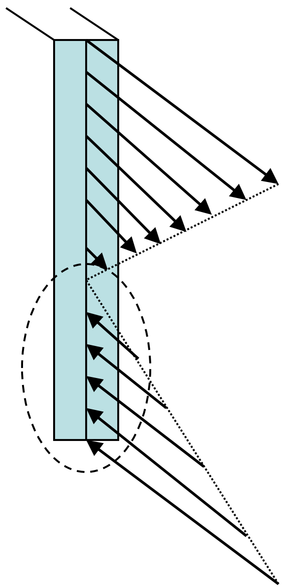
# Crippling of flanges in bending

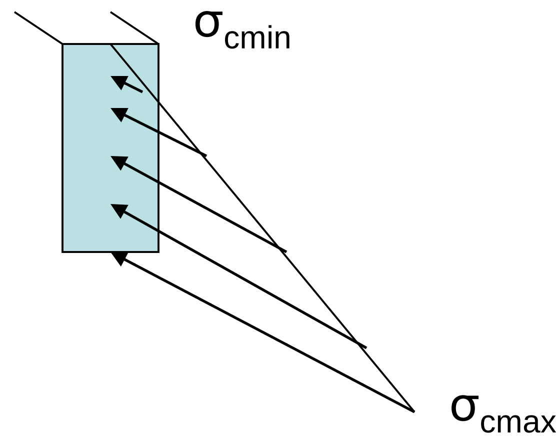
• determine the normal stress distribution on the flange of interest



# Crippling of flanges in bending

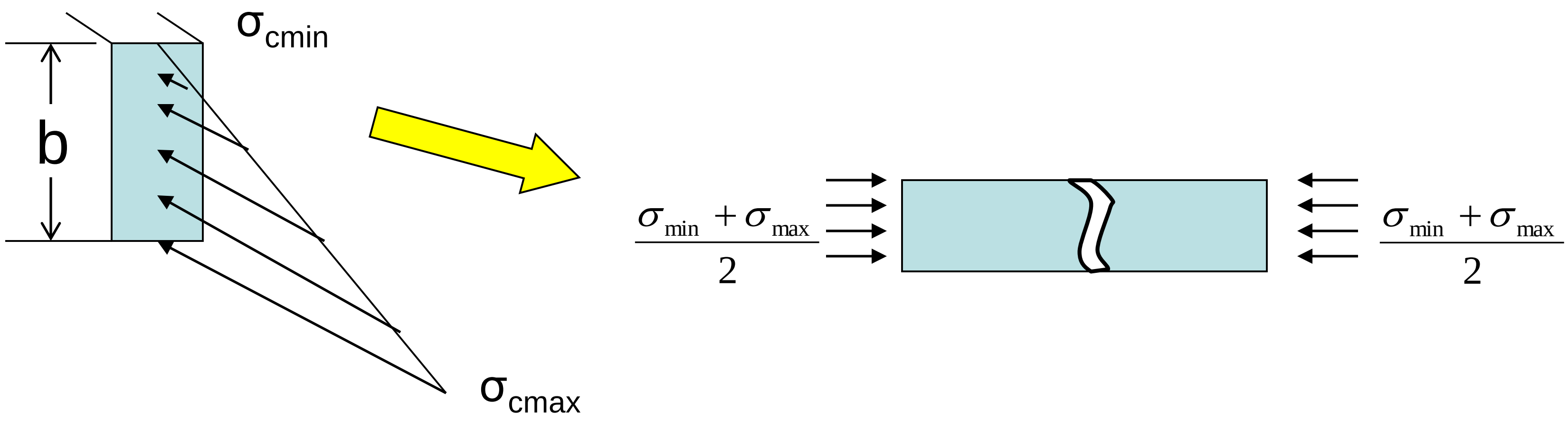
- determine the portion that is under compression and find the extreme stresses $\sigma_{cmax}$ $\sigma_{cmin}$



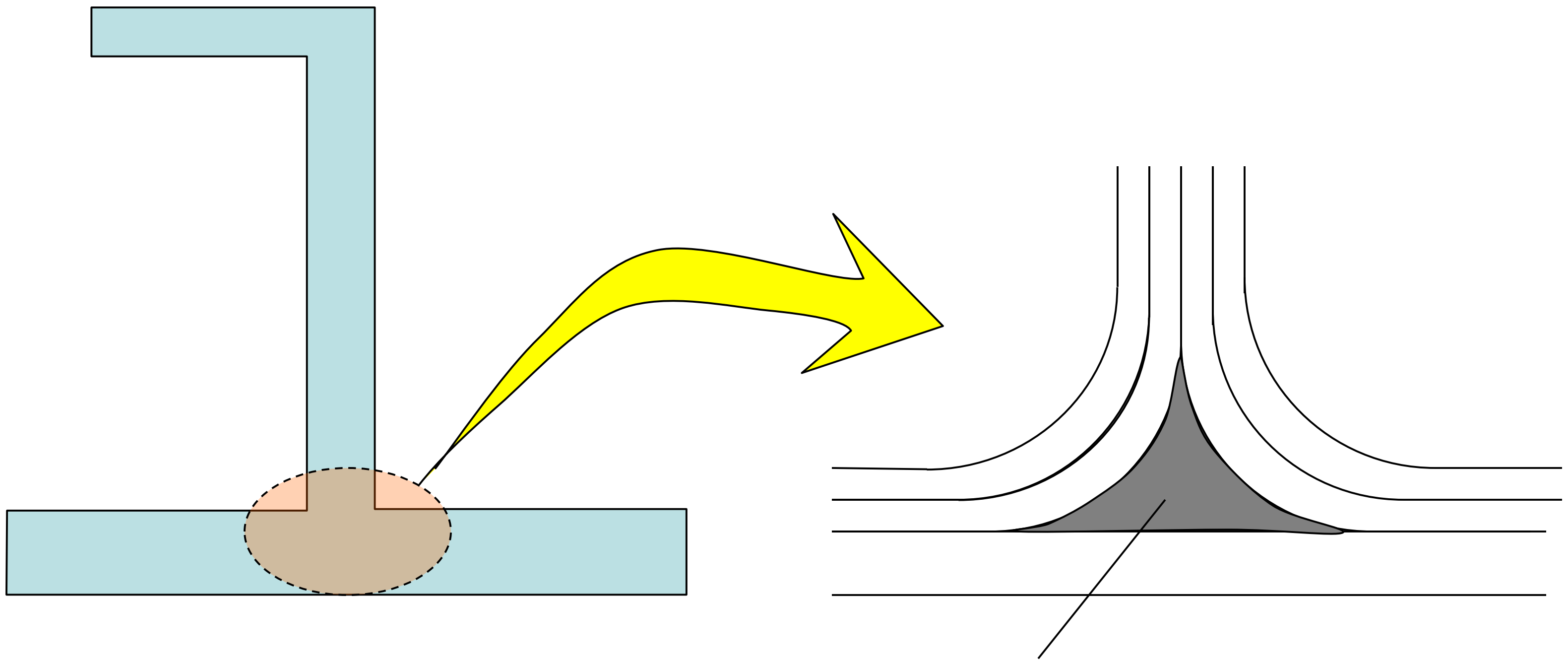


# Crippling of flanges in bending

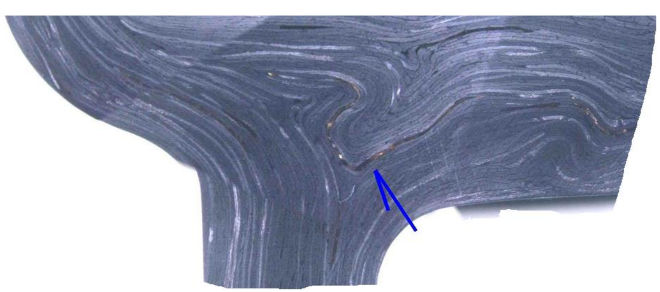
• determine average compr. stress and analyze as a flange in compression



# Crippling: importance of radius regions



without special provisions,
this region fills with resin
=> weaker

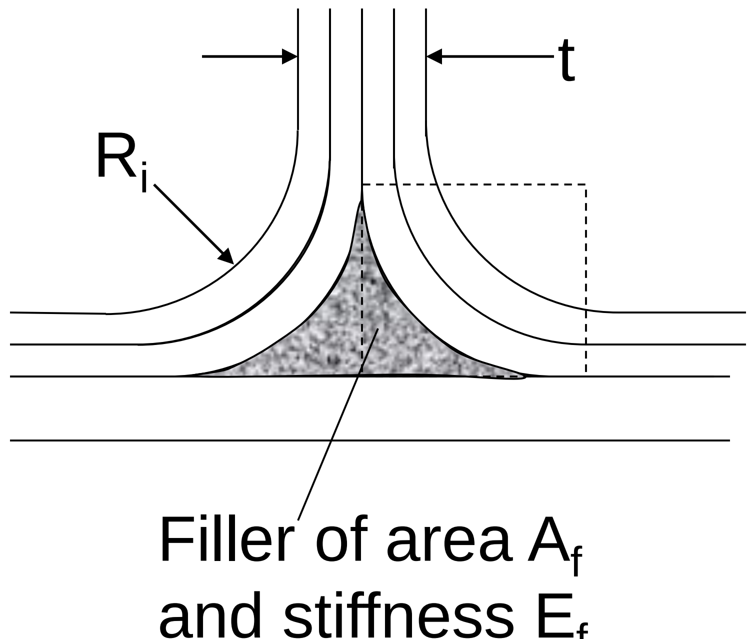


# Crippling: importance of radius regions

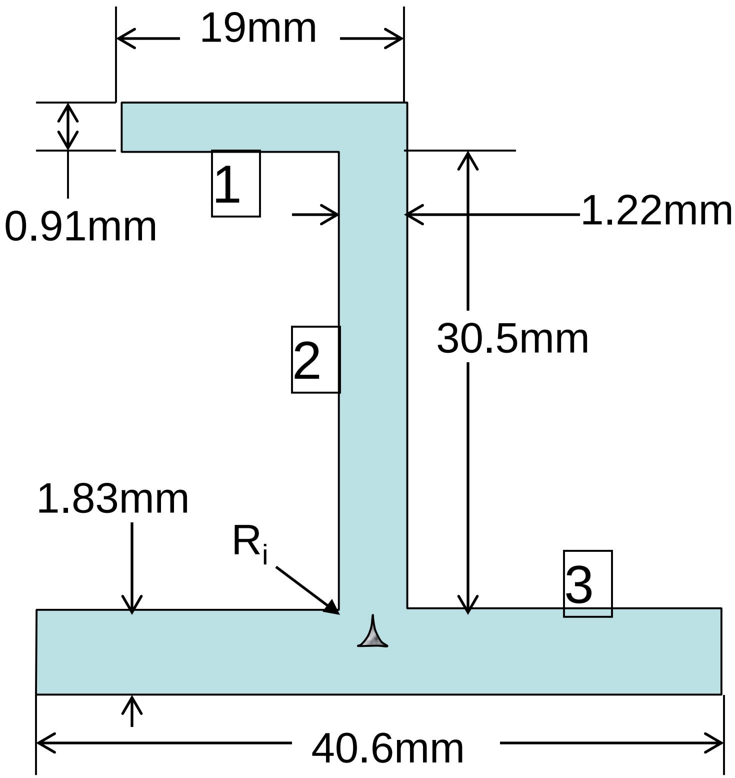
• significant improvement is obtained by filling the radius region with stiff material (uni-directional tape for example)

$$A_f = 2\left[R_i + \frac{t}{2}\right]^2 \left(1 - \frac{\pi}{4}\right)$$

$$F_f = \frac{E_f A_f}{\sum E_j A_j} F_{TOT}$$



# Crippling: Effect of filler material in radius regions




$E_f$ ranges from 3GPa (pure resin) to 138 GPA (0 degree tape)

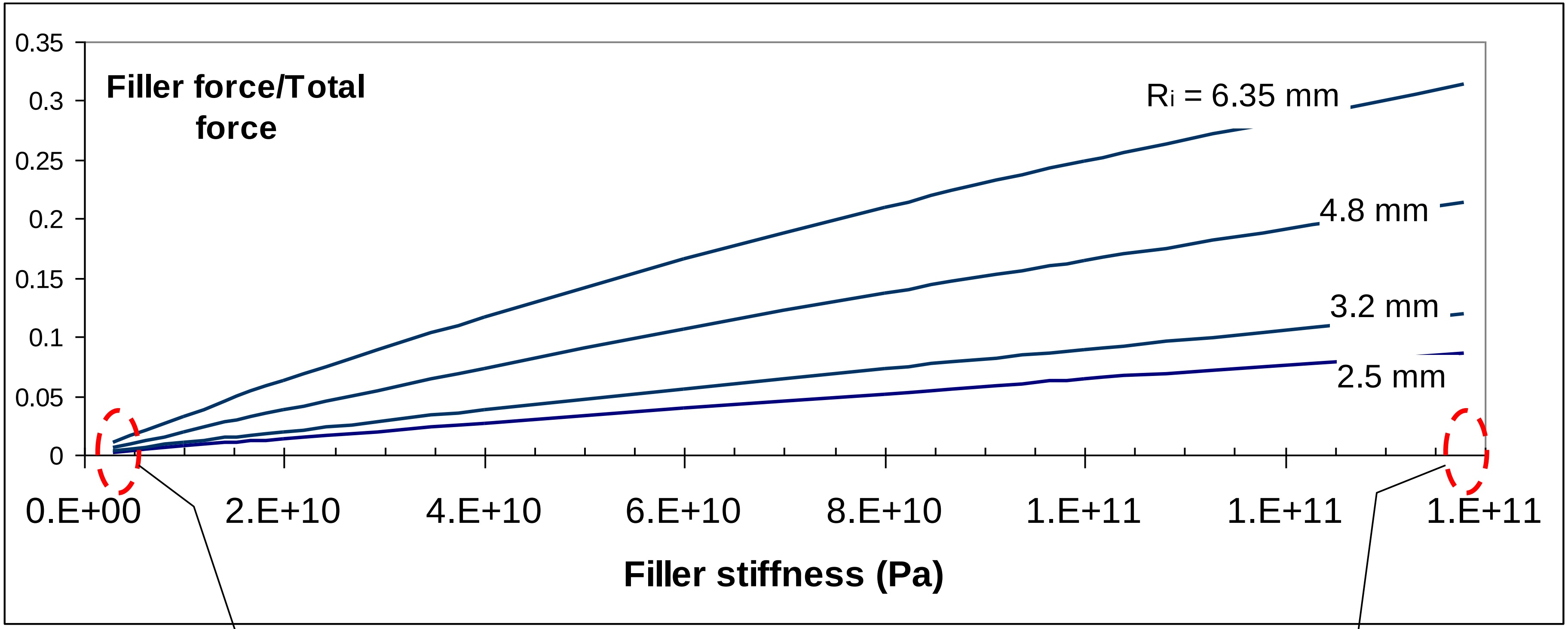
$R_i$ ranges from 2.5mm-6.35mm

$E_1$=89.6GPa

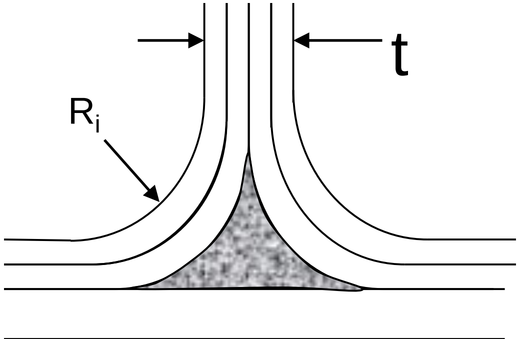
$E_2$= 31.0GPa

$E_3$=48.3GPa

# Crippling: Effect of filler stiffness and radius




filler material is all resin




filler material is UD tape