Offshore Hydromechanics Part 2

Ir. Peter Naaijen

1. Introduction and Ship Motion

TU Delft

OE 4630 2012 - 2013
Offshore Hydromechanics, lecture 1

Take your laptop, i- or whatever smart-phone and go to:
www.rwpoll.com
Login with session ID

Teacher module II:
- Ir. Peter Naaijen
- p.naaijen@tudelft.nl
- Room 34 B-0-360 (next to towing tank)

Book:
- Offshore Hydromechanics, by J.M.J. Journee & W.W.Massie

Useful weblinks:
- http://www.shipmotions.nl
- Blackboard

OE4630 module II course content

- +/- 7 Lectures
- Bonus assignments (optional, contributes 20% of your exam grade)
- Laboratory Excercise (starting 30 nov)
  - 1 of the bonus assignments is dedicated to this exercise
  - Groups of 7 students
  - Subscription available soon on BB
- Written exam

Schedule OE4630 D2, Offshore Hydromechanics Pt 2, 2012-2013 **Version 1 (9-11-2012)**
Disclaimer: always track for (last minute) changes in location at huidigeroosters.tudelft.nl!!!

Date :	Time:	Type:	Teacher:	Location
Wed 14 Nov	13.30 – 16.30	Lecture	Peter Naaijen	3mE-CZ D (James Watt)
Wed 14 Nov	16.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ D (James Watt)
Fri 16 Nov	10.30 – 12.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 19 Nov	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Tue 20 Nov	13.30 – 15.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ C (Daniel Bernoulli)
Wed 28 Nov	13.30 – 15.30	Lecture	Peter Naaijen	3mE-CZ D (James Watt)
Wed 28 Nov	15.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ D (James Watt)
Fri 30 Nov	10.30 – 13.00	Lab session	Peter Naaijen	Towing Tank
Mon 3 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Tue 4 Dec	13.30 – 16.00	Lab session	Gideon Hertzberger	Towing Tank
Tue 4 Dec	16.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	Room Peter Naaijen (34 B 0 360)
Mon 10 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 17 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 7 Jan	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)

Lecture notes:

- Disclaimer: Not everything you (should) learn is in the lecture notes (lees: niet alles wat op het tentamen gevraagd kan worden staat in diktaat…) →

Make personal notes during lectures!!

- Don't save your questions 'till the break →

Ask if anything is unclear

Learning goals Module II, behavior of floating bodies in waves

• Definition of ship motions **Motion Response in regular waves:** • How to use RAO's • Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces • How to solve RAO's from the equation of motion **Motion Response in irregular waves:** •How to determine response in irregular waves from RAO's and wave spectrum without forward speed	
3D linear Potential Theory •How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential •How to determine Velocity Potential	
Motion Response in irregular waves: • How to determine response in irregular waves from RAO's and wave spectrum with forward speed • Make down time analysis using wave spectra, scatter diagram and RAO's	Ch. 8
Structural aspects: • Calculate internal forces and bending moments due to waves	
Nonlinear behavior: • Calculate mean horizontal wave force on wall • Use of time domain motion equation	Ch.6

Introduction


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Introduction

Offshore → oil resources have to be explored in deeper water → floating structures instead of bottom founded


[4]

Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Inertia forces on sea fastening due to accelerations:

Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Direct wave induced structural loads

Minimum required air gap to avoid wave damage

Introduction

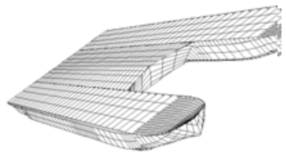
Reasons to study waves and ship behavior in waves:

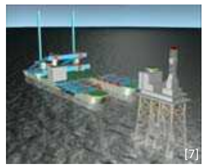
- Determine allowable / survival conditions for offshore operations

Introduction

Decommissioning / Installation / Pipe laying → Excalibur / Allseas 'Pieter Schelte'

- Motion Analysis





Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Forces on mooring system, motion envelopes loading arms

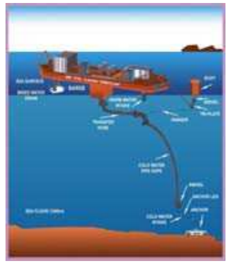
[8]

[2]

Introduction

Floating Offshore: More than just oil

Floating wind farm [9]



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Introduction

Floating Offshore: More than just oil

[11]

[12]

Wave energy conversion

Introduction

Floating Offshore: More than just oil

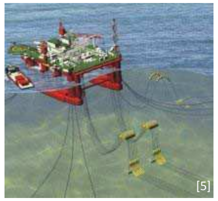
Mega Floaters

Introduction

Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations
- Downtime analysis

Winter Data of Areas 8, 9, 15 and 16 of the North Atlantic (Global Wave Statistics)												
	T_2 (s)											
Hs (m)	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	Total
14.5	0	0	0	0	2	30	154	362	466	370	202	1586
13.5	0	0	0	0	3	33	145	293	322	219	101	1116
12.5	0	0	0	0	7	72	289	539	548	345	149	1949
11.5	0	0	0	0	17	160	585	996	931	543	217	3449
10.5	0	0	0	1	41	363	1200	1852	1579	843	310	6189
9.5	0	0	0	4	109	845	2485	3443	2648	1283	432	11249
8.5	0	0	0	12	295	1996	5157	6323	4333	1882	572	20570
7.5	0	0	0	41	818	4723	10537	11242	6755	2594	703	37413
6.5	0	0	1	138	2273	10967	20620	18718	9665	3222	767	66371
5.5	0	0	7	471	6187	24075	36940	27702	11969	3387	694	111432
4.5	0	0	31	1586	15757	47072	56347	33539	11710	2731	471	169244
3.5	0	0	148	5017	34720	74007	64809	28964	7804	1444	202	217115
2.5	0	4	681	13441	56847	77259	45013	13962	2725	381	41	210354
1.5	0	40	2699	23284	47839	34532	11554	2208	282	27	2	122467
0.5	5	350	3314	8131	5858	1598	216	18	1	0	0	19491
Total	5	394	6881	52126	170773	277732	256051	150161	61738	19271	4863	999995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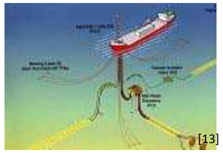
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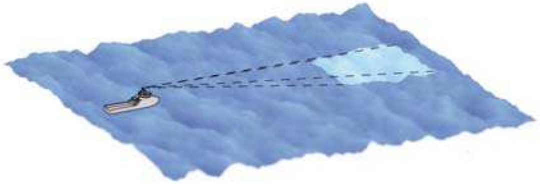
Real-time motion prediction
Using X-band radar remote wave observation



Definitions & Conventions
Regular waves
Ship motions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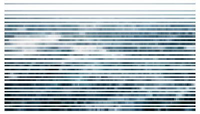
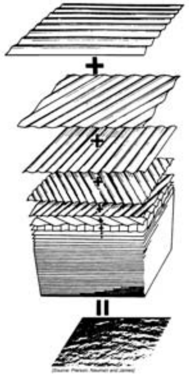
Irregular wind waves

apparently irregular but can be considered as a superposition of a finite number of regular waves, each having own frequency, amplitude and propagation direction

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Regular waves

(Ch.5 revisited)

Regular waves

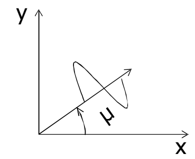
regular wave propagating in direction μ:

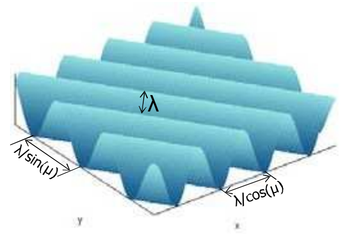
$$\zeta(t,x) = \zeta_a \cos(\omega t - kx\cos\mu - ky\sin\mu)$$

$$k = 2\pi / \lambda$$

$$\omega = 2\pi / T$$

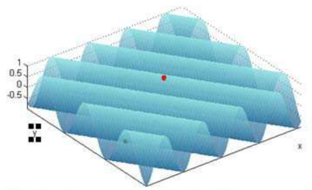
Linear solution Laplace equation

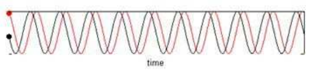




- Regular waves
- regular wave propagating in direction μ

$$\zeta(t,x) = \zeta_a \cos(\omega t \underbrace{- kx\cos\mu - ky\sin\mu}_{\text{Phase angle } \varepsilon})$$





Phase angle wave at black dot with respect to wave at red dot:

$$\varepsilon_{\zeta,\zeta} = -k(x - x)\cos\mu - k(y - y)\sin\mu$$

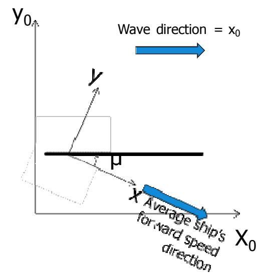
Co-ordinate systems

Definition of systems of axes

Earth fixed: (x_0, y_0, z_0)

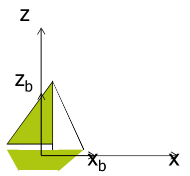
Following average ship position: (x,y,z)

wave direction with respect to ship's exes system: μ



Behavior of structures in waves

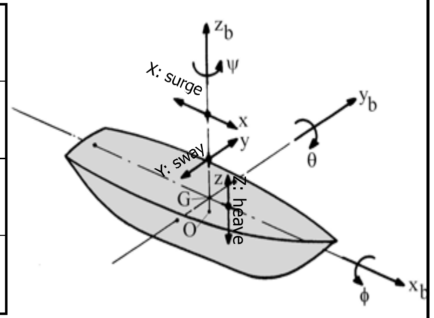
Ship's body bound axes system (x_b,y_b,z_b) follows all ship motions



Behavior of structures in waves

Definition of translations

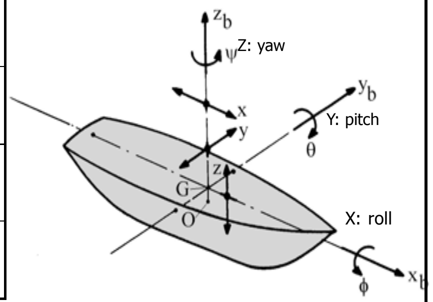
		NE	EN
1	x	Schrikken	Surge
2	y	Verzetten	Sway
3	z	Dompen	Heave



Behavior of structures in waves

Definition of rotations

		NE	EN
4	x	Slingeren	Roll
5	y	Stampen	Pitch
6	z	Gieren	Yaw



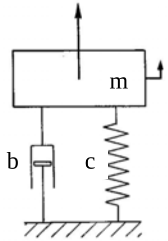
How do we describe ship motion response?

Rao's
Phase angles

Mass-Spring system:

$m\ddot{z} + b\dot{z} + cz = F_a \cos(\omega t)$ Motion equation

$z(t) = z_a \cos(\omega t + \varepsilon)$ Steady state solution



Motions of and about COG

Amplitude Phase angle

$Surge(schrikken): \; x = x_a \cos(\omega t + \varepsilon_{x\zeta})$

$Sway(verzetten): \; y = y_a \cos(\omega t + \varepsilon_{y\zeta})$

$Heave(dompen): \; z = z_a \cos(\omega t + \varepsilon_{z\zeta})$

$Roll(rollen): \; \langle phi \rangle \phi = \phi_a \cos(\omega t + \varepsilon_{\phi\zeta})$

$Pitch(stampen): \; \langle theta \rangle \theta = \theta_a \cos(\omega t + \varepsilon_{\theta\zeta})$

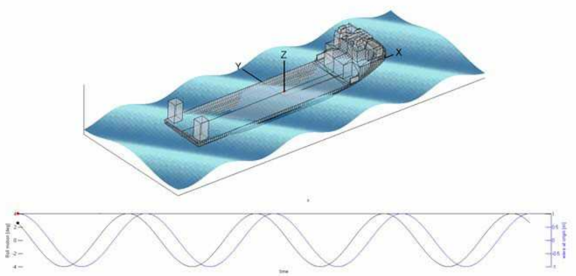
$Yaw(gieren): \; \langle psi \rangle \psi = \psi_a \cos(\omega t + \varepsilon_{\psi\zeta})$

Phase angles ε are related to undisturbed wave at origin of steadily translating ship-bound system of axes (→COG)

Motions of and about COG

Phase angles ε are related to undisturbed wave at origin of steadily

translating ship-bound system of axes (→COG)



Motions of and about COG

$Surge(schrikken): \; x = x_a \cos(\omega t + \varepsilon_{x\zeta})$ $RAOSurge: \frac{x_a}{\zeta_a}(\omega, \mu)$

$Sway(verzetten): \; y = y_a \cos(\omega t + \varepsilon_{y\zeta})$ $RAOSway: \frac{y_a}{\zeta_a}(\omega, \mu)$

$Heave(dompen): \; z = z_a \cos(\omega t + \varepsilon_{z\zeta})$ $RAOHeave: \frac{z_a}{\zeta_a}(\omega, \mu)$

$Roll(rollen): \; \langle phi \rangle \phi = \phi_a \cos(\omega t + \varepsilon_{\phi\zeta})$ $RAORoll: \frac{\phi_a}{\zeta_a}(\omega, \mu)$

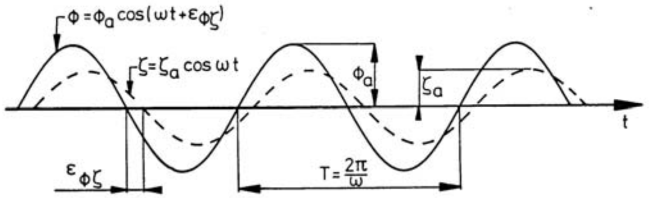
$Pitch(stampen): \; \langle theta \rangle \theta = \theta_a \cos(\omega t + \varepsilon_{\theta\zeta})$ $RAOPitch: \frac{\theta_a}{\zeta_a}(\omega, \mu)$

$Yaw(gieren): \; \langle psi \rangle \psi = \psi_a \cos(\omega t + \varepsilon_{\psi\zeta})$ $RAOYaw: \frac{\psi_a}{\zeta_a}(\omega, \mu)$

RAO and phase depend on:

- Wave frequency
- Wave direction

Example: roll signal



Displacement $\phi = \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right)$

Velocity... $\dot{\phi} = -\omega_e \phi_a \sin\left(\omega_e t + \varepsilon_{\phi\zeta}\right) = \omega_e \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta} + \pi/2\right)$

Acceleration... $\ddot{\phi} = -\omega_e^2 \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right) = \omega_e^2 \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta} + \pi\right)$

Motions of and about COG

1 $Surge(schrikken): \; x = x_a \cos\left(\omega_e t + \varepsilon_{x\zeta}\right)$

2 $Sway(verzetten): \; y = y_a \cos\left(\omega_e t + \varepsilon_{y\zeta}\right)$

3 $Heave(dompen): \; z = z_a \cos\left(\omega_e t + \varepsilon_{z\zeta}\right)$

4 $Roll(rollen): \; (phi)\, \phi = \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right)$

5 $Pitch(stampen): \; (theta)\, \theta = \theta_a \cos\left(\omega_e t + \varepsilon_{\theta\zeta}\right)$

6 $Yaw(gieren): \; (psi)\, \psi = \psi_a \cos\left(\omega_e t + \varepsilon_{\psi\zeta}\right)$

- Frequency of input (regular wave) and output (motion) is ALWAYS THE SAME !!
- Phase can be positive ! (shipmotion ahead of wave elevation at COG)
- Due to symmetry: some of the motions will be zero
- Ratio of motion amplitude / wave amplitude = RAO (Response Amplitude Operator)
- RAO's and phase angles depend on wave frequency and wave direction
- RAO's and phase angles must be calculated by dedicated software or measured by experiments
- Only some special cases in which 'common sense' is enough:

Consider Long waves relative to ship dimensions

What is the RAO of pitch in head waves ?

- Phase angle heave in head waves ?...
- RAO pitch in head waves ?...
- Phase angle pitch in head waves ?...
- Phase angle pitch in following waves ?...

Local motions (in steadily translating axes system)

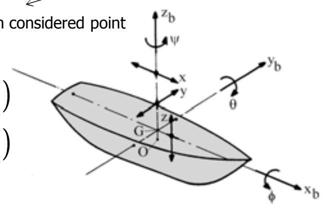
- **Only variations!!**
- **Linearized!!**

$$\begin{pmatrix} x_P(t) \\ y_P(t) \\ z_P(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} + \begin{pmatrix} 0 & -\psi(t) & \theta(t) \\ \psi(t) & 0 & -\phi(t) \\ -\theta(t) & \phi(t) & 0 \end{pmatrix} \cdot \begin{pmatrix} x_{bP} \\ y_{bP} \\ z_{bP} \end{pmatrix}$$

6 DOF Ship motions

Location considered point

$$x_P(t) = x(t) - y_{bP}\psi(t) + z_{bP}\theta(t)$$
$$y_P(t) = y(t) + x_{bP}\psi(t) - z_{bP}\phi(t)$$
$$z_P(t) = z(t) - x_{bP}\theta(t) + y_{bP}\phi(t)$$



Local Motions

Example 3: horizontal crane tip motions

The tip of an onboard crane, location:
xb,yb,zb = -40, -9.8, 25.0

For a frequency ω=0.6 the RAO's and phase angles of the ship motions are:

SURGE		SWAY		HEAVE		ROLL		PITCH		YAW	
RAO	phase	RAO	phase	RAO	phase	RAO	phase	RAO	phase	RAO	phase
-	degr	-	degr	-	degr	deg/m	degr	deg/m	degr	deg/m	degr
1.014E-03	3.421E+02	5.992E-01	2.811E+02	9.991E-01	3.580E+02	2.590E+00	1.002E+02	2.424E-03	1.922E+02	2.102E-04	5.686E+01

Calculate the RAO and phase angle of the transverse horizontal motion (y-direction) of the crane tip.

Complex notation of harmonic functions

$$1\; Surge(schrikken): \quad x = x_a \cos\left(\omega_e t + \varepsilon_{x\zeta}\right)$$

$$= \mathrm{Re}\left(x_a e^{i\left(\omega t + \varepsilon_{x\zeta}\right)} \right)$$

$$= \mathrm{Re}\left(x_a e^{\varepsilon_{x\zeta}} \cdot e^{i\omega t} \right)$$

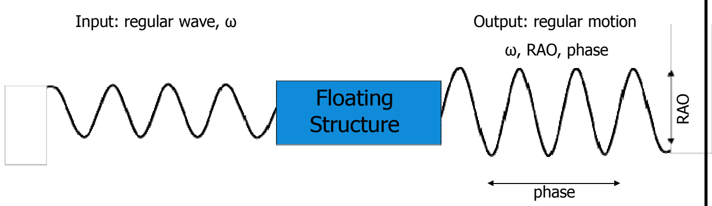
Complex motion amplitude

$$= \mathrm{Re}\left(\hat{x}_a \cdot e^{i\omega t} \right)$$

- :

Relation between Motions and Waves

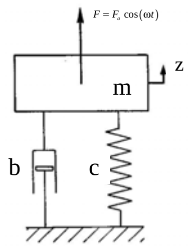
How to calculate RAO's and phases ?



Mass-Spring system:

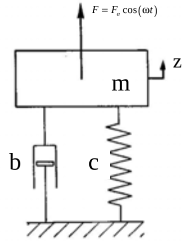
Forces acting on body:

...?



Mass-Spring system:

$$m\ddot{z} + b\dot{z} + cz = F_a \cos(\omega t)$$



Transient solution

$$z(t) = A_t e^{-\zeta\omega_0 t} \sin\left(\sqrt{1-\zeta^2}\,\omega_0 t + \varphi_t\right)$$

$$\left(\zeta = \frac{b}{2\sqrt{mc}}\right)$$ Damping ratio

Steady state solution:

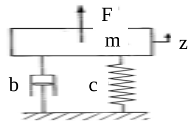
$$z(t) = z_a \cos(\omega t + \varepsilon)$$

$$\varepsilon = a\tan\left(\frac{(-b\omega)}{(-\omega^2(m) + c)}\right)$$

$$z_a = \sqrt{\frac{F_a^2}{\left((-(m)\omega^2 + c)^2 + (b\omega)^2\right)}}$$

Moving ship in waves:

[14]



$$m_3\ddot{z} + b_3\dot{z} + c_3 z = F_{a3}\cos(\omega t)$$

Restoring coefficient for **heave** ?

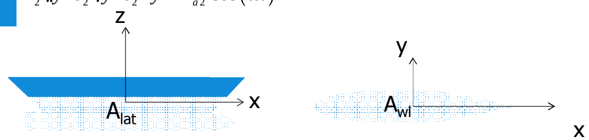
$$m_4\ddot{\varphi} + b_4\dot{\varphi} + c_4\varphi = F_{a4}\cos(\omega t)$$

Restoring coefficient for **roll** ?

m for **roll** ?

What is the hydrostatic spring coefficient for the sway motion ?

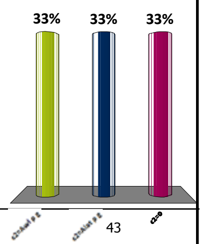
$$m_2 \cdot \ddot{y} + b_2 \cdot \dot{y} + c_2 \cdot y = F_{a2}\cos(\omega t)$$



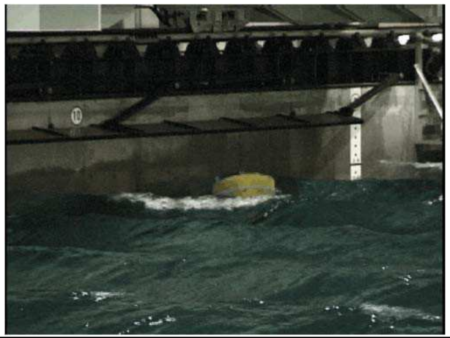
A. $c_2 = A_{wl}\,\rho\,g$

B. $c_2 = A_{lat}\,\rho\,g$

C. $c_2 = 0$



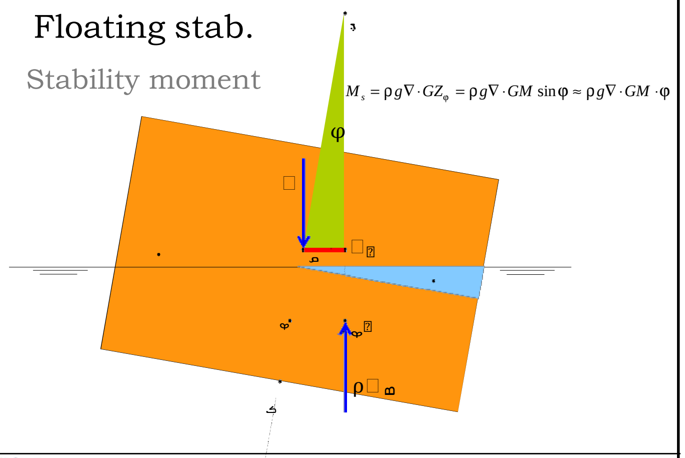
Non linear stability issue…



Floating stab.

Stability moment

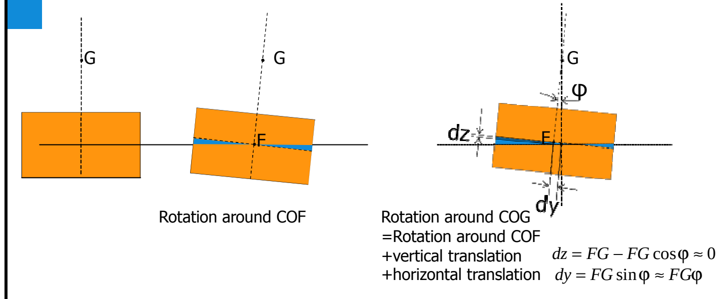
$$M_s = \rho g \nabla \cdot GZ_\varphi = \rho g \nabla \cdot GM \sin\varphi \approx \rho g \nabla \cdot GM \cdot \varphi$$



Moving ship in waves:

$$m_4\ddot{\varphi} + b_4\dot{\varphi} + c_4\varphi = F_{a4}\cos(\omega t)$$

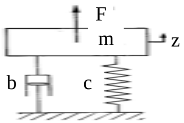
Restoring coefficient for **roll** ?



$dz = FG - FG\cos\varphi \approx 0$

$dy = FG\sin\varphi \approx FG\varphi$

Moving ship in waves:
Not in air but in water!



$F = m\ddot{z}$

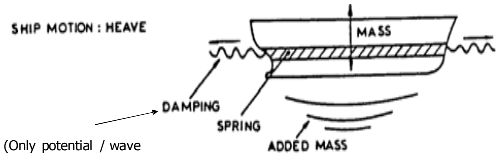
→ F_w

→ $-c \cdot z$

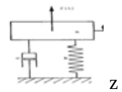
→ $-b \cdot \dot{z}$

→ $-a \cdot \ddot{z}$

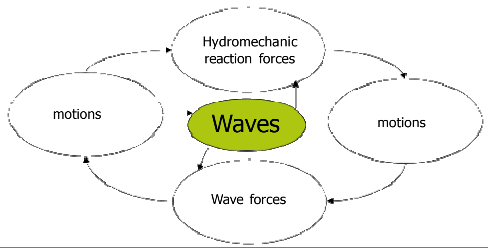
(Only potential / wave damping)



Moving ship in waves:



$$(m+a)\ddot{z} + b\dot{z} + c \cdot z = F_w$$

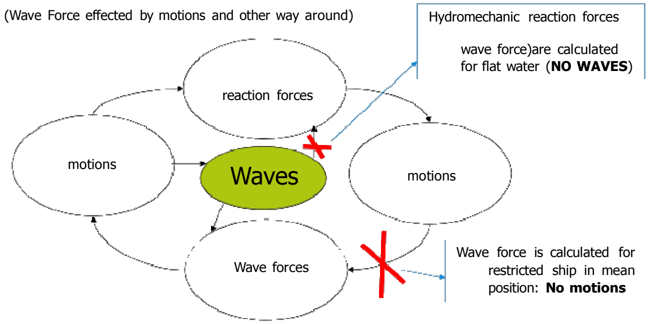


Moving ship in waves:

Solutions

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$

2. (Wave Force effected by motions and other way around)



Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x)=\zeta_a\cos(\omega t-kx\cos\mu-ky\sin\mu)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x)=\zeta_a\cos(\omega t-kx\cos\mu-ky\sin\mu)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

Potential Theory

What is potential theory ?:
way to give a mathematical description of flowfield

Most complete mathematical description of flow is viscous Navier-Stokes equation:

Navier-Stokes vergelijkingen:

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\lambda\nabla\cdot V+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]$$

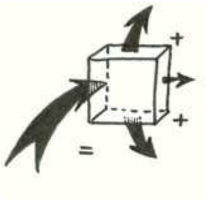
$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left(\lambda\nabla\cdot V+2\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]$$

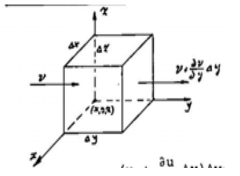
$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left(\lambda\nabla\cdot V+2\mu\frac{\partial w}{\partial z}\right)$$

(A-relaxed !)

$\rightarrow$

Apply principle of continuity on control volume:





Continuity: what comes in, must go out

This results in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

If in addition the flow is considered to be irrotational and non viscous $\rightarrow$

Velocity potential function can be used to describe water motions
Main property of velocity potential function:

for potential flow, a function $\Phi(x,y,z,t)$ exists whose derivative in a certain arbitrary direction equals the flow velocity in that direction. This function is called the velocity potential.

From definition of velocity potential:

$$u = \frac{\partial \Phi}{\partial x}, v = \frac{\partial \Phi}{\partial v}, w = \frac{\partial \Phi}{\partial z}$$

Substituting in continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Results in Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

Summary

- Potential theory is mathematical way to describe flow

Important facts about velocity potential function Φ:

- definition: Φ is a function whose derivative in any direction equals the flow velocity in that direction
- Φ describes non-viscous flow
- Φ is a scalar function of space and time (NOT a vector!)

Summary

- Velocity potential for regular wave is obtained by
  - Solving Laplace equation satisfying:
    1. Seabed boundary condition
    2. Dynamic free surface condition

$$\Phi(x, y, z, t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx \cos\mu + ky \sin\mu - \omega t)$$

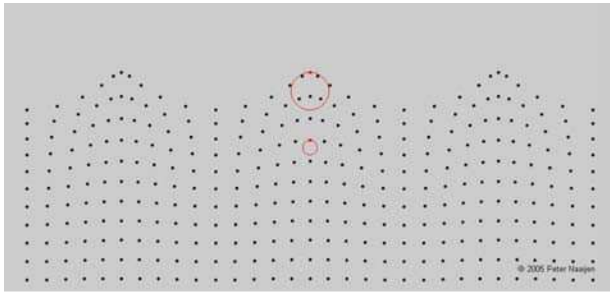
$$\Phi(x, y, z, t) = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh(k(h+z))}{\cosh(kh)} \cdot \sin(kx \cos\mu + ky \sin\mu - \omega t)$$

3. Kinematic free surface boundary condition results in:
   Dispersion relation = relation between wave frequency and wave length

$$\omega^2 = kg \tanh(kh)$$

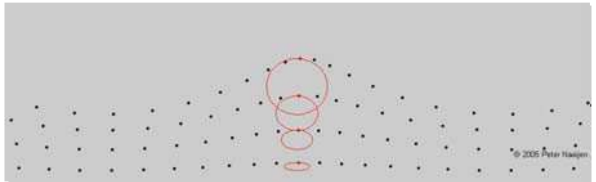
Water Particle Kinematics

trajectories of water particles in infinite water depth



Water Particle Kinematics

trajectories of water particles in finite water depth



Pressure

Pressure in the fluid can be found using Bernouilli equation for unsteady flow:

$$\frac{\partial \Phi}{\partial t} + \tfrac{1}{2}(u^2 + w^2) + \frac{p}{\rho} + gz = 0$$

$$p = -\rho \frac{\partial \Phi}{\partial t} - \tfrac{1}{2}\rho(u^2 + w^2) - \rho g z$$

1st order fluctuating pressure

2nd order (small quantity squared=small enough to neglect)

Hydrostatic pressure (Archimedes)

Potential Theory

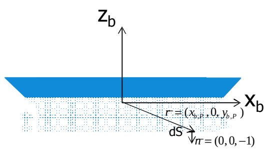
From all these velocity potentials we can derive:

- Pressure
- Forces and moments can be derived from pressures:

$$\bar{F} = -\iint_S (p \cdot \bar{n})\, dS$$

$$\bar{M} = -\iint_S p \cdot (\bar{r} \times \bar{n})\, dS$$

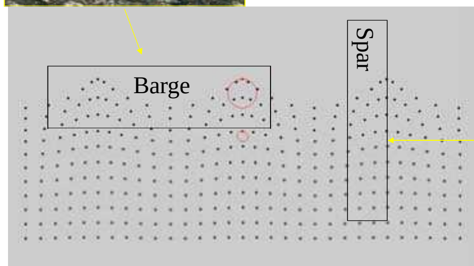
Verify these formulae (incl the signs!) yourself in order to understand them. Just check e.g. the force in heave direction (F_3) and the pitch moment (M_5) induced by a pressure on an infinite piece of hull surface dS at location $\bar{r}$:



Wave Force

Determination F_w

- Froude Krilov
- Diffraction



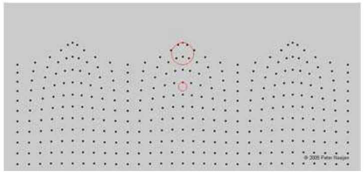
Flow superposition

$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w$

Considering a fixed structure (ignoring the motions) we will try to find a description of the disturbance of the flow by the presence of the structure in the form of a velocity potential. We will call this one the diffraction potential and added to the undisturbed wave potential (for which we have an analytical expression) it will describe the total flow due to the waves.

1. Flow due to Undisturbed wave

$$\Phi_0=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(\omega t-kx\cos\mu-ky\sin\mu)$$



2. Flow due to Diffraction

Has to be solved. What is boundary condition at body surface ?

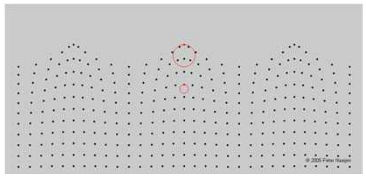
Φ_7

Exciting force due to waves

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w$$
$$=F_{FK}+F_D$$

1. Undisturbed wave force (Froude-Krilov)

$$\Phi_0=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(\omega t-kx\cos\mu-ky\sin\mu+\varepsilon)$$

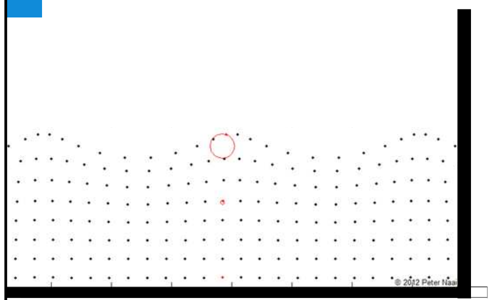


2. Diffraction force

Has to be solved. What is boundary condition at body surface ?

Φ_7

Wave Forces
Wave force acting on vertical wall



Force on the wall

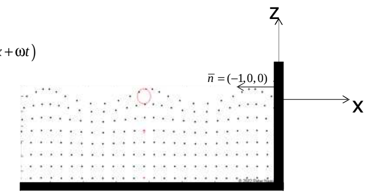
$$\bar{F}=-\int_{-\infty}^{0}p\cdot\bar{n}\,dz$$

$$\Phi_0=\frac{\zeta_a g}{\omega}e^{kz}\sin(kx-\omega t),\quad\Phi_7=-\frac{\zeta_a g}{\omega}e^{kz}\sin(kx+\omega t)$$

$$p=-\rho\frac{\partial\Phi}{\partial t}=-\rho\frac{\partial(\Phi_0+\Phi_7)}{\partial t}=$$

$$-\rho\frac{\partial\left(-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)\right)}{\partial t}=$$

$$2\rho\zeta_a g\cdot e^{kz}\cos(kx)\cos(\omega t)$$



$$\bar{n}=(-1,0,0)$$
$$x=0$$

$$F_x=\int_{-\infty}^{0}2\rho\zeta_a g\cdot e^{kz}\cos(\omega t)\,dz=\left[2\rho\frac{\zeta_a g}{k}\cdot e^{kz}\cos(\omega t)\right]_{-\infty}^{0}=$$

$$2\rho\frac{\zeta_a g}{k}\cdot\cos(\omega t)-0$$

Sources images

[1] Towage of SSDR Transocean Amirante, source: Transocean
[2] Tower Mooring, source: unknown
[3] Rogue waves, source: unknown
[4] Bluewater Rig No. 1, source: Friede & Goldman, LTD/GNU General Public License
[5] Source: unknown
[6] Rig Neptune, source: Seafarer Media
[7] Pieter Schelte vessel, source: Excalibur
[8] FPSO design basis, source: Statoil
[9] Floating wind turbines, source: Principle Power Inc.
[10] Ocean Thermal Energy Conversion (OTEC), source: Institute of Ocean Energy/Saga University
[11] ABB generator, source: ABB
[12] A Pelamis installed at the Agucadoura Wave Park off Portugal, source: S.Portland/Wikipedia
[13] Schematic of Curlew Field, United Kingdom, source: offshore-technology.com
[14] Ocean Quest Brave Sea, source: Zamakona Yards
[15] Medusa, A Floating SPAR Production Platform, source: Murphy USA