ME1633: Integration of Physics, Motion and Metrology

Assignment 7: Motion control

12 February 2014

- You can do this assignment on your own or in groups, as long as you hand in your own solutions and it becomes clear that you understand your solutions. Formulate your solutions step-by-step, carefully pointing out the logical structure of your answer, but keep your answers brief.
- If in some question you happen to need an answer from a previous question that you don't know the answer to, assume an answer or at least explain the method you would use when you would have had that previous answer.
- Only for the students who follow the PME track's course ME1633: Your solution to this assignment must be submitted in the lecture or via Blackboard (in pdf format) before 25 February 2014, 23:59h.
- Your solution is preferred to be formulated in *English*.

PCB drilling machine6

A student builds himself a PCB drilling machine as shown in Figure 1a. The PCB (green part) is carried by a table that is able to move in *z*-direction. The driller (black) is connected through a slider beam to the actuator (blue) actuating in *x*-direction (generating force without adding stiffness to the motion direction). In this assignment, only *x*-direction actuation is considered, see Figure 1b. The mass of the actuator and the driller are $m_1 = 2 \text{ kg}$, $m_2 = 0.5 \text{ kg}$ respectively.

1. Considering the two bodies (actuator and driller) as connected rigidly through the beam ($x_1 = x_2 = x$), the student wants to control the driller's position working in a known environment. Please help him to do that.



Figure 1

1a. In the working environment, the carrying table might vibrate in *x*-direction up to 4 Hz with the amplitude of 1 mm. The maximum allowed error of the hole's position on the PCB board is 0.01 mm. What is the (undamped) natural frequency of a proportional controller that satisfies the requirement? (5 points)

1b. Firstly, a P-controller is applied as shown in Figure 2, calculate the k_p for the controller to realise the natural frequency found in Question 1a. Derive the **closed loop transfer function** x/x_{ref} , remember to write down the intermediate steps you need to obtain this transfer function. (5 points)



Figure 2: The closed loop system.

1c. Consider the equivalent mechanical system of the closed loop system of Question 1b. It is shown in Figure 3 that the body mass $m_1 + m_2$ is connected to a **displacement actuator** using a spring that has stiffness k_p . Also with intermediate steps, please derive the **open loop transfer function** x/x_{ref} of this system, using only the mechanical equivalent parts, to see that the P-controller in Question 1a works as a spring. (5 points)



Figure 3: Equivalent system with **displacement actuator** x_{ref} .

1d. Describe the response of the system in Question 1b when we give a command to move the driller to a new position (step response).(5 points)

1e. How can the controller be modified to include a damper? Design a suitable controller with the rules of thumb given in the book. Sketch the equivalent mechanical system. (10 points)

1f. Answer briefly what would be the benefit of integrating I-control in this case? *Hint for this question*: Think of some possible kinds of disturbance that can act on the system. (5 points)

2. Unfortunately, the connecting beam between the two masses is not rigid. Let's investigate how the system is affected by this internal flexibility. In this part, the actuator applies the force on m_1 , a sensor is used to measure the position of m_2 . The beam is made of acrylic and has the dimensions in x, y, z as follows $l_x = l_z = 40 \text{ mm}$ and $l_y = 200 \text{ mm}$ (square cross section). The Young's modulus of acrylic is known as $E = 3.2 \cdot 10^9 \text{N/m}^2$.



Figure 4: Simplified model.

2a. The system now can be modelled as in Figure 4. What is the most important assumption if we want to use this model? Please sketch by hand the Bode plot from *F* to x_2 (calculate the magnitude at $1 \frac{rad}{s}$ and resonance frequencies if there are, note the slope). (15 points)

2b. Assuming that there is a little damping in the connecting beam, such that the resonance has a factor Q of 10, what is the highest achievable unity-gain cross-over frequency for a stable system if we only apply a P controller? (The answer is not the resonance frequency) (10 points)

2c. The P controller is not sufficiently damped, so following the rule of thumb, the student designed a PD-controller for this system with the required unity-gain cross-over frequency calculated in Question 1a. He found out the controlled system became unstable. Can you write down that PD-controller's transfer function? Draw the Nyquist plot that **shows** why it is unstable (You are encouraged to derive the Nyquist plot from the Bode plot by sketching it by hand. But to be more precise, you are allowed to use MATLAB or another program for that purpose). (10 points)

2d. As shown in Figure 5, adding a low-pass filter is one way to solve the unstable problem in Question 2c. Choose an appropriate cutoff frequency for the low-pass filter so that the system becomes stable. Draw the Nyquist plot of the improved system, remember to indicate the vector margin to show how good the stability is. (10 points)



Figure 5: Adding a Low-pass filter.

2e. Implementing the controller designed above, when the student gives a step command to x_{ref} , the time for the driller head m_2 to be stable at the expected position is much longer than his simulation result. That is caused by the limited force generated by the actuator. Instead of using a step command, he developed an input profile generator that creates a smooth profile to move to the expected position as quick as possible. The maximum actuator force is 25 N, and the driller needs to move 4 mm. Can you calculate t_{min} and fill in the box that has a question mark in Figure 6. (10 points)

2f. The performance can still be improved by adding a feedforward controller. Adding to Figure 6 to show how your feedforward is implemented, remember to write down what is inside the feedforward block . (10 points)



Figure 6: With input profile generator.

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