CIE4801 Transportation and spatial modelling
Modal split

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Content

- Nested logit part 2
- Modelling component 3: Modal split
  - Your comments/questions on Chapter 6
  - Practical issues

1.

Comments/questions last lecture

Comments/questions

- Trip distribution
  - Trip distribution models
    - Growth factor
    - Gravity model and Entropy model
    - Choice modelling
  - Iterative algorithm
    - Singly, doubly and triply constrained
  - Calibration methods
    - Hyman, Poisson

Estimation of the distribution function

Hyman's method

Given:

- Cost matrix
- Production and atraction
- Mean trip length

Assumed:

- Type of distribution function

Estimated:

- Q_i, X_j and parameter distribution function

Poisson model

Given:

- Cost matrix
- (partial) observed OD-matrix

Thus also known:

- (partial) production and attraction
- Totals per "bin" for the travel costs

Estimated:

Q_i, X_j and F_k

Solution methods: overview

Hyman's method

1. Set parameter α of the distribution function equal to 1/MTL
2. Determine values for $f(c_{ij})$
3. Balance the matrix for the productions and attractions (i.e. apply gravity model)
4. Determine new estimate for α based on observed MTL and computed MTL and go to step 2 until convergence is achieved

Poisson model

1. Set values for F_k equal to 1
2. Balance the (partial) matrix for the (partial) production
3. Balance the (partial) matrix for the (partial) attraction
4. Balance the (partial) matrix for the totals per cost class (i.e. correction in iteration i for estimate of F_k)
5. Go to step 2 until convergence is achieved

$$F_k^i = \prod_j \frac{observed\ total_k}{computed\ total_k^j}$$

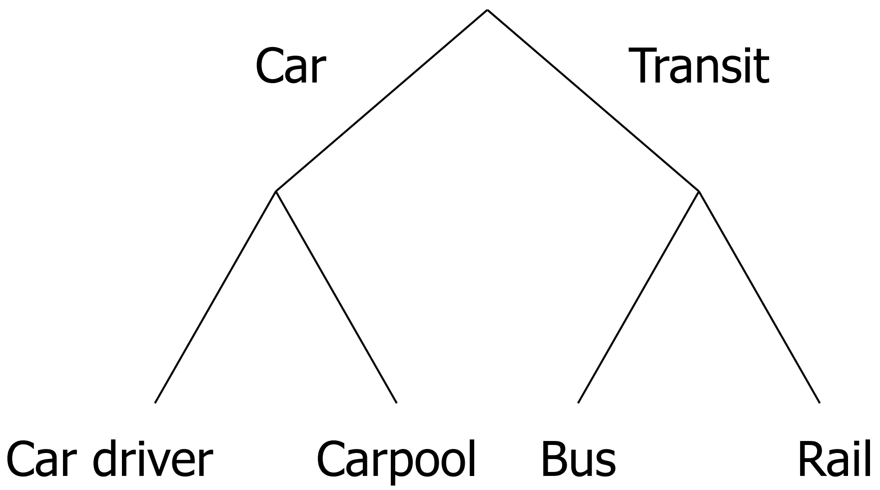
Practical issues

- Distribution function and trip length distribution
- Intra-zonal trips
- External zones: through traffic
- All trips or single mode?

2.

Nested logit part 2

Recall the example



Nests k
Scale parameter β

Alternatives i
Scale parameter λ_k

$$P_i = \frac{e^{\beta V_i}}{\sum_j e^{\beta V_j}} \Rightarrow P(i) = P(i \mid k) P(k) = \frac{e^{\lambda_k V_{i|k}}}{\sum_{j \in k} e^{\lambda_k V_{j|k}}} \cdot \frac{e^{\beta V_k}}{\sum_{l \in K} e^{\beta V_l}}$$

Decomposition in two logits

Split utility in two parts:

- variables describing attributes for nests (aggregate level): W_k
- variables describing attributes within nest: Y_j

$$U_i = W_k + Y_i + \varepsilon_i \quad i \in B_k$$

Probability alternative is product of probability of alternative within nest and probability of nest

$$P_i = P_{i|B_k} P_{B_k}$$

Decomposition in two logits
Resulting formulas

$$P_{B_k} = \frac{e^{\beta\cdot(W_k+I_k)}}{\sum_{l=1}^{K} e^{\beta\cdot(W_l+I_l)}}$$

Probability of choosing a nest

$$P_{i|B_k} = \frac{e^{\lambda_k \cdot Y_i}}{\sum_{j\in B_k} e^{\lambda_k \cdot Y_j}}$$

Probability of an alternative within a nest

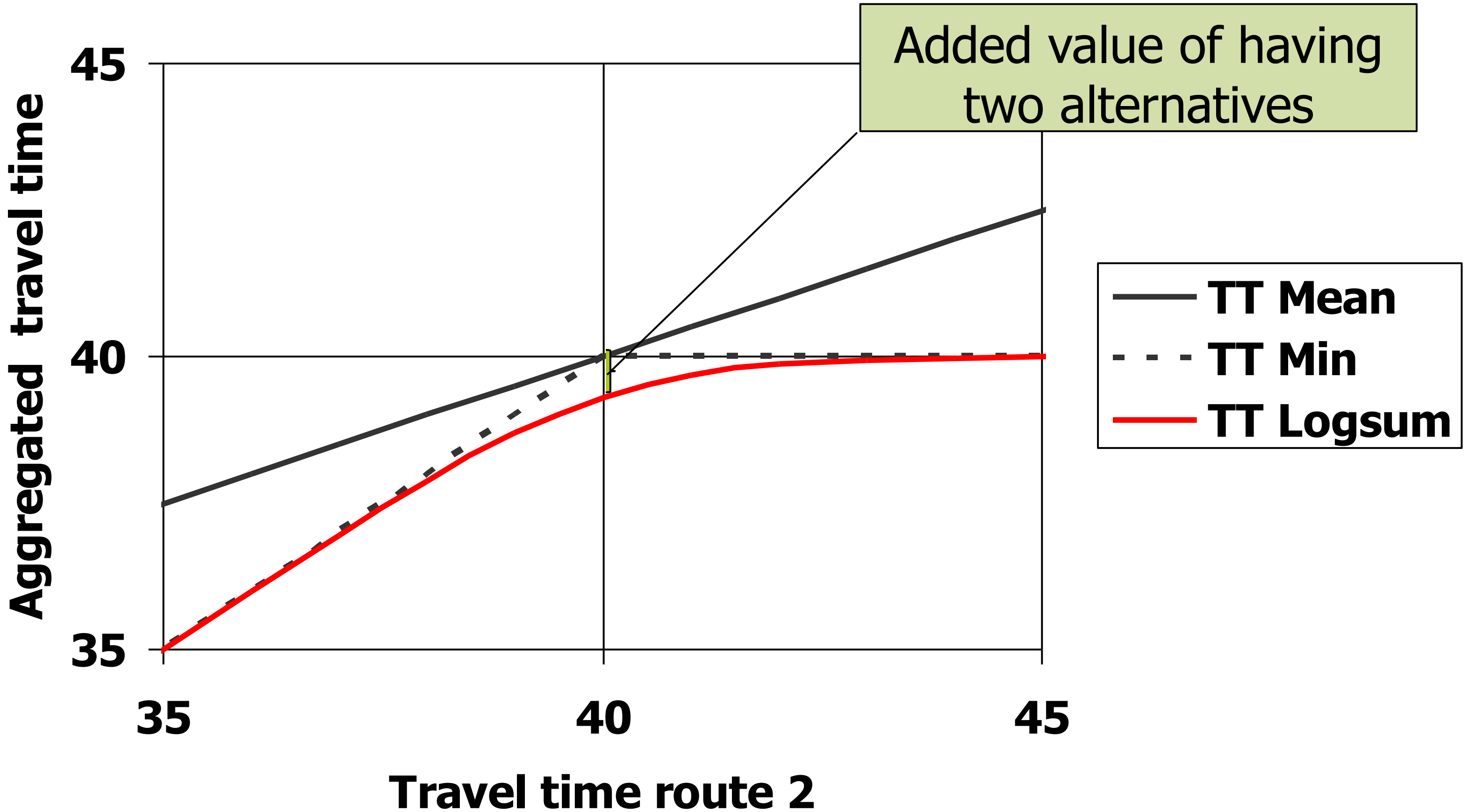
$$I_k = \frac{1}{\lambda_k} \ln \sum_{j\in B_k} e^{\lambda_k \cdot Y_j}$$

Utility of a nest

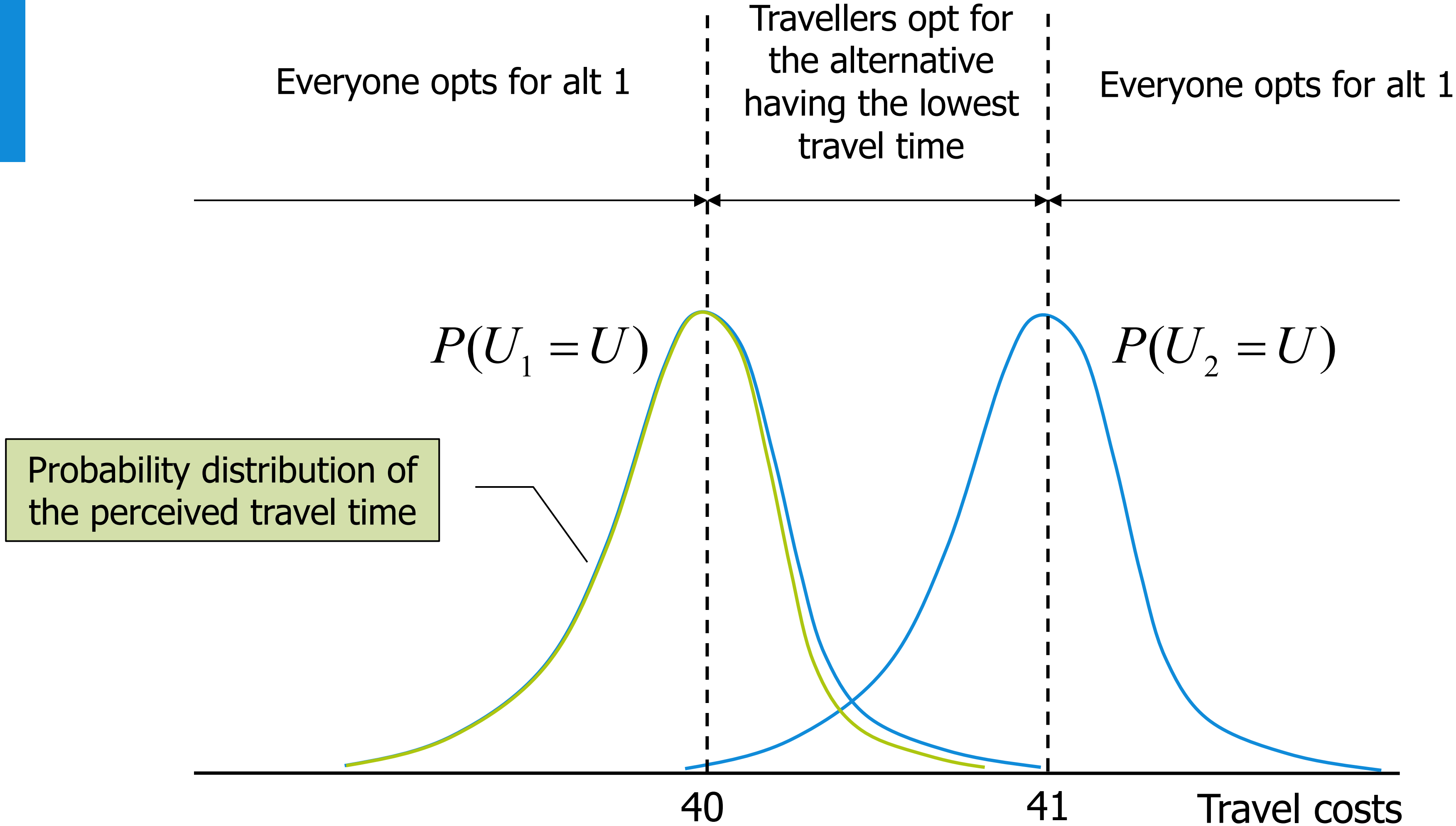
Main concept is $I_k \geq \max(Y_i)$, thus having alternatives can have benefits

Example route choice with 2 routes

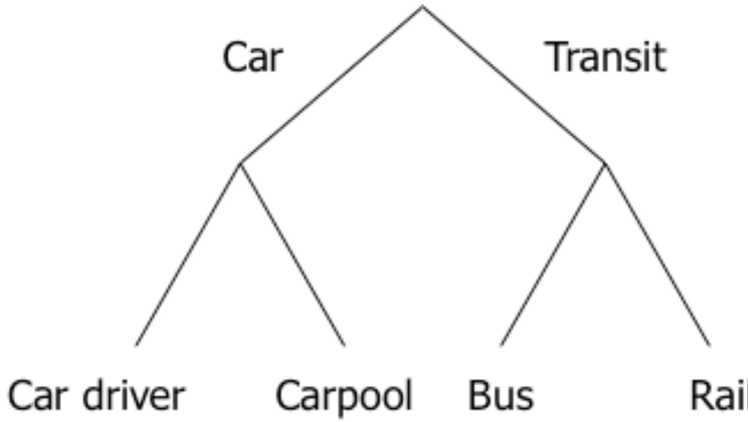
Travel time route 1 is 40 minutes, travel time route 2 varies



Why is there an added value?



Typical conditions for nested logit



Nests k
Scale parameter β

Alternatives i
Scale parameter λ_k

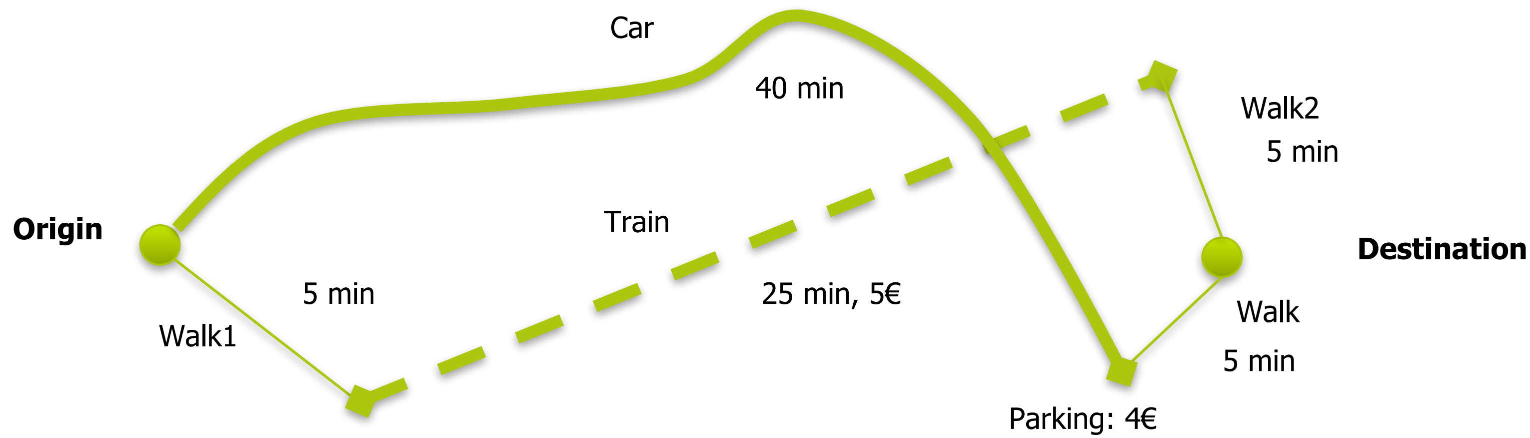
$$P_{i|B_k} \cdot P_{B_k} = \frac{e^{\lambda_k \cdot Y_i}}{\sum_{j\in B_k} e^{\lambda_k \cdot Y_j}} \cdot \frac{e^{\beta\cdot\left(W_k + \frac{1}{\lambda_k}\ln\sum_{j\in B_k} e^{\lambda_k\cdot Y_j}\right)}}{\sum_{l=1}^{K} e^{\beta\cdot\left(W_l+\frac{1}{\lambda_l}\ln\sum_{j\in B_l} e^{\lambda_l\cdot Y_j}\right)}}$$

Define the parameter

$$\mu_k = \frac{\beta}{\lambda_k}$$

- It is required that $\mu_k \leq 1$
- If $\mu_k = 1$ this expression collapses to the standard logit model
- If $\mu_k \rightarrow 0$, the nest is reduced to the alternative having the highest utility, i.e. the other alternatives in the nest have no additional value

Example for P+R facility



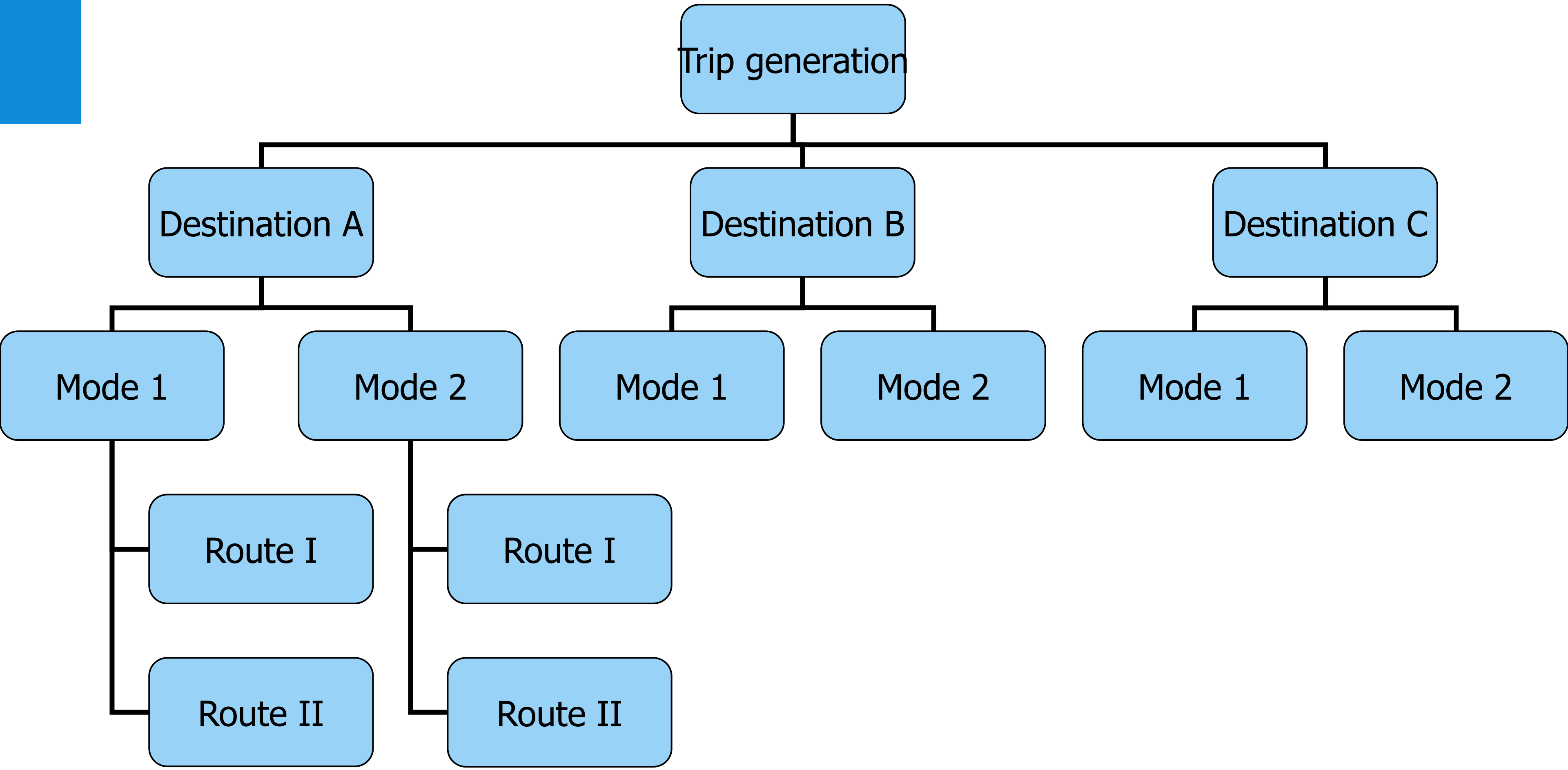
- See spreadsheet on Blackboard

  Analyse the spreadsheet and experiment with the values of β and λ

Other examples of nested models

- Logsum over routes in mode choice
- Logsum over modes in destination choice
- Dutch National Model considers nesting when modelling destination and mode choice (and tour generation)

Four stage model and logsums



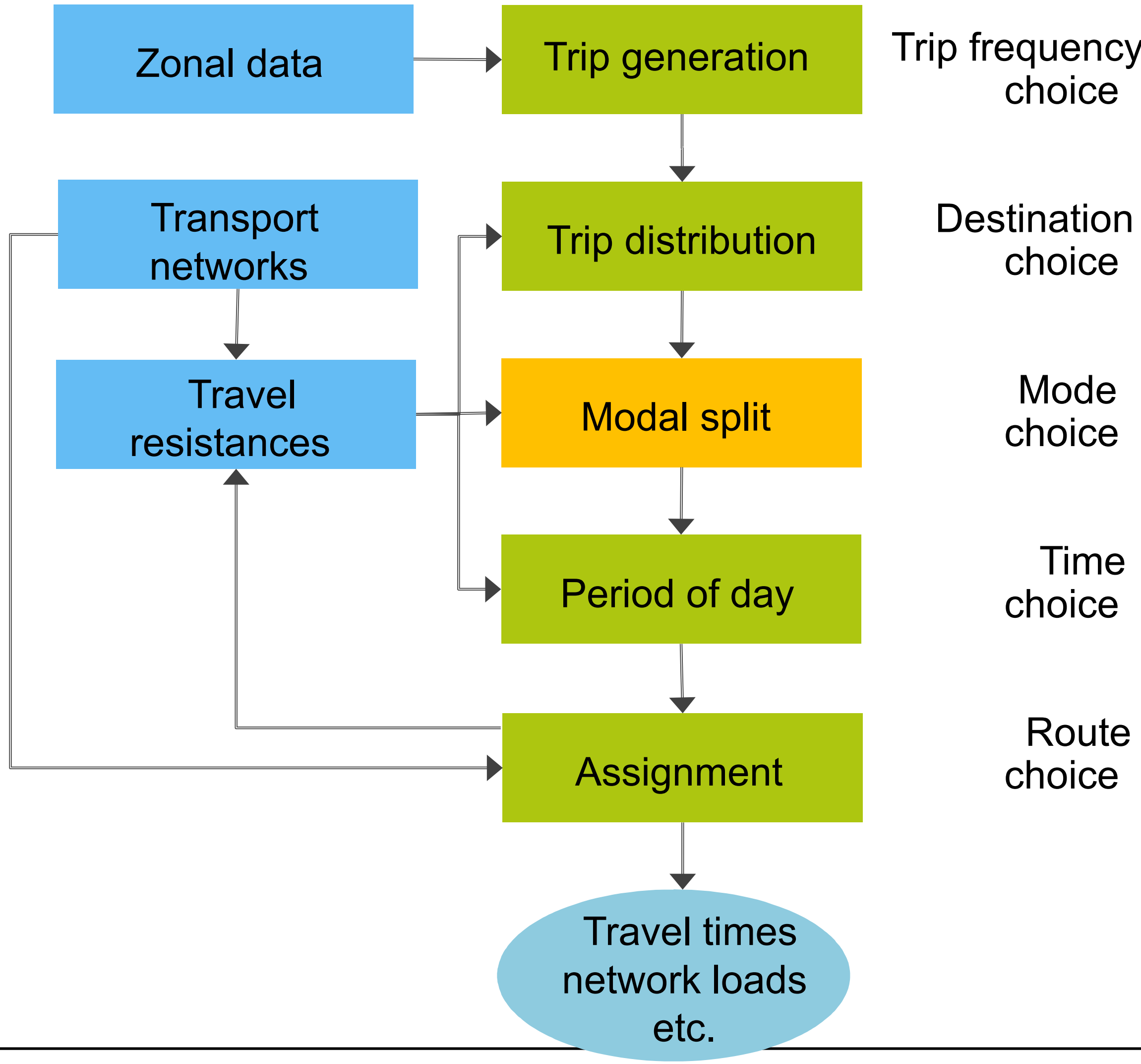
Nested logit: to conclude

- Nested logit modelling proved to be a powerful tool for travel behaviour modelling
- Limitations: an alternative can only be allocated to a specific nest
- Possible extensions:
  - Cross-nested logit
  - Generalised nested logit
  - Network GEV (Generalised Extreme Value)

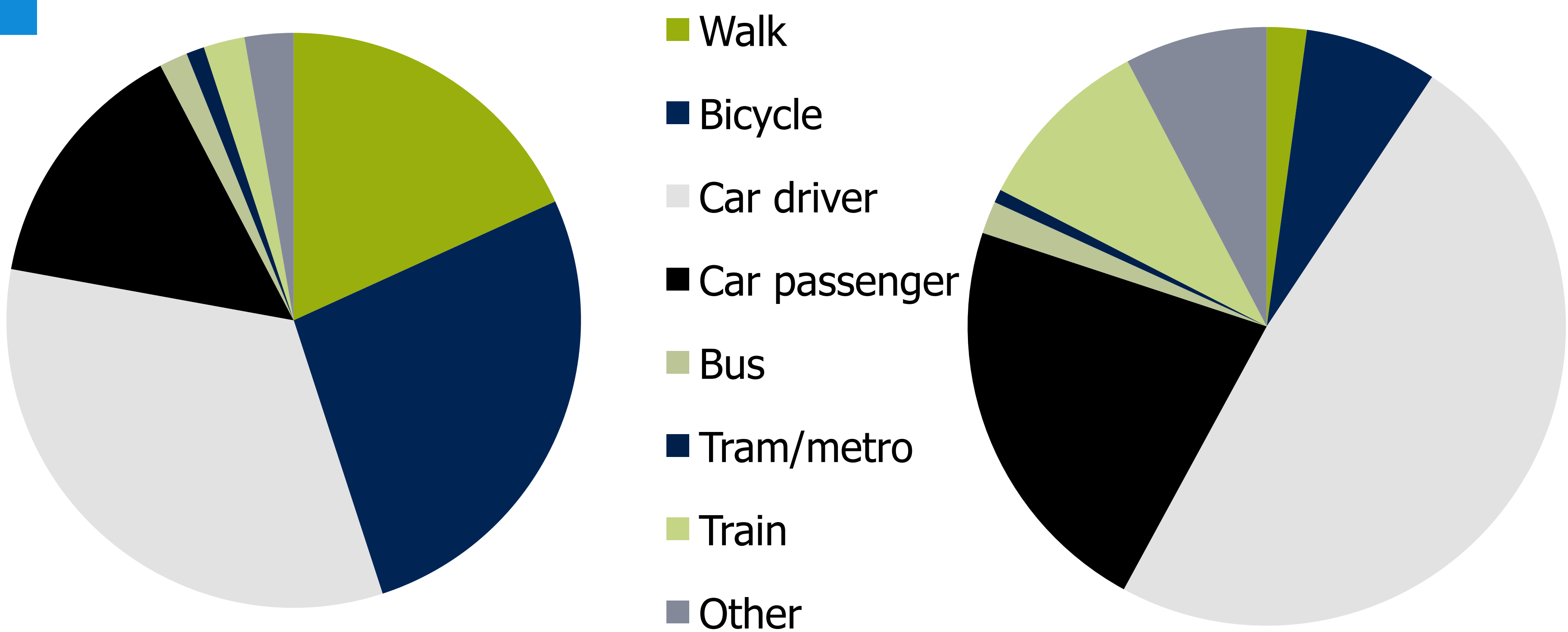
3.

Mode choice: what's it about?

Introduction to modal split



Modal split (Netherlands)
Trips and trip kilometres for an average day



Topics to study sections 6.1-6.5

- What does this modelling component do? What’s its output and what’s its input? How does it fit in the framework?
- Do you agree with the influencing factors?
  - For the trip maker, trip, and transport service
- Do you understand the modelling methods?
  - Empirical curves
  - Entropy based simultaneous distribution/modal split model
    - For a method to solve it, see these slides!
  - Choice model approach
    - Nested logit model
- Are these models appropriate?

3.1.1

Modal split models
Method 1: Empirical curves

Travel time ratio (VF)

- Ratio between travel time by public transport and by car:

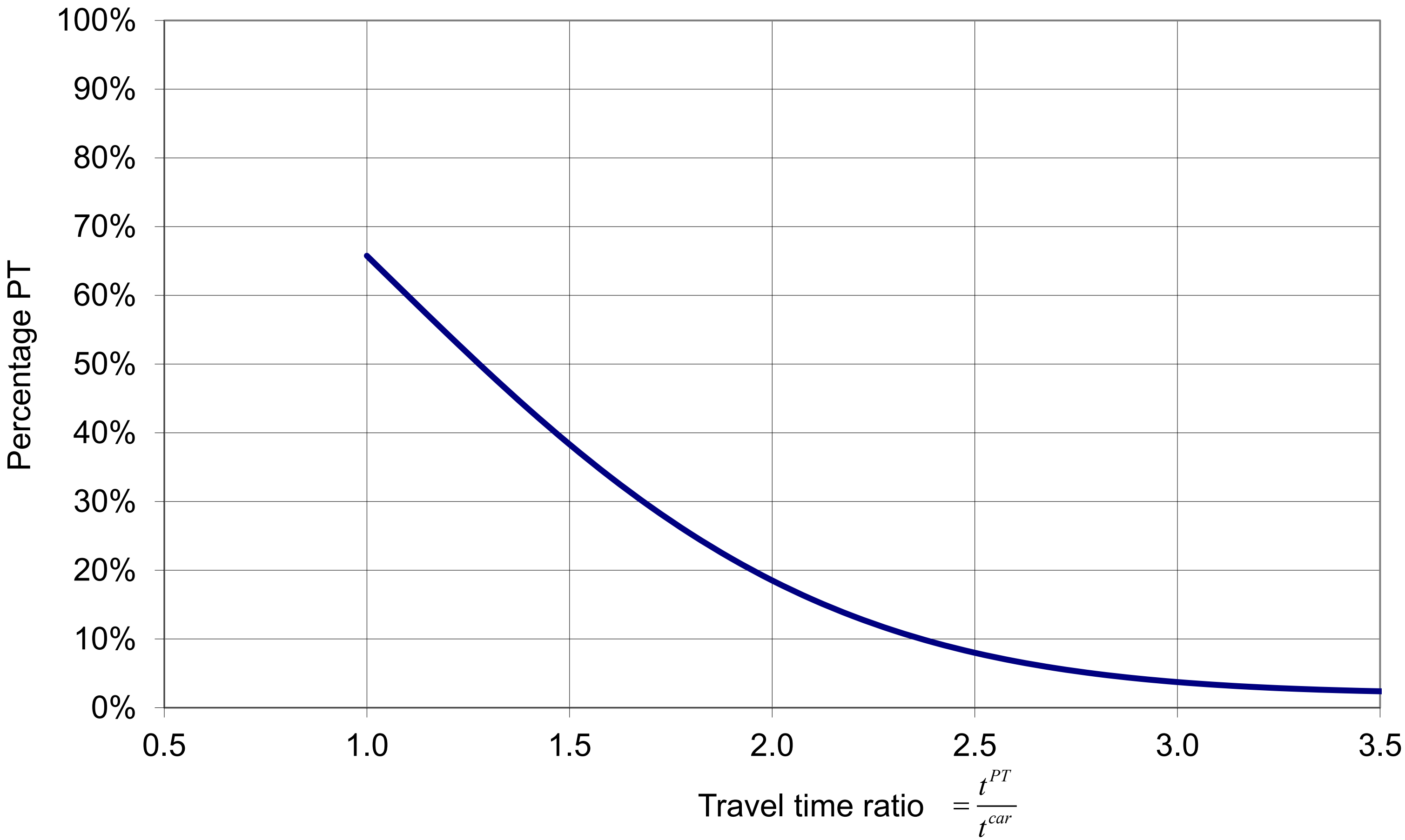
$$VF = \frac{t^{PT}}{t^{car}}$$

- Data used: observed trips where PT might be attractive
  - i.e. train service or 'express' service available
  - Public transport: Access and egress time to main PT mode, waiting time first stop, in-vehicle time of main PT mode, waiting time transfer
  - Car: travel time based on fixed speeds per area and period of day, parking

- Basic version: $P_{pt} = e^{-0.45 \cdot VF^2} + 0.02$

- Elaborate model: $P_{pt} = e^{-0.36 \cdot VF^2 - 0.17 \cdot N_t - \frac{1.35}{F} + 0.23} + 0.03$ N_t=transfers, F=frequency

Travel time ratio (basic version)



3.1.2

Modal split models
Method 2: Simultaneous distribution-modal split

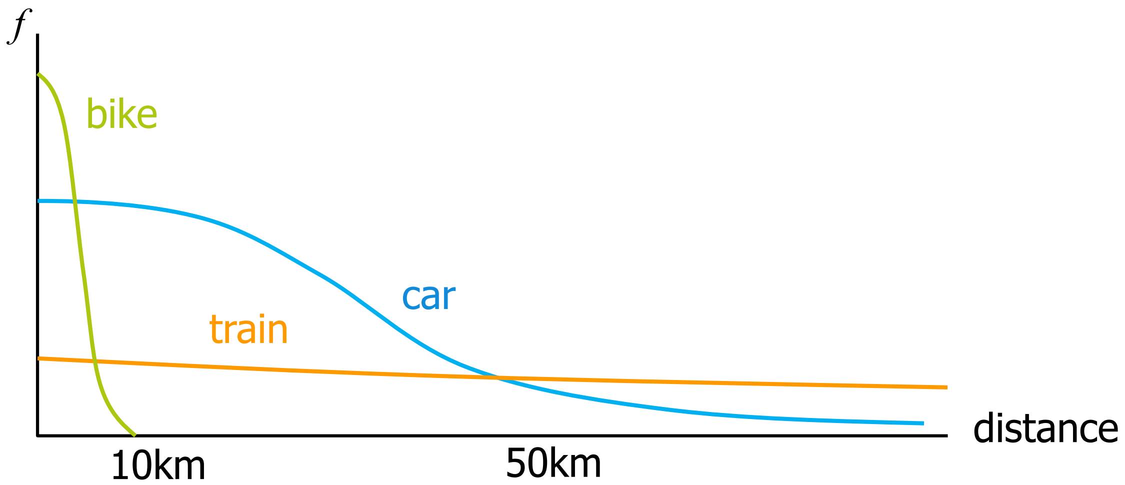
Gravity model and distribution functions

- Using distribution functions per mode

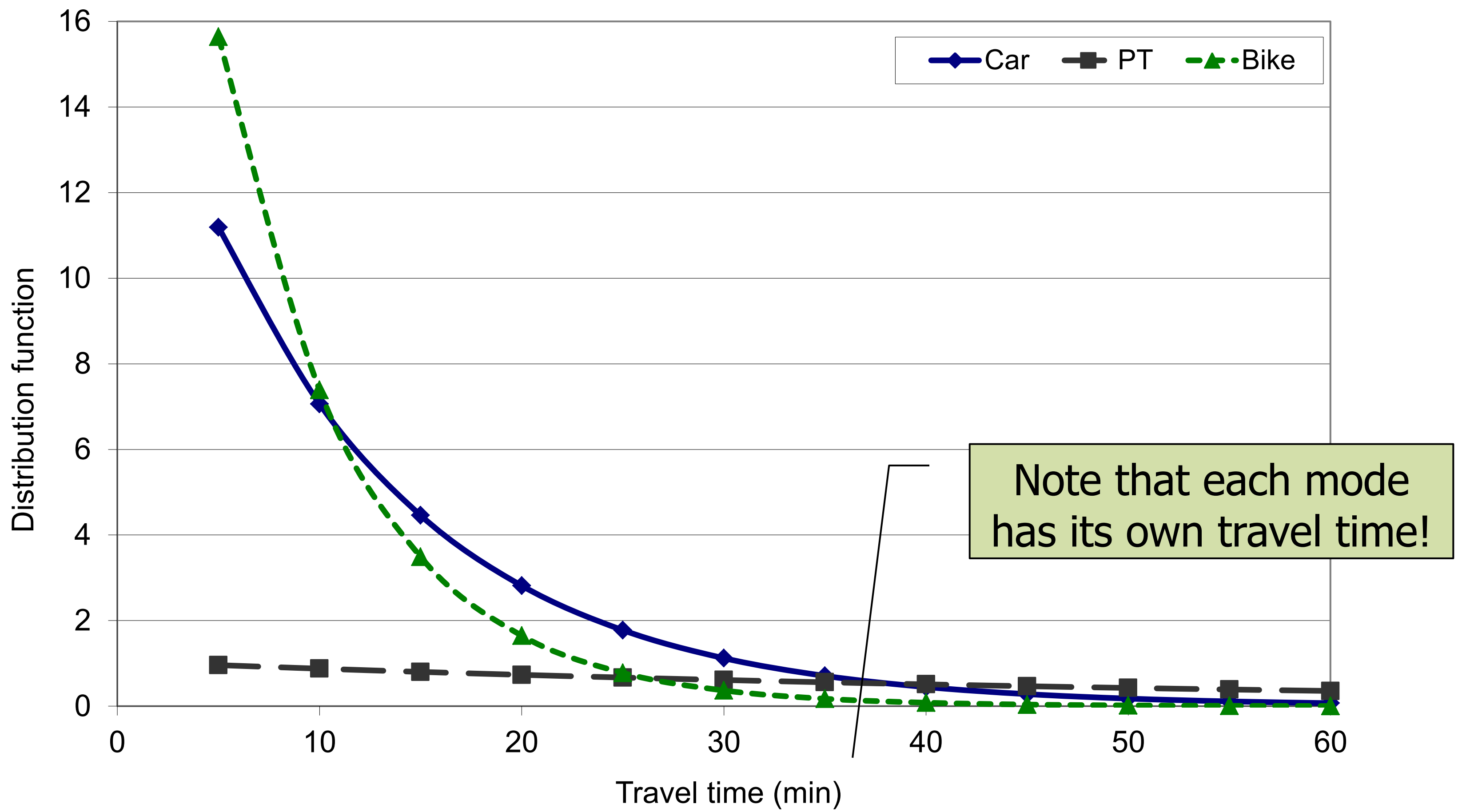
$$T_{ij} = \rho Q_i X_j F_{ij} \quad \text{with} \quad F_{ij} = \sum_v f_v(c_{ijv}) = \sum_v F_{ijv}$$

- Note that this formula also holds per mode

$$T_{ij} = \rho Q_i X_j F_{ij} = \rho Q_i X_j \sum_v f_v(c_{ijv}) = \sum_v \rho Q_i X_j f_v(c_{ijv}) = \sum_v T_{ijv}$$



Distribution functions per mode



Example mode choice

Zoetermeer - TU Delft

- Car: 30 min
- PT: 50 min
- Bike: 45 min

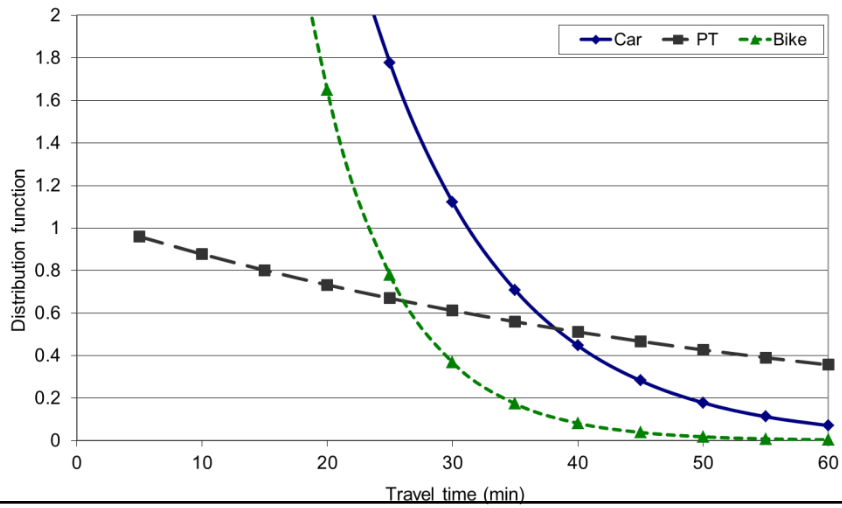
Function values

- Car: 1.12
- PT: 0.43
- Bike: 0.04

Total: 1.59

Result

- Car: 71%
- PT: 27%
- Bike: 2%



Doubly constrained simultaneous distribution/modal split model

$$T_{ijv} = a_i b_j P_i A_j F_{ijv} \quad \text{with} \quad F_{ijv} = f_v(c_{ijv})$$

$$\sum_j \sum_v T_{ijv} = P_i \quad \text{and} \quad \sum_i \sum_v T_{ijv} = A_j$$

	zone 1		zone 2		zone 3		Σ
	car	PT	car	PT	car	PT	
zone 1	… …	…	…	… …	…	… …	
zone 3	… …	…	…	… …	…	… …	
Σ							

Can be solved in a similar way as the standard doubly constrained model

Simultaneous distribution/modal split model

Most of the time, destination choice and mode choice are made *simultaneously* instead of *sequentially*.

Combined choices (e.g. for going shopping):

- Take the train to the center of Amsterdam
- Take the bike to the center of Delft
- Take the car to the center of Rotterdam

General gravity model for simultaneous trip distribution/modal split:

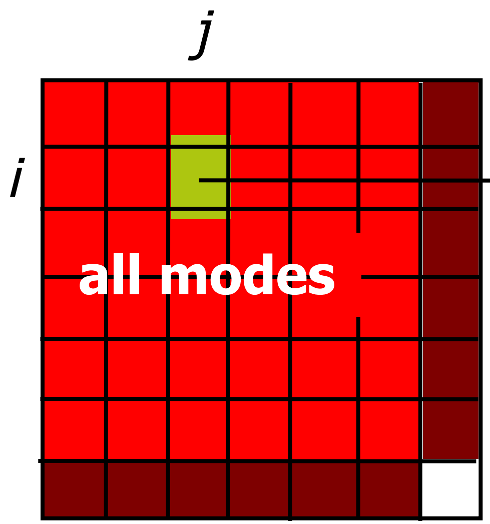
$$T_{ij} = \rho Q_i X_j F_{ij} \text{ with } F_{ij} = \sum_v f\left(c_{ijv}\right)$$

Does simultaneous imply that you cannot compute it sequentially?

3.1.3

Modal split models
Method 3: Choice modelling

Mode choice model



car share: P_{ij1}

PT share: P_{ij2}

bike share: P_{ij3}

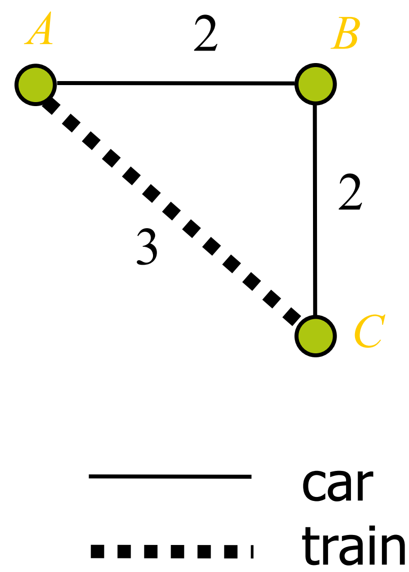
$$P_{ijv} = \frac{e^{\beta V_{ijv}}}{\sum_w e^{\beta V_{ijw}}}$$

$$V_{ijv} = \theta_0^v + \theta_1^v X_{ij1}^v + \theta_2^v X_{ij2}^v + \theta_3^v X_{ij3}^v$$

Mode choice model

	A	B	C
A	0	200	150
B	200	0	200
C	150	200	0

car 93 train 57



$$V_{CA}^{car} = -0.6 \cdot c_{CA}^{car}$$

$$V_{CA}^{train} = -0.5 - 0.8 \cdot c_{CA}^{train}$$

$$P_{ijv} = \frac{e^{V_{ijv}}}{\sum_w \mathrm{e}^{V_{ijw}}}, \quad (\text{i.e. } \beta = 1)$$

0	200	93
200	0	200
93	200	0

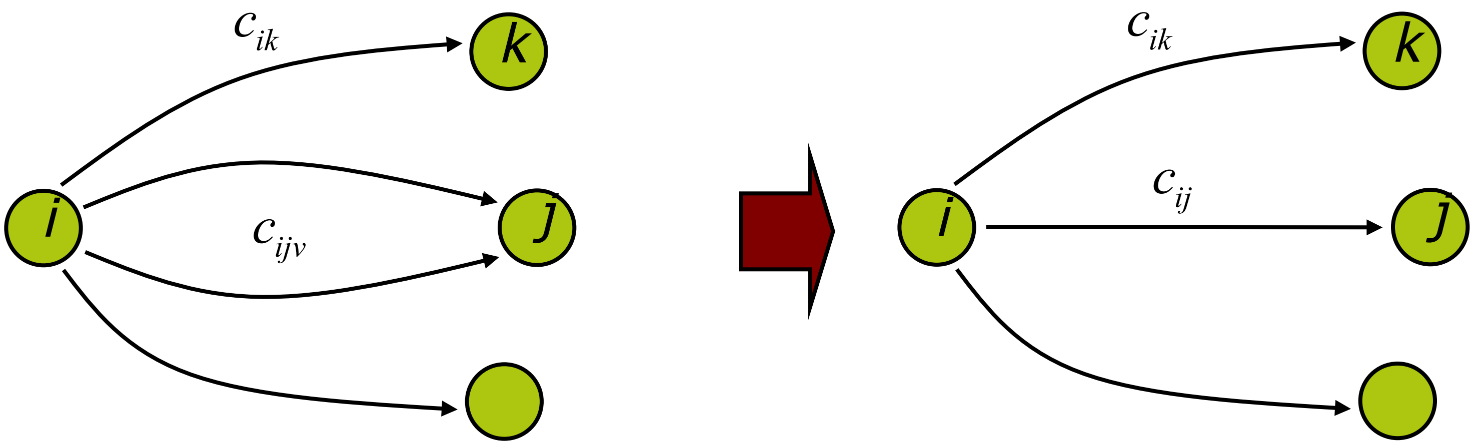
car

0	0	57
0	0	0
57	0	0

train

$$P_{CA}^{car} = \frac{e^{-0.6 \cdot 4}}{e^{-0.6 \cdot 4} + e^{-0.5 - 0.8 \cdot 3}} = 62\%$$

Derivation nested distribution-modal split model



Probability of choosing alternative *j* ?

Let's first look at the **mode choice** for *j*

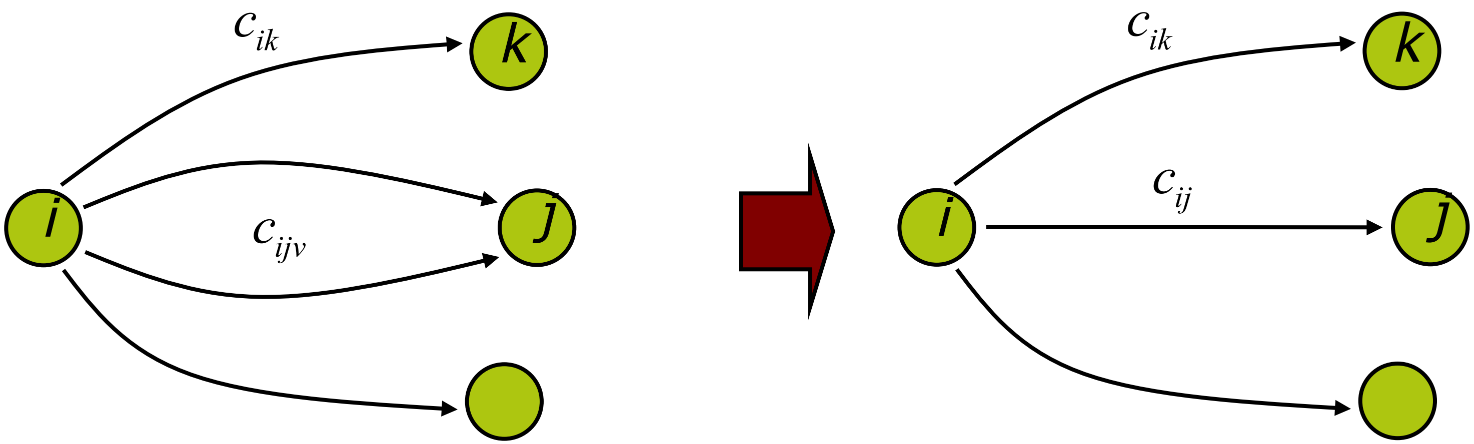
$$P(w \mid ij) = \frac{\exp(-\beta_m c_{ijw})}{\sum_v \exp(-\beta_m c_{ijv})}$$

→ Attractiveness of alternative w

→ Attractiveness of all alternatives

Translate attractiveness of all alternatives back into costs $\Rightarrow \quad c_{ij} = -\frac{1}{\beta_m} \ln \sum_v \exp(-\beta_m c_{ijv})$

Derivation nested distribution-modal split model



Probability of choosing alternative j :

$$P(j)=\frac{\exp\left(\beta_d\cdot\left(\sum_a\left(\theta_a\cdot X_{ja}\right)-c_{ij}\right)\right)}{\exp\left(\beta_d\cdot\left(\sum_a\left(\theta_a\cdot X_{ja}\right)-c_{ij}\right)\right)+\sum_{k\neq j}\exp\left(\beta_d\cdot\left(\sum_a\left(\theta_a\cdot X_{ka}\right)-c_{ik}\right)\right)}$$

$$c_{ij}=-\tfrac{1}{\beta_m}\ln\sum_v\exp\left(-\beta_m c_{ijv}\right)$$

Note: $\beta_d / \beta_m = \mu \leq 1$

Difference simultaneous distribution model and nested logit model?

- Assume exponential deterrence functions: $F_v\left(c_{ijv}\right) = e^{\beta \cdot -c_{ijv}}$
- For trip distribution you use: $F_{ij} = \sum_v F_v\left(c_{ijv}\right) = \sum_v e^{\beta \cdot -c_{ijv}}$
- For nested logit you use: $F_{ij} = F_d\left(\frac{-1}{\beta}\ln\left(\sum_v e^{\beta \cdot -c_{ijv}}\right)\right)$
- With F_d being an exponential function as well:

$$F_{ij} = F_d\left(\frac{-1}{\beta}\ln\left(\sum_v e^{\beta \cdot -c_{ijv}}\right)\right) = e^{\beta_d \cdot -\frac{-1}{\beta}\ln\left(\sum_v e^{\beta \cdot -c_{ijv}}\right)}$$

$$= \left(e^{\ln\left(\sum_v e^{\beta \cdot -c_{ijv}}\right)}\right)^{\frac{\beta_d}{\beta}} = \left(\sum_v e^{\beta \cdot -c_{ijv}}\right)^{\frac{\beta_d}{\beta}}$$

Thus only similar if scale parameters are identical

3.2

Practical topics

Practical topics

- Role of constraints: Car ownership
- Car passenger
- Parking
- What comes first: destination choice or mode choice?
- Multimodality

Car ownership

- How is that accounted for so far?

- Implicit
  - Mode specific constant
  - Distribution functions

- Explicit approach
  - Split demand matrix in matrix for people having a car and people not having a car available and apply appropriate functions

Car passenger

- Why is this relevant?
- Simple approach
  - Exogenous car occupancy rate per trip purpose
- Comprehensive approach
  - Car passenger and car driver as separate modes
- Problem in both cases?

  No guarantee for consistency in trip patterns car driver and car passenger

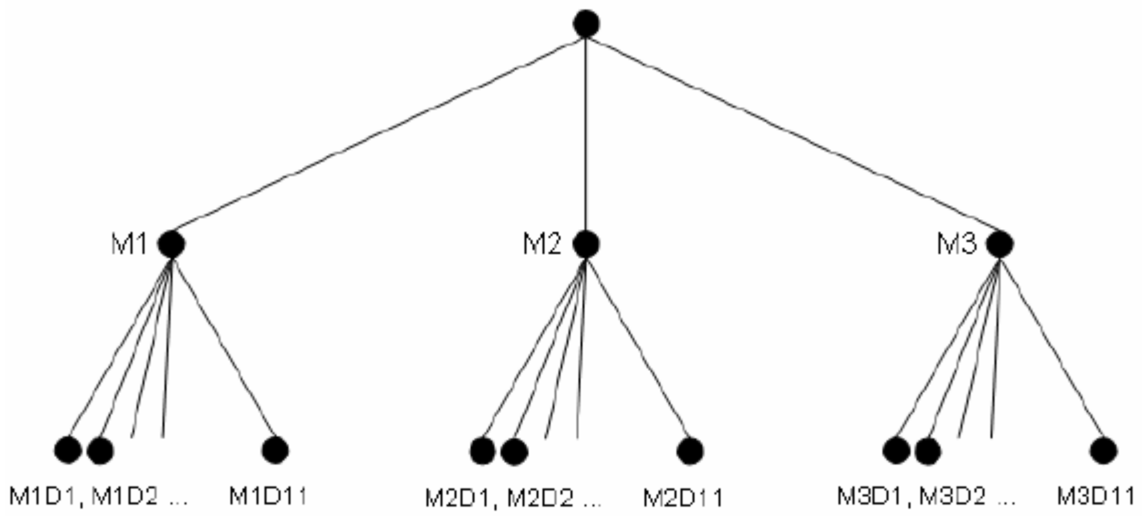
Parking

- Would you include it, and if so, how?
- How would you do it in case of tours?
- How would you do it in case of trips?
- What about morning and evening peak?

  Assign half of parking costs to a trip (origin and destination)?

What comes first: trip distribution or mode choice?

- What is the order we discussed so far?
- Swiss model:

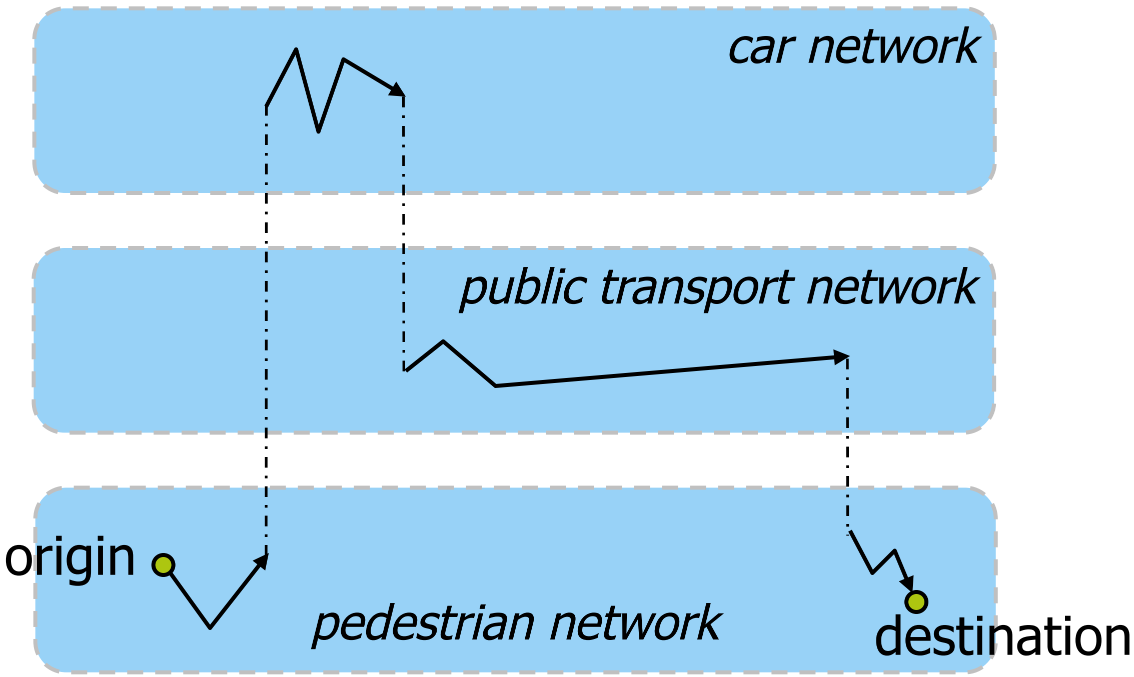


- Mode preferences appear to be pretty strong

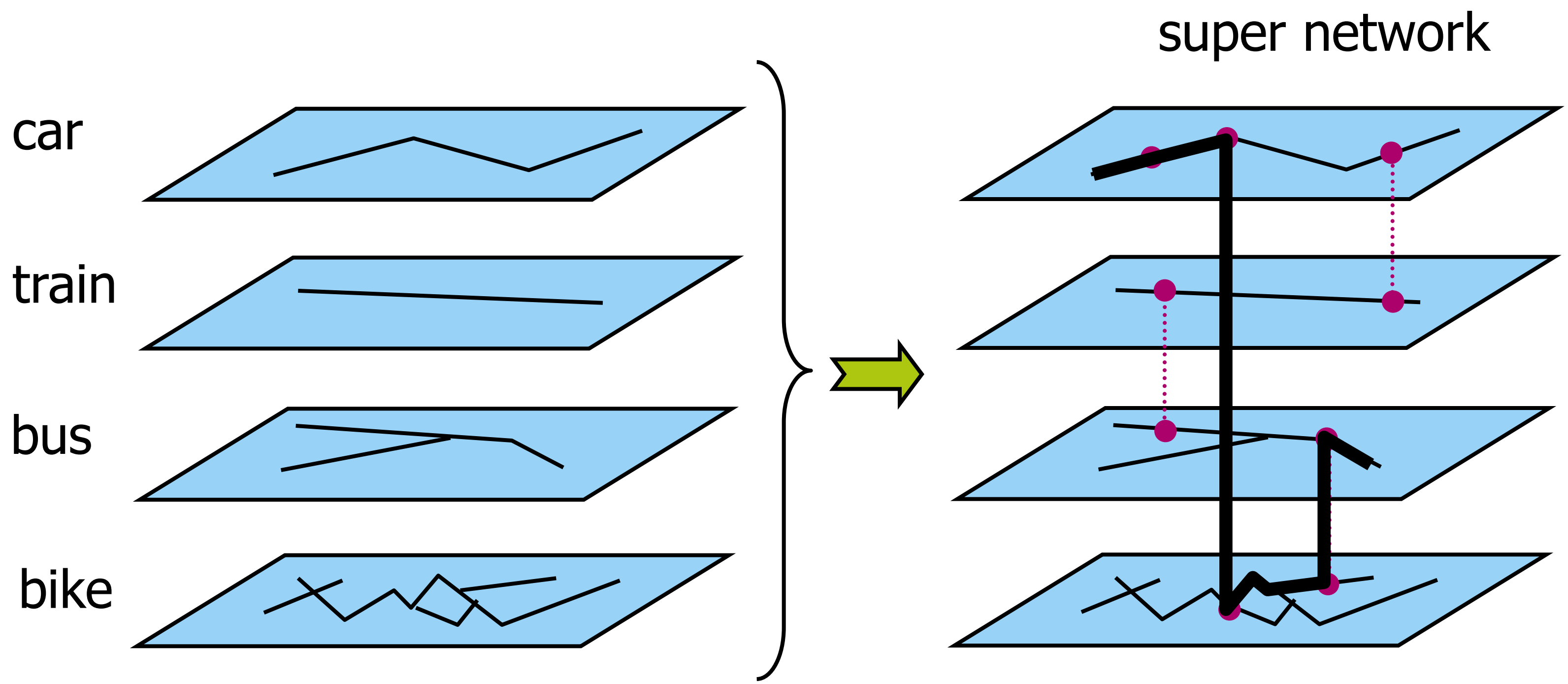
Multimodality?

- Approach so far: clear split between car, public transport (and slow modes)
- Special case: separate modes for train and BTM
- 80% of train travellers use other modes for access and/or egress
  - Bus/tram/metro, but also bike and car
- So where do we find the cyclist to the station?

Multimodal trips



Simultaneous route/mode choice



Alternative model structures