Traffic Flow Theory and Simulation

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Lecture 2
Arrival patterns and cumulative curves

Arrival patterns

From microscopic to macroscopic

24-3-2014

Recap traffic flow variables

Microscopic (vehicle-based)	Macroscopic (flow-based)
Space headway (s [m])	Density (k [veh/km])
Time headway (h [s])	Flow (q [veh/h])
Speed (v [m/s])	Average speed (u [km/h])

s=h*v

q=k*u

Excercise– a task for you!

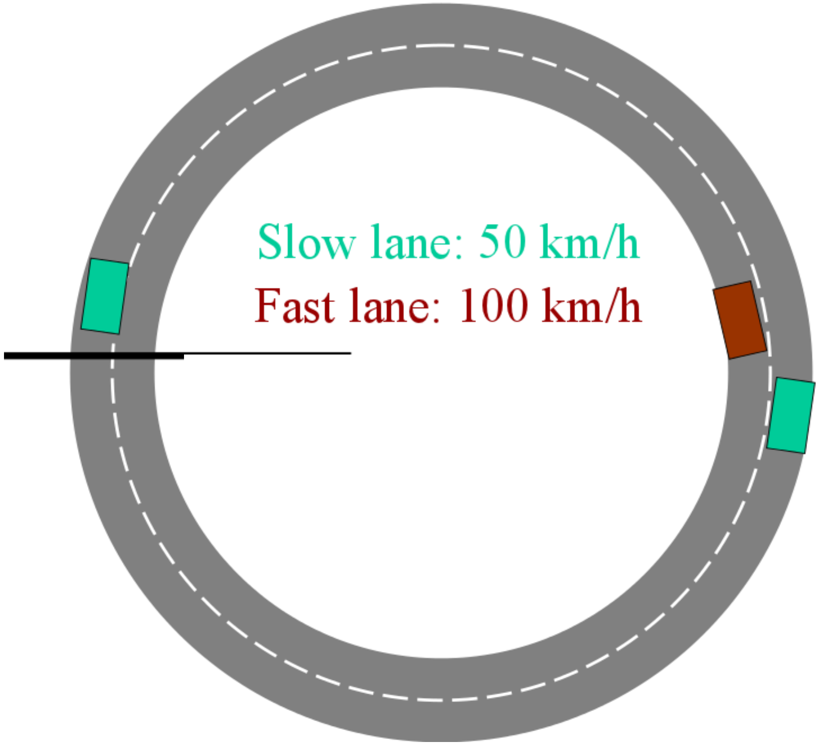
- What is the average speed if you travel
  - 10 km/h from home to university
  - 20 km/h on the way back

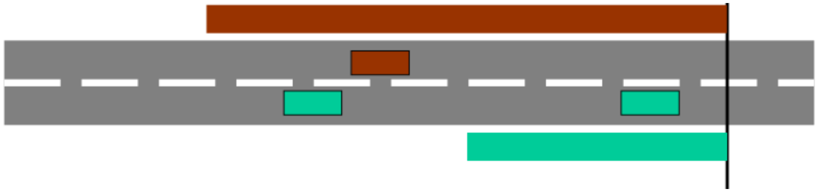
- What is the average speed if you make a trip with speed
  - v1 on the outbound trip
  - v2 on the inbound trip

- What is the average speed if you split the trip in n equidistant sections, which you travel in v_i

Similar problems arise in traffic

- Local measurements, spatial average speed needed





- Weigh speed measurements
- Inversely proportional to speed

The math…

- Weigh speed measurements (w)
- Inversely proportional to speed: wi=1/vi

$$\langle v \rangle_{\text{space}} = \frac{\sum_i w_i v_i}{\sum_i w_i} = \frac{\sum_i \frac{1}{v_i} v_i}{\sum_i \frac{1}{v_i}}$$

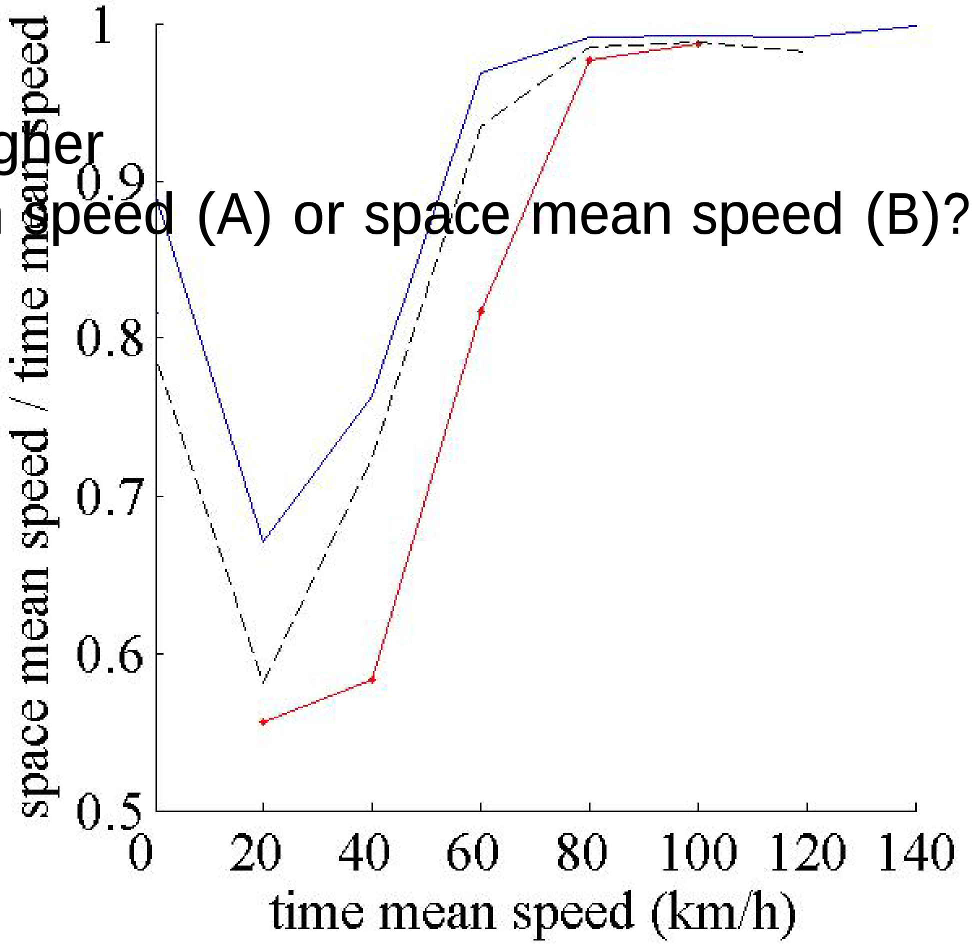
$$= \frac{n}{\sum_i \frac{1}{v_i}}$$

$$= \frac{1}{\sum_i \frac{1}{v_i}} n = 1 \bigg/ \left\langle \frac{1}{v_i} \right\rangle_{\text{time}}$$

Time-average of pace (1/v)

… and the effect

What is higher
time mean speed (A) or space mean speed (B)?



Summary

- Speed averaging is not trivial
- Obtain time mean speed from loop detector data by harmonic average (i.e., averaging 1/v)
- Space mean speed is lower, with differences in practice up to factor 2

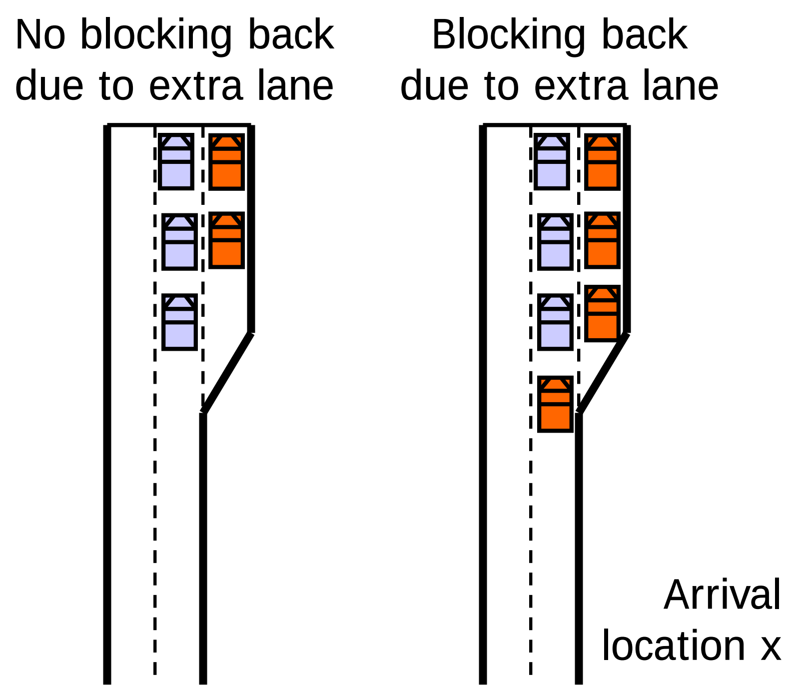
Overview of remainder of lecture

- Arrival process and relating probability distribution functions
    - Poisson process (independent arrivals)
    - Neg. binomial distribution and binomial distribution
    - Applications facility design (determine length right- or left-turn lane)
- Time headway distributions
    - Distribution functions and headway models
    - Applications
- Speed distribution and free speed distributions

Arrival processes

- Insight into probability distribution of the number of vehicles arriving in a short time interval (e.g. 15 seconds) is important for several applications

- Example:
  - Length of extra lane for left-turning vehicles at an intersection
  - Probability that queue exceeds roadway space is limited (e.g. 5%)
- Models for the distribution of vehicles arriving in a short period of time
  - Poisson process
  - Binomial process
  - Negative binomial process



Poisson process

- Number of vehicles passing *x* during certain period of length *h*
- *N*(*h*) can be described by a *stochastic variable*
- Assume **independent arrivals**
  - Dilute traffic operations w. sufficient passing opportunities
  - No upstream disturbances (e.g. signalized intersection)

Then nr of arrivals *N*(*h*) is **Poisson**:

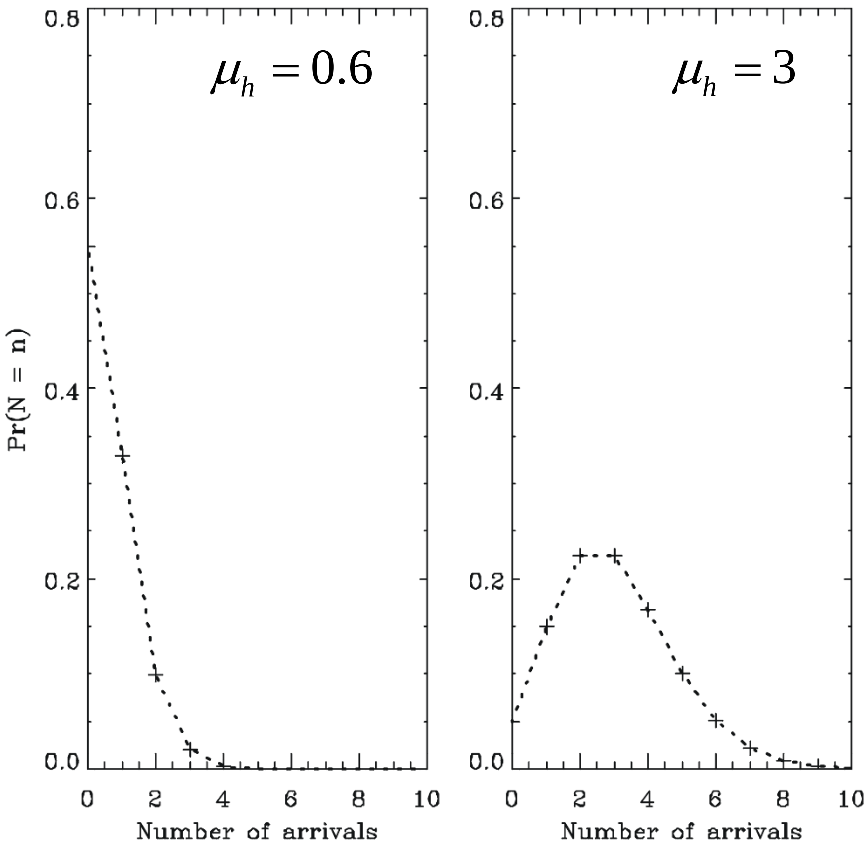
$$f(k;l) = \frac{\lambda^k \exp - \lambda}{k!}$$

Properties of the Poisson distribution

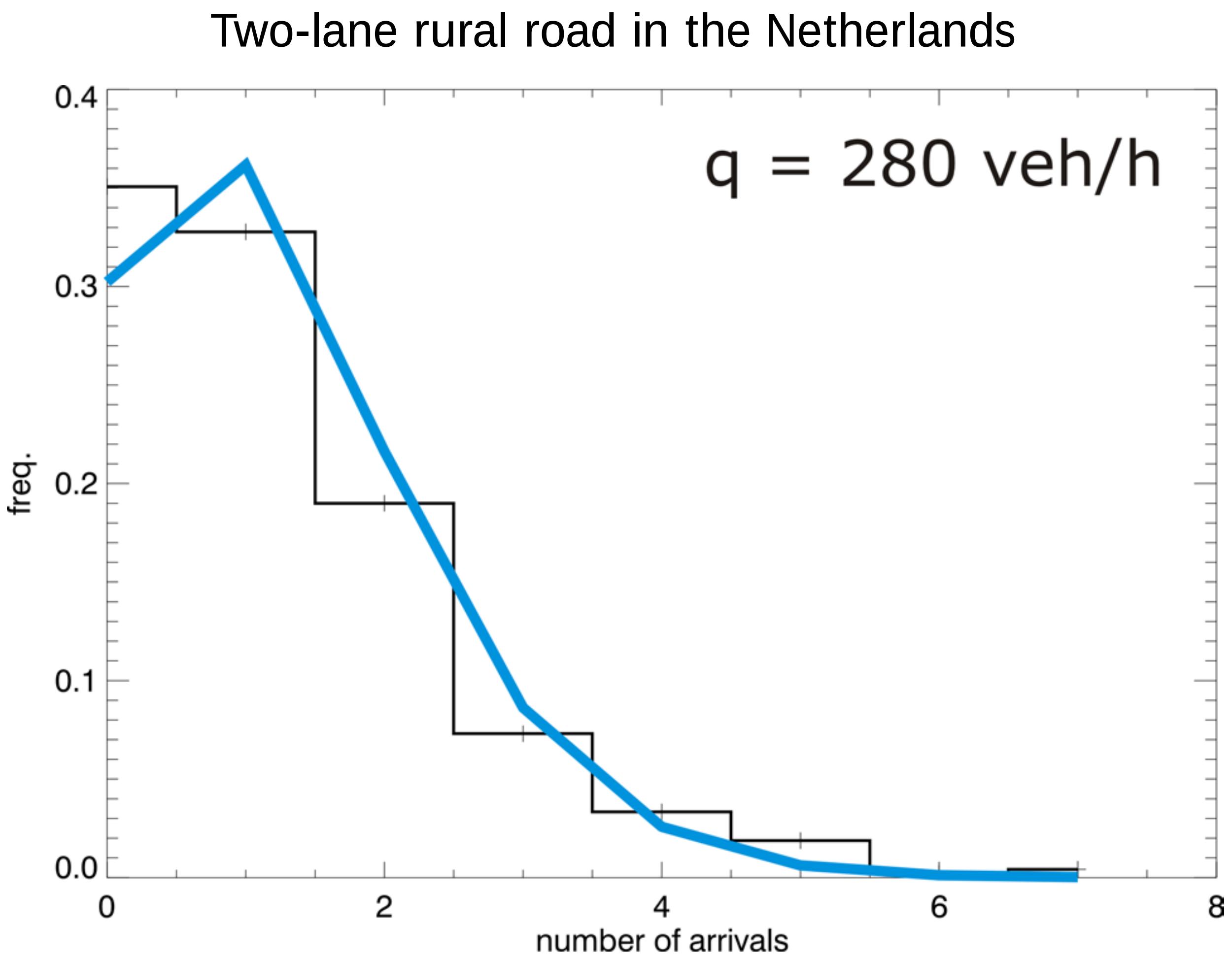
- **Mean** and **variance** are equal (= μ_h) for a certain period of length h
- If "mean = variance" then Poisson is likely
- And thus independence of arrivals is a good assumptions

Poisson process (2)

- Examples of Poisson distributions for different periods / intensities

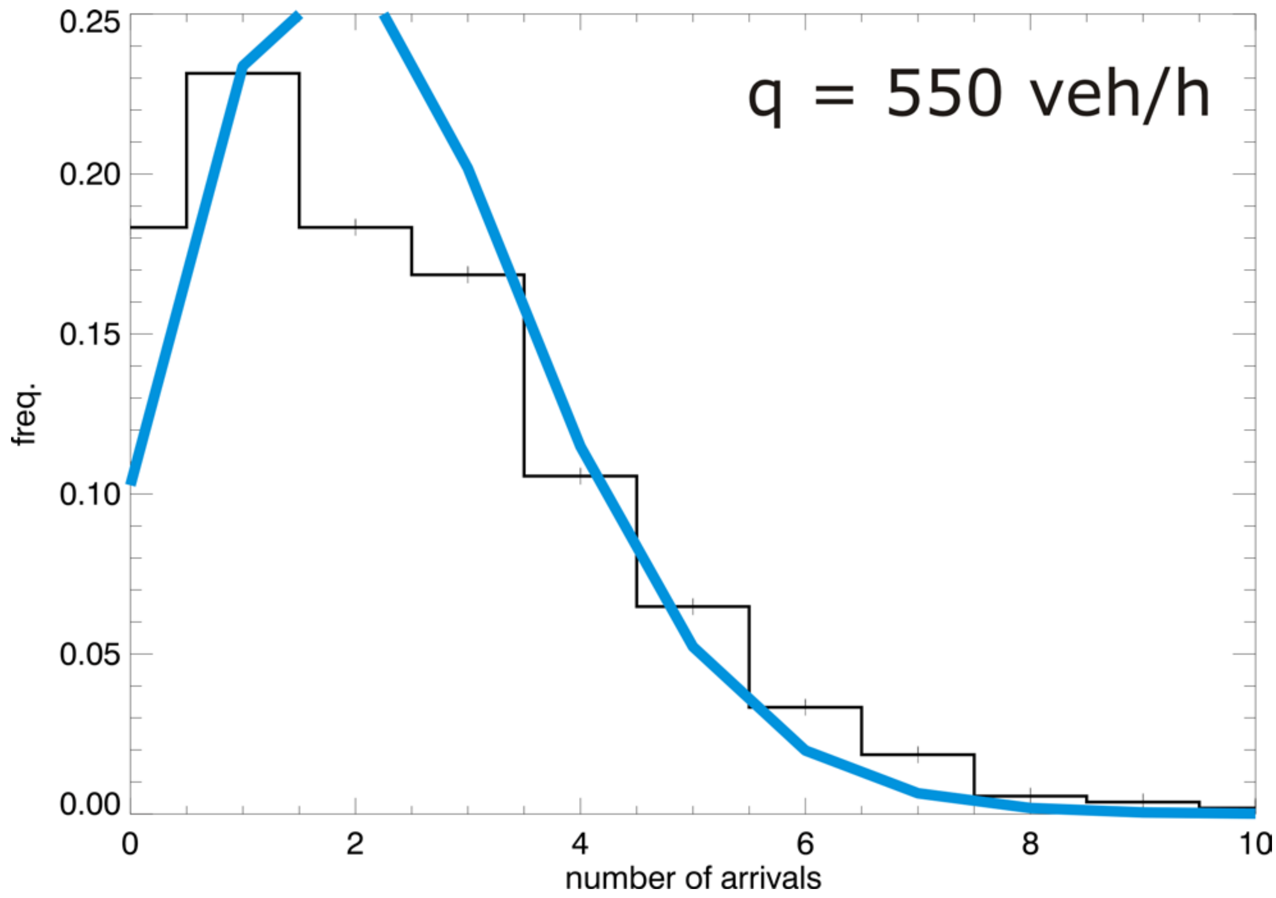


Poisson and real data (arrivals / 15s)



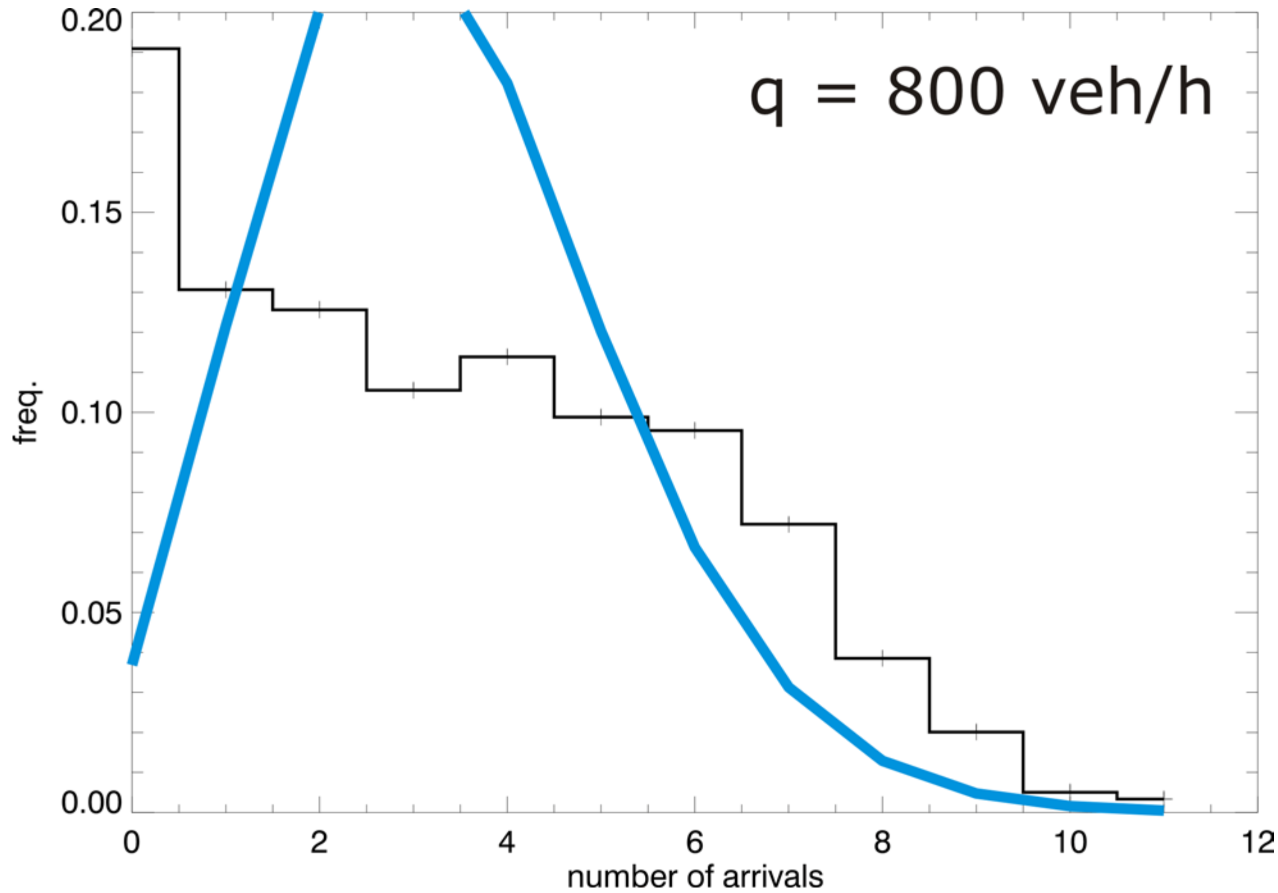
Poisson and real data (arrivals / 15s)

Two-lane rural road in the Netherlands



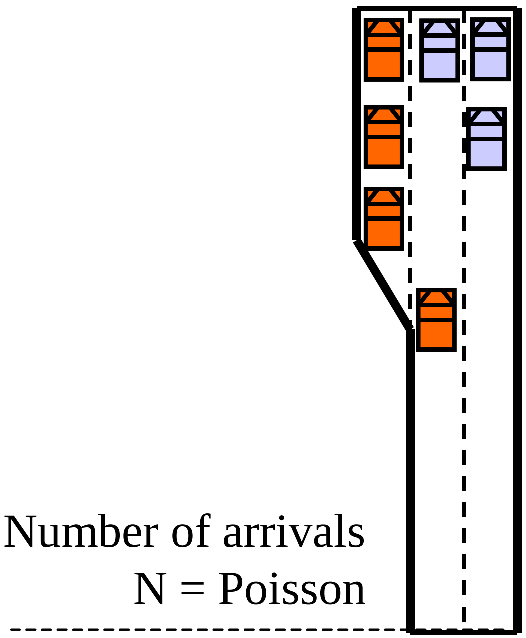
Poisson and real data (arrivals / 15s)

Two-lane rural road in the Netherlands



Poisson process (3): exercise

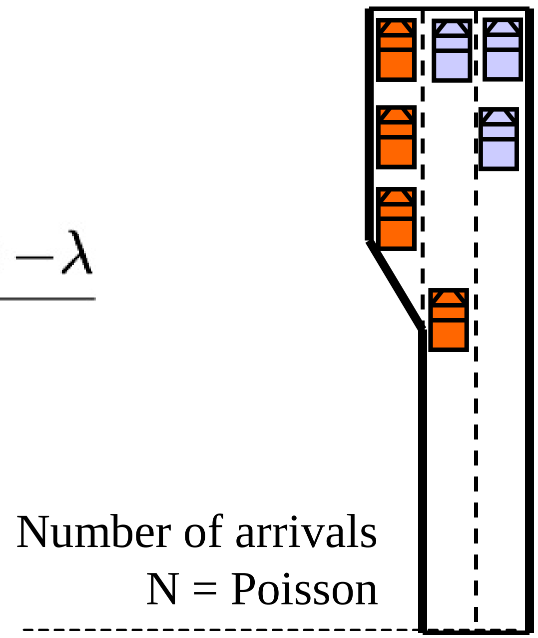
- Application to left-turn lane design problem
- Intensity for **left-turning lane** during peak-hour = 360 veh/h
- Duration of the red-phase 50s
- In 95% of the cycles must be undersaturated
- How long must the left-lane be?



Poisson process (3): exercise

- Application to left-turn lane design problem
- Intensity for **left-turning lane** during peak-hour = 360 veh/h
- Duration of the red-phase 50s
- In 95% of the cycles must be undersaturated
- How long must the left-lane be?

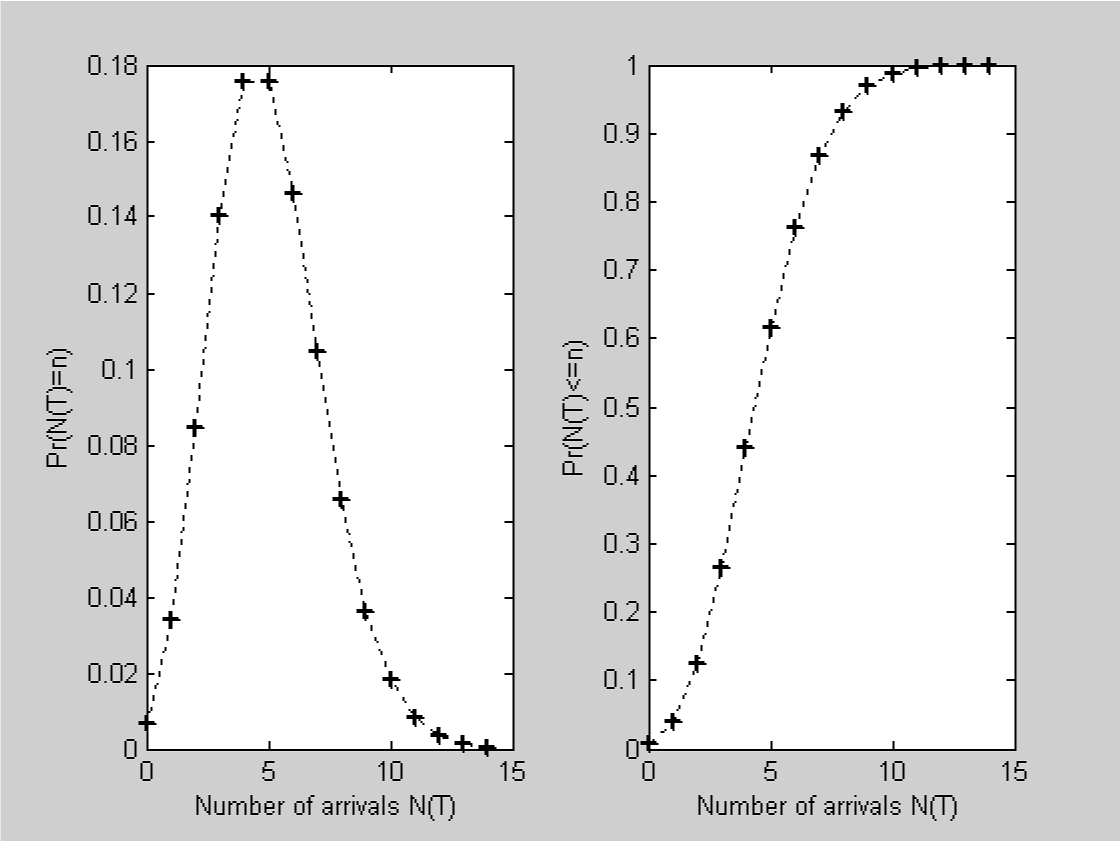
- Assume Poisson process: $f(k;l) = \dfrac{\lambda^k \exp -\lambda}{k!}$
- Mean number of arrivals 50·(360/3600) = 5
- Thus λ = 5 veh per aggregation interval (= qh with q = 360 and h = 50/3600)



Poisson process (4)

- From graph:
  Pr(N≥8) = 0.9319
  Pr(N≥9) = 0.9682

- If the length of the left-turn lane can accommodate 9 vehicles, the probability of blocking back = 3.18%



Two other possibilities

1. Vehicles are (mainly) following
   => binomial distribution

2. Downstream of a regulated intersection
   => negative binomial distribtution

Binomial process

- Increase traffic flow yields formation of platoons (interaction)
- Poisson is no longer valid description
- Alternative: Binomial process

$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Binomial distribution describes the probability of *n* successful, independent trials; the probability of success equals p; n_0 = max. number of arrivals within period h

Properties of binomial distribution

- Mean: $n_0 p$ and variance: $n_0(1 - p)p$
- Note that **variance < mean**
- No rationale why approach yields reasonable results
- Choose appropriate model by statistical testing!

Binomial process (3)

- Left-turn example revisited
- Note: variance < mean!
  - Assume a variance of 2.5 veh^2 / cycle (=50 sec)
- Determine how long the left-turning lane must be under these assumptions
- **Hint**: use the recursive formula in reader
  (Home work)

Downstream of controlled intersection

- More likely to have bunched vehicles *(Why?)*
- Negative binomial distribution

How to determine model to use?

Some guidelines:

- Are arrivals independent => Poisson distribution
    - Low traffic volumes
    - Consider traffic conditions upstream (but also downstream)
    - Consider mean and variance of arrival process
- Platooning due to regular increase in intensity? => Binomial
- Downstream of signalized intersection? => Negative binomial

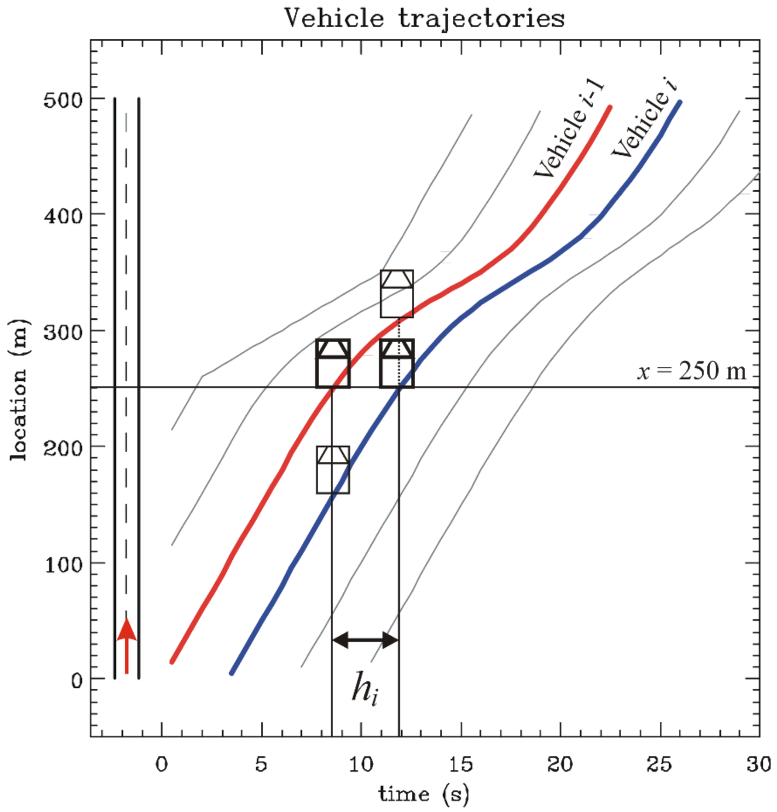
Definitive answer?

- Use statistical tests (Chi-square tests) to figure out which distribution is best

Time headways

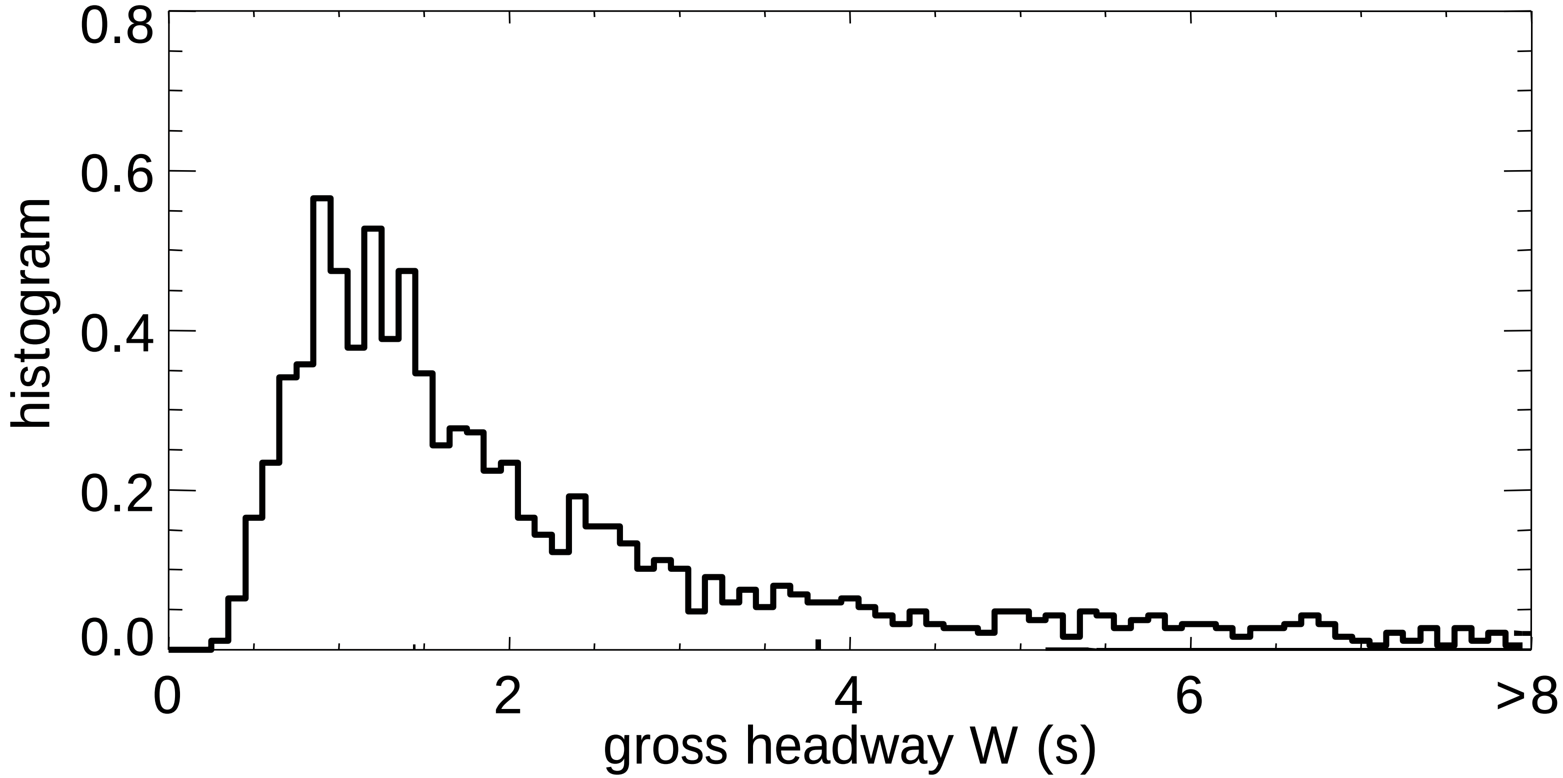
- A time headway of a vehicle is defined by the period between the passing moment of the preceding vehicle and the vehicle considered

- Distinction between
  - Net headway (gap): rear bumper – front bumper
  - Gross headway: rear bumper – rear bumper
- Headway distribution models generally pertain to a single lane

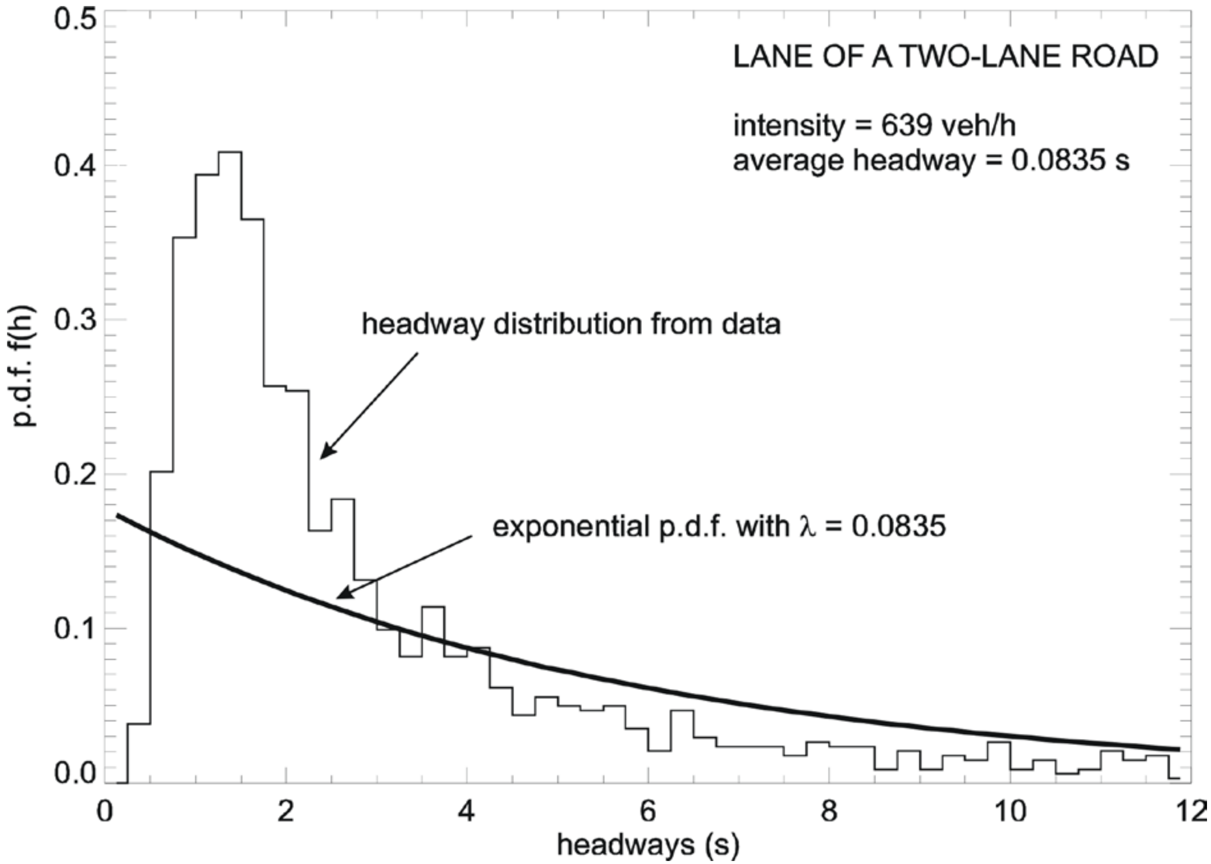


Example headway distribution two-lane road

- Site: Doenkade



Exponential distribution



Maximum likelihood

- Maximum-likelihood method aims to *maximize the 'probability' of the parameters given the sample (observation)*
- The likelihood of a single observation h_i equals (lambda parameter)

$$P(H = h_i) = f(h_i; \lambda)$$

- The likelihood of an entire sample $\{h_i\}$ equals

$$\mathcal{L} = \prod_i P(H = h_i) = \prod_i f(h_i; \lambda)$$

Optimisation of parameters

- The probability of an entire sample $\{h_i\}$ equals

$$\mathcal{L} = \prod_i P(H = h_i) = \prod_i f(h_i; \lambda)$$

- ML entails maximizing this likelihood, i.e.

$$\lambda^* = \underset{\lambda}{\operatorname{argmax}} \left(\mathcal{L}(\lambda) \right)$$

Composite headway models

Composite headway models distinguish

- Vehicles that **are driving freely** (and thus arrive according to some Poisson process, or whose headways are exponentially distributed)
- Vehicles that **are following at some minimum headway** (so-called empty zone, or constrained headway)
  which distribution is suitable?

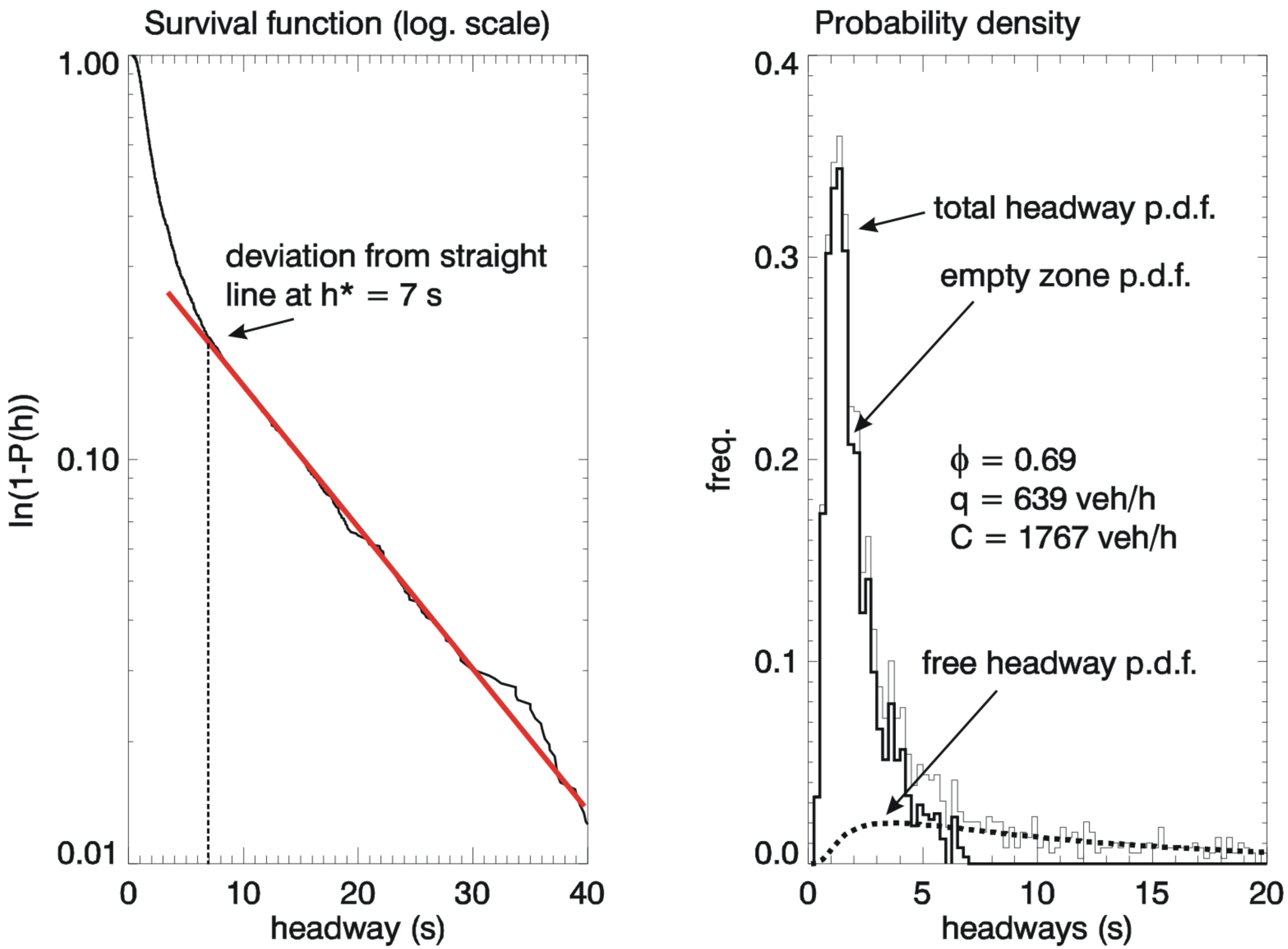
Basic idea

- Combine two distribution functions
  Contstraint: fraction phi
  Free: fraction (1-phi)

$$P(H = h) = \phi\left(P_{\text{constraint}}(h)\right) + (1 - \phi)\left(P_{\text{free}}(h)\right)$$

- Application: capactiy estimation
  How?

Estimation: likelyhood or graphically



Summary

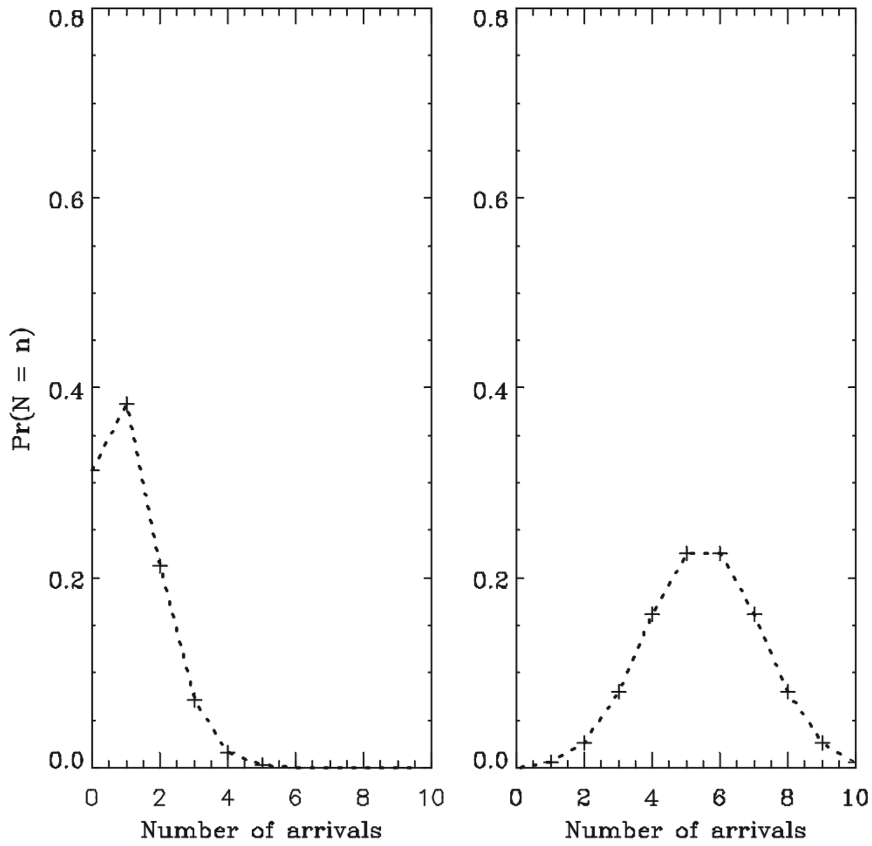
- Arrival distribution:
  Free flow: headways exponential,
  nr of per aggregation time: Poisson
  Following: binomial

- Calculation of required dedicated lane length

- Composite headway models => capacity estimation

Binomial process (2)

- Examples of the binomial process



Cowan's M3 model

- Drivers have a certain **minimum headway** *x* which can be described by a **deterministic variable**
- Minimum headway *x* describes the headway a driver needs for safe and comfortable following the vehicle in front
- If a driver is **not following**, we assume that he / she is driving at a headway *u* which can be described by a random variate $U \sim p_{free}(u)$
- The headway *h* of a driver is **the sum** of the free headway *u* and the minimum headway *x*, and thus $H = x+U$

- *The free headway is assumed distributed according to the exponential distribution (motivation?)*

Cowan's M3 model

- Probability density function of *H*

$$p(h) = \phi\delta(x-h) + (1-\phi)\, p_{free}(h) = p_1(h) + p_2(h)$$

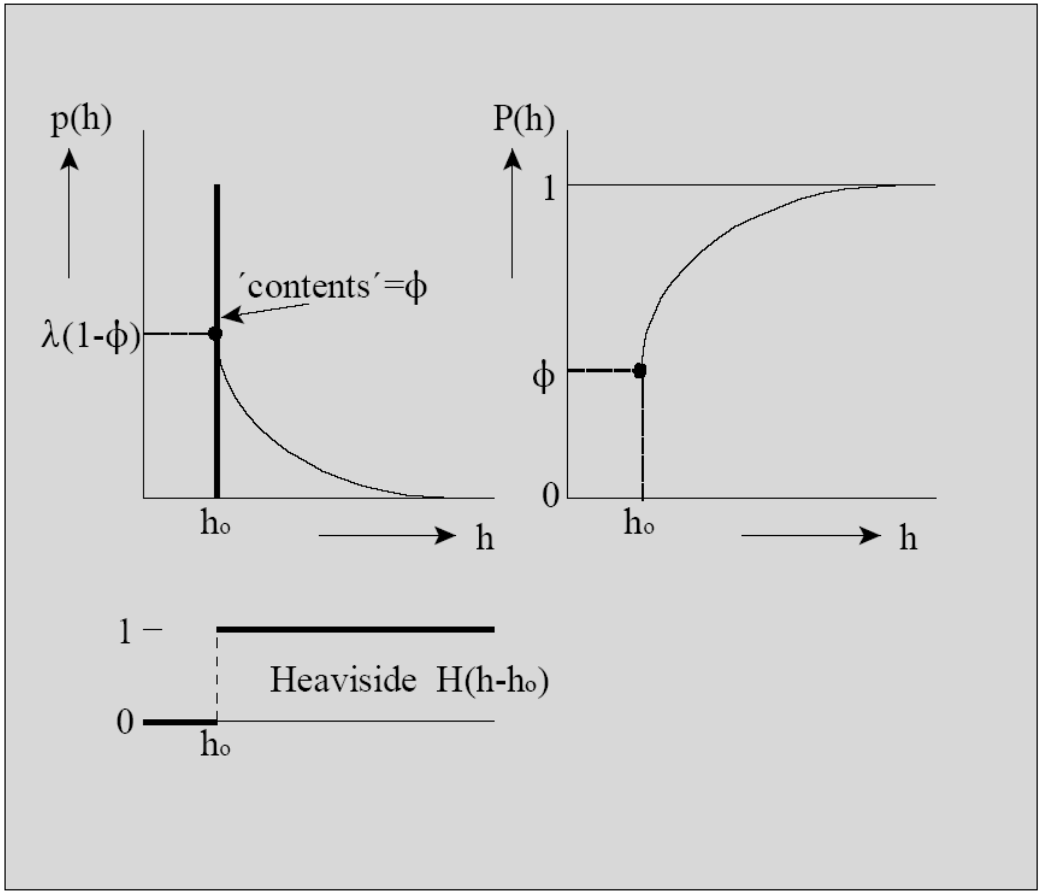
- Unless there are major upstream disturbances, the free headways are **exponentially distributed**, i.e.

$$p_2(h) = (1-\phi)\, p_{free}(h) = (1-\phi) H(h-x)\lambda e^{-\lambda(h-x)} \quad \text{for } h > x$$

- The model thus assumes that all constrained drivers maintain headway *x*, while the headway of the unconstrained drivers is distributed according to a shifted exponential distribution

Cowan's M3 model

- Example of Cowan's M3 model
- (note that $h_0 = x$)



Branston's headway distribution model

- Rather that assuming one fixed x for all drivers, we can assume that different drivers maintain different minimum headways
- Inter-driver differences are reflected by assuming that x is also a stochastic variable X with a specific distribution function $p_{follow}(x)$
- This yields the composite headway distribution model of Branston:

$$\begin{aligned} p(h) &= \phi p_{follow}(h) + (1-\phi) p_{free}(h) \\ &= \phi p_{follow}(h) + (1-\phi)\lambda e^{-\lambda h} \int_0^h p_{follow}(s) e^{-\lambda s} ds \end{aligned}$$

Buckley's composite model

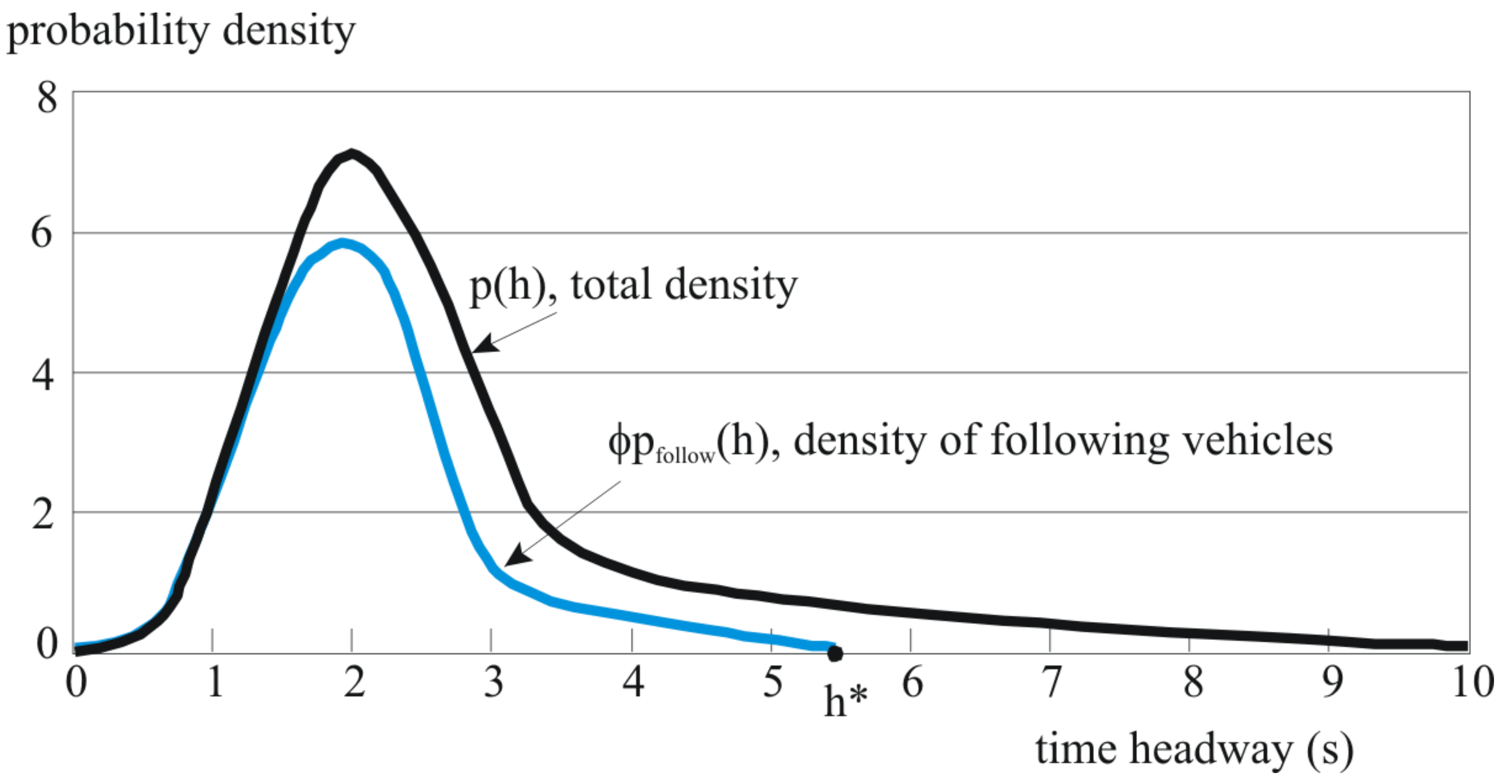
- Drivers have a certain **minimum headway** x which can be described by a random variable $X \sim p_{follow}(x)$
- Minimum headway describes the headway a driver needs for safe and comfortable following the vehicle in front
- X describes differences between drivers and within a single driver

- If a driver is **not following**, we assume that he / she is driving at a headway u which can be described by a random variate $U \sim p_{free}(u)$

- The headway h of a driver is then the minimum between the free headway u and the minimum headway x, and thus $H = \min\{X,U\}$

Buckley's composite model[3]

- Consider a threshold value h^* such that all observed headways h are larger than h^* are free headways, i.e. $p_{follow}(h) = 0,\ h > h^*$



Buckley's composite model[4]

- Unless there are major upstream disturbances, the free headways are **exponentially distributed**, i.e.

$$p(h) = (1-\phi)\, p_{free}(h) = A\lambda e^{-\lambda h} \quad \text{for} \;\; h > h^*$$

- For headways h smaller than h^*, we need to correct the total headway. This is done by *removing from the exponential distribution the fraction of vehicles that have preferred following times larger than h*
- The fraction of drivers with headway h that are not following equals

$$(1-\theta(h)) = (1-\phi)\int_0^h p_{follow}(\eta)\,d\eta$$

- Then for $h < h^*$ we have for the **free headway part**

$$p_2(h) = (1-\phi)\, p_{free}(h) = (1-\theta(h))A\lambda e^{-\lambda h} = A\lambda e^{-\lambda h}(1-\phi)\int_0^h p_{follow}(\eta)\,d\eta$$

Buckley's composite model[5]

- For the total headway distribution, we then get

$$\boxed{p(h) = \phi p_{follow}(h) + (1-\phi) A \lambda e^{-\lambda h} \int_0^h p_{follow}(\eta) d\eta}$$

where A denotes the **normalization constant** and can be determined from

$$\int_0^\infty p(\eta) d\eta = 1 \quad \Rightarrow \quad A = \int_0^\infty \lambda e^{-\lambda \eta} p_{follow}(\eta) d\eta$$

- Special estimation procedures exist to determine λ, ϕ and p.d.f. $p_{follow}(h)$
  - Parametric: specify p.d.f. $p_{follow}(h)$ with unknown parameters
  - Distribution free, non-parametric, i.e. without explicit specification $p_{follow}(h)$

Determination of h^*

- For $h > h^*$, distribution is exponential, i.e.

$$p(h) = A\lambda e^{-\lambda h} \quad \text{for} \quad h > h^*$$

- Consider survival function $S(h)$

$$S(h) = \Pr(H > h) = \int_h^\infty A\lambda e^{-\lambda\eta} d\eta = Ae^{-\lambda h}$$

- Obviously, for $h > h^*$, we have $\ln[S(h)] = \ln\left[Ae^{-\lambda h}\right] = \ln(A) - \lambda h$
- Suppose we have headway sample $\{h_i\}$
- Consider the *empirical distribution function*

$$\hat{S}_n(h) = 1 - \frac{1}{n}\sum H(h - h_i)$$

- Now draw $\ln\left[\hat{S}_n(h)\right]$ and determine h^*, A, and λ

Applications of headway distributions

- Analysis of crossing a street / gap-acceptance
- Calculate waiting time or delay
- Capacity estimation (without capacity observations) using composite headway models of Cowan / Branston / Buckley
  - Estimate **all parameters** of composite headway distribution
  - Assume all vehicles are following (ϕ = 1) to see that ($p(h) = p_{follow}(h)$), i.e. headway is minimal: we have mean(H) = mean(X)
  - Recall that $q = 1/\text{mean}(H)$
  - Estimate mean headway (mean empty zone) and capacity
- Vehicle generation for micro-simulation

$$C = \frac{1}{mean(X)} = \frac{1}{\int_0^\infty x p_{follow}(x) dx}$$

Binomial process (3)

- Left-turn example revisited
- Assume a variance of 2.5 veh^2 / cycle (=50 sec)
- Determine how long the left-turning lane must be under these assumptions

- Mean number of arrivals 5 : $n_0 p = 5$
- 'Assume' variance of 2.5 veh^2 : $n_0(1 - p)p = 2.5$
- Then we can determine:

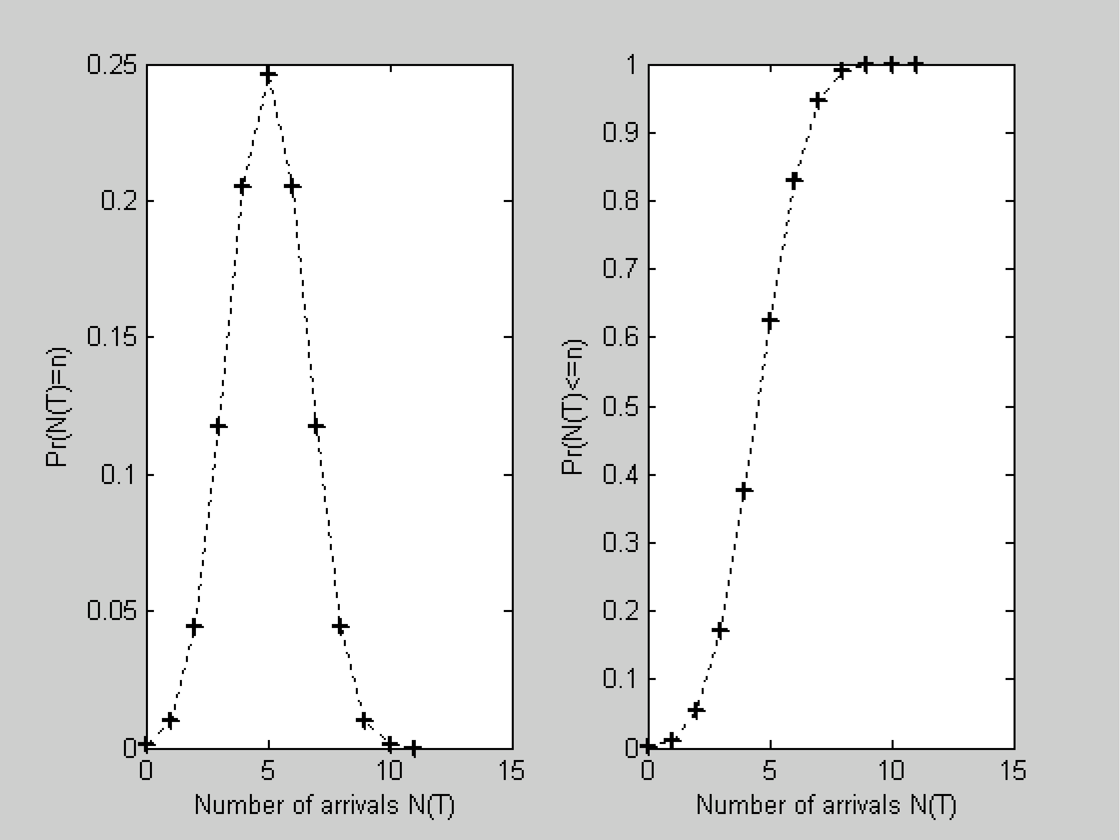
$$p = 0.5 \text{ and } n_0 = 10$$

- Results are very sensitive to variance!!

Binomial process (4)

- From graph:
  Pr(N≥7) = 0.9453
  Pr(N≥8) = 0.9893

- If the length of the left-turn lane can accommodate 7 vehicles, the probability of blocking back = 5.47%



Negative Binomial distribution

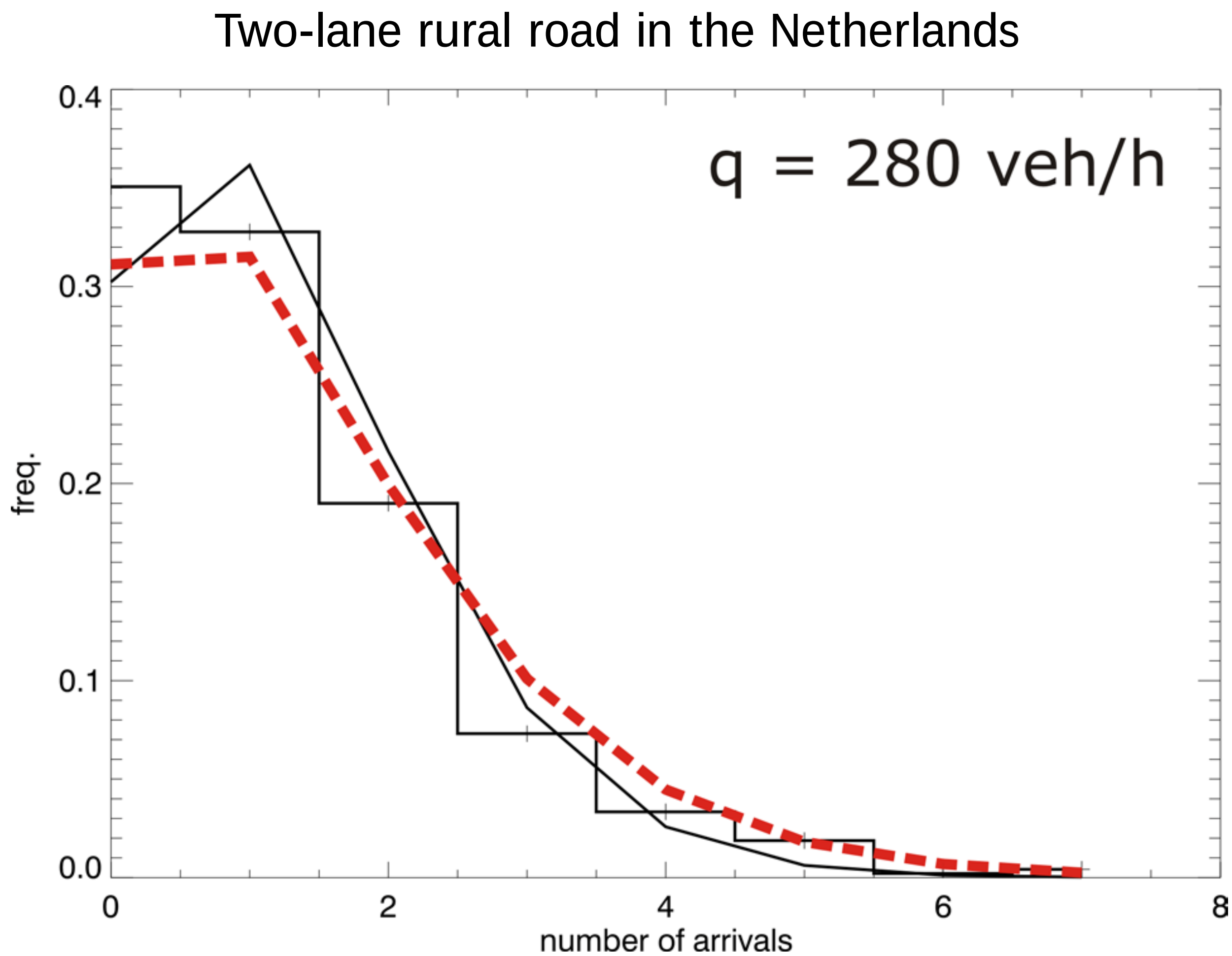
- Also consider **negative Binomial distribution** (see course notes)

$$\Pr\left(N(h)=n\right)=\binom{n_0+n-1}{n}p^{n_0}\left(1-p\right)^n$$

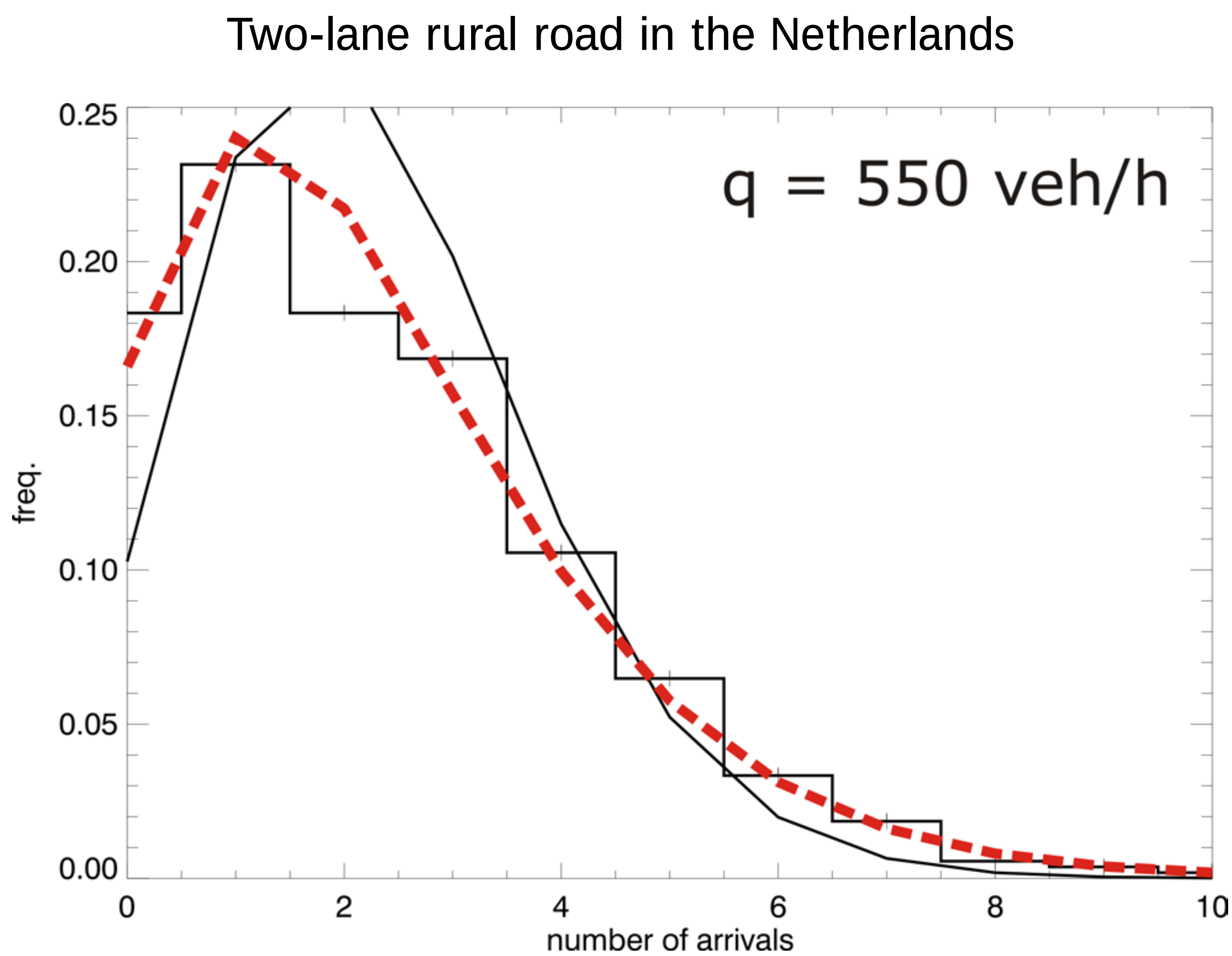
- Appears to hold when upstream disturbances are present

- Choose appropriate model by trial and error and statistical testing

Neg. bin. distribtion & observations

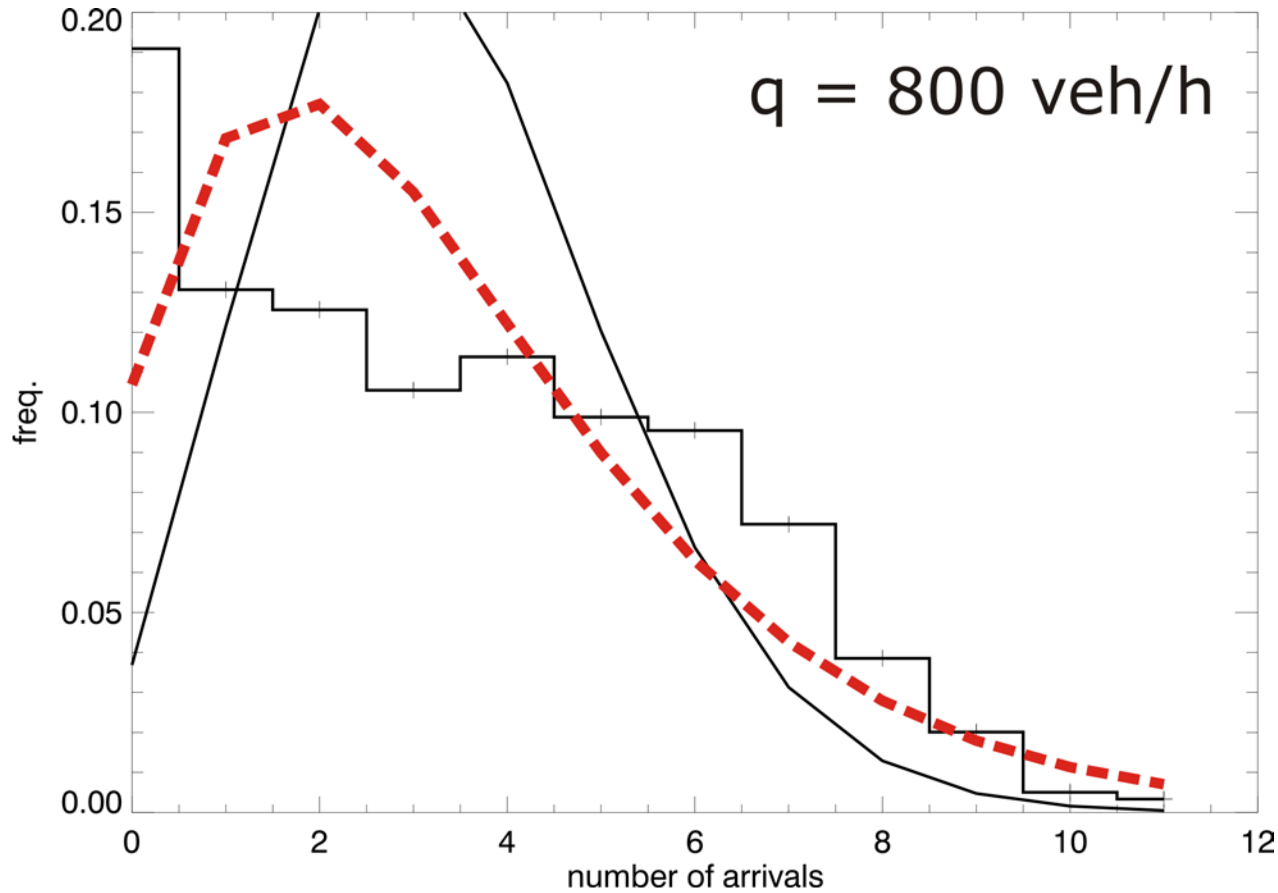


Neg. bin. distribtion & observations



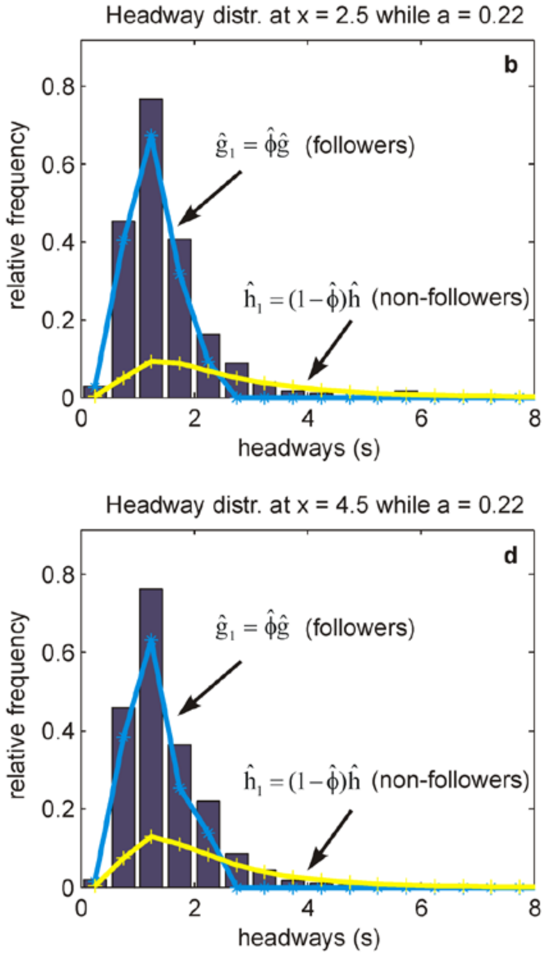
Neg. bin. distribtion & observations

Two-lane rural road in the Netherlands



Example real-life headway distribution

- Pedestrian experiments
- Headways of pedestrians passing a certain cross-section (wide bottleneck scenario)

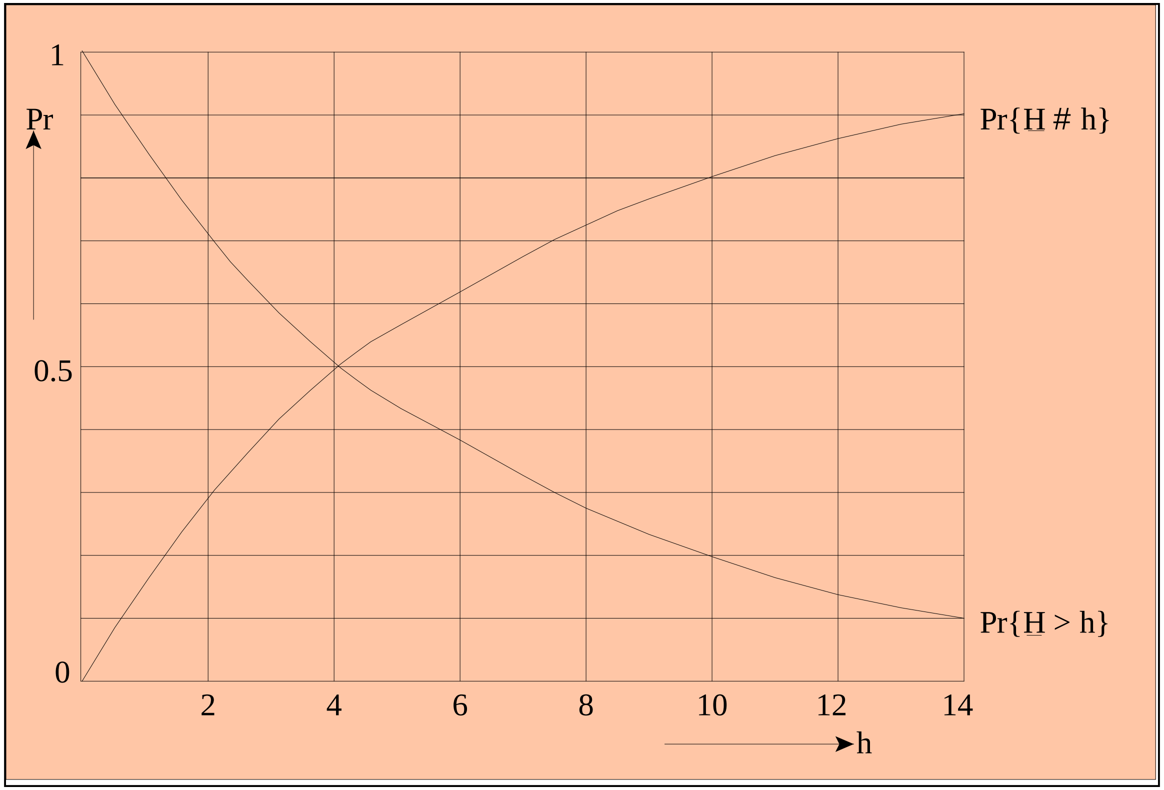


Exponential distribution (2)

- Exponential distribution function / survival function

$$\text{mean}(H) = \bar{H} = \frac{1}{q}$$

$$\text{var}(H) = \frac{1}{q^2}$$



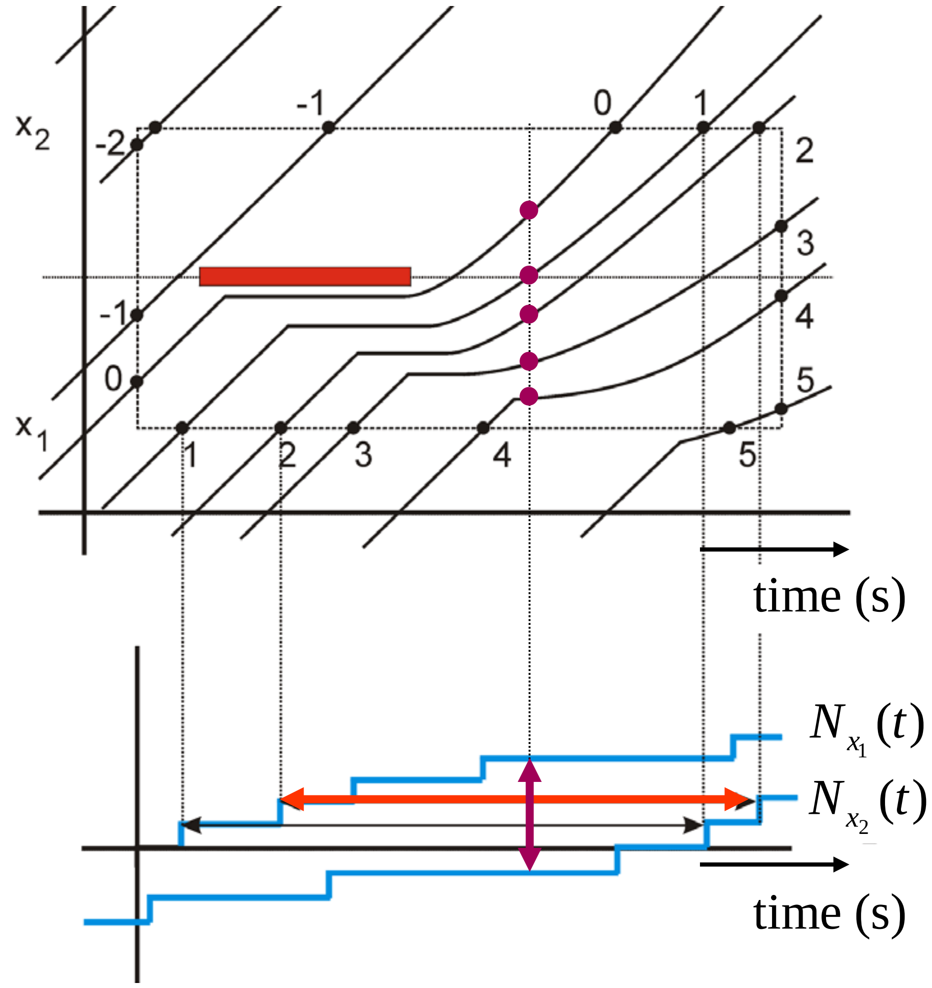
Cumulative curves

Calculation of delays and queues

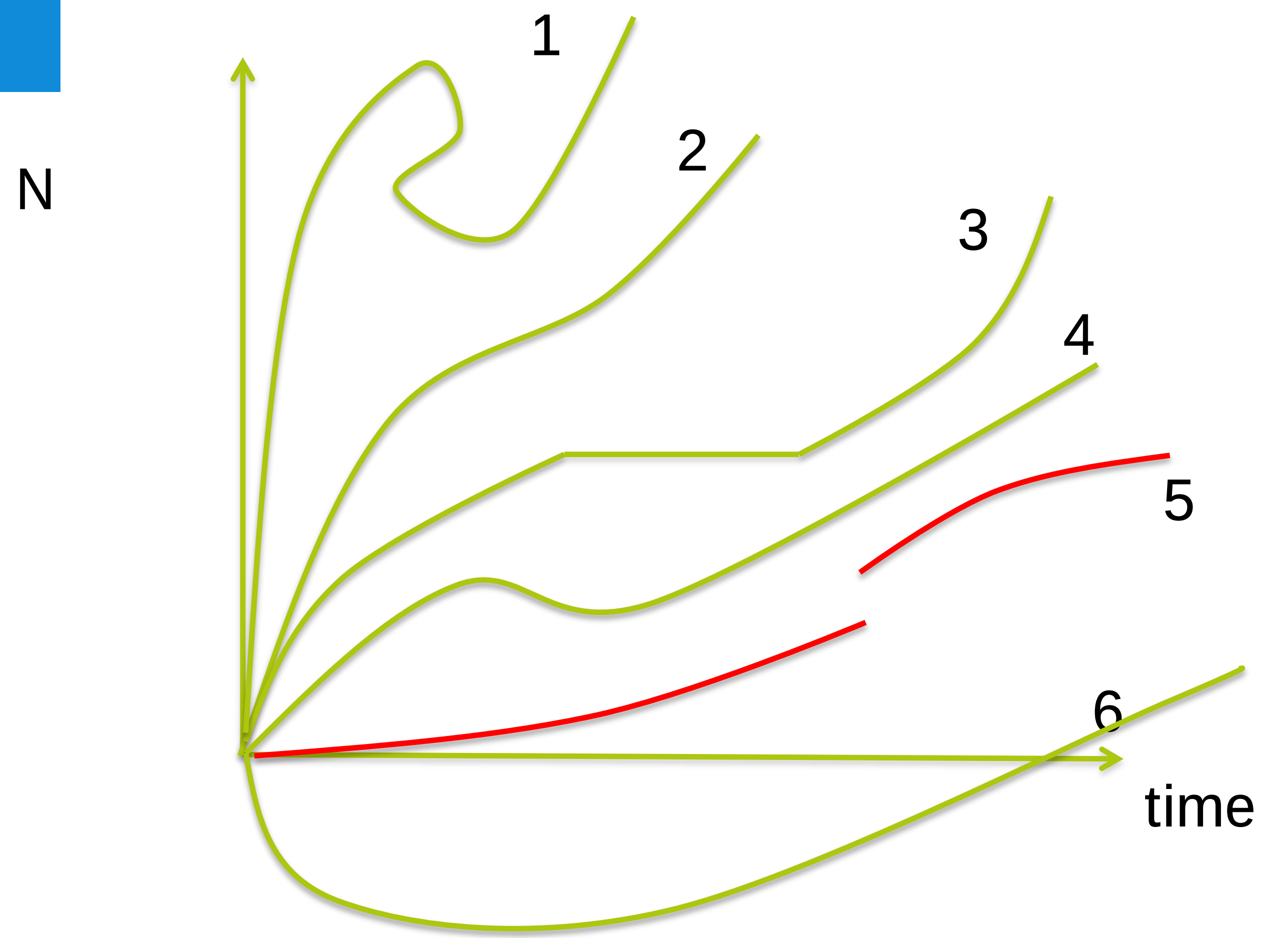
24-3-2014

Cumulative vehicle plots

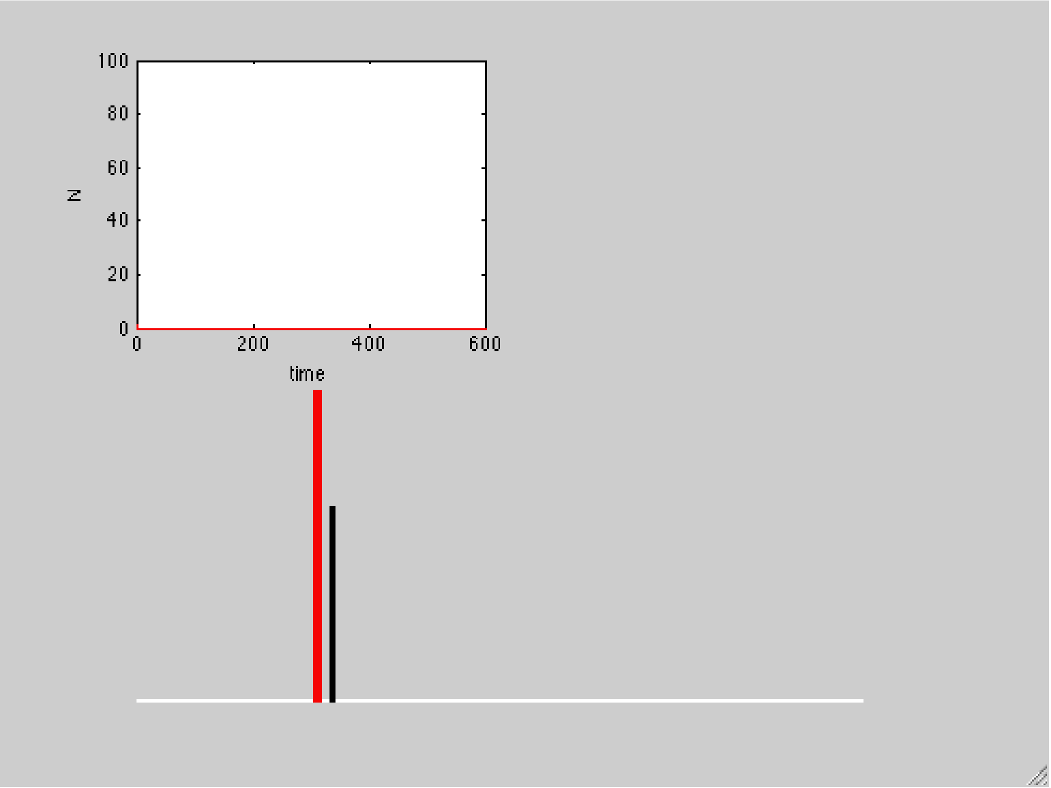
- Cumulative flow function $N_x(t)$: number of vehicles that have passed cross-section x at time instant t
- $N_x(t)$: step function that increases with 1 each time instant vehicle passes
- Horizontal axis: trip times
- Vertical axis: vehicle count (storage)



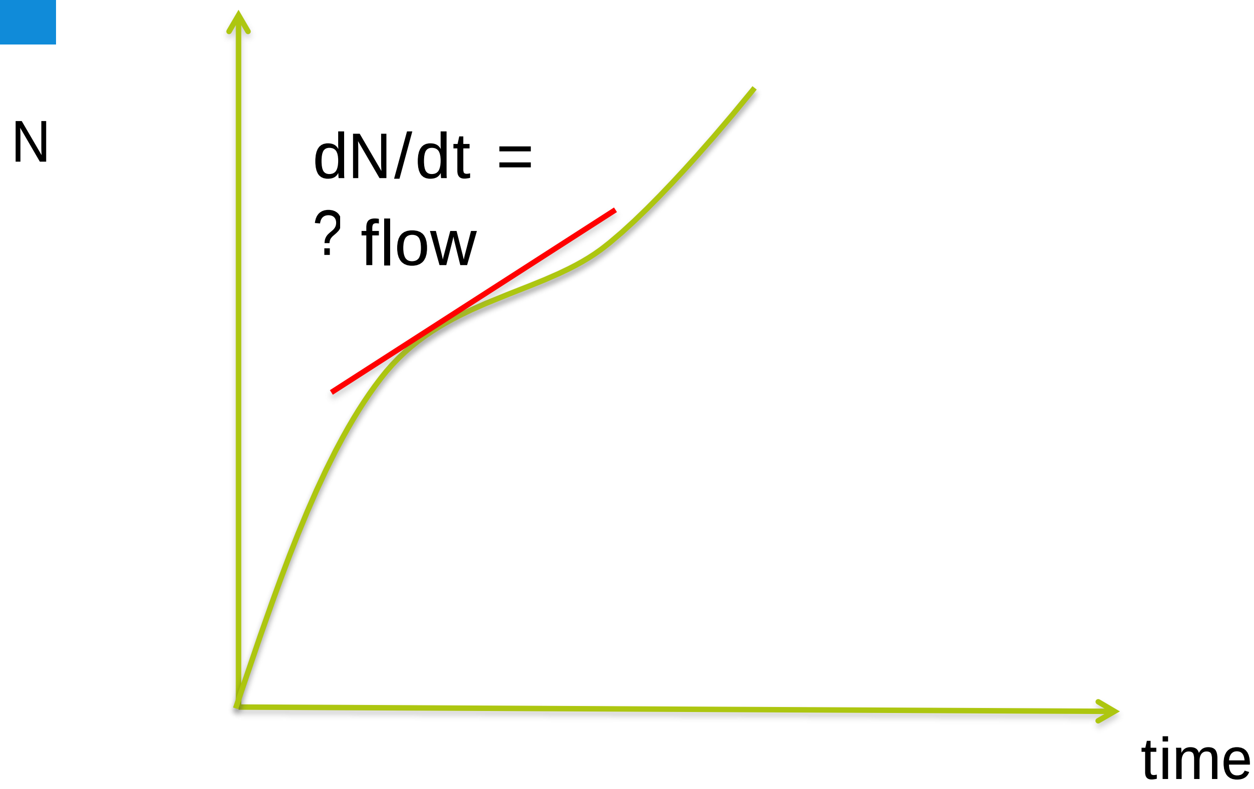
Examples of cumulative curves?



Construction of cumulative curves

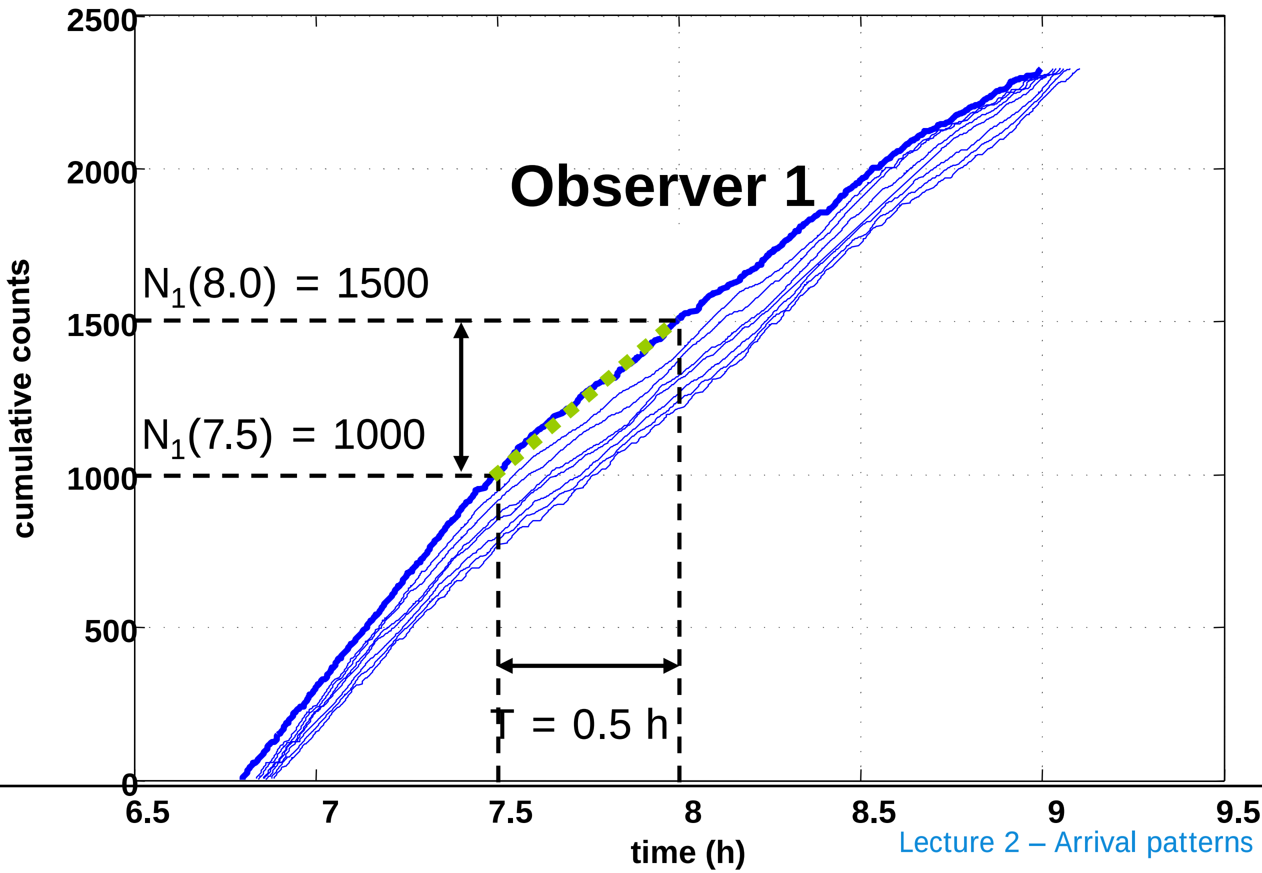


Information in cumulative curves

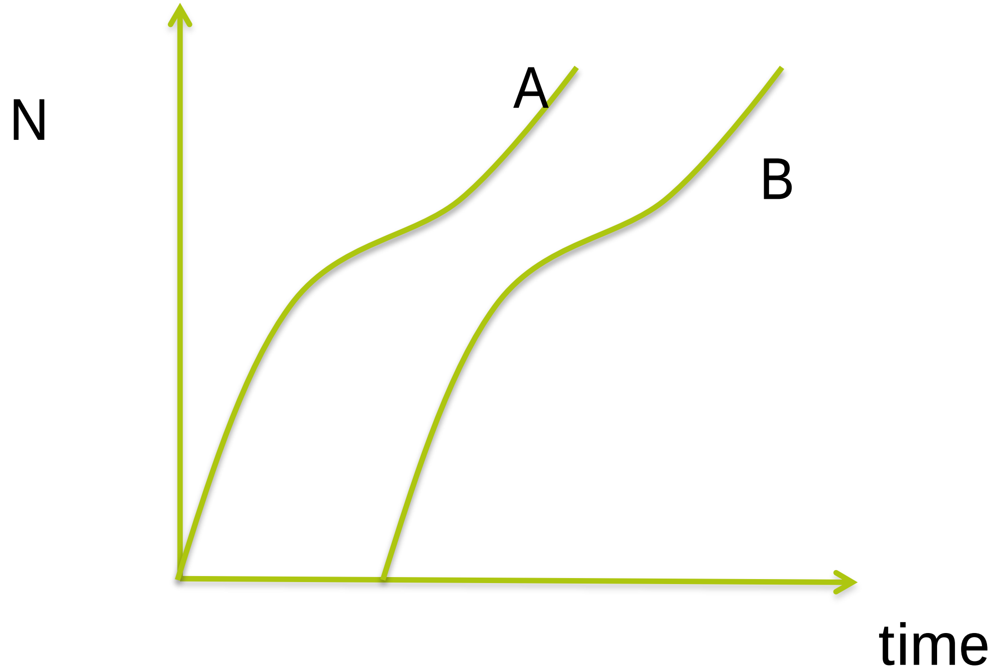


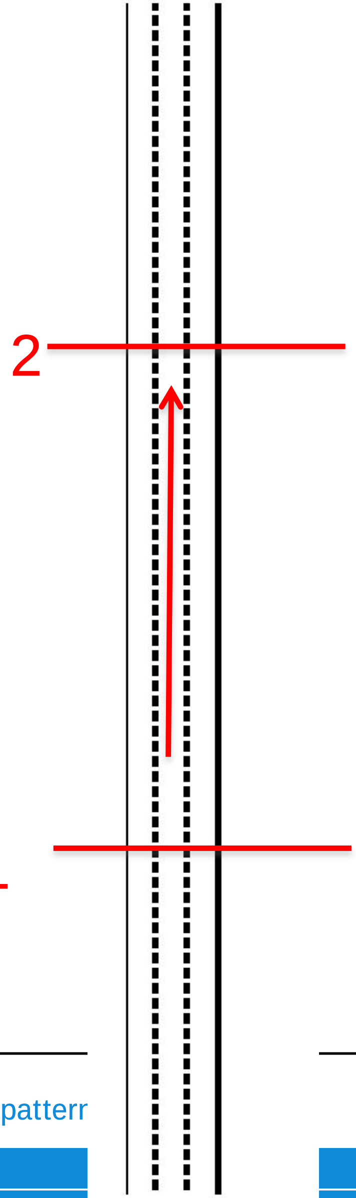
Cumulative vehicle plots[3]

- Flow = number of vehicles passing *x* (observer) during *T*
- *What is the flow in this case?*

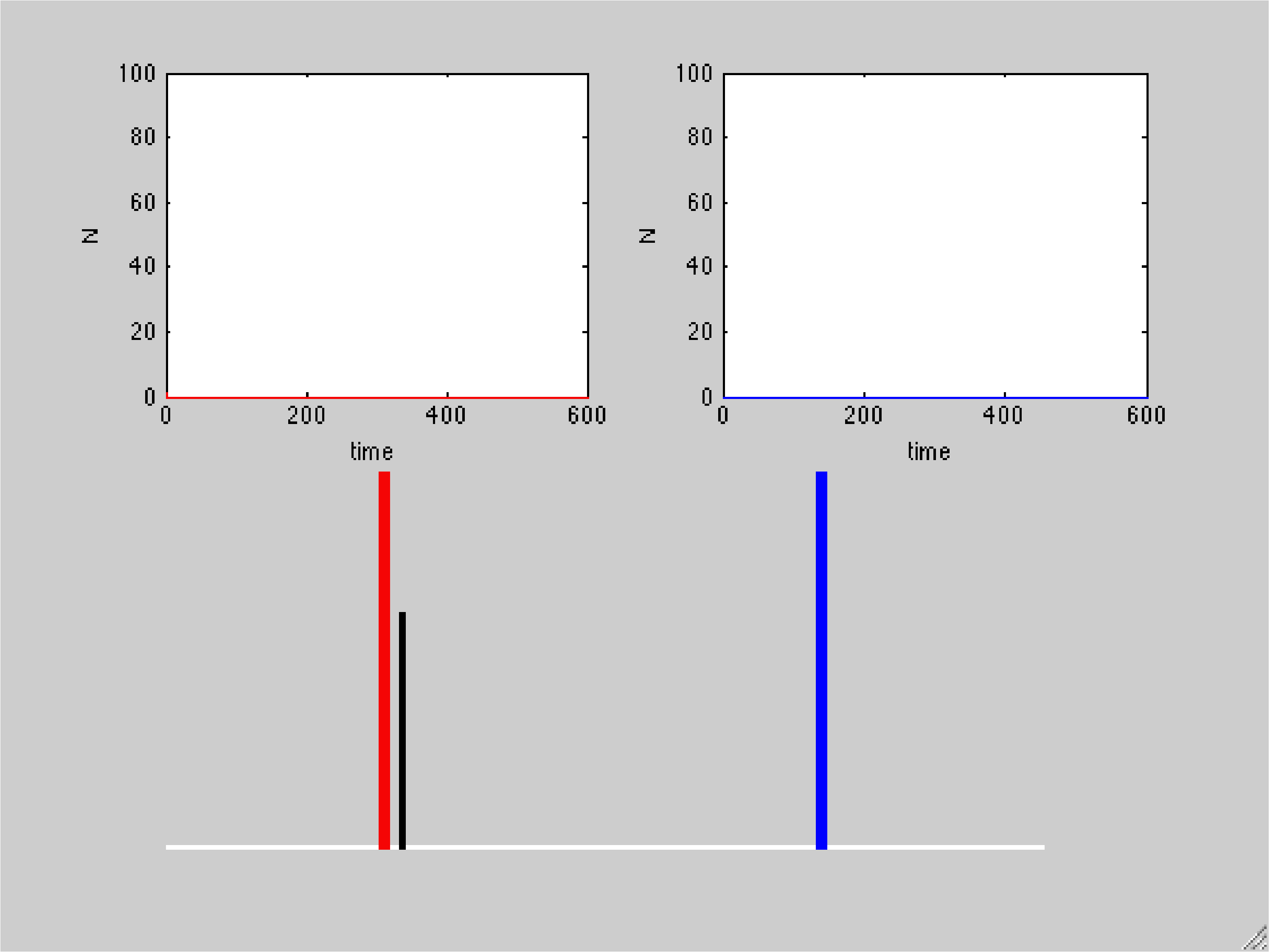


Information in cumulative curves



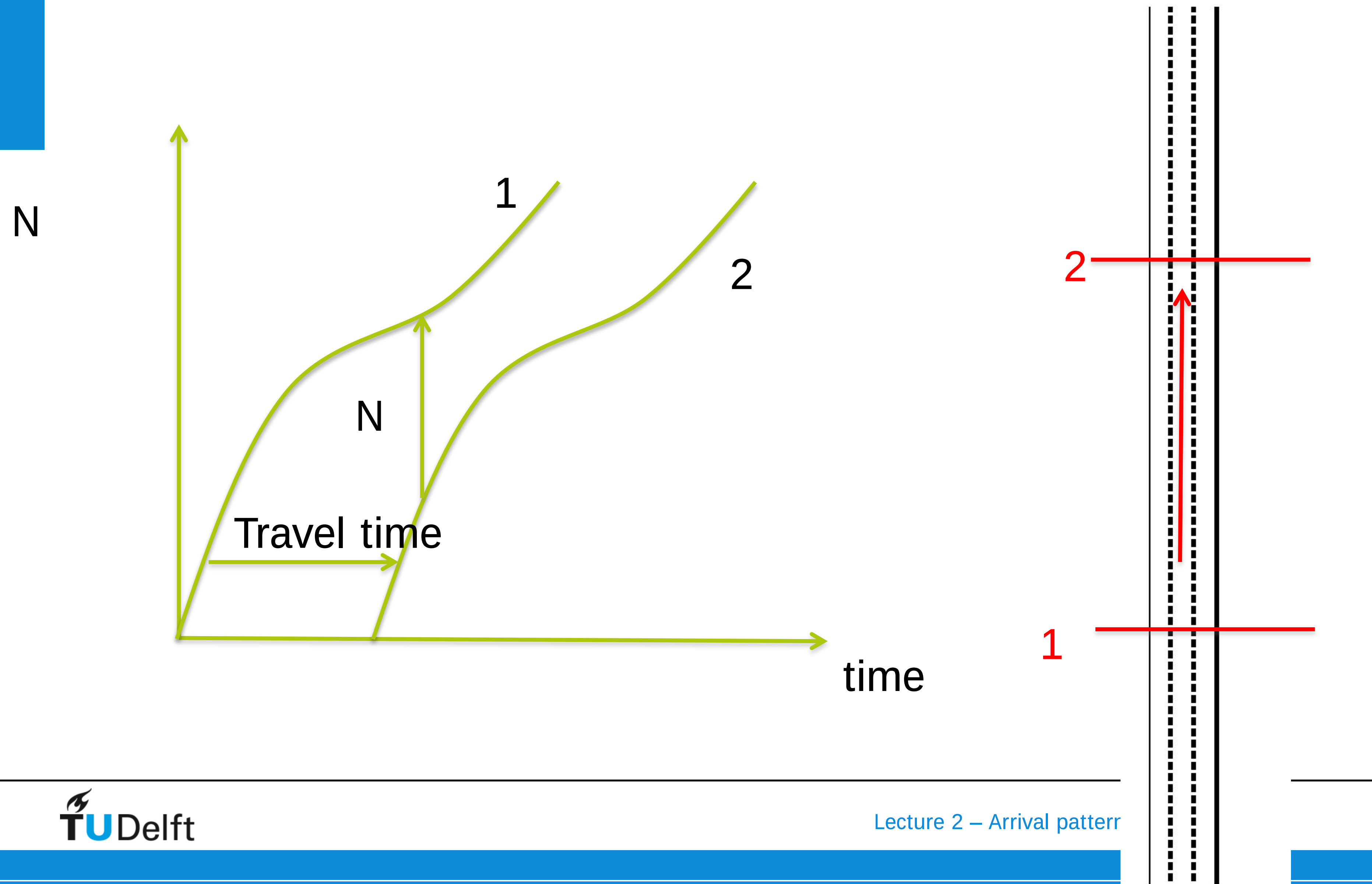


Which line corresponds with detector 1?



100
80
60
N
40
20
0
0
200
400
600
time
100
80
60
N
40
20
0
0
200
400
600
time

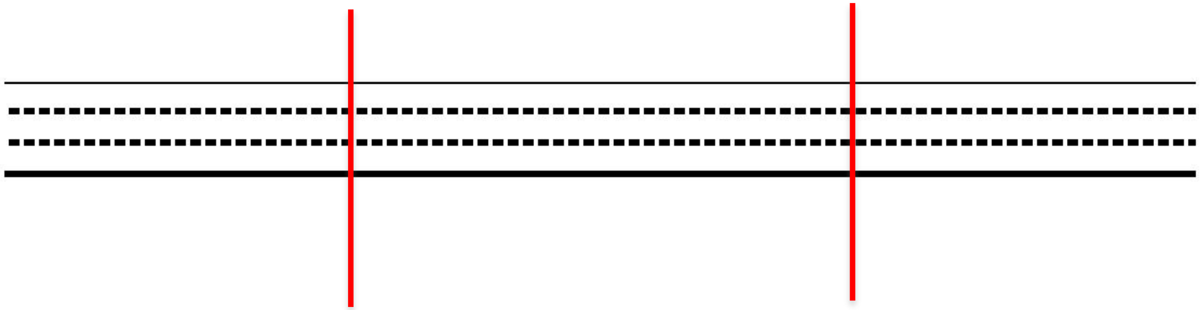
Information in cumulative curves



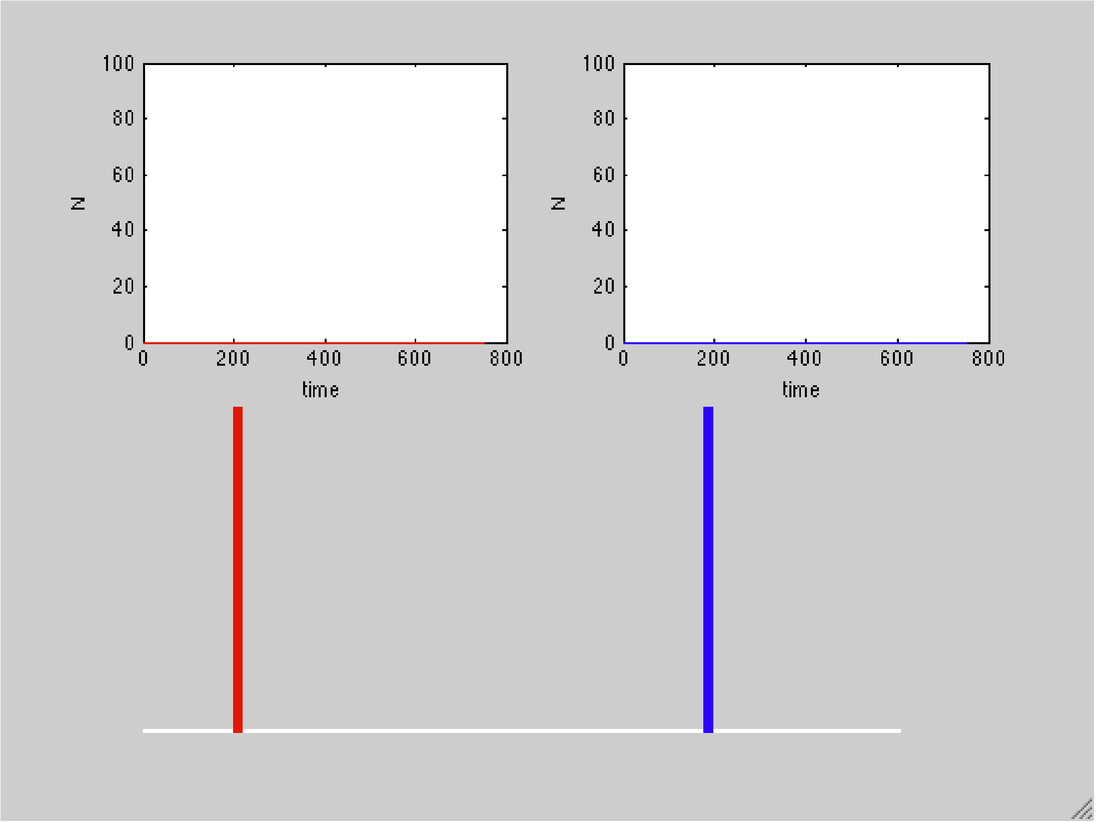
Intermezzo - capacity

- Capacity is the maximum flow on a cross section
- What determines the capacity
- Nr of lanees
- Minimum headway influenced by
- Speed limit
-

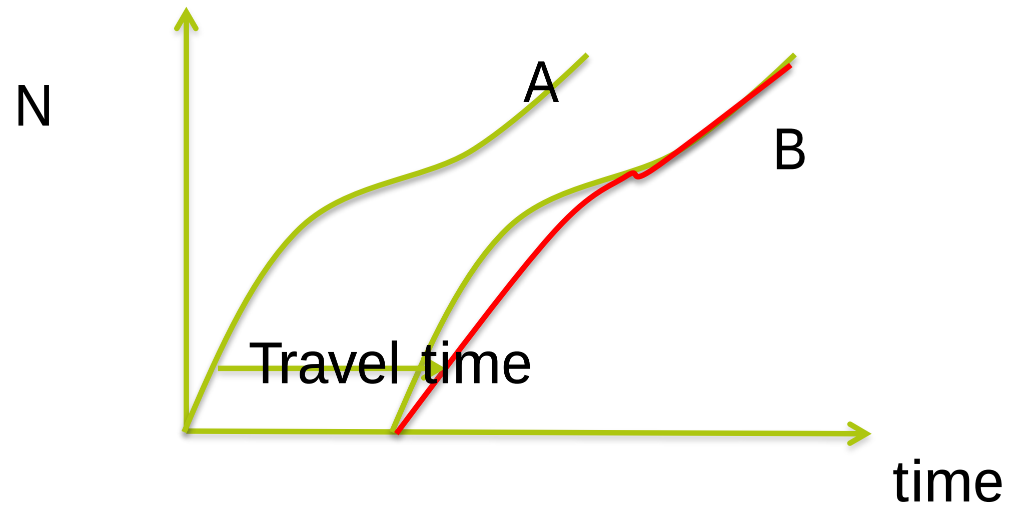
  1.5 s/veh = 2400 veh/h

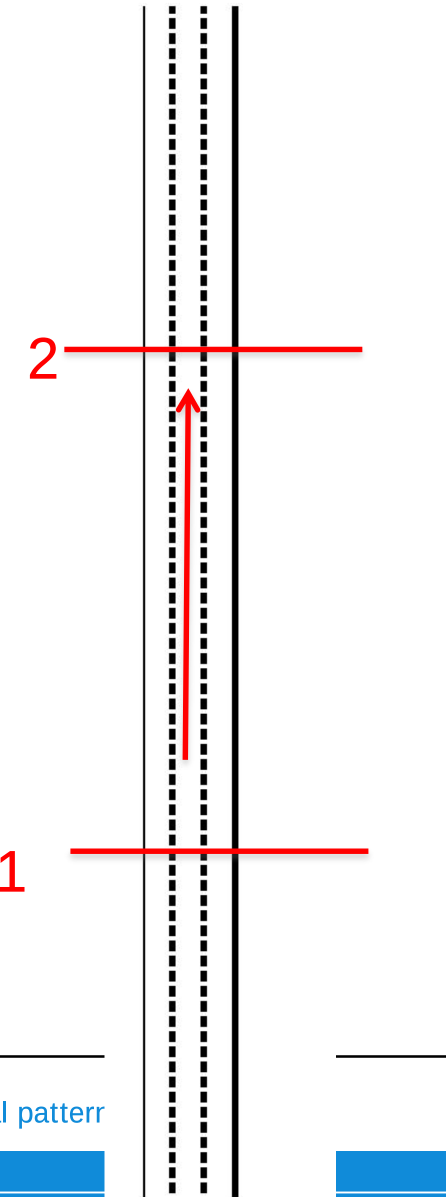


Bottleneck in section



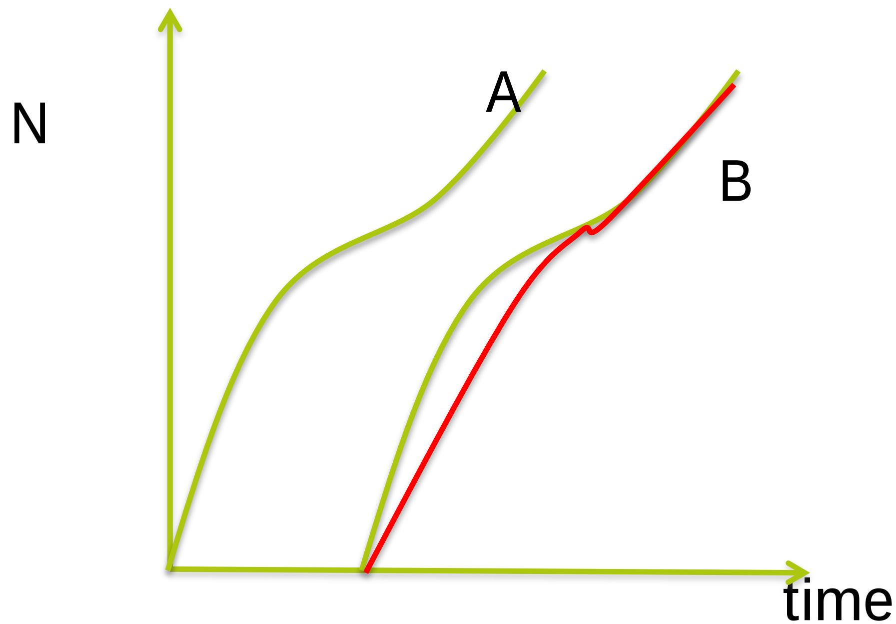
What if bottleneck is present

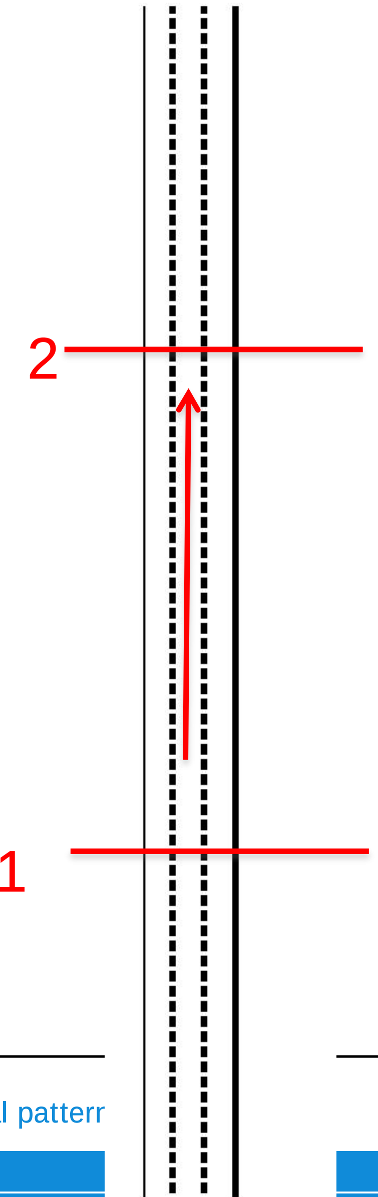




Flow limited to capacity
Travel time increses
More vehicles in the section

Delay = area

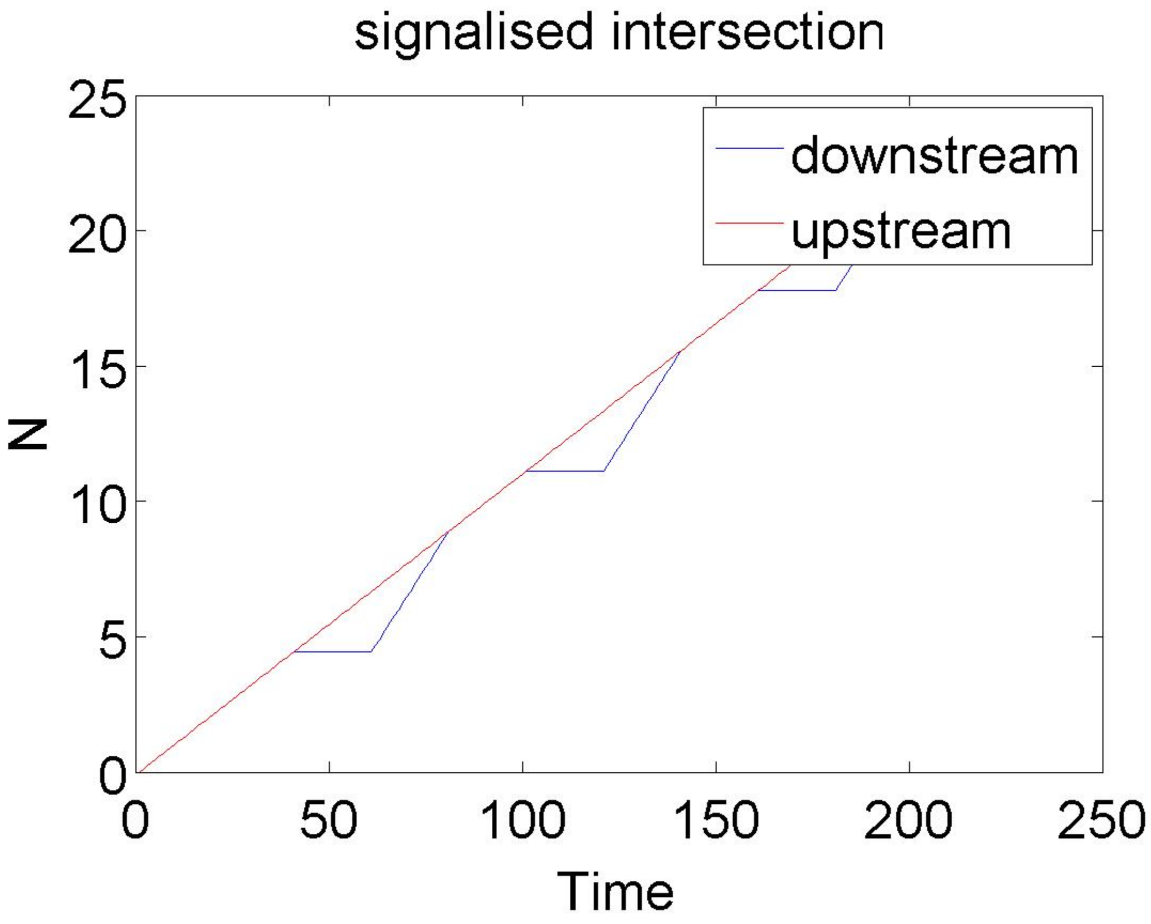




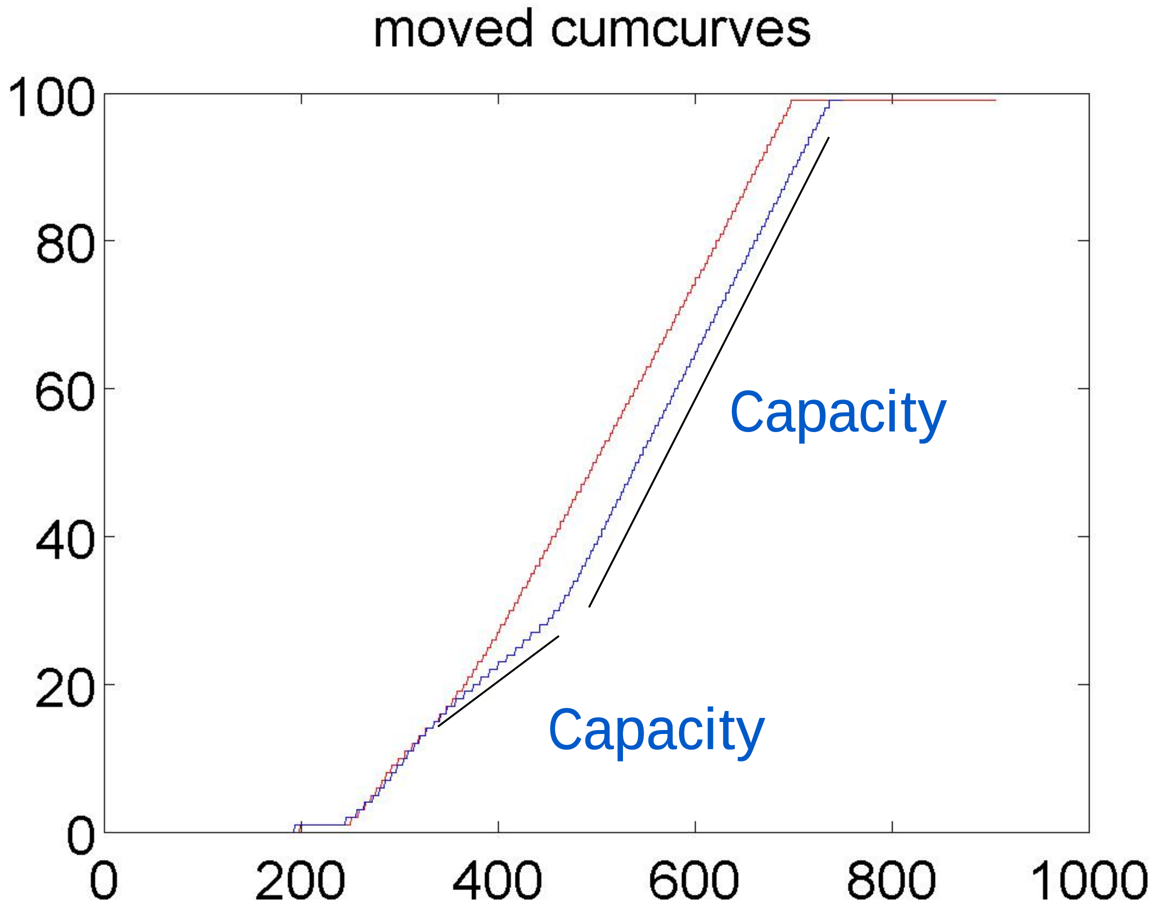
Total delay = sum delay over vehicles

Total delay = sum # extra vehicles in section

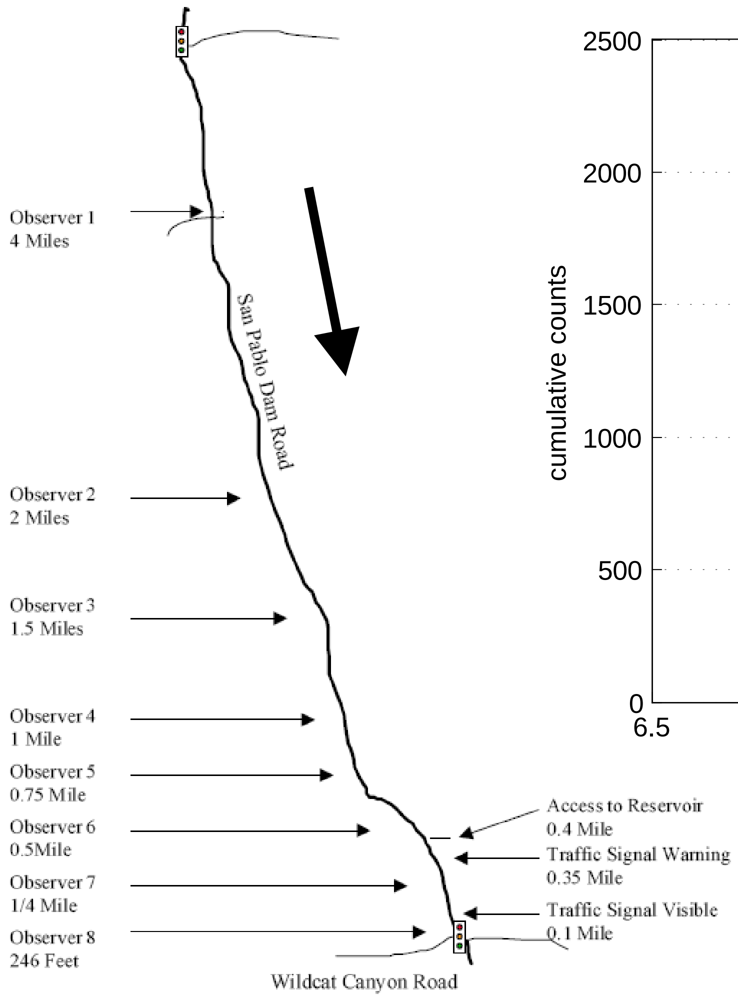
Controlled intersection

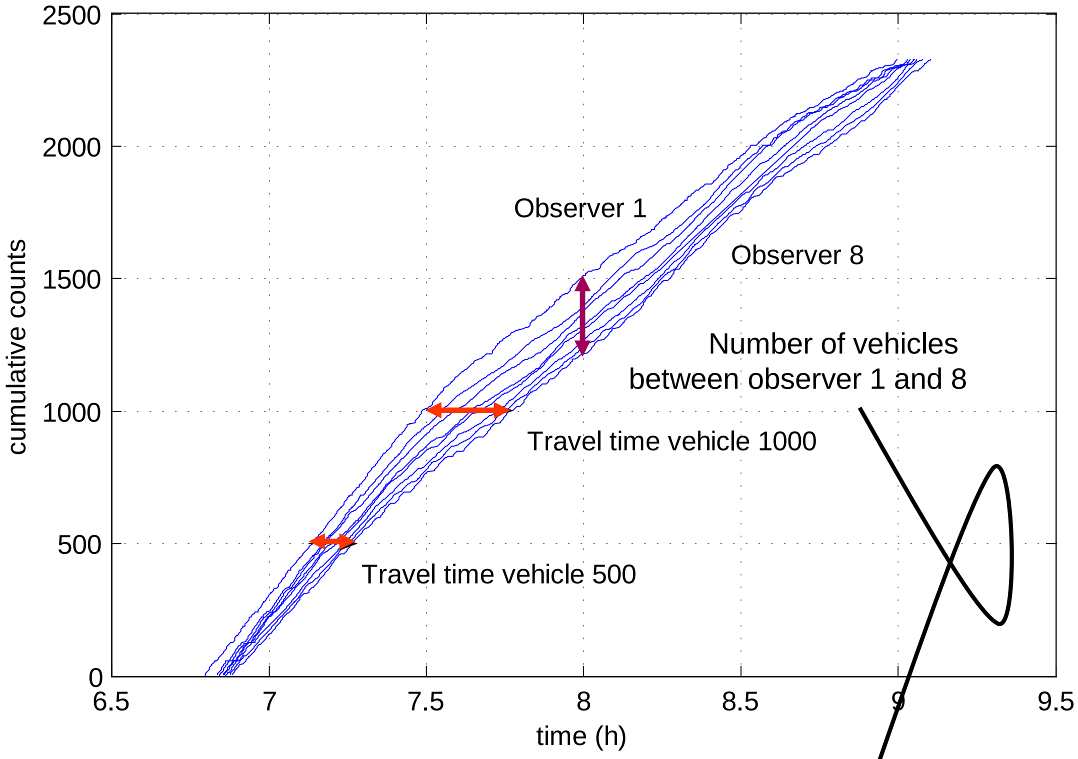


Delay (2)


moved cumcurves
100
80
60
40
20
0
0
200
400
600
800
1000
Capacity
Capacity

Real-life curves

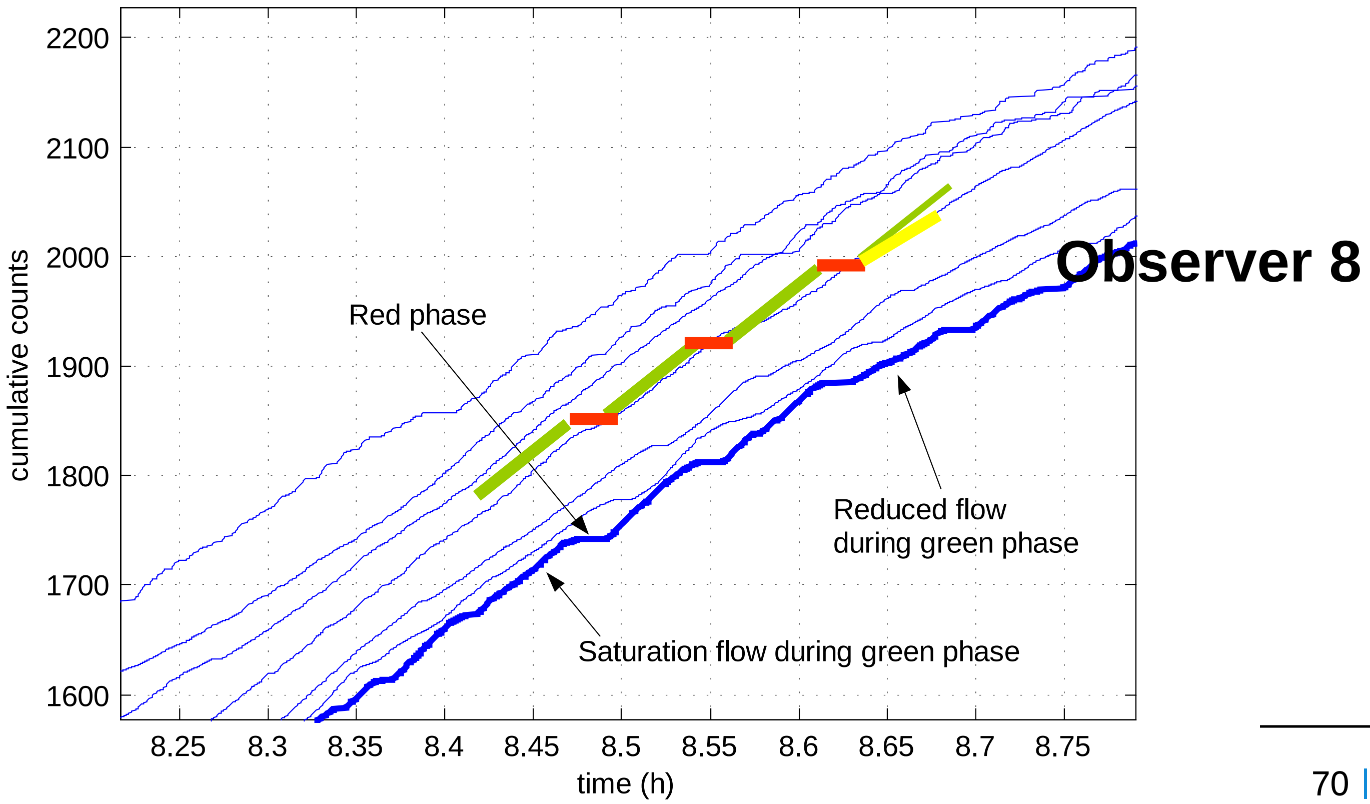




Number of vehicles between observers can be used to determine density!

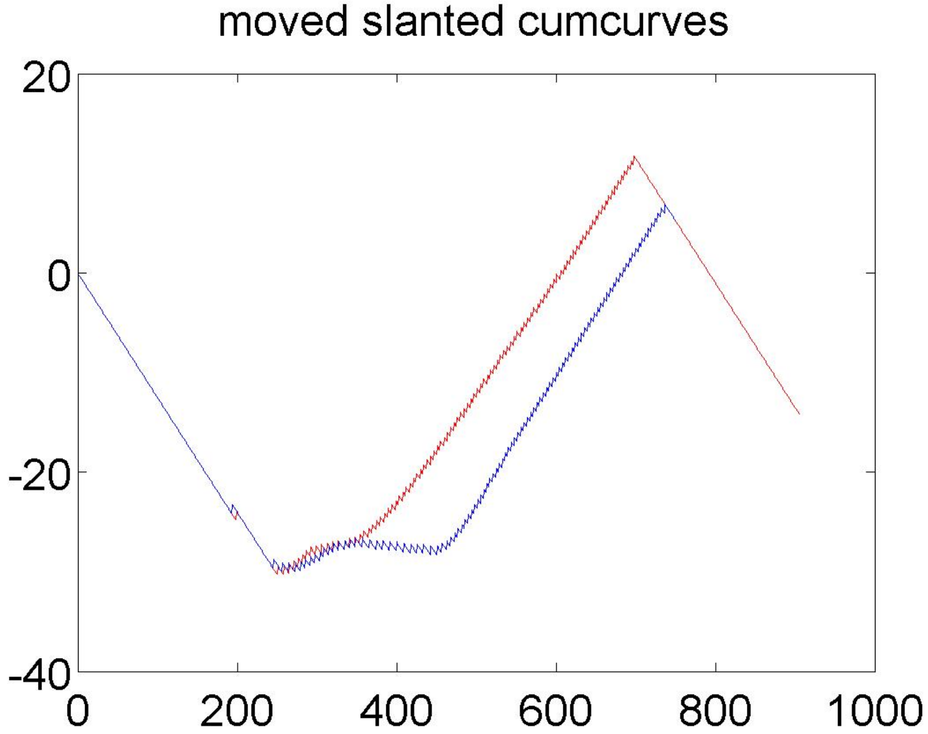
Application

- Applications: identification of stationary periods (constant flow)
- In this case, cumulative curves are (nearly) straight lines



Oblique curves

- Amplify the features of the curves by applying an oblique scaling rate q_0
- Use transformation: $N2=N-q_0t$



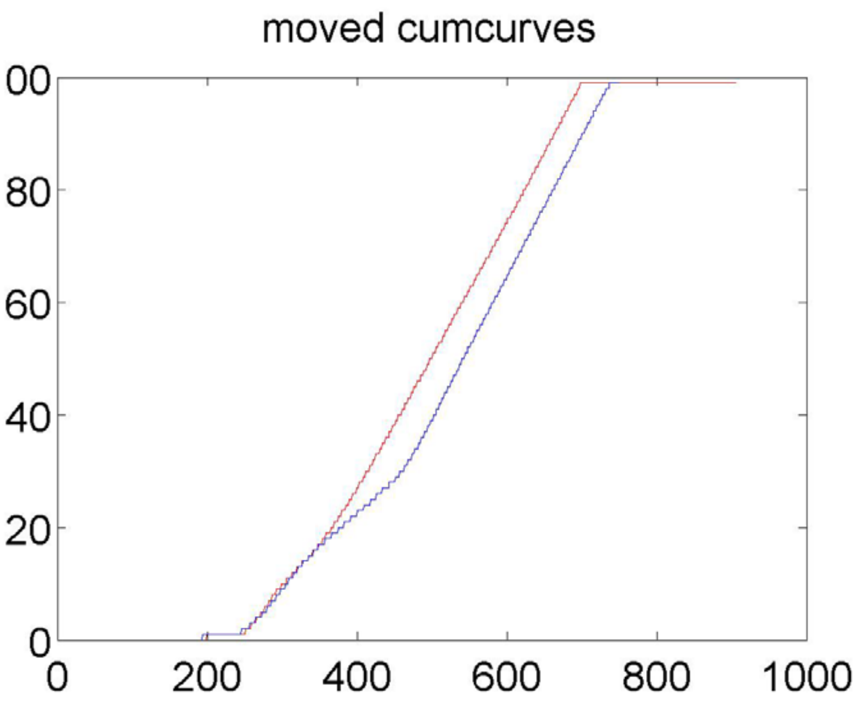
Oblique curves

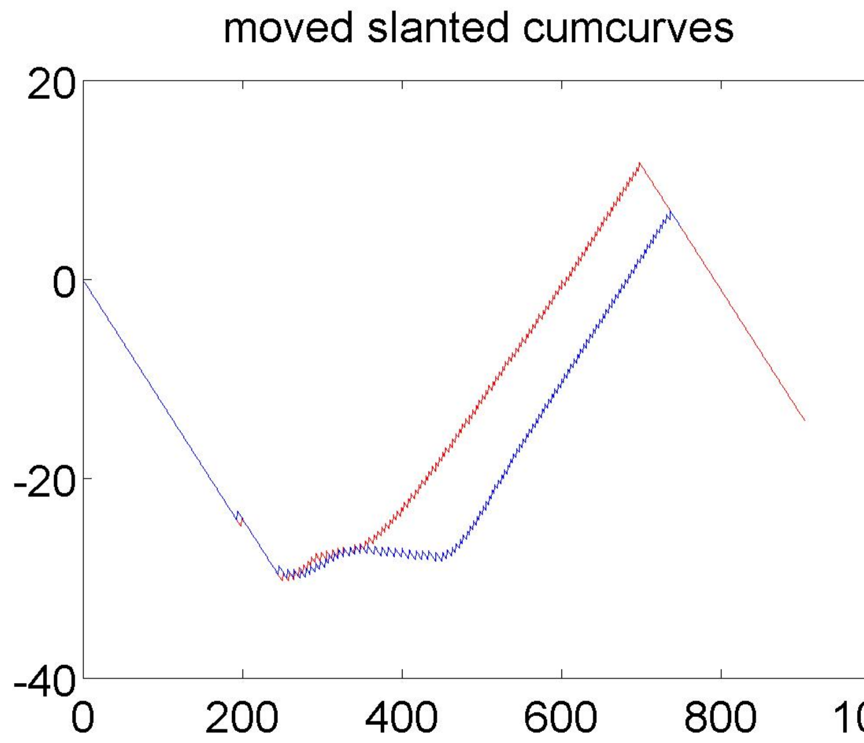
- Notice that density can still be determined directly from the graph; accumulation of vehicles becomes more pronounced
- This holds equally for stationary periods
- For flows note that q_0 needs to be added to the flow determined from the graph by considering the slope op the slanted cumulative curve

Use of oblique curves

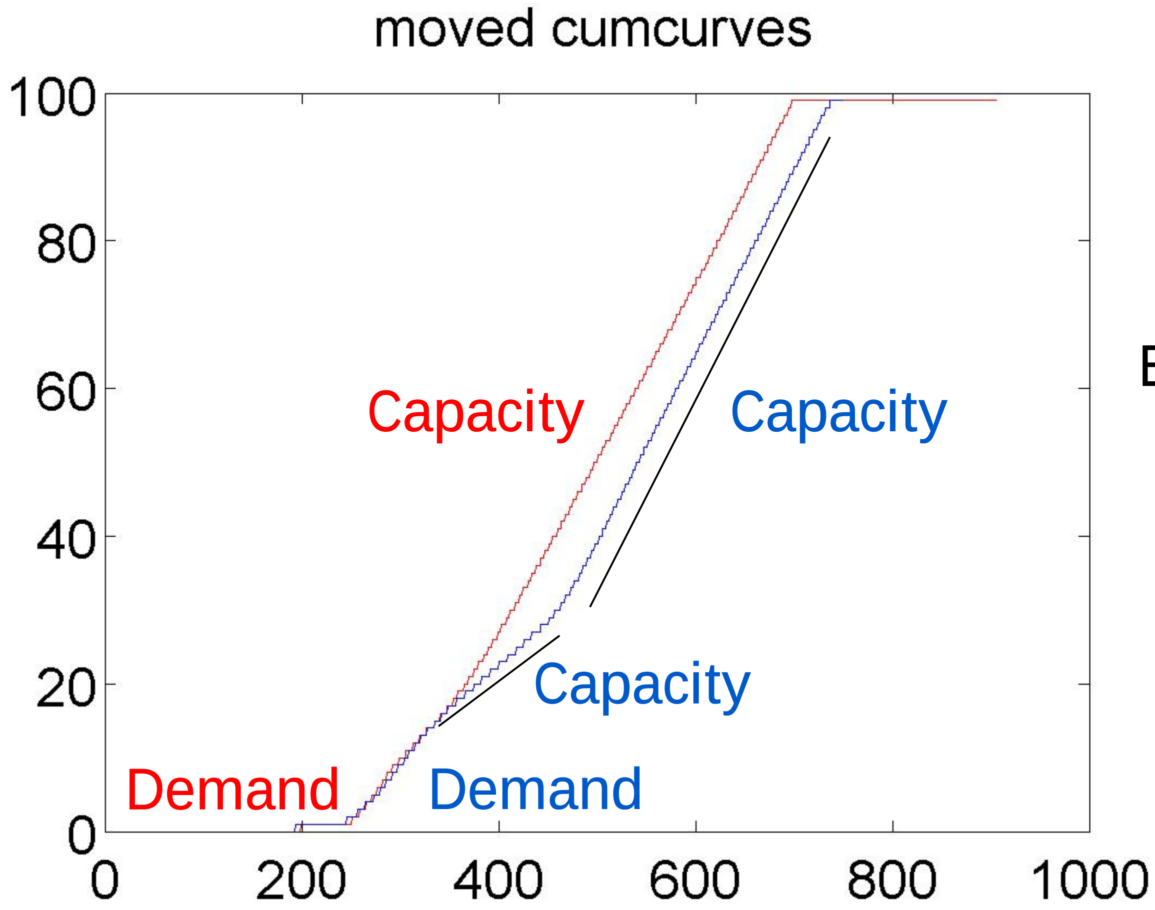
- Can delay be read directly from oblique cumulative curves
  A=yes
  B=no
- Can travel times be read directly from oblique cumulative curves?
  A=yes
  B=no

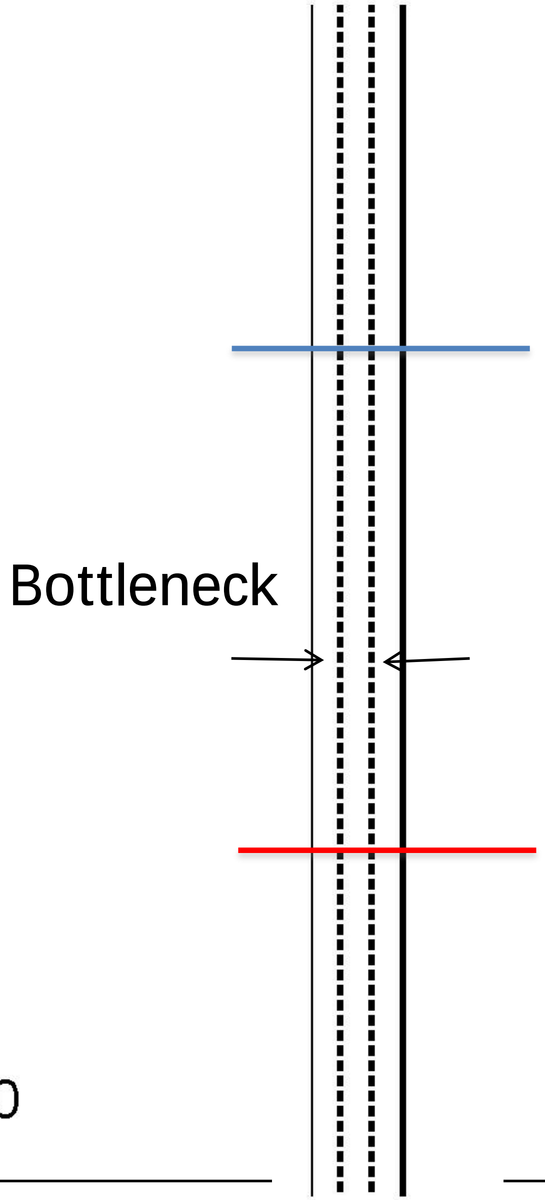
Delay determination simpeler





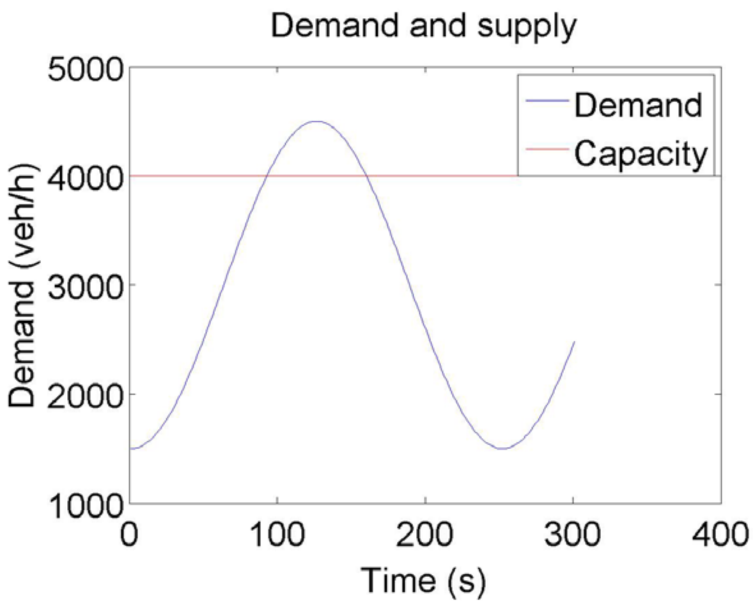
Deeper analysis



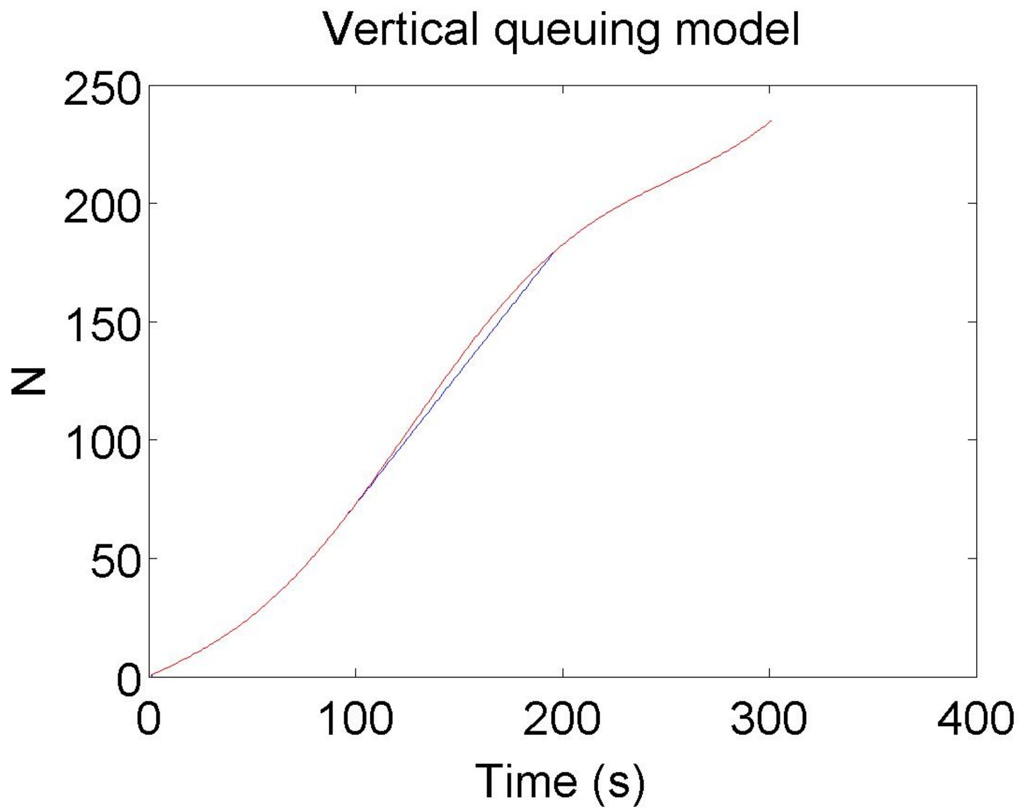


Simplest queuing model

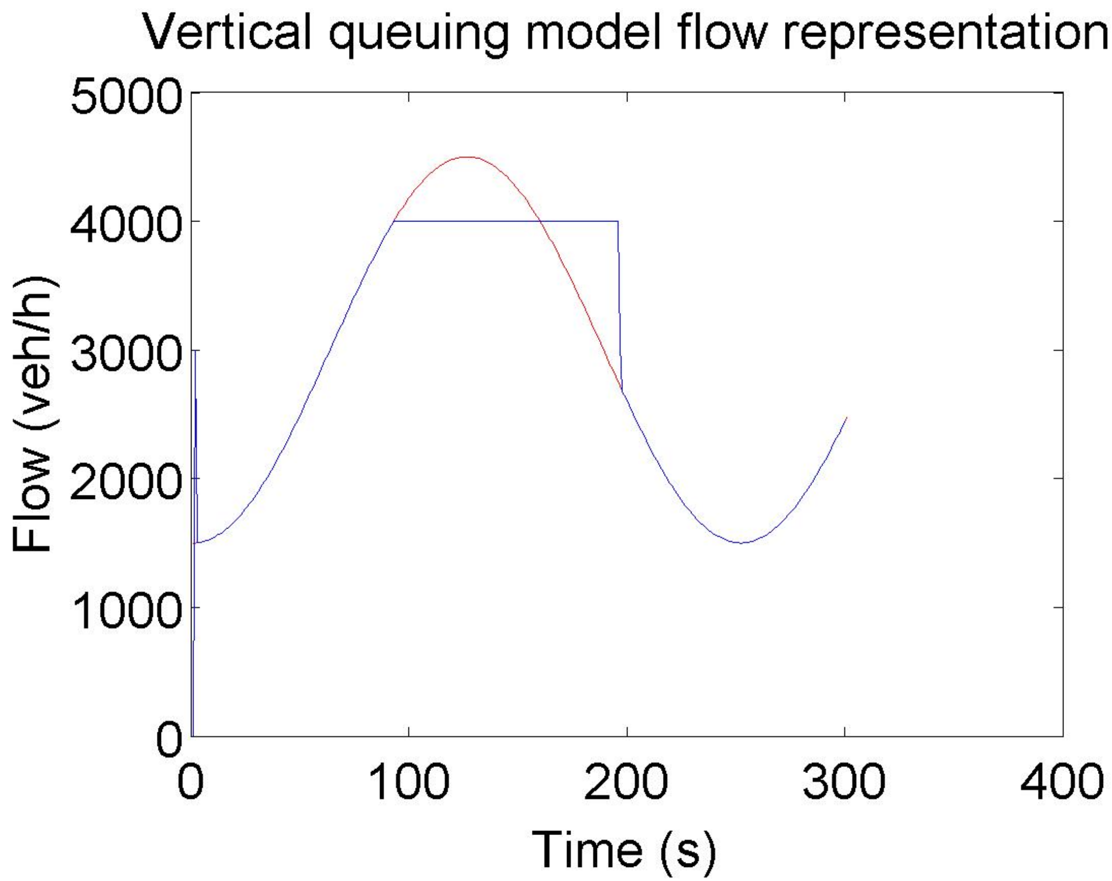
- “Vertical queuing model”
- Given the following demand profile, and capacity, give the (translated) cumulative curves (and determine the delay)



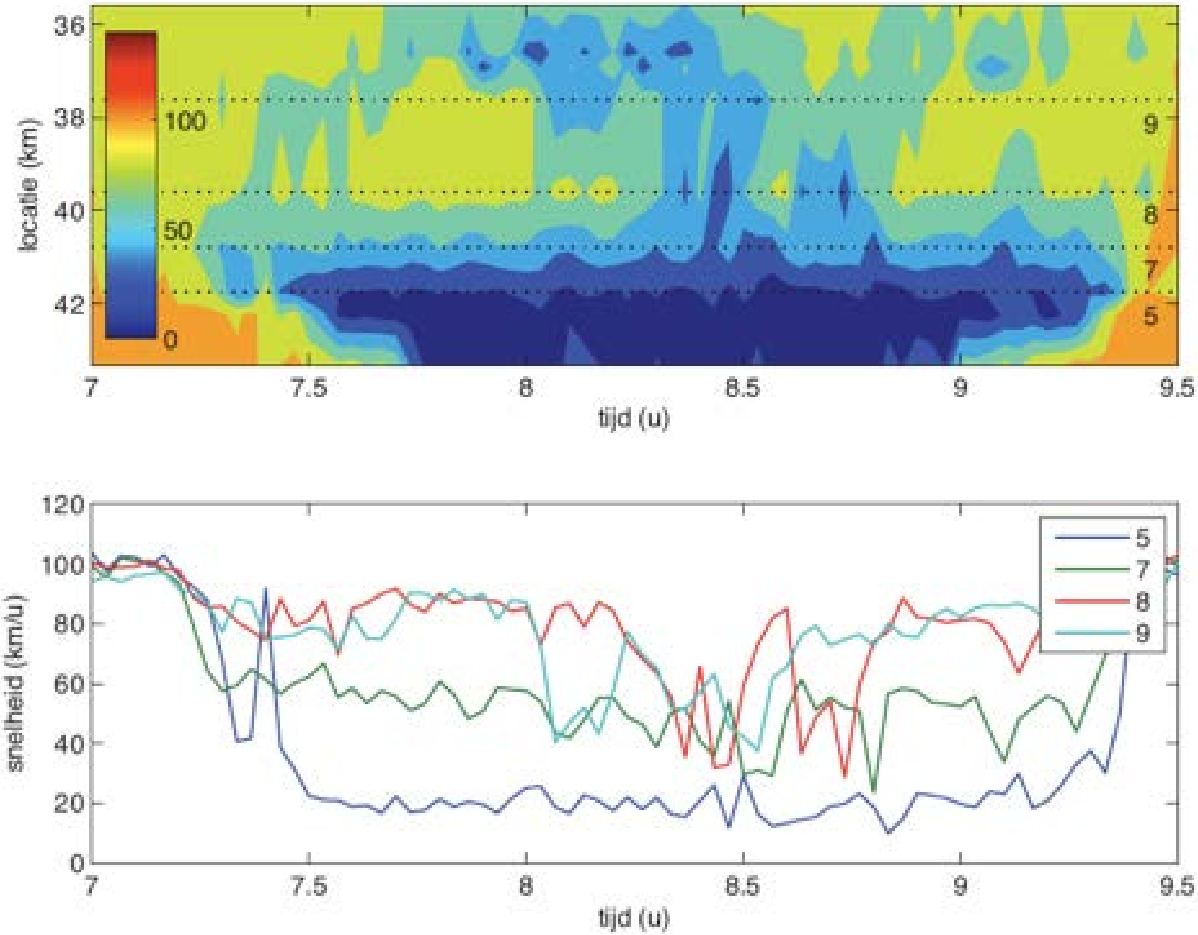
Answer: cumulative curves



Answer: flow representation

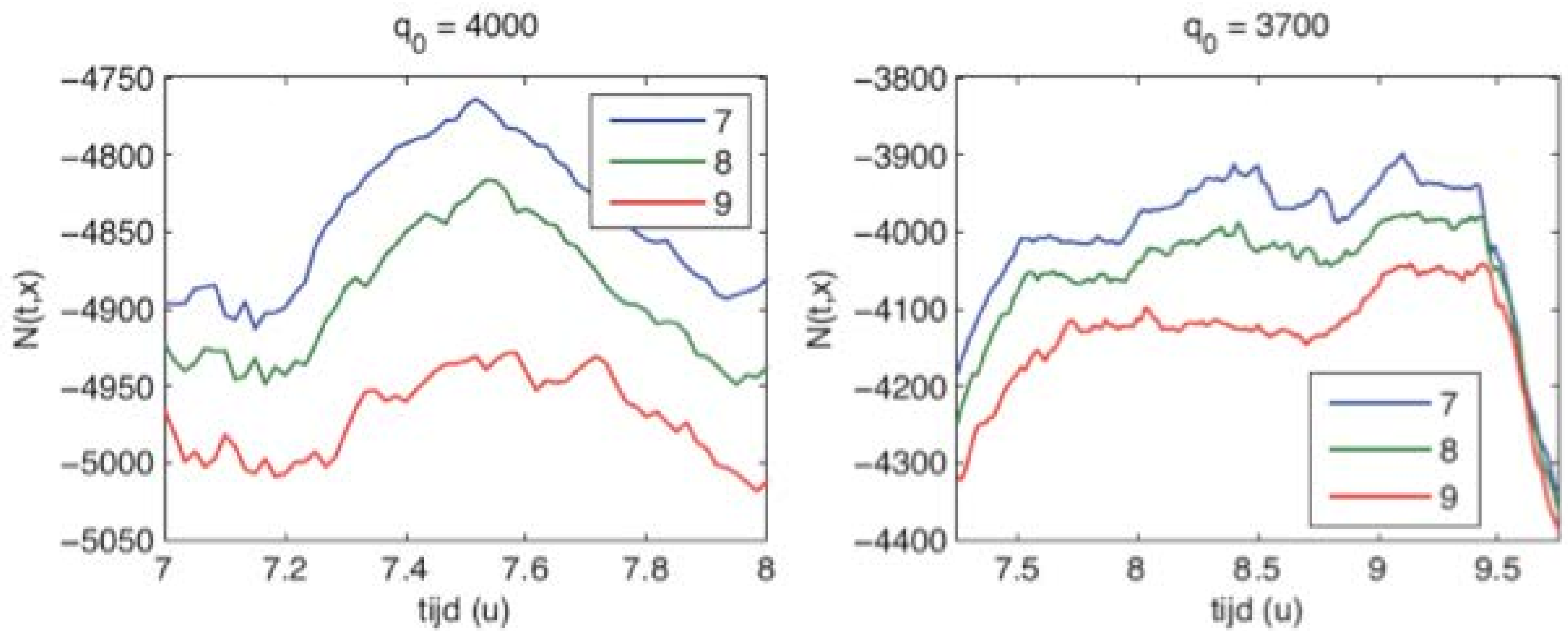


Identifying capacity drop



Identifying capacity drop

- Using oblique (slanted) cumulative curves show that capacity



Learning goals

- You now can:
  - Construct (slanted) fundamental diagrams
  - Use these to calculate:
    delays, travel times, density, flow
- In practice shortcoming:
  Data is corrupt, and errors accumulate

- Test yourself: tomorrow in excersise