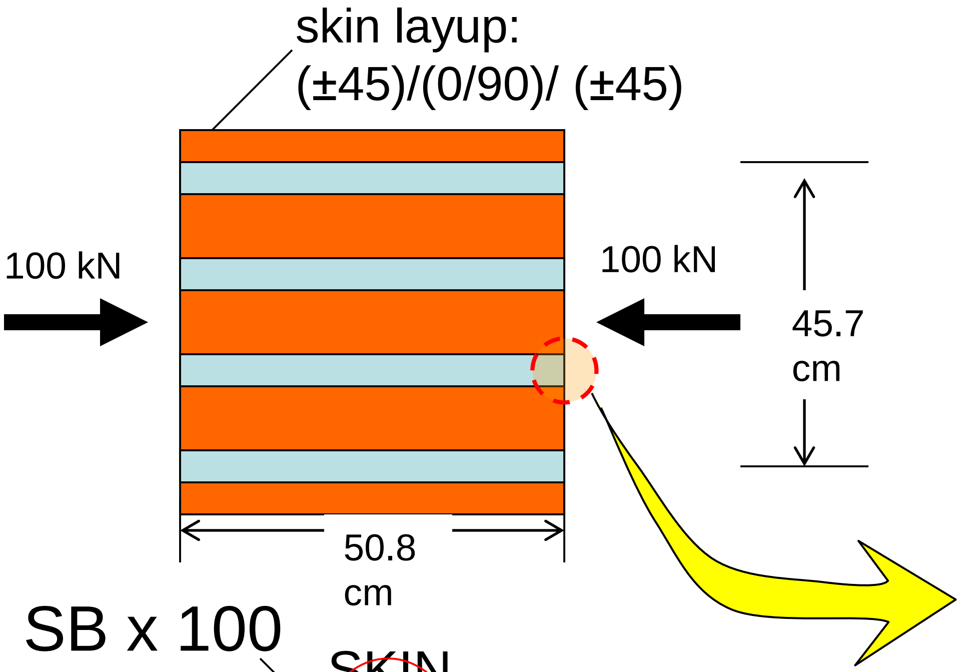
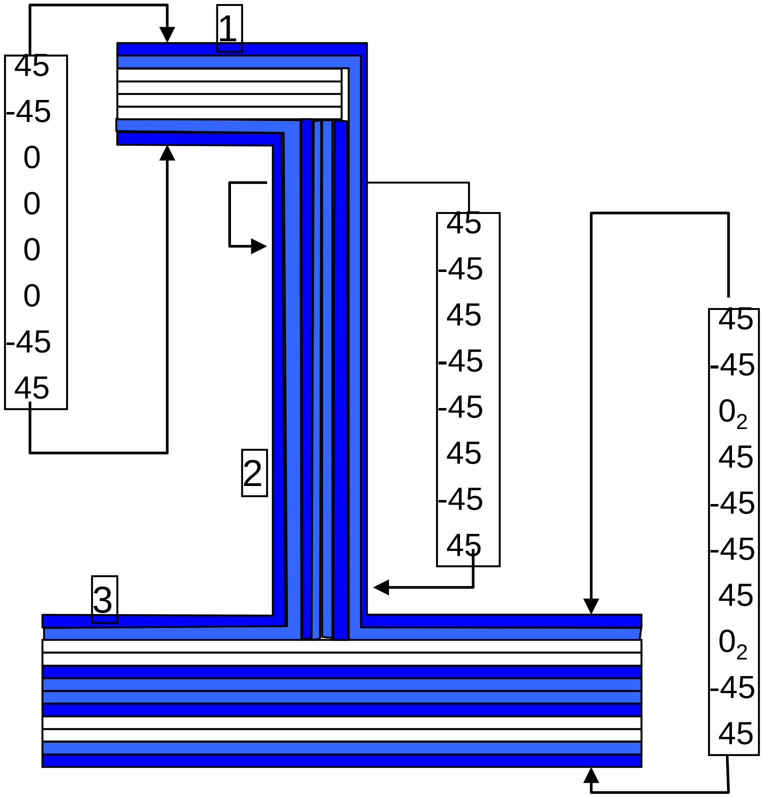
Example: stiffened panel under compression





SB x 100

SKIN

D11	659.7	Nmm
D12	466.9	Nmm
D22	659.7	Nmm
D66	494.0	Nmm

skin thickness=0.57 mm

A11	28912.44	N/mm
A12	12491.43	N/mm
A22	28912.44	N/mm
A66	13468.58	N/mm

Member	b (mm)	t (mm)	Em (GPa)	Eb (GPa)
1	12.7	1.2192	75.6	32.4
2	31.75	1.2192	18.2	17.9
3	38.1	1.8288	56.5	47.9

Example: Stiffened panel under compression

- assume skin between stiffeners (and frames at the ends) is simply supported; then, the15.2 cm x 50.8 cm skin buckles when

$$N_o = \frac{\pi^2\left[D_{11}m^4 + 2(D_{12}+2D_{66})m^2(AR)^2 + D_{22}(AR)^4\right]}{a^2m^2}$$ =182N/mm

(m=3)

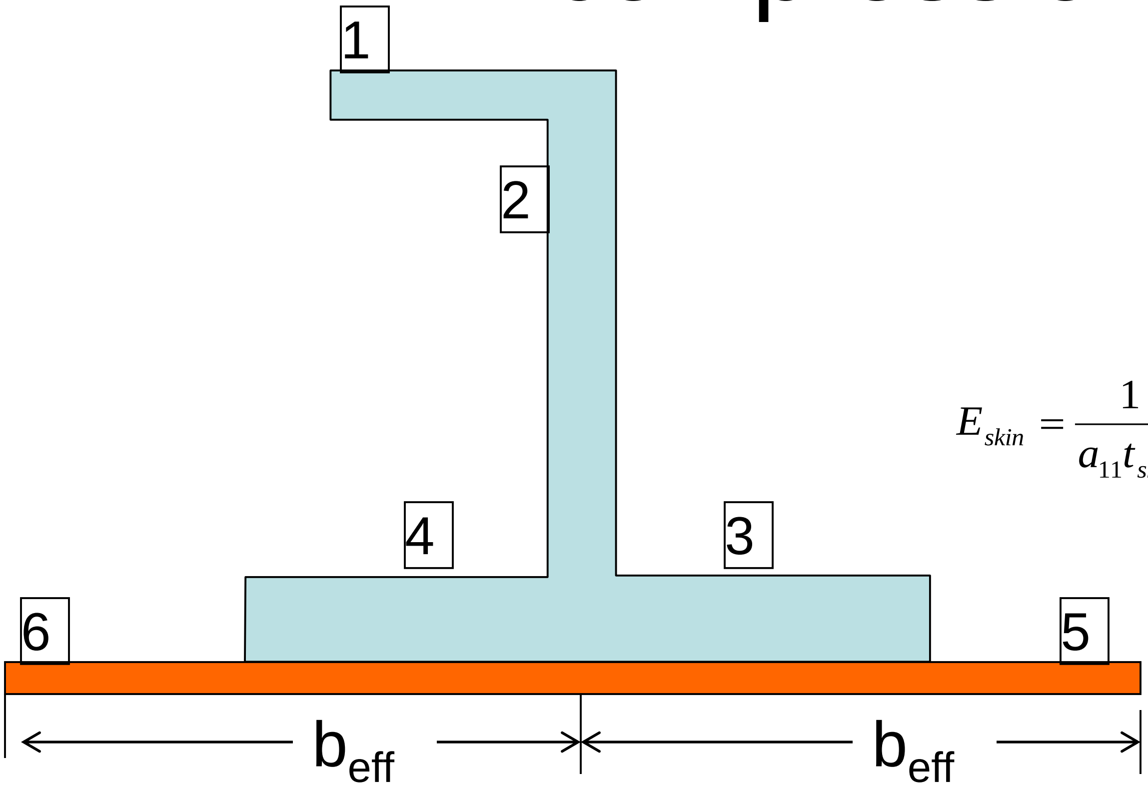
PB= (100000/457)/182 = 1.20

- assume that after buckling the load in the skin stays constant and equal to the buckling load
- Load per stiffener-skin combination:

– stiffener =(100000-182x457)/4=4207 N

– skin only = 182 x 457/4= 20794 N

Example: Stiffened panel under compression



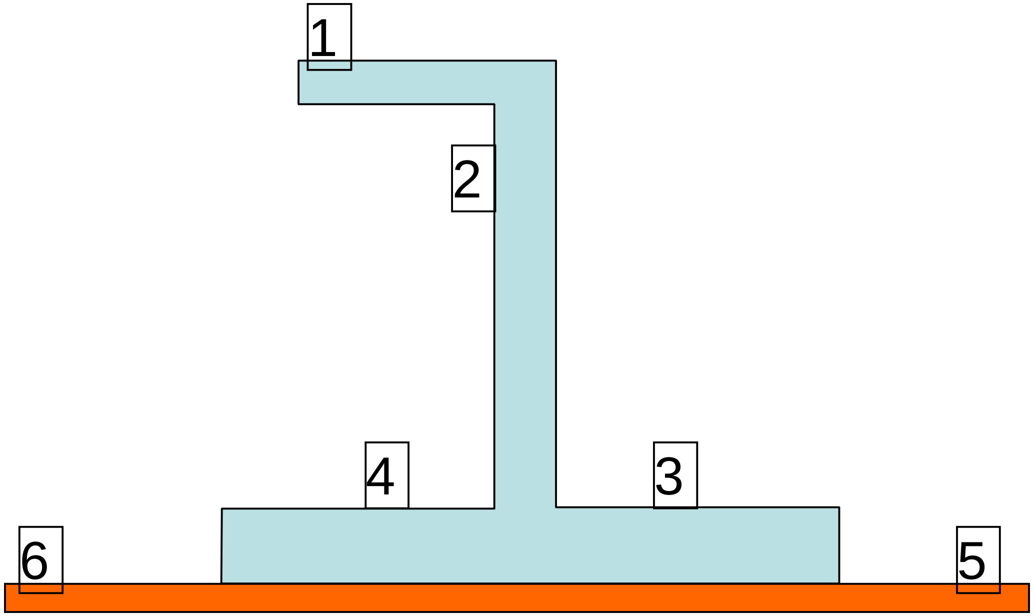
$$E_{skin} = \frac{1}{a_{11} t_{skin}} = \frac{\left(A_{11}A_{22} - A_{12}^{\;2}\right)}{A_{22} t_{skin}} = 41.15 GPa$$

$$b_{eff} = a \frac{1}{2\left(1 + 2\left(1 + \frac{A_{12}}{A_{11}}\right)\left(1 - \frac{P_{cr}}{P_x}\right)\frac{A_{11}}{A_{11} + 3A_{22}}\right)} = 0.292a = \underline{4.45\text{ cm}}$$

note small inconsistency: buckling calc’s done with ss edges but b_{eff} assumes immovable edges

Example: Stiffened panel under compression

Member	b (mm)	t(mm)	E (N/m^2)	A (mm^2)	EA (N)	Fi/FTOT
1	12.7	1.22	7.56E+10	15.48	1.17E+05	0.148
2	31.75	1.22	1.82E+10	38.71	7.05E+04	0.089
3	19.05	1.83	5.65E+10	34.84	1.97E+05	0.249
4	19.05	1.83	5.65E+10	34.84	1.97E+05	0.249
5	44.5	0.57	4.12E+10	25.37	1.04E+05	0.132
6	44.5	0.57	4.12E+10	25.37	1.04E+05	0.132
				Total	7.90E+05	



Example: Stiffened panel under compression

F_i/F_{TOT}	Applied F (N)	$\sigma_{applied}$ (N/mm^2)
0.148	623.42	40.26
0.089	375.20	9.69
0.249	1048.31	30.09
0.249	1048.31	30.09
0.132	10952.9	431.8
0.132	10952.9	431.8

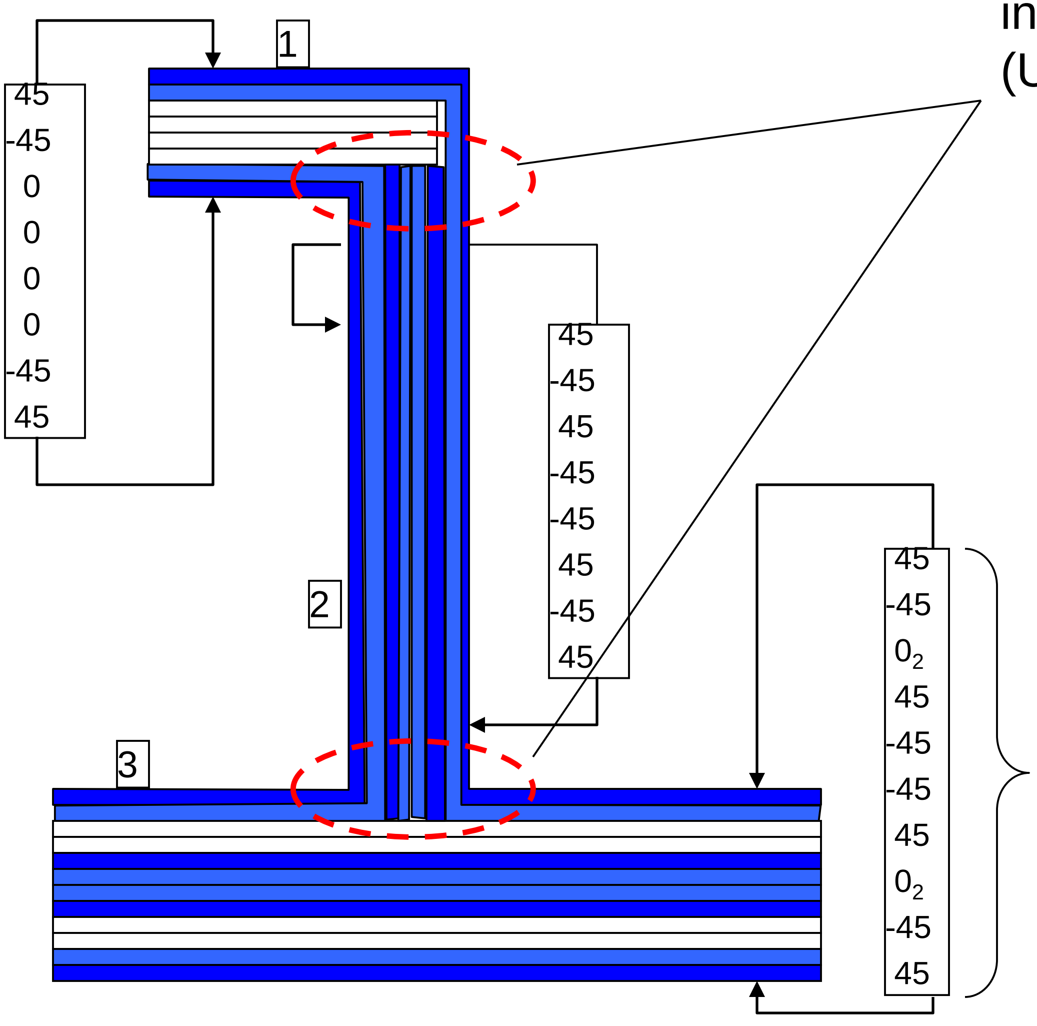
applied/ allowable
0.268
0.121
0.282
0.282
9.942
9.942

Member	b (mm)	t(mm)	OEF/NEF	b/t	$\sigma_{crip}/\sigma_{cu}$	σ_{cu}(N/mm^2)	σ_{fail} (N/mm^2)
1	12.7	1.22	OEF	10.42	0.304	494.64	150.2344
2	31.75	1.22	NEF	26.04	0.282	283.88	80.04247
3	19.05	1.83	OEF	10.42	0.304	351.75	106.8331
4	19.05	1.83	OEF	10.42	0.304	351.75	106.8331
5	44.5	0.57	NEF	78.07	0.082	529.14	43.43236
6	44.5	0.57	NEF	78.07	0.082	529.14	43.43236

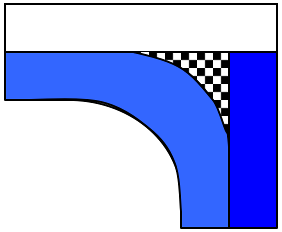
assumption based on load distribution
but probably too conservative

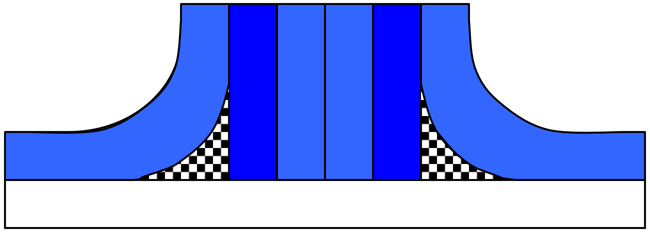
- skin fails! stiffener does not; since stiffener has high margin, perhaps can add to it load from skin to see if it is OK

Revisiting the layup



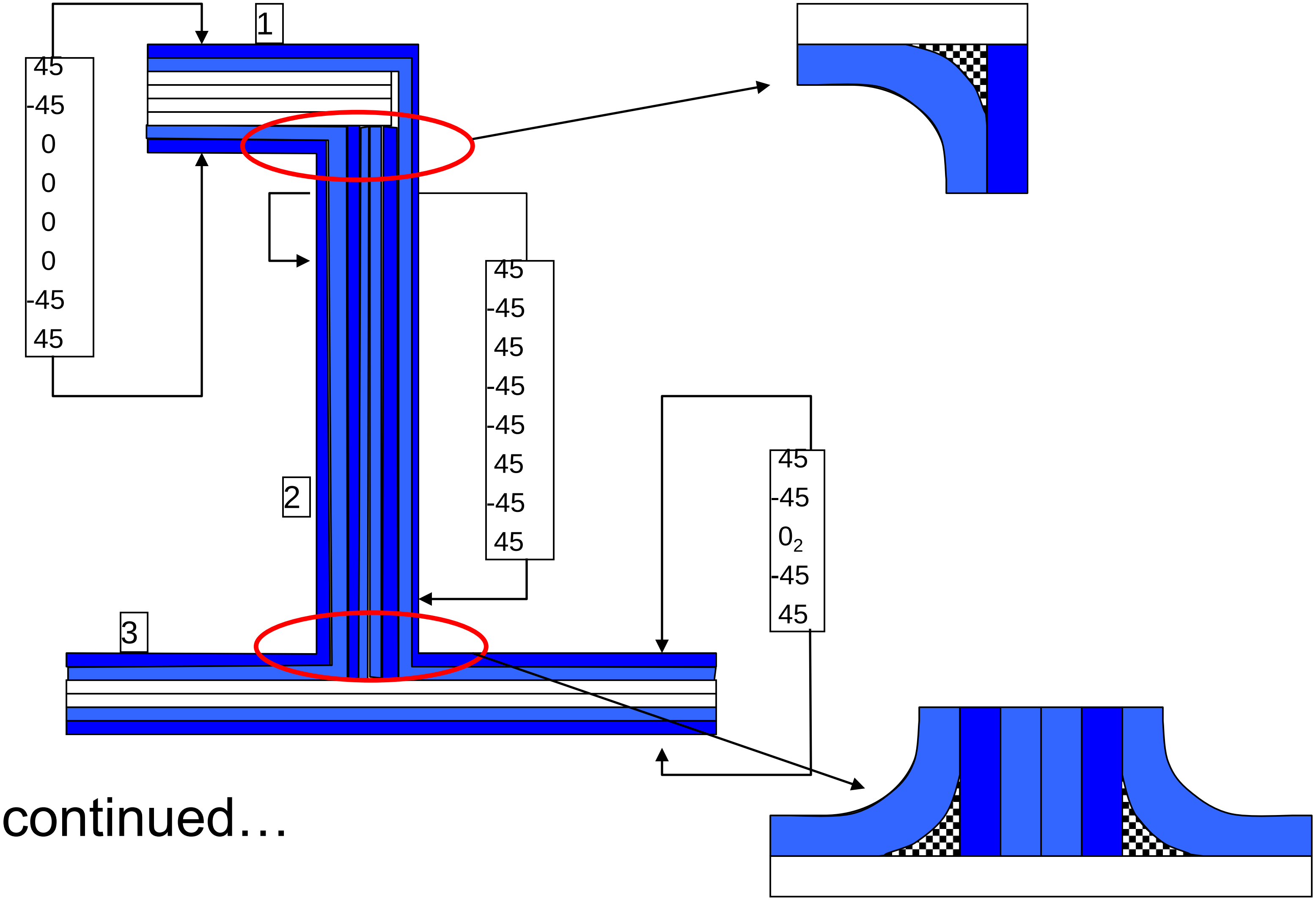
include filler material (UD tape strips)





too stiff, especially compared to the 3-ply skin of the example; also must move ±45 away from mid-plane

Revisiting the layup



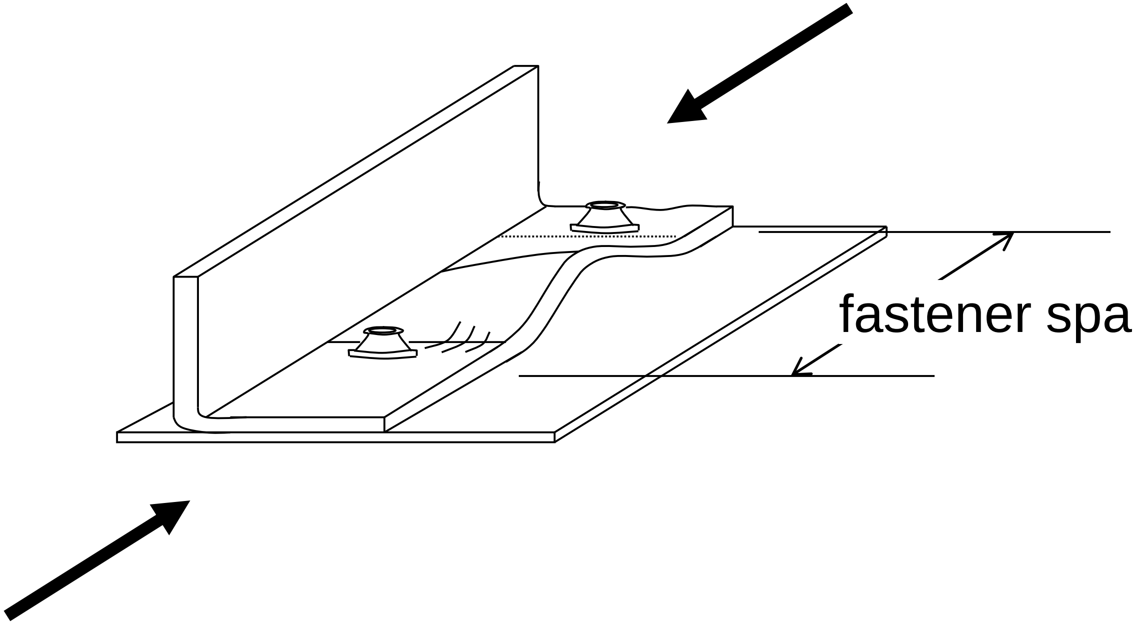
45
-45
0
0
0
0
-45
45

45
-45
45
-45
-45
45
-45
45

45
-45
0_2
-45
45

to be continued…

Inter-rivet buckling of stiffener flanges



- flange under compression buckles with a half-wave length equal to the fastener spacing
- fasteners are used if the skin/stiffener combination is not co-cured; also to keep stiffener from pulling-off the skin during post-buckling

Inter-rivet buckling: Design Eqn.

- conservatively, treat flange as a beam (neglect constraint at the flange/web interface) under compression

- the governing equation we had before

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\bcancel{\frac{\partial^4 w}{\partial x^2\partial y^2}}+D_{22}\bcancel{\frac{\partial^4 w}{\partial y^4}}=N_x\frac{\partial^2 w}{\partial x^2}+N_y\bcancel{\frac{\partial^2 w}{\partial y^2}}$$

- becomes

$$D_{11}\frac{\partial^4 w}{\partial x^4}+N_o\frac{\partial^2 w}{\partial x^2}=0 \qquad (N_o=-N_x)$$

Inter-rivet buckling: Design Eqn

- which has the general solution

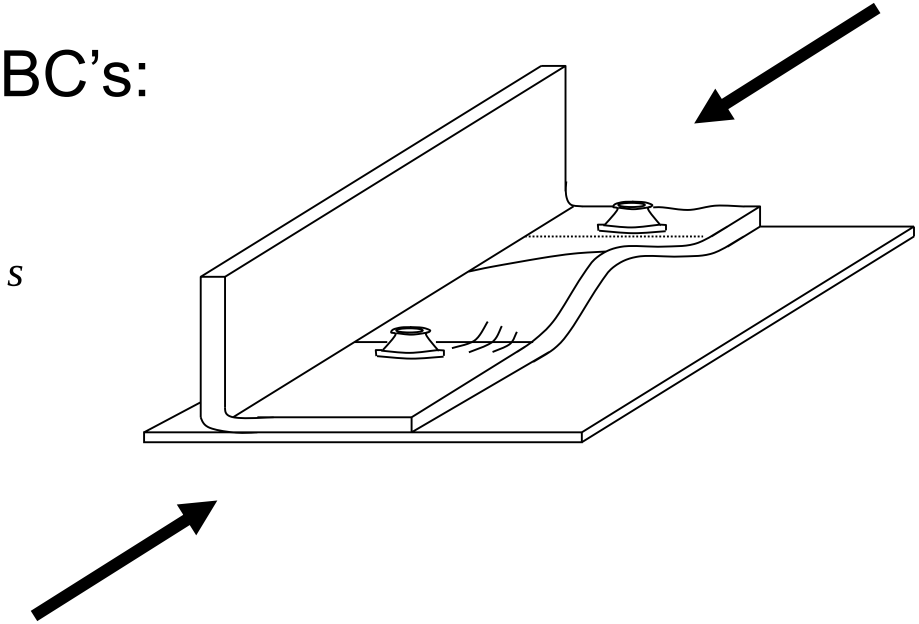
$$w = C_o + C_1 x + C_2 \sin\left(\sqrt{\frac{N_o}{D_{11}}}x\right) + C_3 \cos\left(\sqrt{\frac{N_o}{D_{11}}}x\right)$$

C_i to be determined from BC's

- for simply supported BC's:

$$w(x=0) = w(x=s) = 0$$

$$-D_{11}\frac{d^2 w}{dx^2} = M = 0 \quad at \quad x = 0, x = s$$



Inter-rivet buckling: Design Eqn

- substituting in the BC's:

$$w(x=0)=0 \Rightarrow C_o + C_3 = 0 \qquad (1)$$

$$w(x=s)=0 => C_o + C_1 s + C_2 \sin\left(\sqrt{\frac{N_o}{D_{11}}}s\right) + C_3 \cos\left(\sqrt{\frac{N_o}{D_{11}}}s\right) = 0 \qquad (2)$$

$$-D_{11}\frac{d^2w}{dx^2}(x=0) \Rightarrow C_3\frac{N_0}{D_{11}} = 0 \qquad (3)$$

$$-D_{11}\frac{d^2w}{dx^2}(x=s) \Rightarrow C_2\frac{N_0}{D_{11}}\sin\left(\sqrt{\frac{N_o}{D_{11}}}s\right) + C_3\frac{N_0}{D_{11}}\cos\left(\sqrt{\frac{N_o}{D_{11}}}s\right) = 0 \qquad (4)$$

(3)=> C_3=0

(1)=> C_o=0

(4)=> $\sin\left(\sqrt{\frac{N_o}{D_{11}}}s\right) = 0 \Rightarrow \sqrt{\frac{N_o}{D_{11}}}s = n\pi \Rightarrow N_o = \frac{n^2\pi^2}{s^2}D_{11}$

(use eq (2) to get a relation between C_1 and C_2)

Inter-rivet buckling: Governing Eq

- finally, the inter-rivet buckling stress is

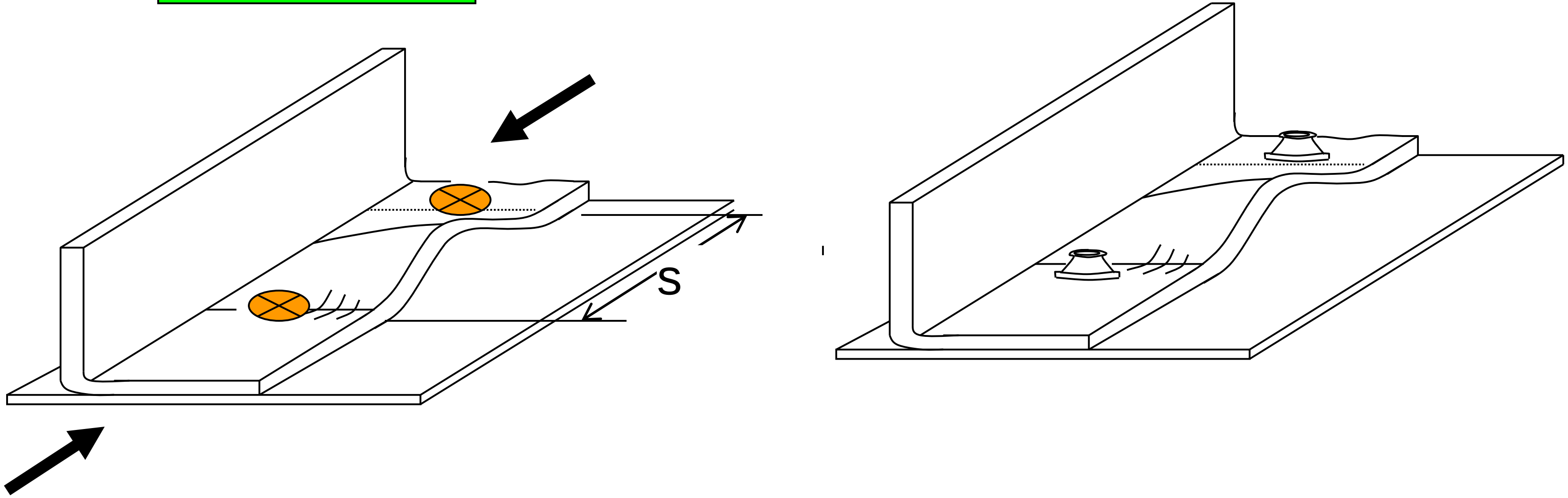
$$\sigma_{ir} = \frac{N_o}{t} = \frac{\pi^2 D_{11}}{ts^2}$$

- in general, for any type of BC at the fastener locations

$$\boxed{\sigma_{ir} = \frac{c\pi^2 D_{11}}{ts^2}}$$

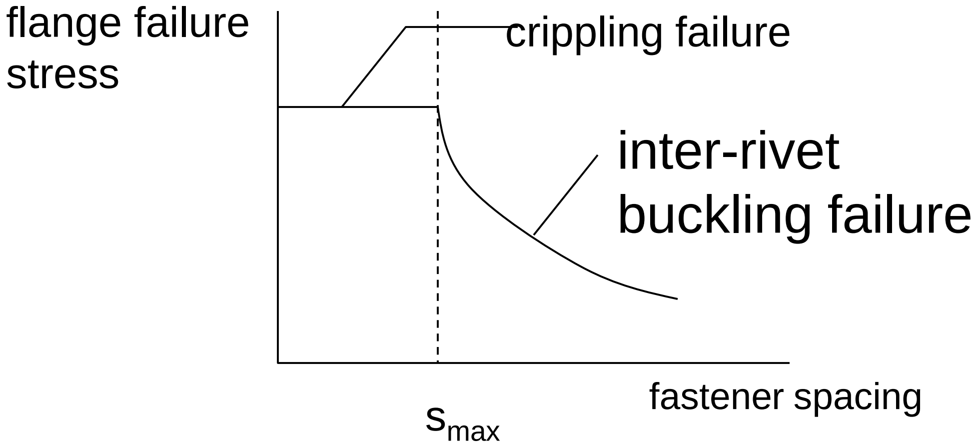
c=1 for counter-sunk fasteners

c=3 for protruding-head fasteners



Implications for fastener spacing

- if the fasteners are far from each other, flange fails in inter-rivet buckling
- if they are close to each other, flange fails in crippling
- there is, therefore, a maximum allowable fastener spacing beyond which inter-rivet buckling is important (for lower values crippling is the mode of failure)



Implications for fastener spacing

- equating the inter-rivet buckling stress to the crippling stress (OEF case)

$$s_{\max} = \sqrt{\frac{c\pi^2 D_{11}}{1.63 t \sigma_c^{\;u}} \left(\frac{b}{t}\right)^{0.717}}$$

fastener spacing must not exceed $s_{\max}$ to avoid inter-rivet buckling

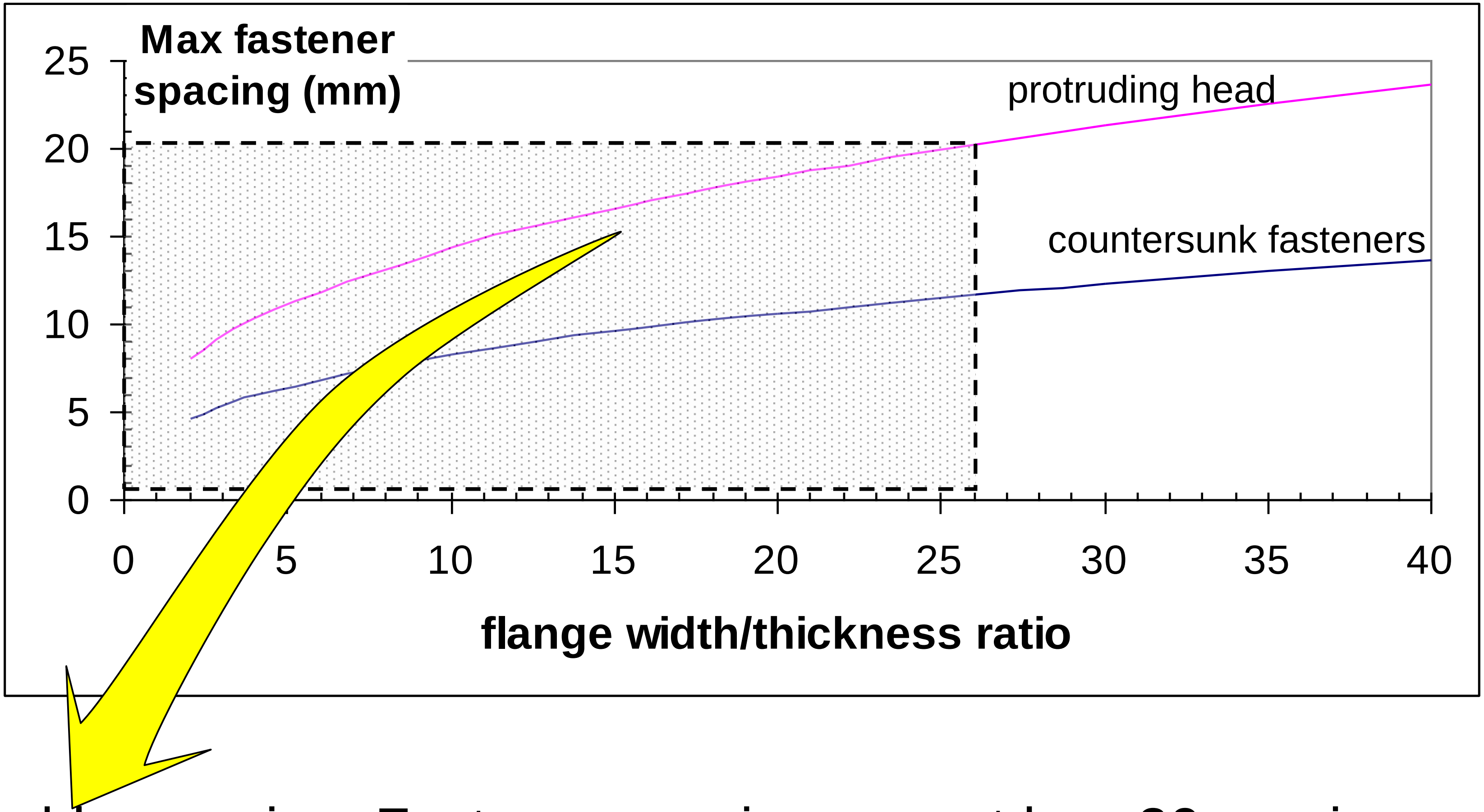
Implications for fastener spacing: Examples

Layup	$[45/0_2/-45/0_4]s$ (1)	[(±45)/(0/90)/ [(±45)]
σ_c^u (MPa)	762	529
D_{11} (Nm)	67.5	0.66
t (mm)	2.032	0.572

(1) this layup has too many 0 degree plies stacked next to each other => microcracking issues

Implications for fastener spacing

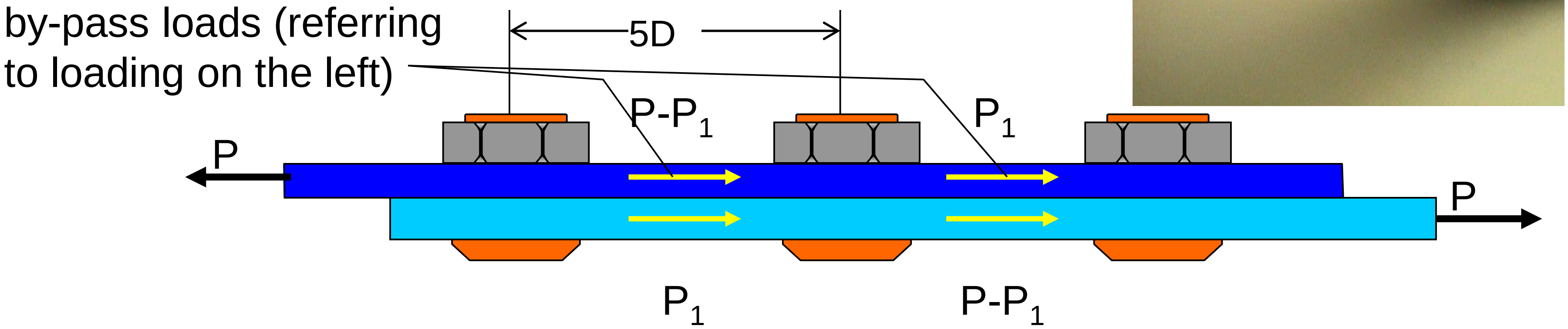
[(±45)/(0/90)/ [(±45)]



Problem region: Fastener spacing cannot be <20 mm in most applications!

Implications for fastener spacing

- to avoid interaction between fasteners and to allow the full by-pass load to develop, fastener spacing should be at least 20 mm (recommended value is 5D)

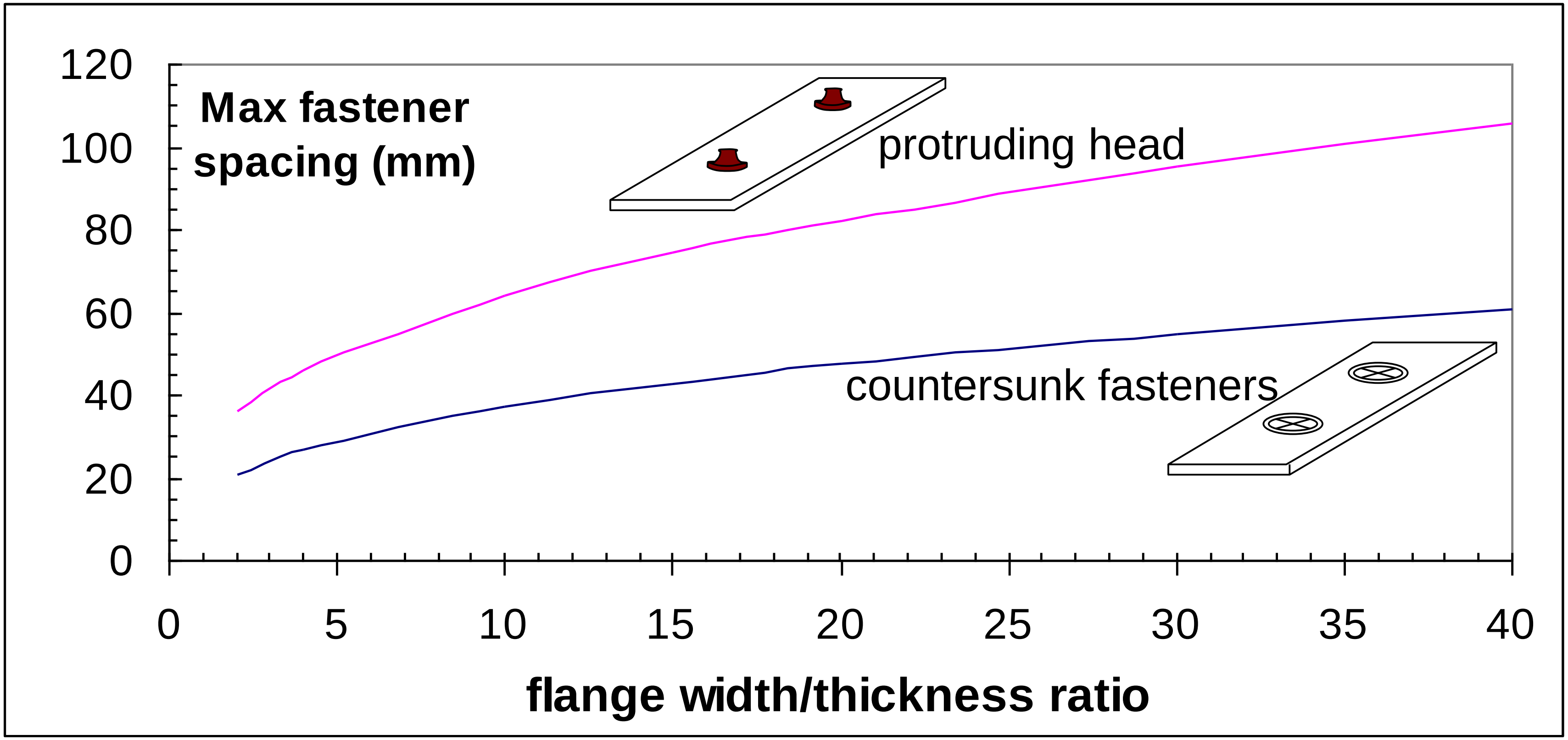


- for thin, soft flanges the max allowable fastener spacing is less than 20 mm => bad design

- therefore making the flange too compliant to match skin stiffness may lead to problems with fastener spacing

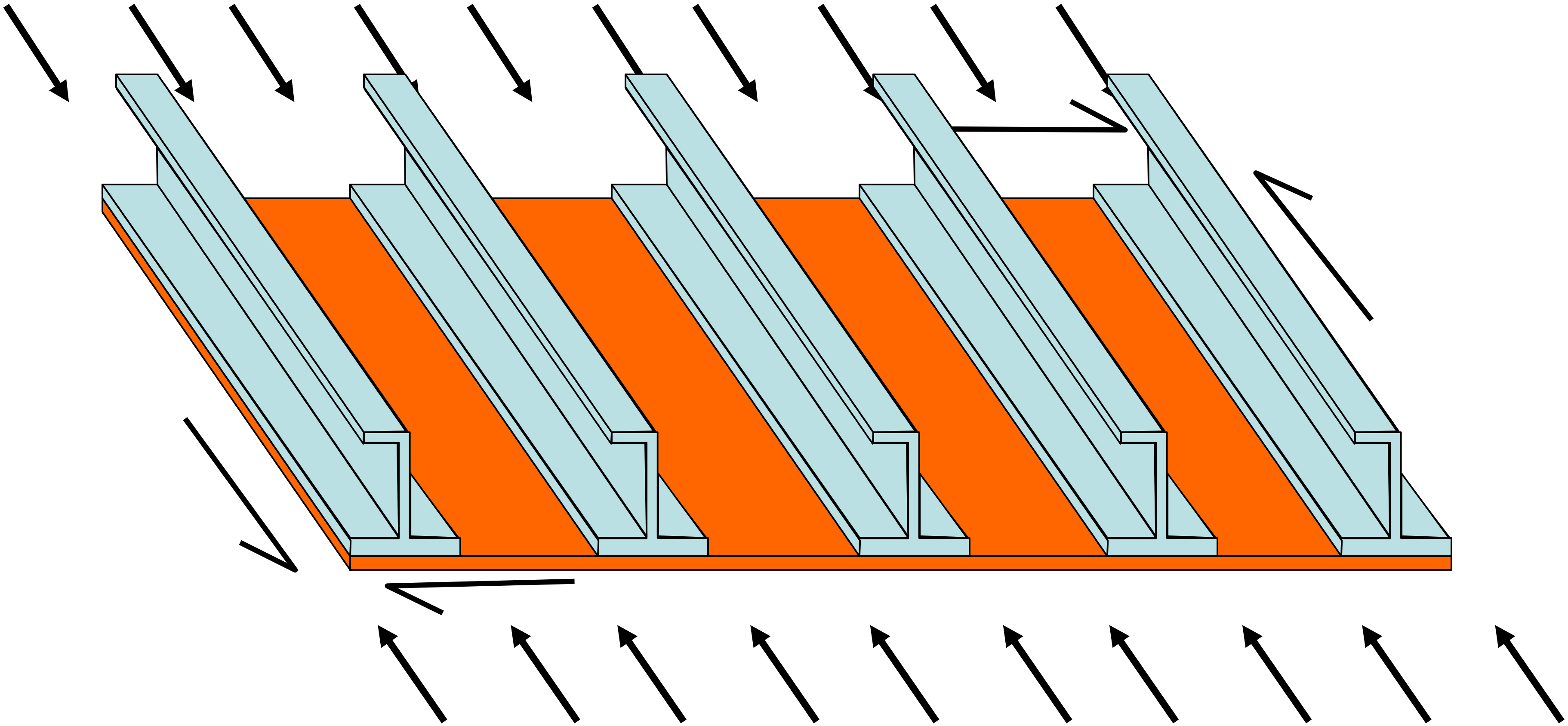
Implications for fastener spacing

$[45/0_2/-45/0_4]s$



- no issues with fastener spacing for this layup

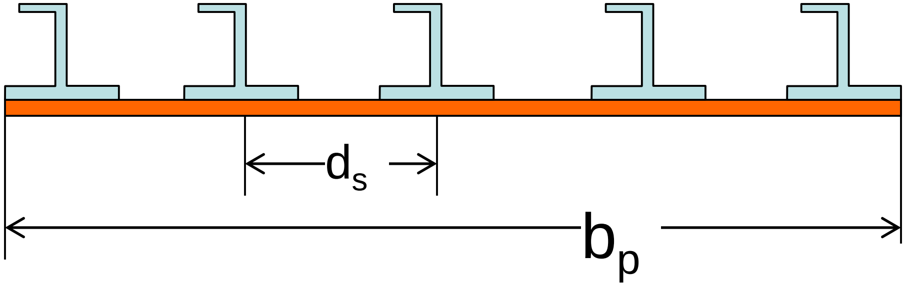
Skin-stiffened structure



Skin-stiffened structure

- skin takes pressure loads (membrane action)
- skin takes shear loads
- skin takes compression loads up to skin buckling (more if post-buckling is allowed)
- stiffeners take bending loads
- stiffeners take compression loads

Equivalent stiffness (membrane)

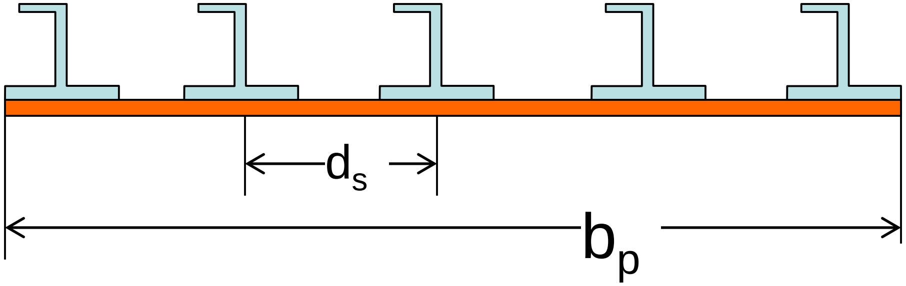


number of stiffeners, n_s:

$$n_s \approx \frac{b_p}{d_s}$$

(expression for n_s becomes more accurate as the number of stiffeners increases)

Equivalent stiffness (membrane)



$$\left.\begin{array}{l}
(A_{ij})_{eq} = (A_{ij})_{skin} + (A_{ij})_{stiffeners} \\[2ex]
(A_{ij})_{stiffeners} = n_s (A_{ij})_{\sin glestiff} \\[2ex]
(A_{ij})_{\sin glestiff} = \dfrac{(EA)_{stiff}}{b_p}
\end{array}\right\}$$

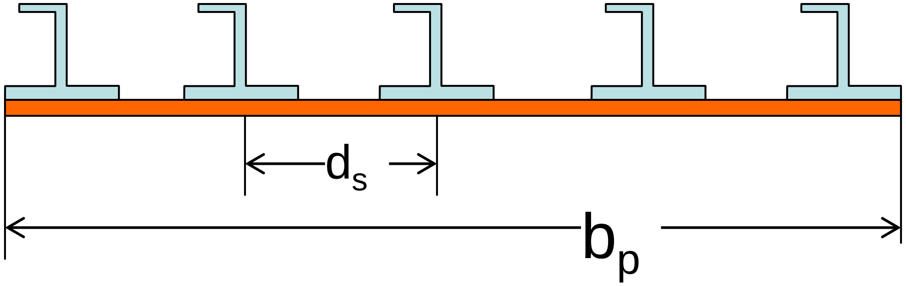
$$(A_{11})_{eq} \approx (A_{11})_{skin} + \frac{(EA)_{stiff}}{d_s}$$

$$(A_{12})_{eq} \approx (A_{12})_{skin}$$

$$(A_{22})_{eq} \approx (A_{22})_{skin}$$

$$(A_{66})_{eq} \approx (A_{66})_{skin}$$

Equivalent stiffness (bending)



- in a manner analogous to the membrane stiffness,

$$(D_{ij})_{eq} = (D_{ij})_{skin} + (D_{ij})_{stiffeners}$$

$$(D_{ij})_{stiffeners} = n_s (D_{ij})_{\sin glestiff}$$

$$(D_{11})_{\sin glestiff} = \frac{(EI)_{stif}}{b_p}$$

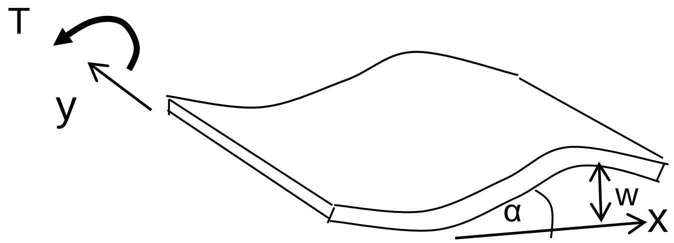
(1 is aligned with the stiffener axis)

- but not for D_{66} ! D_{66} relates applied torque M_{xy} to twisting curvature κ_{xy}

Equivalent stiffness (bending)

- the angle of twist is given by (small angles):

$$\alpha = \frac{\partial w}{\partial x}$$



- from torsion theory, the rate of twist is given by

$$\frac{d\alpha}{dy} = \frac{T}{GJ}$$

T=applied torque (Nm)

G=Shear modulus (N/m^2)

J= Torsional rigidity (m^4)

- combining:

$$\frac{d\alpha}{dy} = \frac{\partial^2 w}{\partial x \partial y} = \frac{T}{GJ} \tag{5.4.1.1}$$

Equivalent stiffness (bending)

- but from plate theory,

$$M_{xy} = -2D_{66}\frac{\partial^2 w}{\partial x \partial y}$$

(assuming D_{16}=D_{26}=0; Note units of M_{xy} are N-m/m) (5.4.1.2)

- and since M_{xy} is torque per unit width, for a plate of width b_p

$$\frac{T}{b_p} = -M_{xy}$$

(5.4.1.3)

- combining eqs 5.4.1.1-5.4.1.3 and solving for D_{66}, the contribution of a single stiffener is

$$\left(D_{66}\right)_{\sin glestiff} = \frac{\left(GJ\right)_{stif}}{2b_p}$$

(5.4.1.4)

Equivalent stiffness (bending)

- summing the contribution from all stiffeners,

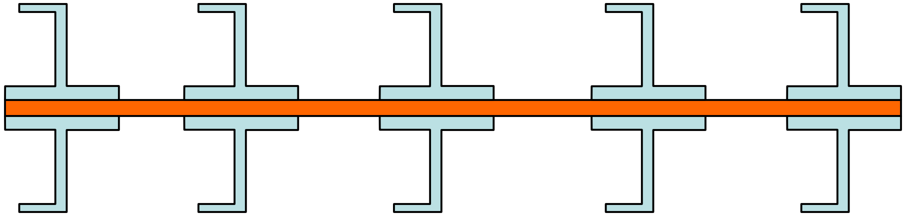
$$D_{66} = n_s \left(D_{66}\right)_{\sin glestiff} = \frac{\left(GJ\right)_{stif}}{2d_s}$$

- summarizing,

$$\boxed{\begin{aligned}
\left(D_{11}\right)_{eq} &\approx \left(D_{11}\right)_{skin} + \frac{\left(EI\right)_{stif}}{d_s} \\
\left(D_{12}\right)_{eq} &\approx \left(D_{12}\right)_{skin} \\
\left(D_{22}\right)_{eq} &\approx \left(D_{22}\right)_{skin} \\
\left(D_{66}\right)_{eq} &\approx \left(D_{66}\right)_{skin} + \frac{\left(GJ\right)_{stif}}{2d_s}
\end{aligned}}$$

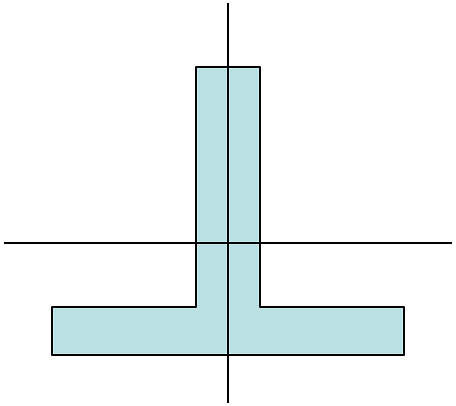
Equivalent stiffness- Notes

- number of stiffeners used in the expressions is approximate; the fewer the stiffeners the bigger the error
- the derivation neglects coupling; in general, unless the stiffeners are mirrored on the other side of the skin, there is a B-matrix present



B matrix=0

Note on membrane vs bending stiffness for a cross-section

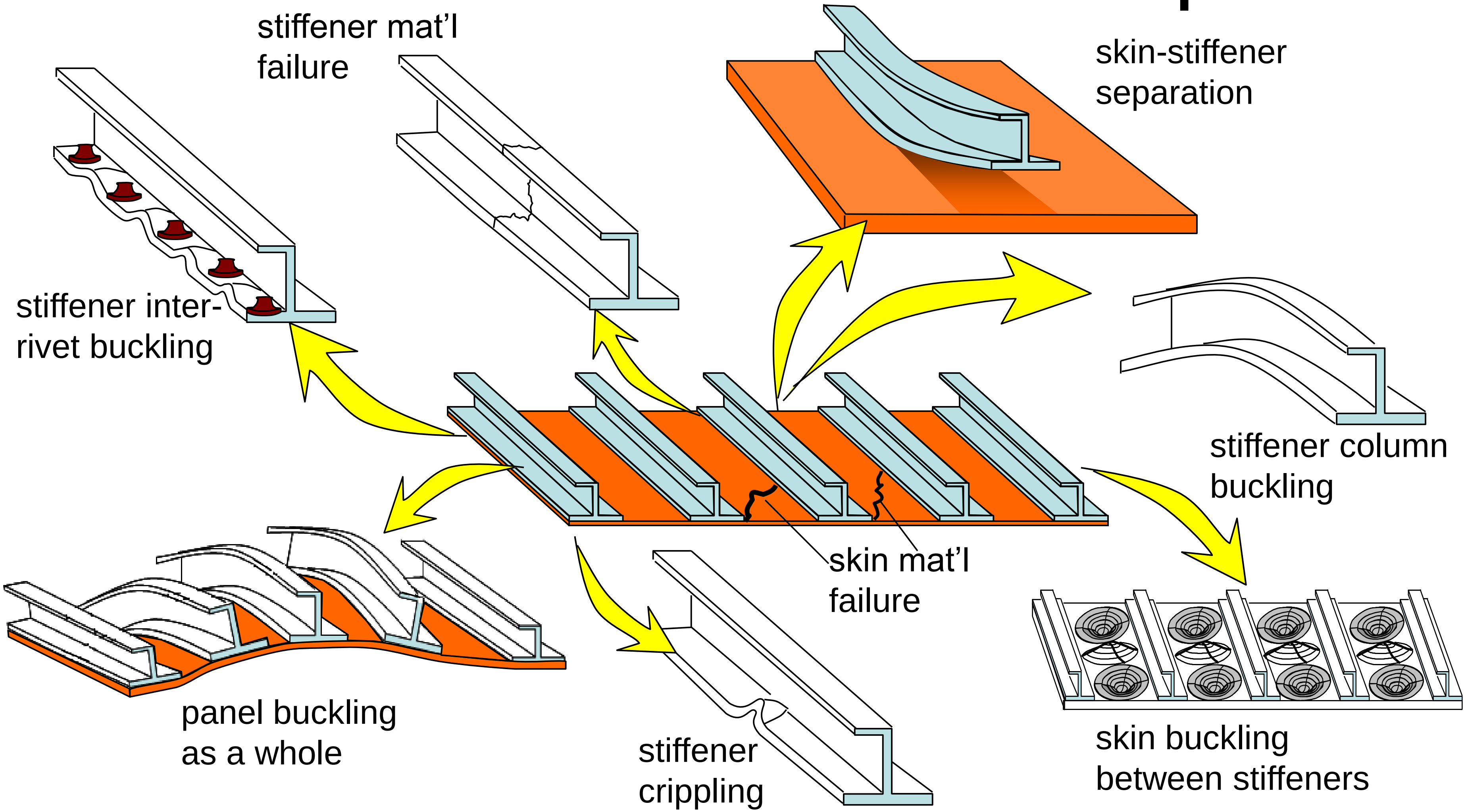


membrane $E_i = \frac{1}{(a_{11})_i t_i}$

bending $E_{bi} = \frac{12}{t_i^{\,3}(d_{11})_i}$

- axial loading up to buckling: use membrane
- bending loading or post-buckling: use bending for short stiffeners, otherwise membrane
- when in doubt use the average of the two or, whichever gives you the more conservative answer

Failure modes of stiffened panel



stiffener mat'l failure

skin-stiffener separation

stiffener inter-rivet buckling

stiffener column buckling

skin mat'l failure

panel buckling as a whole

stiffener crippling

skin buckling between stiffeners

- depending on the load, any of the constituents can fail in a variety of failure modes

Failure modes of stiffened panel

- failure modes do not occur simultaneously: some failure modes are more critical than others (loading and geometry dependent)
- in some cases, having more than one failure modes occur simultaneously is efficient since no component is overdesigned (this requires “independent” failure of components)
  - e.g. buckling between stiffeners and stiffener crippling occurring simultaneously

Failure modes of stiffened panel

- it is important and useful to know when failure switches from one mode to another
  - inter-rivet buckling switching to flange crippling

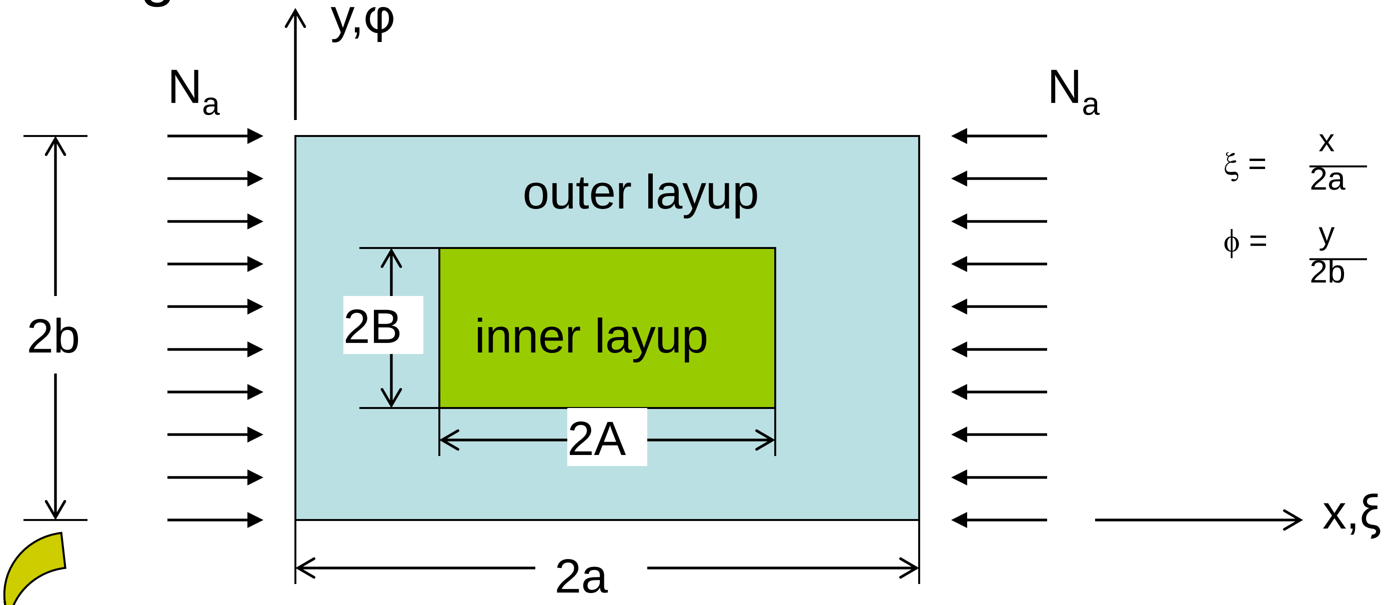
    (examined before)
  - inter-rivet buckling or flange crippling switching to column buckling
  - skin buckling between stiffeners switching to panel buckling as a whole (=> panel breaker condition)

    (coming up)
  - any stability failure switching to material failure (very rare)

Panel breaker condition

- what is the condition the stiffeners have to fulfill to keep the skin from (global) buckling as a whole? (i.e. skin buckles between stiffeners)

- one way to deal with this is to use the solution we saw at the beginning of the course:



$$\xi = \frac{x}{2a}$$

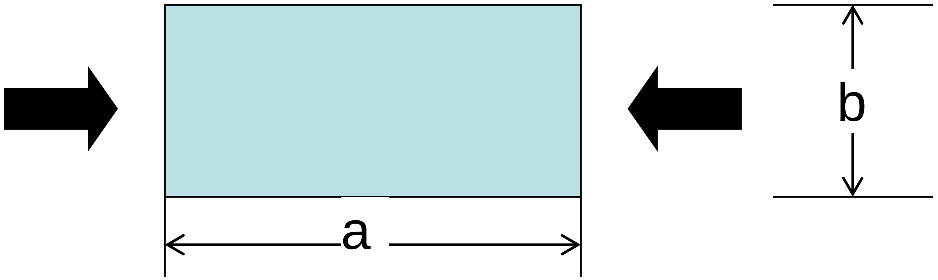
$$\phi = \frac{y}{2b}$$

led to system of equations: $[E]\{H\} = \{R\}$

Panel breaker condition

- another (simpler) approach uses the buckling solution and the equivalent stiffness expressions for a stiffened panel combined with some post-buckling requirement
- buckling load under compression (from before):

$$N_o = \frac{\pi^2 \left[D_{11} m^4 + 2(D_{12} + 2D_{66}) m^2 (AR)^2 + D_{22} (AR)^4 \right]}{a^2 m^2}$$

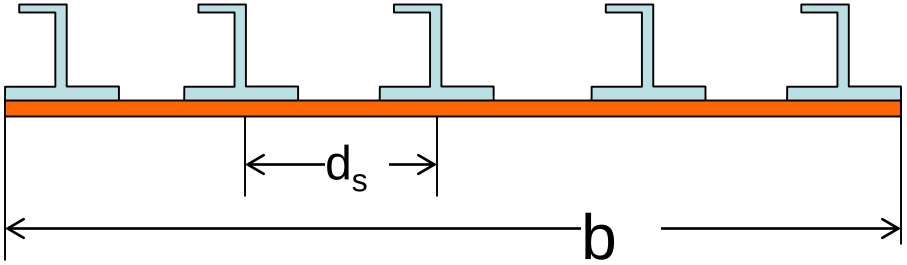


(AR)=aspect ratio=a/b

simply-supported plate

Panel breaker condition

- require that the buckling load of the panel as a whole equals the buckling load of the skin between stiffeners



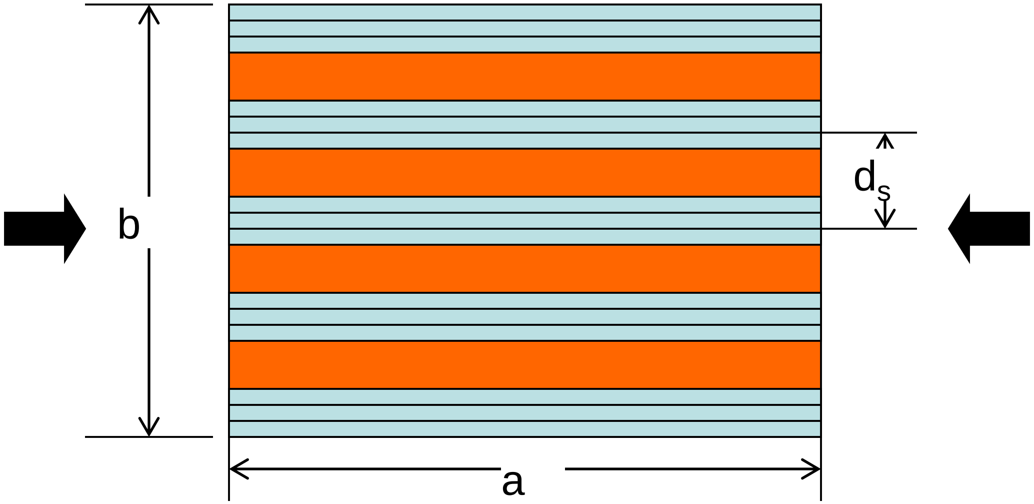
panel length a

$$\left(N_o\right)_{stiffenedpanel} = \left(N_o\right)_{skinbetweenstiffeners}$$

(corrected for the presence of stiffeners)

Panel breaker condition

- if F_{TOT} is the total applied force, then, prior to any of the components buckling, the forces in the skin and stiffeners can be determined from strain compatibility (see our derivation for EA for a stiffener cross-section)



Panel breaker condition

- impose two requirements:
  - the entire cross-section (skin and stiffeners smeared together) does not buckle before the skin between stiffeners buckles
  - no stiffener can buckle (column buckling) before the ultimate load is reached; note that the skin between stiffeners buckles before ultimate load and the ultimate load = skin buckling load x PB (Post Buckling ratio)

Panel breaker condition

- First requirement (panel buckling=bay buckling)

- force in the skin

$$F_{skin} = \frac{bA_{11}}{bA_{11} + b\frac{EA}{d_s}} F_{TOT} = \frac{A_{11}}{A_{11} + \frac{EA}{d_s}} F_{TOT} \qquad (5.4.2.1)$$

Number of stiffeners (from before)

A_{ij} refer to skin membrane stiffness; EA is stiffener axial stiffness

- and

$$N_{xskin} = \frac{F_{skin}}{b} \qquad (5.4.2.2)$$