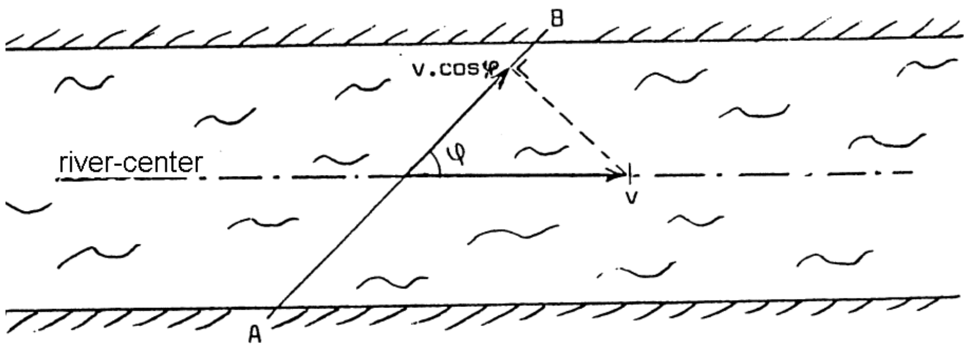
# Hydrological Measurements

Wim Luxemburg

2. Ultra-sonic stream flow measurement
   Integrating rising bubble technique
   Discharge dilution gauging
   Moving boat
   Discharge structures
   Theory of errors
   Heat as a tracer

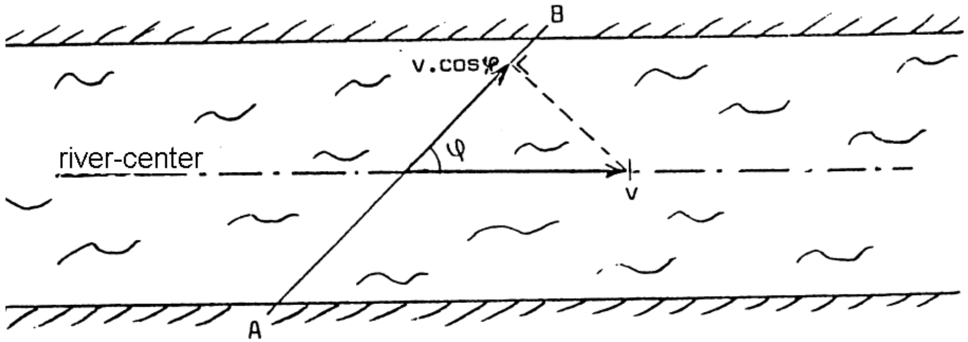
# Ultra-sonic streamflow measurement




Faster with current :

$$L_{AB} = (c + v \cdot \cos\varphi) \cdot T_{AB}$$

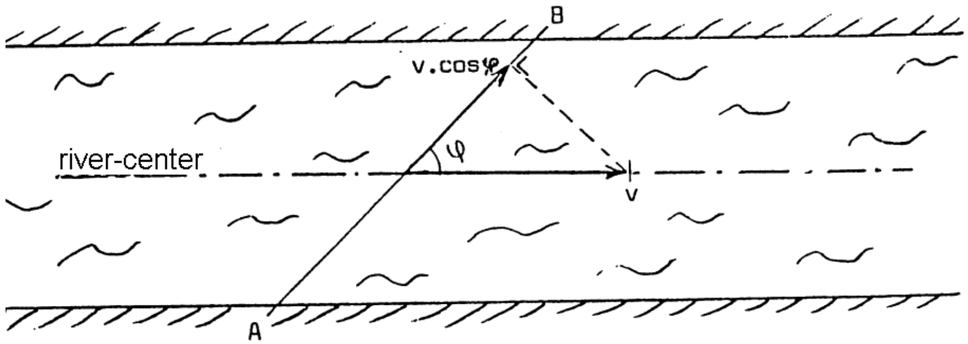
Ultra-sonic streamflow measurement




Slower against current:

$$L_{BA} = (c - v \cdot \cos\varphi) \cdot T_{BA}$$

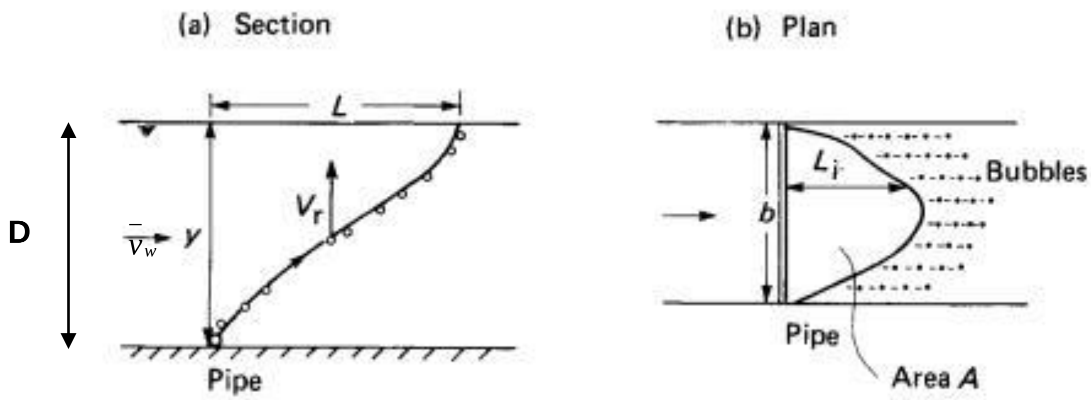
# Ultra-sonic streamflow measurement




Solve and eliminate c:

$$v = \frac{L}{2 \cdot \cos\varphi} \cdot \left( \frac{1}{T_{AB}} - \frac{1}{T_{BA}} \right)$$

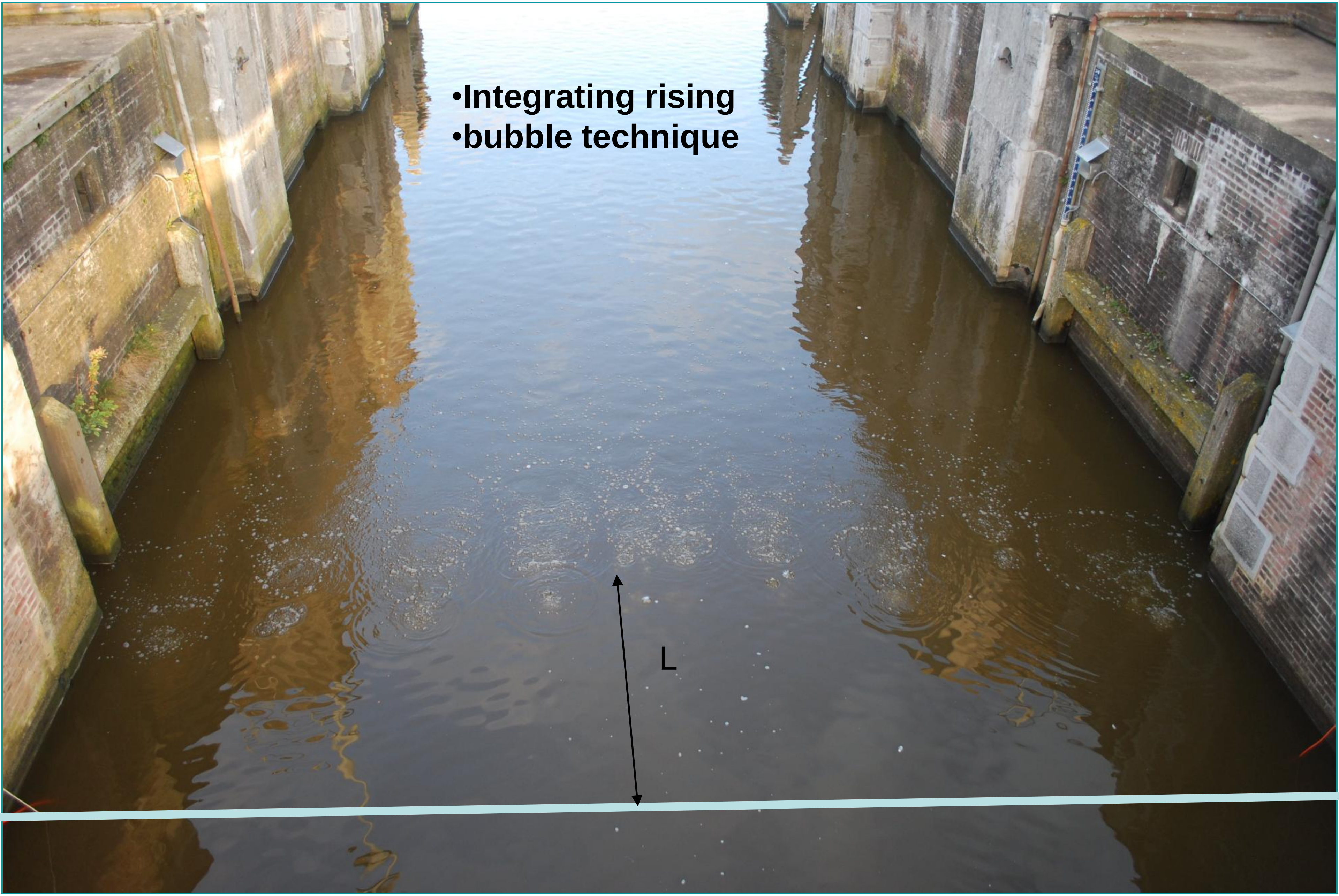
# Integrating rising bubble technique




$$T = \frac{L}{\bar{v}_w} = \frac{D}{v_r}$$

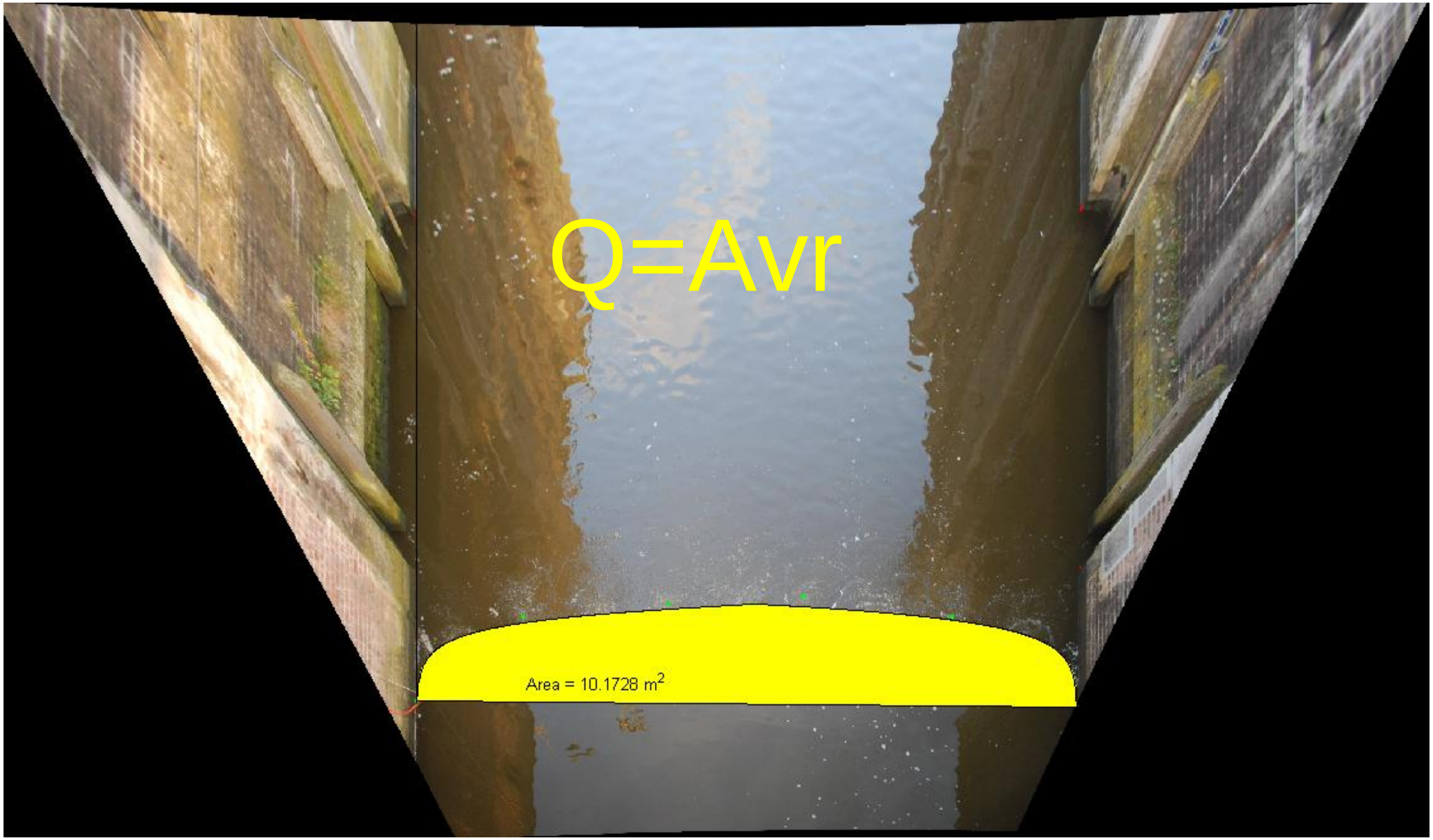
$$q = \bar{v}_w D = v_r L$$

$$Q = v_r A$$


•Integrating rising
•bubble technique
L

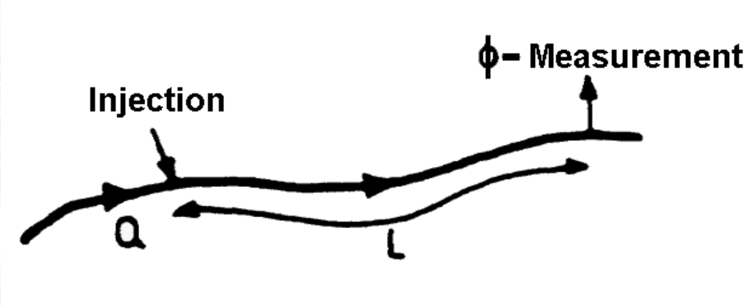

L

# Bubble gauging
## Digital picture corrected for distortions



Discharge & Streamflow measurements

- Dilution gauging




Principle:

-) Adding of known amount of tracer to the stream
   Method 1: Constant rate injection
   Method 2: Sudden injection

-) Measurement of concentration downstream

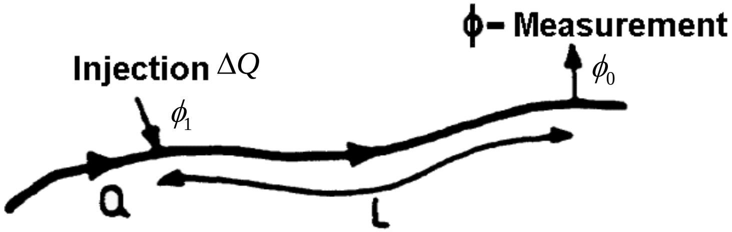
$$L_{0,95} = \frac{0,4}{\alpha} \cdot \frac{C}{\sqrt{g}} \cdot \frac{B^2}{a}$$

Discharge & Streamflow measurements

- Dilution gauging, **constant rate injection**

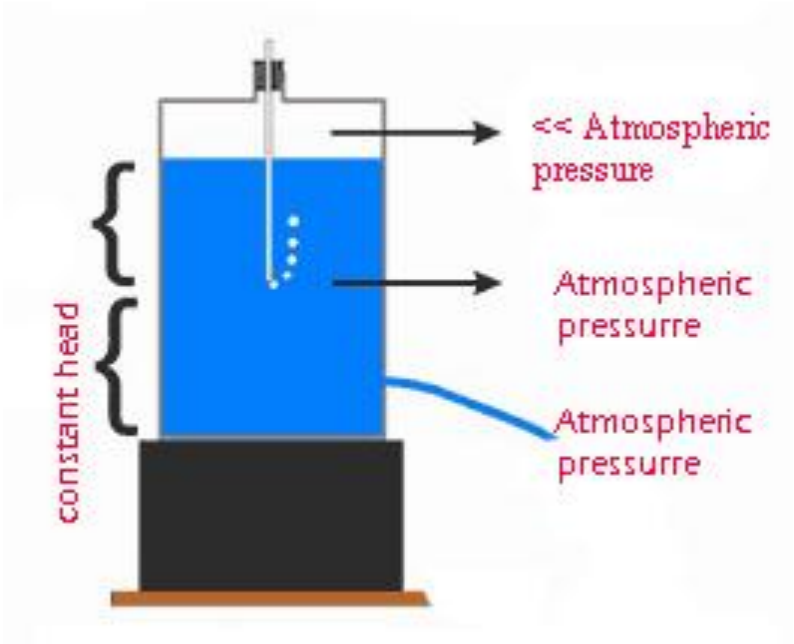
Mass-balance of the concentrations when $\Delta Q \ll Q$:

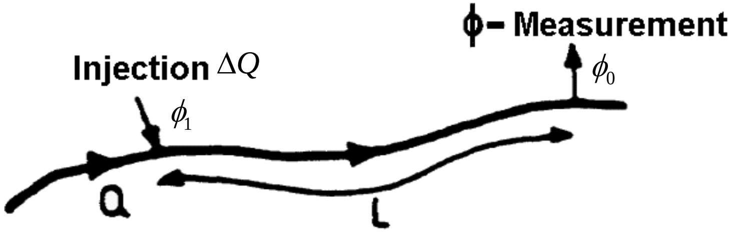
$$\phi_0 = \frac{\Delta Q}{Q} \phi_1$$



Discharge & Streamflow measurements

- Dilution gauging, **constant rate injection**

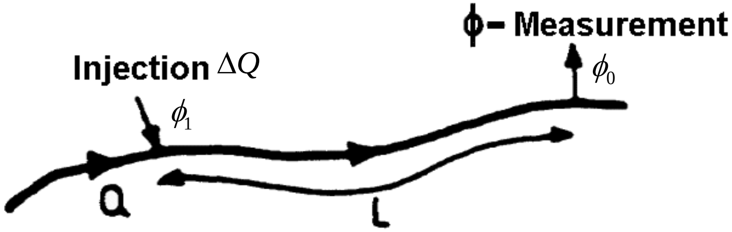


Discharge & Streamflow measurements

- Dilution gauging, **mixing requirements**

$$L_{0,95} = \frac{0,4}{\alpha} \cdot \frac{C}{\sqrt{g}} \cdot \frac{B^2}{a}$$

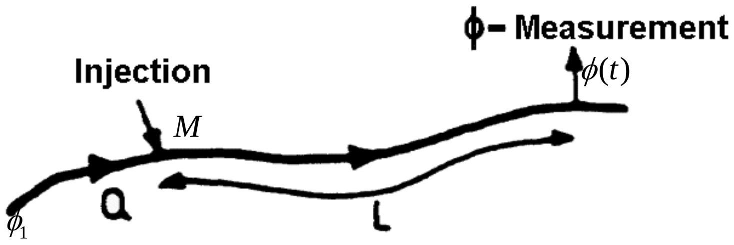



Discharge & Streamflow measurements

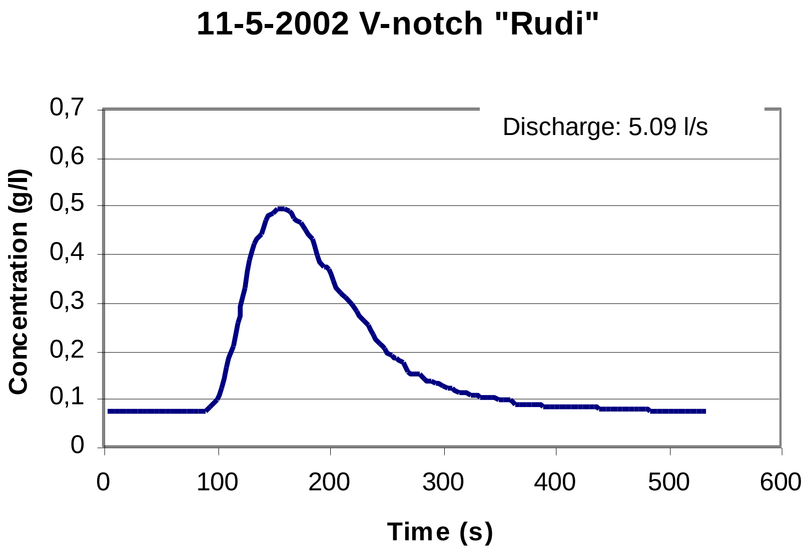
- Dilution gauging, **<u>sudden injection</u>**


Discharge & Streamflow measurements

- Dilution gauging, **sudden injection**

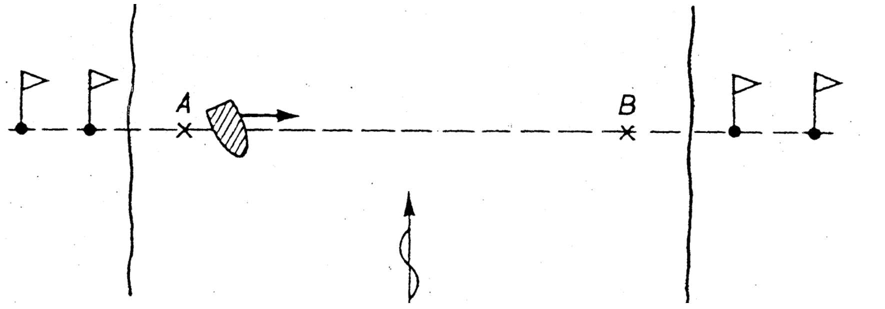



$$M = Q\int_{0}^{\infty}\{\phi(t) - \phi_1\}dt$$



# Moving boat discharge measurements:

Discharge measurement
whilst crossing a river in a boat



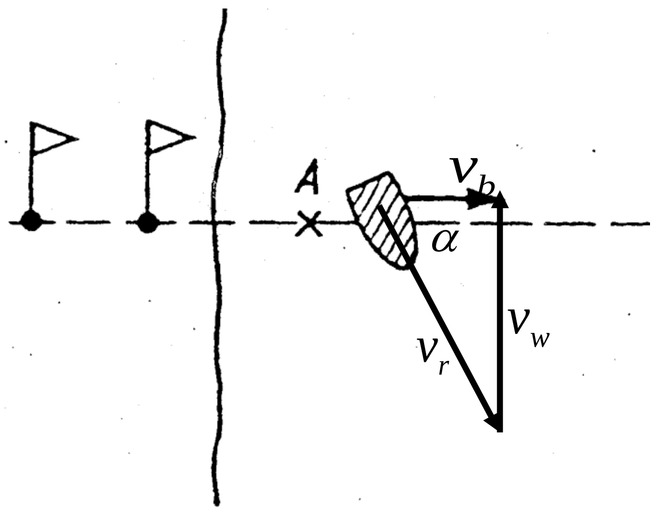

Measurement of:

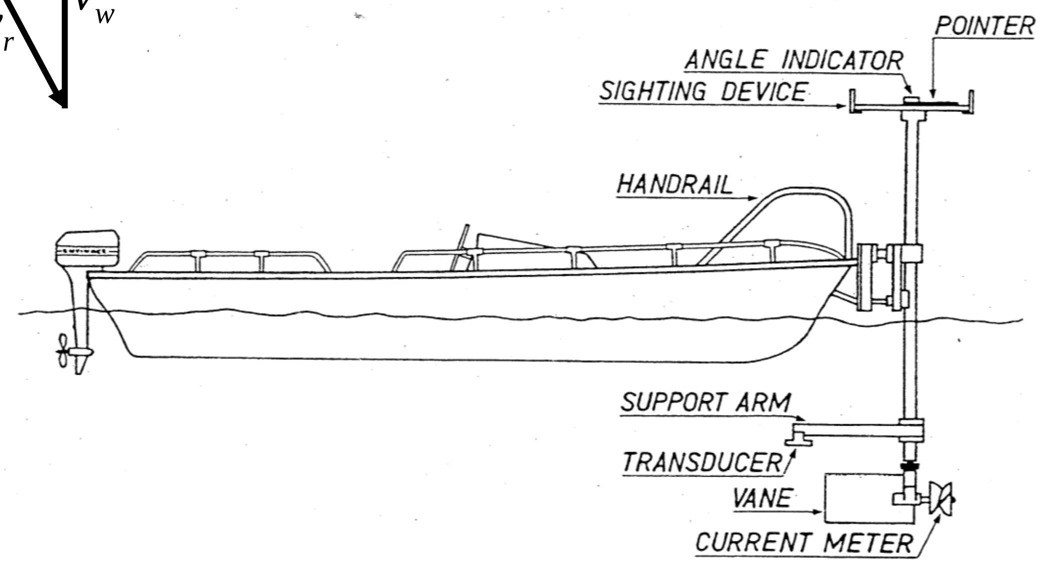
1) position in the stream (two methods)
2) stream depth (echo depth sounder)
3) velocity (e.g. current meter)

Moving boat discharge measurements

Method 1

- Positioning by measuring angle between boat and cross section
  (at time interval Δt)



Moving boat discharge measurements

Method 1

- Positioning by measuring angle between boat and cross section (at time interval Δt)

calculation steps:

$$v_b = v_r * \cos\alpha$$

$$\Delta L = v_b * \Delta t$$

$$v_w = v_r * \sin\alpha$$

$$\Delta A = \Delta L * d$$

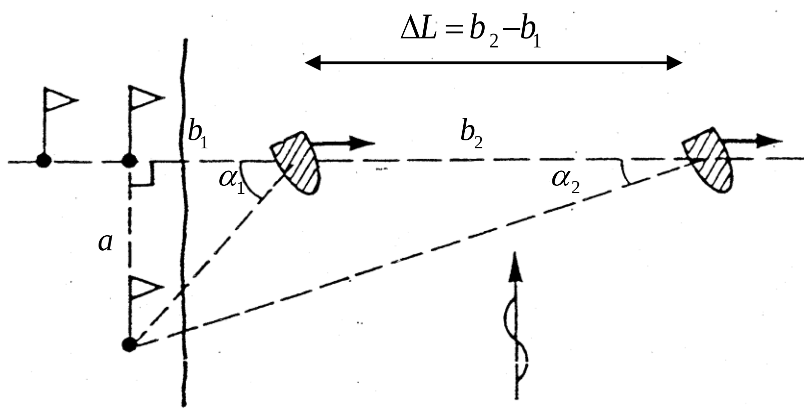
$$\Delta Q = \Delta A * k_v * v_w$$

$$Q_{AB} = k_l * \sum_n \Delta Q$$

Moving boat discharge measurements

Method 2

- Positioning relative to beacons on the shore
  (at time interval Δt)

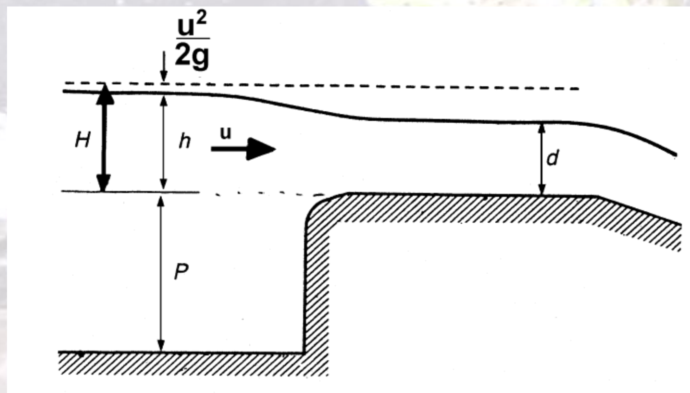



$$v_b = \frac{\Delta L}{\Delta t}$$

$$v_w = \sqrt{{v_r}^2 - {v_b}^2}$$

# DISCHARGE STRUCTURES

Principles free (critical) flow




$$H = d_c + \frac{u^2}{2 \cdot g} \qquad Q = b.d_c.\bar{u} \qquad d_c = \frac{2}{3} H$$

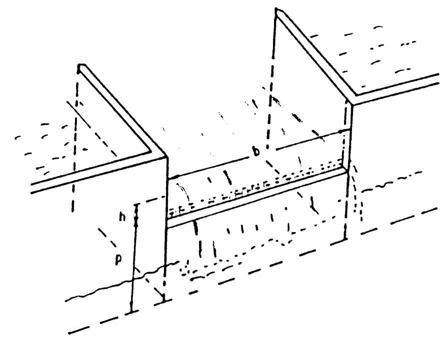
$$\longrightarrow \quad Q = \left(\frac{2}{3}\right)^{3/2} \cdot b \cdot \sqrt{g} \cdot H^{3/2}$$

Categories discharge structures

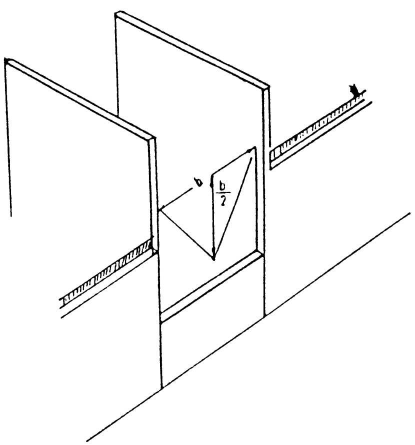
- Thin/sharp crested weirs
- Broad crested weirs
- Flumes
- Compound measuring structures
- Non-standard weirs.

# Examples sharp crested weirs:

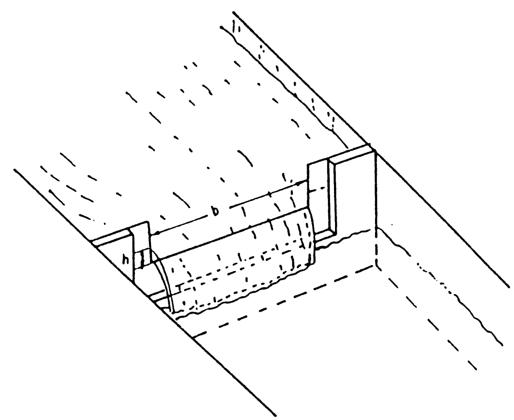
V- notch




Rectangular suppressed notch



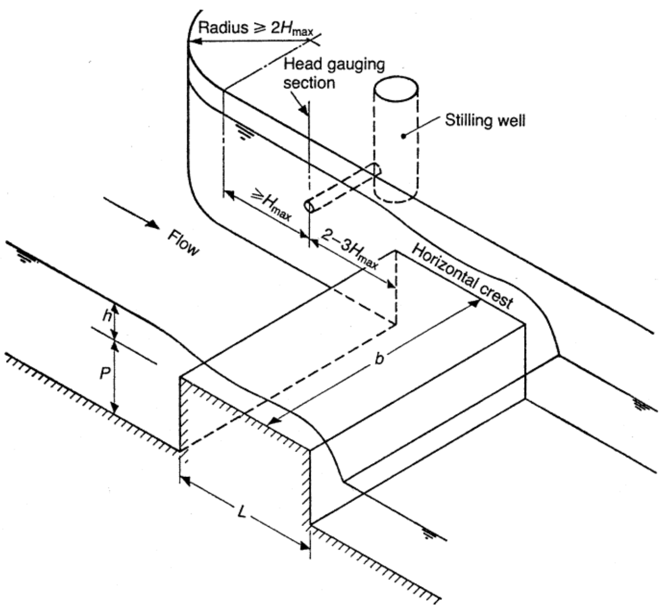

V notch installed with training walls

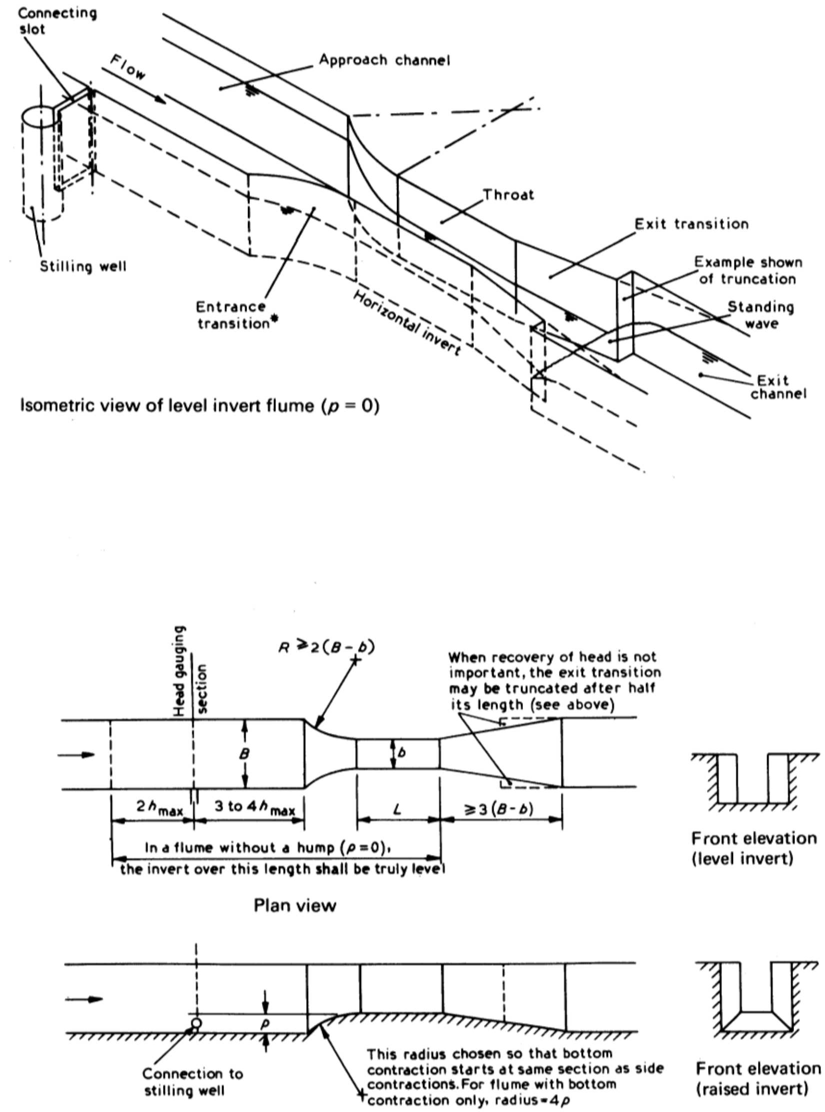



Rectangular compressed notch

# Flume

# Broad crested weir



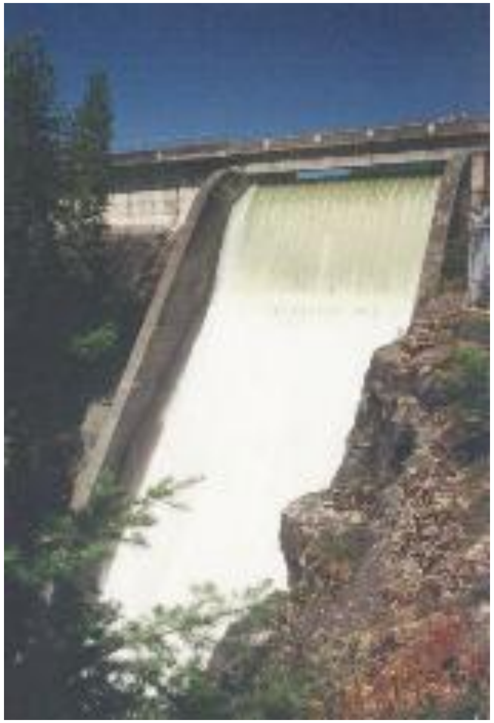

Isometric view of level invert flume ($p = 0$)

Plan view

Longitudinal section of flume with raised invert (hump)

# Compound structures

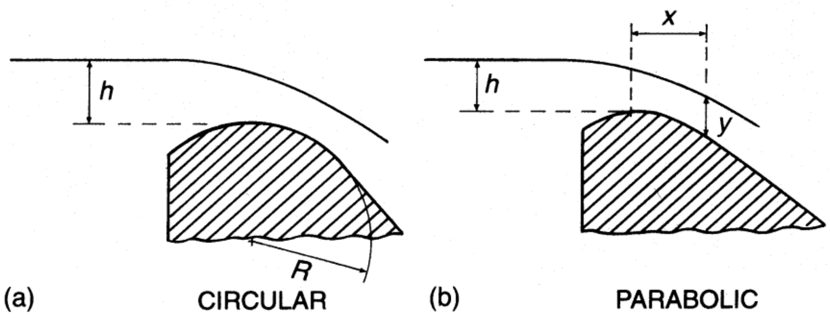

## Discharge measurement over a spillway



$$Q = Cbh^{3/2}$$

C for circular weirs = $2.03(h/R)^{0.07}$

C for parabolic weirs = $1.86h^{0.01}$




# Drowned condition sharp weirs

$$Q = Q_0 \left[ 1 - \left( \frac{h_2}{h_1} \right)^n \right]^{0.385}$$

$Q$ = discharge when submerged

$Q_0$ = discharge under free-flow conditions at the same upstream head $h_1$

$h_2$ = tail water level, relative to the vertex of the notch

$h_1$ = elevation of the upstream water surface relative to the vertex of the notch

$n$ = exponent of the basic flow equation, for example 1.5 for rectangular weirs and 2.5 for V-notches

Theory of errors:

<u>Nature of errors</u>

Random errors

Systematic errors

Spurious errors

Theory of (random) errors:

Why
1) Indication of accuracy
2) Find most critical parameter

Measure of error: $\sigma$ (standard deviation)

Propagation of errors from mathematical relations

General:

$$\sigma_q^{\ 2} = \left(\frac{\partial q}{\partial x}\right)^2 \sigma_x^{\ 2} + \left(\frac{\partial q}{\partial y}\right)^2 \sigma_y^{\ 2} + 2\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}\sigma_{xy}$$

Theory of (random) errors:

Independent relations: $$\sigma_q{}^2 = \left(\frac{\partial q}{\partial x}\right)^2 \sigma_x{}^2 + \left(\frac{\partial q}{\partial y}\right)^2 \sigma_y{}^2$$

e.g. for: $$q(x) = ax + b \qquad \sigma_q{}^2 = a^2 * \sigma_x{}^2$$

$$q(x, y) = a_1 x + a_2 y$$

Independent relation

$$\sigma_q{}^2 = a_1{}^2 * \sigma_x{}^2 + a_2{}^2 * \sigma_y{}^2$$ or

dependent relation

$$\sigma_q{}^2 = a_1{}^2 * \sigma_x{}^2 + a_2{}^2 * \sigma_y{}^2 + 2a_1 a_2 \sigma_{xy}$$

Theory of (random) errors:

e.g. for:

$$q(x,y) = a * x^b * y^c$$

$$\sigma_q^{\,2} = \left(a.b.x^{b-1}.y^c\right)^2.\sigma_x^{\,2} + \left(a.c.x^b.y^{c-1}\right)^2.\sigma_y^{\,2}$$

definition relative errors: $\frac{\sigma_q^{\,2}}{\bar{q}^2} = r_q^{\,2}$ $\frac{\sigma_x^{\,2}}{\bar{x}^2} = r_x^{\,2}$ $\frac{\sigma_y^{\,2}}{\bar{y}^2} = r_y^{\,2}$

$$r_q^{\,2} = b^2 r_x^{\,2} + c^2 r_y^{\,2}$$

Example errors:

Discharge over a crested weir:

$$Q = \left(\frac{2}{3}\right)^{3/2} C.g^{1/2}.b.h^{3/2}$$

$r_C$=3%
b=8.50m (width)
h=0.30m (level above weir)

Give a realistic value for the relative error in Q

Example errors:

$$Q = \left(\frac{2}{3}\right)^{3/2} C.g^{1/2}.b.h^{3/2}$$

$$r_Q^{\ 2} = r_C^{\ 2} + r_b^{\ 2} + (\frac{3}{2})^2 * r_h^{\ 2}$$

$r_C$=3%
b=8.50m (width)
h=0.30m (level above weir)

$$r_Q^{\ 2} = 3^2 + (\frac{0.01}{8.5}*100)^2 + (\frac{3}{2})^2 * (\frac{0.01}{0.3}*100)^2$$

$$r_Q^{\ 2} = (3)^2 + (0.11)^2 + (4.5)^2$$

Example errors:

One way to reduce errors: repetition

$$y = \frac{x_1 + x_2 + ......x_n}{n} = \frac{x_1}{n} + \frac{x_2}{n} + ........ + \frac{x_n}{n}$$

$$\sigma_y{}^2 = \frac{1}{n^2}\sigma_{x_1}{}^2 + ....... + \frac{1}{n^2}\sigma_{x_n}{}^2 = n * \frac{1}{n^2}\sigma_x{}^2 = \frac{1}{n}\sigma_x{}^2$$

$$\sigma_y = \frac{1}{\sqrt{n}}\sigma_x$$

# Heat as a tracer

## Distributed temperature sensing

28-2-2013

# Heat as a tracer

- Why heat
- Distribute Temperature Sensing
- Examples (qualitative)
- Examples (quantitative)
- Propagation of errors

# Why heat

- Not conservative

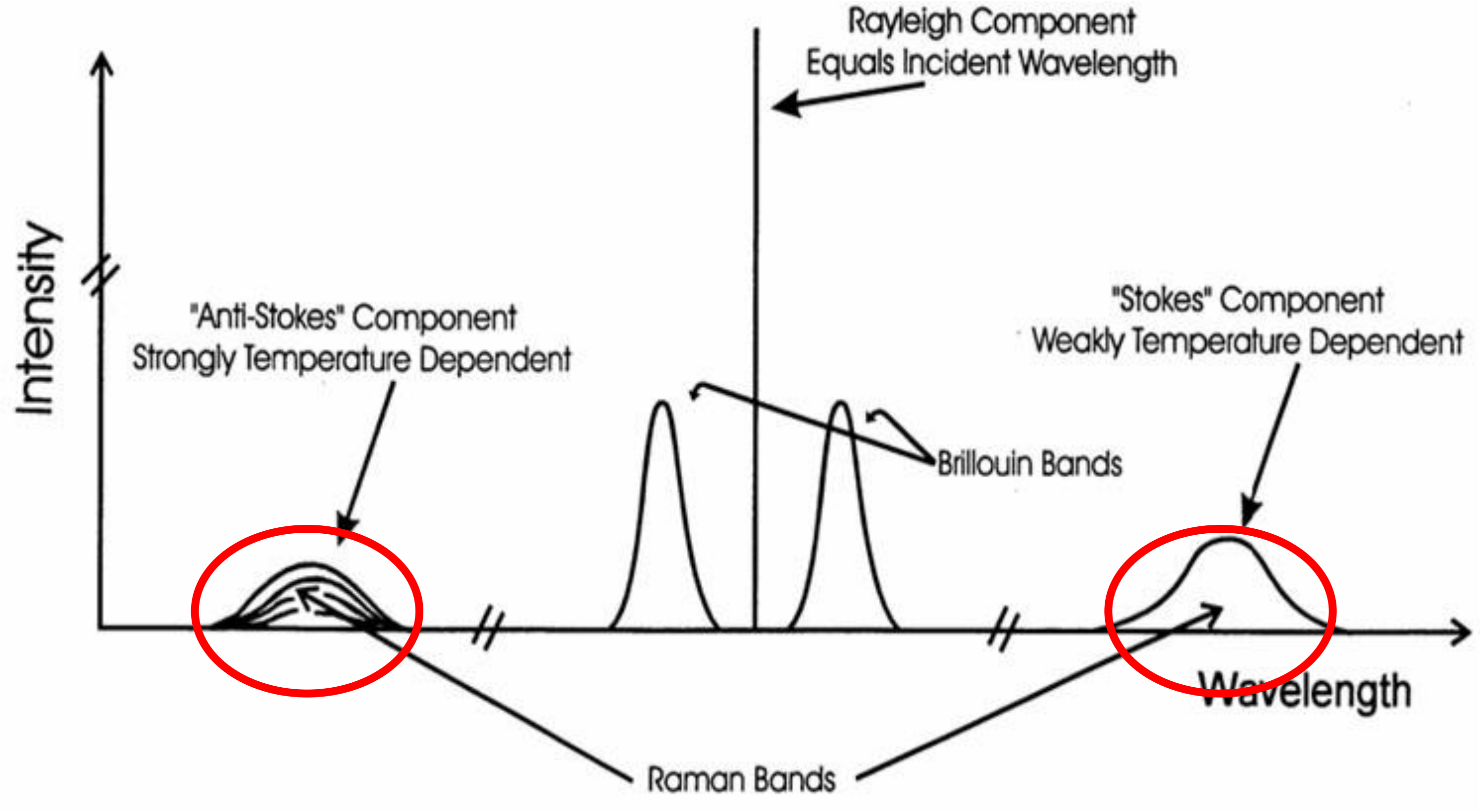
  BUT

- easy to measure at high resolution (with DTS)

# Distribute Temperature Sensing

- Fiber optic cable
- Laser pulse (~ ns)
- Reflections
- Time of flight
  $v = c/n = (3 \times 10^8)/1.5 = 2 \times 10^8$ m/s

# Distribute Temperature Sensing

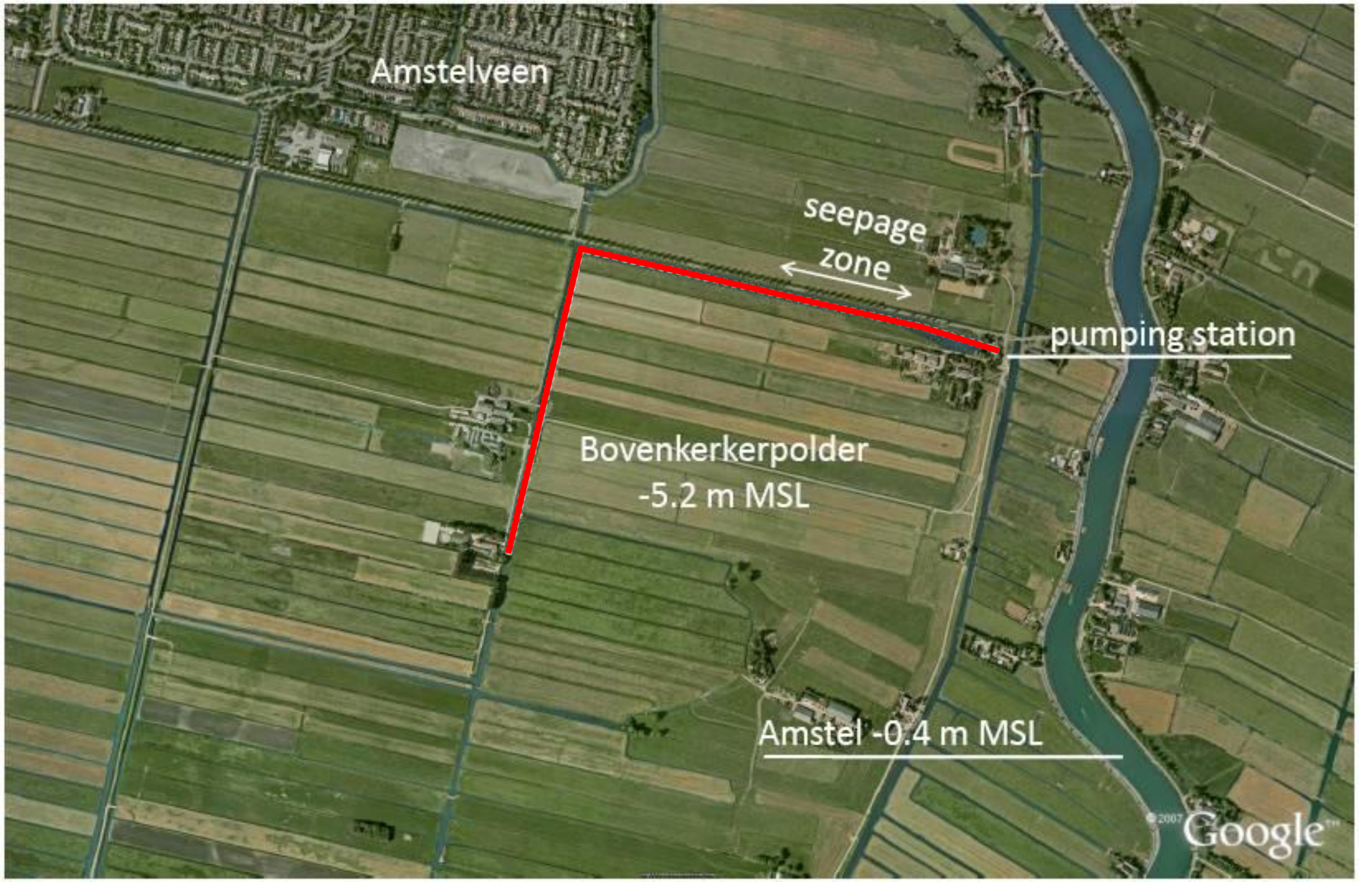


# Distribute Temperature Sensing

- up to 30 km long
- resolution: 1m<br>3min
- accuracy: 0.1°C

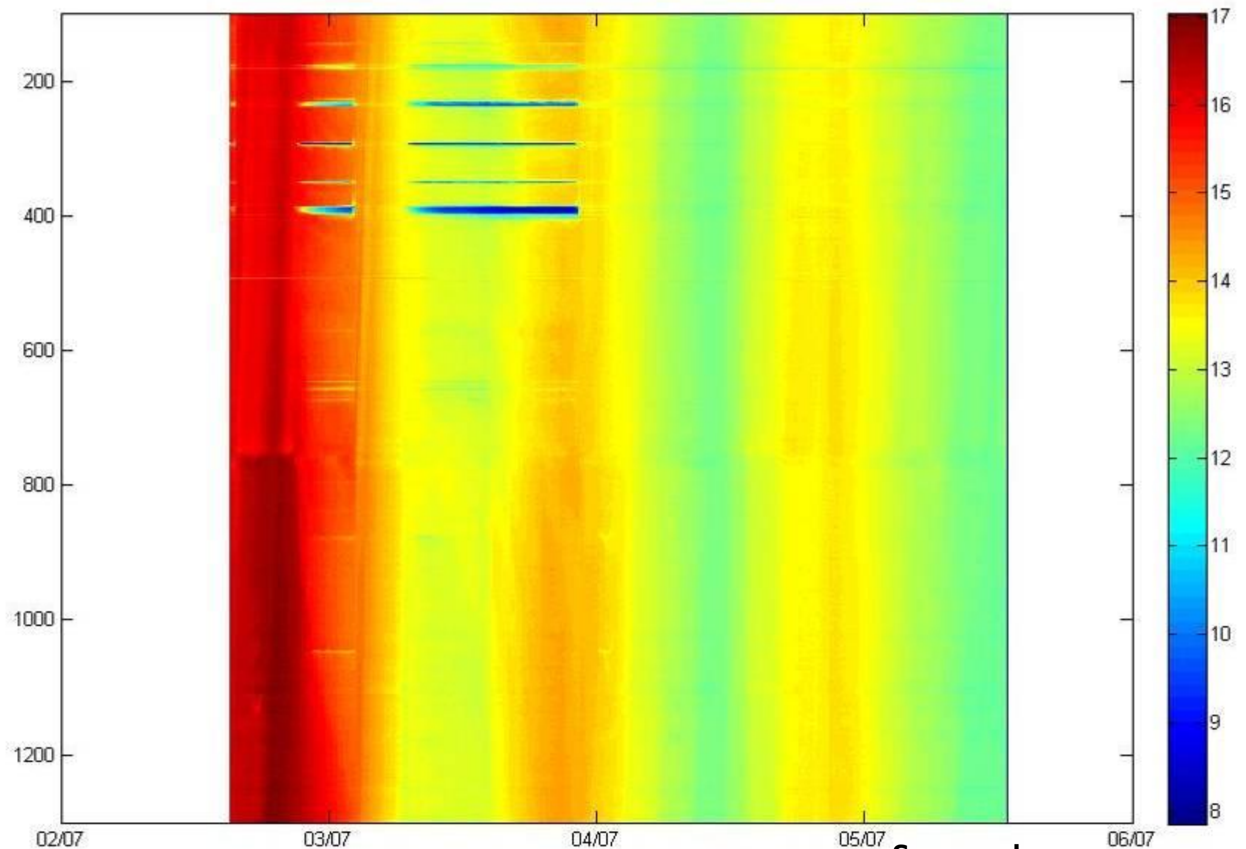
# Examples (qualitative)

- Determination of seepage in a polder



# Examples (qualitative)

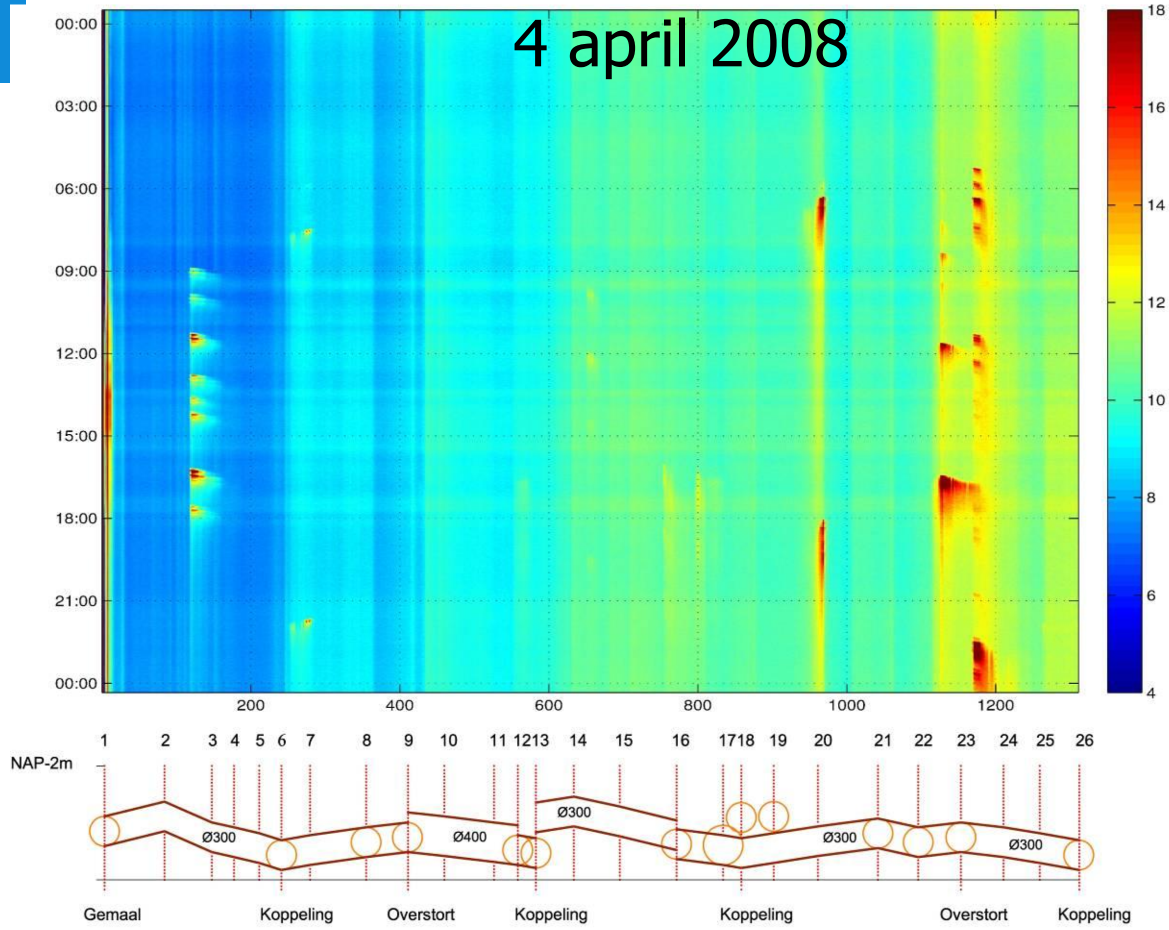
- Determination of seepage in a polder




Source unknown

# Examples (qualitative)

- Wrong connections to a rain water sewerage

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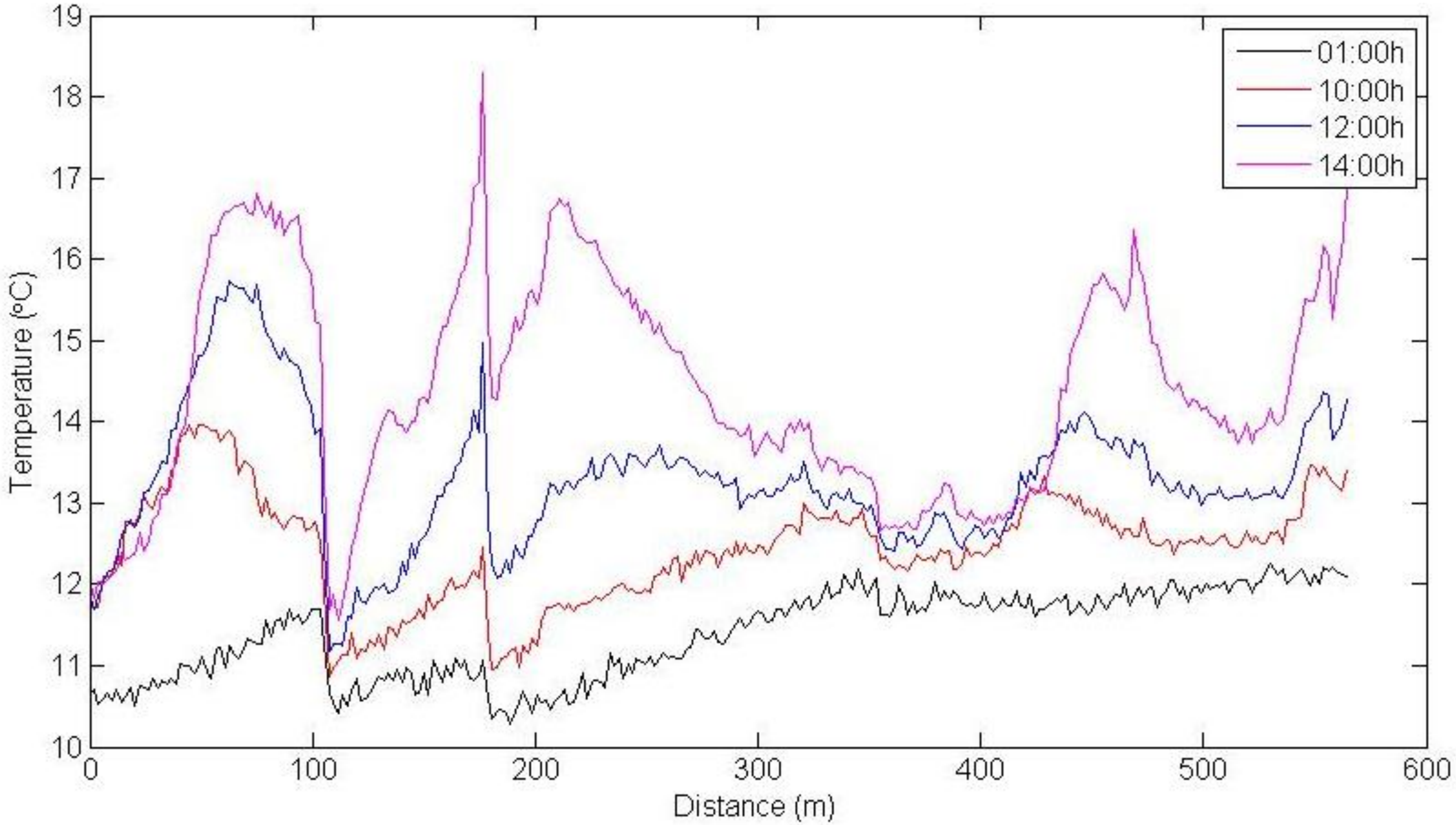
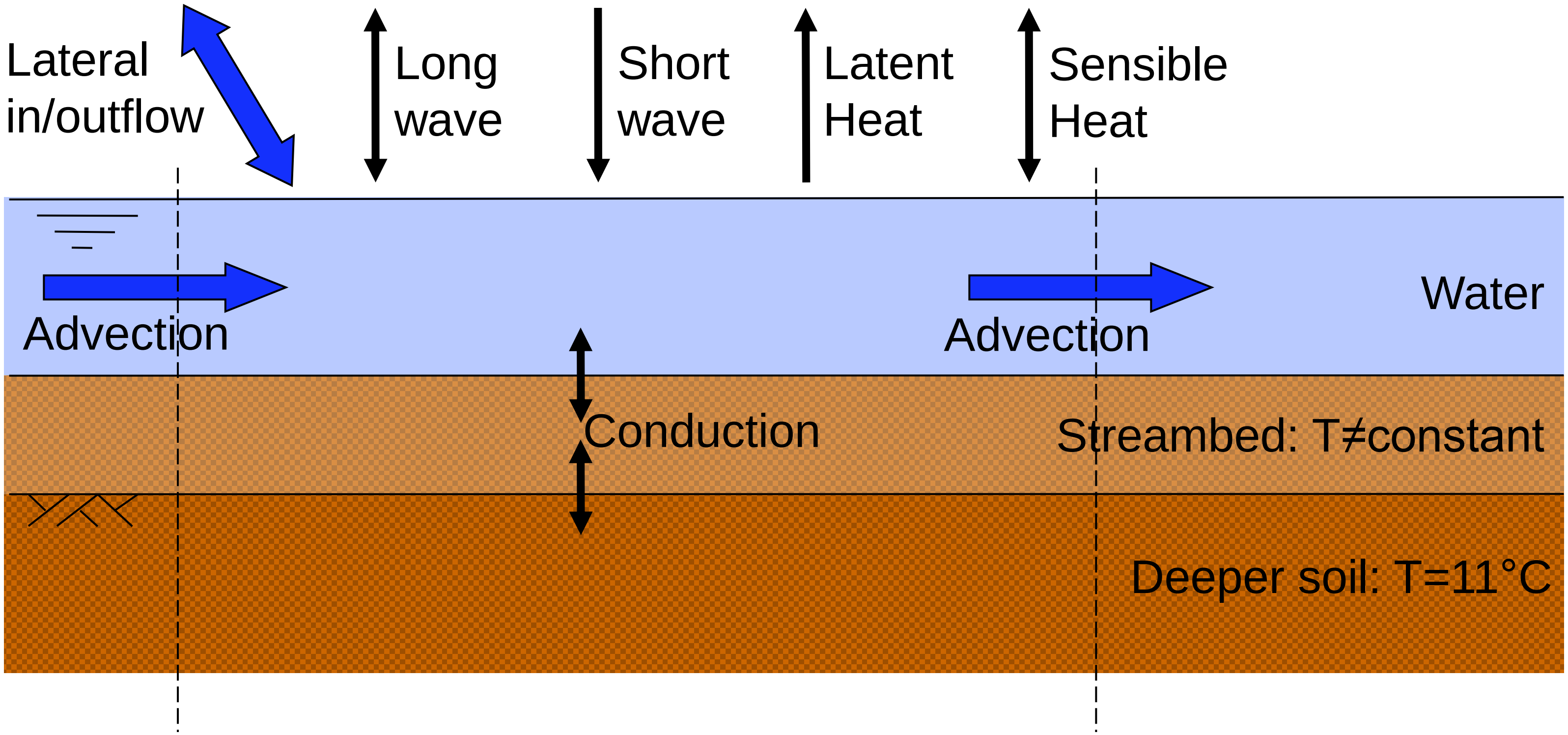
# Examples (quantitative)

- Maisbich



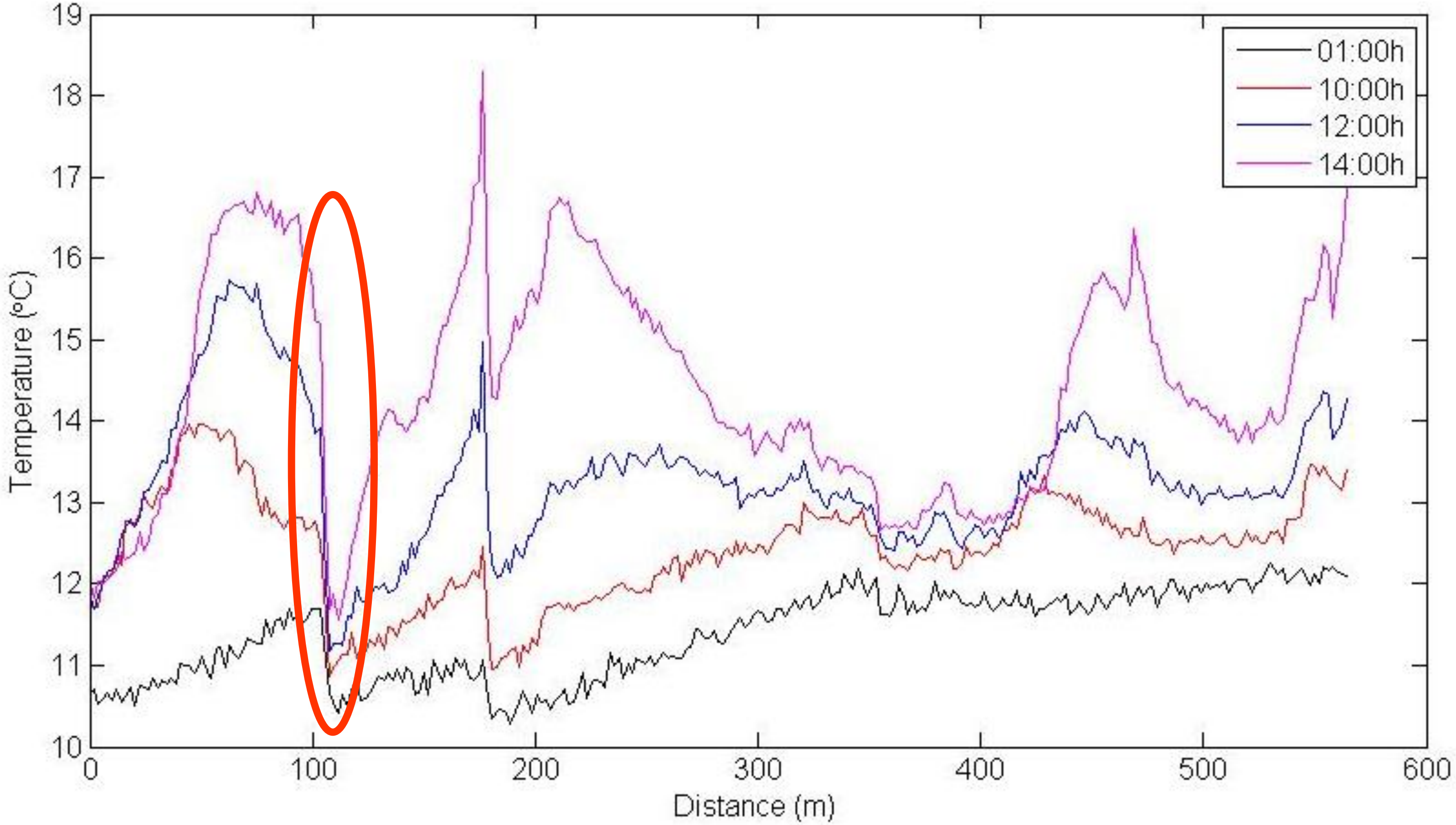
# Examples (quantitative)

- Maisbich



# Examples (quantitative)

- Maisbich



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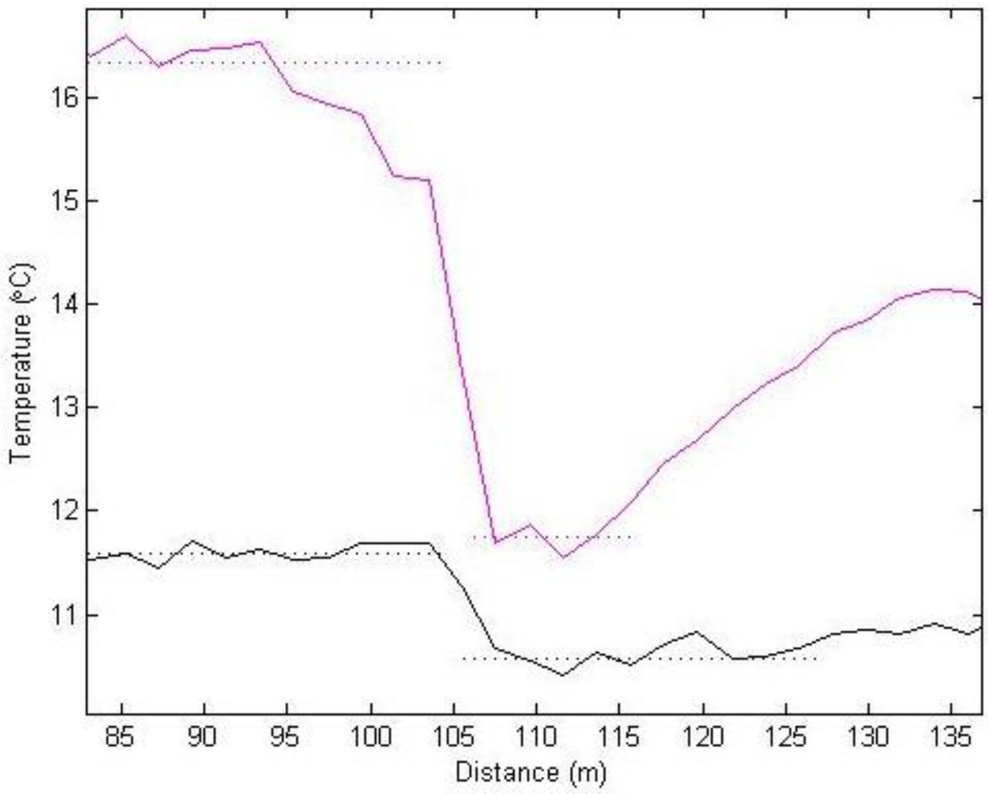


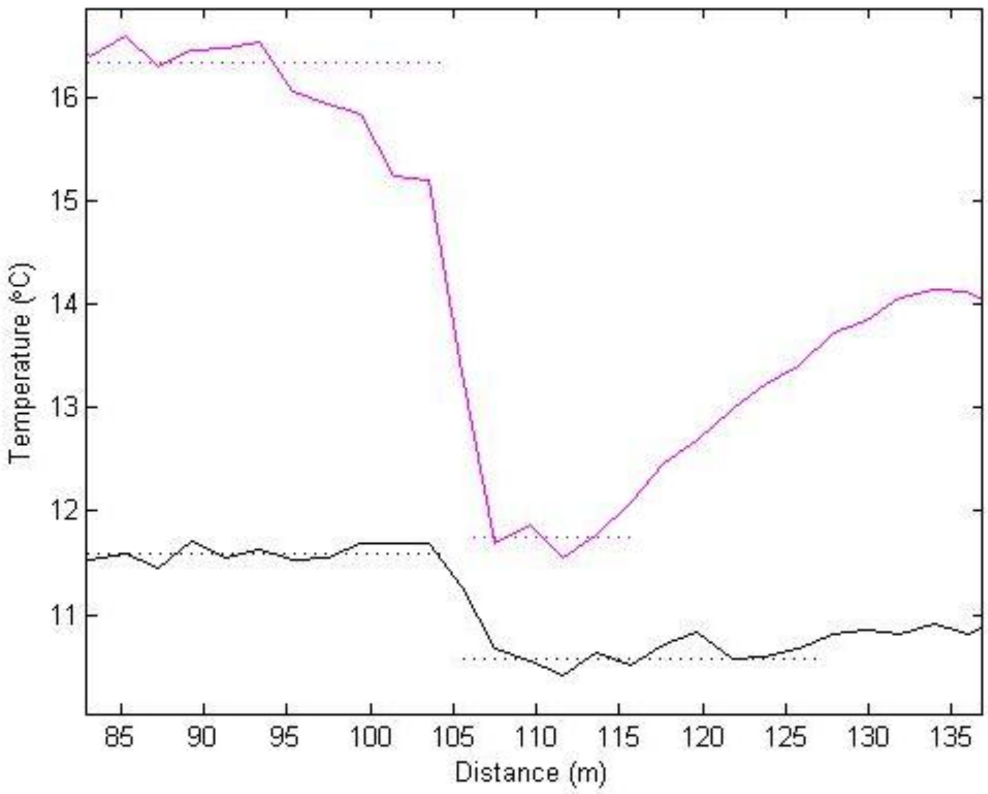

Mass balance

$$Q_d = Q_u + Q_L$$

$$T_d Q_d = T_u Q_u + T_L Q_L$$

$$\frac{Q_L}{Q_d} = \frac{T_d - T_u}{T_L - T_u}$$

# Examples (quantitative)




$$\frac{Q_L}{Q_d} = \frac{T_{d_1} - T_{u_1}}{T_L - T_{u_1}}$$

AND

$$\frac{Q_L}{Q_d} = \frac{T_{d_2} - T_{u_2}}{T_L - T_{u_2}}$$

# Propagation of errors

$$\frac{Q_L}{Q_d} = \frac{T_{d_2} - T_{u_2} - T_{d_1} + T_{u_1}}{T_{u_1} - T_{u_2}}$$

---

$$q = a x^b y^c$$

$$r_q^2 = \frac{\sigma_q^2}{\bar{q}^2}$$

$$r_q^2 = b^2 r_x^2 + c^2 r_y^2$$

$$\sigma_q^2 = \left( \frac{\partial q}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial q}{\partial y} \right)^2 \sigma_y^2$$

# Propagation of errors

$$r^2_{\frac{Q_L}{100\%}} = \frac{\sigma^2_{T_{d_2}} + \sigma^2_{T_{u_1}} + \sigma^2_{T_{u_2}} + \sigma^2_{T_{d_1}}}{\overline{\left(T_{d_2} + T_{u_1} - T_{u_2} - T_{d_1}\right)}^2} + \frac{\sigma^2_{T_{u_1}} + \sigma^2_{T_{u_2}}}{\overline{\left(T_{u_1} - T_{u_2}\right)}^2}$$