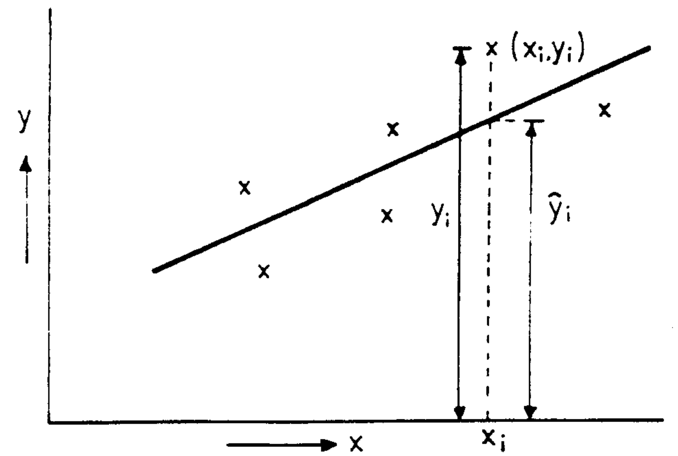
Hydrological Measurements

Wim Luxemburg

1. Linear regression, stage discharge relation and backwater

Linear regression

$$\hat{y} = ax + b$$



Least square method
(minimize sum of squared differences)

$$S = \sum_{i=1}^{n} \left\{ y_i - \hat{y}_i \right\}^2$$

$$a = \frac{\sum^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum^{n} (x_i - \bar{x})^2}$$

$$b = \bar{y} - a\bar{x}$$

or

$$a = \frac{s_{xy}}{s_x^{\ 2}}$$

s_{xy}

is the covariance from the data and

$s_x^{\ 2}$

the variance from the x-values.

Bivariate linear correlation coefficient

$$r = \frac{\sum^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum^{n}(x_i - \bar{x})^2}\sqrt{\sum^{n}(y_i - \bar{y})^2}}$$

$$r = \frac{s_{xy}}{s_x s_y}$$

$$-1 < r < 1$$

Regression for non linear relations

$$Q = m.H^{5/2}$$

$$Log(Q) = Log(m) + 5/2 * Log(H)$$

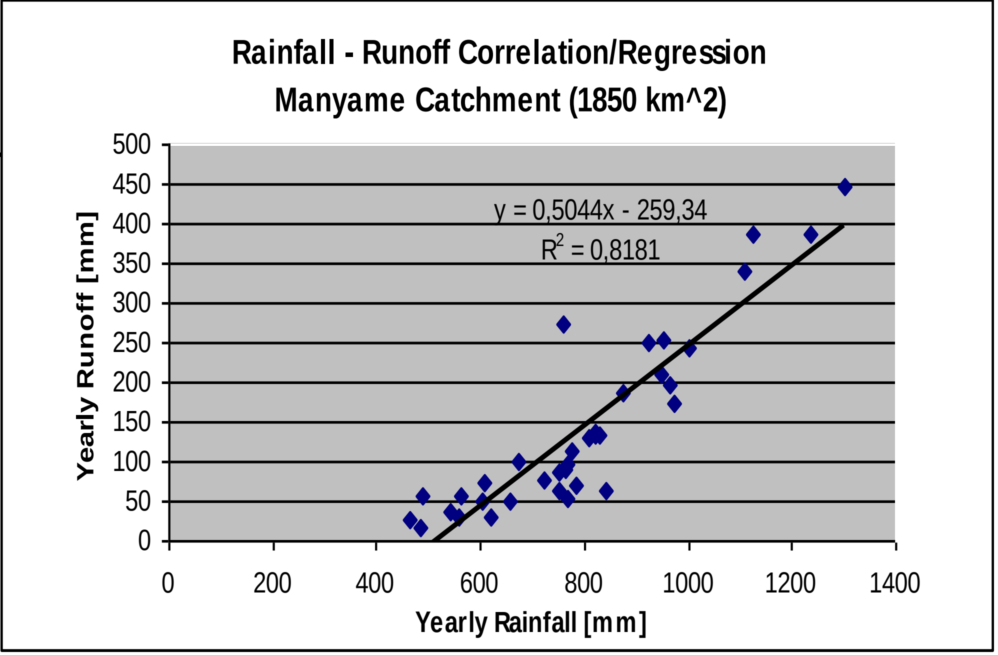
Example Linear Regression

Year	Annual Rainfall [mm] x	Annual Runoff [mm] y
1958	925	251,53
1959	760	273,35
1960	657	49,47
1961	876	188,12
1962	672	98,46
1963	1109	340,05
1964	542	36,35
1965	764	89,33
1966	841	64,11
1967	753	87,55
1968	486	17,01
1969	823	133,08
1970	753	64,70
1971	769	52,93
1972	965	195,41
1973	464	28,19
1974	1300	446,88
1975	1001	242,18

Year	Annual Rainfall [mm] x	Annual Runoff [mm] y
1976	777	111,96
1977	954	251,69
1978	1237	387,53
1979	609	74,80
1980	768	97,27
1981	1126	385,69
1982	722	76,70
1983	558	28,57
1984	618	30,41
1985	975	171,97
1986	949	209,89
1987	488	56,55
1988	809	128,49
1989	831	134,22
1990	823	135,03
1991	562	55,95
1992	603	49,91
1993	785	71,29

x_{avg}	795,9
y_{avg}	142,1
s_x	204,5
s_x^2	41813,6
s_y	114
s_{xy}	21090

a	0,504
b	-259,3
r	0,904
r^2	0,8181



Stage-discharge relation

in natural control sections

Ideally:

-) unique rating curve (Q-H)
-) Chezy applies

Chezy:

$$Q = C * B * (h - h_0)^{1.5} * i_b^{0.5}$$

$\Rightarrow$

Ratinge curve

$$Q = a * (h - h_0)^b$$

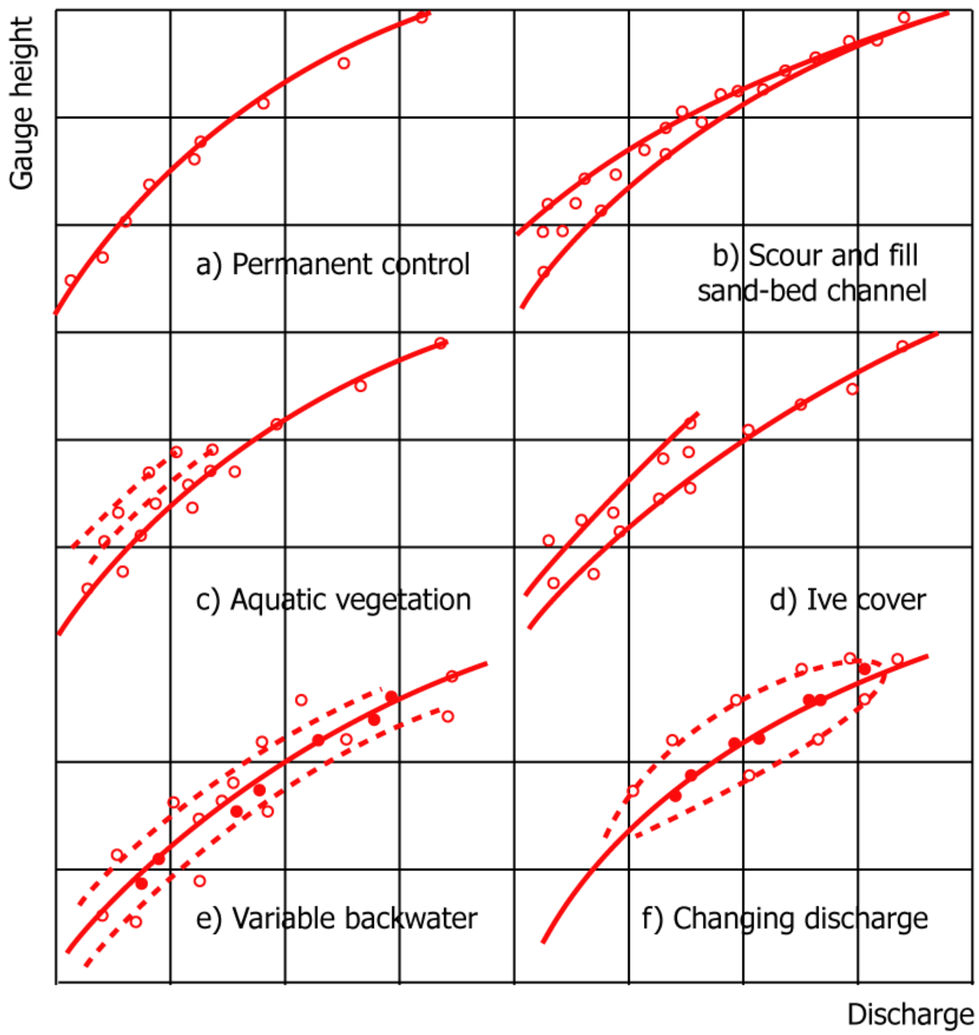
a,b are site specific

b relates to shape of the channel.

Stage-discharge relation

in natural control sections

examples non-unique relations



Composition of rating curve

e.g. linear regression

$$Q = a * (h - h_0)^b \quad \Rightarrow \quad Log(Q) = Log(a) + b * Log(h - h_0)$$

How to determine:

a
b
h_0

Composition of rating curve at Boane 1991/1992 (shifting control)

determine h_0

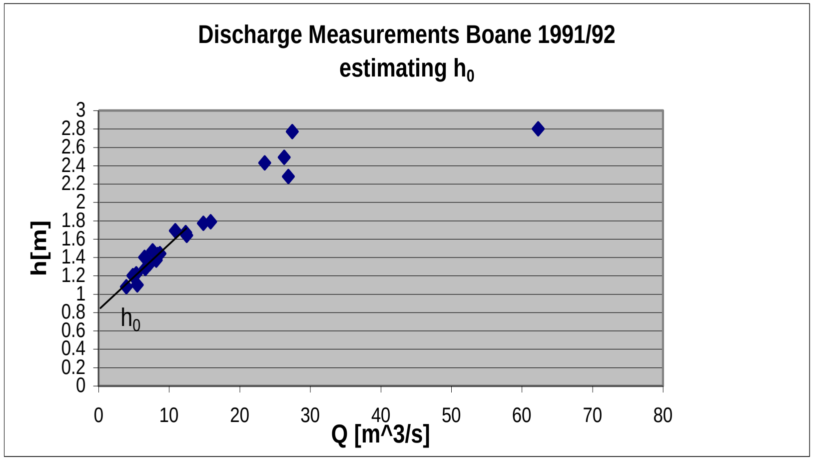


Figure 4.1

Discharge measurements 1991/92

H	Q	H	Q
1.47	7.652	1.67	12.338
4.67	88.339	1.37	8.175
3.35	35.599	1.77	14.857
2.77	27.45	1.79	15.879
2.43	23.536	1.4	6.539
2.28	26.916	1.28	6.617
3.16	37.279	1.33	7.231
2.49	26.302	1.22	5.344
1.69	10.865	1.2	4.853
1.64	12.505	1.08	3.931
1.44	8.7	1.1	5.479
1.43	8.383	2.8	62.297

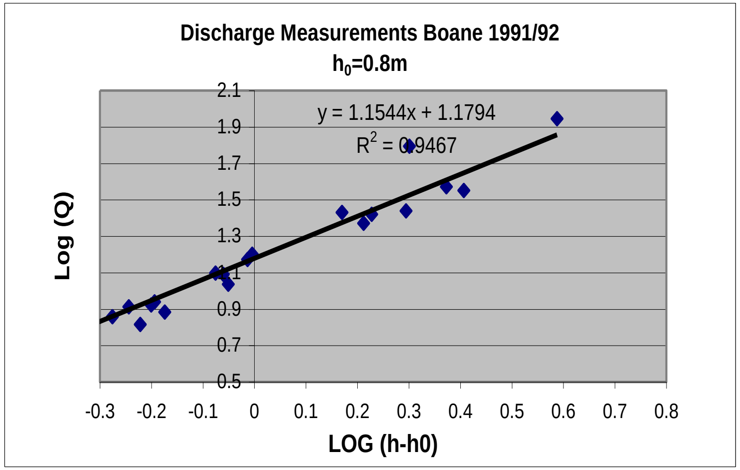


Figure 4.2

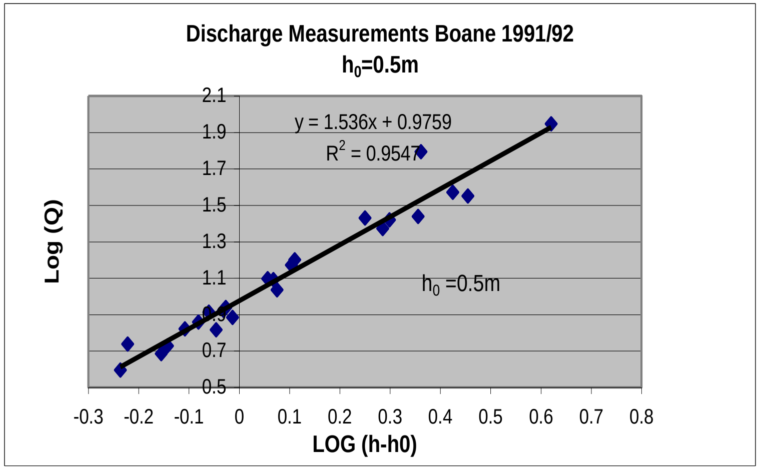
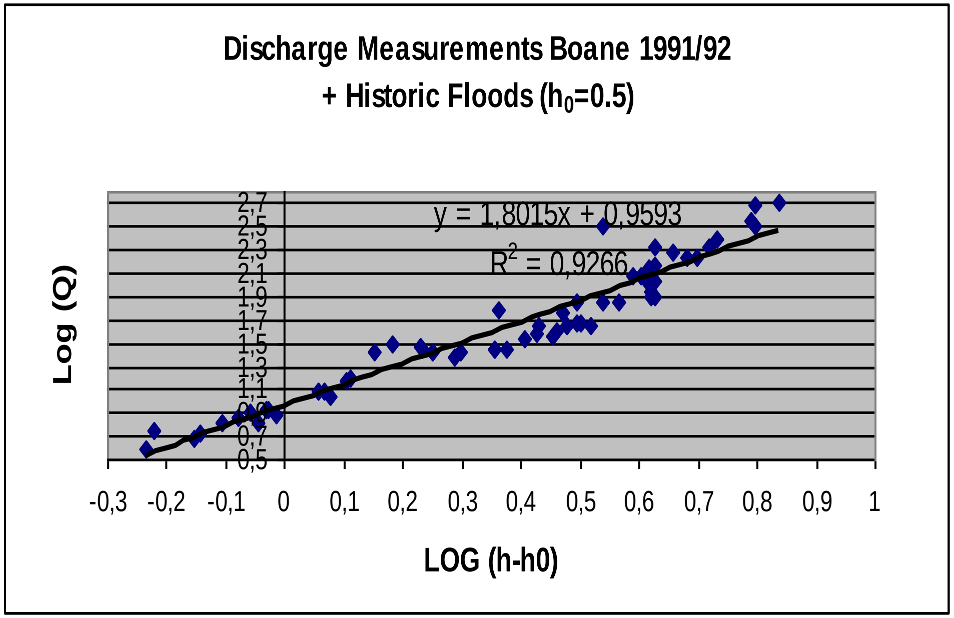


Figure 4.3

Composition of rating curve at Boane 1991/1992 (shifting control)

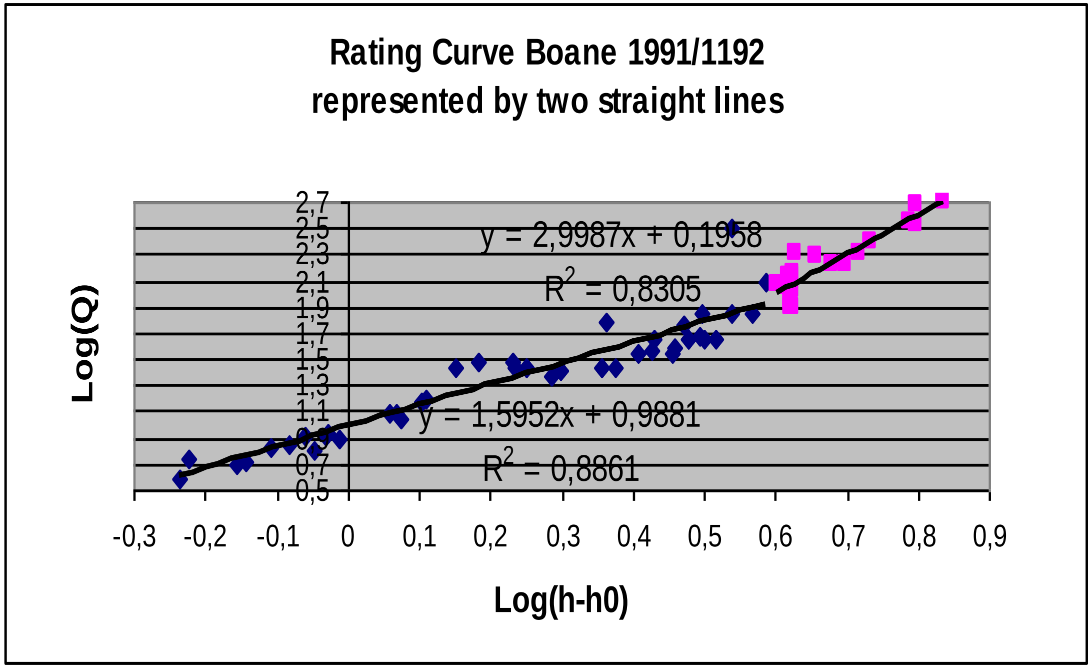
Include historic floods



Discharges Historic floods					
Date	H	Q	Date	H	Q
18-feb-75	7,38	513,44	26-feb-75	4,18	71,934
11-feb-77	6,77	485,986	10-feb-55	3,63	70,95
22-mrt-72	6,66	355,859	9-feb-55	3,96	69,743
22-dec-73	6,76	328,384	25-feb-55	3,45	57,52
13-feb-85	3,95	316	16-mrt-78	3,61	47,497
1-feb-74	5,9	244,743	28-nov-69	3,67	46,111
6-dec-89	4,74	208	12-dec-69	3,5	45,574
21-feb-67	5,72	207,928	12-jan-78	3,5	45,218
10-jan-66	5,05	189,027	26-feb-55	3,18	44,656
28-feb-67	5,48	169,81	14-jan-72	3,79	44,637
6-feb-55	5,28	168,15	3-jan-74	3,38	39,462
4-feb-55	4,73	146,396	21-feb-55	3,05	34,938
15-feb-55	4,7	137,749	12-jan-55	2,02	30,728
28-dec-73	4,65	137,482	5-feb-58	2,2	30,022
12-feb-55	4,37	121,037	15-apr-55	2,21	27,756
8-feb-55	4,5	118,335	22-nov-76	2,88	27,488
18-dec-80	4,61	109	17-mrt-81	2,77	27,45
3-mrt-75	4,72	105,974	27-mrt-81	2,28	26,916
10-feb-81	4,67	88,339	8-jan-59	1,92	26,797
18-feb-67	4,67	79,589	3-apr-81	2,49	26,302
21-okt-69	4,73	77,608			

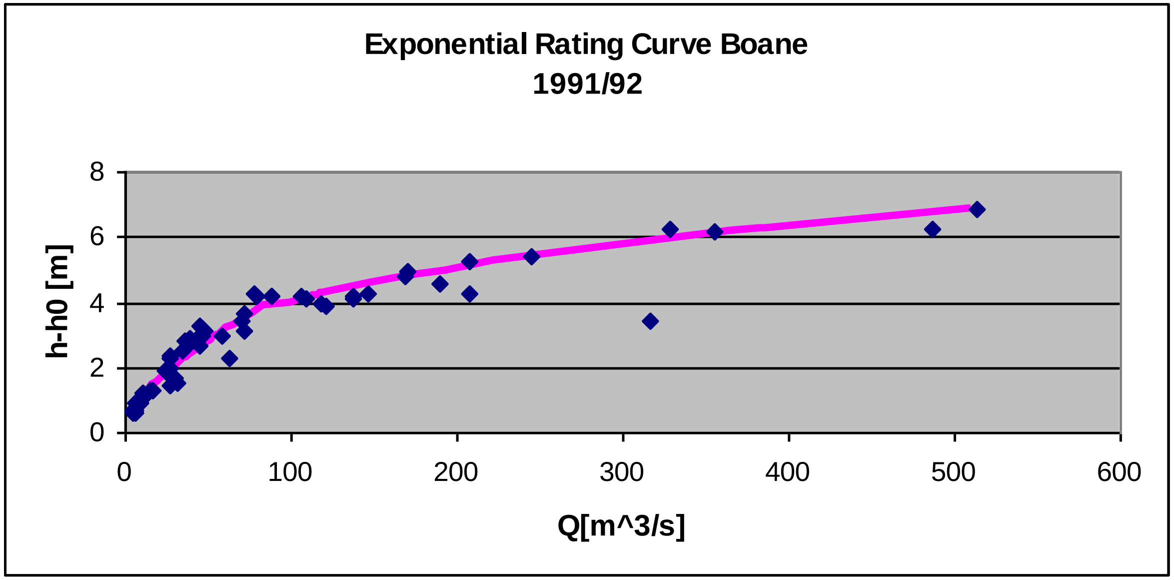
Composition of rating curve at Boane 1991/1992 (shifting control)

Correct for effect of floodplains



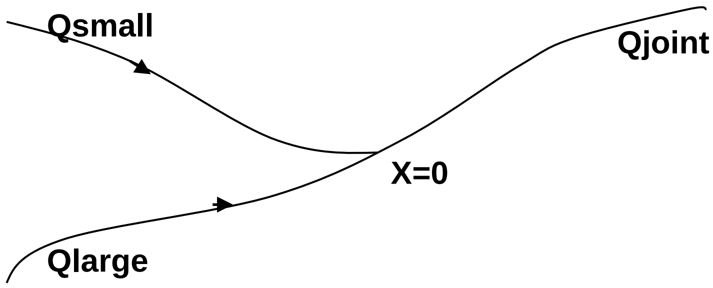
Composition of rating curve at Boane 1991/1992 (shifting control)

	0.5m<h<4.5m	h>4.5m
Log(a)	0.9881	0.1958
A	9.7297	1.5696
B	1.5952	2.9987
H_0	0.5	0.5



Rating curve and non uniform flow

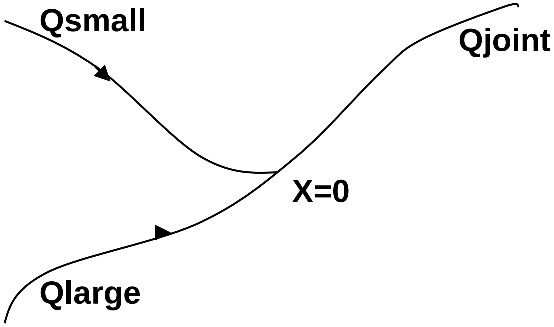
Variable backwater $\Rightarrow$ non-unique relation Q-h



Backwater from tributaries with variable discharge

Rating curve and non uniform flow

Backwater from tributary with variable discharge



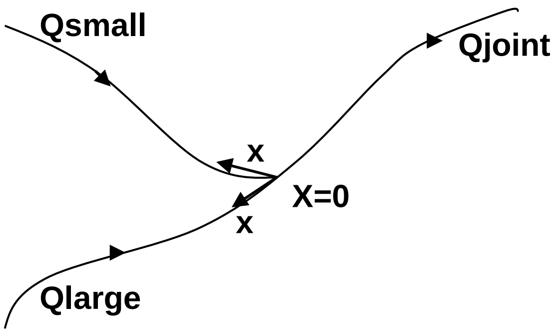
General equation for a backwater curve:

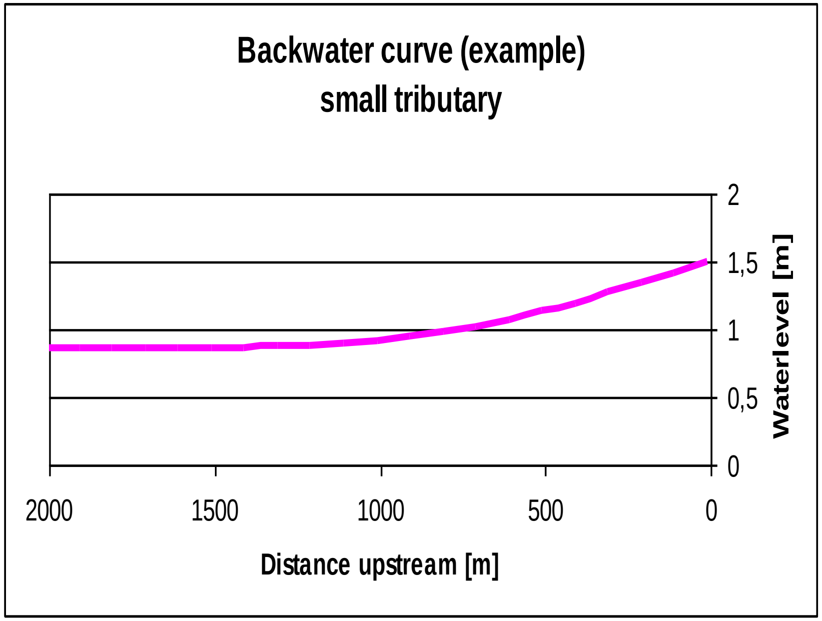
$$\frac{\partial h}{\partial x} = i_b \, \frac{h^3 - h_e^{\,3}}{h^3 - h_c^{\,3}}$$

with: $h_e = \left(\dfrac{Q\,|\,Q\,|}{C^2 B^2 i_b} \right)^{1/3}$ and $h_c = \left(\dfrac{Q^2}{g B_s^{\,2}} \right)^{1/3}$

Rating curve and non uniform flow

Backwater caused by tributary





		SMALL TRIBUTARY	LARGE TRIBUTARY	JOINT RIVER
Discharge	Q	2	33	35
Width	B	4	30	30
Chezy	C	20	20	20
Bottom Slope	ib	0,001	0,001	0,001
Critical depth	hc	0,29	0,50	0,52
Equilibrium depth	he	0,86	1,45	1,50