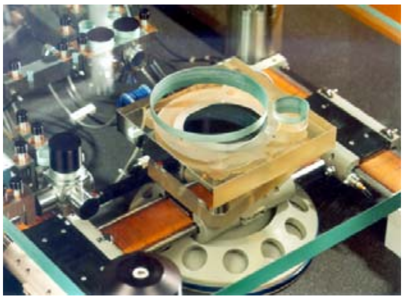
Mechatronic system design

Mechatronic system design wb2414-2013/2014
Course part 2



Signals and dynamic plots

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Mechatronic System Design

Contents

- Signals
  - Periodic, Harmonic signals
  - Fourier analysis
- Dynamic plots
  - Laplace & Fourier transform
  - Time domain
  - Frequency domain

Signals

- A Signal is a function that conveys information about the behavior or attributes of some phenomenon.
- DC (Direct current) signal remains constant in a measured time period. It equals the average value of any signal.
- AC (Alternating current) has periodically alternating negative and positive values at a certain **frequency** (f) It's average value is zero.

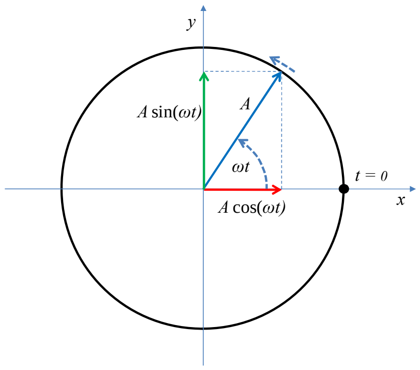
$$f = \text{ amount of events per time (Hz)}$$

$$\text{Period: } T = \frac{1}{f} \quad \text{(s)}$$

- Any signal can be described as a combination of different AC and DC signals, varying over time or space.

Special periodic function: Harmonic oscillations (sine-cosine) Angular frequency

$$\omega = 2\pi f$$



cos and sin have 90 degrees different phase angle

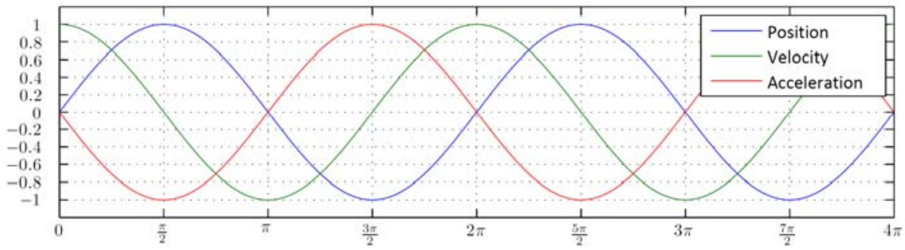
Harmonic movement

$$x(t) = A\sin(\omega t)$$

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) = A\omega\cos(\omega t)$$

$$\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = \ddot{x}(t) = -A\omega^2\sin(\omega t)$$

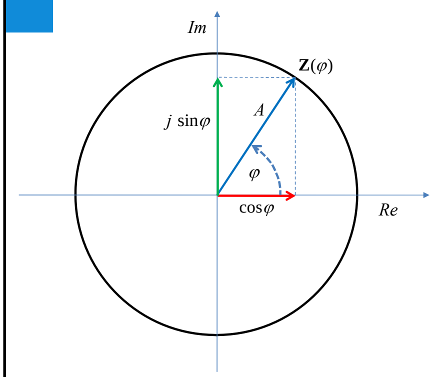
$x(\phi) = x(\omega t)$



$\omega = A = 1$

Derivative gives phase lead → $\phi = \omega t$

Phase relation by means of complex number



Eulers formula

$$Z(\varphi) = A(\cos\varphi + j\sin\varphi) = Ae^{j\varphi}$$

$$Z(\omega t) = A(\cos\omega t + j\sin\omega t) = Ae^{j\omega t}$$

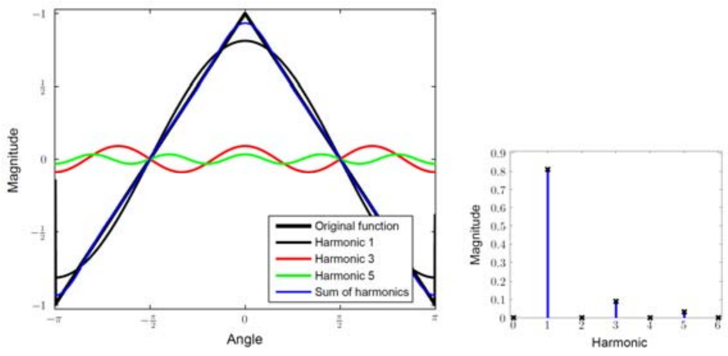
Fourier decomposition of signals

- Periodic signals can be seen as a combination of harmonically related sinusoidal functions
- Fourier decomposition is based on the mathematical fact that the multiplication of one sinusoidal function with another can only give a non-zero average value over time when they have an equal frequency and phase

$$\sin(\omega_1 t)\sin(\omega_2 t) = \frac{-\cos(\omega_1 t + \omega_2 t) + \cos(\omega_1 t - \omega_2 t)}{2}$$

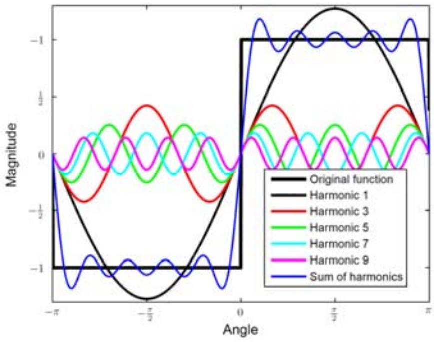
Triangle waveform

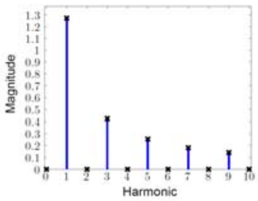
$$\hat{f}(t) = \frac{8}{\pi^2}\left(\cos(\omega t) + \frac{1}{3^2}\cos(3\omega t) + \frac{1}{5^2}\cos(5\omega t) + ...\right)$$



Square wave form

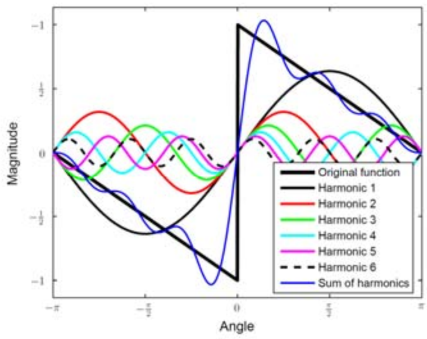
$$\hat{f}(t)=\frac{4}{\pi}\left(\sin(\omega t)+\frac{1}{3}\sin(3\omega t)+\frac{1}{5}\sin(5\omega t)+.....\right)$$

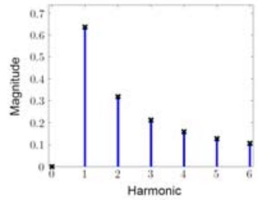




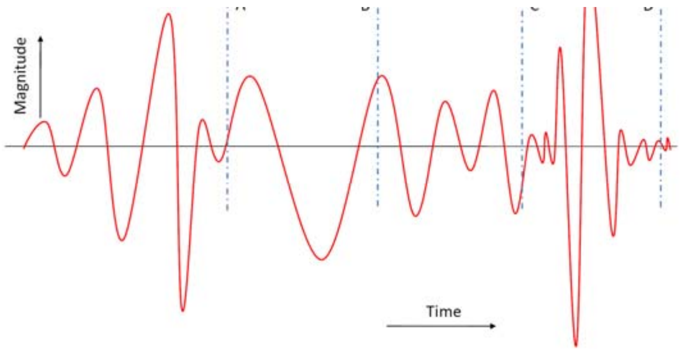
Sawtooth waveform

$$\hat{f}(t)=\frac{2}{\pi}\left(\sin(\omega t)+\frac{1}{2}\sin(2\omega t)+\frac{1}{3}\sin(3\omega t)+.....\right)$$



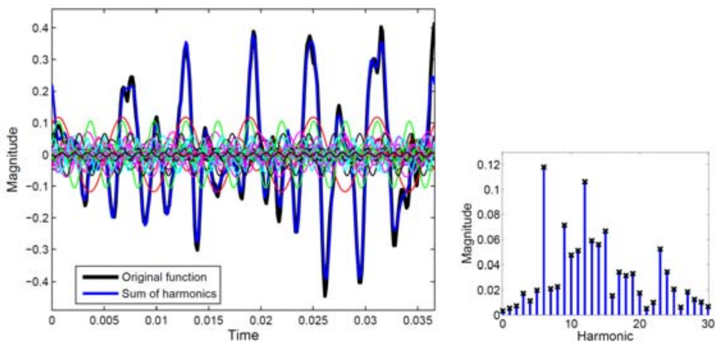


Random signals

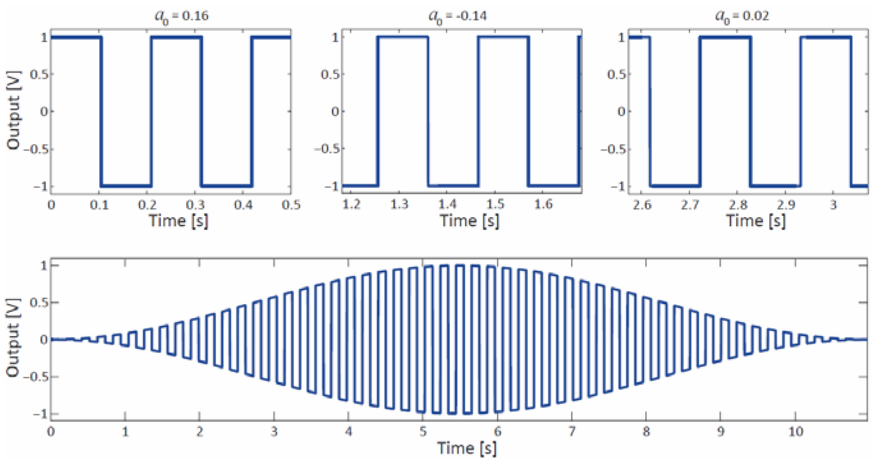


Fast Fourier Transform

Mind the window!



Window functions



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Laplace transform, first step from the time into the frequency domain via the transfer function

$$f(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt \qquad s = \sigma + j\omega$$

$$\mathrm{F_d}(s) = \mathcal{L}\left\{\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right\} = sx(s) - x(0)$$

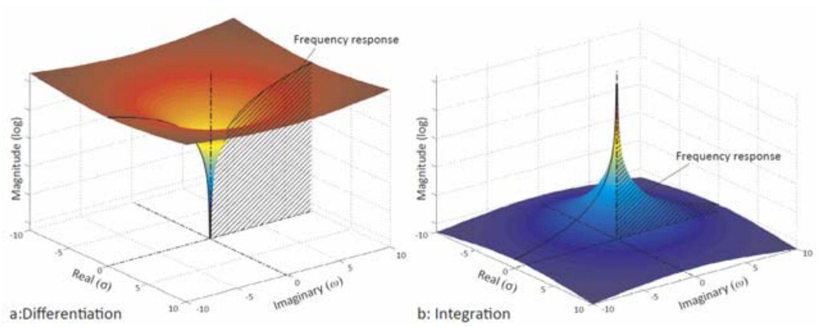
$$\mathrm{F_i}(s) = \mathcal{L}\left\{\int_0^{t'} x(t)\mathrm{d}t\right\} = \frac{x(s)}{s}$$

Laplace domain = complex plane with poles and zeros. Fourier transform gives frequency response

$$s = \sigma + j\omega$$

$$\mathrm{F_d}(\omega) = \mathcal{F}\left\{\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right\} = j\omega x(\omega)$$

$$\mathrm{F_i}(\omega) = \mathcal{F}\left\{\int_0^{t'} x(t)\mathrm{d}t\right\} = \frac{x(\omega)}{j\omega} = -j\frac{x(\omega)}{\omega}$$



With a phase change depending on the location of s

$$F_d(s) = \mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sx(s) - x(0)$$

$$F_d(\omega) = \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega x(\omega)$$

$$F_i(s) = \mathcal{L}\left\{\int_0^{t'} x(t)dt\right\} = \frac{x(s)}{s}$$

$$F_i(\omega) = \mathcal{L}\left\{\int_0^{t'} x(t)dt\right\} = \frac{x(\omega)}{j\omega} = -j\frac{x(\omega)}{\omega}$$

Derivative gives phase lead (+*j*), Integral gives phase lag (-*j*)

Multiply with *s* ➔ 90° phase lead, divide by *s* ➔ 90° phase lag

Test and analysis stimuli of dynamic systems

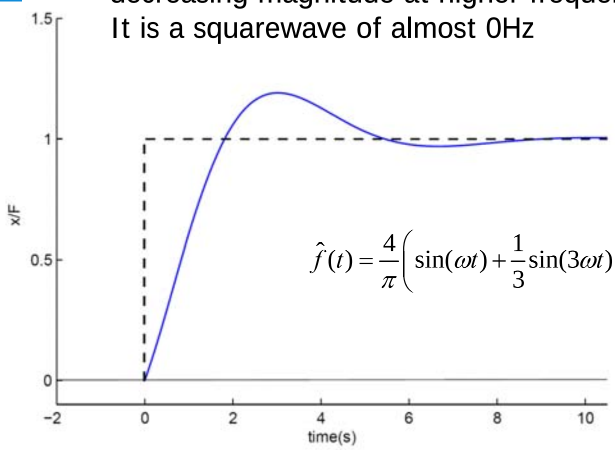
- Frequency domain related responses
  - Step and impulse
  - Frequency sweep
  - Noise
  - Multi-sines

Stimuli in the frequency domain

- Step and impulse
  - Need for limitation of force
- Frequency sweep
  - Not too fast to enable stabilising of resonances (energy)
- Noise
  - White noise, constant “Power Spectral Density”
- Multi-sines
  - Tuneable to frequency area of interest

Step response

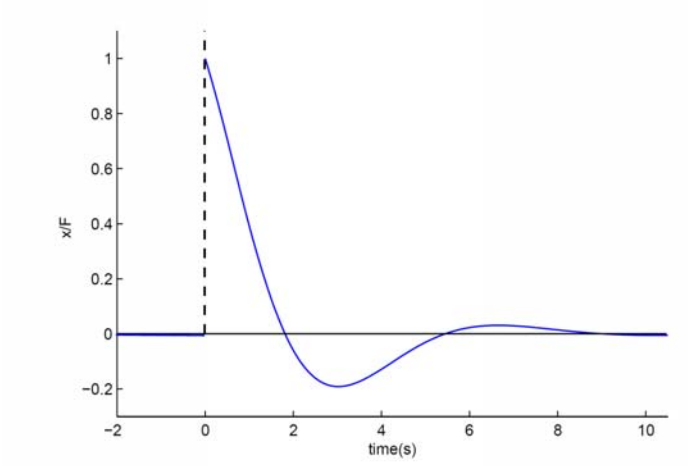
Fourier analysis of a unit step gives wide spectrum with decreasing magnitude at higher frequencies
It is a squarewave of almost 0Hz



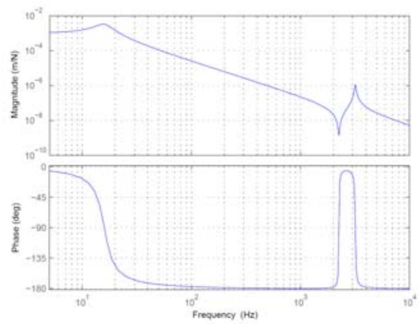
$$\hat{f}(t) = \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + ...\right)$$

Impulse time response

Fourier analysis of impulse is constant amplitude density over all frequencies (linear scale).

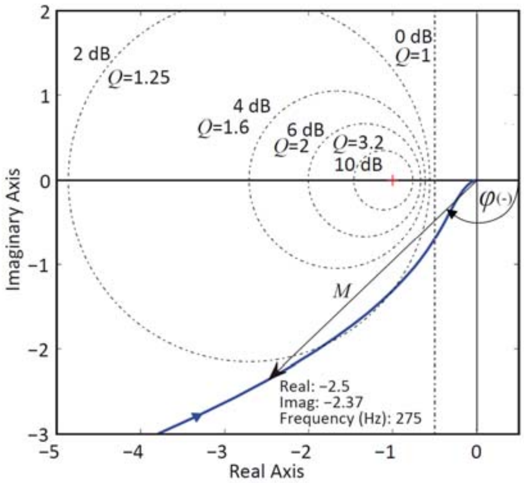


Bode plot



- Magnitude and phase
- Output versus input
- Output and input can be different (Force vs displacement)
- Double logarithmic scale loglog 1/x = straight line -1
- Horizontal frequency axis in <u>Hz</u> or rad/s
- Vertical axis magnitude <u>abs</u> or in dB
- Vertical axis phase in degrees

Nyquist plot, the most important of all!



And don't show the negative frequencies!