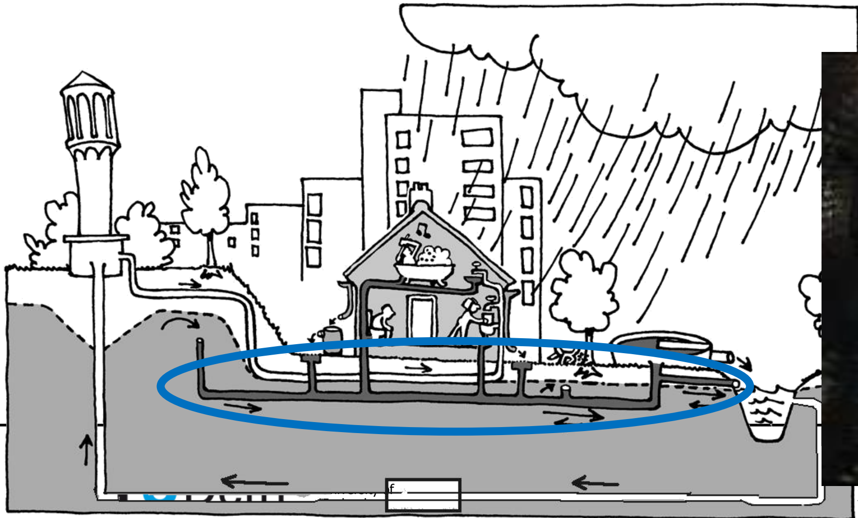
CIE4491

Lecture. Hydraulic design

Marie-claire ten Veldhuis

19-9-2013



Source: news.bbc.co.uk

Hydraulic design of urban stormwater systems

- Focus on sewer pipes

- Pressurized and part-full pipe flow
  - Saint-Venant equations and its simplifications
  - Differences between part-full and pipe flow
  - Part-full flow in pipes

- Local losses in sewer networks
  - Orifice
  - Gully pot, manhole
  - Hydraulic jump
  - Weir

Stormwater systems – design steps

- Quantify stormwater design flow rate
  - Run-off model, rational method

- Determine pipe capacity based on full pipe flow
  - Computationally easier
  - Max Capacity is reached at 85% filling rate
  - Actual capacity is only 5% greater than for fully pipe, but not stable

- Determine required D_h
- Select standard $D > D_h$

$$D_h = \frac{4A}{\Omega} = 4R_h$$

- Account for full pipe flow condition at maximum capacity
- Account for part-full flow for all other conditions (esp. combined systems – dry weather flow)

De Saint-Venant momentum equation

$$\underbrace{\frac{\partial Q}{\partial t}}_{I} + \underbrace{\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)}_{II} + \underbrace{g \cdot A \cdot \frac{\partial\left(y+z\right)}{\partial x}}_{III} + \underbrace{\frac{\tau}{\rho} \cdot \Omega}_{IV} = 0$$

Q, A, z, Ω, τ, ρ are discharge, flow area, bottom elevation, wet perimeter, wall shear stress and density.

y is the water depth (part-full flow) or pressure head (pipe flow)

Applicable for part-full flow and pressurized pipe flow!

It comprises 4 parts:

I. Advective acceleration (inertia)

II. Convective acceleration (changes in flow cross-section)

III. Gravity force

IV. Friction force

De Saint-Venant continuity equation

$$B\frac{\partial y}{\partial t}+\frac{\partial Q}{\partial x}=0$$

B is the surface width, y is water depth (part-full flow) or pressure head (pressurised flow)

- Part full flow: Internal storage at the free surface.
  Each element behaves like a storage tank with surface area B*Δx.
  The net discharge ΔQ is stored in the element.

- Full pipe flow: B = 0, pipe elasticity and liquid compressibility are neglected

→ $$\frac{\partial Q}{\partial x}=0$$

Typical applications of De Saint Venant terms momentum equations

a. Rainfall event (sewer network filling and draining)	
b. Rapid filling event at maximum hydraulic grade line	
c. Steady varied flow (in part-full pipe)	
d. Steady full pipe flow	
e. Part-full flow at normal depth	

Typical applications of De Saint Venant terms momentum equations

a. Rainfall event (sewer network filling and draining)	Complete De Saint-Venant equations: Terms I, II, II, IV
b. Rapid filling event at maximum hydraulic grade line	$\frac{\partial Q}{\partial t} + gA\frac{\partial (y+z)}{\partial x} = 0$ Terms I, III
c. Steady varied flow (in part-full pipe)	$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial (y+z)}{\partial x} + \frac{\tau}{\rho}\Omega = 0$ Terms II, III, IV
d. Steady full pipe flow	$gA\frac{\partial y}{\partial x} + \frac{\tau}{\rho}\Omega = 0 \quad , y > D$ Terms III, IV
e. Part-full flow at normal depth	$gA\frac{\partial z}{\partial x} + \frac{\tau}{\rho}\Omega = 0$ Terms III, IV

Typical applications of De Saint Venant terms

Rainfall event (sewer network filling and draining)	Complete De Saint-Venant equations
Rapid filling event at maximum hydraulic grade line	$\frac{\partial Q}{\partial t} + gA\frac{\partial (y+z)}{\partial x} = 0$
Steady varied flow (in part-full pipe)	$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial (y+z)}{\partial x} + \frac{\tau}{\rho}\Omega = 0$
Steady full pipe flow **Independent of bottom slope!**	$gA\frac{\partial y}{\partial x} + \frac{\tau}{\rho}\Omega = 0 \quad , y > D$
Part-full flow at normal depth **Friction slope=bottom slope!**	$gA\frac{\partial z}{\partial x} + \frac{\tau}{\rho}\Omega = 0$

Friction term relations

$$D_h = \frac{4A}{\Omega} = 4R_h$$

- Colebrook-White/Darcy-Weisbach
  - Typical values λ = 0.01 - 0.04

$$\Delta H = \lambda \frac{L}{D_h} . \frac{v^2}{2g}$$

- Darcy/Fanning friction factor (UK)
  - Typical values f = 0.003 - 0.012

$$\Delta H = f \frac{L}{R_h} . \frac{v^2}{2g} \qquad f = \frac{1}{4} . \lambda$$

- Chezy friction factor
  - Typical values C = 40 – 85

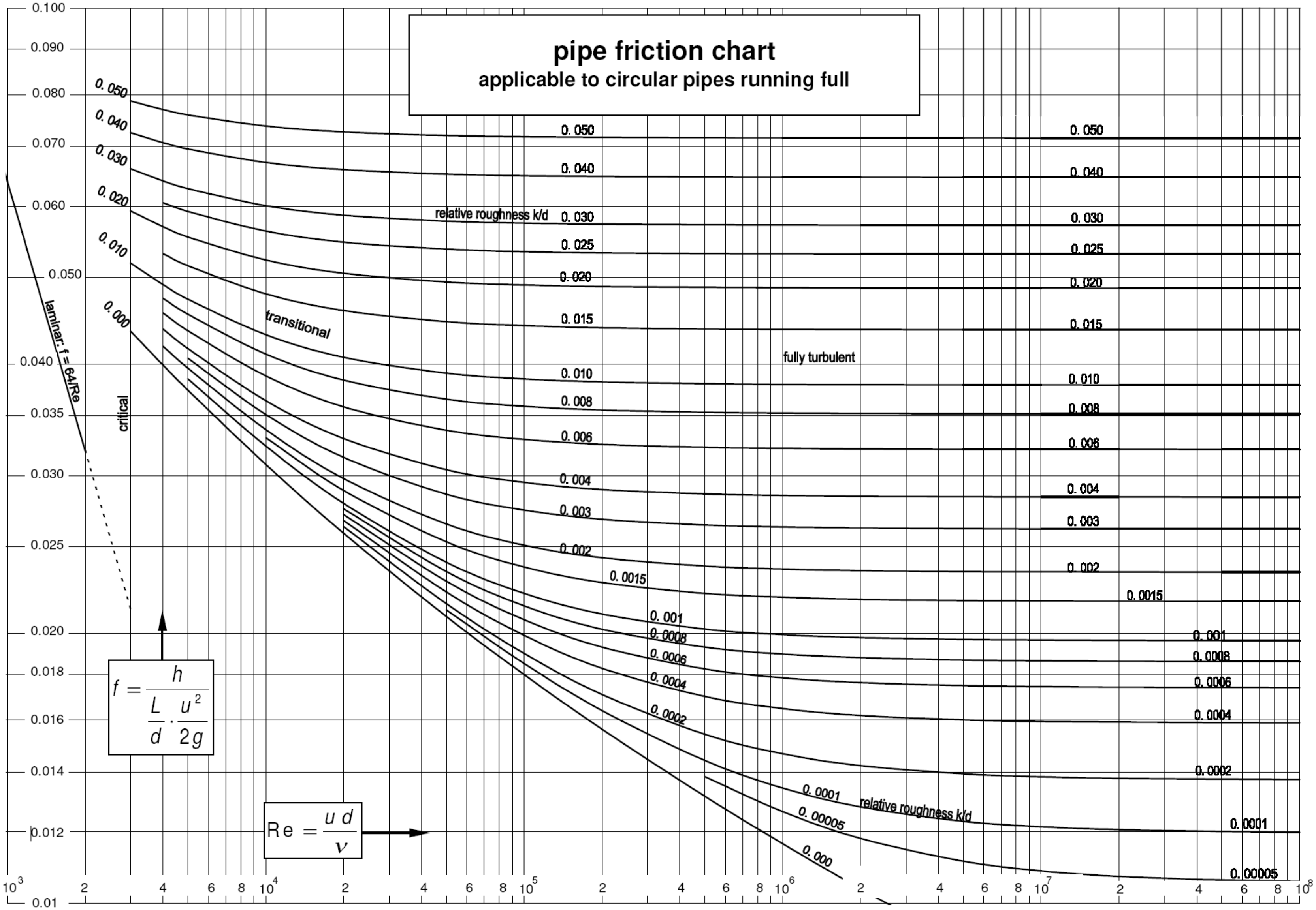
$$v = C\sqrt{R_h \frac{\Delta H}{L}} \qquad C = \sqrt{\frac{8g}{\lambda}}$$

- Manning
  - Typical values n = 0.009 - 0.022 (SI units)
  - n is dependent of units used

$$v = \frac{1}{n} R_h^{0.667} \left(\frac{\Delta H}{L} \right)^{0.5}$$

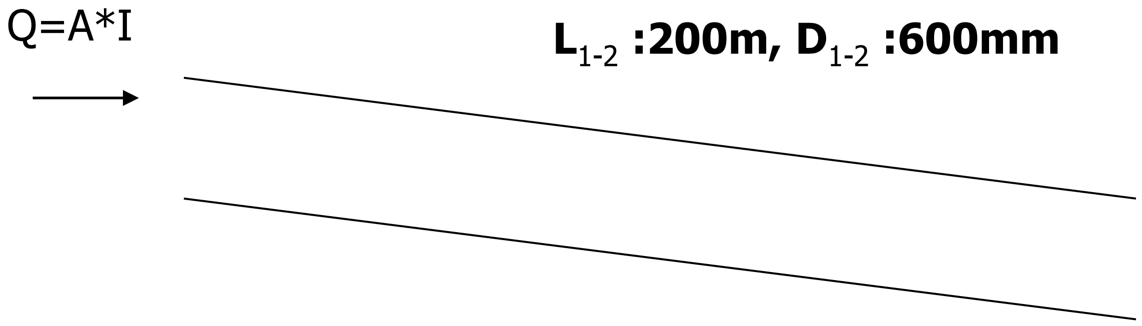
Wall shear stress correlations
(turbulent conditions)

Colebrook-White	$\tau = \frac{\lambda}{8}\rho v^2 \quad ; \quad \frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2{,}51}{Re\sqrt{\lambda}} + \frac{k}{3{,}71D}\right)$
Darcy/Fanning	$\tau = \frac{f}{2}\rho v^2$
Chezy	$\tau = \rho g \frac{v^2}{C^2}$
Manning	$\tau = \rho g \frac{v^2 n^2}{R_h^{1/3}}$


pipe friction chart
applicable to circular pipes running full
relative roughness k/d
fully turbulent
transitional
laminar: f = 64/Re
critical
$f = \frac{h}{\frac{L}{d} \cdot \frac{u^2}{2g}}$
$Re = \frac{u\,d}{\nu}$
0.050
0.040
0.030
0.025
0.020
0.015
0.010
0.008
0.006
0.004
0.003
0.002
0.0015
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.000
0.100
0.090
0.080
0.070
0.060
0.050
0.040
0.035
0.030
0.025
0.020
0.018
0.016
0.014
0.012
0.01
10^3 2 4 6 8 10^4 2 4 6 8 10^5 2 4 6 8 10^6 2 4 6 8 10^7 2 4 6 8 10^8

Hydraulic sewer design

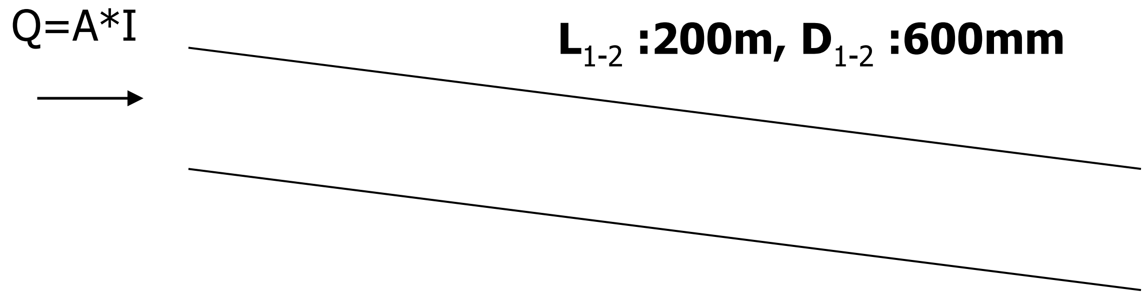
Given a sewer pipeline with the following dimensions:



Connected surface area: 2 ha (70% impervious)
Design rainfall intensity: 90 l/s/ha

Hydraulic sewer design

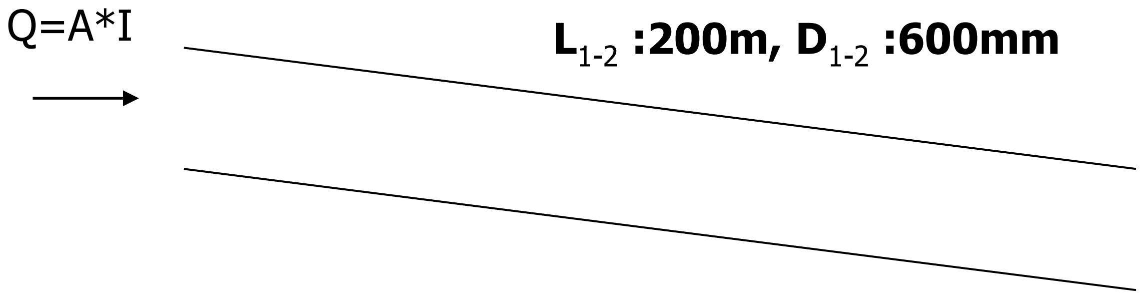
Question: what is the design discharge?



Connected surface area: 2 ha (70% impervious)
Design rainfall intensity: 90 l/s/ha

Hydraulic sewer design

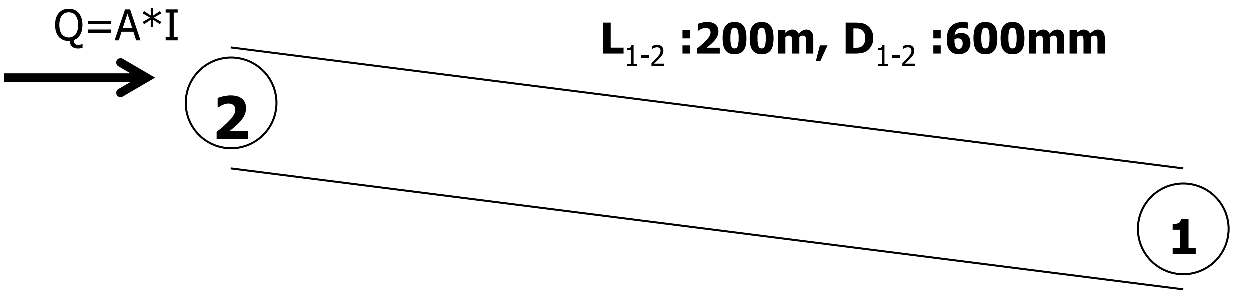
Answer: $Q=A*I*C_{runoff}=2*0.7*90=126$ l/s or $0.126 m^3/s$



Connected surface area: 2 ha (70% impervious)
Design rainfall intensity: 90 l/s/ha

Hydraulic sewer design

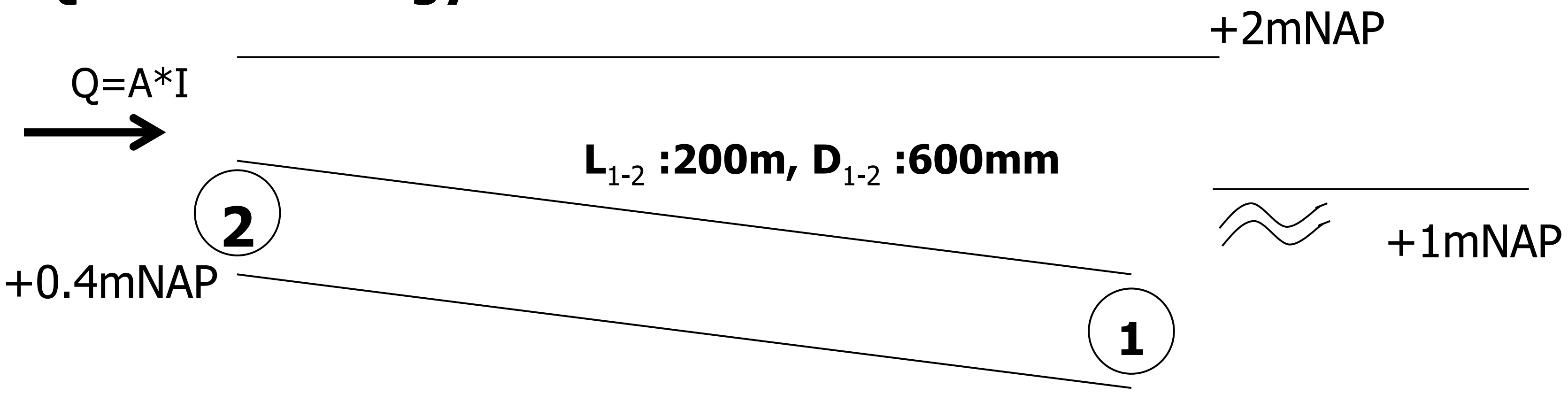
Question: Calculate the energy levels in nodes 1 and 2?



Connected surface area: 2 ha (70% impervious)
Design rainfall intensity: 90 l/s/ha

Hydraulic sewer design

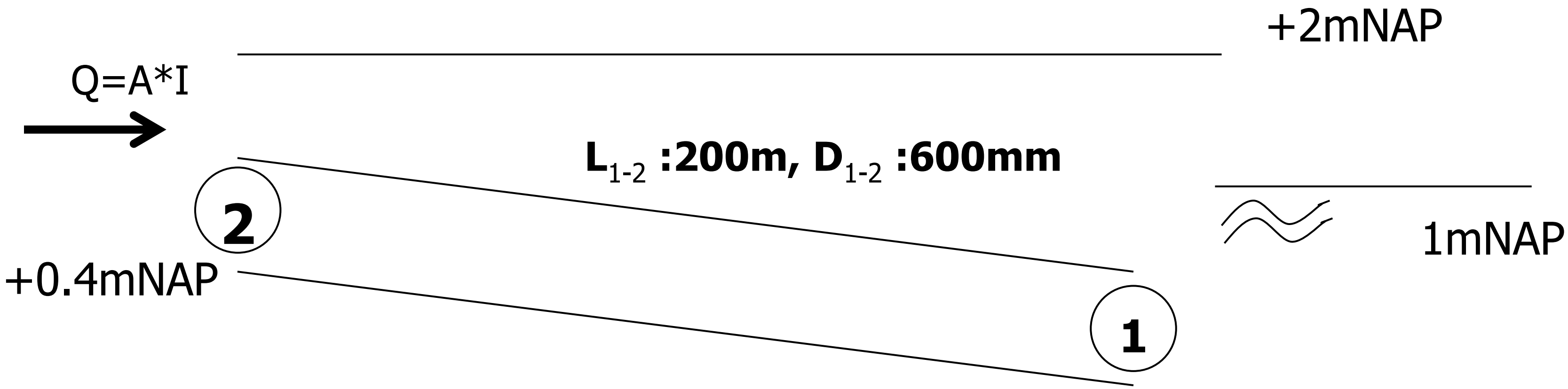
Question: energy levels in nodes 1 and 2?



Connected surface area: 2 ha (70% impervious)
Design rainfall intensity: 90 l/s/ha
⇒ **Additional data:**
Groundlevel: +2mNAP; Waterlevel at outflow: +1mNAP

Hydraulic sewer design

Answer: energy levels in nodes 1 and 2



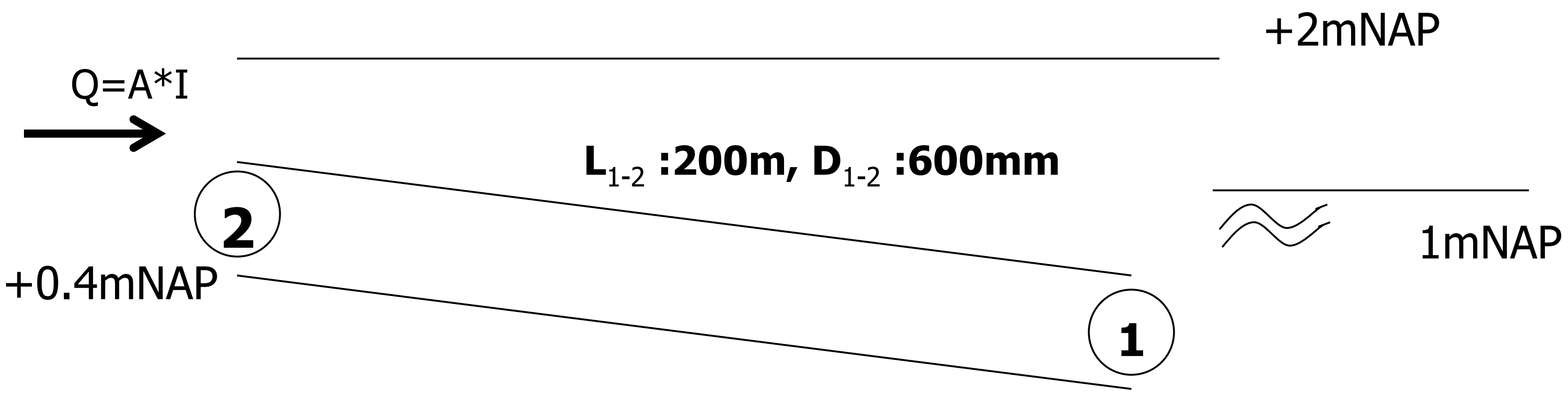
Note: pipe outflow below surface water level ⇒ Flow condition:
full pipe, pressurised flow

⇒ Apply Darcy Weisbach: $dH = \lambda \frac{L}{D} \frac{v^2}{2g}$; λ=0.02[-]; A=0.28m^2

(De Saint-Venant terms III and IV: gravity and friction)

Hydraulic sewer design

Answer: energy levels in nodes 1 and 2



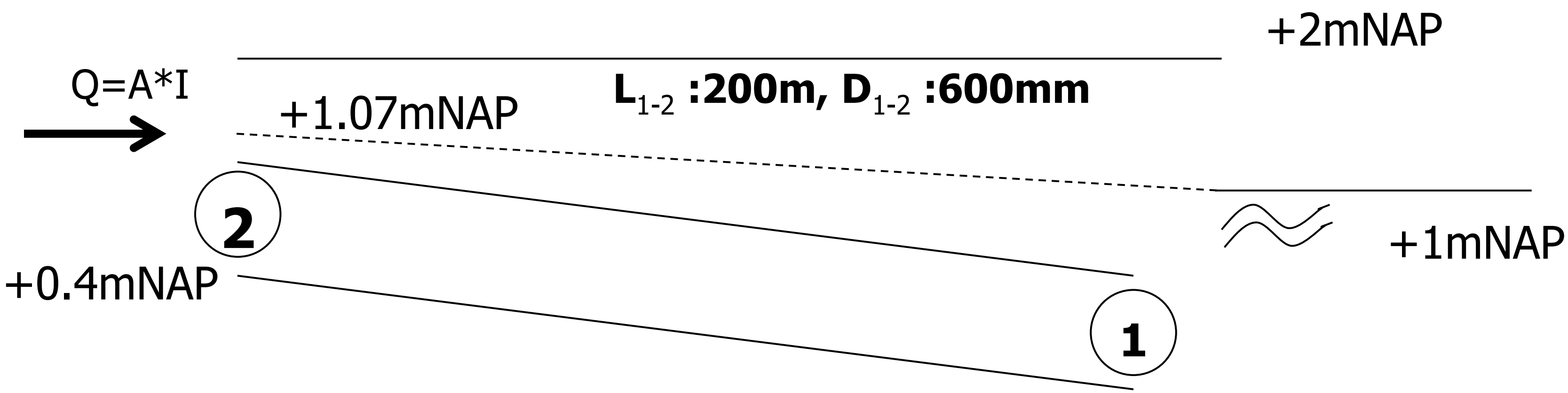
Apply Darcy Weisbach: $dH = \lambda \frac{L}{D}\frac{v^2}{2g}$; λ=0.02[-]; A=0.28m^2; v=0,45m/s

$\Rightarrow$ dH=0,07m

Energy levels: ?

Hydraulic sewer design

Answer: energy levels in nodes 1 and 2

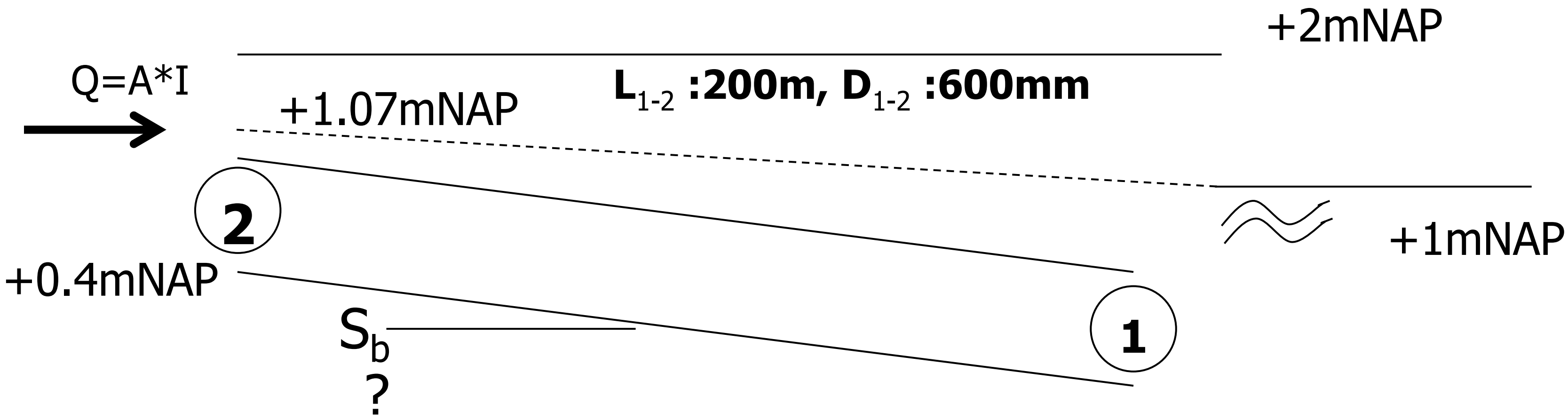


Energy loss according to Darcy Weisbach: dH=0,07m

Energy levels: node 1: H_1=+1mNAP; node 2: H_2=+1.07m

Hydraulic sewer design

NB: we have not defined a bottom slope!
Is this relevant?



Energy loss according to Darcy Weisbach: dH=0,07m
Energy levels: node 1: H_1=+1mNAP; node 2: H_2=+1.07m

Hydraulic sewer design, part-full flow

Question: What is the expected water depth in the sewer?

Given following conditions :

Flow rate = 640 m3/h

Sewer diameter = 500 mm

Slope = 1:200

Friction Factor = 0.015

Hydraulic sewer design, part-full flow

Question: What is the expected water depth in the sewer?

Given following conditions :
Flow rate = 640 m3/h
Sewer diameter = 500 mm
Slope = 1:200
Friction Factor = 0.015

Check: flow rate for full pipe flow conditions, assuming flow at normal depth

Hydraulic sewer design, part-full flow

Answer: step 1, determine full-pipe flow

Given following conditions :
Flow rate = 640 m3/h
Sewer diameter = 500 mm
Slope = 1:200
Friction Factor = 0.015

$$\frac{dH}{L} = \lambda \frac{v^2}{2gD}$$

From Darcy-Weisbach, assuming flow at normal depth (hydraulic gradient equals bottom slope 1:200):
v_{full} = 1.81 m/s; Q_{full} = 1280 m3/h

Note: part-full flow conditions for given flow rate (640 m3/h)

Part-full geometric formulae

Water depth y, angle α

$$\cos\alpha = 1 - \frac{y}{R}$$

Width water surface, B

$$B = 2R\sin\alpha = 2R\sqrt{2\frac{y}{R} - \left(\frac{y}{R}\right)^2}$$

Wet area, A

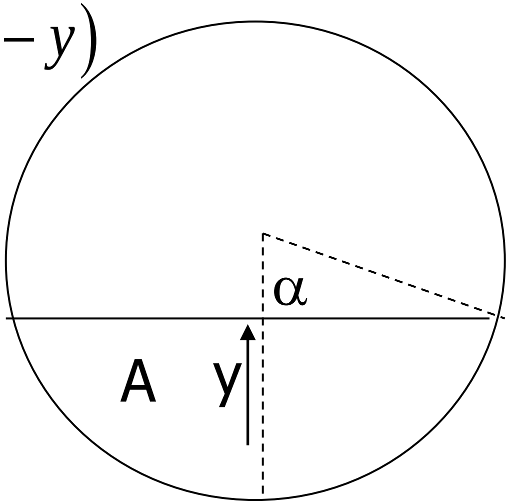
$$A = \alpha R^2 - \frac{B}{2}\left(R - y\right)$$

Wetted perimeter, Ω

$$\Omega = 2\alpha R$$

Hydraulic diameter, D_h

$$D_h = \frac{4A}{\Omega}$$



Part-full vs full pipe geometric relations

Wet area ratio

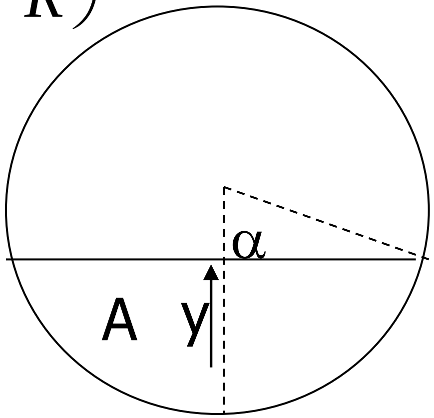
$$\frac{A}{A_f} = \frac{1}{\pi}\left[\cos^{-1}\left(1-\frac{y}{R}\right) - \frac{B}{D}\left(1-\frac{y}{R}\right)\right]$$

Wetted perimeter ratio

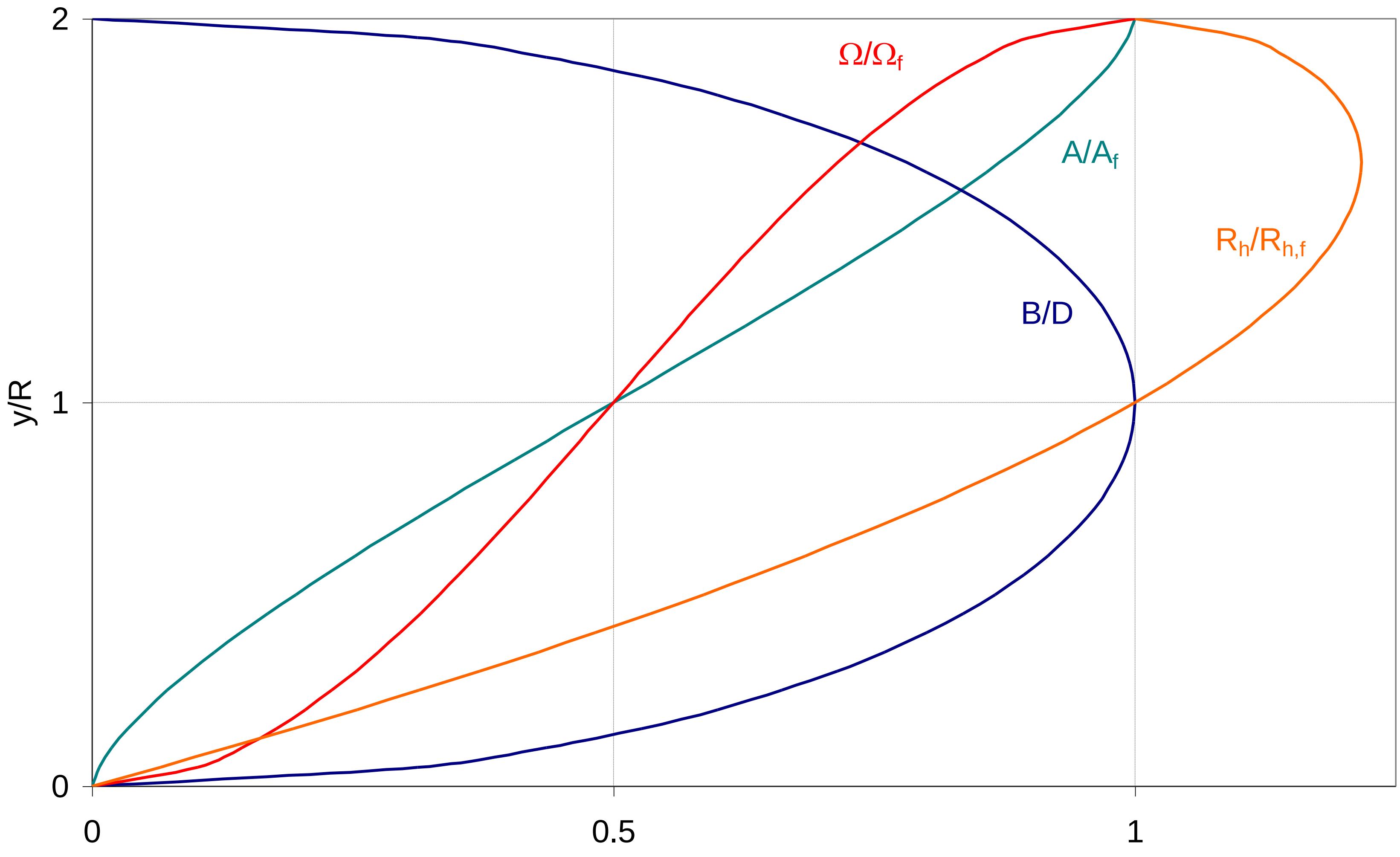
$$\frac{\Omega}{\Omega_f} = \frac{1}{\pi}\cos^{-1}\left(1-\frac{y}{R}\right)$$

Hydraulic diameter ratio

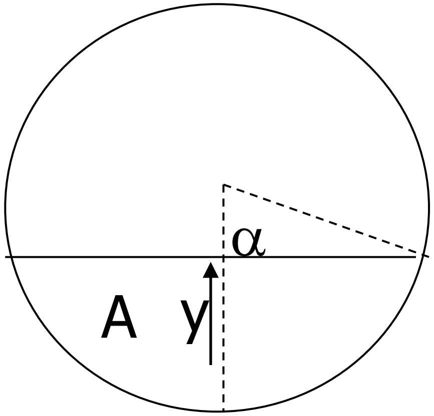
$$\frac{R_h}{R_{h,f}} = \frac{A/A_f}{\Omega/\Omega_f}$$



Part-full vs full pipe flow



Part-full vs full pipe flow properties



Assuming:

- Constant dH/dx equal to bottom slope
- Normal flow in part-full situation
- Constant pipe friction factor

Velocity ratio

$$\frac{v}{v_f} = \sqrt{\frac{R_h}{R_{h,f}}}$$

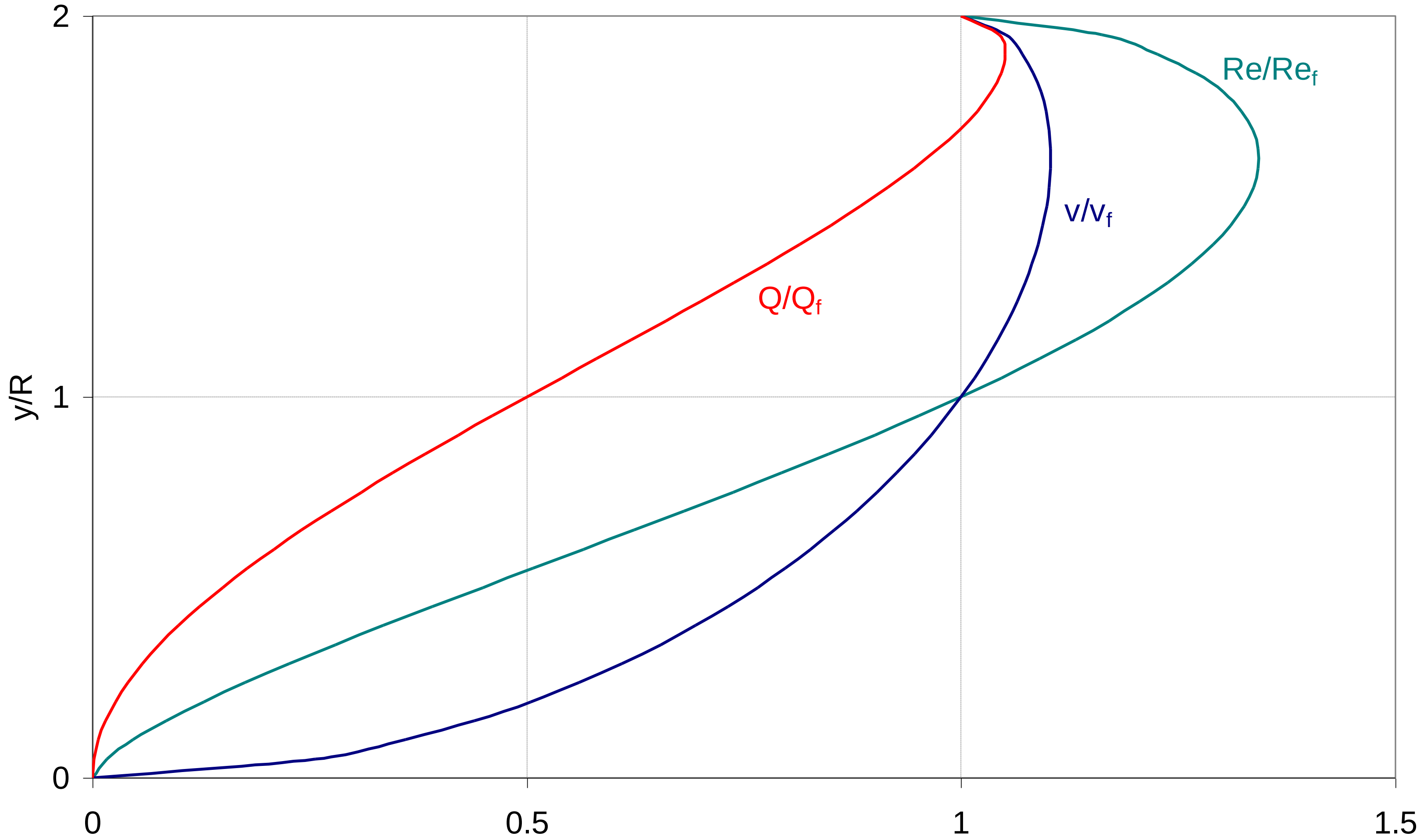
Discharge ratio

$$\frac{Q}{Q_f} = \frac{A}{A_f}\sqrt{\frac{R_h}{R_{h,f}}}$$

Reynolds number ratio

$$\frac{\mathrm{Re}}{\mathrm{Re}_f} = \frac{R_h}{R_{h,f}}\sqrt{\frac{R_h}{R_{h,f}}}$$

Part-full vs pipe flow properties



Hydraulic sewer design, part-full flow

Answer: step 2, determine filling rate Q/Q_f

Given following conditions :

Flow rate = 640 m3/h

Sewer diameter = 500 mm

Slope = 1:200

Friction Factor = 0.015

$$\frac{dH}{L} = \lambda \frac{v^2}{2gD}$$

From Darcy-Weisbach, assuming flow at normal depth (hydraulic gradient equals bottom slope 1:200):

v_{full} = 1.81 m/s; Q_{full} = 1280 m3/h

Q / Q_f = 0.5. Pipe is half full - use: Q/Q_f in previous slides

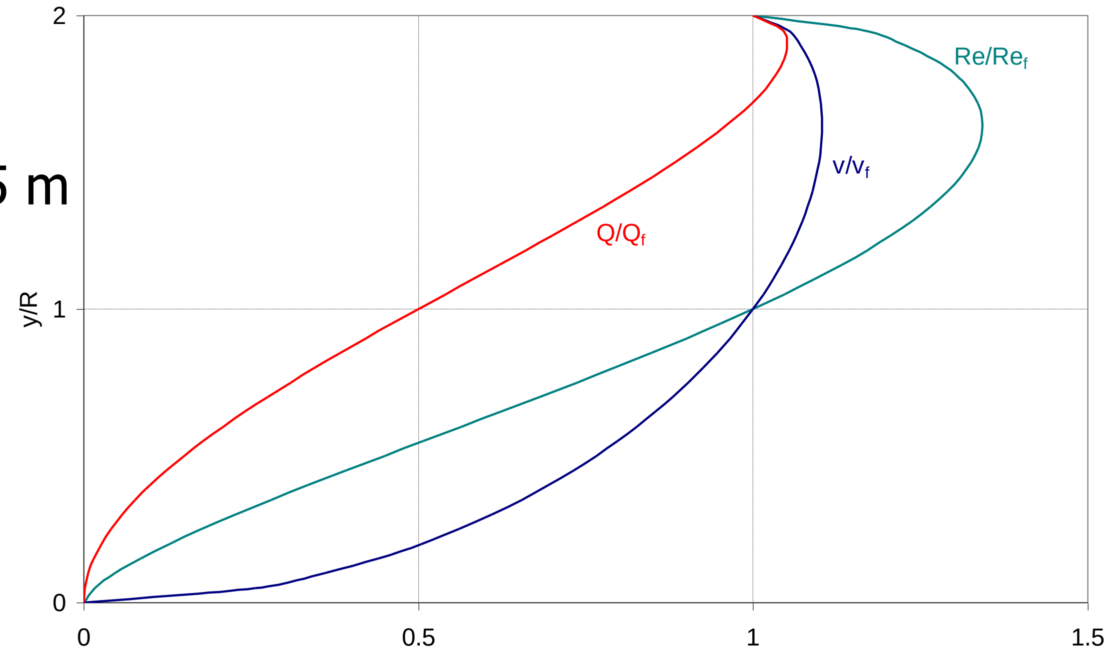
Hydraulic sewer design, part-full flow

Answer: step 3, determine flow depth in sewer

Pipe is half full:
Flow depth = 0.5D = 0.25 m

Flow velocity:
v_{full} = 1.81 m/s;
Q_{full} = 1280 m3/h

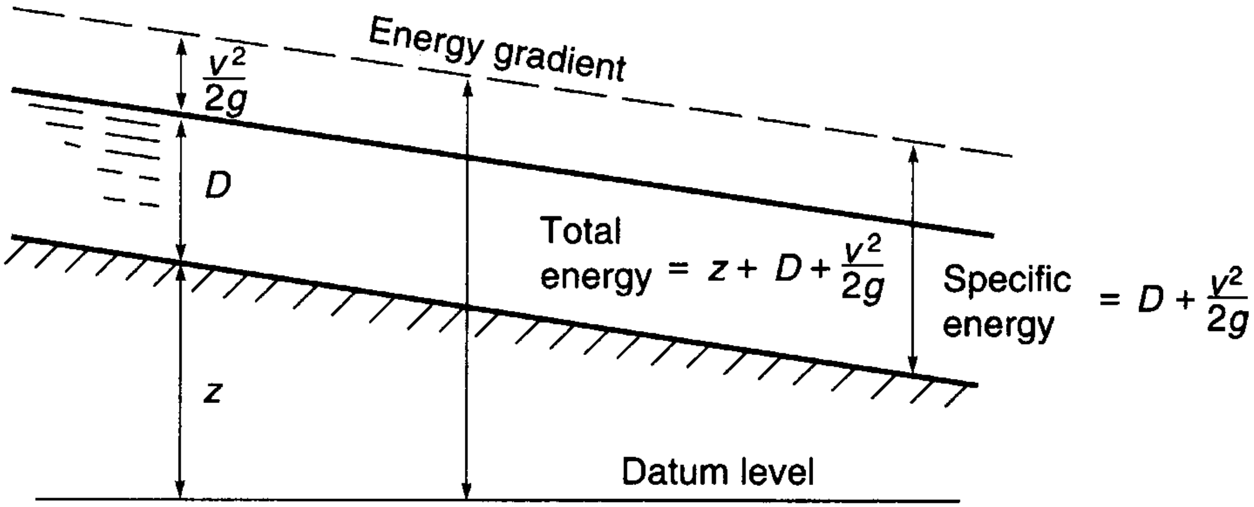
Q / Q_f = 0.5.
v / v_f = 1; v = 1.81 m/s



Hydraulic sewer design, special structures (weirs, valves etc.)

- Specific energy E: water depth + velocity head
- Specific energy E defined for free-surface flows
- Specific energy is energy relative to the bottom/bed

$$E = \frac{v^2}{2g} + y$$



Relevance of specific energy

E drives steady, varied flow equation

Bottom slope $-S_b$

$$\frac{\partial}{\partial x}\left(\frac{v^2}{2g}\right)+\frac{\partial y}{\partial x}+\frac{\partial z}{\partial x}+\frac{\tau}{\rho}\frac{\Omega}{gA}=0 \quad \longleftrightarrow \quad \frac{\partial}{\partial x}\left(\frac{v^2}{2g}+y\right)=S_b-S_f$$

Friction slope S_f

At a given discharge, E is minimum at the critical depth $\longleftrightarrow$ $\left.\frac{\partial E}{\partial y}\right|_{y_c}=0$

At a given E, the discharge is maximum at the critical depth $\longleftrightarrow$ $\left.\frac{\partial Q}{\partial y}\right|_{y_c}=0$

E in rectangular and circular conduit

$$\left.\frac{\partial E}{\partial y}\right|_{y_c} = 0$$

$$\frac{dE}{dy} = \frac{-Q^2}{gA^3}\frac{dA}{dy} + 1 = 0 \qquad A_c = y_c B_c$$

Rectangular conduit

Circular conduit

$$v_c = \sqrt{g y_c}$$

$$Fr_c = 1$$

$$v_c = \sqrt{g \frac{A_c}{B_c}}$$

$$E_{\min} = \frac{3}{2} y_c$$

$$E_{\min} = \frac{A_c}{2B_c} + y_c$$

Shooting Flow, Tranquil Flow

Critical depth marks transition from tranquil to shooting flow
At critical depth Fr = 1

If $y > y_c$ then: tranquil flow or subcritical flow

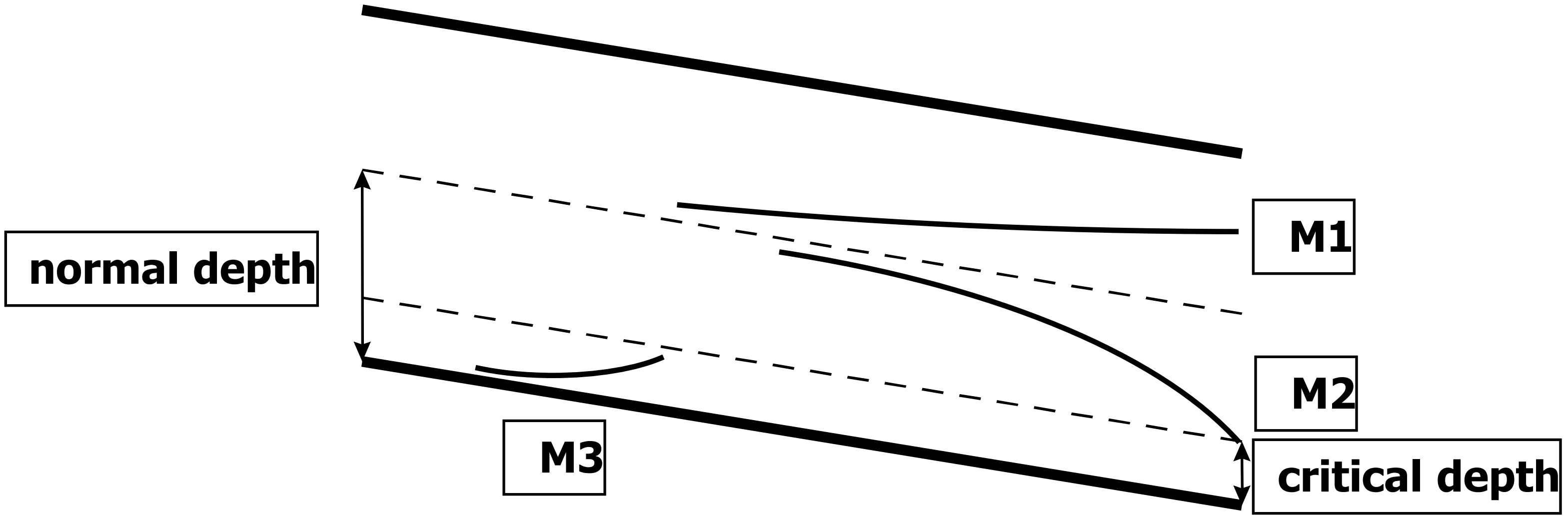
Information can be propagated upstream and downstream

If $y < y_c$ then: shooting flow or supercritical flow

Information can only be propagated downstream

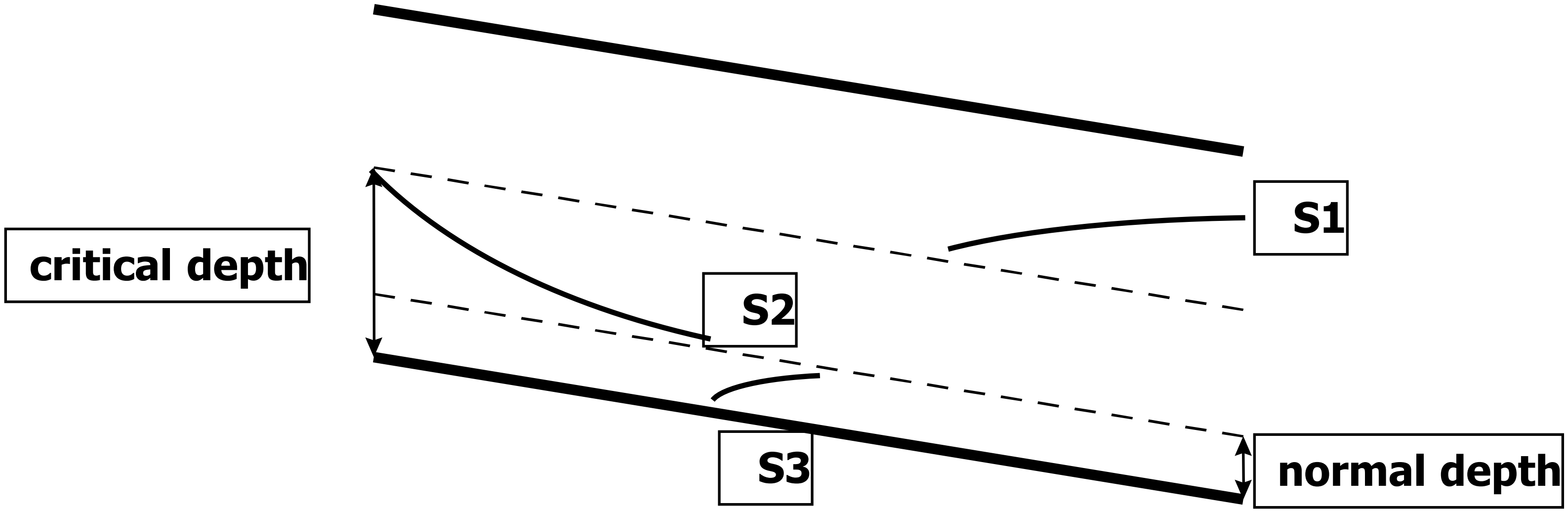
Varied flow profiles (mild slope)

- Mild slope: $y_n > y_c$
- M1 backwater curve (during filling of pump pit and sewers)
- M2 drawdown curve (during dry-weather-flow towards pit)

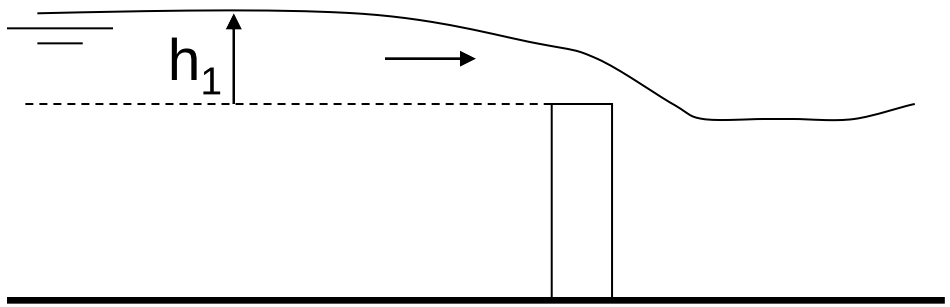


Varied flow (steep slopes)

- Steep slope $y_n < y_c$
- S1 approaches the horizontal
- S2 drawdown curve in steep slope (transition mild → steep)
- S3 may occur downstream of a gate valve in a steep slope



Weir

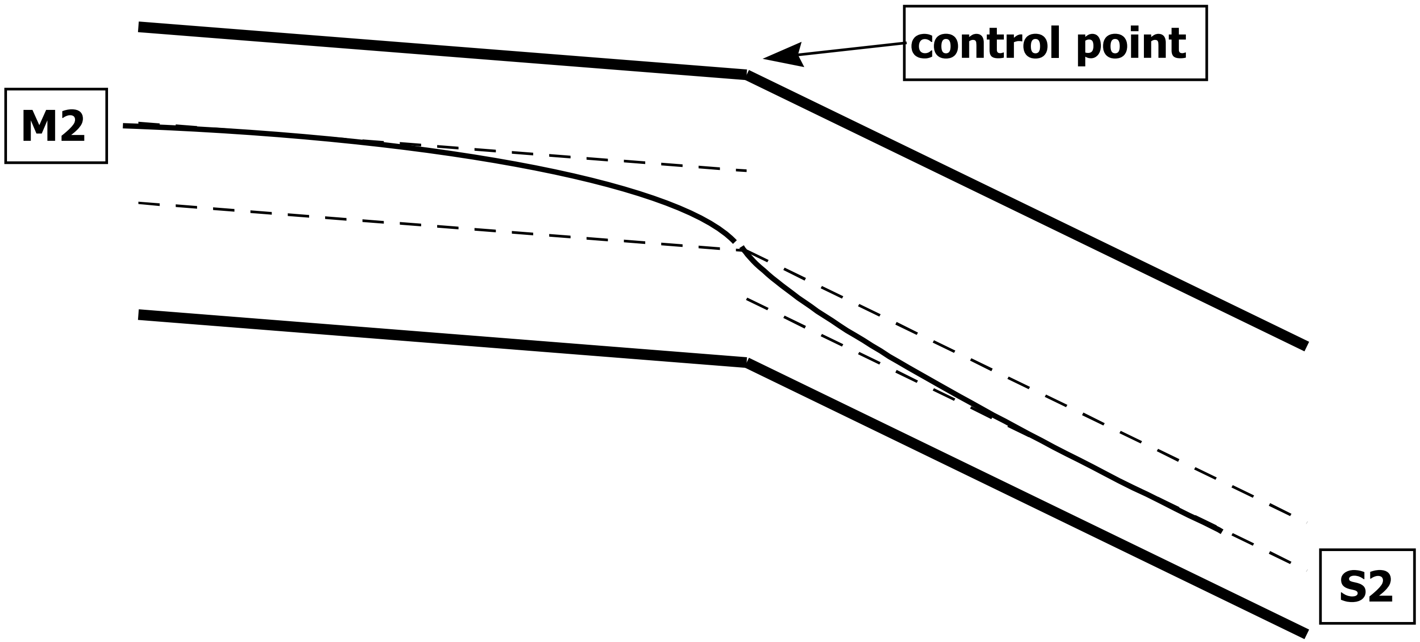


Water levels h_1 always relative to weir crest

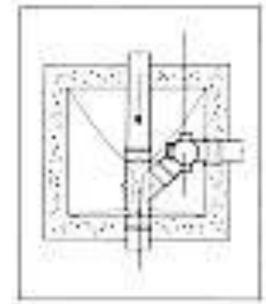
- short crested weir $$Q = \frac{2}{3} b \sqrt{2g} \cdot h_1^{3/2}$$
- Broad-crested weir (critical) $$Q = 1.7 \cdot b \cdot C \cdot h_1^{3/2} \quad , h_2 < \frac{2}{3} h_1$$
- Broad-crested weir (subcritical) $$Q = b \cdot C \cdot h_2 \sqrt{2g\left(h_2 - h_1\right)} \quad , h_2 \geq \frac{2}{3} h_1$$

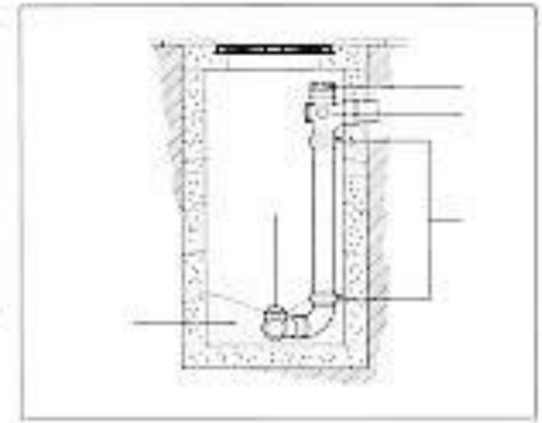
Weir

Depth at control point determines discharge (critical flow)



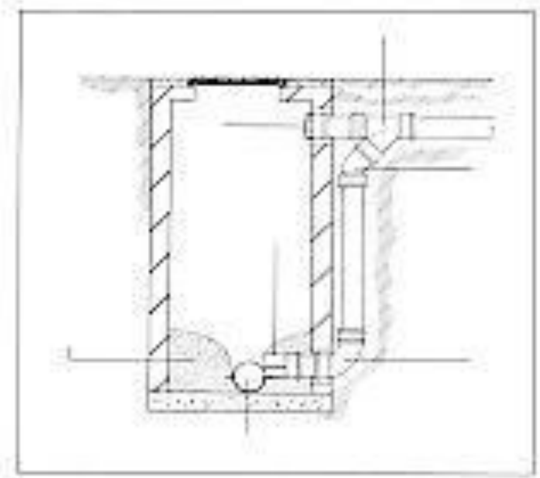
Manhole





Entrance point
Multiple pipes connected
Vertical transport

Typical head losses
(for smooth bottom dz = 0)



Inflow from manhole $$h_L \approx 0.5\,\frac{V^2}{2g}$$

Outflow from pipe into manhole $$h_L \approx \frac{V^2}{2g}$$

Local losses in sewer networks

Local losses are NOT minor losses in sewer networks

General concept of deceleration losses

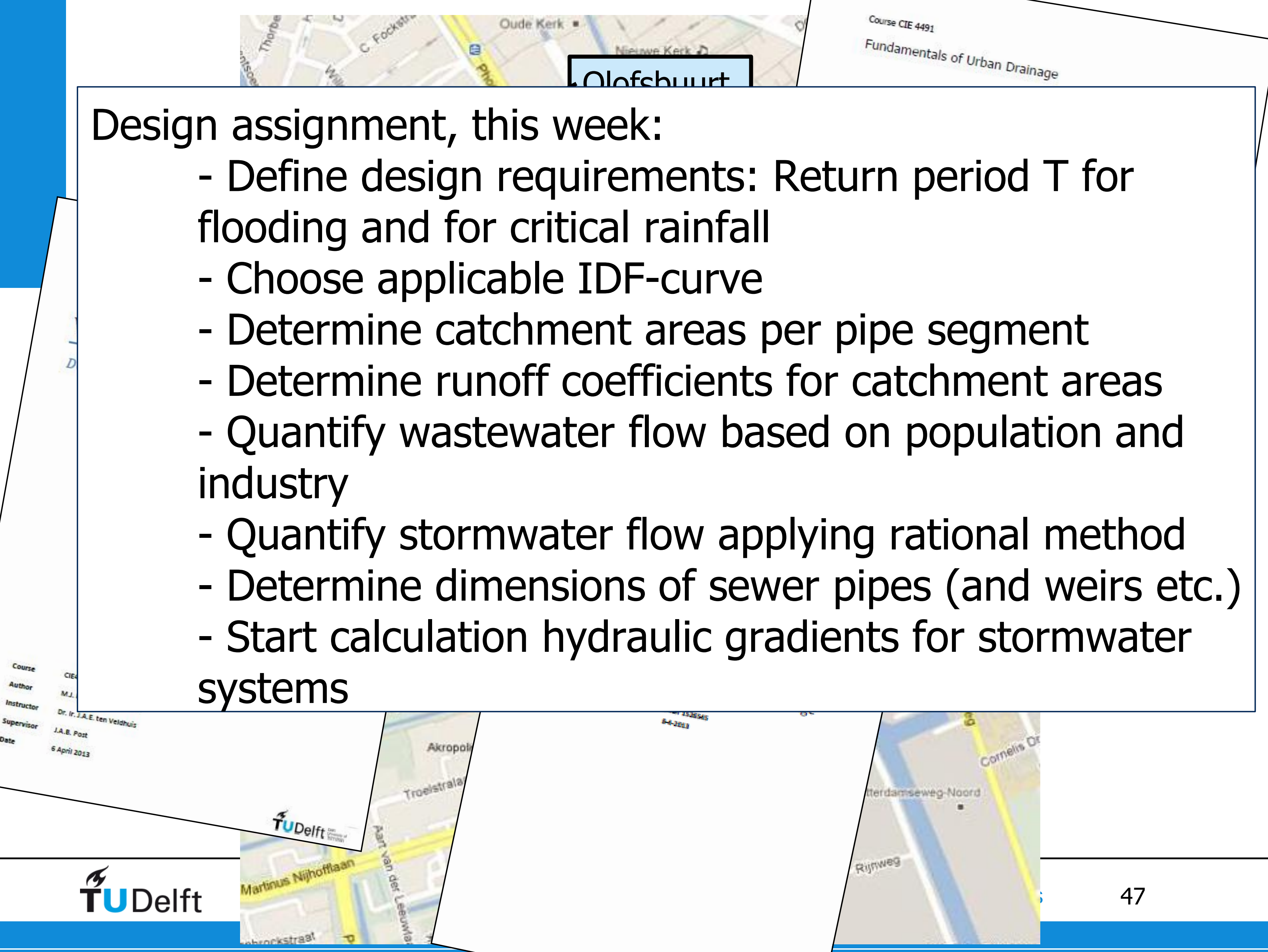
- No energy loss in accelerating flow → Bernoulli
- The energy is lost in decelerating flow → Momentum balance

Example: contribution of local losses

Question: What is the relative contribution from the local manhole losses to the total head loss?

If:

- 1 manhole every 50 metres; local loss factor k = 1
- Sewer diameter 500 mm, flow velocity 1 m/s
- friction factor 0.015



Design assignment, this week:

- Define design requirements: Return period T for flooding and for critical rainfall
- Choose applicable IDF-curve
- Determine catchment areas per pipe segment
- Determine runoff coefficients for catchment areas
- Quantify wastewater flow based on population and industry
- Quantify stormwater flow applying rational method
- Determine dimensions of sewer pipes (and weirs etc.)
- Start calculation hydraulic gradients for stormwater systems