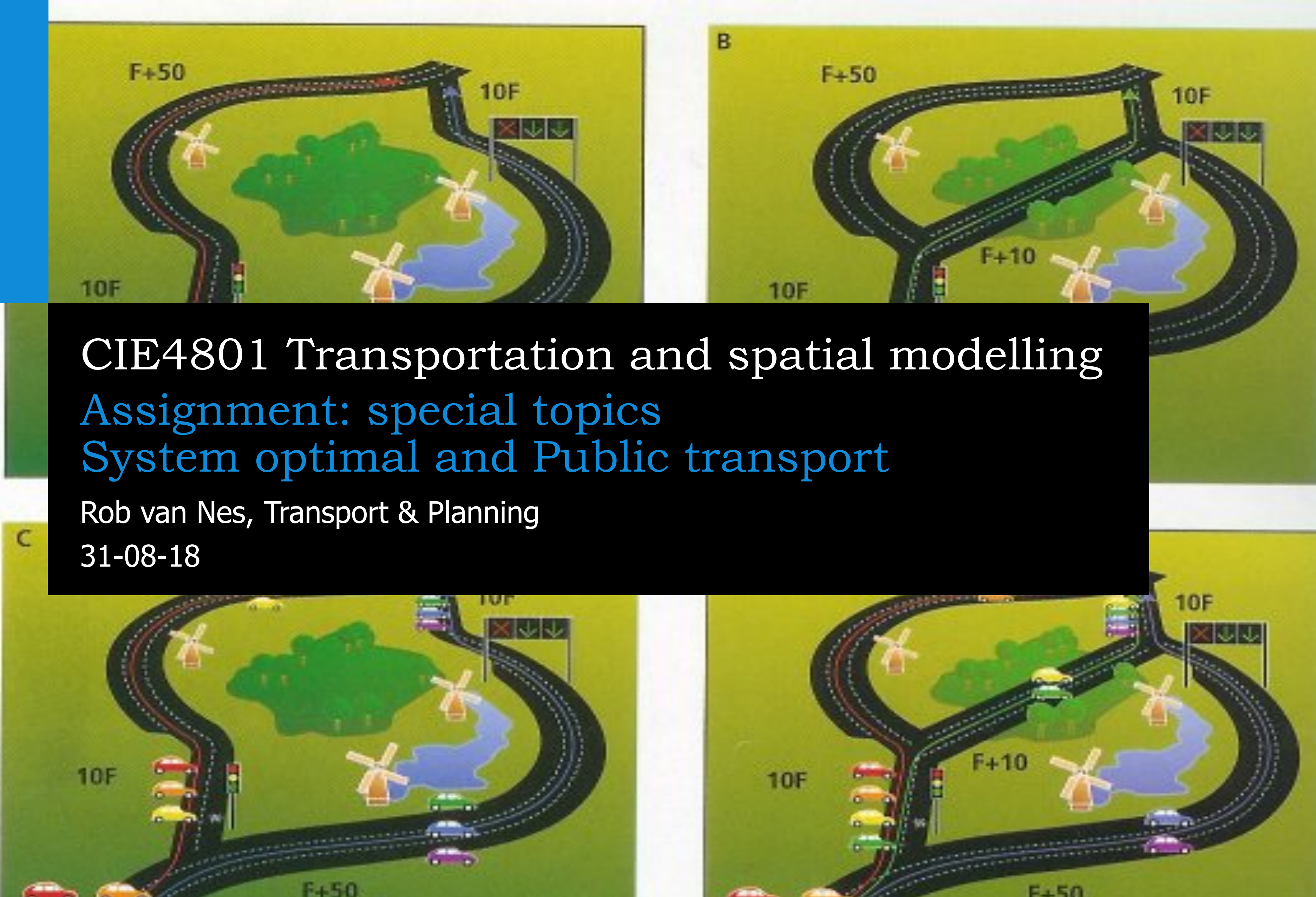


CIE4801 Transportation and spatial modelling

Assignment: special topics
System optimal and Public transport

Rob van Nes, Transport & Planning

31-08-18

Content

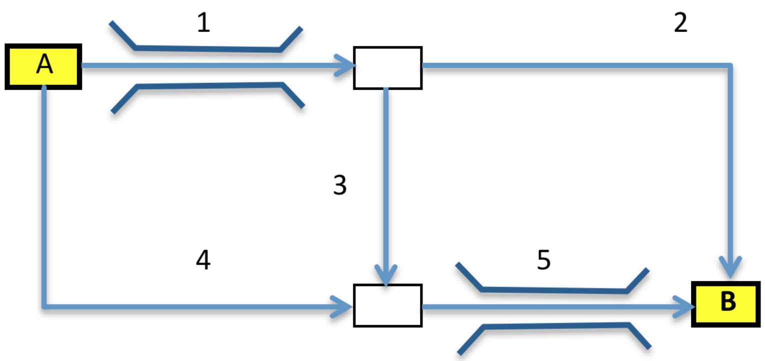
- Comments/questions congested assignment
  - Spreadsheet Braess

- Modelling component 5: Assignment: Special topics
  - Your comments/questions on Chapter 10 and 11
    - Social equilibrium or system optimum (11.2.2)
    - Public transport assignment (10.6)
      - Choice modelling and public transport assignment (e.g. Omnitrans approach)

1.

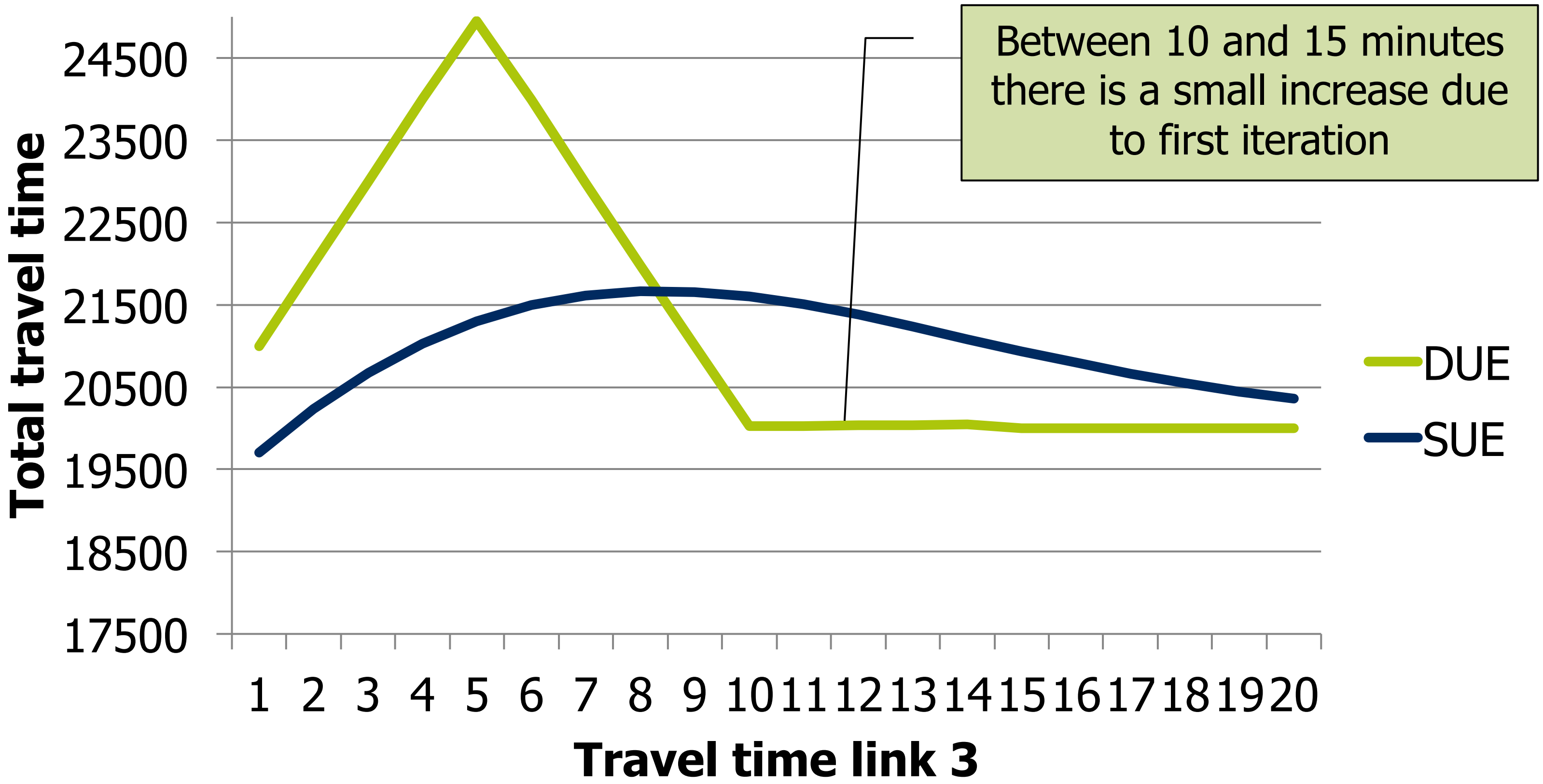
Comments/questions congested assignment

Spreadsheet Braess



- What did you see?
- Adding link 3 can result in higher travel times!
- What happens in case of a substantially lower or higher demand?
- What is the difference between using DUE and SUE assignment?

Difference between DUE and SUE
(demand = 1000)



Comments/questions

- The main concepts
  - Wardrop’s equilibrium, speed-flow curves
- The modelling methods
  - Iterative scheme: Method successive averages (MSA)
  - Convergence criteria: Relative or Duality Gap
  - Mathematical programming approach
  - Frank-Wolfe algorithm versus MSA
  - Route based assignment
  - Stochastic equilibrium assignment (SUE)
  - Route based algorithm
  - DUE versus SUE

1.2

Comments/questions congested assignment: DUE versus SUE

Solution principle DUE

1. Set flows of all links equal to 0
2. Determine link costs based on the link flows
   - In first iteration use free flow travel times
3. Perform an assignment (AON)
4. Determine link flows
5. Return to step 2

Apply either route flow or link flow averaging

Solution principle SUE

1. Set flows of all links equal to 0
2. Determine link costs based on the link flows
   - In first iteration use free flow travel times
3. Perform an assignment (Probit or Logit)
4. Determine link flows
5. Return to step 2

Apply either route flow or link flow averaging

- Stochastic assignment already spreads traffic over routes in a single DUE-iteration => a potential benefit
- Probit implies more AON assignments within a single DUE-iteration => longer computation times
- Logit requires a route based approach (having an additional benefit!), however there's the theoretical drawback: overlap

Duality gap SUE

- Duality gap in words:
  total travel time based on links minus total travel time based on (latest) shortest paths

- For DUE the duality gap should become zero (Wardrop principle)

- In a SUE travellers opt routes that are longer but they perceive to be shortest
  => Total travel time for SUE is higher than for DUE
  => Duality gap > 0

2.

Social equilibrium or System optimum

Assignment: optimisation of objectives

- Objective AON $\min_{q_a} Z = \sum_a t_a q_a$

- Objective DUE $\min_{q_a} \sum_a \int_{x=0}^{q_a} t_a(x)dx$

- Similarity or difference?

- Both minimise the surface under function t_a, but…….

- AON: additional traveller doesn't affect other travellers' travel times
  DUE: additional traveller affects travel time of all travellers

DSO assignment

Deterministic user-equilibrium (DUE):

Assignment in which each traveller minimises his/her own travel time

Deterministic system optimum (DSO):

Assignment in which the total travel time for all travellers is minimised

Analogy with Game Theory: "Prisoners' dilemma"

Individuals X and Y committed a crime
They are in jail and are asked individually to confess

Depending on the confession of both X and Y, they face a prison time

Nash-equilibrium

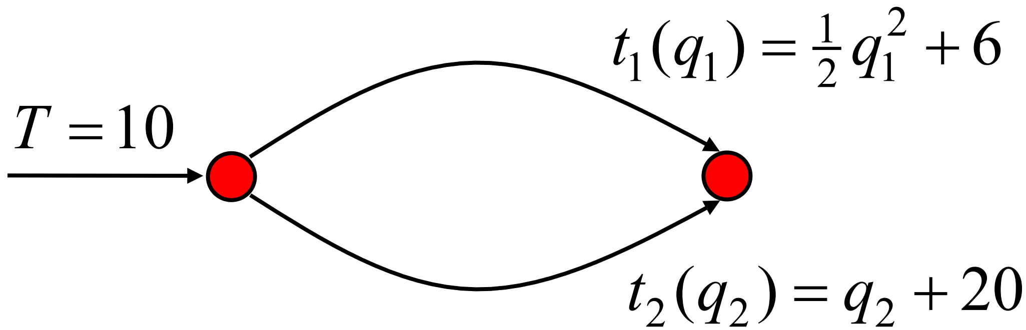
(# years in prison)		Y: confess	Y: don't confess
X	confess	(5), (5)	(0), 20
X	don't confess	20, (0)	1, 1

Decision rule: Confessing yields the best result independent of the response of the other

Better off if cooperating!

Pareto-optimum

Example DSO assignment



<u>DUE assignment</u>

$$q_1 = 6, \; t_1 = 24$$
$$q_2 = 4, \; t_2 = 24$$

total travel time:
240

<u>DSO assignment</u>

$$q_1 = 4.14, \; t_1 = 14.57$$
$$q_2 = 5.86, \; t_2 = 25.86$$

total travel time:
211.8

DSO assignment: mathematical formulation

DUE

$$\min_{q_a} Z_1 = \sum_a \int_{x=0}^{q_a} t_a(x)dx$$

DSO

$$\min_{q_a} Z_2 = \sum_a t_a(q_a) \cdot q_a$$

subject to:

$$\sum_r T_{ijr} = T_{ij} \quad \forall i, j$$

$$q_a = \sum_i \sum_j \sum_r \alpha_{ijr}^a T_{ijr} \quad \forall a$$

$$T_{ijr} \geq 0 \quad \forall i, j, r$$

How to solve for a DSO assignment?

Trick: We can use the same algorithm for solving for a DUE assignment by *replacing the travel time function!*

$$\min_{q_a} \sum_a t_a(q_a) \cdot q_a \quad \Rightarrow \quad \min_{q_a} \sum_a \int_{x=0}^{q_a} t_a^*(x) dx$$

What should we choose for $t_a^*(q_a)$?

$$f(x) = \int f'(x) dx$$

$$f(x) \rightarrow t_a(q_a) \cdot q_a$$

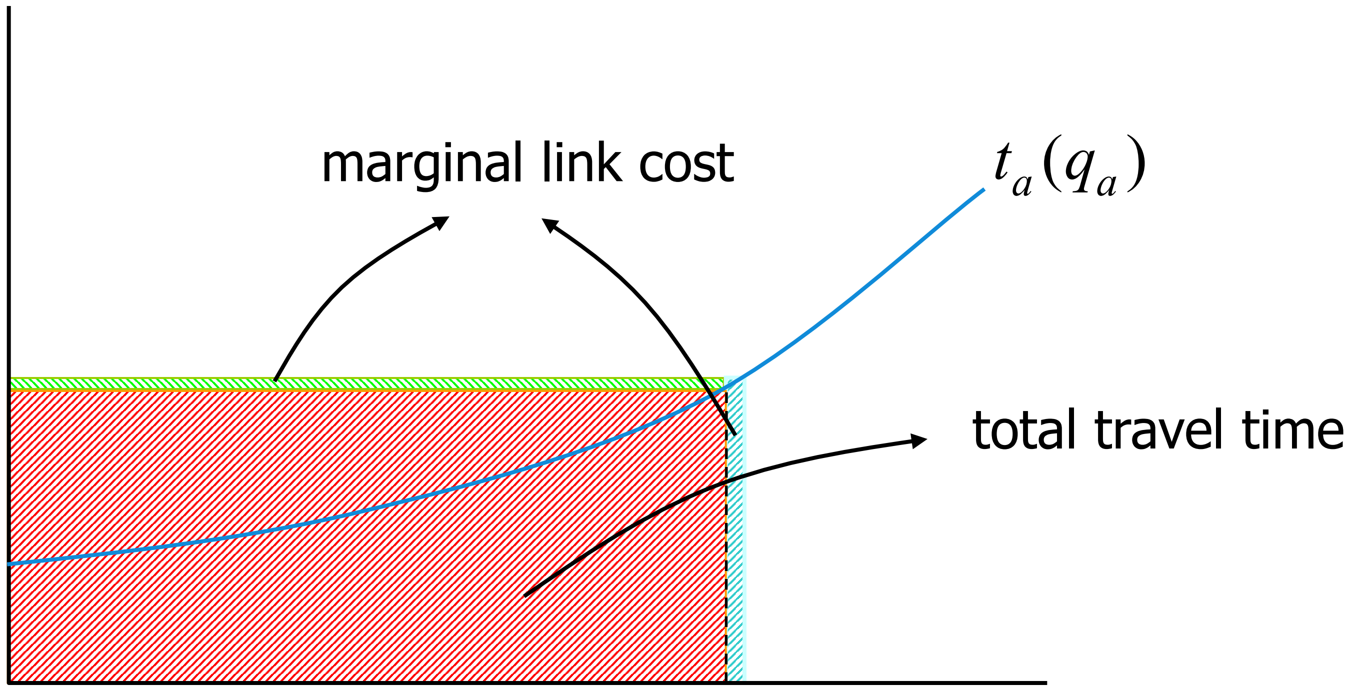
$$f'(x) \rightarrow \frac{d}{dq_a}\left(t_a(q_a) \cdot q_a\right) = t_a(q_a) \cdot \frac{dq_a}{dq_a} + \frac{dt_a(q_a)}{dq_a} \cdot q_a$$

$$= t_a(q_a) + t'_a(q_a) \cdot q_a \equiv t_a^*(q_a)$$

$t'_a(q_a) \cdot q_a$ = marginal link cost

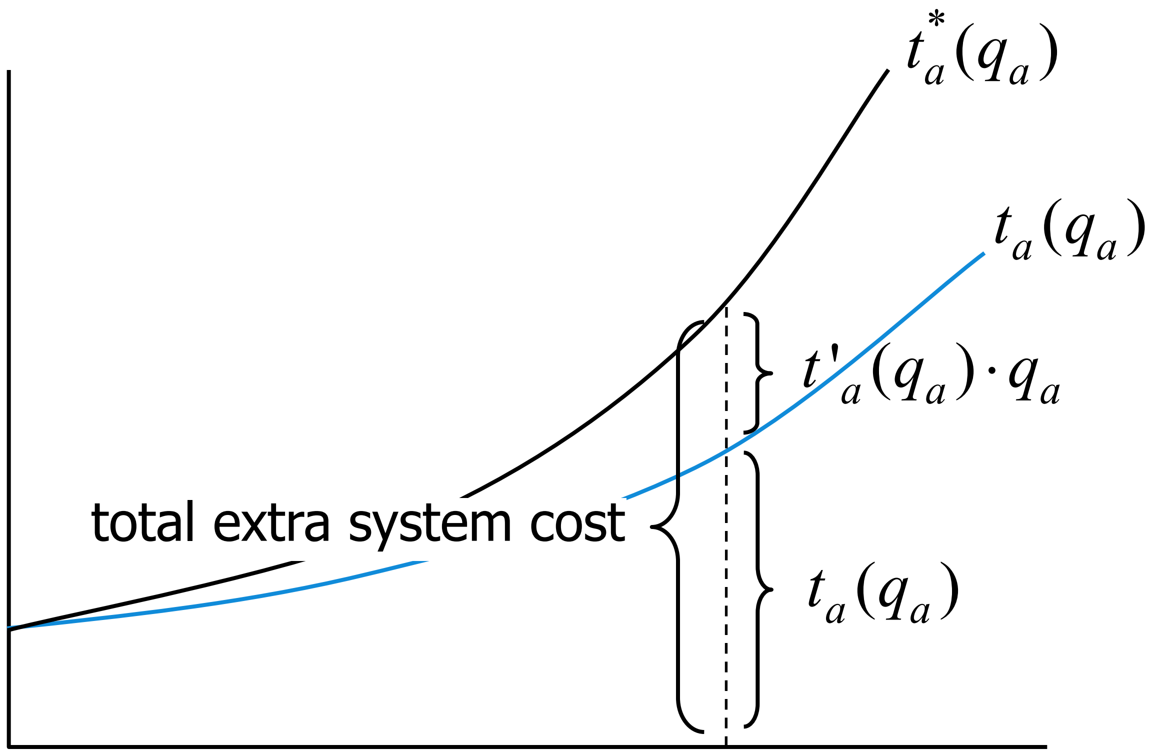
DSO assignment

Marginal link cost =
total extra cost of one extra vehicle on the link
(when there are already q_a vehicles)



Marginal link cost concept

Marginal link cost =
total extra cost of one extra vehicle on the link
(when there are already q_a vehicles)



extra cost for
all other drivers
(external costs)

extra cost for
the extra driver

DSO assignment: Example

Suppose the link travel times are described by the BPR function:

$$t_a(q_a) = t_a^0\left[1+\alpha\left(\frac{q_a}{C_a}\right)^{\beta}\right]$$

Which link travel time function should be used such that we can solve for a DSO assignment by using a DUE algorithm (such as Frank-Wolfe) ?

$$t_a(q_a)+t'_a(q_a)\cdot q_a = \; t_a^0\left[1+\alpha\left(\frac{q_a}{C_a}\right)^{\beta}\right]+t_a^0\,\frac{\alpha\beta}{C_a^{\beta}}(q_a)^{\beta-1}\cdot q_a$$

$$= t_a^0\left[1+\alpha(1+\beta)\left(\frac{q_a}{C_a}\right)^{\beta}\right]$$

DSO assignment in practice?

How to reach a system optimal assignment in practice?

⇒ road pricing ('rekeningrijden', 'kilometerheffing')

How much should the road pricing toll be on each link?

$$t_a^*(q_a) \equiv t_a(q_a) + \underbrace{t'_a(q_a) \cdot q_a}_{\approx \text{ link toll}}$$

N.B. Remember VoT!

3.

Public transport assignment

Topics for discussion

- Main differences between public transport and private modes
  - Generalised costs, common lines, frequency versus schedule based
- Do you understand the network representation?
  - Trunk line
  - Line specific
  - Route sections
- Do you understand the assignment methods?
  - Strategy concept
  - Choice modelling approach
- Are these models appropriate?

3.1

Public transport assignment: Difference between car and public transport assignment

Differences between car & PT assignment

- Network itself
- Congestion effects
- Route travel costs

Network itself

Car: Physical infrastructure network

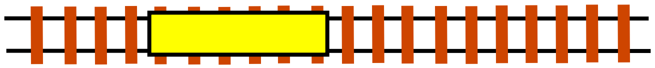
PT: Network of services

Space accessibility	
• Stops/stations	Access and egress links (network)
• Lines	In-vehicle links and transfers
Time accessibility	
• Frequencies	Waiting time (first stop, transfer)
• Operating hours	Mode specific constant
Public transport modes	Modal preferences
Fares	Value of time, fare systems

Link travel times

Car: $t_a = t_a(q_a)$

PT: $t_a = t_a^0$



Bus travel times?

Function of car travel times?

Crowding?

Route travel costs

Car: $c_{ijr} = \sum_a \alpha_{ijr}^a c_a \qquad (c_a = t_a)$

Additive costs

PT: $c_{ijr} = \ldots$

$$c_{ijr} = \gamma_1 t_{ijr}^{\text{access}} + \gamma_2 t_{ijr}^{\text{waiting}} + \gamma_3 t_{ijr}^{\text{in-vehicle}} + \gamma_4 t_{ijr}^{\text{egress}} + {} + \gamma_5 t_{ijr}^{\text{walking\_int}} + \gamma_6 t_{ijr}^{\text{waiting\_int}} + \gamma_7 N$$

γ_1: 2.2, γ_2: 1.5, γ_3: 1.0, γ_4: 1.1, γ_5: 2.3, γ_6: 1.3, γ_7: 5.7

Non-additive costs (?)

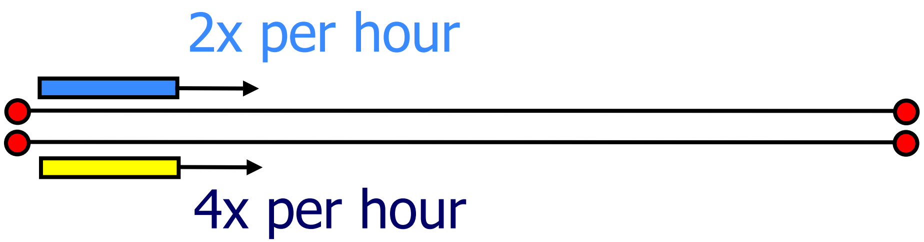
Consequences for assignment?

Non-additive costs would mean that a route based approach using route set generation (e.g. enumeration) is necessary.

Once the routes have been generated, the route costs can be computed.

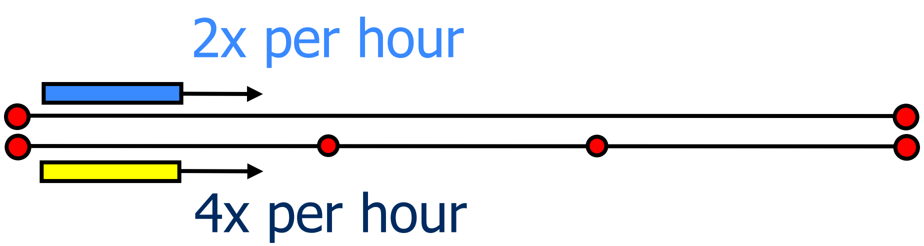
How can *frequencies*, *waiting times*, and *PT costs* be determined for specific routes?

Common lines: frequencies



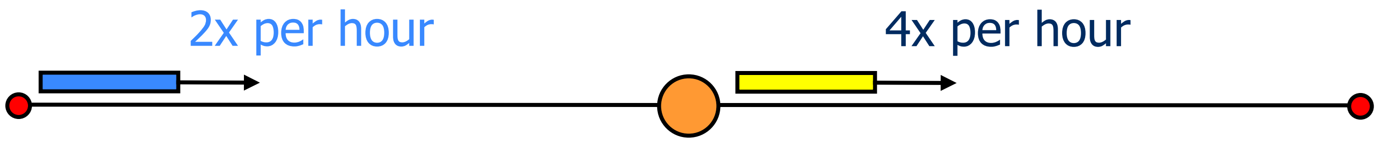
$$\max\{f_1, f_2\} \leq f \leq f_1 + f_2$$

What is the frequency along this route? $4 \leq f \leq 4+2$



What is the frequency along this route? ???

Examples waiting times and costs



What is the waiting time at the transfer node?

- based on frequency of the yellow train: $\tau^{\text{waiting}} \approx 7.5 \ \text{min.}$
- assuming the lines are synchronized: $\tau^{\text{waiting}} \approx 0.0 \ \text{min.}$
- assuming the time schedules are known: ...

How much are the PT costs?
Depends on:
- zone based, distance based, boarding penalty
- single trip or round trip
- combined ticket
- discount pass
- timing of the trip

3.2

Public transport assignment: Modelling approaches

Criteria for modelling approach

- Network
  - complexity and size
- Algorithm
  - complexity
- Level of detail of results
  - modal split
  - average or maximum line occupancies
  - impact of ITS
- Time dimension
  - long term
  - short term

Algorithms

- Shortest path
- Multi-routing (i.e. between pairs of stops only)
- Multiple routing (between origin and destination)
  - Feasible strategy
  - Enumeration of route sets and choice modelling

Representations & algorithms

Representations

- Line based
  - Trunk lines
  - Line specific
- Route sections

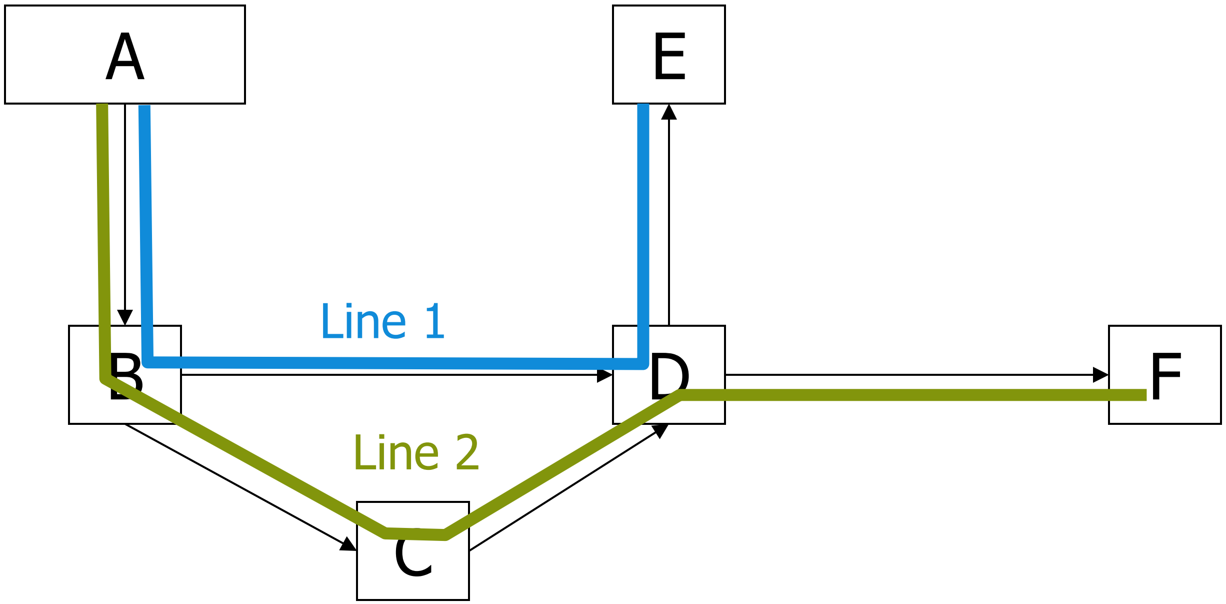
Algorithms

- (Floyd-Warshall)
- Strategies
- Choice modelling

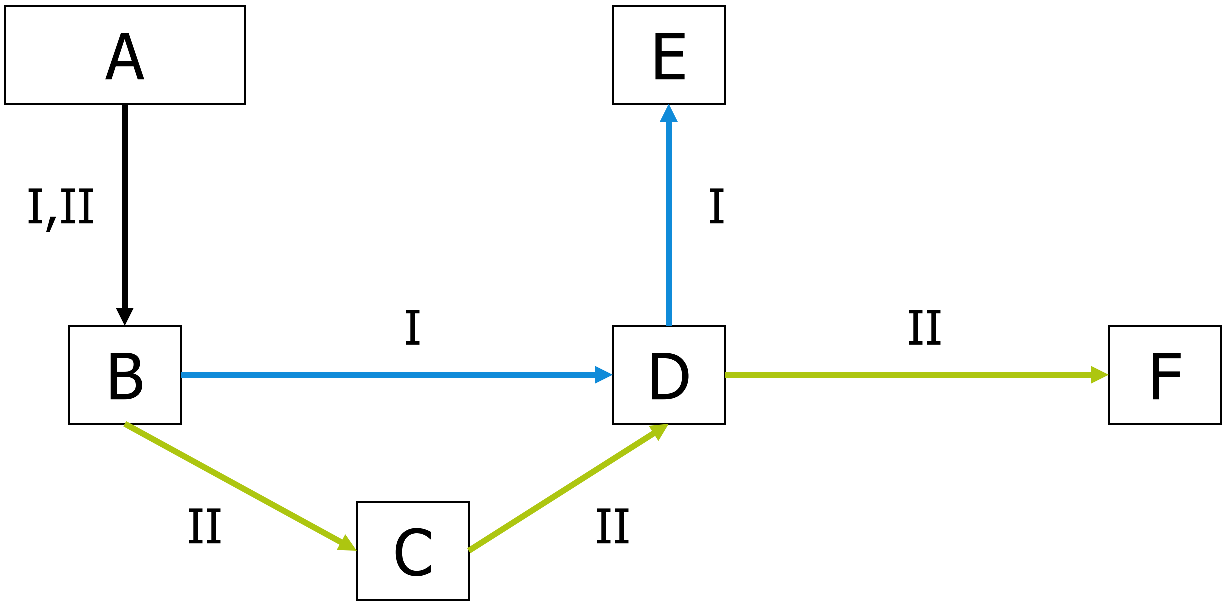
3.3

Public transport assignment: Network representations

Example with 2 transit lines

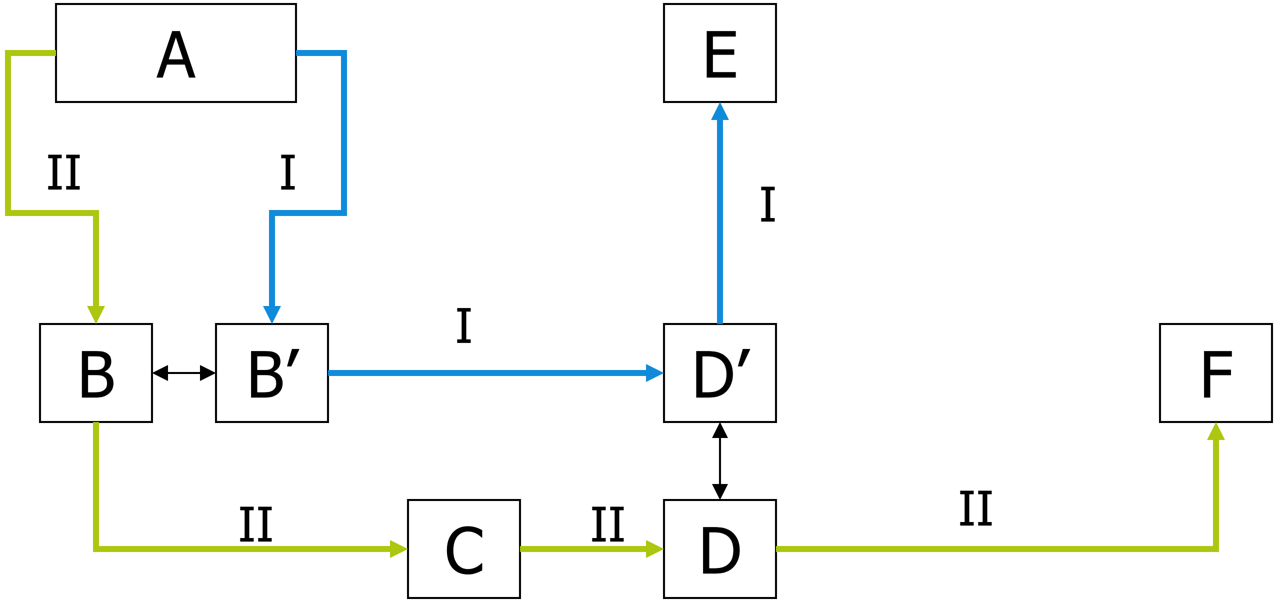


Trunk line links



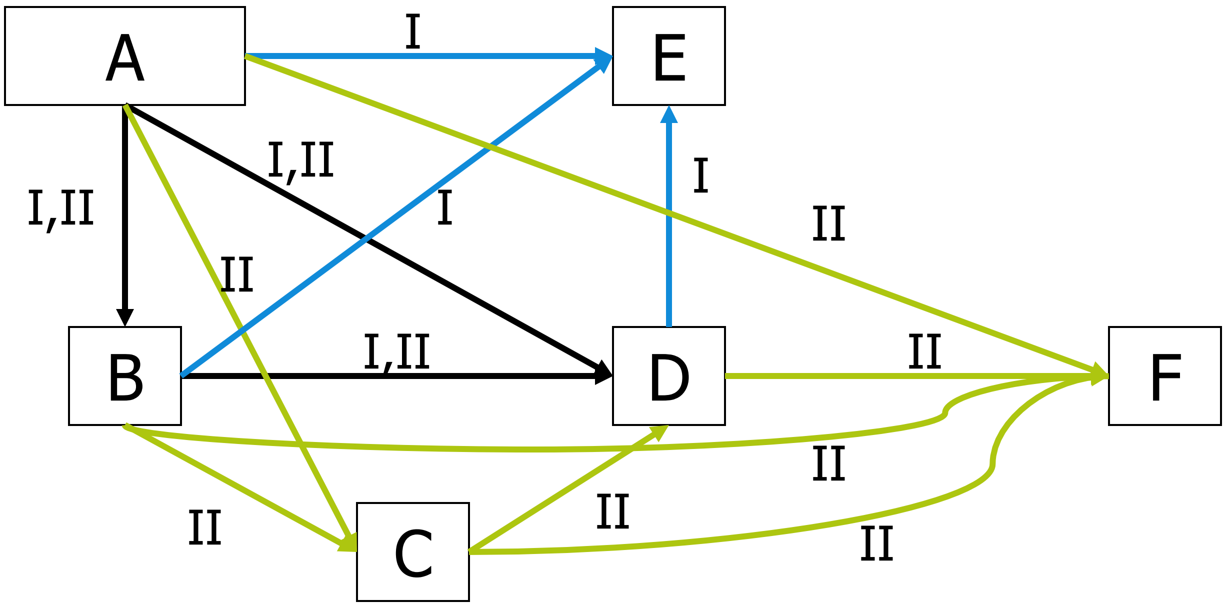
Transfer in algorithm

Line specific network



Transfer links

Route Sections

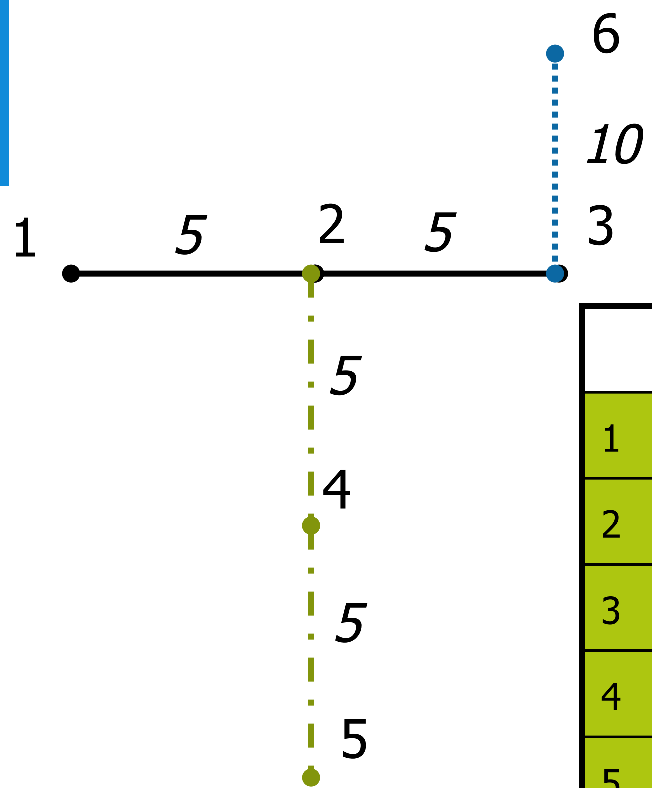


"Non-transfer links"

3.4

Public transport assignment: Algorithms

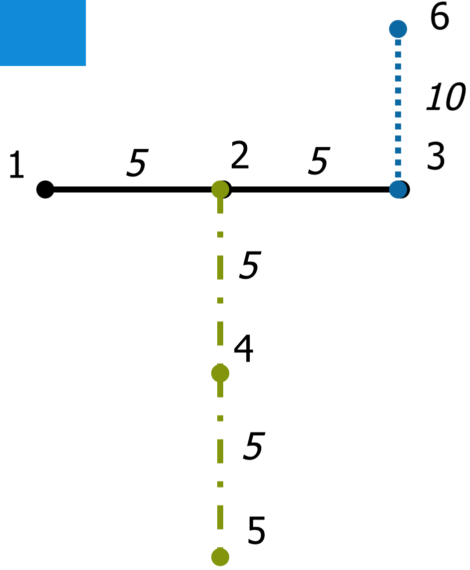
Floyd-Warschall (1/2)



	1	2	3	4	5	6
1	X	5 | 1	10-| 1	∞	∞	∞
2	5 | 2	X	5 | 2	5 | 2	10-| 2	∞
3	10-| 3	5 | 3	X	∞	∞	10-| 3
4	∞	5 | 4	∞	X	5 | 4	∞
5	∞	10-| 5	∞	5 | 5	X	∞
6	∞	∞	10-| 6	∞	∞	X

Time | back node

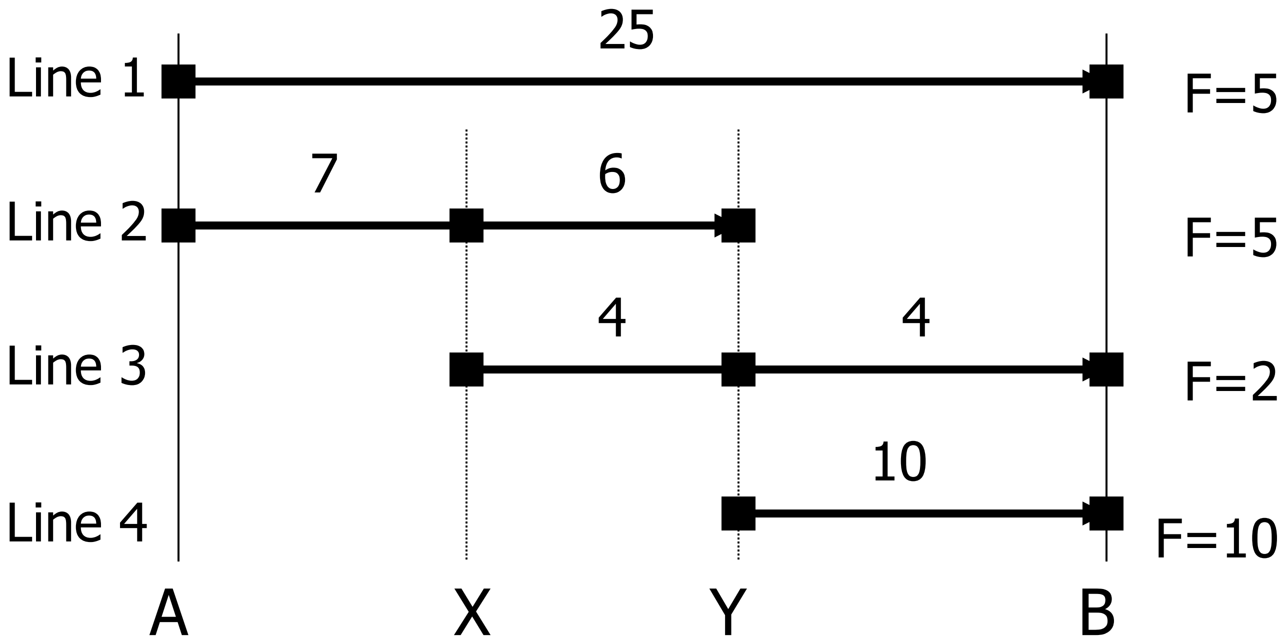
Floyd-Warschall (2/2)



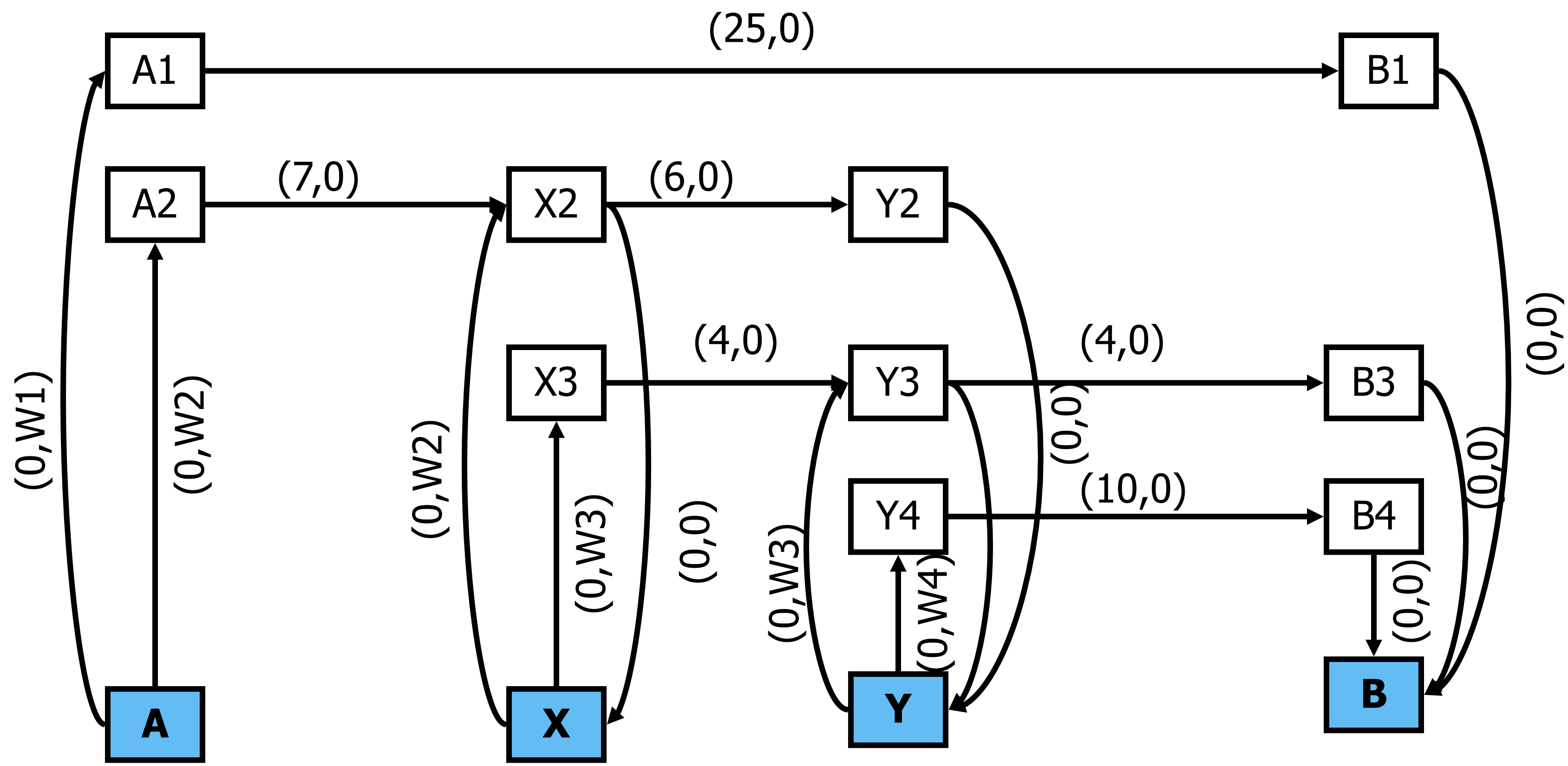
	1	2	3	4	5	6
1	X	5 | 1	10 | 1	10+Pt| 2	15+Pt| 2	20+Pt| 3
2	5 | 2	X	5 | 2	5 | 2	10 | 2	∞
3	10 | 3	5 | 3	X	∞	∞	10 | 3
4	∞	5 | 4	∞	X	5 | 4	∞
5	∞	10 | 5	∞	5 | 5	X	∞
6	∞	∞	10 | 6	∞	∞	X

For i = 1, N
for j = 1, N
for k=1, N
If $Z_{ikj}=Z_{ik}+P_t+Z_{kj}<Z_{ij}$
$Z_{ij}=Z_{ikj}$ *, back node*(Z_{ij}) = *back node* (Z_{kj})

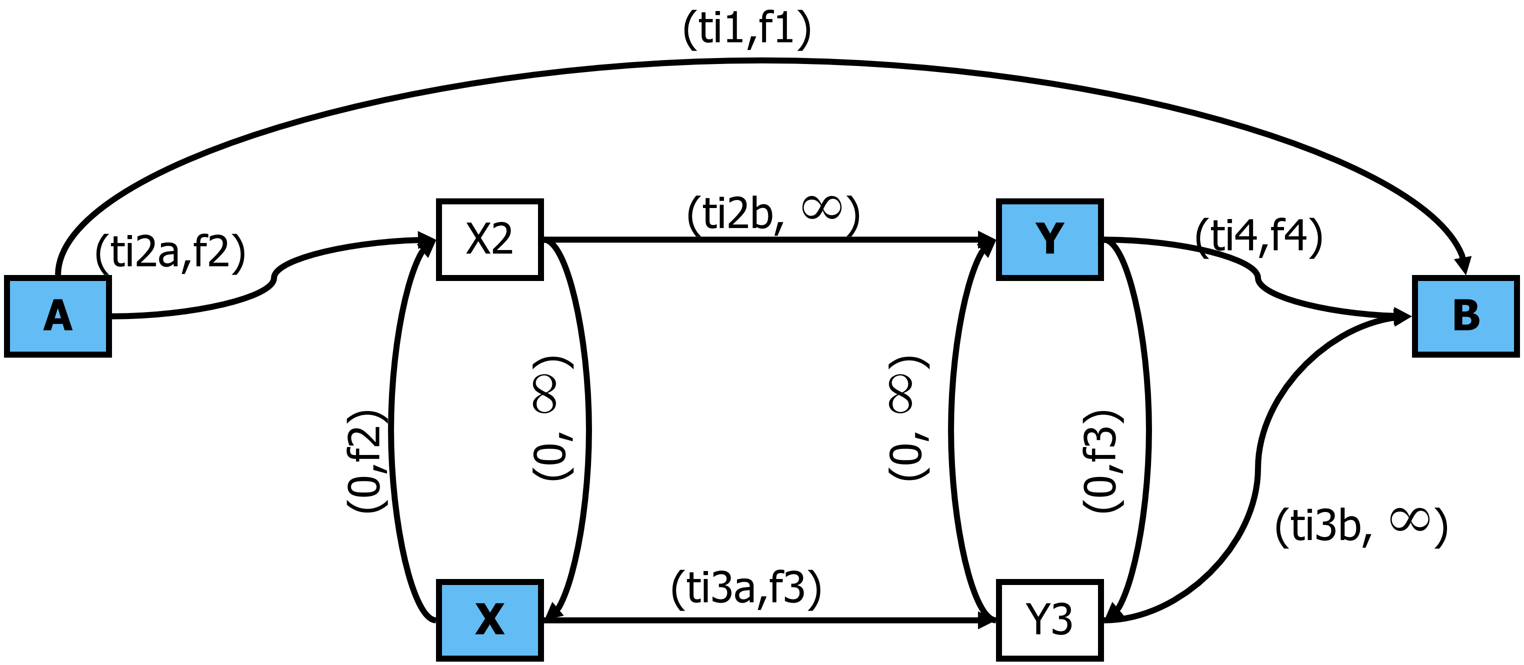
Strategies: network example
(Spiess & Florian, 1989)



Strategies: extended network



Strategies: reduced version

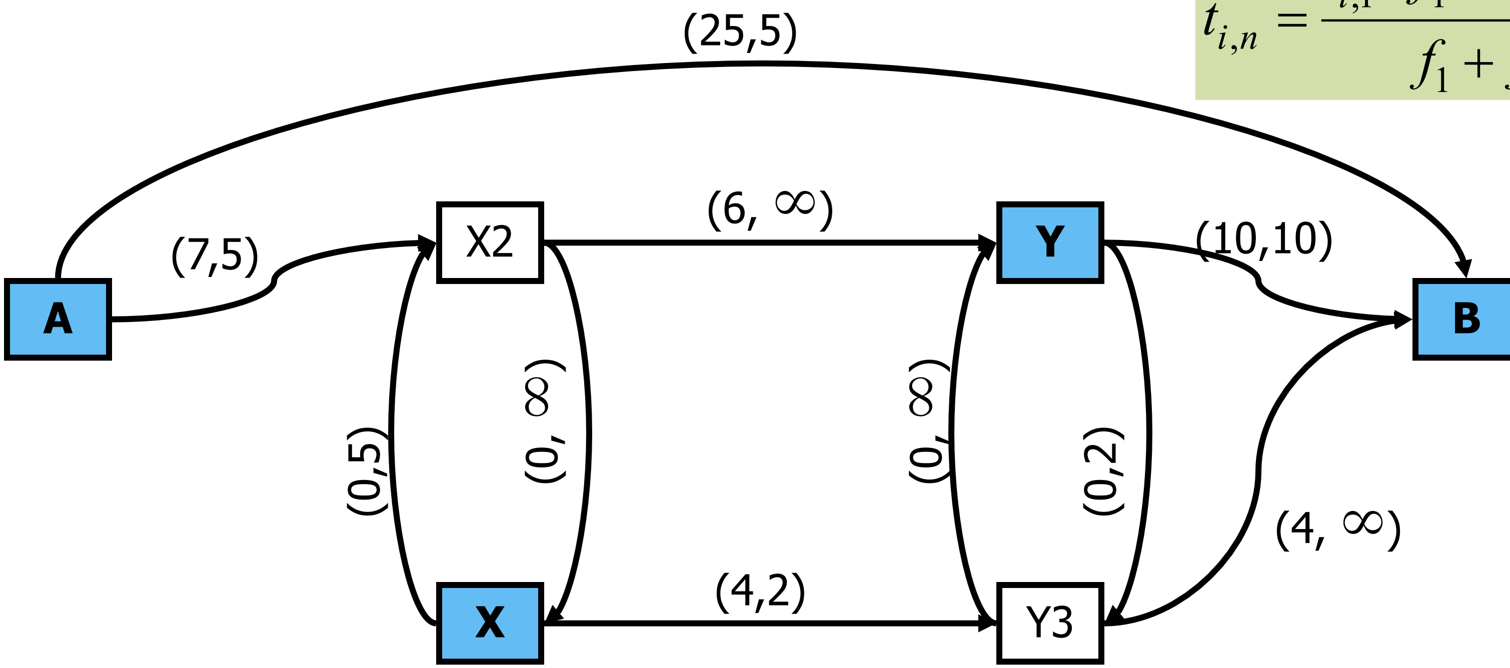


Strategies: reduced version

(time, freq)

$$f_n = f_1 + f_2 \Rightarrow t_w = \frac{30}{f}$$

$$t_{i,n} = \frac{t_{i,1} \cdot f_1 + t_{i,2} \cdot f_2}{f_1 + f_2}$$

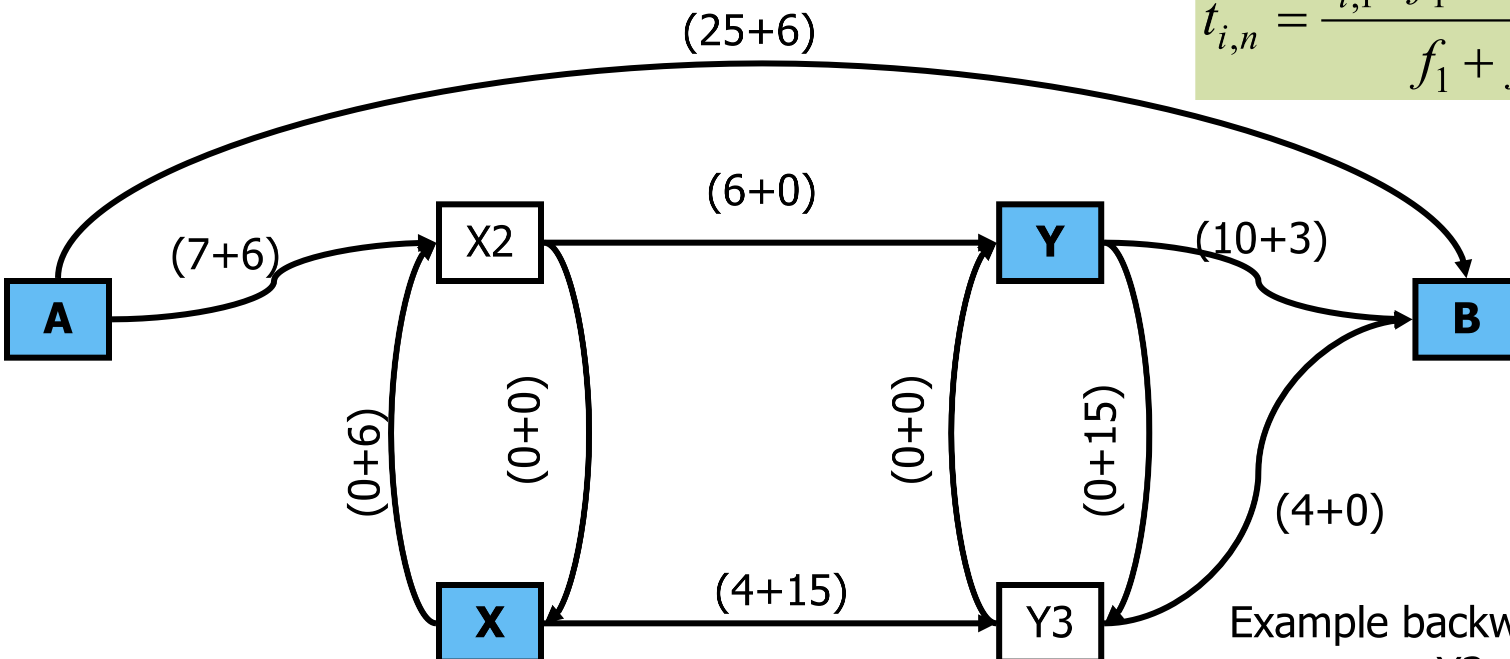


Strategies step 1: Backward search 1

(time + wait)

$$f_n = f_1 + f_2 \Rightarrow t_w = \frac{30}{f}$$

$$t_{i,n} = \frac{t_{i,1} \cdot f_1 + t_{i,2} \cdot f_2}{f_1 + f_2}$$



Example backward search

Y3->B: 4+0=4

Y->B: 10+3=13

Y->B: 4+15=19

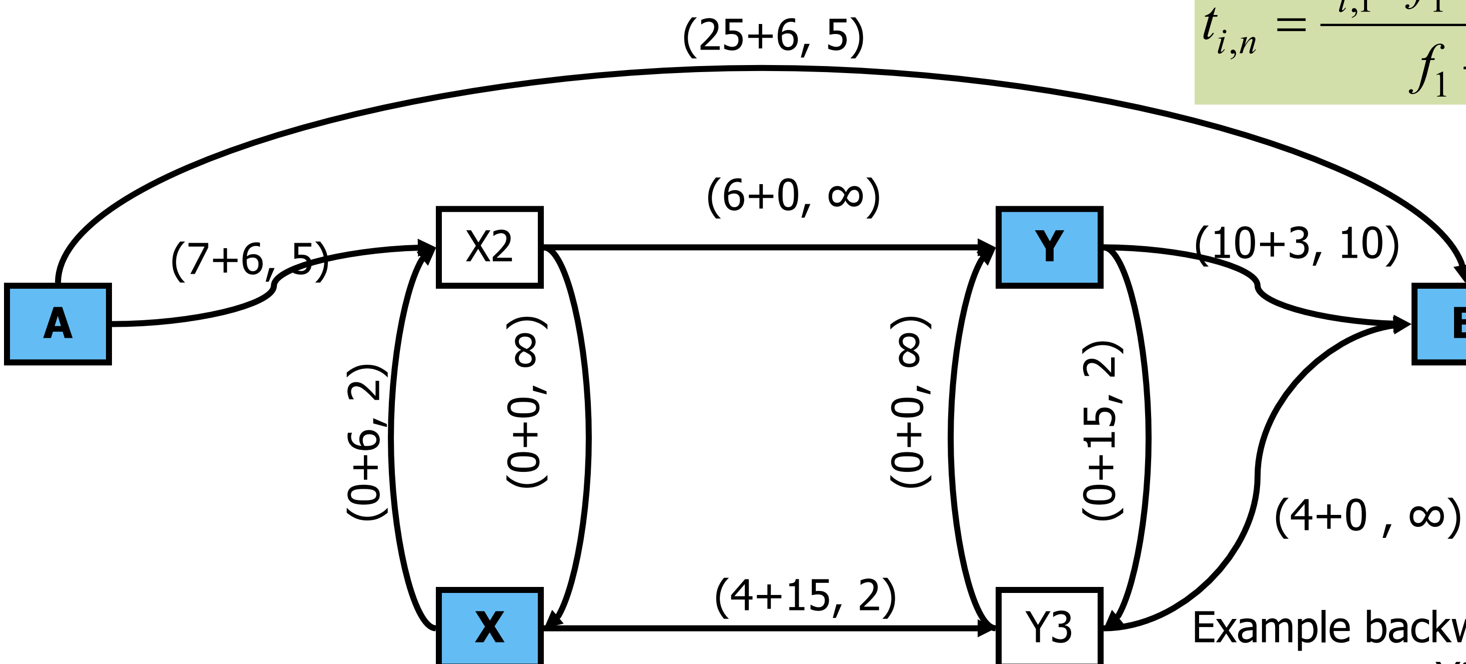
Y->B: f_n=10+2, $t_{i,n}$=(10*10+2*4)/12=9 =>9+2.5=11.5 **<13**!

Strategies step 1: Backward search 2

(time + wait, freq)

$$f_n = f_1 + f_2 \Rightarrow t_w = \frac{30}{f}$$

$$t_{i,n} = \frac{t_{i,1} \cdot f_1 + t_{i,2} \cdot f_2}{f_1 + f_2}$$

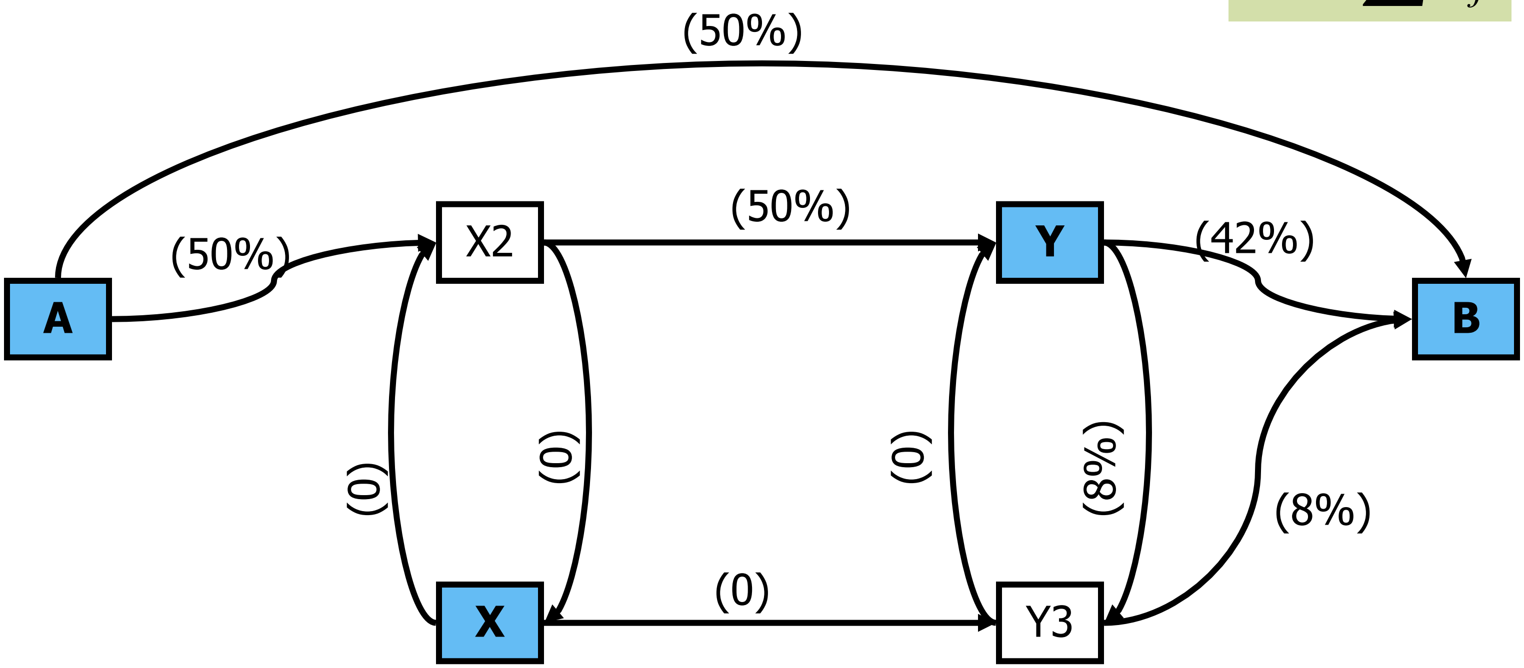


Example backward search 2
Y3->B: 4+0=4
Y->B: aggregate of two routes: 11.5
X->B: 4+4+15=23
X2-B=11.4+6=17.5

So once at X2 you don't transfer to X, since the travel time via X is 5.5 minutes higher and there's no waiting time for travelling to Y (you're in the vehicle!)

Strategies step 2: Forward assignment

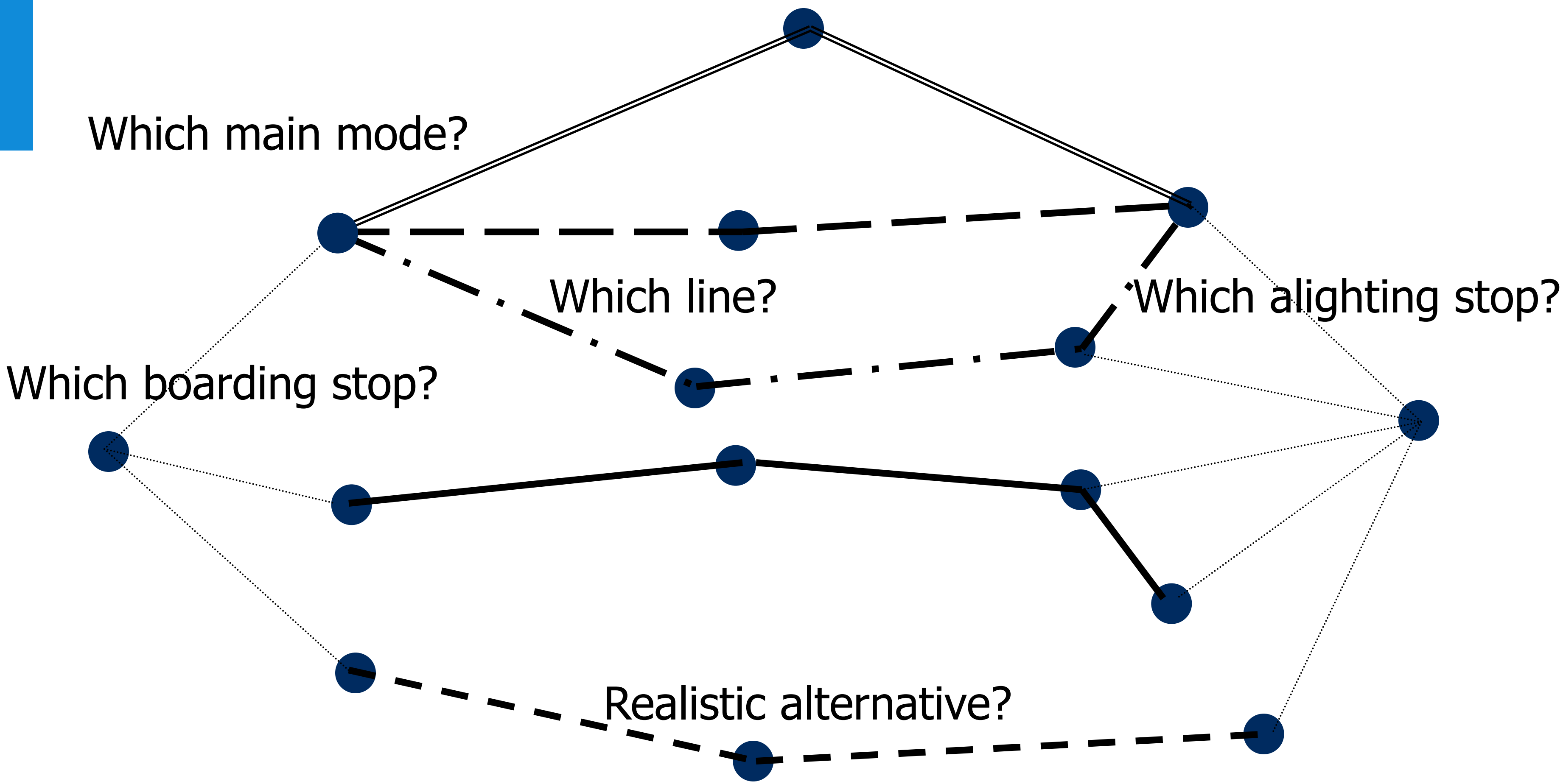
$$p_i = \frac{f_i}{\sum f_j}$$



3.5

Public transport assignment: Choice modelling approach

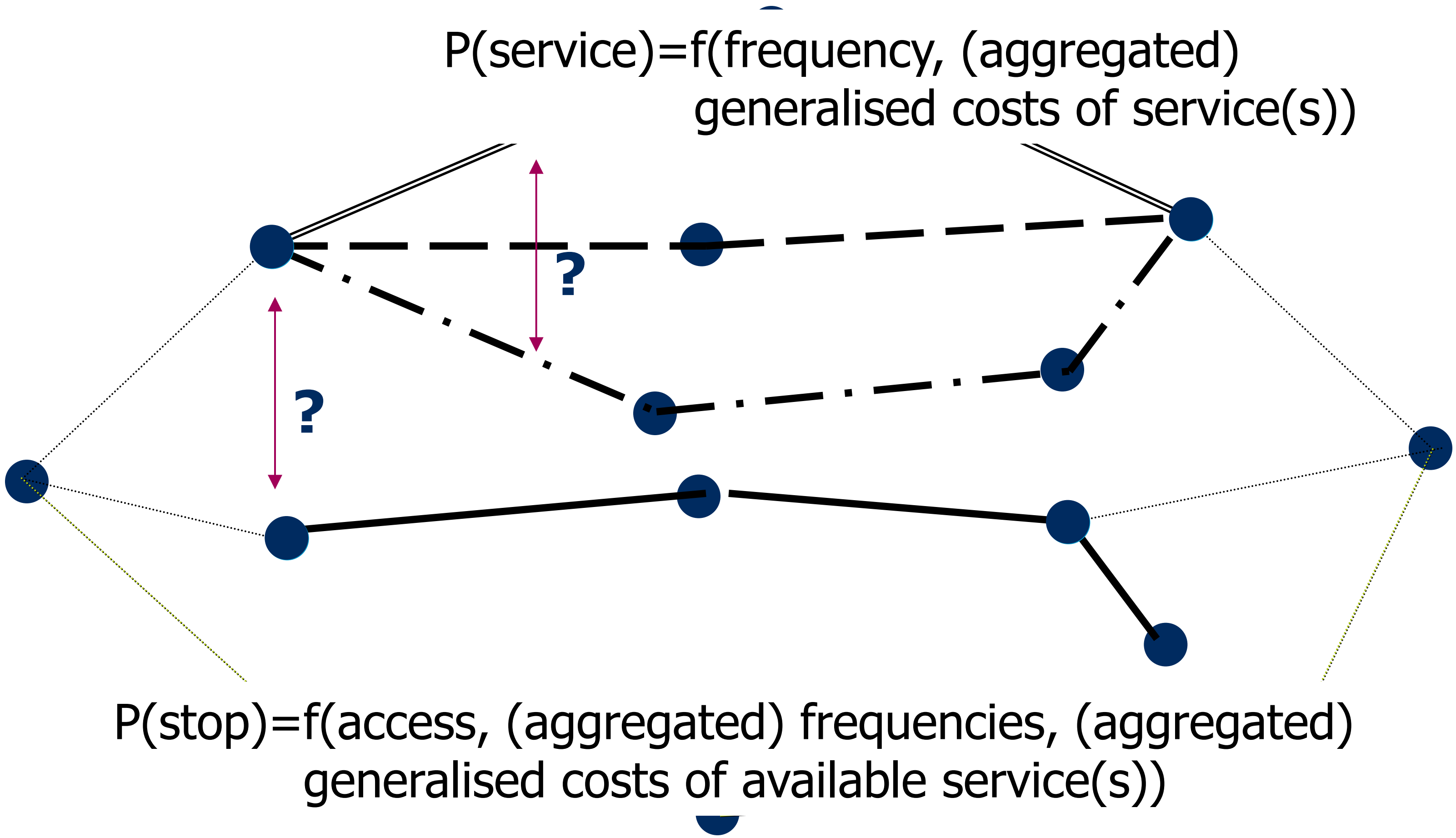
Sequential choices: network example



Omnitrans: Route search

- Determine alighting stop(s) at destination (maximum access distance)
- Determine boarding stop(s) at origin (maximum access distance)
- Start at destination, for each alighting stop
- Determine boarding stops without transfer
  - Determine stops that can be reached with one transfer
    - Determine stops that can be reached with two transfers
      - ….
        - Determine stops that can be reached with N_{max} transfers
        - Check logic of routes (detour)

Omnitrans: Sequential choices



Basic function for line choice

$$P_{i,s} = \frac{F_i \cdot e^{\beta_l \cdot GC_{i,s}}}{\sum_{j \in L_s} F_j \cdot e^{\beta_l \cdot GC_{j,s}}}$$

This models every vehicle as a separate alternative and groups them per line

$P_{i,s}$ = probability boarding line i at stop s

F_i = frequency of line i

β_l = scale parameter line choice

GC_i = generalised costs from stop s to destination, excluding waiting time (negative!)

L_s = set of line at stop s to travel to the destination

This function can also be applied for line choice at transfer nodes (different scale parameter is possible)

Basic function for stop choice

$$F_s = \sum_{j \in L_s} F_l \cdot \frac{e^{\beta_l \cdot GC_{j,s}}}{\max_{k \in L_s} e^{\beta_l \cdot GC_{k,s}}}$$

$$GC_s = \theta_a \cdot ta_s + \theta_w \cdot f_w \cdot \frac{1}{F_s} + \sum_{j \in L_s} P_{j,s} \cdot GC_{j,s}$$

$$P_s = \frac{e^{\beta_{bs} \cdot GC_s}}{\sum_{j \in S} e^{\beta_{bs} \cdot GC_j}}$$

F_s = aggregate frequency at stop s

GC_s = utility for boarding at stop s

ta_s = access time at stop s

f_w = factor waiting time

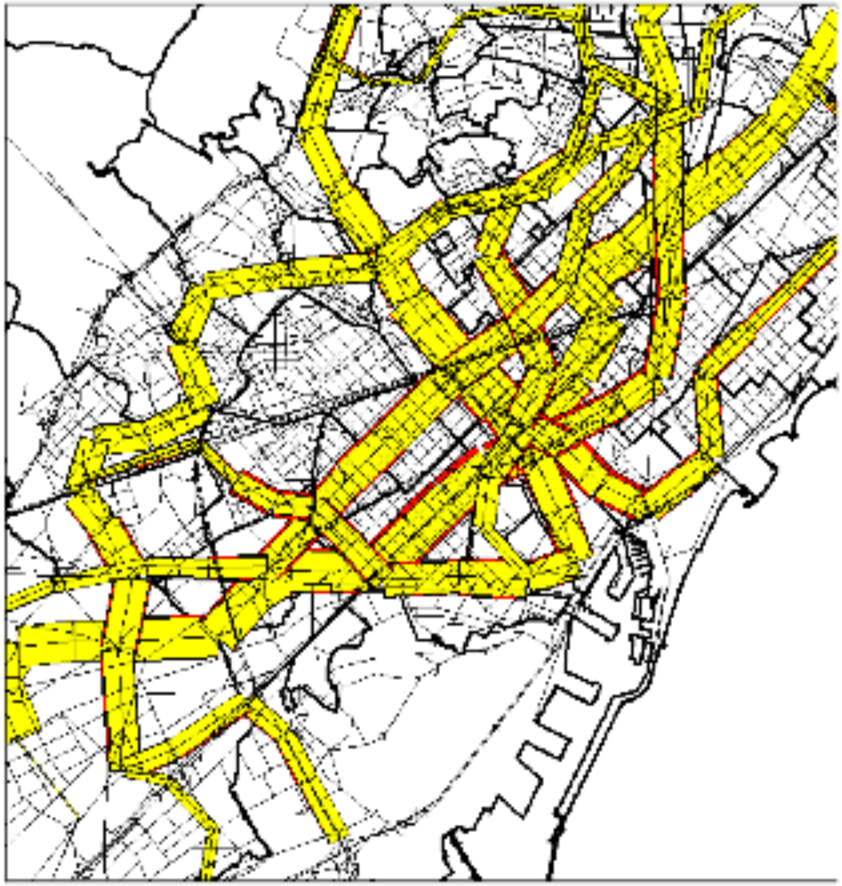
β_{bs} = scale parameter for boarding stop choice

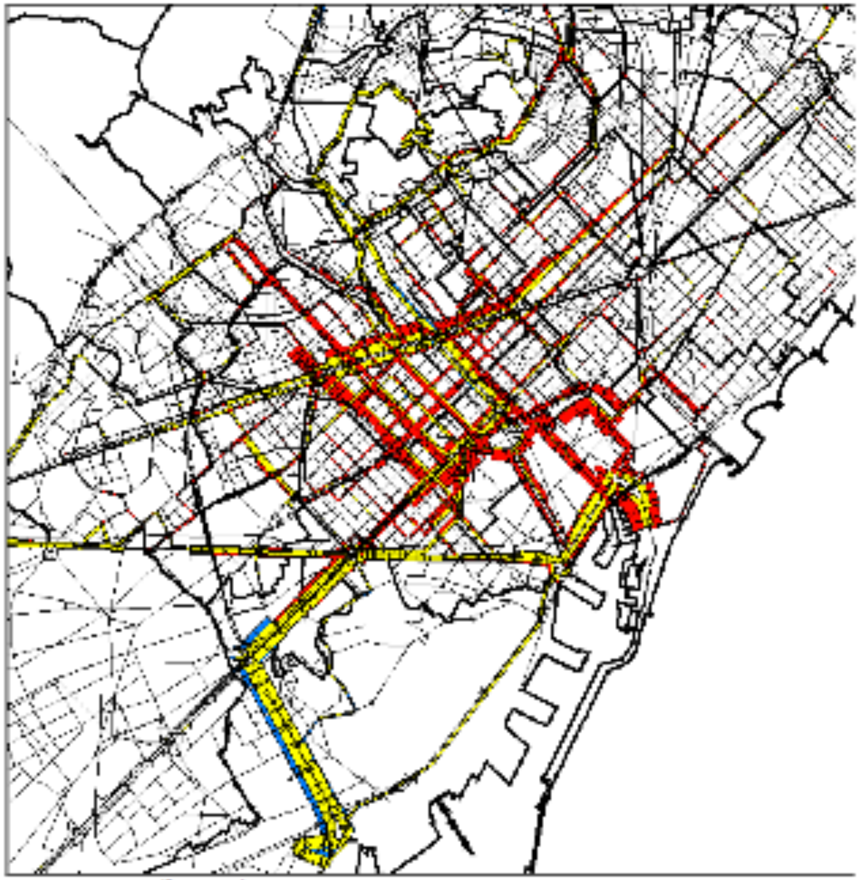
The frequency of the most attractive line is accounted for in full, frequencies of less attractive lines are reduced

Note that the in-vehicle time is based on a weighted average and not on a logsum

This function can also be applied for choice of transfer nodes using walk links

Impact on ridership metro and bus
(example RET Bus Barcelona)





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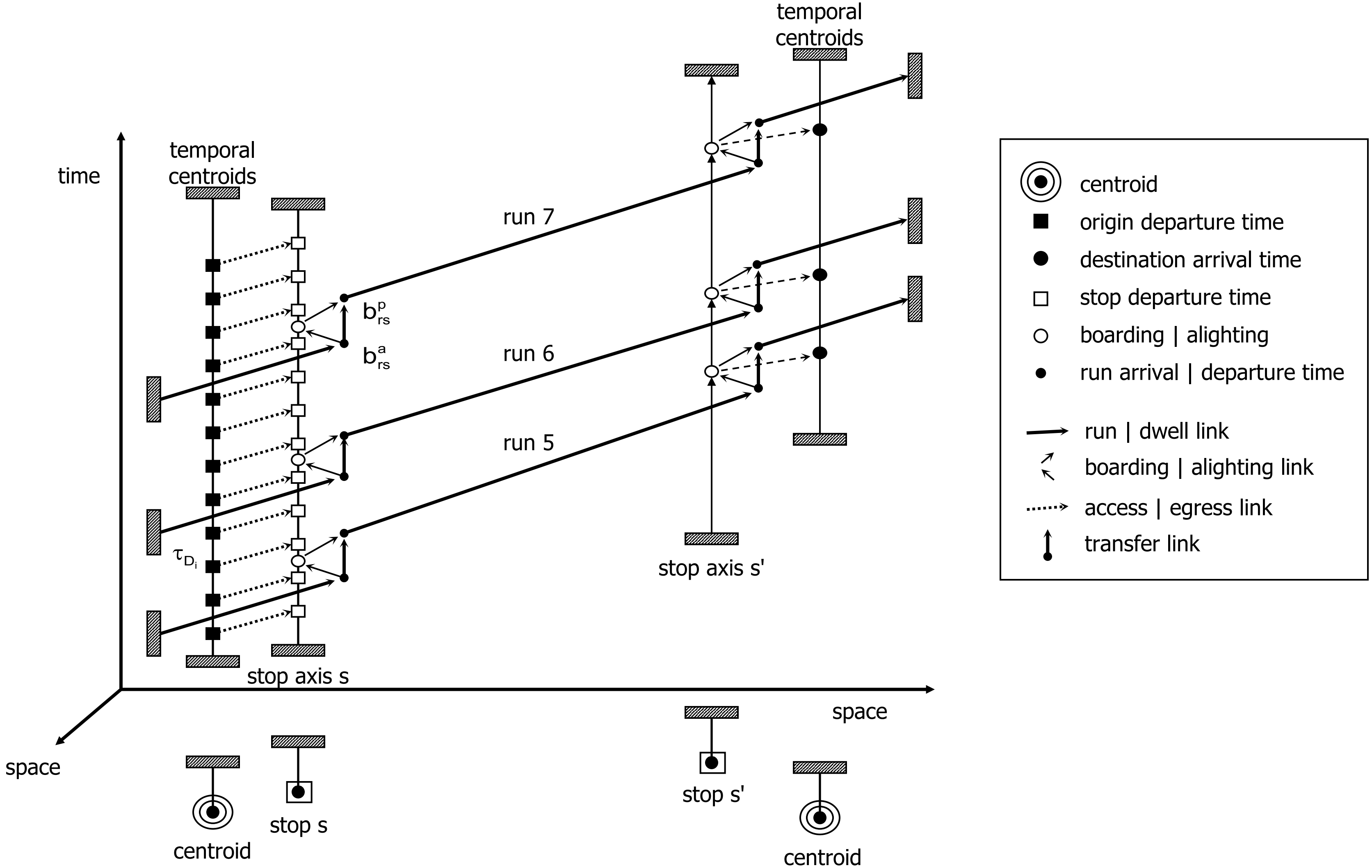
Capacity and (un)reliability

- Higher subjective in-vehicle time (‘BPR’)
- Operational frequency
- Perceived frequency
- Full vehicles

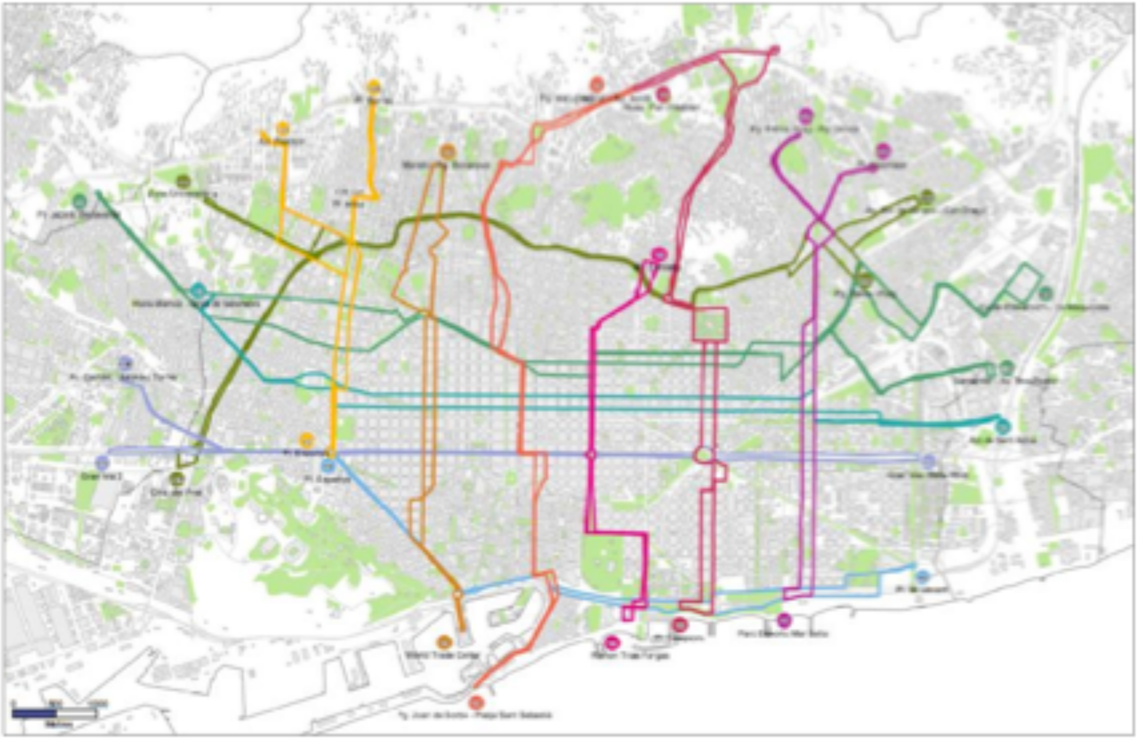
Run based (=> dynamic assignment)



3.6

Case: Modelling PT in Barcelona with Omnitrans

Case: RET-Bus network Barcelona



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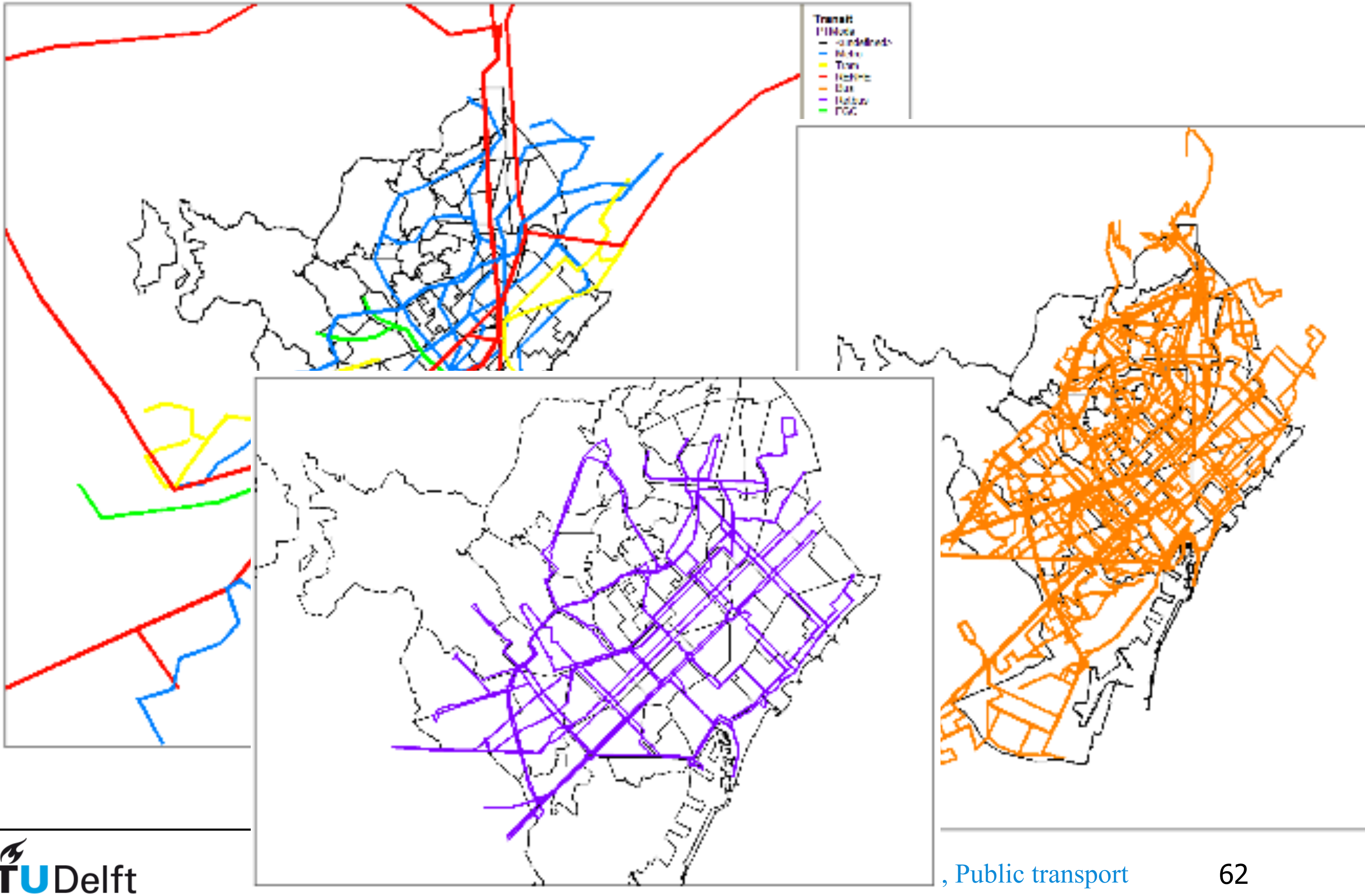
Existing PT network



How to model the RET-Bus system?

- Input
  - OD matrices car, bus and metro (based on surveys)
  - Networks car and PT
- Mode choice
  - Car and PT or Car, metro and bus?
- Assignment

$$V_{PT} = \sum_{n} \left[\left(2.0 \cdot t_a + 2.5 \cdot t_w + t_v + BP_m \right) \cdot VoT \right] + \sum_{n-1} \left(TP_m \cdot VoT \right) + 2.0 \cdot t_e \cdot VoT + Fare$$

  - Difference metro and bus: boarding penalty and transfer penalty
  - Philosophy: metro is preferred over bus, metro stations are more fuzz => what's the net effect?
- RET-bus is either bus, metro or in between

Validation PT assignment

	Model 1	Model 2	Model 3	Model 4
Parameter ϕ	3,0	2,0	3,0	2,0
Parameter BP_{bus}	10,0	10,0	7,0	7,0
Parameter BP_{metro}	5,0	5,0	5,0	5,0
Parameter TP_{bus}	5,0	5,0	3,0	3,0
Parameter TP_{metro}	0,0	0,0	0,0	0,0
Difference in bus trip legs 2007 (predicted-observed) (%)	-20,2	-15,0	+16,5	+23,0
Difference in metro trip legs 2007 (predicted-observed) (%)	+50,0	+48,0	+26,9	+25,2

NB: Data is in trips per mode, Omnitrans computes legs per mode!

Impact on mode choice

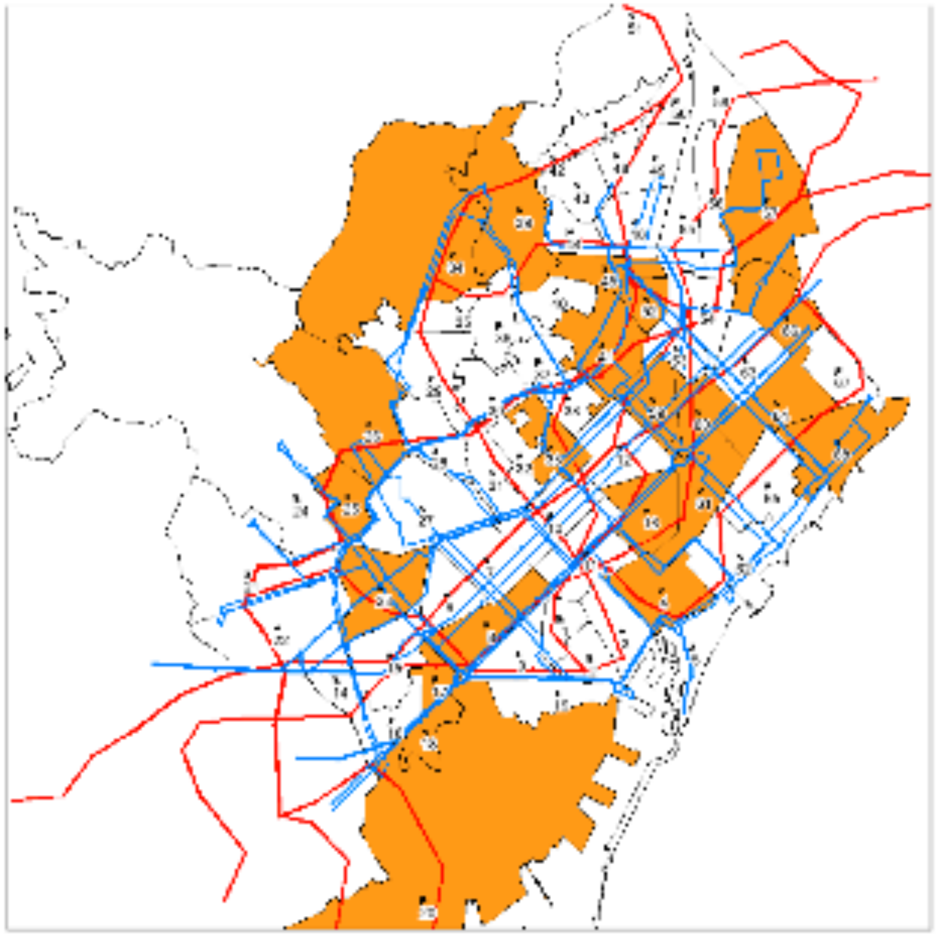


Figure 5-7: Zones that produce more than +1,50% additional transit trips in Alternative 1 (TMB plan) compared with the base case in 2016 (in orange). The metro and BRT networks are shown in red and blue, respectively.

Impact on ridership metro and bus