2. Motion Response in (ir)regular waves
Offshore Hydromechanics, lecture 1

Teacher module II:
• Ir. Peter Naaijen
• p.naaijen@tudelft.nl
• Room 34 B-0-360 (next to towing tank)

Book:
• Offshore Hydromechanics, by J.M.J. Journee & W.W. Massie

Useful weblinks:
• http://www.shipmotions.nl
• Blackboard

Take your laptop, I- or whatever smart-phone and go to:
www.rwpoll.com
Login with session ID

OE4630 module II course content
• +/- 7 Lectures
• Bonus assignments (optional, contributes 20% of your exam grade)
• Laboratory Exercise (starting 30 nov)
  • 1 of the bonus assignments is dedicated to this exercise
  • Groups of 7 students
  • Subscription available soon on BB
• Written exam
Learning goals Module II, behavior of floating bodies in waves

- Definition of ship motions
  - Motion Response in regular waves:
    - How to use RAO's
    - Understand the terms in the equation of motion: hydrodynamic reaction forces, wave exciting forces
  - Motion Response in irregular waves:
    - How to determine response in irregular waves from RAO's and wave spectrum without forward speed
- Motion Potential Theory:
  - How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential
  - How to determine Velocity Potential

Motion Response in irregular waves:
- How to determine response in irregular waves from RAO's and wave spectrum with forward speed
- Make down time analysis using wave spectra, scatter diagram and RAO's

Structural aspects:
- Calculate internal forces and bending moments due to waves
- Calculate mean horizontal wave force on wall
- Use of time domain motion equation

Introduction

Offshore oil resources have to be explored in deeper water floating structures instead of bottom founded
Introduction

- the dynamic loads on the floating structure, its cargo or its equipment:
- Inertia forces on sea fastening due to accelerations.
- Direct wave induced structural loads

Reasons to study waves and ship behavior in waves:
- Determine allowable / survival conditions for offshore operations
- Decommissioning / Installation / Pipe laying: Excalibur / Allseas 'Pieter Schelte'
- Motion Analysis

Minimum required air gap to avoid wave damage
Introduction

Reasons to study waves and ship behavior in waves:
- the dynamic loads on the floating structure, its cargo or its equipment:
  - Forces on mooring system, motion envelopes loading arms

Floating Offshore: More than just oil

Wave energy conversion

Floating wind farm

OTECE

Mega Floaters
Introduction

Reasons to study waves and ship behavior in waves:
- Determine allowable / survival conditions for offshore operations
- Downtime analysis

Real-time motion prediction
Using X-band radar remote wave observation

Definitions & Conventions

Regular waves
Ship motions

apparently irregular but can be considered as a superposition of a finite number of regular waves, each having own frequency, amplitude and propagation direction
Regular waves

Regular waves

regular wave propagating in direction $\mu$: 

$$(t, x) = \cos t \ k_x \cos k_y \sin$$

Linear solution Laplace equation

Co-ordinate systems

Definition of systems of axes

Earth fixed: $(x_0, y_0, z_0)$

wave direction with respect to ship's exes system:

Phase angle wave at black dot with respect to wave at red dot:

$\cos k_x x \cos k_y y \sin$
Behavior of structures in waves
Ship's body bound axes system ($x_b, y_b, z_b$) follows all ship motions

Behavior of structures in waves
Definition of rotations

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<tr>
<td>5</td>
<td>y</td>
<td>Stampen Pitch</td>
</tr>
<tr>
<td>6</td>
<td>z</td>
<td>Gieren Yaw</td>
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How do we describe ship motion response?
Rao's Phase angles

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<tr>
<td>1</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>y</td>
</tr>
<tr>
<td>3</td>
<td>z</td>
</tr>
</tbody>
</table>
Mass-Spring system:

\[ m \ddot{z} + b \dot{z} + cz = F_x \cos t \]

Motion equation

\[ z(t) = z_c \cos t \]

Steady state solution

Motions of and about COG

\[ \text{Amplitude Phase angle} \]

\[ \begin{align*}
\text{Surge (schrikken)} : & \quad x \quad \omega_x \cos t \\
\text{Sway (verzetten)} : & \quad y \quad \omega_y \cos t \\
\text{Heave (dompen)} : & \quad z \quad \omega_z \cos t \\
\text{Roll (rollen)} : & \quad \phi \quad \omega \cos t \\
\text{Pitch (stapen)} : & \quad \theta \quad \omega \cos t \\
\text{Yaw (gieren)} : & \quad \psi \quad \omega \cos t
\end{align*} \]

Phase angles are related to undisturbed wave at origin of steadily translating ship-bound system of axes (COG)

RAO and phase depend on:

- Wave frequency
- Wave direction
Consider Long waves relative to ship dimensions

What is the RAO of pitch in head waves?

- Phase angle heave in head waves?
- RAO pitch in head waves?
- Phase angle pitch in head waves?
- Phase angle pitch in following waves?

Example: roll signal

\[ \phi = \phi_a \cos(\omega_e t + \epsilon_\phi) \]

\[ \phi' = -\omega \phi \sin(\omega t + \epsilon_\phi) = \omega \phi \cos(\omega t + \epsilon_\phi + \pi / 2) \]

\[ \phi'' = -\omega^2 \phi \cos(\omega t + \epsilon_\phi + \pi / 2) \]

Displacement:
\[ \phi_a \]

Velocity:
\[ \phi' \]

Acceleration:
\[ \phi'' \]

Motions of and about COG

1. Surge (translation):
   \[ s = x_a \cos(\omega_e t + \epsilon_s) \]

2. Sway (translation):
   \[ y = y_a \cos(\omega_e t + \epsilon_s) \]

3. Heave (translation):
   \[ z = z_a \cos(\omega_e t + \epsilon_s) \]

4. Roll (rotation):
   \[ \phi = \phi_a \cos(\omega_e t + \epsilon_\phi) \]

5. Pitch (rotation):
   \[ \theta = \theta_a \cos(\omega_e t + \epsilon_\theta) \]

6. Yaw (rotation):
   \[ \psi = \psi_a \cos(\omega_e t + \epsilon_\psi) \]

- Frequency of input (regular wave) and output (motion) is ALWAYS THE SAME!!
- Phase can be positive! (motion ahead of wave elevation at COG)
- Due to symmetry: some of the motions will be zero
- Ratio of motion amplitude / wave amplitude = RAO (Response Amplitude Operator)
- RAO's and phase angles depend on wave frequency and wave direction
- RAO's and phase angles must be calculated by dedicated software or measured by experiments
- Only some special cases in which 'common sense' is enough:

Consider very long waves compared to ship dimensions

What is the RAO for heave in head waves?

A. 0
B. \( \infty \)
C. 1
D. 42
Consider very long waves compared to ship dimensions

What is the phase for heave in head waves?
A. 0 deg
B. 180 deg
C. 90 deg
D. I have no clue

What is the RAO for pitch in head waves?
A. $\infty$ rad/m
B. 1 rad/m
C. 0 rad/m
D. k rad/m
E. I have no clue
Consider very long waves compared to ship dimensions

What is the phase for pitch in head waves?

A. 0 deg
B. 180 deg
C. -90 deg
D. 90 deg
E. I have no clue again

Local motions (in steadily translating axes system)
- Only variations
- Linearized

\[
\begin{pmatrix}
    x_p(t) \\
    y_p(t) \\
    z_p(t)
\end{pmatrix} = \begin{pmatrix}
    0 & -\psi(t) & \theta(t) \\
    \psi(t) & 0 & -\phi(t) \\
    -\theta(t) & \phi(t) & 0
\end{pmatrix} \begin{pmatrix}
    x_{bp} \\
    y_{bp} \\
    z_{bp}
\end{pmatrix}
\]

For a frequency \(\omega = 0.6\) the RAO's and phase angles of the ship motions are:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
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<tbody>
<tr>
<td>RAO</td>
<td>1.014</td>
<td>3.421</td>
<td>5.992</td>
<td>2.811</td>
<td>9.991</td>
<td>3.580</td>
</tr>
<tr>
<td>Phase</td>
<td>degr</td>
<td>degr</td>
<td>degr</td>
<td>deg/m</td>
<td>degr</td>
<td>deg/m</td>
</tr>
</tbody>
</table>

Calculate the RAO and phase angle of the transverse horizontal motion (y-direction)
Complex notation of harmonic functions

1 Surge (schrikken): \( x = x_a \cos(\omega t + \phi_x) \)
   \[ = \text{Re}\left( x_a e^{(\omega t + \phi_x)} \right) \]
   \[ = \text{Re}\left( x_a e^{i\omega t} e^{i\phi_x} \right) \]
   Complex motion amplitude
   \[ = \text{Re}\left( x_a e^{i\phi_x} \right) \]

Relation between Motions and Waves

How to calculate RAO’s and phases?

Input: regular wave, \( \omega \)
Output: regular motion \( u_0, \text{RAO}, \text{phase} \)

Floating Structure

Mass-Spring system:

Forces acting on body:

\[ \mathbf{F} = - \mathbf{K} \mathbf{z} - \mathbf{C} \dot{\mathbf{z}} - \mathbf{D} \mathbf{z} \]

Transient solution:

\[ z(t) = z_a e^{i\omega t} \sin \left( \sqrt{\frac{b}{mc}} t + \phi \right) \]

\[ \left[ z = \frac{b}{\sqrt{mc}} \right] \] Damping ratio

Steady state solution:

\[ z(t) = z_a \cos(\omega t + \phi) \]

\[ E = \frac{(-b\omega)}{(-b\omega)^2 + (2mc\omega)^2)} \]

\[ z_a = \frac{E}{\sqrt{-(m\omega^2 + c^2)}} \]
Roll restoring

Roll restoring coefficient:

\[ c_4 = \rho g \nabla \cdot GM \]

What is the point the ship rotates around statically speaking? (Ch 2)
Moving ship in waves:

- Mass-spring system: $m \ddot{\phi} + b \dot{\phi} + c \phi = F_a \cos(\omega t)$

- Restoring coefficient for roll: $G G G \phi$

- Rotation around COF: $F$

- Rotation around COG: $G$

- Vertical translation: $dz = FG - F G \cos \phi \approx 0$

- Horizontal translation: $dy = FG \sin \phi - FG \phi \approx 0$

Floating stab.
Stability moment

$M = \rho g \nabla \cdot G Z = \rho g \nabla \cdot GM \sin \phi \approx \rho g \nabla \cdot GM \cdot \phi$

Moving ship in waves: Not in air but in water!

$F = m_\text{z}$

- $F_a$
- $c \cdot z$
- $b \cdot z$
- $a \cdot z$

$(m + a) \ddot{z} + b \dot{z} + c \cdot z = F_a$

Moving analogy / differences with mass-spring system:

- External force $F(\ell)$: Wave exciting force
- Restoring force $c^2 z$: Archimedes' buoyancy
- Damping force $b \ddot{z} / \ell$: Hydrodynamic damping
- Inertia force $m_\ell \ddot{\ell}$: Mass + Hydrodynamic Mass

Depend on frequency!
Moving ship in waves:

\[(m + a) \ddot{z} + b \dot{z} + c \cdot z = F_w\]

Hydromechanic reaction forces

Waves

Right hand side of m.e.:
Wave Exciting Forces

- Incoming: regular wave with given frequency and propagation direction
- Assuming the vessel is not moving

Back to Regular waves

regular wave propagating in direction \( \mu \)

\[ \zeta(t, x) = \zeta_0 \cos(\omega t - kx \cos \mu - ky \sin \mu) \]

Linear solution Laplace equation

In order to calculate forces on immersed bodies:
What happens underneath free surface?
Potential Theory

What is potential theory?:
way to give a mathematical description of flowfield

Most complete mathematical description of flow is
viscous Navier-Stokes equation:

Navier-Stokes vergelijkingen:

\[
\begin{align*}
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} &= -\frac{\partial p}{\partial x} + \lambda \nabla \cdot V + 2\mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} &= -\frac{\partial p}{\partial y} + \lambda \nabla \cdot V + 2\mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} &= -\frac{\partial p}{\partial z} + \lambda \nabla \cdot V + 2\mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
\end{align*}
\]

(not relaxed.)

Apply principle of continuity on control volume:

Continuity: what comes in, must go out
If in addition the flow is considered to be irrotational and non viscous →

Velocity potential function can be used to describe water motions

Main property of velocity potential function:

for potential flow, a function \( \Phi(x,y,z,t) \) exists whose derivative in a certain arbitrary direction equals the flow velocity in that direction. This function is called the velocity potential.

Summary

• Potential theory is mathematical way to describe flow

Important facts about velocity potential function \( \Phi \):

• definition: \( \Phi \) is a function whose derivative in any direction equals the flow velocity in that direction

• \( \Phi \) describes non-viscous flow

• \( \Phi \) is a scalar function of space and time (NOT a vector!)
Summary

- Velocity potential for regular wave is obtained by
  - Solving Laplace equation satisfying:
    1. Seabed boundary condition
    2. Dynamic free surface condition

\[
\Phi(x, y, z, t) = \frac{\zeta}{g} e^{\nu t} \sin(kx \cos \mu + ky \sin \mu - \omega t)
\]

\[
\Phi(x, y, z, t) = \frac{\zeta}{g} \frac{\cosh(k(h+z))}{\cosh(kh)} \sin(kx \cos \mu + ky \sin \mu - \omega t)
\]

3. Kinematic free surface boundary condition results in:
   - Dispersion relation = relation between wave frequency and wave length
   \[
   \omega^2 = \frac{kg}{\tanh(kh)}
   \]

Water Particle Kinematics

- Trajectories of water particles in infinite water depth

\[
\Phi(x, y, z, t) = \frac{\zeta}{g} e^{\nu t} \sin(kx \cos \mu + ky \sin \mu - \omega t)
\]

- Trajectories of water particles in finite water depth

\[
\Phi(x, y, z, t) = \frac{\zeta}{g} \frac{\cosh(k(h+z))}{\cosh(kh)} \sin(kx \cos \mu + ky \sin \mu - \omega t)
\]

Pressure

Pressure in the fluid can be found using Bernoulli equation for unsteady flow:

\[
\frac{\partial \Phi}{\partial t} + \frac{1}{2} (u^2 + w^2) + \frac{p}{\rho} + gz = 0
\]

\[
p = \rho \frac{\partial \Phi}{\partial t} - \frac{1}{2} \rho (u^2 + w^2) - \rho gz
\]

1st order fluctuating pressure

2nd order (small quantity squared = small enough to neglect)

Hydrostatic pressure (Archimedes)
Potential Theory

From all these velocity potentials we can derive:

- Pressure
- Forces and moments can be derived from pressures:

\[
\begin{align*}
F &= -\iint (\mathbf{p} \cdot \mathbf{n}) \, dS \\
M &= -\iint \mathbf{p} \cdot (\mathbf{r} \times \mathbf{n}) \, dS
\end{align*}
\]

Verifying these formulae (including the signs) yourself in order to understand them. Just check up the force in heave direction (\(F_z\)) and the pitch moment (\(M_{\phi}\)) induced by a pressure on an infinite piece of hull surface \(dS\) at location \(P\).

Flow superposition

\[(m + a) \mathbf{\tau} \mathbf{b} \times \mathbf{c} \cdot \mathbf{z} = F\]

1. Flow due to Undisturbed wave
   \[
   \Phi_u = \frac{1}{2 \mu} e^{\omega t} \sin (\omega \tau - k_x \cos \mu - k_y \sin \mu) 
   \]

   Has to be solved. What is boundary condition at body surface?

2. Flow due to Diffraction
   \[
   \Phi_D = \frac{1}{2 \mu} e^{\omega t} \sin (\omega \tau - k_x \cos \mu - k_y \sin \mu + \varepsilon)
   \]

   Has to be solved. What is boundary condition at body surface?

Exciting force due to waves

\[(m + a) \mathbf{\tau} \mathbf{b} \times \mathbf{c} \cdot \mathbf{z} = \mathbf{F}_w\]

1. Undisturbed wave force (Froude-Krilov)
   \[
   \Phi_u = \frac{1}{2 \mu} e^{\omega t} \sin (\omega \tau - k_x \cos \mu - k_y \sin \mu) 
   \]

2. Diffraction force
   \[
   \Phi_D = \frac{1}{2 \mu} e^{\omega t} \sin (\omega \tau - k_x \cos \mu - k_y \sin \mu + \varepsilon)
   \]

   Has to be solved. What is boundary condition at body surface?
Pressure due to undisturbed incoming wave
\[ T = 4 \text{ s} \]

Pressure due to undisturbed incoming wave
\[ T = 10 \text{ s} \]

Wave Forces
Wave force acting on vertical wall

Calculating hydrodynamic coefficient and diffraction force
\[
(m + a) \cdot z + b \cdot x + c \cdot z = F_w = F_{FR} + F_h
\]
\[
\Phi_x = \frac{k \cdot x}{\omega} \cdot e^{ix} \cdot \sin (kx - \omega t)
\]
\[
\Phi_x = -\frac{k \cdot x}{\omega} \cdot e^{ix} \cdot \sin (kx + \omega t)
\]
\[
\Phi_x + \Phi_x = \frac{k \cdot x}{\omega} \cdot e^{ix} \cdot \sin (kx)
\]
**Force on the wall**

\[ F = \int p \eta \, d\xi \]

\[ \Phi_1 = \frac{1}{2} \sigma^2 \eta \sin (\xi - \omega t), \Phi_2 = \frac{1}{2} \sigma^2 \eta \sin (\xi + \omega t) \]

\[ p = p_0 - \frac{2}{\eta} \eta \Phi_1 - \frac{3}{\eta} \eta \Phi_2 \]

\[ = 2 \rho c \sigma^2 \eta \sin (\xi) \cos (\omega t) \]

\[ \pi = (1, 0, 0) \]

\[ \lambda = 0 \]

\[ F = \int 2 \rho c \sigma^2 \eta \cos (\xi) \cos (\omega t) \, d\xi = 2 \rho \sigma^2 \cos (\omega t) \int_0^\pi \eta \cos (\xi) \, d\xi = 2 \rho \sigma^2 \cos (\omega t) \phi \]

**Left hand side of m.e.: Hydromechanic reaction forces**

- NO incoming waves:
- Vessel moves with given frequency

\[ (m + a) \ddot{z} + b \dot{z} + c \cdot z = F_K + F_D = F_W \]

**Hydrodynamic coefficients**

Determination of \( a \) and \( b \):
- Forced oscillation with known frequency and amplitude
- Measure force needed to oscillate the model
Equation of motion

\[(m + a)\ddot{z} + b\dot{z} + c\cdot z = F_D + F_K = F_w\]

Hydrodynamic coefficients:

\[a = \text{added mass coefficient} = \text{force on ship per } 1 \text{ m/s}^2 \text{ acceleration} \rightarrow a\ \text{ acceleration} = \text{hydrodynamic inertia force}\]

\[b = \text{damping coefficient} = \text{force on ship per } 1 \text{ m/s velocity} \rightarrow b\ \text{velocity} = \text{hydrodynamic damping force}\]

Equation of motion

\[(m + a)\ddot{z} + b\dot{z} + c\cdot z = +F_K + F_D = F_w\]

To solve equation of motion for certain frequency:

- Determine some coefficient:
  - \(c\) follows from geometry of vessel

- Determine required hydrodynamic coefficients for desired frequency:
  - \(a, b\) \rightarrow computer / experiment

- Determine amplitude and phase of \(F_w\) of regular wave with amplitude \(=1:\)
  - Computer / experiment: \(F_w = F\cos(\omega + \xi)\)

- As we consider the response to a regular wave with frequency \(\omega:\)
  - Assume steady state response: \(z = z_w(\cos(\omega + \xi))\)
  - And substitute in equation of motion:
Equation of motion

\[(m + a) \ddot{z} + b \dot{z} + c z = F_w\]

Now solve the equation for the unknown motion amplitude \(z\) and phase angle \(\xi\) for 1 frequency

System is linear

If wave amplitude doubles \(\rightarrow\) wave force doubles \(\rightarrow\) motion doubles

\[(m + a) \ddot{z} + b \dot{z} + c z = \frac{F_w}{z}\]

Substitute solution \(z = \frac{F_w}{z} \cos(\omega t + \xi)\) and solve RAO and phase

Calculated RAO spar with potential theory

**RAO**

\[(m + a) \ddot{z} + b \dot{z} + c z = F_w\]

**Frequency Response of semi-submersible**

Motion dominated by spring terms

Motion dominated by damping terms

Motion dominated by mass terms

Frequency (rad/s)

0 1 2 3 4 5

0 0.5 1 1.5 2 2.5 3

Frequency (rad/s)

0 1 2 3 4 5

0 0.5 1 1.5 2 2.5 3

(From: McDermid, 1972)
Bonus Assignment

Bonus Question 1, 2, 3

Deadline, 28 November 13.45 (beginning of lecture)

Deliver hard copy, properly stapled / binded, with names and student numbers

Schedule?

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<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Type</th>
<th>Teacher</th>
<th>Location</th>
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<tbody>
<tr>
<td>Wed 4 Nov</td>
<td>15.30-16.30</td>
<td>Lecture</td>
<td>Peter Neejum</td>
<td>CA1-C20 (Jenius)</td>
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<tr>
<td>Wed 14 Nov</td>
<td>15.30-17.30</td>
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<td>Peter Neejum</td>
<td>CA1-C20 (Jenius)</td>
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<td>CA1-C29 (Isaac Newton)</td>
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</table>
Sources images

[1] Towage of SSDR Transocean Amirante, source: Transocean
[4] Bluewater Rig No. 1, source: Friede & Goldman, LTD/GNU General Public License
[5] Source: unknown
[6] Rig Neptune, source: Seafarer Media
[7] Pieter Schelte vessel, source: Excalibur
[8] FPSO design basis, source: Statoil
[13] Schematic of Curlew Field, United Kingdom, source: offshore-technology.com
[14] Ocean Quest Brave Sea, source: Zamakona Yards
[15] Medusa, A Floating SPAR Production Platform, source: Murphy USA