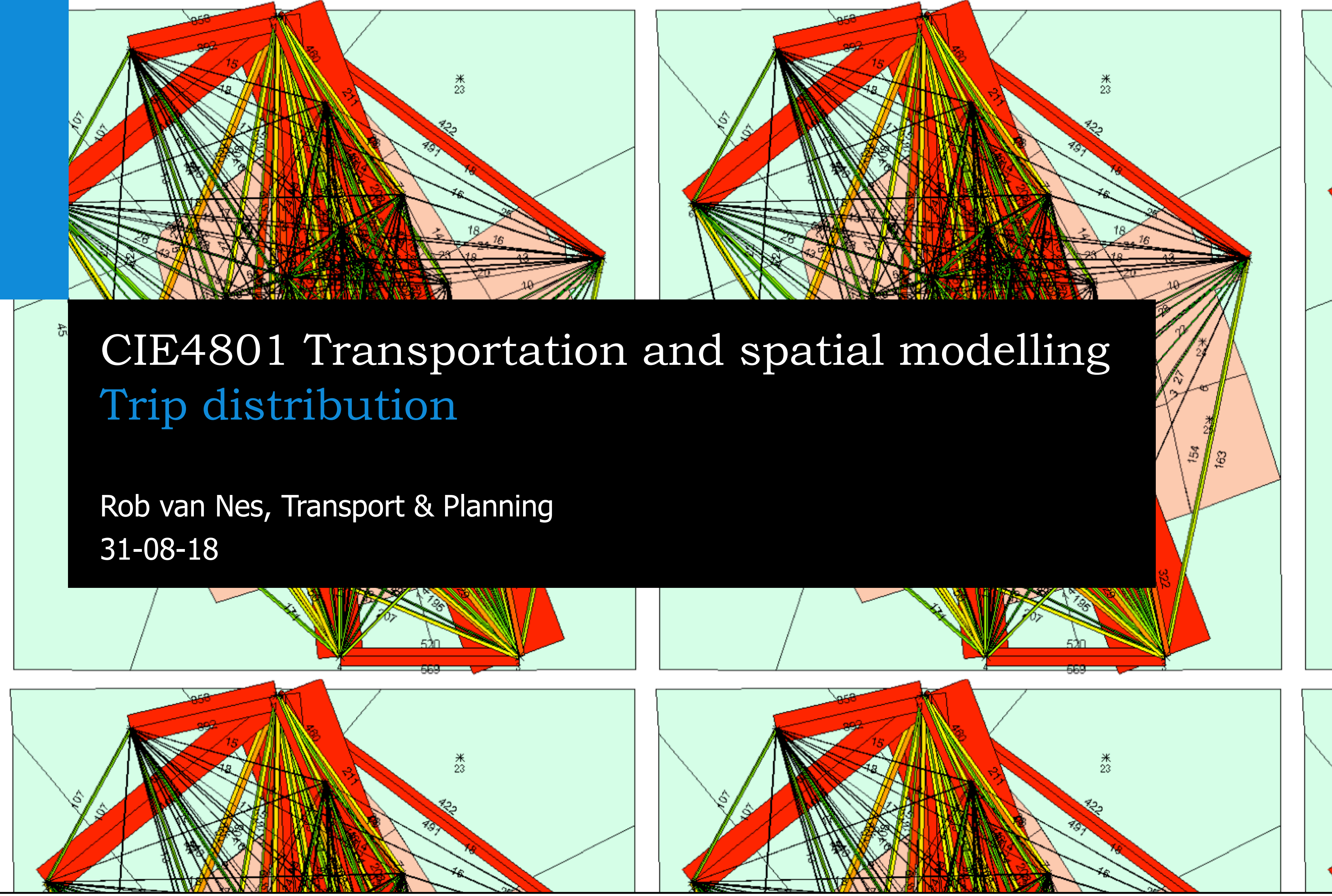

CIE4801 Transportation and spatial modelling
Trip distribution

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31-08-18

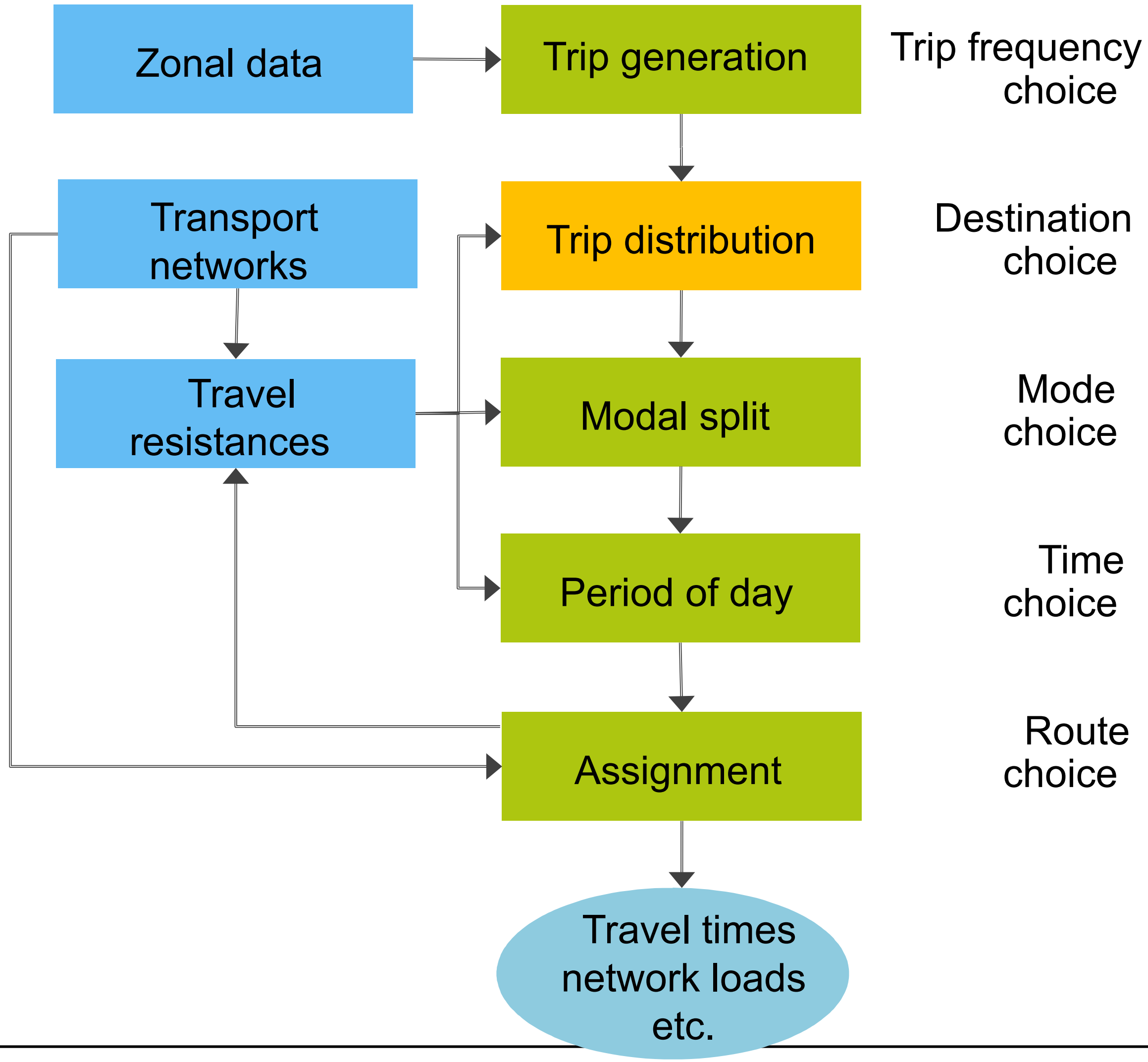
Content

- Question on choice modelling
- Modelling component 2: Trip distribution
  - Your comments/questions on Chapter 5
  - Additional topics
    - Choice models
    - Poisson model
  - Practical issues

2.

Trip distribution

Introduction to trip distribution



Topics to study sections 5.1-5.6

- What does this modelling component do? What's its output and what's its input? How does it fit in the framework?
- Do you understand the definitions?
  - OD-matrix, production, attraction, generalised costs, deterrence function
- Do you understand the modelling methods?
  - Growth factor: singly and doubly constrained
  - Gravity model
    - Logic for "borrowing" from Newton or Entropy maximisation?
- Do you understand the iterative algorithm?
- Do you understand the calibration of the model?
  - Deterrence function (Hymann's method)
- Do you understand tri-proportional fitting ("bins")?
- Are these models appropriate?

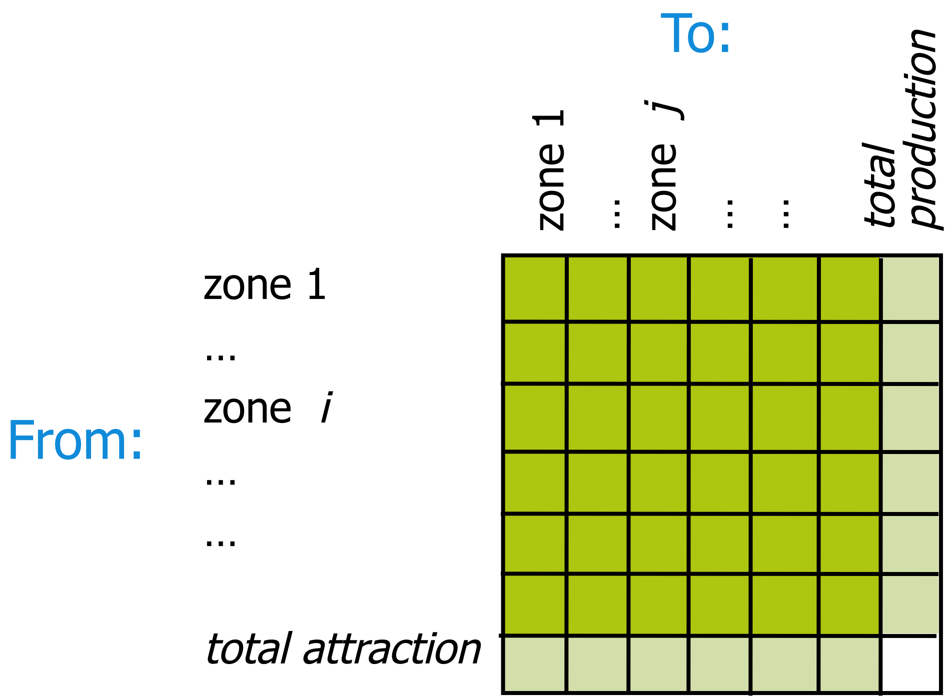
2.1

Trip distribution: definitions

Introduction to trip distribution

<u>Given:</u> Productions and attractions for each zone
(i.e. departures and arrivals)

<u>Determine:</u> The number of trips from each zone to all other zones
(i.e. fill in the OD-matrix)



Two key principles in trip distribution

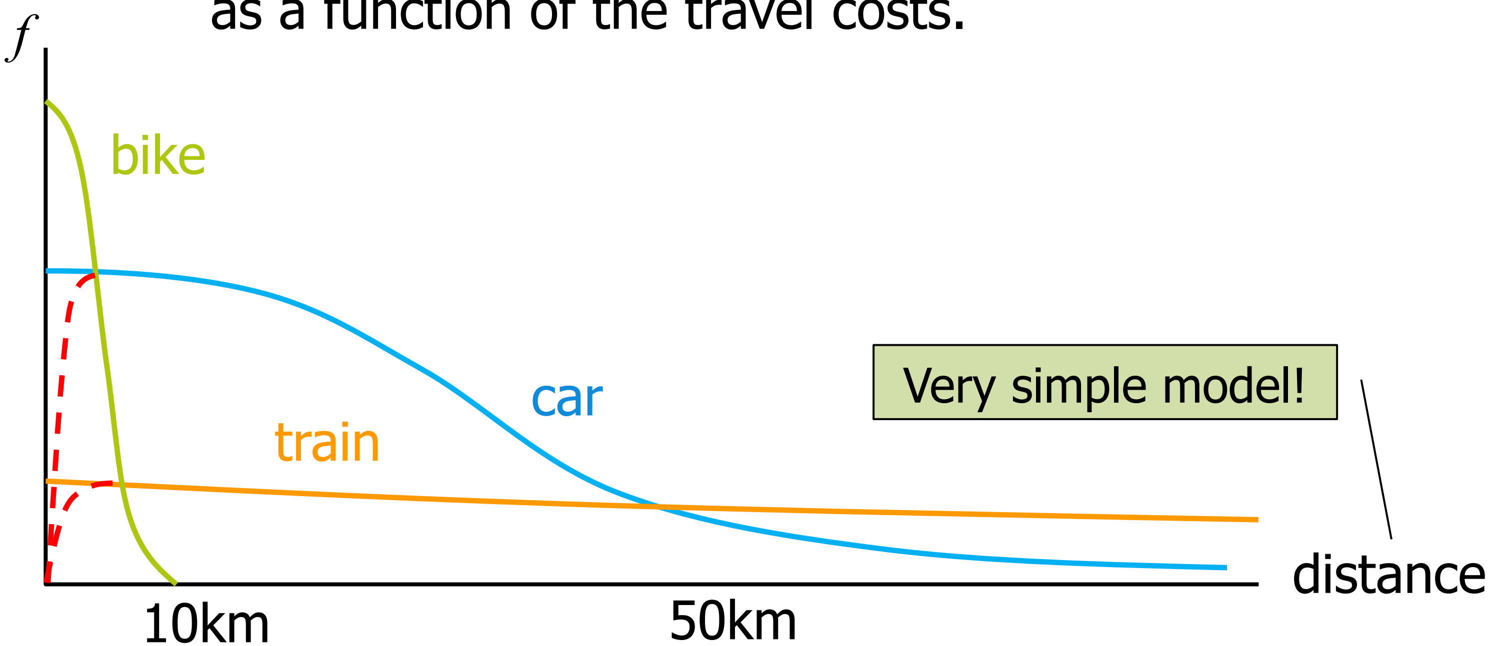
- Big produces/attracts more
- Nearby attracts more
- These principles remain, independent of the analogy or framework chosen

Far away is less: Distribution or deterrence functions

$$\underbrace{F_{ij} = f(c_{ij})}_{}$$

$c_{ij} =$ travel costs from zone *i* to zone *j*

distribution function
describes the relative **willingness** to make a trip as a function of the travel costs.



Requirements for distribution functions

- Decreasing with travel costs
- Integral should be finite
- Fraction $\dfrac{F\left(a \cdot c_{ij}\right)}{F\left(c_{ij}\right)}$ depends on value of c_{ij}
- Fixed changes should have a diminishing relative impact:

$$\frac{F\left(c_{ij} + \Delta c\right)}{F\left(c_{ij}\right)} > \frac{F\left(c_{ij} + A + \Delta c\right)}{F\left(c_{ij} + A\right)}$$

Distribution functions

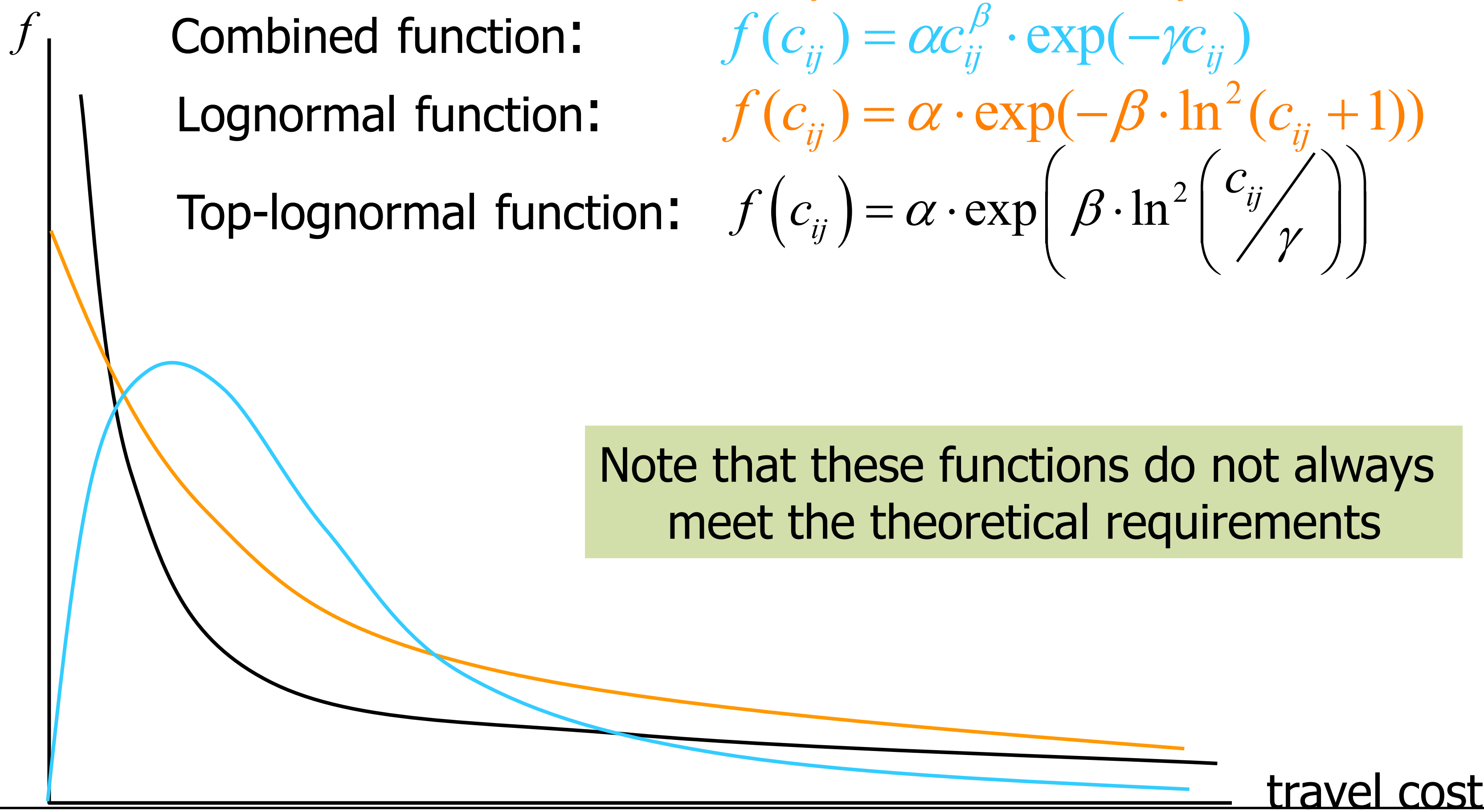
Power function: $f(c_{ij}) = \alpha \cdot c_{ij}^{-\beta}$

Exponential function: $f(c_{ij}) = \alpha \cdot \exp(-\beta c_{ij})$

Combined function: $f(c_{ij}) = \alpha c_{ij}^{\beta} \cdot \exp(-\gamma c_{ij})$

Lognormal function: $f(c_{ij}) = \alpha \cdot \exp(-\beta \cdot \ln^2(c_{ij} + 1))$

Top-lognormal function: $f\left(c_{ij}\right) = \alpha \cdot \exp\left(\beta \cdot \ln^2\left({c_{ij}}/{\gamma} \right)\right)$

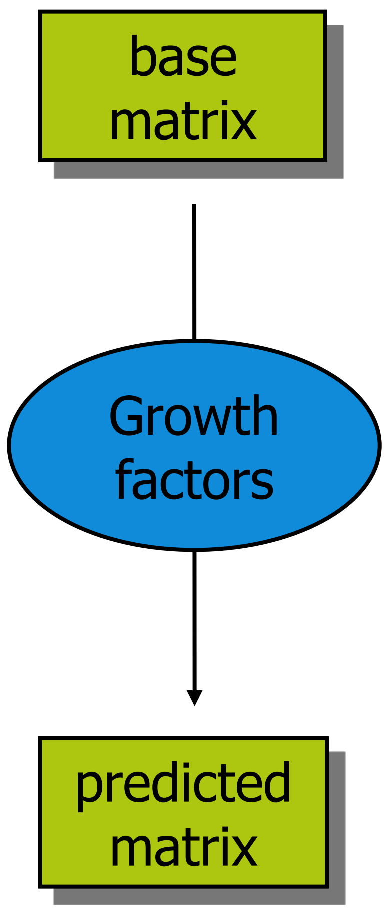


Note that these functions do not always meet the theoretical requirements

2.2.1

Trip distribution models: Method 1: Growth models

Growth factor models



- trip observations
- outcomes of (older) model
- calibrated using traffic counts

- outcomes base year and future trip generation model
- outcomes base year and future gravity model

Growth factor models

Advantages

- Network specific peculiarities can be captured by observations
- A base matrix is more understandable and verifiable than a model

Disadvantages

- New residential zones are difficult to capture
- Historical patterns may change over time

Growth factor models

$T_{ij}^0 \longrightarrow g \longrightarrow T_{ij} = gT_{ij}^0$

base matrix cell | growth factor | predicted matrix cell

$T_{ij} = gT_{ij}^0$ Network independent, general factor

$T_{ij} = g_i T_{ij}^0$ or $T_{ij} = g_j T_{ij}^0$ Network independent, origin or destination specific factor

$T_{ij} = g_i g_j T_{ij}^0$ Network independent, factors for origins and destinations

$T_{ij} = g_{ij} T_{ij}^0$ Network dependent, OD-specific factors

Common application

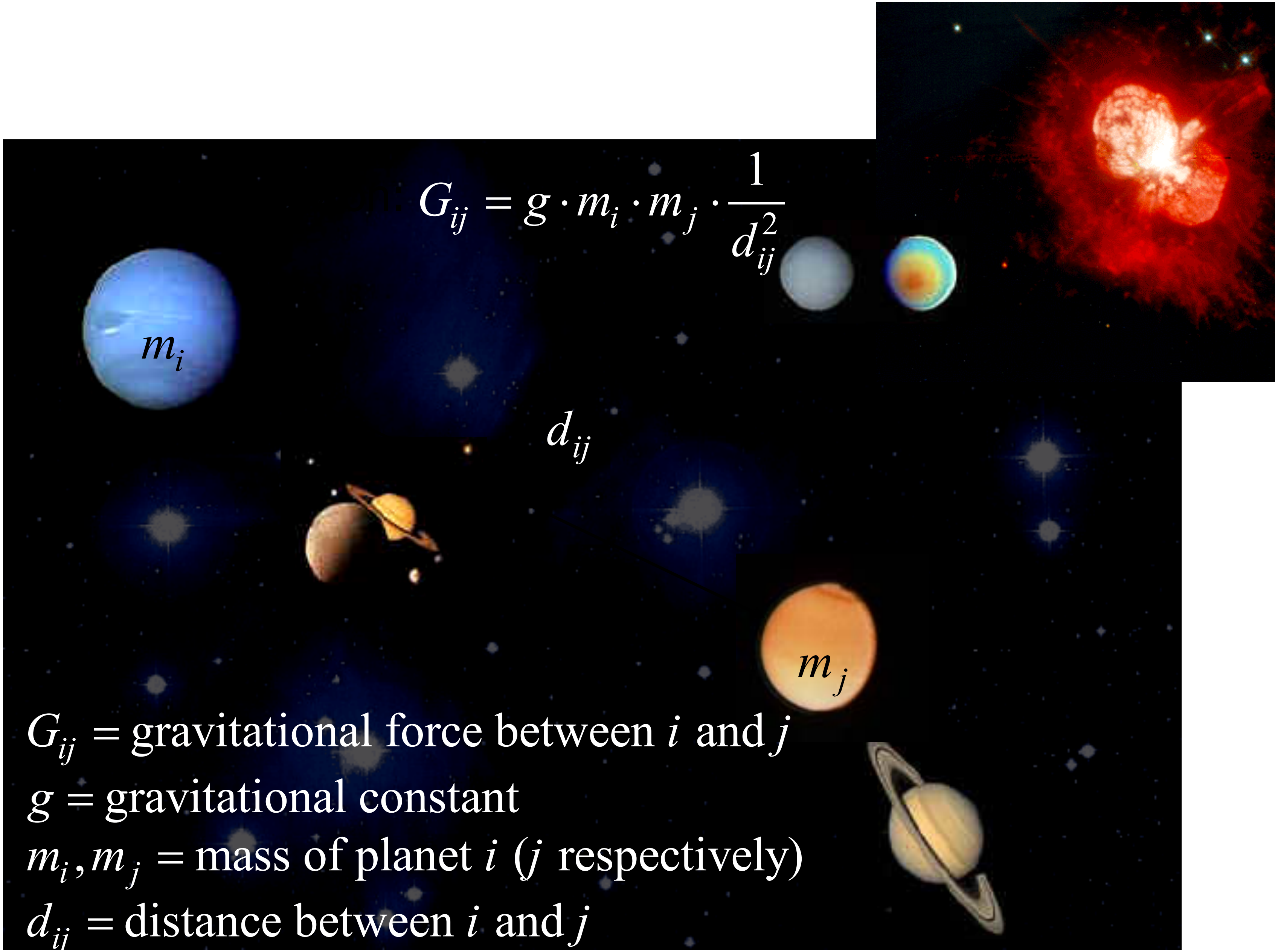
- Given expected spatial development (short term)
  - Future production (departures)
  - Future attraction (arrivals)

- Fill in new areas by copying columns and rows of nearby (and look alike!) zones from the base year matrix
  - Pay attention to interaction of original and copy!

- Scale this adapted base year matrix to the future production and attraction using appropriate factors a_i and b_j
  (see also slides 2.2.2, replace F_{ij} by T_{ij} (i.e prior OD-matrix), and example on slides 2.3)

2.2.2

Trip distribution models
Method 2: Gravity model



$$G_{ij} = g \cdot m_i \cdot m_j \cdot \frac{1}{d_{ij}^2}$$

G_{ij} = gravitational force between i and j

g = gravitational constant

m_i, m_j = mass of planet i (j respectively)

d_{ij} = distance between i and j

The gravity model

Assumptions:

Number of trips between an origin and a destination zone is proportional to:

- a production ability factor for the origin zone
- an attraction ability factor for the destination zone
- a factor depending on the travel costs between the zones

Mathematical formulation:

$$T_{ij} = \rho Q_i X_j F_{ij}$$

T_{ij} = # trips from zone *i* to zone *j*

ρ = measure of average trip intensity

Q_i = production potential of zone *i*

X_j = attraction potential of zone *j*

F_{ij} = willingness to travel from *i to j*

Possible interpretations of Q_i and X_j: populations, production & attraction, …

Singly constrained model

Basic gravity model:

$$T_{ij} = \rho Q_i X_j F_{ij}$$

T_{ij} = # trips from zone *i* to zone *j*

ρ = measure of average trip intensity

Q_i = production potential of zone j

X_j = attraction potential of zone *i*

F_{ij} = willingness to travel from *i to j*

If the trip productions P_i are known: $\sum_j T_{ij} = P_i$

If the trip attractions A_j are known: $\sum_i T_{ij} = A_j$

Singly constrained: origin

$$\begin{cases} T_{ij} = \rho Q_i X_j F_{ij} \\ \sum_j T_{ij} = P_i \end{cases}$$

$$\sum_j T_{ij} = \sum_j \left(\rho Q_i X_j F_{ij}\right) = \rho Q_i \sum_j \left(X_j F_{ij}\right) = P_i$$

$$\Rightarrow \qquad Q_i = \frac{P_i}{\rho \sum_j X_j F_{ij}}$$

$$\Rightarrow \qquad T_{ij} = \rho \frac{P_i}{\rho \sum_j X_j F_{ij}} X_j F_{ij} = a_i P_i X_j F_{ij}$$

(a_i = balancing factor)

Singly constrained origin based model: $T_{ij} = a_i P_i X_j F_{ij}$

Singly constrained: destination

$$\begin{cases} T_{ij} = \rho Q_i X_j F_{ij} \\ \sum_i T_{ij} = A_j \end{cases}$$

$$\sum_i T_{ij} = \sum_i \left(\rho Q_i X_j F_{ij}\right) = \rho X_j \sum_i \left(Q_i F_{ij}\right) = A_j$$

$$\Rightarrow \qquad X_j = \frac{A_j}{\rho \sum_i Q_i F_{ij}}$$

$$\Rightarrow \qquad T_{ij} = \rho Q_i \frac{A_j}{\rho \sum_i Q_i F_{ij}} F_{ij} = b_j Q_i A_j F_{ij}$$

(b_j = balancing factor)

Singly constrained destination based model: $T_{ij} = b_j Q_i A_j F_{ij}$

Doubly constrained model

Basic gravity model:

$$T_{ij} = \rho Q_i X_j F_{ij}$$

T_{ij} = # trips from zone *i* to zone *j*

ρ = measure of average trip intensity

Q_i = production potential of zone j

X_j = attraction potential of zone *i*

F_{ij} = willingness to travel from *i to j*

Trip productions P_i and trip attractions A_j are known:

$$\sum_j T_{ij} = P_i \quad \text{and} \quad \sum_i T_{ij} = A_j$$

Doubly constrained model

$$\begin{cases} T_{ij} = \rho Q_i X_j F_{ij} \\ \sum_i T_{ij} = A_j \\ \sum_j T_{ij} = P_i \end{cases}$$

$$\sum_j T_{ij} = \sum_j \left(\rho Q_i X_j F_{ij} \right) = \rho Q_i \sum_j \left(X_j F_{ij} \right) = P_i$$

$$\sum_i T_{ij} = \sum_i \left(\rho Q_i X_j F_{ij} \right) = \rho X_j \sum_i \left(Q_i F_{ij} \right) = A_j$$

$$\Rightarrow \qquad Q_i = \frac{P_i}{\rho \sum_j \left(X_j F_{ij} \right)} \qquad \text{and} \qquad X_j = \frac{A_j}{\rho \sum_i \left(Q_i F_{ij} \right)}$$

Doubly constrained model

$$\Rightarrow \quad T_{ij} = \rho \frac{P_i}{\rho \sum_j X_j F_{ij}} \cdot \frac{A_j}{\rho \sum_i Q_i F_{ij}} F_{ij} = a_i b_j P_i A_j F_{ij}$$

a_i = balancing factor
b_j = balancing factor

Doubly constrained model: $T_{ij} = a_i b_j P_i A_j F_{ij}$

Note that X_j is a function of Q_i and vice versa
Solving this model thus requires an iterative approach

2.2.3

Trip distribution models
Method 3: Entropy maximisation

Maximising entropy given constraints

Analogue to the thermodynamic concept of entropy as maximum disorder, the entropy- maximizing procedure seeks the most likely configuration of elements within a constrained situation.

The objective can be formulated as:

$$Max\ w\left(T_{ij}\right) = \frac{T!}{\prod_{ij} T_{ij}!}$$

$$\sum_j T_{ij} = P_i$$

$$\sum_i T_{ij} = A_j$$

$$\sum_i \sum_j T_{ij} \cdot c_{ij} = C$$

Replace by logarithm

$$Max\left(\ln\left(w\left(T_{ij}\right)\right)\right) = Max\left(\ln\left(T!\right) - \sum_{ij} \ln\left(T_{ij}!\right)\right)$$

Illustration entropy principle

- How many ways can you distribute 4 people?

	H	T
C		

- Let's assume we use a coin to decide where a person will go to: thus flip a coin 4 times

- In total there are 16 sequences leading to 5 options:
  - 4H,0T (1) 3H,1T (4), **2H,2T (6)**, 1H,3T (4), 0H,4T (1)

- Weight of each option is determined by $\frac{T!}{\prod_{ij} T_{ij}!}$

Derivation (1/2)

Lagrangian of maximisation objective:

$$\ln(T!) - \sum_{ij} \ln(T_{ij}!) + \sum_i \lambda_i \cdot \left(P_i - \sum_j T_{ij} \right) + \sum_j \lambda_j \cdot \left(A_j - \sum_i T_{ij} \right) + \beta \cdot \left(C - \sum_i \sum_j T_{ij} \cdot c_{ij} \right)$$

Use as approximation

$$\ln(N!) \approx N \cdot \ln(N) - N \Rightarrow \frac{\partial \ln(N!)}{\partial N} \approx \ln(N)$$

Set derivatives equal to zero and solve the equation

$$\frac{\partial L}{\partial T_{ij}} = -\ln(T_{ij}) - \lambda_i - \lambda_j - \beta \cdot c_{ij} = 0$$

$$\Rightarrow T_{ij} = e^{-\lambda_i - \lambda_j - \beta \cdot c_{ij}} = e^{-\lambda_i} \cdot e^{-\lambda_j} \cdot e^{-\beta \cdot c_{ij}}$$

Note that the other derivatives lead to the orginal constraints

Derivation (2/2)

Substitute result in constraints:

$$\sum_j T_{ij} = \sum_j e^{-\lambda_i - \lambda_j - \beta \cdot c_{ij}} = e^{-\lambda_i} \sum_j e^{-\lambda_j - \beta \cdot c_{ij}} = P_i$$

$$\Rightarrow e^{-\lambda_i} = \frac{1}{\sum_j e^{-\lambda_j - \beta \cdot c_{ij}}} \cdot P_i = a_i \cdot P_i$$

Similar for destinations, and substitute in T_{ij}

$$T_{ij} = e^{-\lambda_i} \cdot e^{-\lambda_j} \cdot e^{-\beta \cdot c_{ij}} = a_i \cdot P_i \cdot b_j \cdot A_j \cdot e^{-\beta \cdot c_{ij}}$$

Which is equivalent to the doubly constrained model
Thus different analogies lead to similar formulation

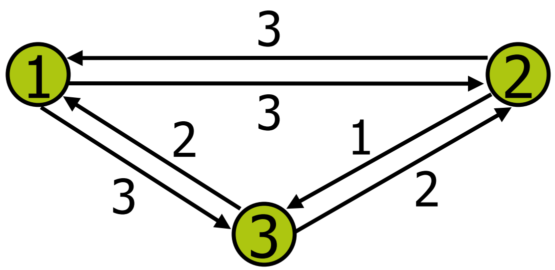
2.3

Matrix balancing algorithm

Key algorithm

- Distributing departures or arrivals over rows or columns based on:
  - Older OD-matrix
  - Willingness to travel
  - Choice probabilities
- Two modelling methods
  - Singly constrained: simply distributing departures over destinations or arrivals over origins
  - Doubly constrained: iteratively distributing departures and arrivals
  - Triply constrained: iteratively distributing departures, arrivals and e.g. distance classes

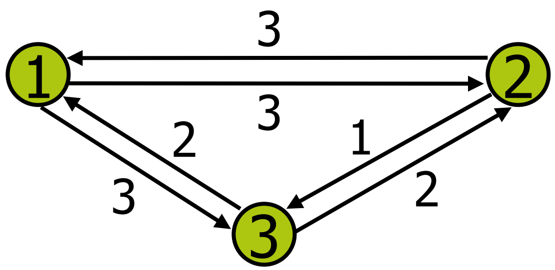
Example doubly constrained model



Trip balancing

				P_i
				100
				200
				250
A_j	220	165	165	

Example doubly constrained model

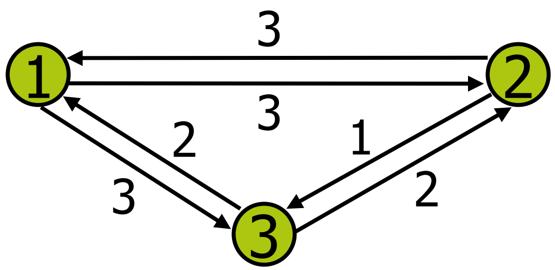


Trip balancing

Travel costs c_{ij}

				P_i
	1	3	3	100
	3	1	1	200
	2	2	1	250
A_j	220	165	165	

Example doubly constrained model



				P_i
	3.0	1.1	1.1	100
	1.1	3.0	3.0	200
	1.8	1.8	3.0	250
A_j	220	165	165	

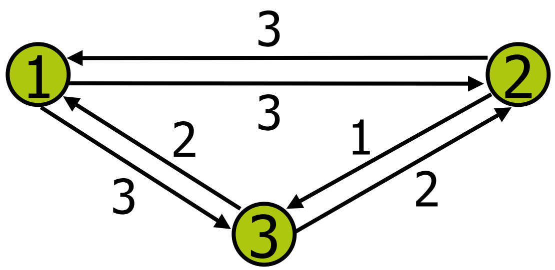
Trip balancing

Travel costs c_{ij}

Willingness F_{ij}

$$F_{ij} = f(c_{ij})$$
$$= 5 \cdot \exp(-0.5 \cdot c_{ij})$$

Example doubly constrained model



				P_i
$\times\frac{100}{5.2}$	3.0	1.1	1.1	100
$\times\frac{200}{7.1}$	1.1	3.0	3.0	200
$\times\frac{250}{6.6}$	1.8	1.8	3.0	250
A_j	220	165	165	

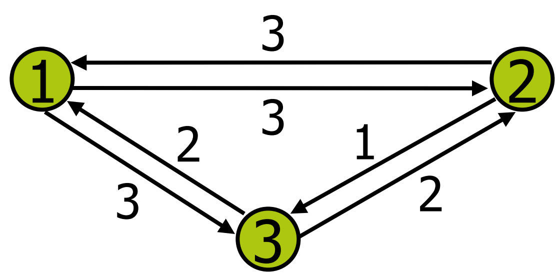
Trip balancing

Travel costs c_{ij}

Willingness F_{ij}

Balancing factors a_i, b_j

Example doubly constrained model



				P_i
	57.6	21.2	21.2	100
	31.0	84.5	84.5	200
	68.2	68.2	113.6	250
A_j	220	165	165	
	$\times\dfrac{220}{156.8}$	$\times\dfrac{165}{173.9}$	$\times\dfrac{165}{219.3}$	

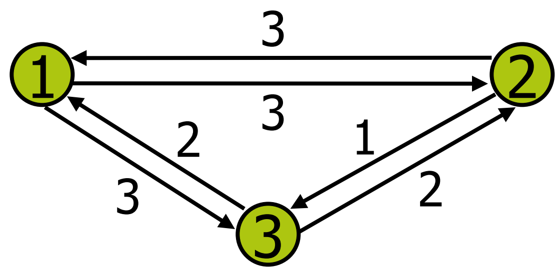
Trip balancing

Travel costs c_{ij}

Willingness F_{ij}

Balancing factors a_i, b_j

Trip distribution using the gravity model



				P_i
	80.8	20.1	15.9	100
	43.5	80.2	63.6	200
	95.7	64.7	85.5	250
A_j	220	165	165	

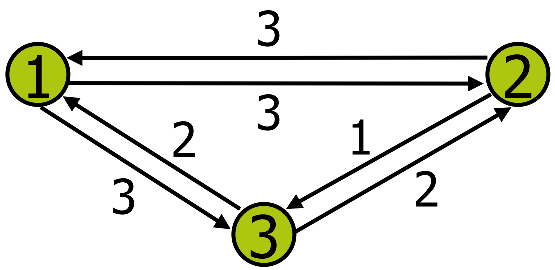
Trip balancing

Travel costs c_{ij}

Willingness F_{ij}

Balancing factors a_i, b_j

Example doubly constrained model



				P_i
	70.5	16.5	13.0	100
	48.7	84.3	67.0	200
	100.8	64.2	85.0	250
A_j	220	165	165	

Trip balancing

Travel costs c_{ij}

Willingness F_{ij}

Balancing factors a_i, b_j

Repeat until there are no changes: =>OD matrix

2.4.1

Calibration of deterrence functions
Hyman's method

Hyman's method

No OD-information available

Given:

- Observed trip production
- Observed trip attraction
- Cost matrix
- Observed mean trip length (MTL)

Assumption: $F(c_{ij}) = \exp(-\alpha c_{ij}), \quad \alpha$ is unknown

OD matrix can then be determined by:
using the doubly constrained gravity model
while updating α to match the MTL.

Example Hyman's method

Given:

$$c_{ij} = \begin{pmatrix} 3 & 11 & 18 & 22 \\ 12 & 3 & 13 & 19 \\ 15 & 13 & 5 & 7 \\ 24 & 18 & 8 & 5 \end{pmatrix} \quad P_i = \begin{pmatrix} 400 \\ 460 \\ 400 \\ 702 \end{pmatrix}$$

$$A_j = \begin{bmatrix} 260 & 400 & 500 & 802 \end{bmatrix}$$

Observed: MTL = 10

$F(c_{ij}) = \exp(-\alpha c_{ij}), \quad \alpha$ is unknown

Compute the OD-matrix and distribution function that fits the MTL.

Example Hyman's method: iteration 1

				P_i
156	99	68	77	400
58	204	103	95	460
26	45	138	191	400
20	52	191	439	702
A_j: 260	400	500	802	

$$c_{ij} = \begin{pmatrix} 3 & 11 & 18 & 22 \\ 12 & 3 & 13 & 19 \\ 15 & 13 & 5 & 7 \\ 24 & 18 & 8 & 5 \end{pmatrix}$$

- First step: Choose $\alpha = 1/\mathrm{MTL} = 0.1$
- Compute trip distribution using a gravity model
- Compute modelled MTL

$$\mathrm{MTL}_1 = \frac{156 \times 3 + 99 \times 11 + 68 \times 18 + 77 \times 22 + 58 \times 12 + \ldots}{156 + 99 + 68 + 77 + 58 + \ldots} = 8.7$$

Example Hyman's method: iterations and result

- Set next α
- Compute OD-matrix
- Set next α , etc.
- After a number of iterations: $\alpha = 0.0586$

$$n = 1 : \alpha_2 = \frac{MTL_1}{MTL} \alpha_1,$$

$$n \geq 1 : \alpha_{n+1} = \frac{(MTL - MTL_{n-1})\alpha_n - (MTL - MTL_n)\alpha_{n-1}}{MTL_n - MTL_{n-1}}$$

					P_i
	112	98	81	109	400
	66	156	109	129	460
	39	60	120	181	400
	43	86	190	383	702
A_j	260	400	500	802	

$$c_{ij} = \begin{pmatrix} 3 & 11 & 18 & 22 \\ 12 & 3 & 13 & 19 \\ 15 & 13 & 5 & 7 \\ 24 & 18 & 8 & 5 \end{pmatrix}$$

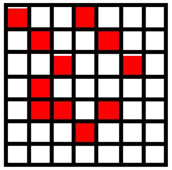
$$\text{MTL} = \frac{112 \times 3 + 98 \times 11 + 81 \times 18 + 109 \times 22 + 66 \times 12 + \ldots}{112 + 98 + 81 + 109 + 66 + \ldots} = 10.0$$

2.4.2

Calibration of deterrence functions Poisson model or Tri-proportional fitting

Poisson model (Tri-proportional problem)

- Observed OD-matrix from a survey
  - Not necessarily complete
  - Usually at an aggregate level (e.g. municipality)



- Cost functions
  - Your definition in time, cost, length plus…..
  - When an aggregate level is used, costs should be aggregated as well
- Discretisation of the cost function $F\left(c_{ij}\right) \Rightarrow F_k\left(c_{ij}\right)$
  - Preferably each "bin" having a similar rate of observations

Mathematical background (1/2)

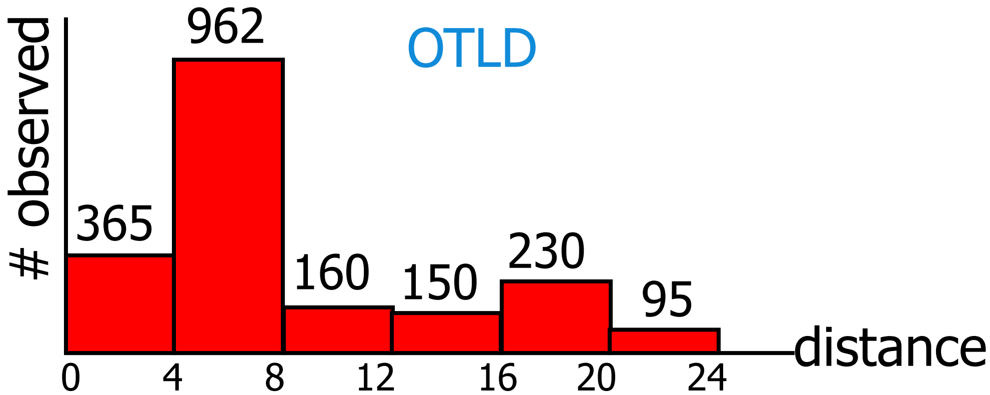
- Key assumption: number of trips per OD-pair is Poisson distributed
- Model formulation: $\hat{T}_{ij} = Q_i \cdot X_j \cdot F_k\left(c_{ij}\right)$
- Poisson model: $P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$
- For $T_{ij} \Rightarrow p\left(T_{ij}\right) = \dfrac{e^{-\left(Q_i \cdot X_j \cdot F_k\left(c_{ij}\right)\right)}\left(Q_i \cdot X_j \cdot F_k\left(c_{ij}\right)\right)^{T_{ij}}}{T_{ij}!}$
- For a set of N observations n_{ij} the likelihood becomes

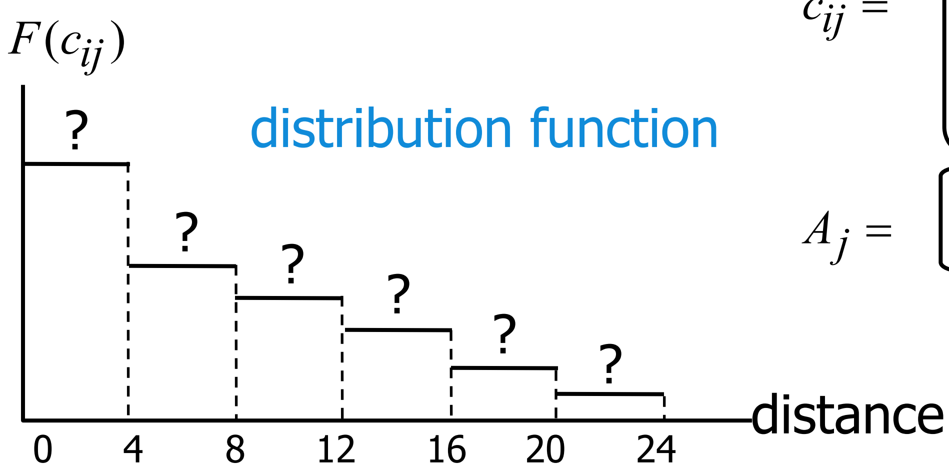
$$p\left(\left\{n_{ij}\right\} \mid Q_i, X_j, F_k\left(c_{ij}\right)\right) = \prod_{i,j\in N} \frac{e^{-\left(c \cdot Q_i \cdot X_j \cdot F_k\left(c_{ij}\right)\right)}\left(c \cdot Q_i \cdot X_j \cdot F_k\left(c_{ij}\right)\right)^{n_{ij}}}{n_{ij}!}, c = \frac{\sum_{ij\in N} n_{ij}}{\hat{T}}$$

Mathematical background (2/2)

- Setting derivatives equal to 0 yields 3 linear equations in which each parameter is function of the other two
- Similar solution procedure as for trip distribution:
- Determine the constraints
  - For each origin i the number of observed trips (departures)
  - For each destination j the number of observed trips (arrivals)
  - For each “bin” k the number of observed trips
- Set all parameters Q_i, X_j and F_k equal to 1
- Determine successively the values for Q_i, X_j and F_k until convergence

Example Poisson estimator





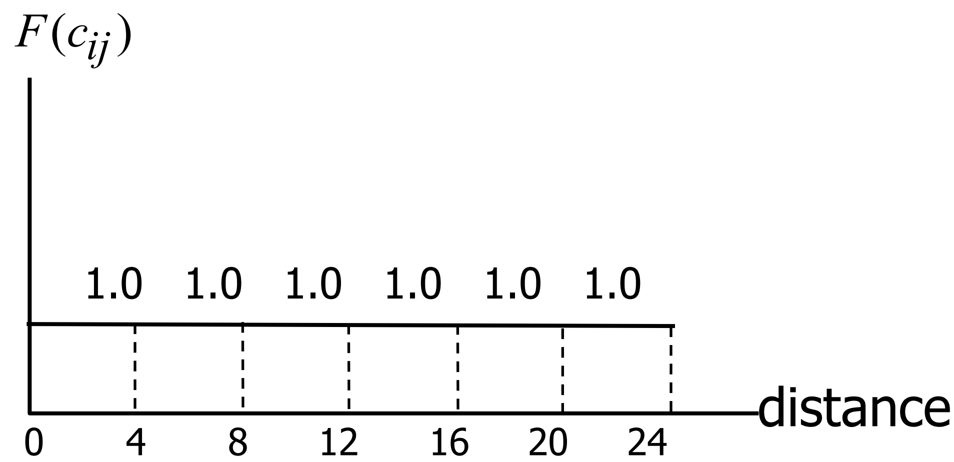
$$c_{ij} = \begin{pmatrix} 3 & 11 & 18 & 22 \\ 10 & 3 & 13 & 19 \\ 15 & 13 & 5 & 7 \\ 24 & 18 & 6 & 5 \end{pmatrix} \quad P_i = \begin{pmatrix} 400 \\ 460 \\ 400 \\ 702 \end{pmatrix}$$

$$A_j = \begin{bmatrix} 260 & 400 & 500 & 802 \end{bmatrix}$$

Example Poisson estimator: step 1

					P_i	
	1	1	1	1	400	$\times 100$
	1	1	1	1	460	$\times 115$
	1	1	1	1	400	$\times 100$
	1	1	1	1	702	$\times 175.5$
A_j	260	400	500	802		

- Start with $F(c_{ij}) = 1$
- Scale to the productions



Example Poisson estimator: step 2

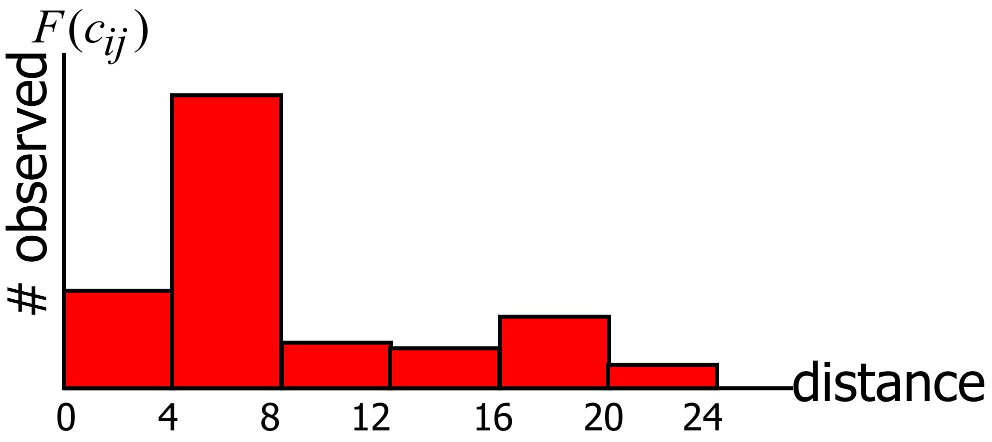
					P_i
	100	100	100	100	400
	115	115	115	115	460
	100	100	100	100	400
	175.5	175.5	175.5	175.5	702
A_j	260	400	500	802	
	$\times 0.53$	$\times 0.82$	$\times 1.01$	$\times 1.63$	

- Scale to the attractions

Example Poisson estimator: step 3

					P_i
	53	82	102	164	400
	61	94	117	188	460
	53	81	102	163	400
	93	143	179	287	702
A_j	260	400	500	802	

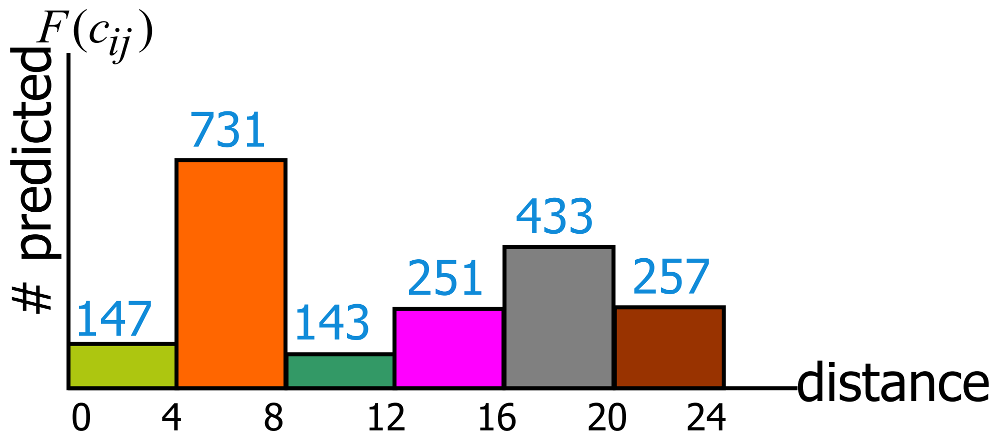
- Scale the distribution values such that they represent the OTLD



Example Poisson estimator: step 3

					P_i
	53	82	102	164	400
	61	94	117	188	460
	53	81	102	163	400
	93	143	179	287	702
A_j	260	400	500	802	

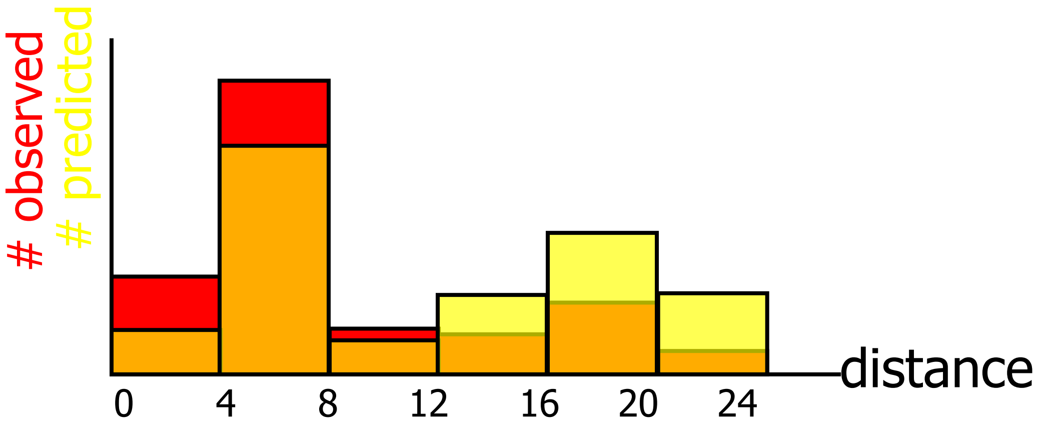
$$c_{ij} = \begin{pmatrix} 3 & 11 & 18 & 22 \\ 10 & 3 & 13 & 19 \\ 15 & 13 & 5 & 7 \\ 24 & 18 & 6 & 5 \end{pmatrix}$$



Example Poisson estimator: step 3

					P_i
	53	82	102	164	400
	61	94	117	188	460
	53	81	102	163	400
	93	143	179	287	702
A_j	260	400	500	802	

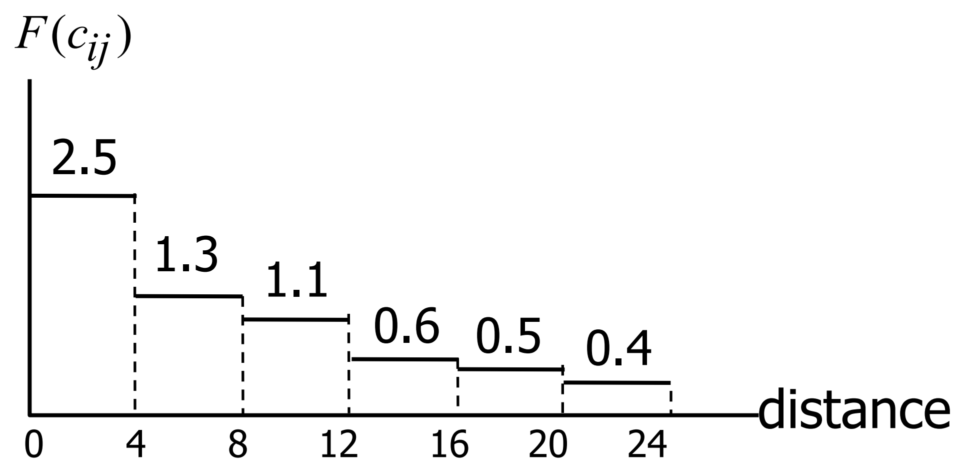
- Scale the distribution values such that they represent the OTLD



Example Poisson estimator: step 3

					P_i
	53 $\times 2.5$	82 $\times 1.1$	102 $\times 0.5$	164 $\times 0.4$	400
	61 $\times 1.1$	94 $\times 2.5$	117 $\times 0.6$	188 $\times 0.5$	460
	53 $\times 0.6$	81 $\times 0.6$	102 $\times 1.3$	163 $\times 1.3$	400
	93 $\times 0.4$	143 $\times 0.5$	179 $\times 1.3$	287 $\times 1.3$	702
A_j	260	400	500	802	

$$c_{ij} = \begin{pmatrix} 3 & 11 & 18 & 22 \\ 10 & 3 & 13 & 19 \\ 15 & 13 & 5 & 7 \\ 24 & 18 & 6 & 5 \end{pmatrix}$$



Example Poisson estimator: result iteration 1

					P_i
	132	92	54	61	400
	68	233	70	100	460
	32	49	134	215	400
	34	76	235	377	702
A_j	260	400	500	802	

- Perform next iteration
  - scale to productions
  - scale to attractions
  - scale distribution values

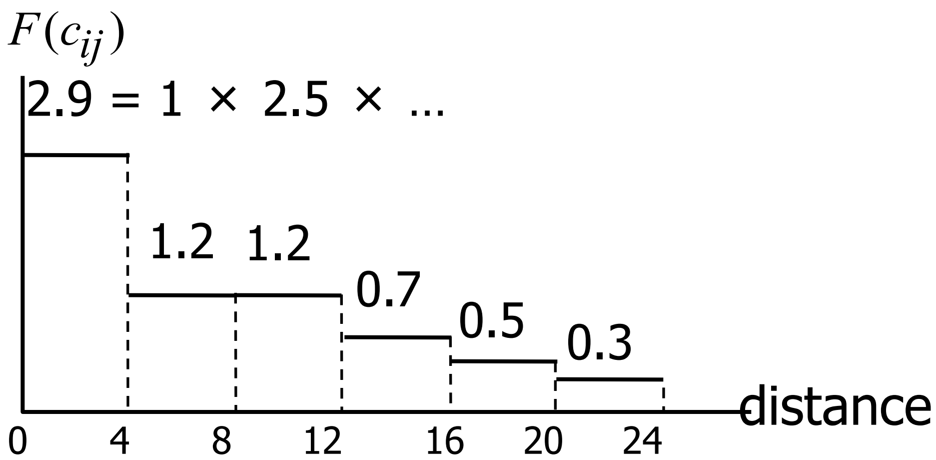
  etc.

Example Poisson estimator: result

				P_i
156	101	69	74	400
58	208	85	109	460
26	39	121	214	400
20	52	225	405	702
A_j: 260	400	500	802	

OD-matrix after 10 iterations

Distribution function:



From discrete distribution function to continuous function

- Just test which function yields the best fit with the function values
  - Function type <u>and</u> parameters

- In practice it's likely that you have to choose for which range of costs the fit is best
  - "One size doesn't fit all"

2.4.3

Calibration of deterrence functions Overview of the 2 methods

Estimation of the distribution function

Hyman's method

Given:

- Cost matrix
- Production and atraction
- Mean trip length

Assumed:

- Type of distribution function

Estimated:

- Q_i, X_j and parameter distribution function

Poisson model

Given:

- Cost matrix
- (partial) observed OD-matrix

Thus also known:

- (partial) production and attraction
- Totals per "bin" for the travel costs

Estimated:

Q_i, X_j and F_k

Solution methods: overview

Hyman's method

1. Set parameter α of the distribution function equal to 1/MTL
2. Determine values for $f(c_{ij})$
3. Balance the matrix for the productions and attractions (i.e. apply gravity model)
4. Determine new estimate for α based on observed MTL and computed MTL and go to step 2 until convergence is achieved

Poisson model

1. Set values for F_k equal to 1
2. Balance the (partial) matrix for the (partial) production
3. Balance the (partial) matrix for the (partial) attraction
4. Balance the (partial) matrix for the totals per cost class (i.e. correction in iteration i for estimate of F_k)
5. Go to step 2 until convergence is achieved

$$F_k^i = \prod_j \frac{observed\ total_k}{computed\ total_k^j}$$

2.5

Trip distribution models
Method 4: Choice modelling

Discrete choice model

$$T_{ij} = P_i \frac{\exp(\beta V_j)}{\sum_k \exp(\beta V_k)}, \qquad V_j = \theta_1 X_j - \theta_2 c_{ij}$$

$T_{ij} =$ number of trips from *i* to *j*

$\theta_1, \theta_2 =$ parameters

$\beta =$ scaling parameter

$P_i =$ trip production at zone *i*

$X_j =$ trip attraction potential at zone *j*

$c_{ij} =$ travel cost from zone *i* to zone *j*

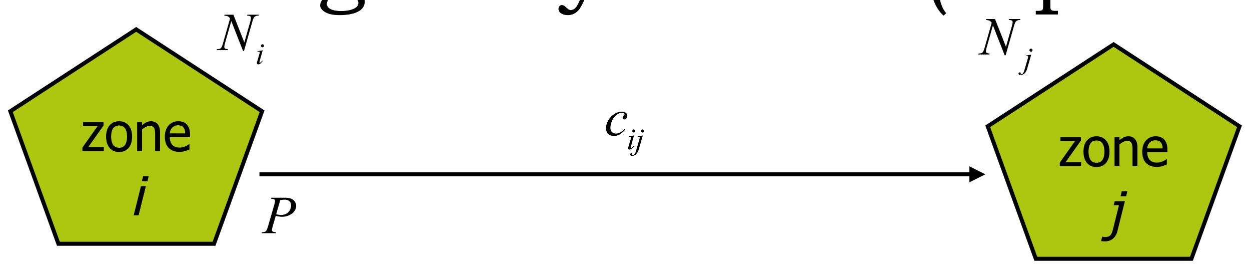
Explanatory variables?

- Inhabitants
- Households
- Jobs
- Retail jobs
- Students
- Densities
- Location types
- Etc.

- Minus travel costs

More suited for trips or for tours?

Derivation of the gravity model (reprise)



Observed utility for activities in zone *i* and zone *j*:

$$V_{ij} = N_j - N_i - \theta_2 \cdot c_{ij}$$

Subjective utility:

$$U_{ij} = V_{ij} + \varepsilon_{ij}$$

Number of people traveling from *i* to *j*:

$$p_{ij} \cdot T = \frac{\mathrm{e}^{\beta V_{ij}}}{\sum_{rs} \mathrm{e}^{\beta V_{rs}}} \cdot T \qquad T_{ij} = \rho Q_i X_j F_{ij}$$

$$= \frac{T}{\sum_{rs} \exp(\beta V_{rs})} \cdot \exp(-\beta N_i) \cdot \exp(\beta N_j) \cdot \exp(-\beta \theta_2 c_{ij})$$

2.7

Trip distribution
Practical issues

Practical issues

- Distribution function and trip length distribution
- Intra-zonal trips
- External zones: through traffic
- All trips or single mode?

Distribution function and trip length distribution

- Similar or different?
- Simply put:
  distribution function is input and trip length distribution is output!
- See also differences in OTLD en distribution function in slides on Poisson estimation

Intrazonal trips

- What's the problem?
- Intrazonal travel costs?
- Rule of thumb: $\frac{1}{3}$ (or $\frac{1}{2}$?) of lowest cost to neighbouring zone
  - True for public transport?
- Alternative: Trip generation for intrazonal only
  - How?

External zones

- Two possible issues

- Size issue
  - Very large zones => high values for production and attraction => intrazonal trips? => small errors lead to large differences

- Cordon models
  - Through traffic follows from other source, e.g. license plate survey or other model => through traffic is thus fixed input and should not be modelled using trip distribution models

Approach for cordon model

- Determine production and attraction for internal zones using e.g. regression analysis
- Determine production and attraction for external zones using e.g. counts
- Derive matrix for through traffic (i.e. from cordon zone to cordon zone) from e.g. a regional model
- Subtract through traffic from production and attraction of the external zones
- Apply gravity model with the resulting production and attraction, while making sure that there is no through traffic, e.g. by setting the travel costs between cordon zones equal to ∞
- Add matrix for through traffic to the resulting matrix of the gravity model

All trips or a single mode?

- Check the slides
- Which parts consider a single mode?