ME1633: Integration of Physics, Motion and Metrology

Assignment 7: Motion control

12 February 2014

- You can do this assignment on your own or in groups, as long as you hand in your own solutions and it becomes clear that you understand your solutions. Formulate your solutions step-by-step, carefully pointing out the logical structure of your answer, but keep your answers brief.

- If in some question you happen to need an answer from a previous question that you don't know the answer to, assume an answer or at least explain the method you would use when you would have had that previous answer.

- *Only for the students who follow the PME track's course ME1633:* Your solution to this assignment must be submitted in the lecture or via Blackboard (in pdf format) *before* 25 February 2014, 23:59h.

- Your solution is preferred to be formulated in *English*.

PCB drilling machine6

A student builds himself a PCB drilling machine as shown in Figure 1a. The PCB (green part) is carried by a table that is able to move in *z*-direction. The driller (black) is connected through a slider beam to the actuator (blue) actuating in *x*-direction (generating force without adding stiffness to the motion direction). In this assignment, only *x*-direction actuation is considered, see Figure 1b. The mass of the actuator and the driller are $m_1 = 2\,\text{kg}$, $m_2 = 0.5\,\text{kg}$ respectively.

1. Considering the two bodies (actuator and driller) as connected rigidly through the beam ($x_1 = x_2 = x$), the student wants to control the driller's position working in a known environment. Please help him to do that.
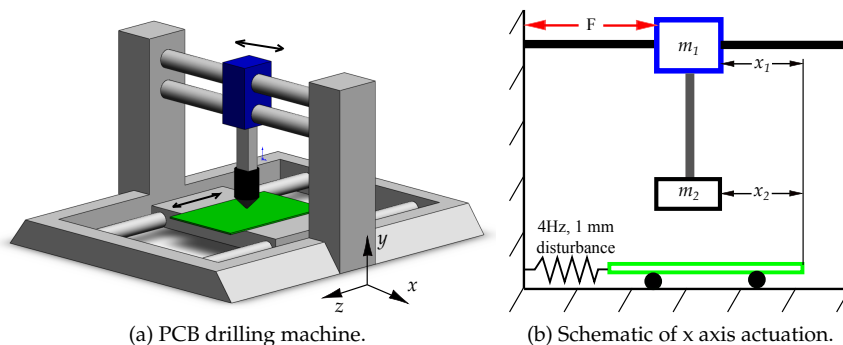

(a) PCB drilling machine.

(b) Schematic of x axis actuation.

Figure 1

1a. In the working environment, the carrying table might vibrate in x-direction up to $4\,\text{Hz}$ with the amplitude of $1\,\text{mm}$. The maximum allowed error of the hole's position on the PCB board is $0.01\,\text{mm}$. What is the (undamped) natural frequency of a proportional controller that satisfies the requirement? (5 points)

Answer

- The (undamped) natural frequency can be calculated:

$$f_{\text{bw}} = f_{\text{dist}} \cdot \sqrt{\frac{\text{dist}}{\text{error}}} = 40\,\text{Hz}$$

1b. Firstly, a P-controller is applied as shown in Figure 2, calculate the k_{p} for the controller to realise the natural frequency found in Question 1a. Derive the **closed loop transfer function** x/x_{ref}, remember to write down the intermediate steps you need to obtain this transfer function. (5 points)
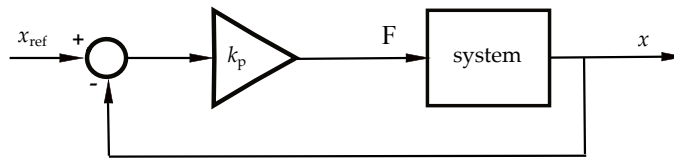

Figure 2: The closed loop system.

Answer

- Since the connecting beam is considered as a rigid body, the transfer function can be written as follows

$$G_1(s) = \frac{x}{F} = \frac{1}{(m_1 + m_2)s^2} = \frac{1}{2.5 \cdot s^2}$$

$f_{\text{bw}} = 40\,\text{Hz}$, k_{p} is calculated as follows (to make the magnitude of G_1 at the frequency ω_{c} equal to 1)

$$k_{\text{p}} = \frac{1}{|G_1(j \cdot 2\pi f_{\text{bw}})|} = 1.58 \cdot 10^5$$

- The closed loop transfer function is:

$$G_{1-closed}(s) = \frac{x}{x_{\text{ref}}} = \frac{k_{\text{p}} \cdot G_1}{1 + k_{\text{p}} \cdot G_1} = \frac{k_{\text{p}}}{(m_1 + m_2)s^2 + k_{\text{p}}}$$

1c. Consider the equivalent mechanical system of the closed loop system of Question 1b. It is shown in Figure 3 that the body mass $m_1 + m_2$ is connected to a **displacement actuator** using a spring that has stiffness k_{p}. Also with intermediate steps, please derive the **open loop transfer function** x/x_{ref}

of this system, using only the mechanical equivalent parts, to see that the P-controller in Question 1a works as a spring. (5 points)
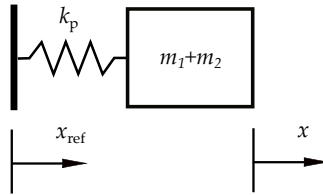

Figure 3: Equivalent system with **displacement actuator** x_{ref}.

Answer

- The open loop transfer function in Figure 3

$$(m_1 + m_2)\ddot{x} = -k_{\text{p}}(x - x_{\text{ref}})$$

$$\Rightarrow \frac{x}{x_{\text{ref}}}(s) = \frac{k_{\text{p}}}{(m_1 + m_2)s^2 + k_{\text{p}}}$$

1d. Describe the response of the system in Question 1b when we give a command to move the driller to a new position (step response).(5 points)

Answer

- The step response of the system with P controller is similar with the response of the equivalent mechanical system when x_{ref} suddenly moves to a new position. The body mass will oscillate around the new equilibrium point with the frequency of $40\,\text{Hz}$.

- Figure of step response
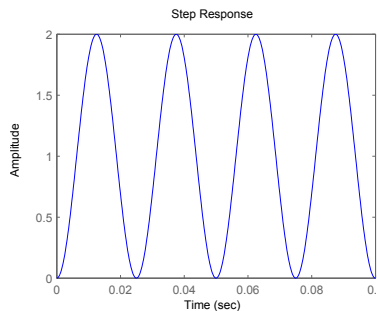
1e. How can the controller be modified to include a damper? Design a suitable controller with the rules of thumb given in the book. Sketch the equivalent mechanical system. (10 points)

Answer

- To include a damper, a D-control should be added. The controller now becomes PD-controller.

- Using the "rule of thumb" the PD-controller can be designed as follows:

  - Because of the gain of D-term, k_p in PD-controller must be reduced 3 times compared with P-controller in question 1a.

  $$k_p = \frac{1.58 \cdot 10^5}{3} = 5.26 \cdot 10^4$$

  - D-controller starts at $\omega_d = \frac{\omega_c}{3}$ and stop at $\omega_t = 3.3 \cdot \omega_c$
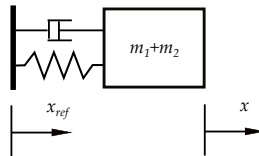
  $$k_d = 3 \cdot \frac{k_p}{\omega_c} = 628.3$$

  $$k_{d\text{-tamed}} = \frac{k_d}{1/\omega_t \cdot s + 1} = \frac{628.3}{1.2 \cdot 10^{-3} \cdot s + 1}$$

  - PD-controller

  $$C_{pd}(s) = k_p + k_{d\text{-tamed}} \cdot s$$

- The equivalent mechanical system is

1f. Answer briefly what would be the benefit of integrating I-control in this case? *Hint for this question*: Think of some possible kinds of disturbance that can act on the system. (5 points)

Answer

- We use I-control to increase the gain at low frequency to reduce the steady state error. For example, if the system is tilted a little bit, there is a constant force applies to the two bodies due to the gravity. The steady state error is no longer zero if we use the PD-controller. The I control should be added in this case.

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2. Unfortunately, the connecting beam between the two masses is not rigid. Let's investigate how the system is affected by this internal flexibility. In this part, the actuator applies the force on m_1, a sensor is used to measure the position of m_2. The beam is made of acrylic and has the dimensions in x, y, z as follows $l_x = l_z = 40\,mm$ and $l_y = 200\,mm$ (square cross section). The Young's modulus of acrylic is known as $E = 3.2 \cdot 10^9 N/m^2$.

2a. The system now can be modelled as in Figure 4. What is the most important assumption if we want to use this model? Please sketch by hand the Bode plot from F to x_2 (calculate the magnitude at $1\,rad/s$ and resonance frequencies if there are, note the slope). (15 points)
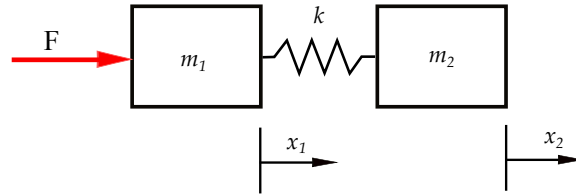

Figure 4: Simplified model.

Answer

- This model can be used when only the first eigen frequency is important in designing a controller. The second eigen frequency must stay far away after the first one so that it does not affect to stability when the controller is applied. When the beam's mass is very small compared to m_1 and m_2, the higher dynamic peaks will stay far away from the first eigen frequency.

- The beam's stiffness value can be calculated as follows:

$$k = \frac{EI}{L^3} = \frac{E \cdot l_z l_x^3/12}{l_y^3} = 2.56 \cdot 10^5 \, \text{Nm}$$
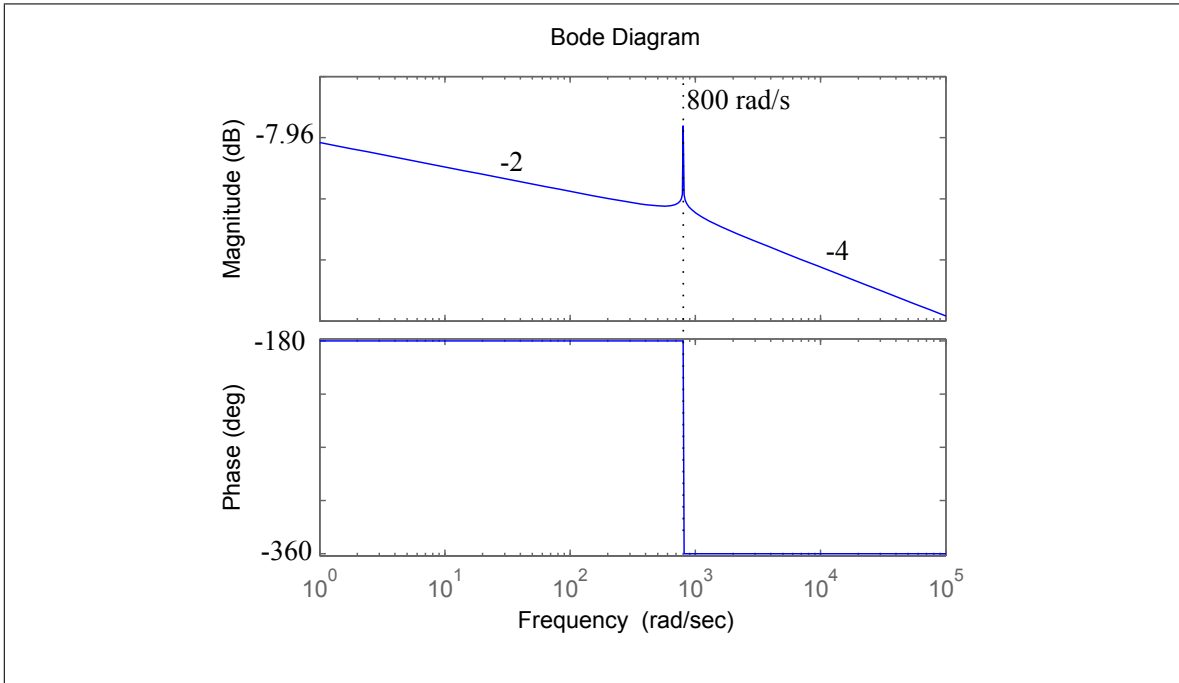
The transfer function is:

$$G_2(s) = \frac{x_2}{F}(s) = \frac{1}{(m_1 + m_2)s^2} \frac{k}{\frac{m_1 m_2}{m_1 + m_2}s^2 + k}$$

Magnitude at $1\,rad/s$:

$$G_2(j \cdot 1) = 20 \cdot log(|G_2(j \cdot 1)|) = -7.96\,\text{dB}$$

Resonance frequency:

$$\omega_{\text{res}} = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} = 800 \, rad/s \text{ (or } 127.3\,\text{Hz)}$$

Bode Diagram

2b. Assuming that there is a little damping in the connecting beam, such that the resonance has a factor Q of 10, what is the highest achievable unity-gain cross-over frequency for a stable system if we only apply a P controller? (The answer is not the resonance frequency) (10 points)

Answer

- With damping, the Bode plot will be:
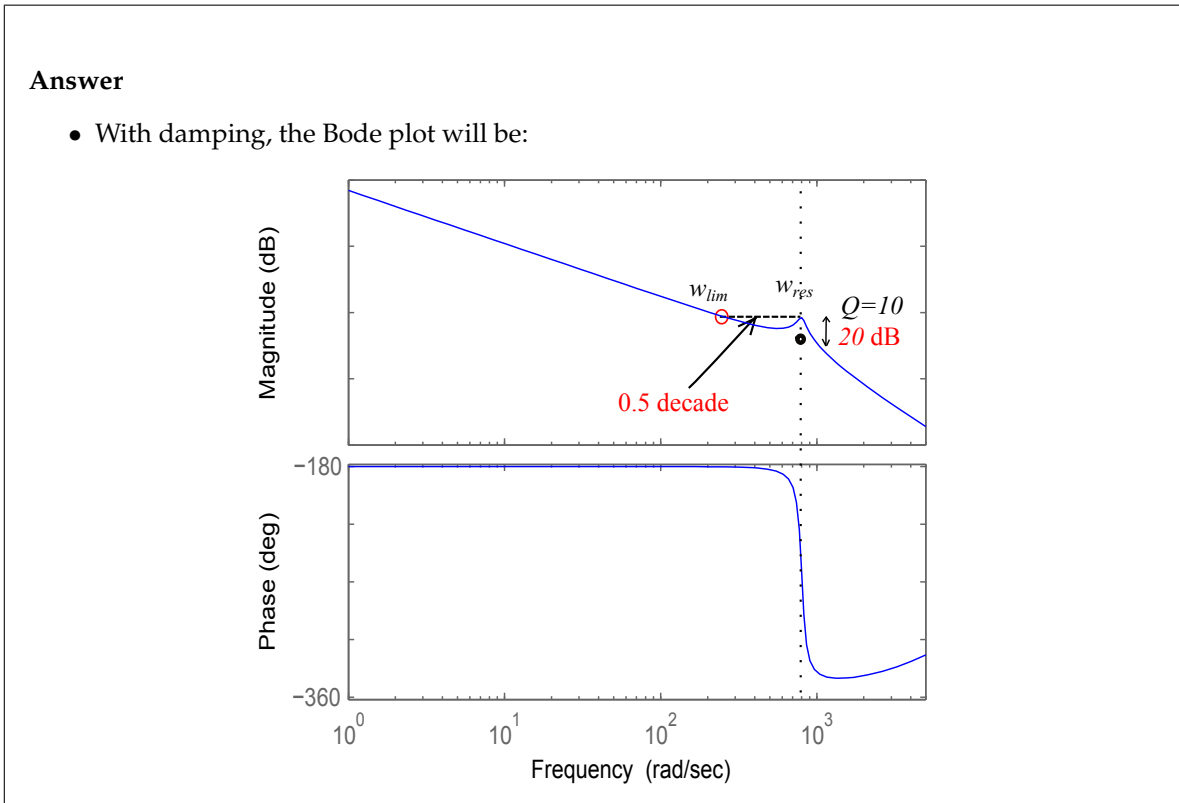
- With $Q = 10$, as noted in the plot, we have

$$\omega_{\text{lim}} = 10^{(\log(\omega_{\text{res}})-0.5)} = 253\,{}^{rad}\!/_{s} \text{ (or 40.2 Hz)}$$

2c. The P controller is not sufficiently damped, so following the rule of thumb, the student designed a PD-controller for this system with the required unity-gain cross-over frequency calculated in Question 1a. He found out the controlled system became unstable. Can you write down that PD-controller's transfer function? Draw the Nyquist plot that **shows** why it is unstable (You are encouraged to derive the Nyquist plot from the Bode plot by sketching it by hand. But to be more precise, you are allowed to use MATLAB or another program for that purpose). (10 points)

Answer

- From the Bode plot, we can find the gain of the controller needed to archive 40 Hz is 103 dB (or $1.4 \cdot 10^5$) , and using the rule of thumb we have$\omega_{\text{d}} = \omega_{bw}/3$,$\omega_{\text{tamed}} = \omega_{\text{bw}} \cdot 3.3$. The PD-controller transfer function will be:

$$C_{2c}(s) = 4.75 \cdot 10^4 \frac{1/\omega_{\text{d}}s + 1}{1/\omega_{\text{tamed}}s + 1}$$

- The Nyquist plot of the open loop clearly shows that the controlled system is unstable:

Nyquist Diagram
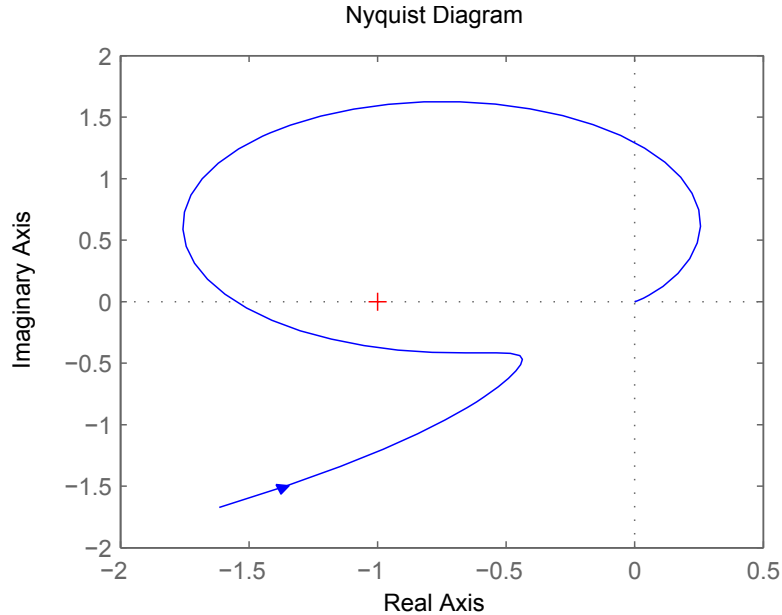
2d. As shown in Figure 5, adding a low-pass filter is one way to solve the unstable problem in Question 2c. Choose an appropriate cutoff frequency for the low-pass filter so that the system becomes stable. Draw the Nyquist plot of the improved system, remember to indicate the vector margin to show how good the stability is. (10 points)
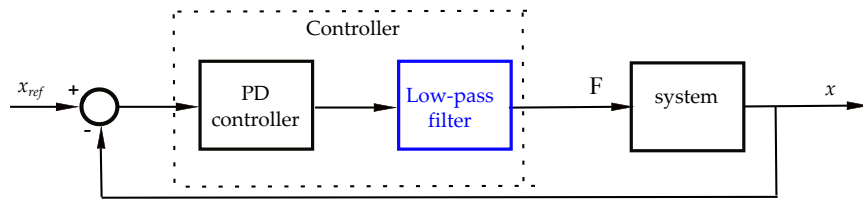
Figure 5: Adding a Low-pass filter.

Answer

- We can choose any values for the cutoff frequency of the low-pass filter that is not too far after ω_{res} because it will not make the system stable if this value is too high. This value should not be smaller than ω_{bw} also because if it is too small, it might counteract your D-controller and the system will still be unstable.

- If we choose $\omega_{\text{low-pass}} = 3 \cdot \omega_{\text{bw}}$, the Nyquist plot shows the system is stable
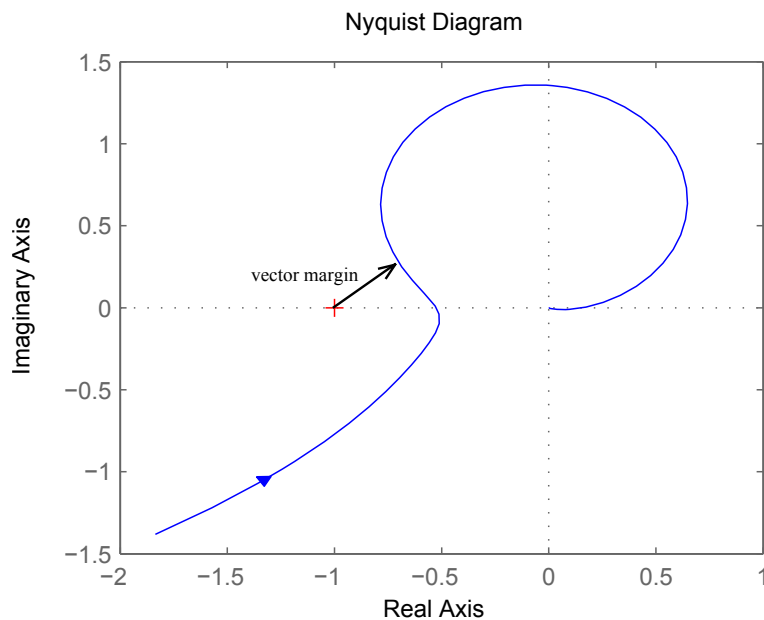
2e. Implementing the controller designed above, when the student gives a step command to x_{ref}, the time for the driller head m_2 to be stable at the expected position is much longer than his simulation result. That is caused by the limited force generated by the actuator. Instead of using a step command, he developed an input profile generator that creates a smooth profile to move to the expected position as quick as possible. The maximum actuator force is $25\,\text{N}$, and the driller needs to move $4\,\text{mm}$. Can you calculate t_{min} and fill in the box that has a question mark in Figure 6. (10 points)
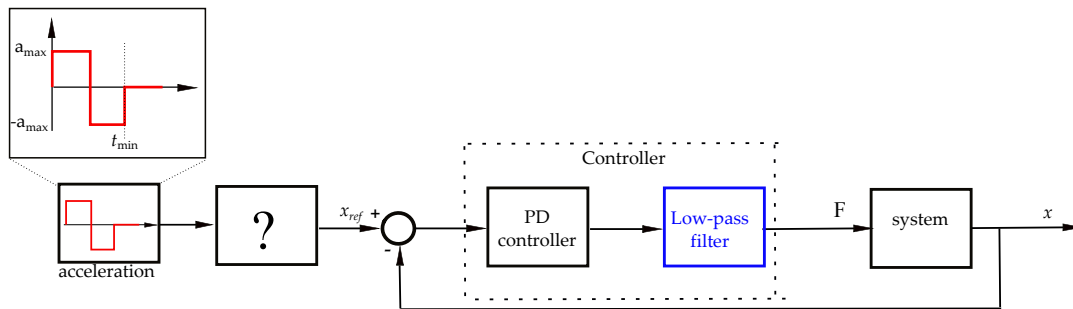
Figure 6: With input profile generator.

Answer

- We have $a_{\max} = \frac{F}{m_1 + m_2} = 10 \, m/s^2$, $x_{\text{required}} = 4 \cdot 10^{-3} \text{m} \Rightarrow t_{min} = 0.04 \, \text{s}$

- The output of the question mark box is the required position x_{ref}. Therefore, that box should contain a second order integration $\frac{1}{s^2}$
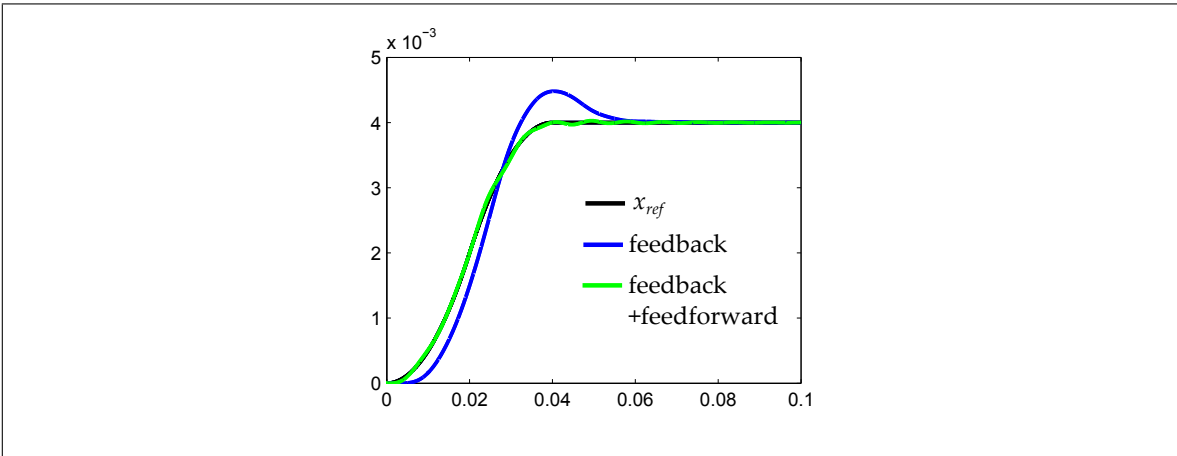

2f. The performance can still be improved by adding a feedforward controller. Adding to Figure 6 to show how your feedforward is implemented, remember to write down what is inside the feedforward block . (10 points)

Answer

- A simple feedfoward controller can be implemented like this
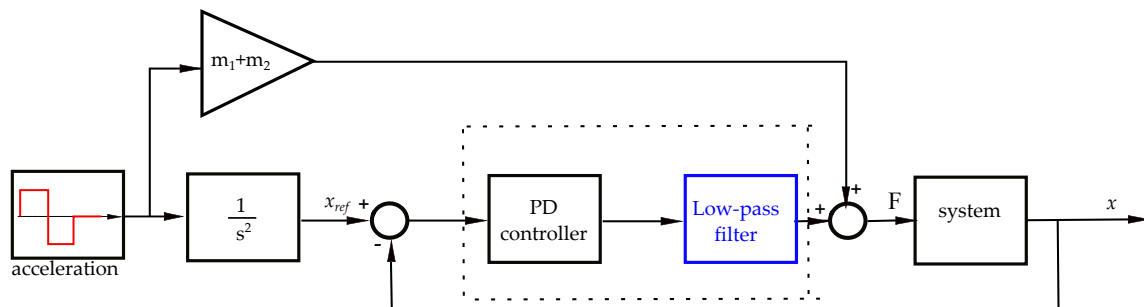
- Comparison:

Please indicate, for each question, the following:

This question was:

Difficult	⟷		Doable
○	○	○	○

I spent:

Hours