

# Dredging Processes

Dr.ir. Sape A. Miedema

3. Cutting Introduction





[1]

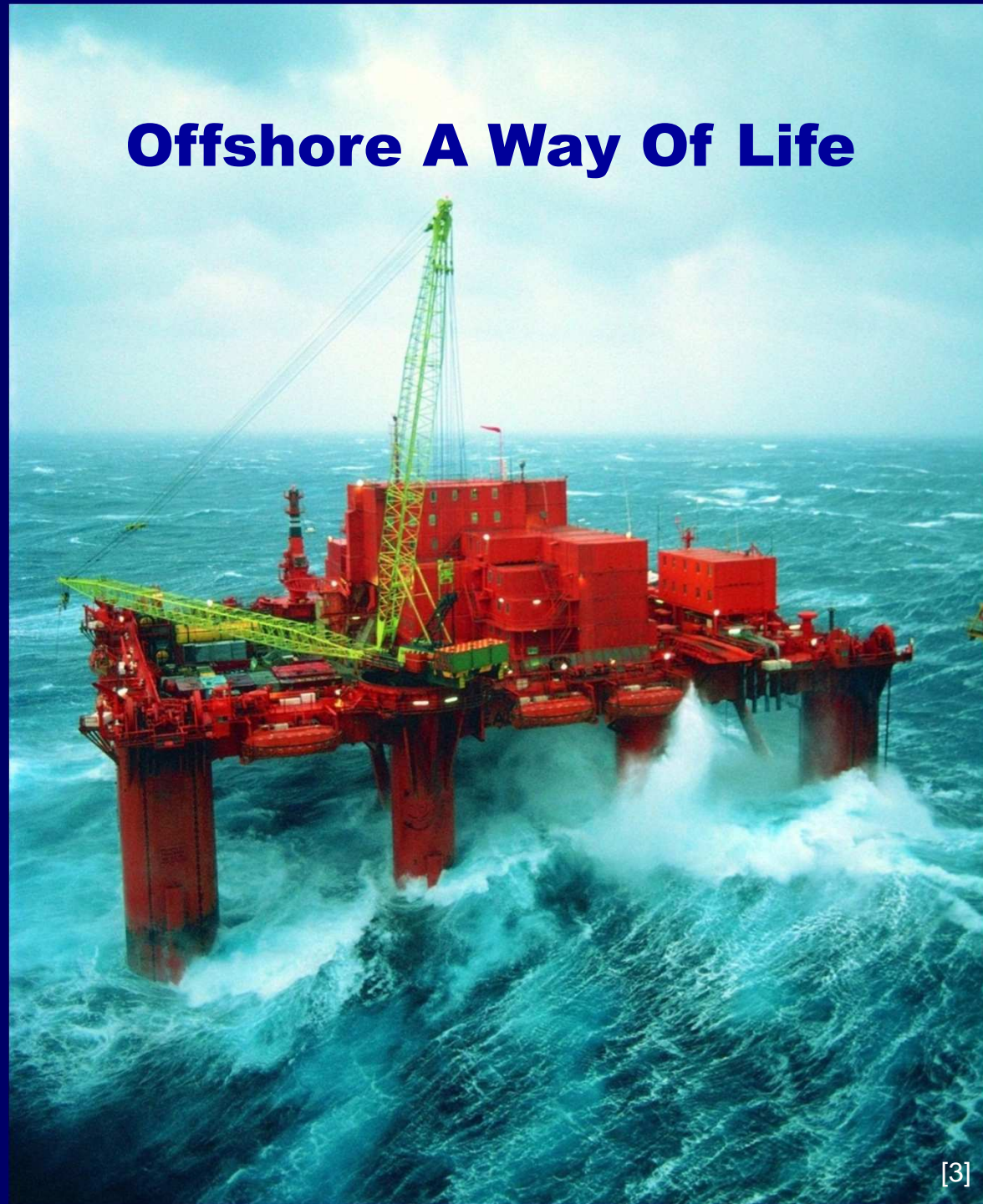
# Dredging A Way Of Life







[2]



[3]

# Offshore A Way Of Life





# Offshore & Dredging Engineering

**Dr.ir. Sape A. Miedema**  
**Educational Director**

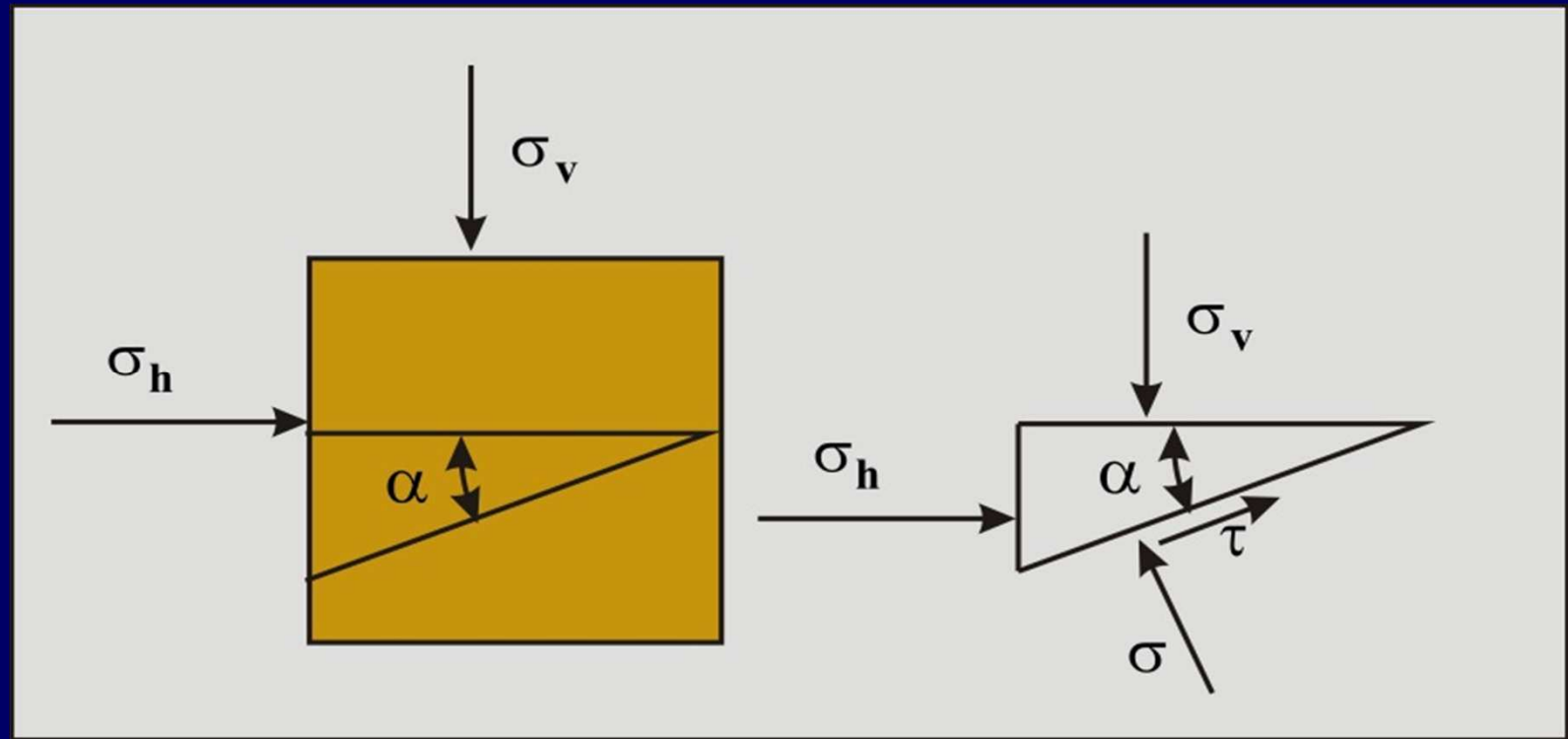






# Mohr Circle

# Mohr Circle 1



Vertical Equilibrium of Forces

$$\sigma_v \cdot \cos(\alpha) = \sigma \cdot \cos(\alpha) + \tau \cdot \sin(\alpha)$$

Horizontal Equilibrium of Forces

$$\sigma_h \cdot \sin(\alpha) = \sigma \cdot \sin(\alpha) - \tau \cdot \cos(\alpha)$$

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## Mohr Circle 2

$$\sigma_v \cdot \cos(\alpha) \cdot \cos(\alpha) = \sigma \cdot \cos(\alpha) \cdot \cos(\alpha) + \tau \cdot \sin(\alpha) \cdot \cos(\alpha)$$

$$\sigma_h \cdot \sin(\alpha) \cdot \sin(\alpha) = \sigma \cdot \sin(\alpha) \cdot \sin(\alpha) - \tau \cdot \cos(\alpha) \cdot \sin(\alpha)$$

$$\sigma_v \cdot \cos^2(\alpha) + \sigma_h \cdot \sin^2(\alpha) = \sigma$$

$$\cos^2(\alpha) = \frac{1 + \cos(2 \cdot \alpha)}{2}$$

$$\sin^2(\alpha) = \frac{1 - \cos(2 \cdot \alpha)}{2}$$

$$\sigma = \left( \frac{\sigma_v + \sigma_h}{2} \right) + \left( \frac{\sigma_v - \sigma_h}{2} \right) \cdot \cos(2 \cdot \alpha)$$



## Mohr Circle 3

$$\sigma_v \cdot \cos(\alpha) \cdot \sin(\alpha) = \sigma \cdot \cos(\alpha) \cdot \sin(\alpha) + \tau \cdot \sin(\alpha) \cdot \sin(\alpha)$$

$$-\sigma_h \cdot \sin(\alpha) \cdot \cos(\alpha) = -\sigma \cdot \sin(\alpha) \cdot \cos(\alpha) + \tau \cdot \cos(\alpha) \cdot \cos(\alpha)$$

$$(\sigma_v - \sigma_h) \cdot \sin(\alpha) \cdot \cos(\alpha) = \tau$$

$$\tau = \left( \frac{\sigma_v - \sigma_h}{2} \right) \cdot \sin(2 \cdot \alpha)$$





# Mohr Circle 4

$$\sigma - \left( \frac{\sigma_v + \sigma_h}{2} \right) = \left( \frac{\sigma_v - \sigma_h}{2} \right) \cdot \cos(2 \cdot \alpha)$$

$$\tau = \left( \frac{\sigma_v - \sigma_h}{2} \right) \cdot \sin(2 \cdot \alpha)$$

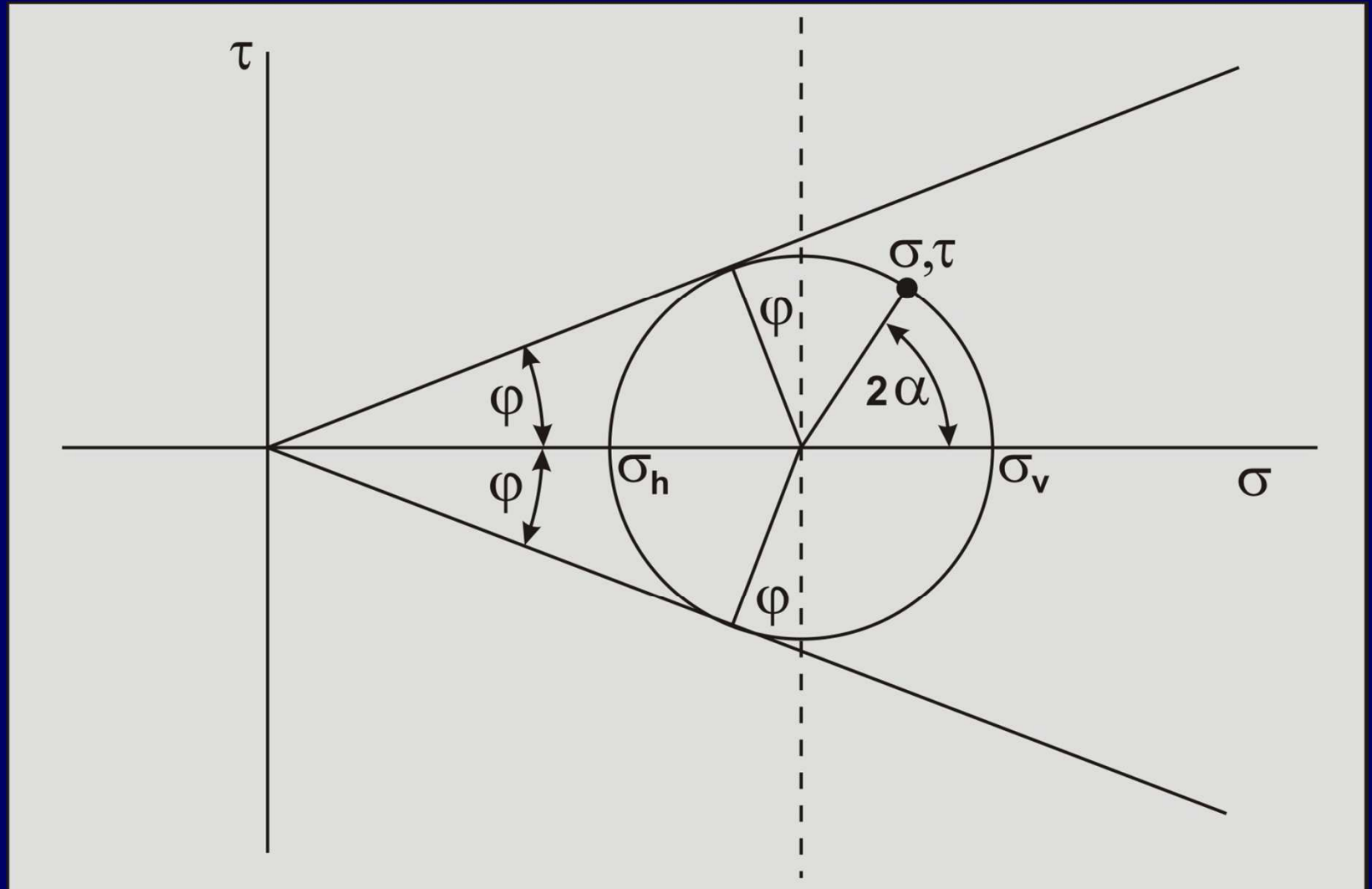
$$\left( \sigma - \left( \frac{\sigma_v + \sigma_h}{2} \right) \right)^2 = \left( \frac{\sigma_v - \sigma_h}{2} \right)^2 \cdot \cos^2(2 \cdot \alpha)$$

$$\tau^2 = \left( \frac{\sigma_v - \sigma_h}{2} \right)^2 \cdot \sin^2(2 \cdot \alpha)$$

$$\left( \sigma - \left( \frac{\sigma_v + \sigma_h}{2} \right) \right)^2 + \tau^2 = \left( \frac{\sigma_v - \sigma_h}{2} \right)^2$$

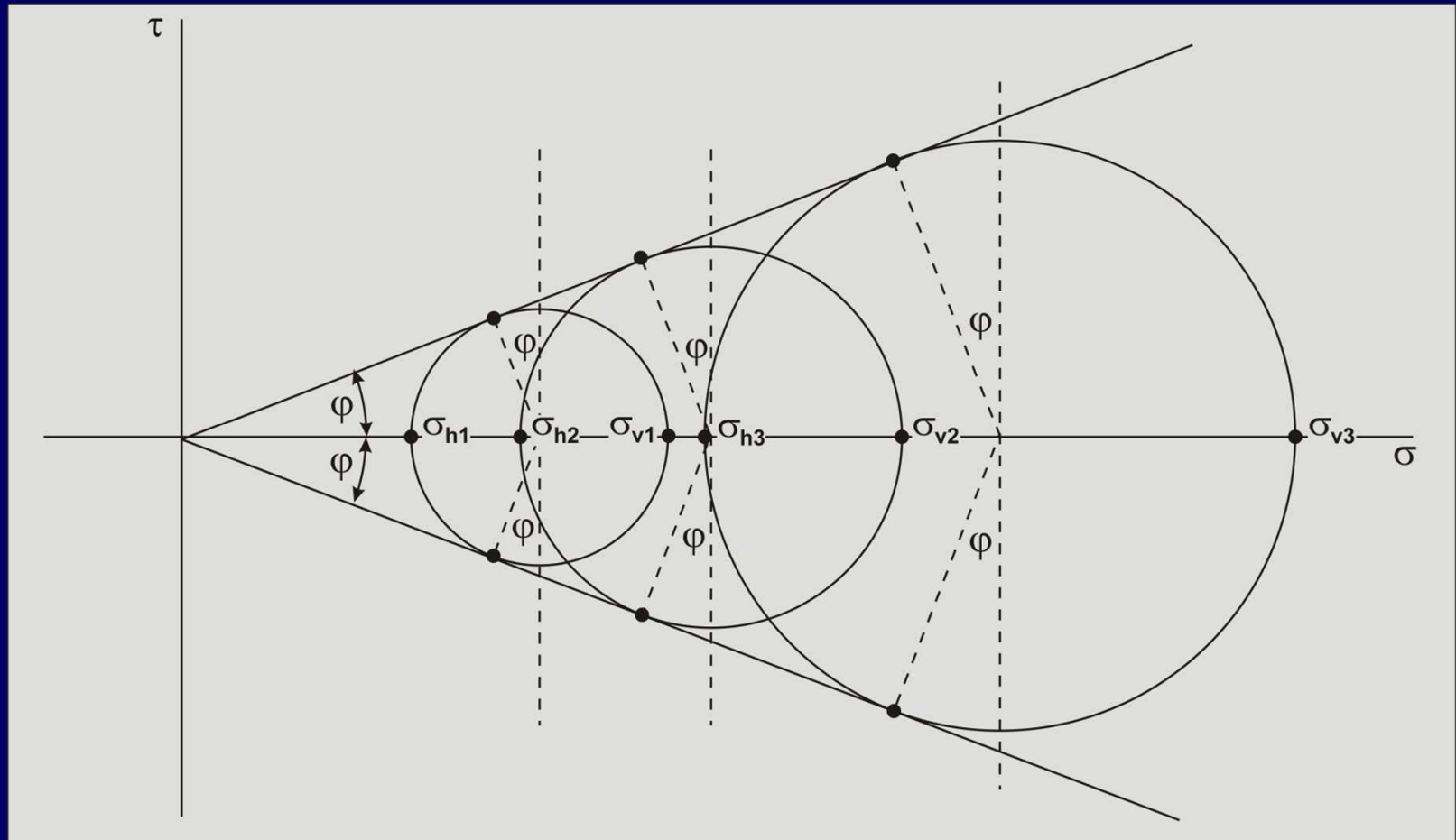


# Mohr Circle 5



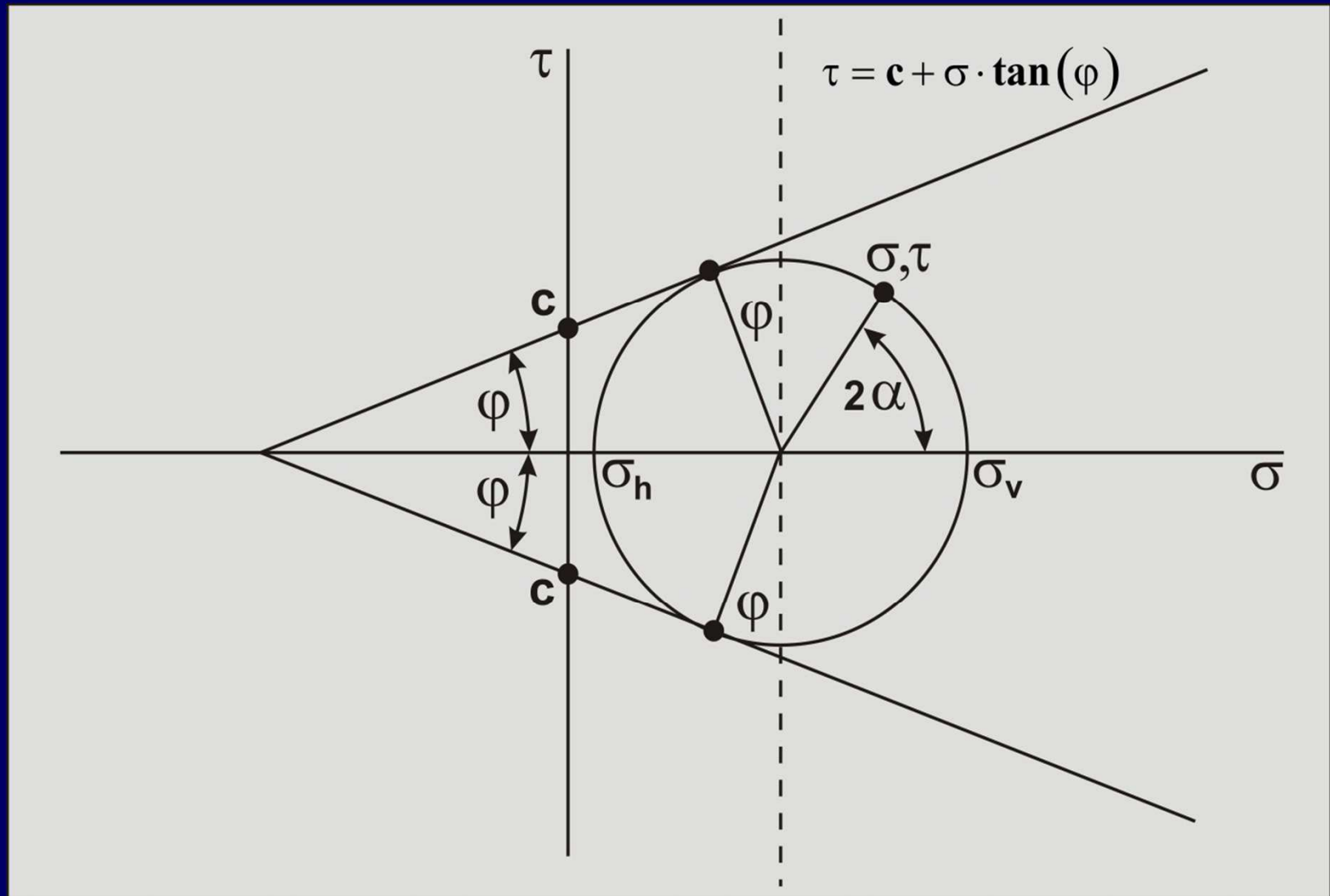


# Mohr Circle From Triaxial Tests



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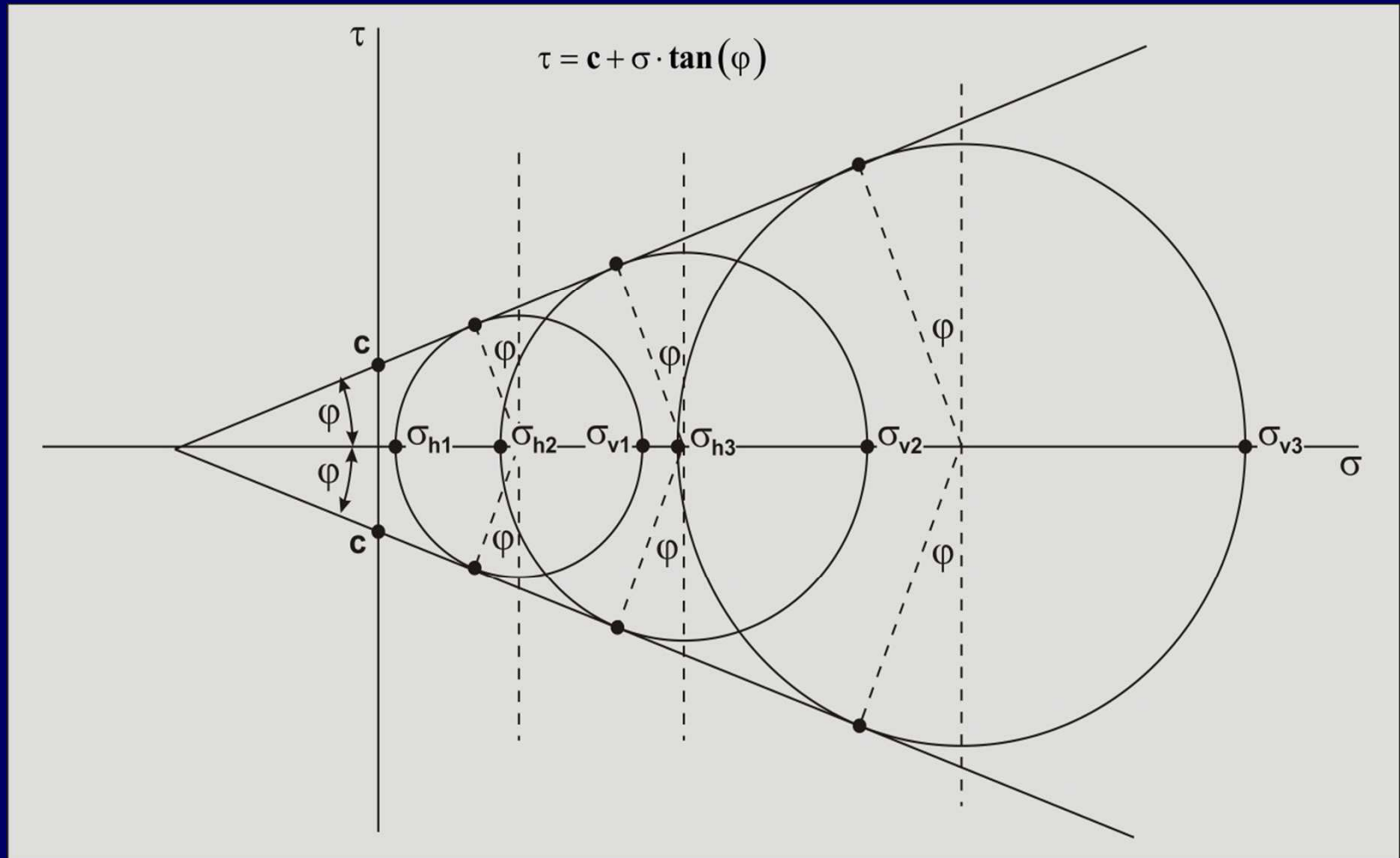
# Mohr Circle With Cohesion



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# Mohr Circle From Triaxial Tests

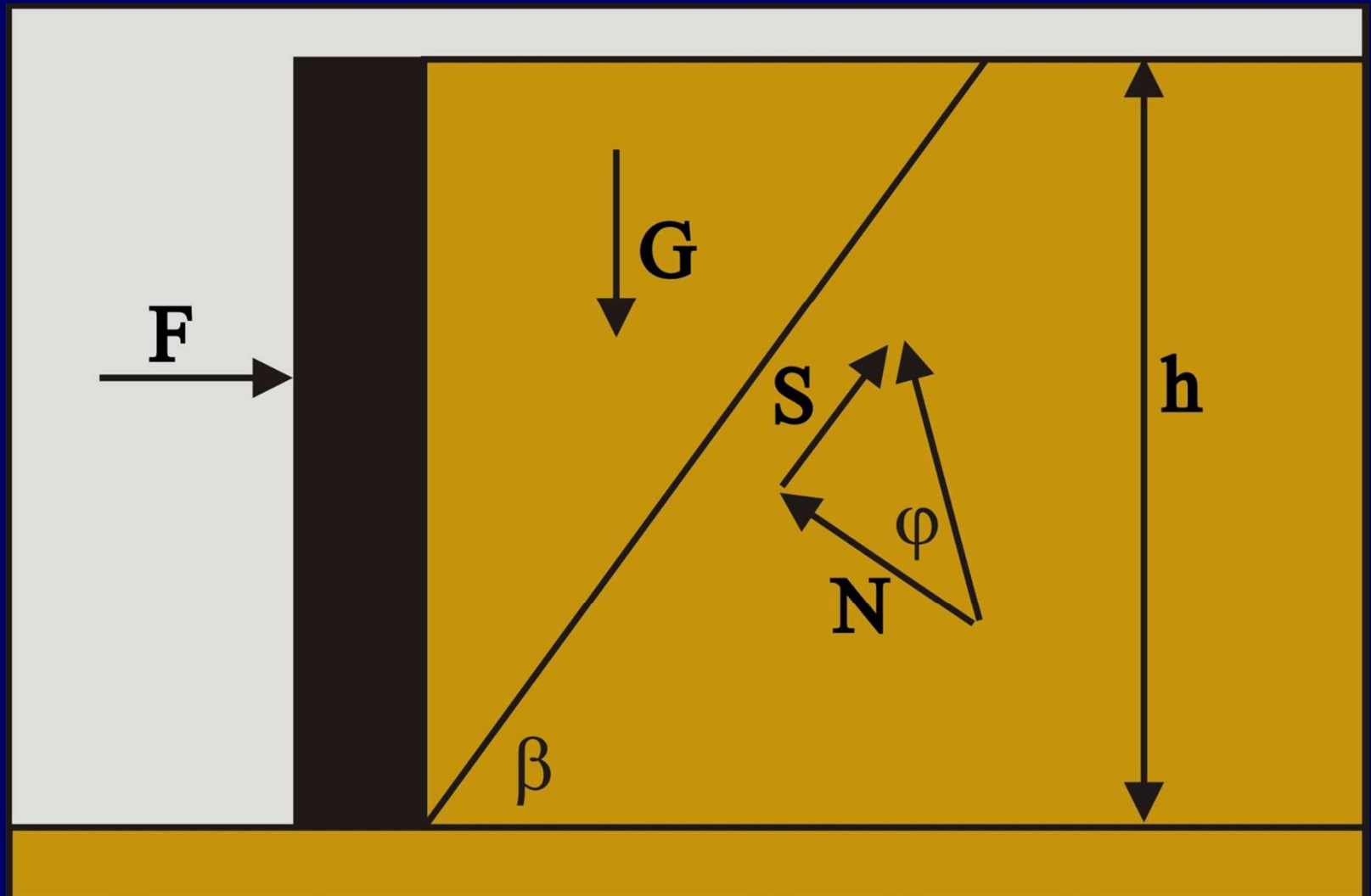


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# Active Soil Failure

# Active Soil Failure 1





## Active Soil Failure 2

$$G = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \cot(\beta)$$

$$S = N \cdot \tan(\varphi)$$

No cohesion  $\Rightarrow c=0$

No adhesion  $\Rightarrow a=0$

Smooth wall  $\Rightarrow \delta=0$

$$\text{Horizontal} \Rightarrow F + S \cdot \cos(\beta) - N \cdot \sin(\beta) = 0$$

$$\text{Vertical} \Rightarrow G - N \cdot \cos(\beta) - S \cdot \sin(\beta) = 0$$

## Active Soil Failure 3

$$F = -G \cdot \tan(\varphi - \beta)$$

$$G = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \cot(\beta)$$

$$F = -\frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \frac{\cos(\beta) \cdot \sin(\varphi - \beta)}{\sin(\beta) \cdot \cos(\varphi - \beta)}$$

F at maximum if:

$$\frac{dF}{d\beta} = 0$$

$$\frac{d^2F}{d\beta^2} < 0$$

$$F = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \left( 1 - \frac{\sin(\varphi)}{\sin(\beta) \cdot \cos(\varphi - \beta)} \right)$$

$$f = \sin(\beta) \cdot \cos(\beta - \varphi) \Rightarrow F \text{ maximum if } f \text{ maximum}$$

## Intermezzo

$$\frac{\cos(\beta) \cdot \sin(\varphi - \beta)}{\sin(\beta) \cdot \cos(\varphi - \beta)} =$$

$$\frac{\cos(\beta) \cdot \sin(\varphi - \beta)}{\sin(\beta) \cdot \cos(\varphi - \beta)} - 1 + 1 =$$

$$\frac{\cos(\beta) \cdot \sin(\varphi - \beta)}{\sin(\beta) \cdot \cos(\varphi - \beta)} - \frac{\sin(\beta) \cdot \cos(\varphi - \beta)}{\sin(\beta) \cdot \cos(\varphi - \beta)} + 1 =$$

$$1 - \frac{\sin(\varphi)}{\sin(\beta) \cdot \cos(\varphi - \beta)}$$



## Active Soil Failure 4

$$\frac{df}{d\beta} = \cos(2 \cdot \beta - \varphi)$$

$$\frac{d^2f}{d\beta^2} = -2 \cdot \sin(2 \cdot \beta - \varphi)$$

$$\frac{df}{d\beta} = 0 \Rightarrow \beta = \frac{\pi}{4} + \frac{1}{2} \cdot \varphi$$

$$\frac{d^2f}{d\beta^2} = -2 \text{ for } \beta = \frac{\pi}{4} + \frac{1}{2} \cdot \varphi$$

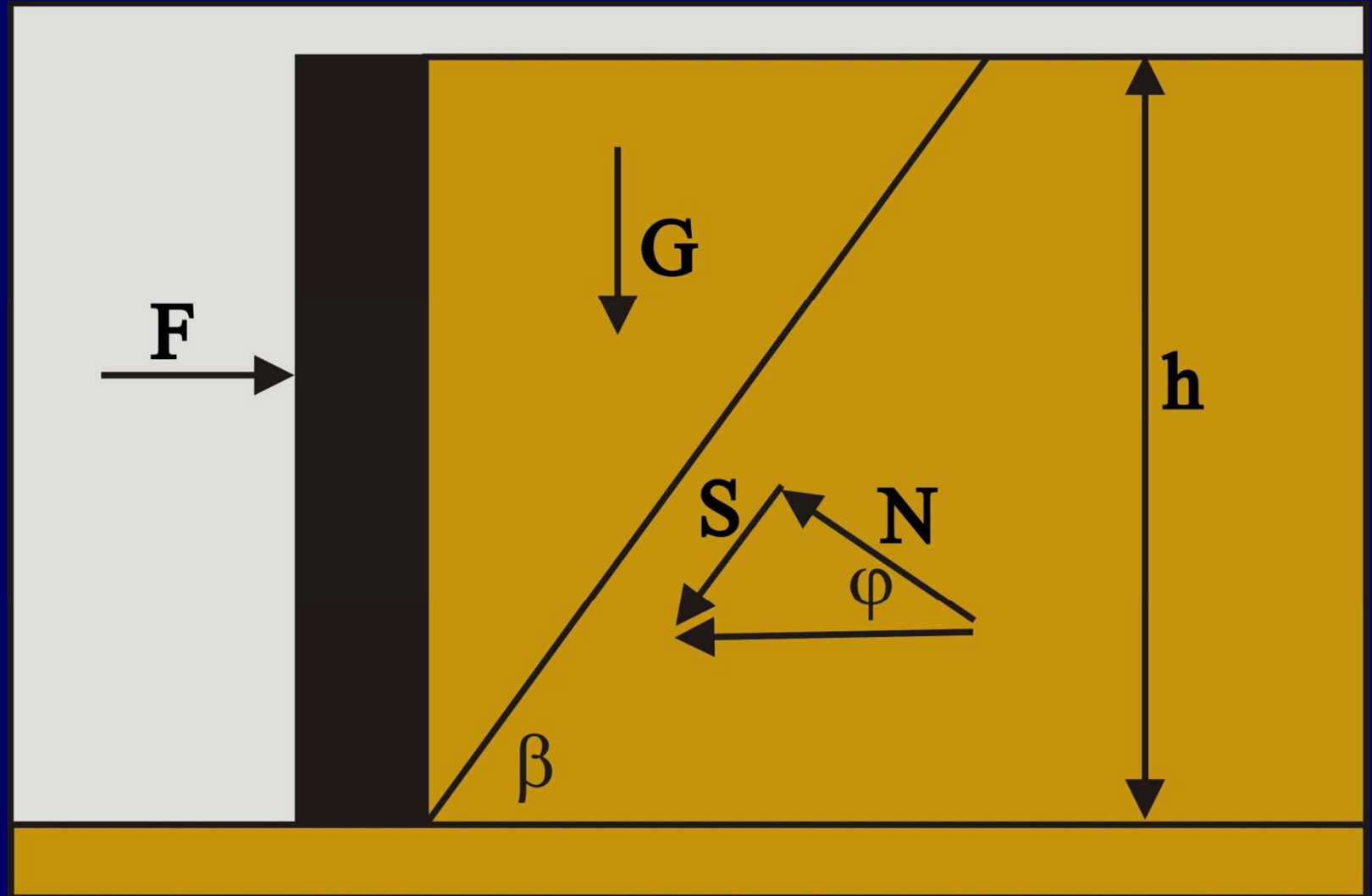
$$F = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \left( \frac{1 - \sin(\varphi)}{1 + \sin(\varphi)} \right) = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot K_a$$

$$K_A = \frac{1 - \sin \varphi}{1 + \sin \varphi} = \tan^2(45 - \varphi / 2)$$

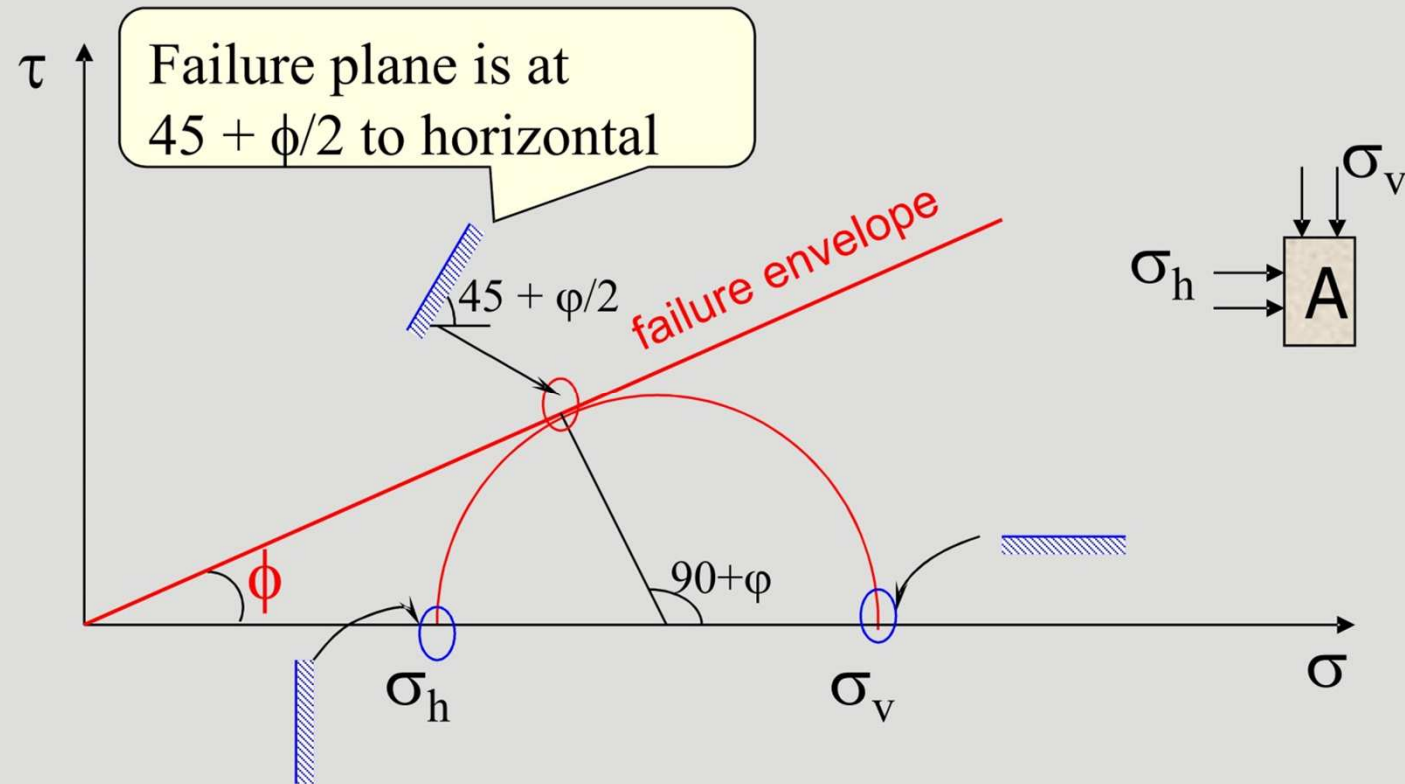
$$\sigma_h = K_A \cdot \sigma_v$$



# Passive Soil Failure 1



# Active Soil Failure 5







# Passive Soil Failure

## Passive Soil Failure 2

$$G = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \cot(\beta)$$

$$S = N \cdot \tan(\varphi)$$

No cohesion  $\Rightarrow c=0$

No adhesion  $\Rightarrow a=0$

Smooth wall  $\Rightarrow \delta=0$

$$\text{Horizontal} \Rightarrow F - S \cdot \cos(\beta) - N \cdot \sin(\beta) = 0$$

$$\text{Vertical} \Rightarrow G - N \cdot \cos(\beta) + S \cdot \sin(\beta) = 0$$

## Passive Soil Failure 3

$$F = G \cdot \tan(\varphi + \beta)$$

$$G = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \cot(\beta)$$

$$F = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \frac{\cos(\beta) \cdot \sin(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)}$$

F at minimum if:

$$\frac{dF}{d\beta} = 0$$

$$\frac{d^2F}{d\beta^2} > 0$$

$$F = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \left( 1 + \frac{\sin(\varphi)}{\sin(\beta) \cdot \cos(\varphi + \beta)} \right)$$

$$f = \sin(\beta) \cdot \cos(\beta + \varphi) \Rightarrow F \text{ minimum if } f \text{ maximum}$$



## Intermezzo

$$\frac{\cos(\beta) \cdot \sin(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)} = \frac{\cos(\beta) \cdot \sin(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)} - 1 + 1 =$$

$$\frac{\cos(\beta) \cdot \sin(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)} - \frac{\sin(\beta) \cdot \cos(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)} + 1 =$$

$$\frac{\cos(-\beta) \cdot \sin(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)} + \frac{\sin(-\beta) \cdot \cos(\varphi + \beta)}{\sin(\beta) \cdot \cos(\varphi + \beta)} + 1 =$$

$$1 + \frac{\sin(\varphi)}{\sin(\beta) \cdot \cos(\varphi + \beta)}$$



## Passive Soil Failure 4

$$\frac{df}{d\beta} = \cos(2 \cdot \beta + \varphi)$$

$$\frac{d^2f}{d\beta^2} = -2 \cdot \sin(2 \cdot \beta + \varphi)$$

$$\frac{df}{d\beta} = 0 \Rightarrow \beta = \frac{\pi}{4} - \frac{1}{2} \cdot \varphi$$

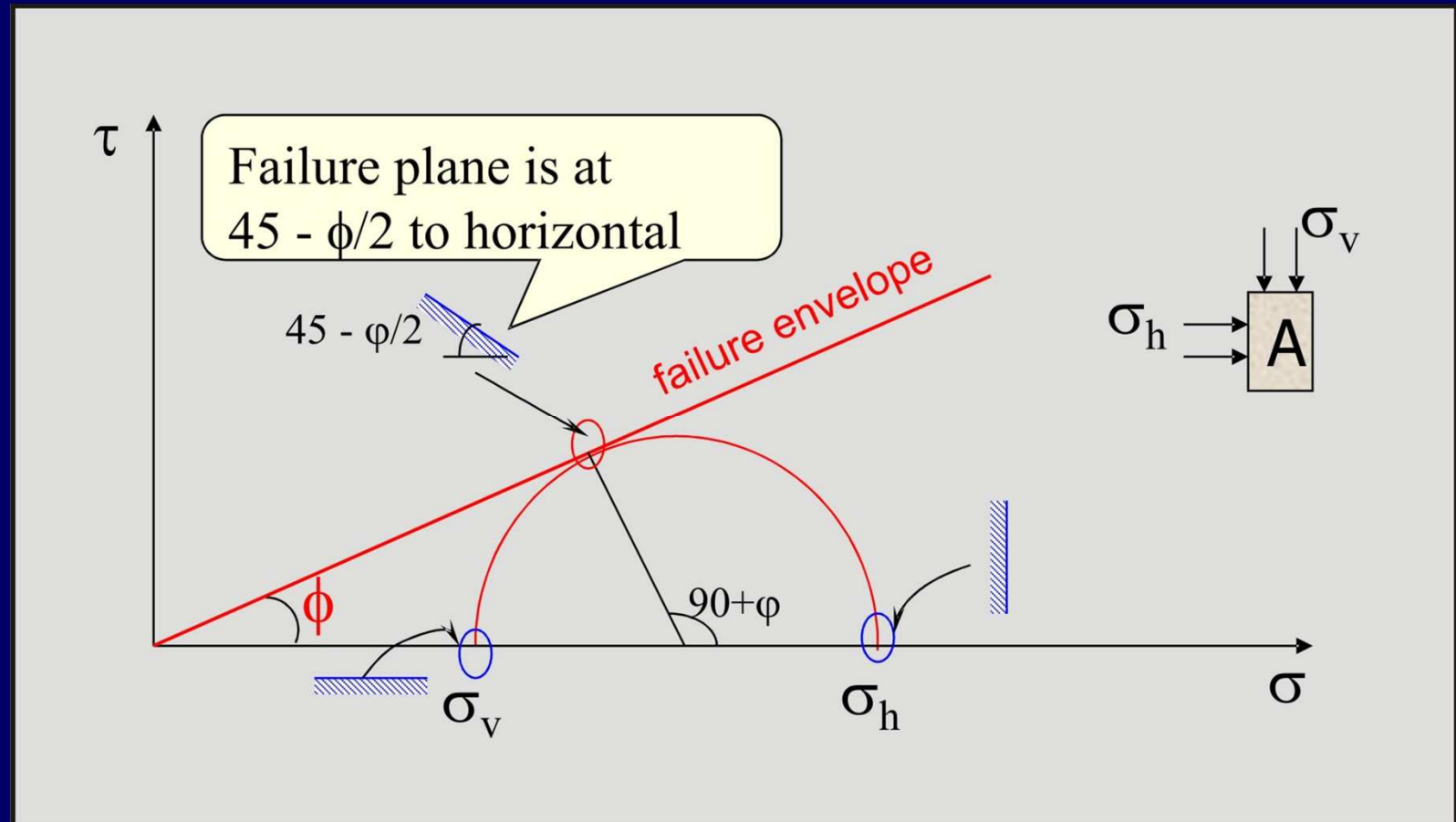
$$\frac{d^2f}{d\beta^2} = -2 \text{ for } \beta = \frac{\pi}{4} - \frac{1}{2} \cdot \varphi$$

$$F = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot \left( \frac{1 + \sin(\varphi)}{1 - \sin(\varphi)} \right) = \frac{1}{2} \cdot \rho_g \cdot g \cdot h^2 \cdot K_p$$

$$K_p = \frac{1 + \sin \varphi}{1 - \sin \varphi} = \tan^2(45 + \varphi / 2)$$

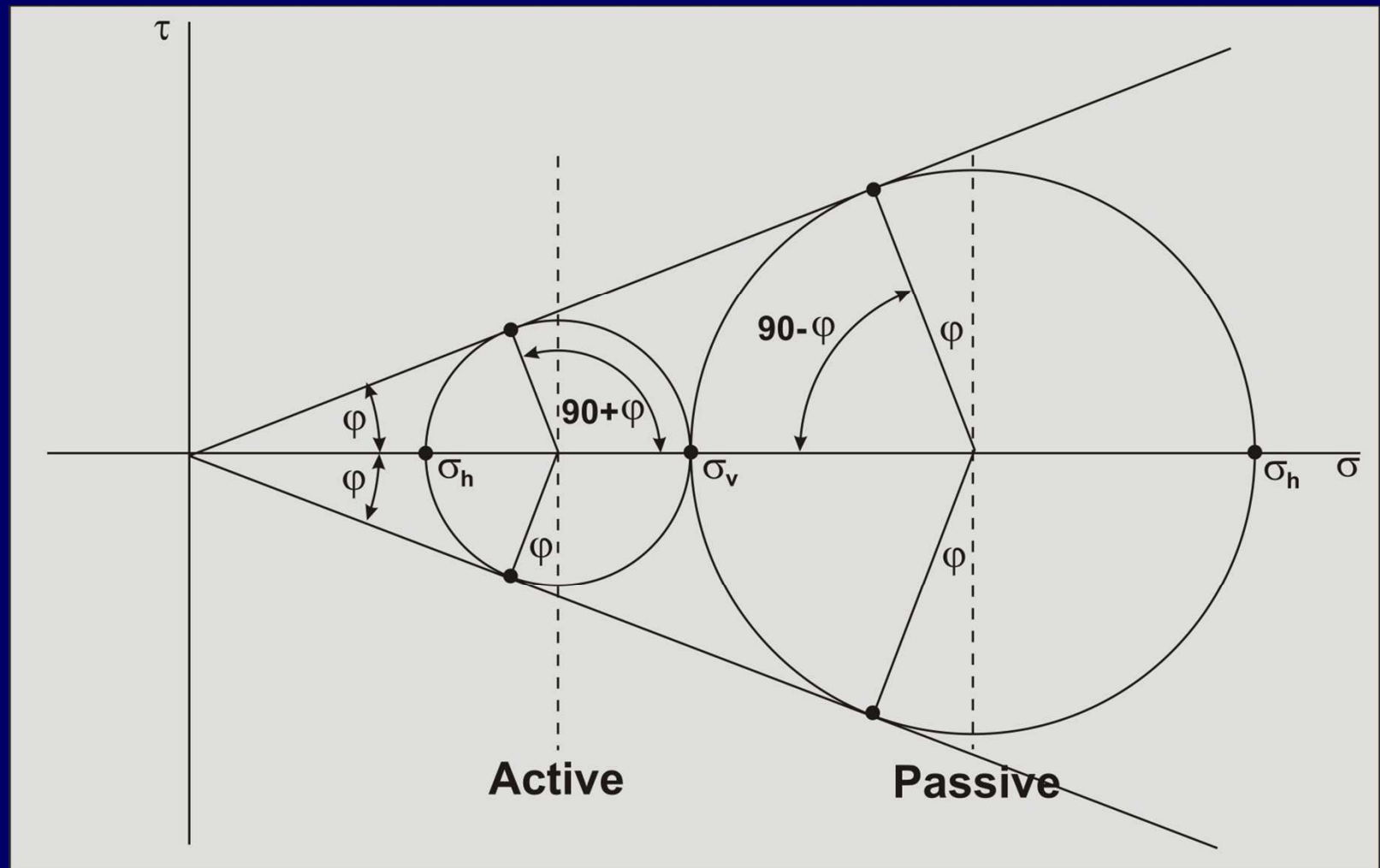
$$\sigma_h = K_p \cdot \sigma_v$$

# Passive Soil Failure 5

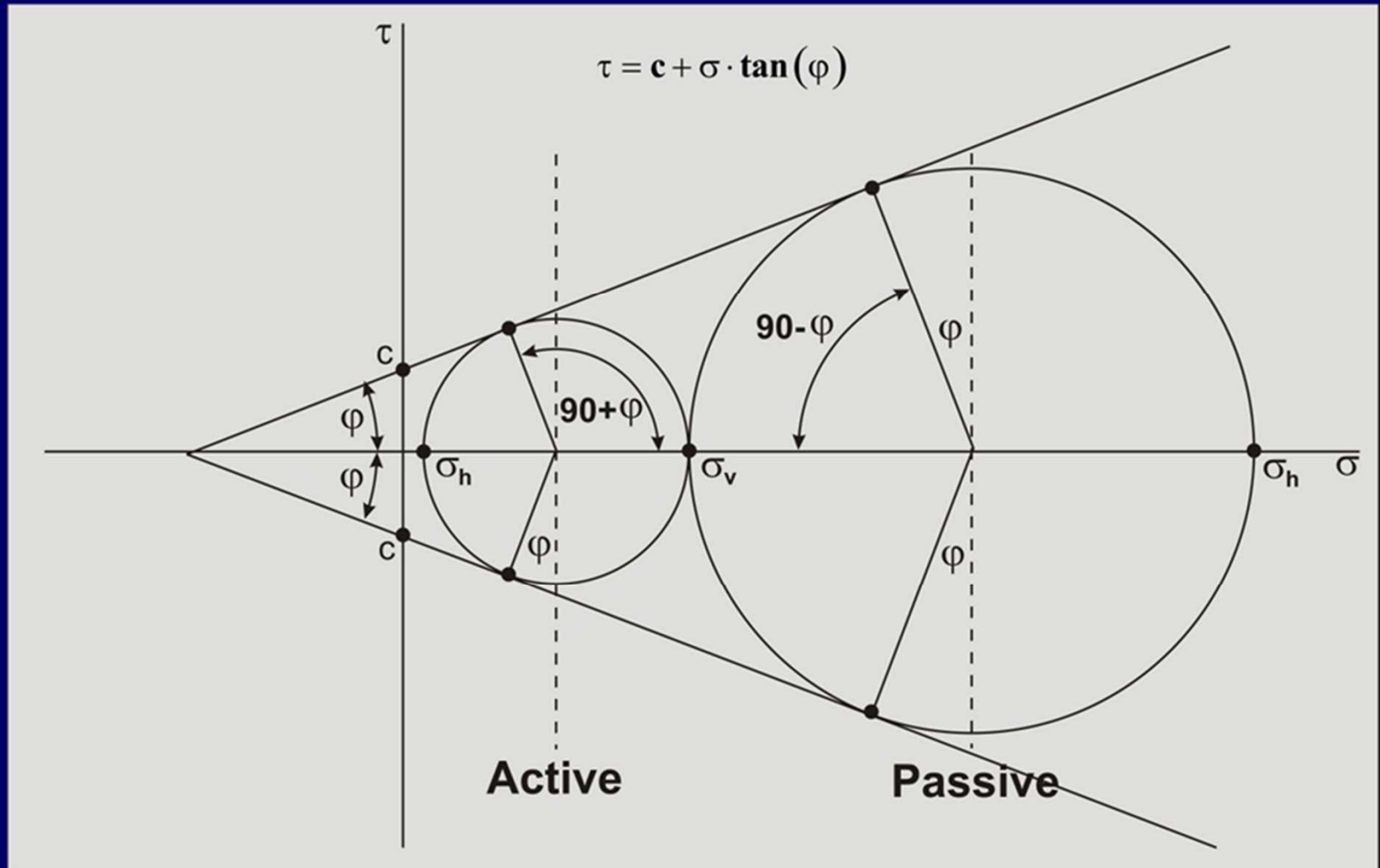




# Active & Passive Soil Failure



# Active & Passive Soil Failure, Cohesion

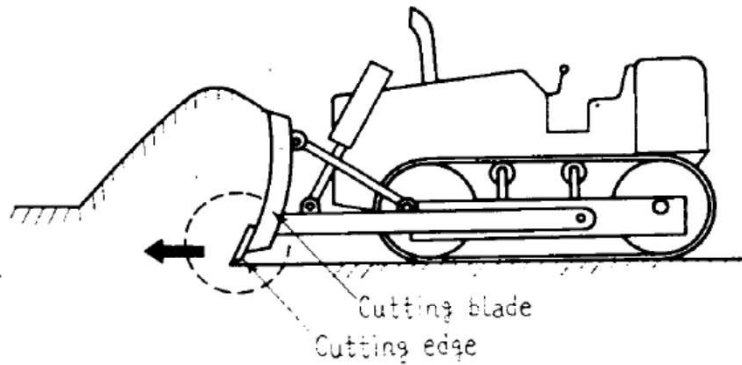




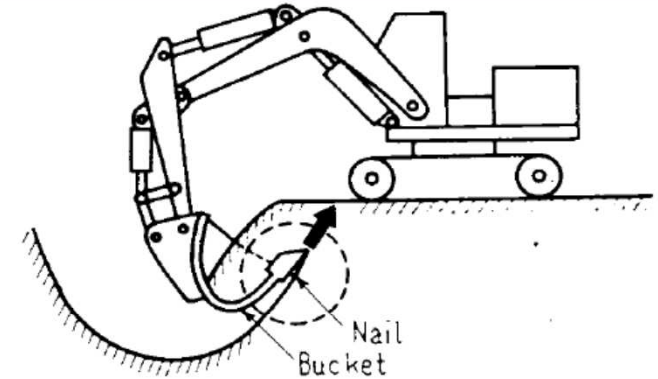
# Cutting Mechanisms



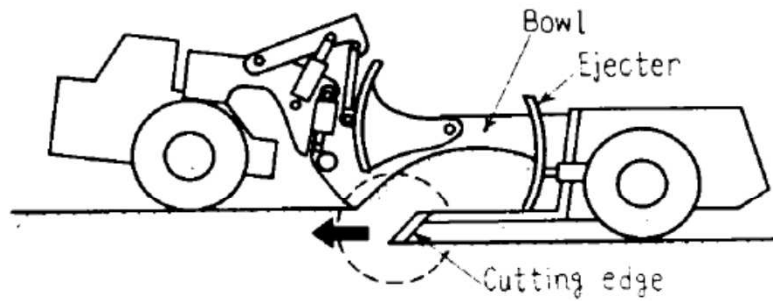
# Hatamura Chijiwa Equipment



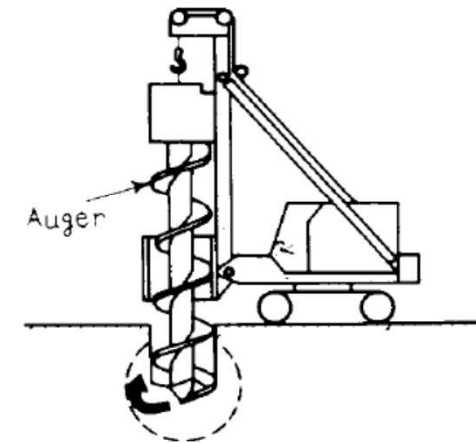
(a) Bulldozer



(b) Power shovel



(c) Scraper



(d) Earth-auger



# Hatamura Chijiwa Test Facility

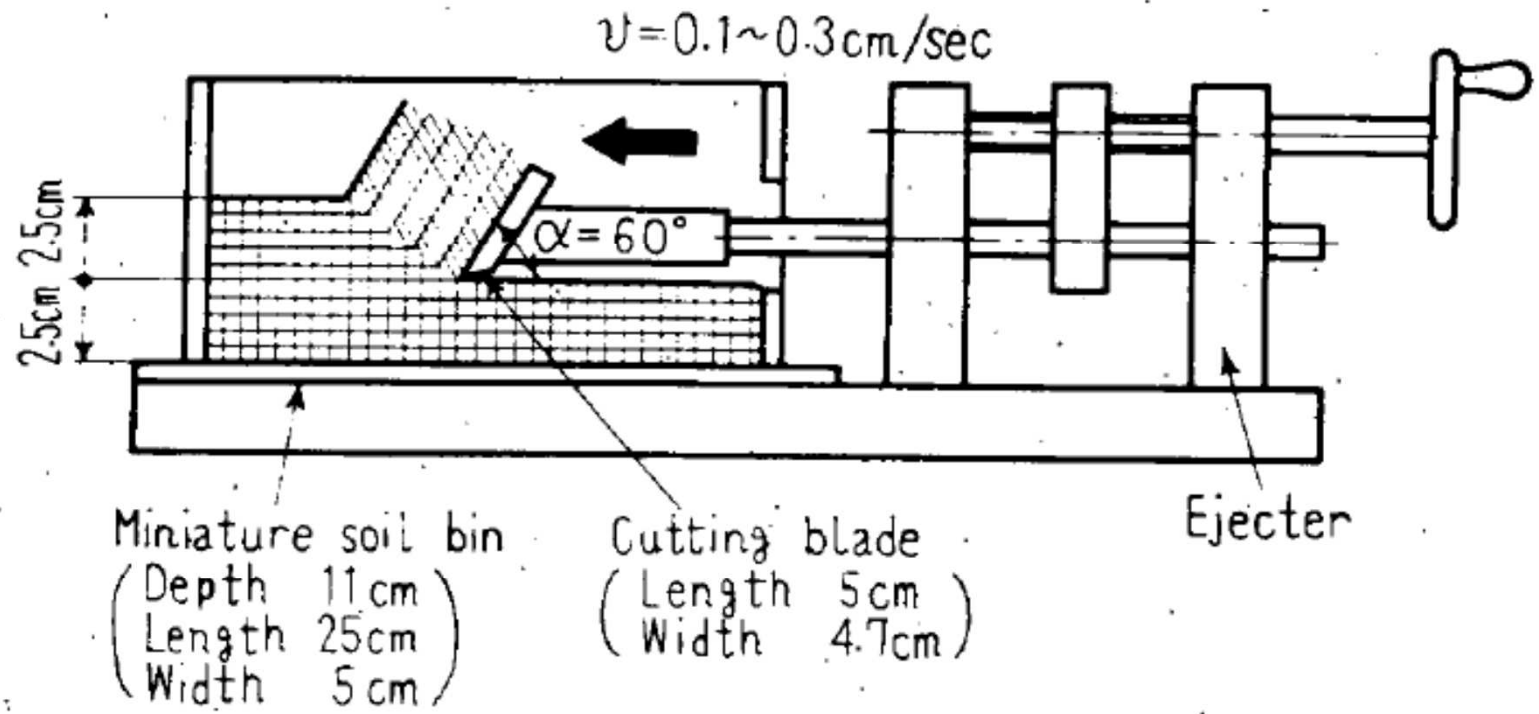
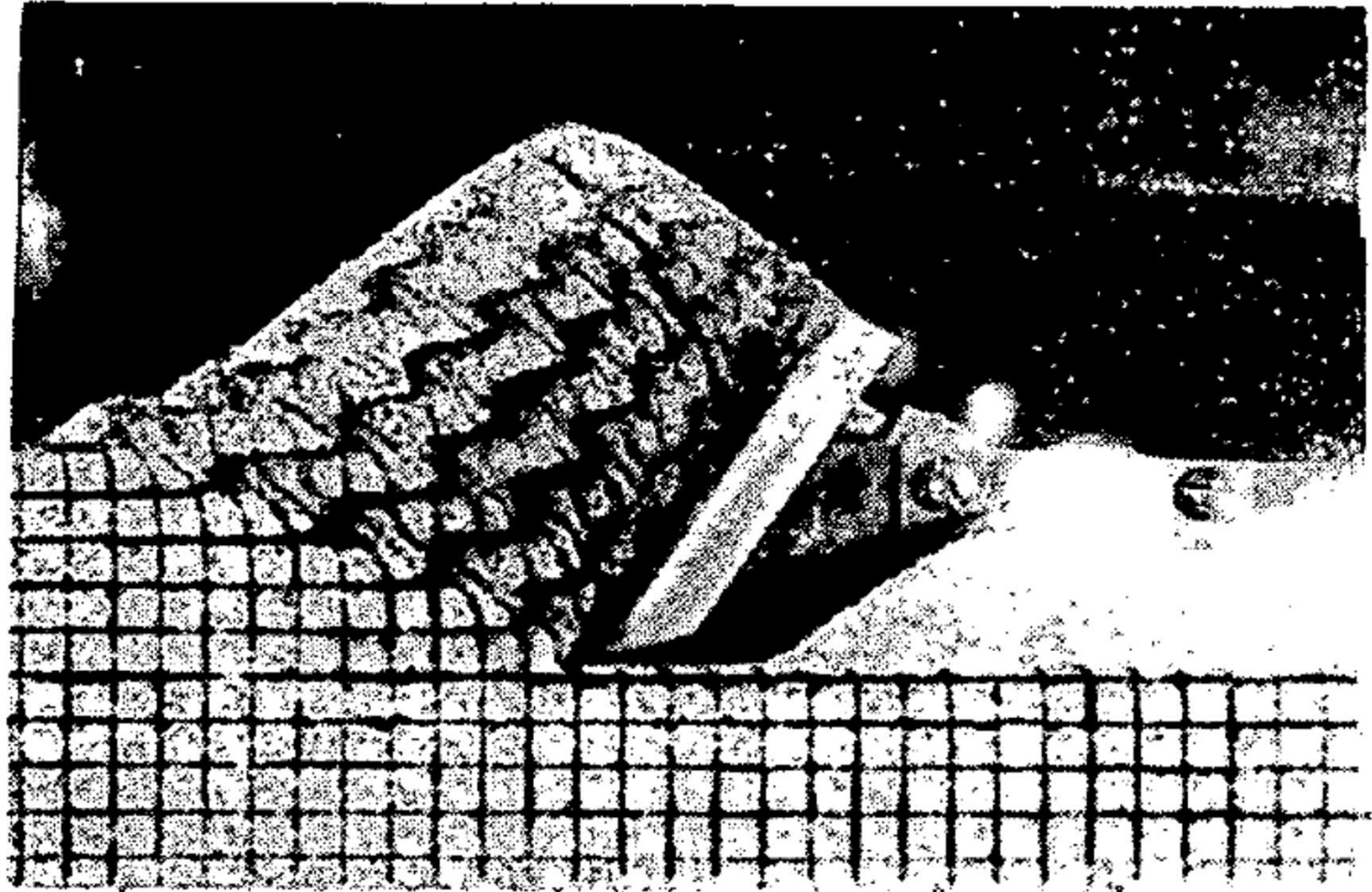


Fig. 2 Apparatus for investigating the deformation of soil by cutting.



# Hatamura Chijiwa Dry Quarts Sand



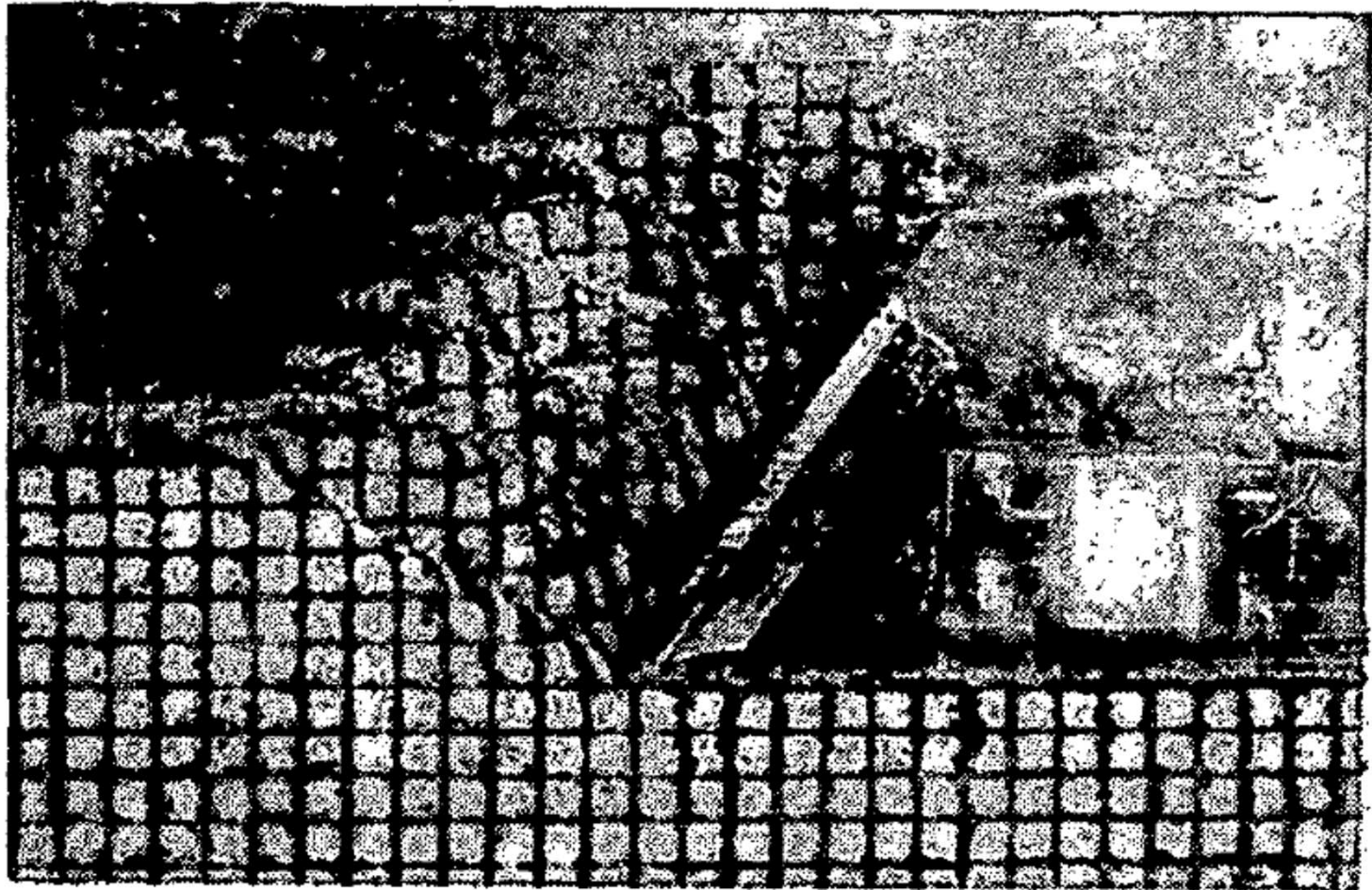
(a) Dry quartz sand

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# Hatamura Chijiwa Wet Quarts Sand

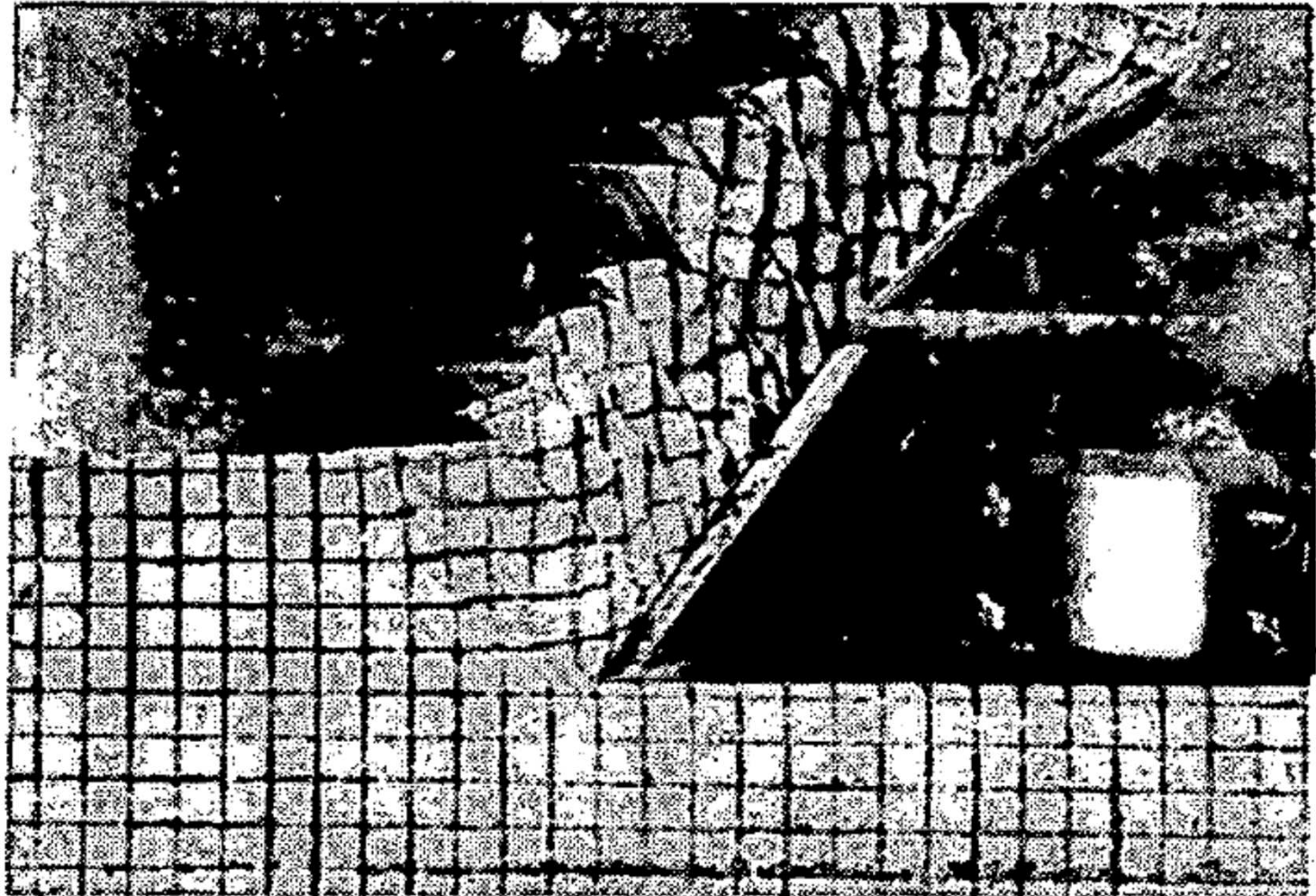


(b) Wet quartz sand





# Hatamura Chijiwa Plastic Bentonite



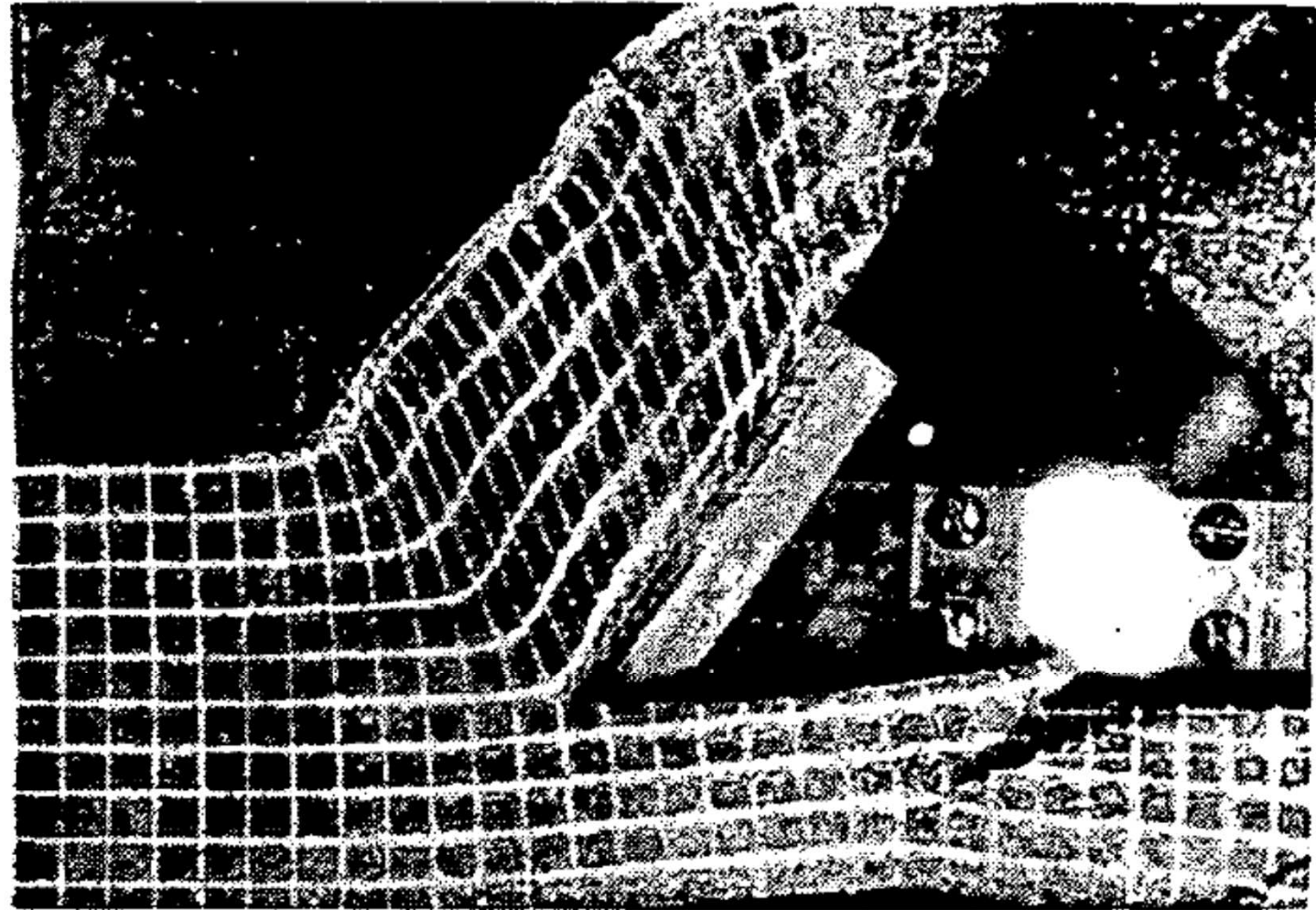
(c) Plastic bentonite

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# Hatamura Chijiwa Plastic Loam

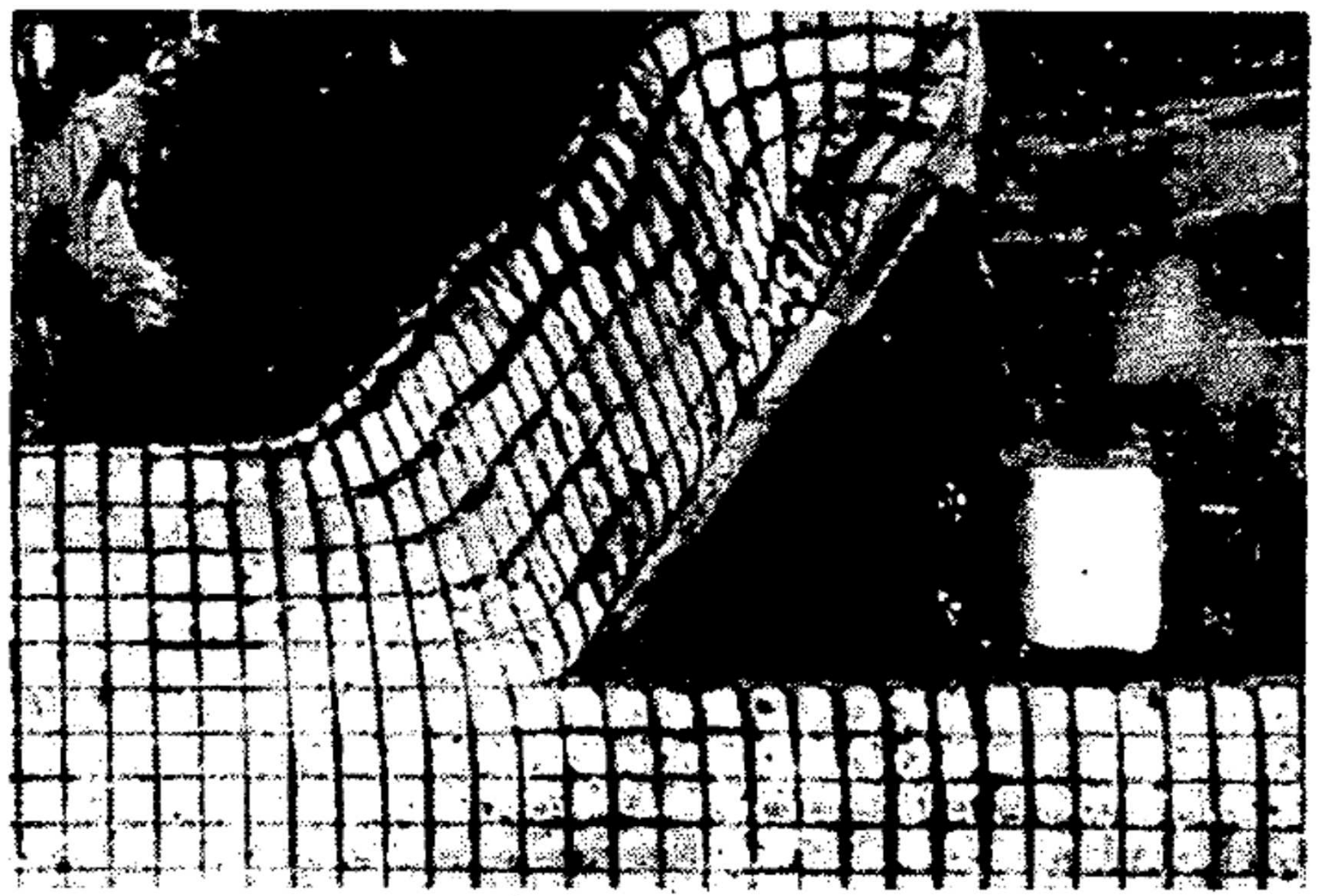


(d) Plastic loam

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# Hatamura Chijiwa Plastic Clay

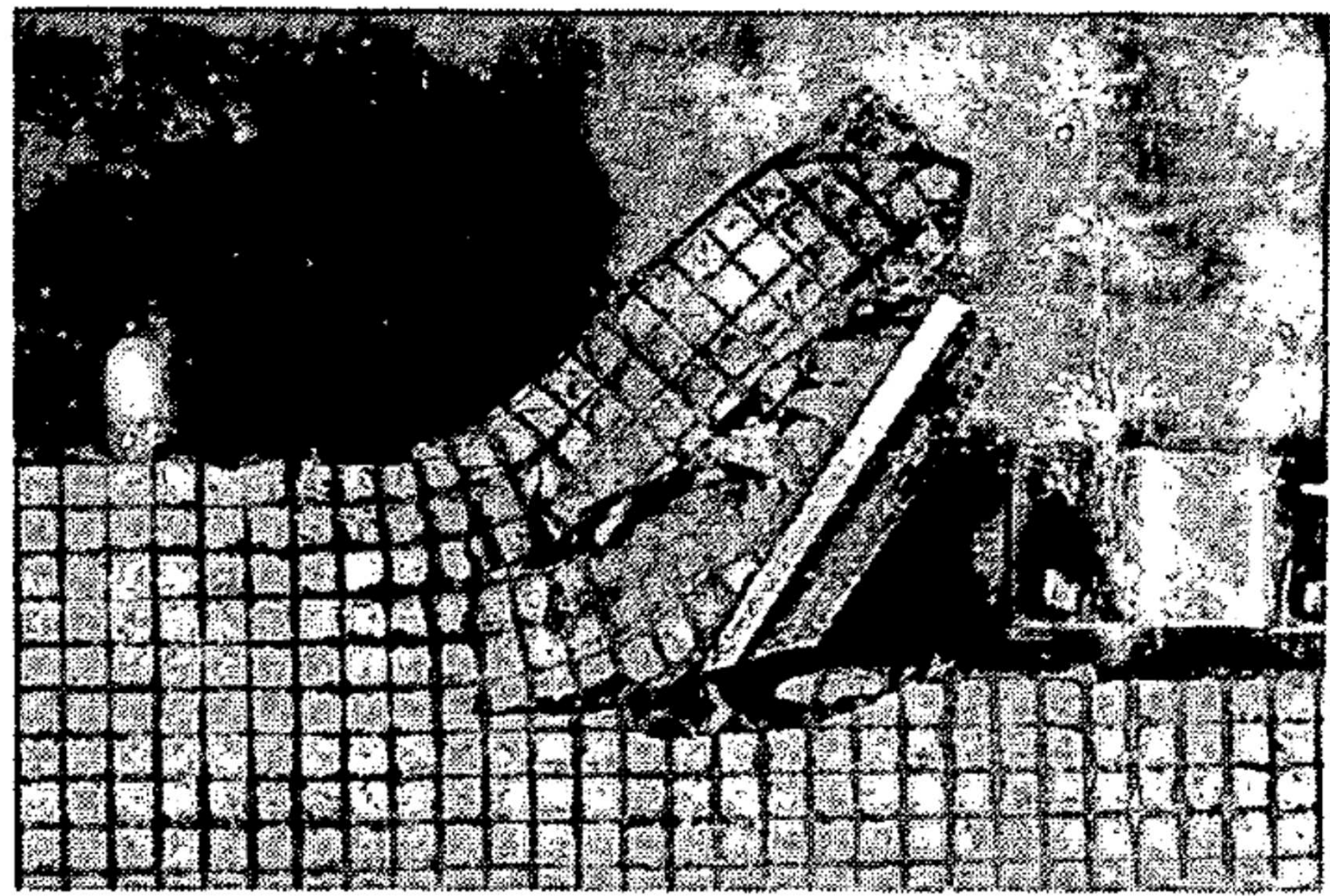


(e) Plastic clay





# Hatamura Chijiwa Compacted Loam



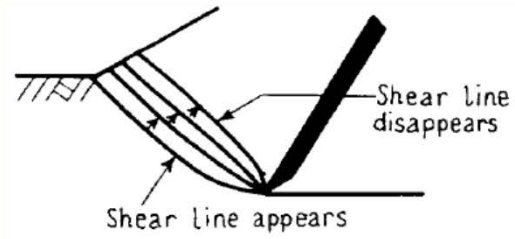
(f) Compacted loam

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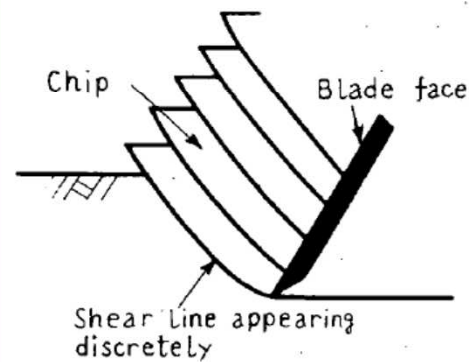




# Hatamura Chijiwa Failure Types



(a) Appearance of shear line



(b) Accumulation of chips

Fig. 4 Shear type

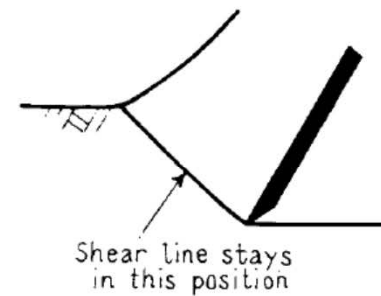


Fig. 5 Flow type

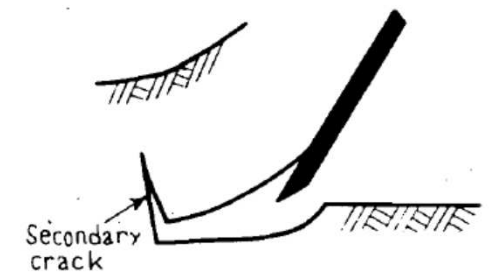
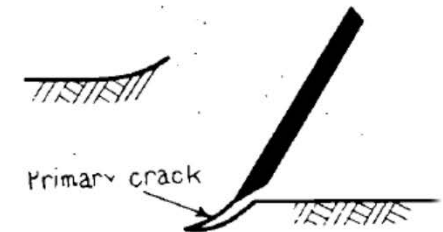
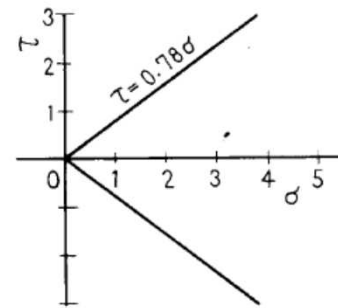


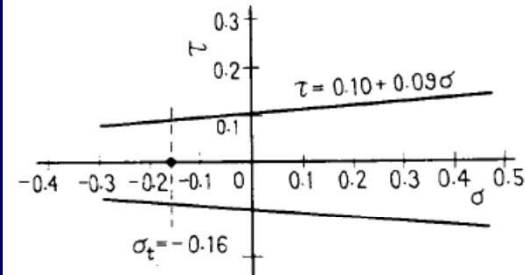
Fig. 6 Tear type



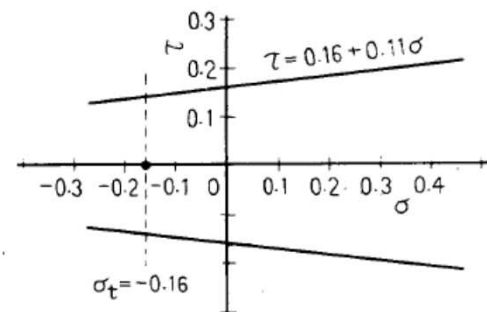
# Hatamura Chijiwa Mohr Circles



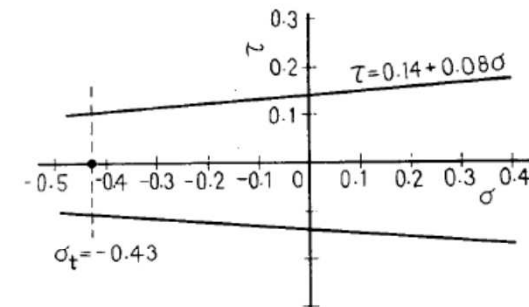
(1) Dry quartz sand



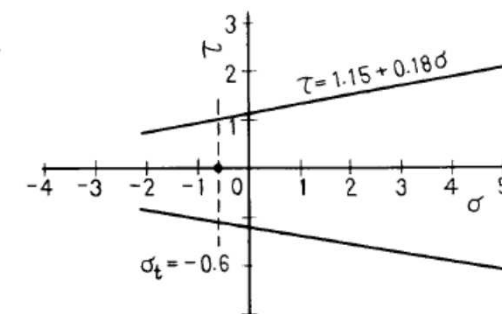
(2) Plastic bentonite



(3) Plastic loam



(4) Plastic clay



(5) Compacted loam

$\tau$  : shearing stress  
 $\sigma$  : normal stress  
 $\sigma_t$  : tensile strength

}  $\text{kg/cm}^2$

Fig. 11 Failure conditions of soils

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# Hatamura Chijiwa Conditions

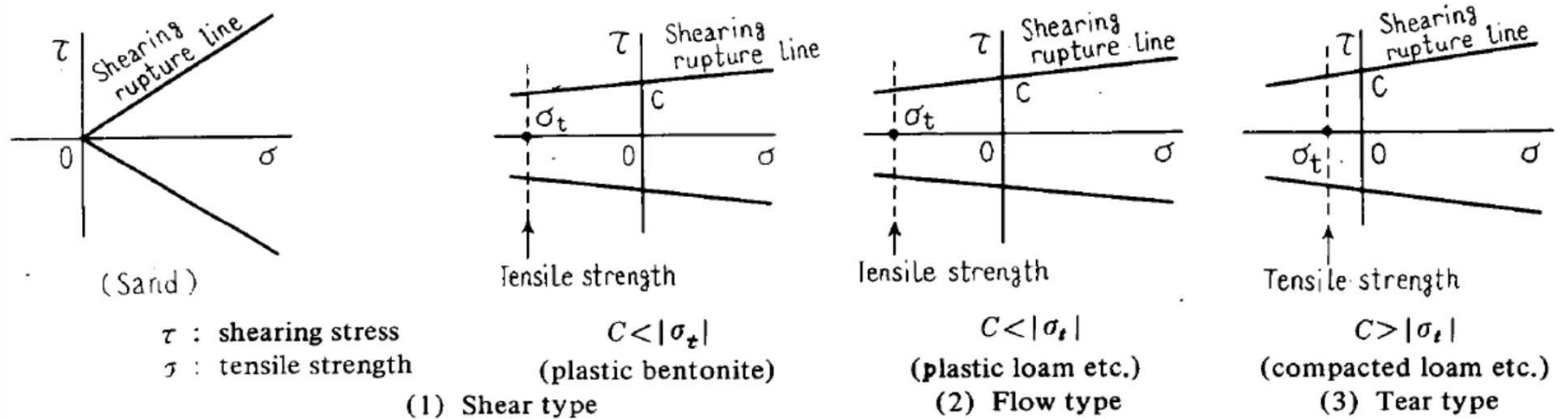


Fig. 12 Relationship between cutting types and failure conditions of soils

# Hatamura Chijiwa Stresses

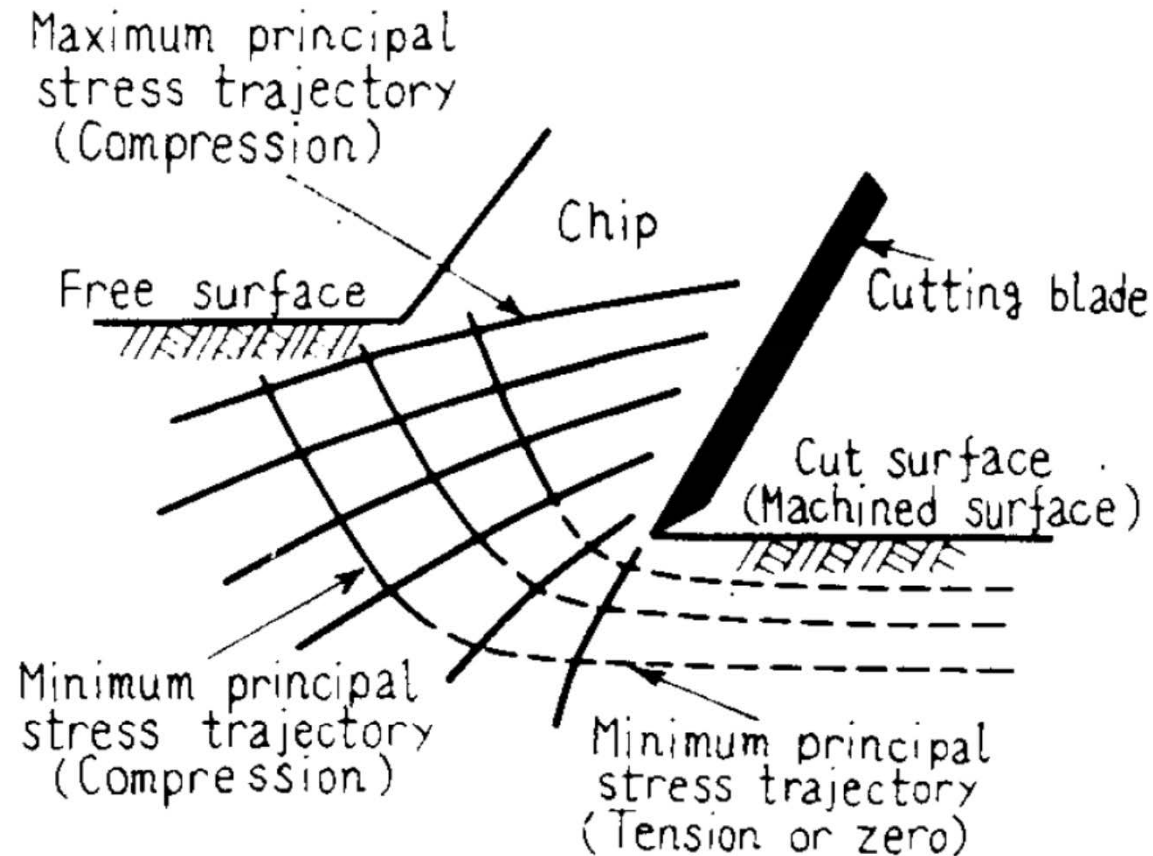
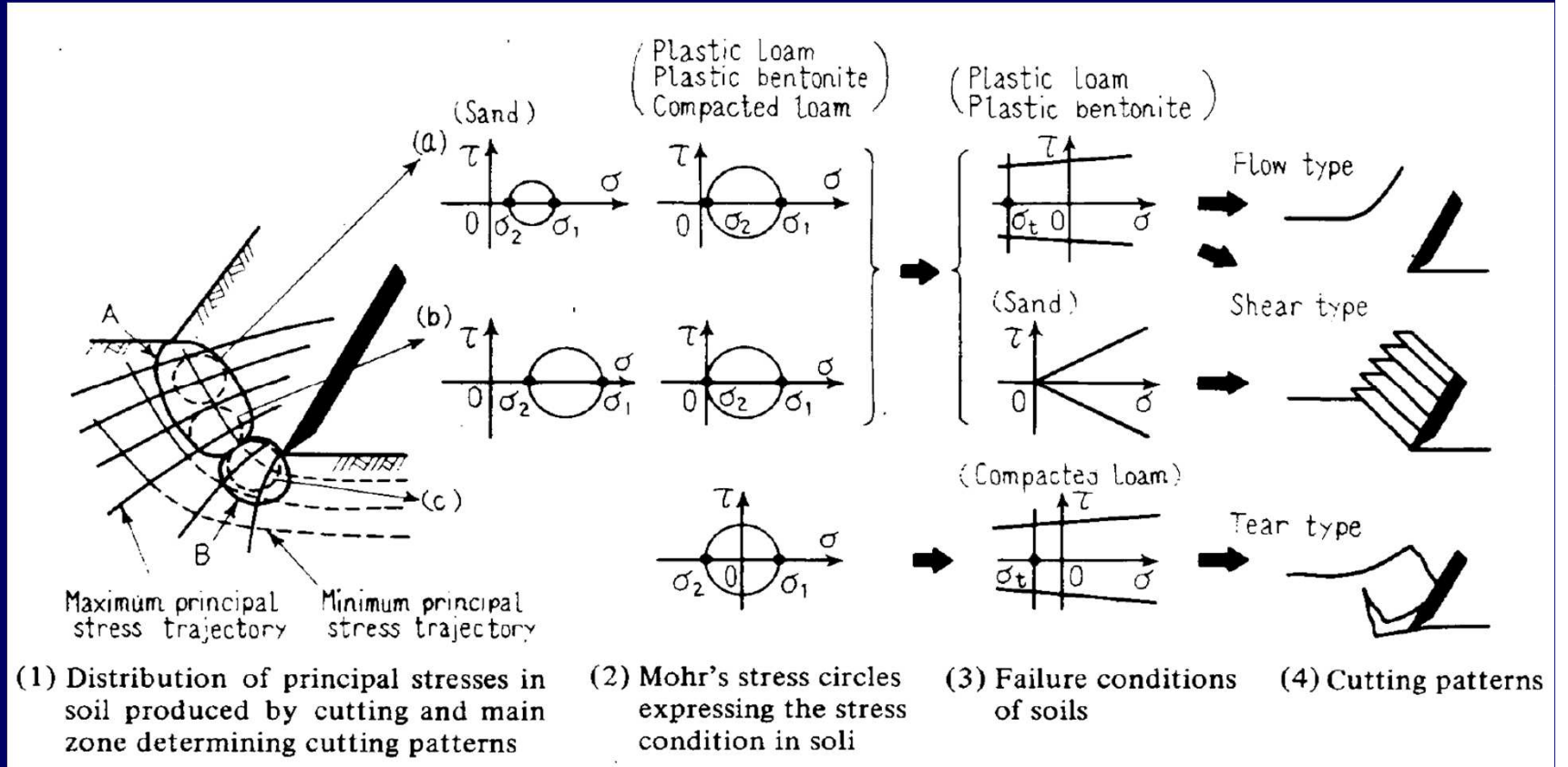


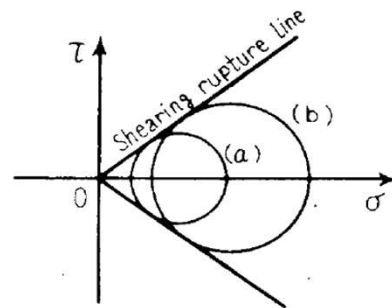
Fig. 14 Idealized distribution of principal stresses in soil produced by cutting



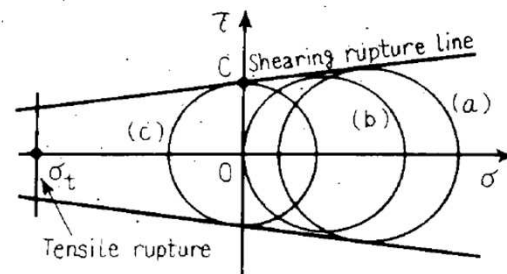
# Hatamura Chijiwa Mechanisms



# Hatamura Chijiwa Types



(a) Sand etc.



(b) Plastic bentonite etc.

Fig. 16 Relationship between failure conditions and stress situations in soil presenting shear type

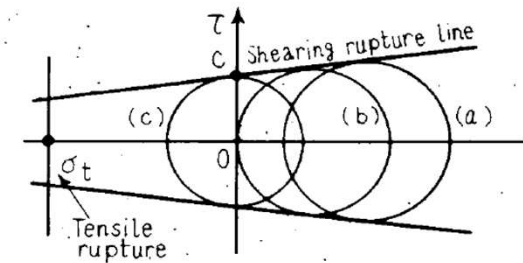


Fig. 17 Relationship between rupture conditions and stress situations in soil presenting flow type

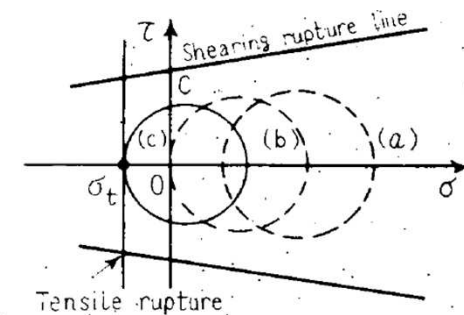
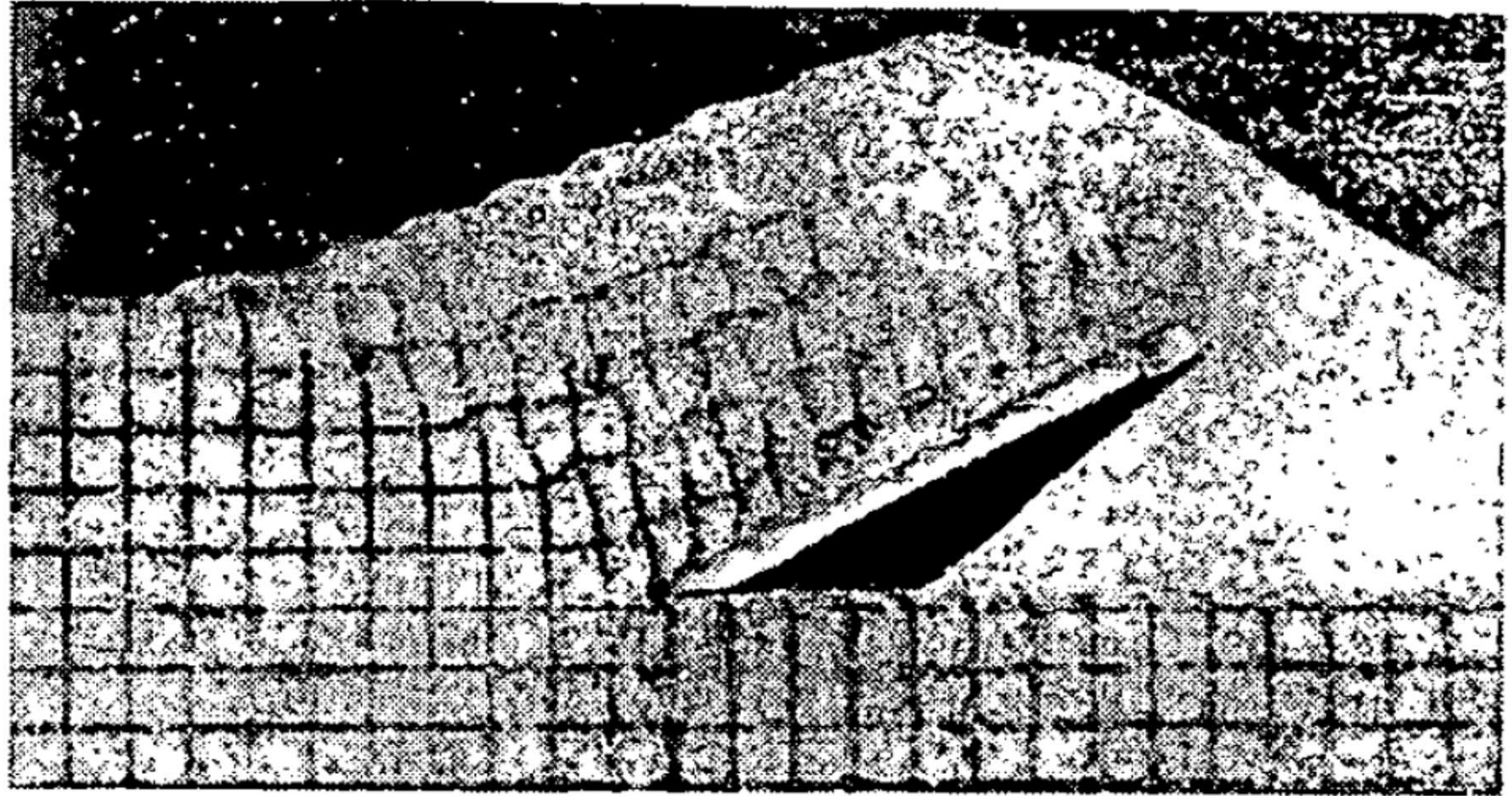


Fig. 18 Relationship between failure conditions and stress situations in soil presenting tear type



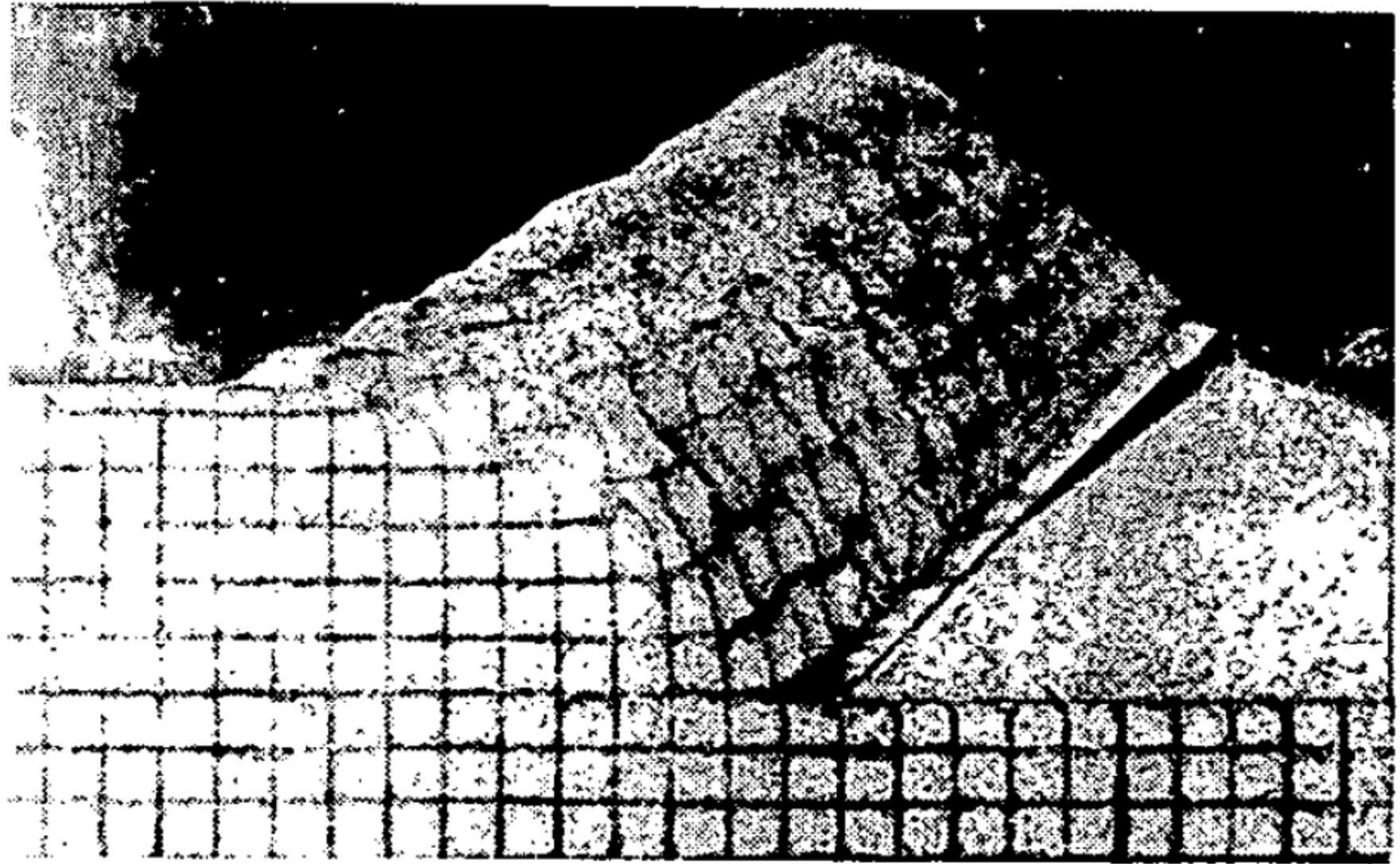
# Hatamura Chijiwa Dry Sand 30 deg.



(a) Cutting angle  $\alpha = 30^\circ$



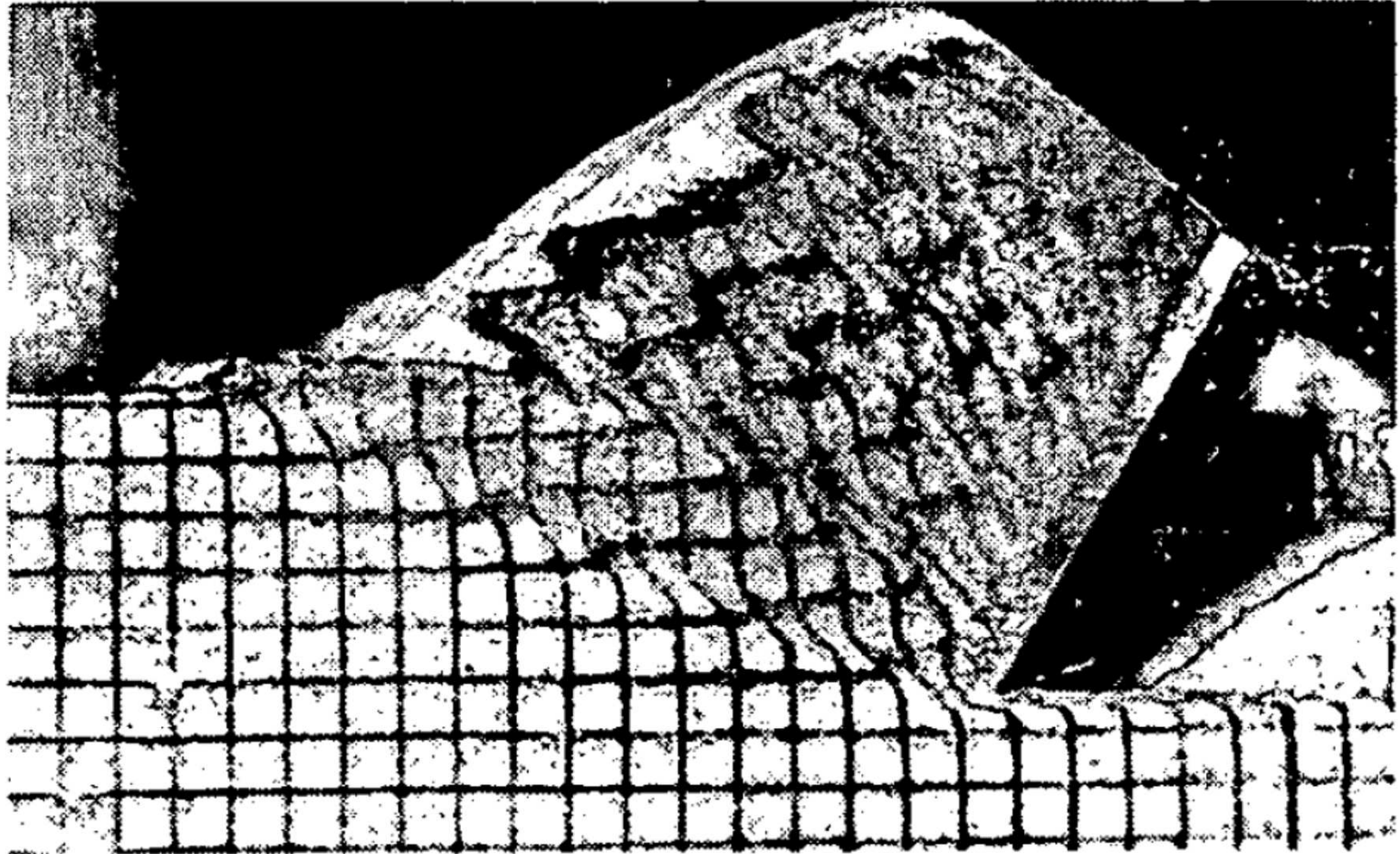
# Hatamura Chijiwa Dry Sand 45 deg.



(b) Cutting angle  $\alpha = 45^\circ$ .



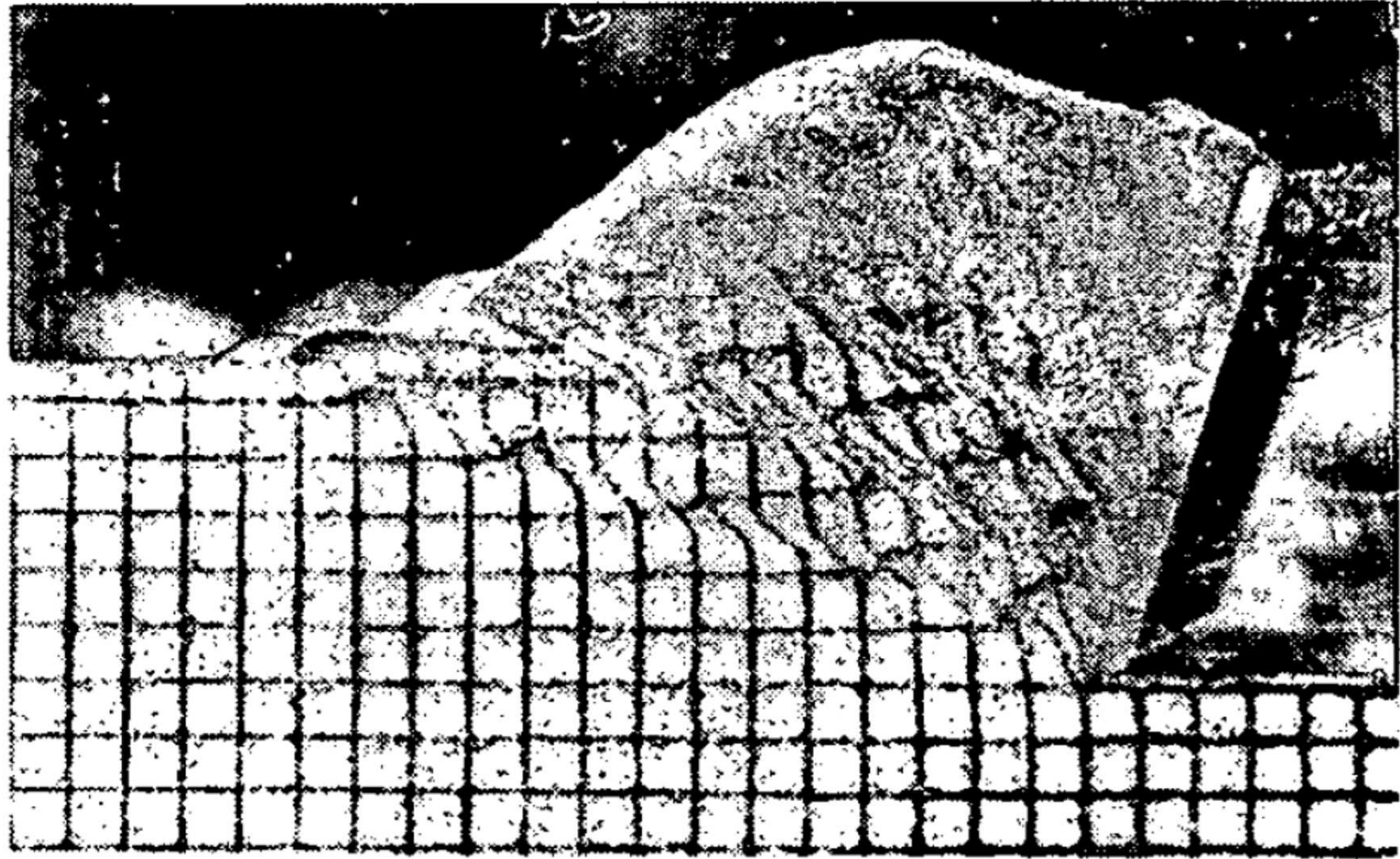
# Hatamura Chijiwa Dry Sand 60 deg.



(c) Cutting angle  $\alpha = 60^\circ$



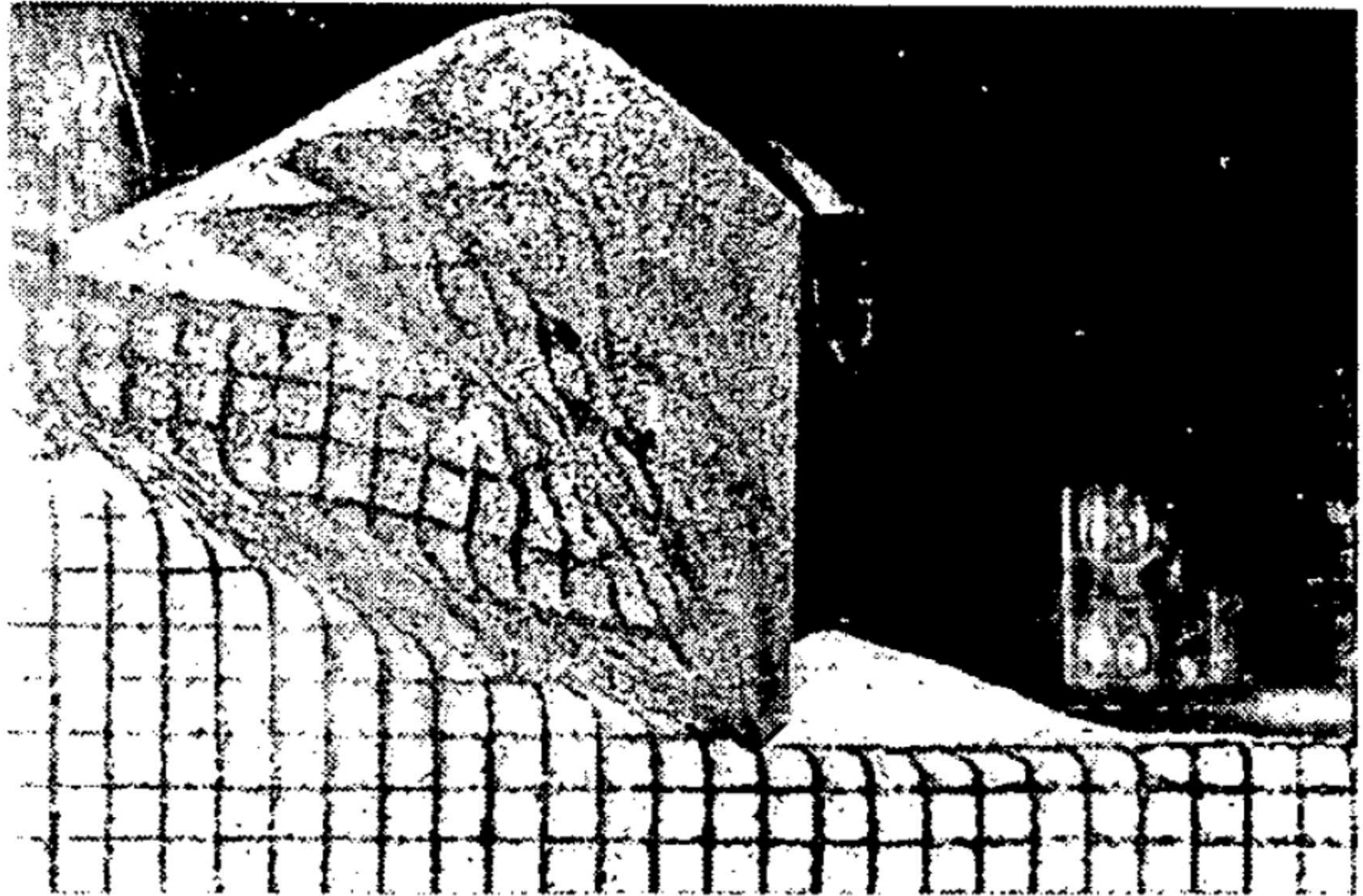
# Hatamura Chijiwa Dry Sand 75 deg.



(d) Cutting angle  $\alpha = 75^\circ$



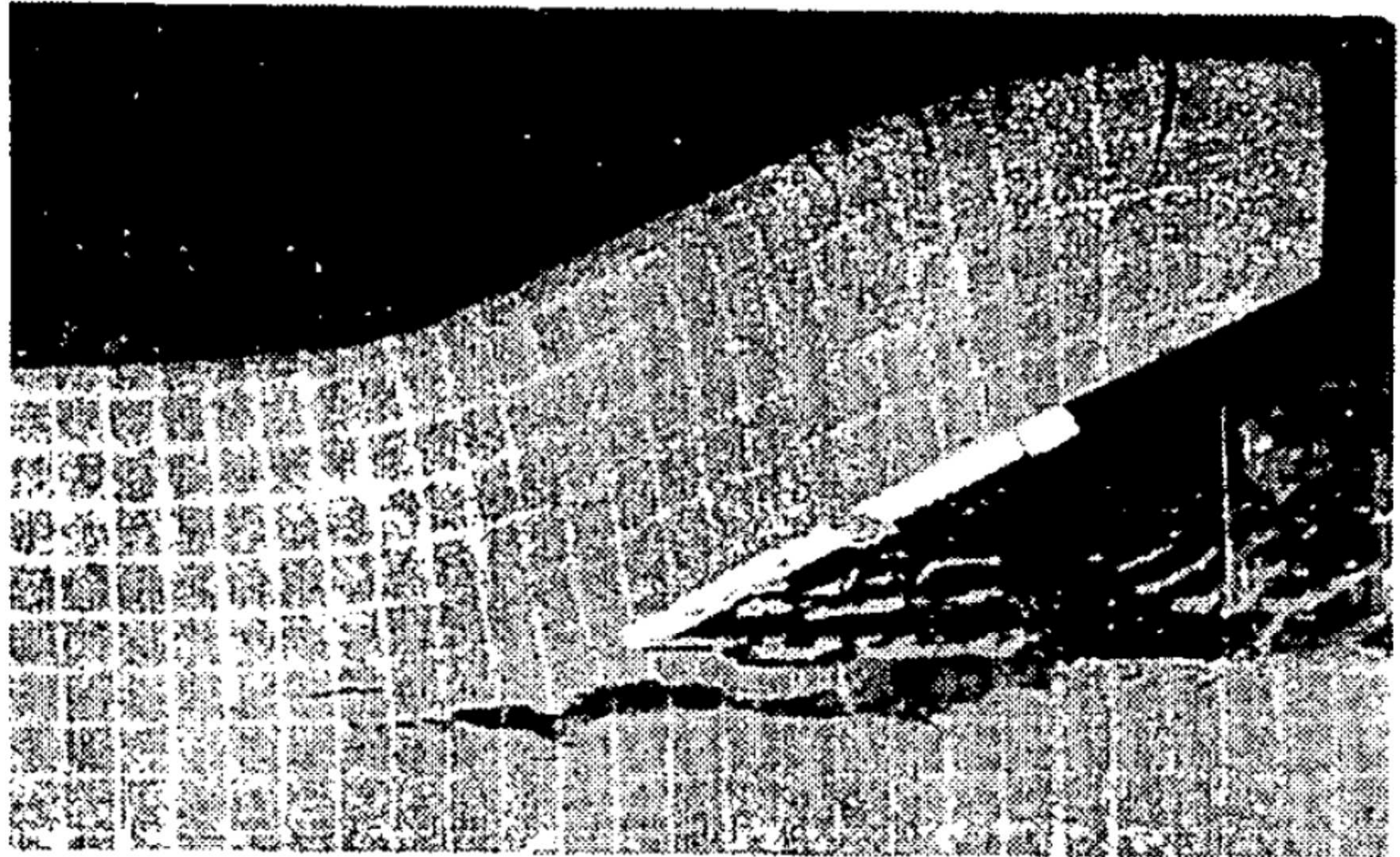
# Hatamura Chijiwa Dry Sand 90 deg.



(e) Cutting angle  $\alpha = 90^\circ$



# Hatamura Chijiwa Plastic Loam 30 deg.

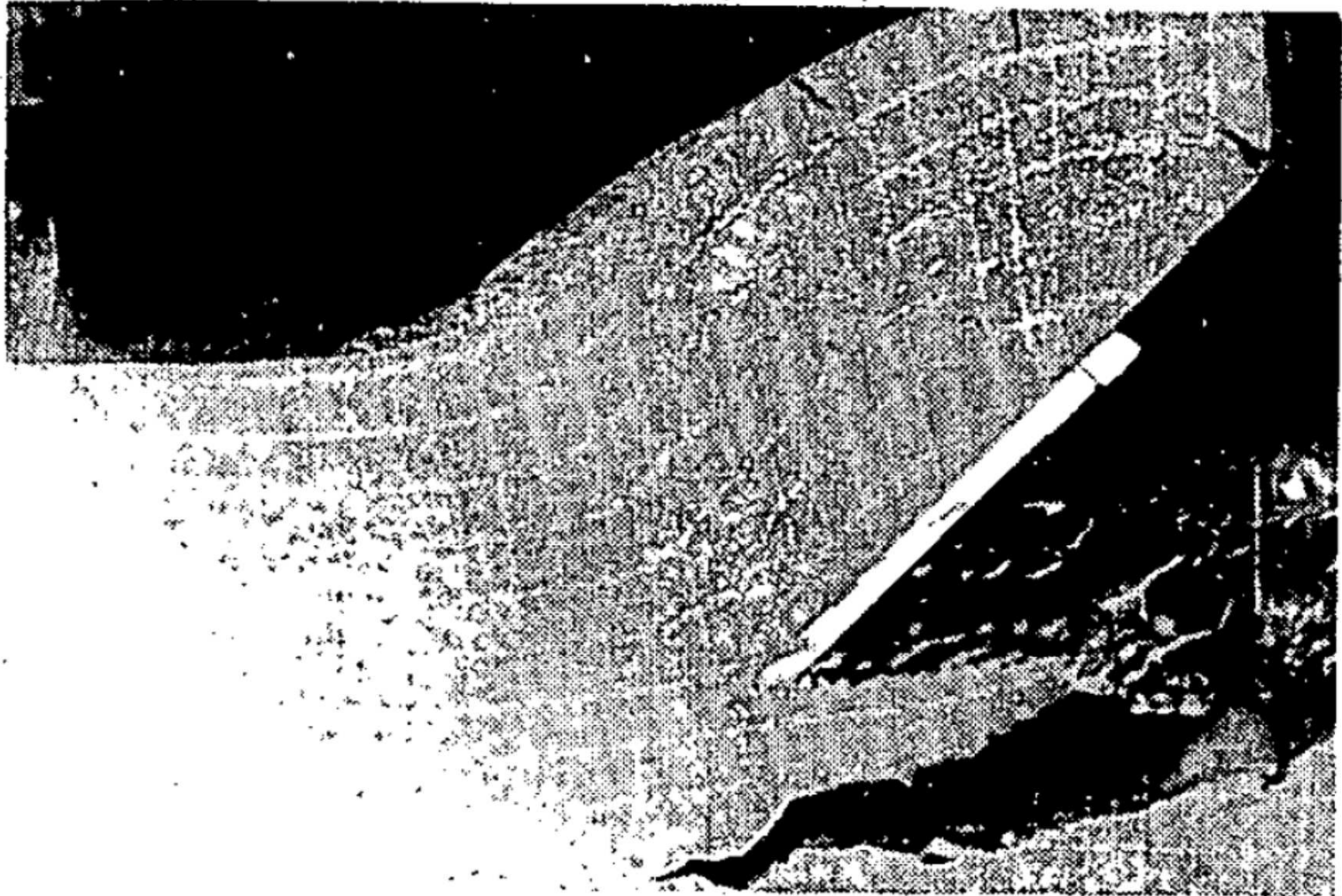


(a) Cutting angle  $\alpha = 30^\circ$

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# Hatamura Chijiwa Plastic Loam 45 deg.

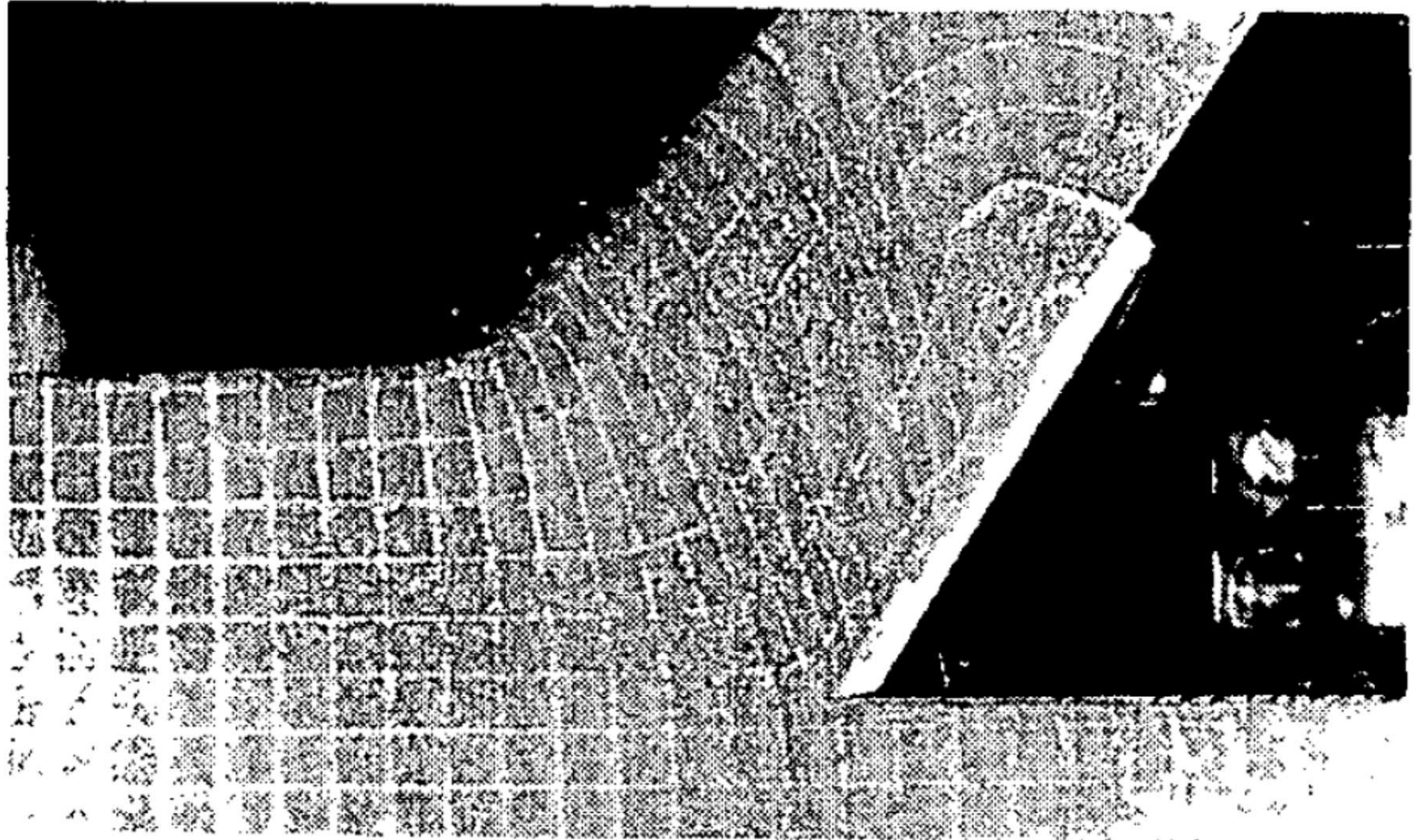


(b) Cutting angle  $\alpha = 45^\circ$

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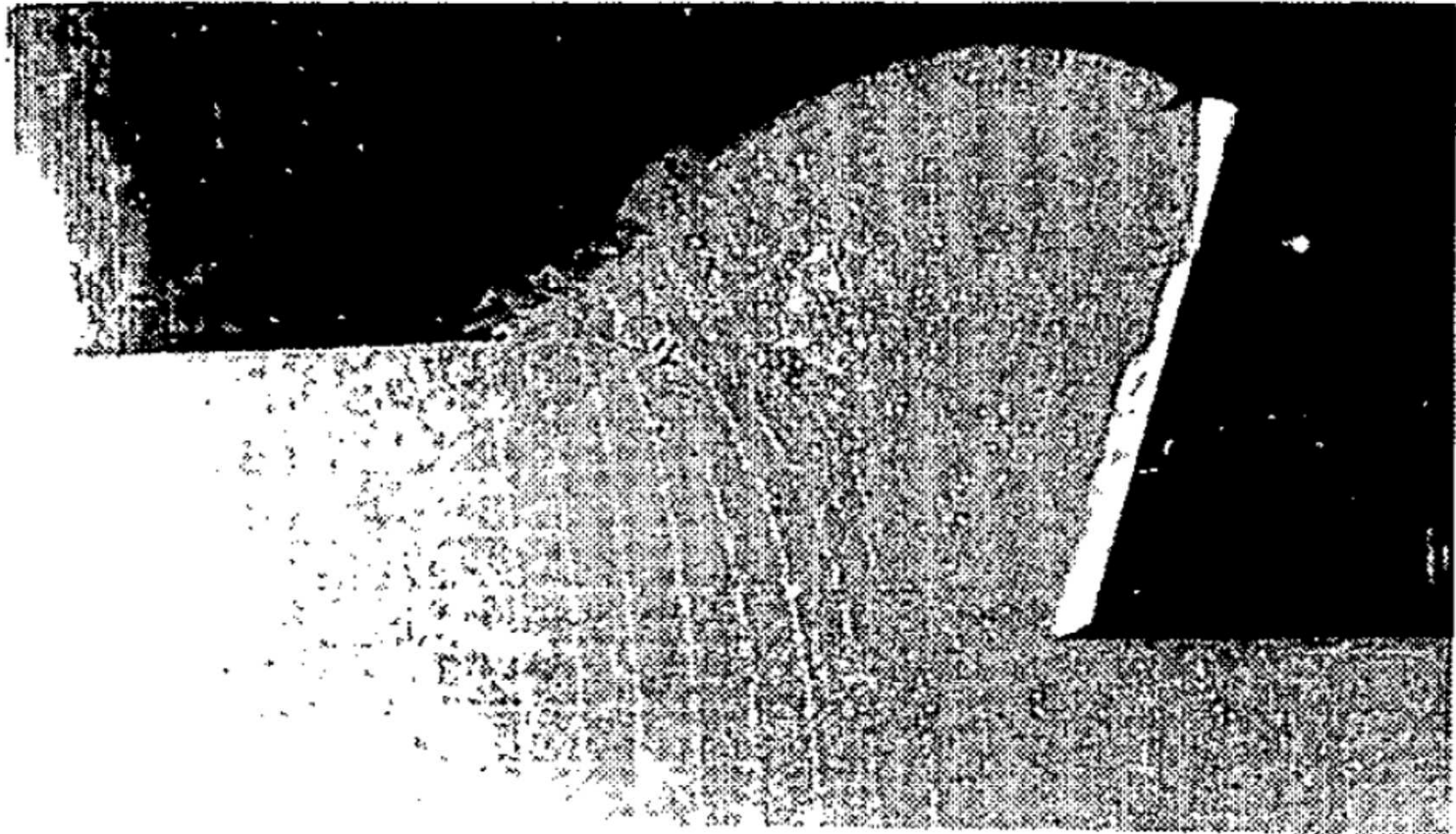
# Hatamura Chijiwa Plastic Loam 60 deg.



(c) Cutting angle  $\alpha = 60^\circ$



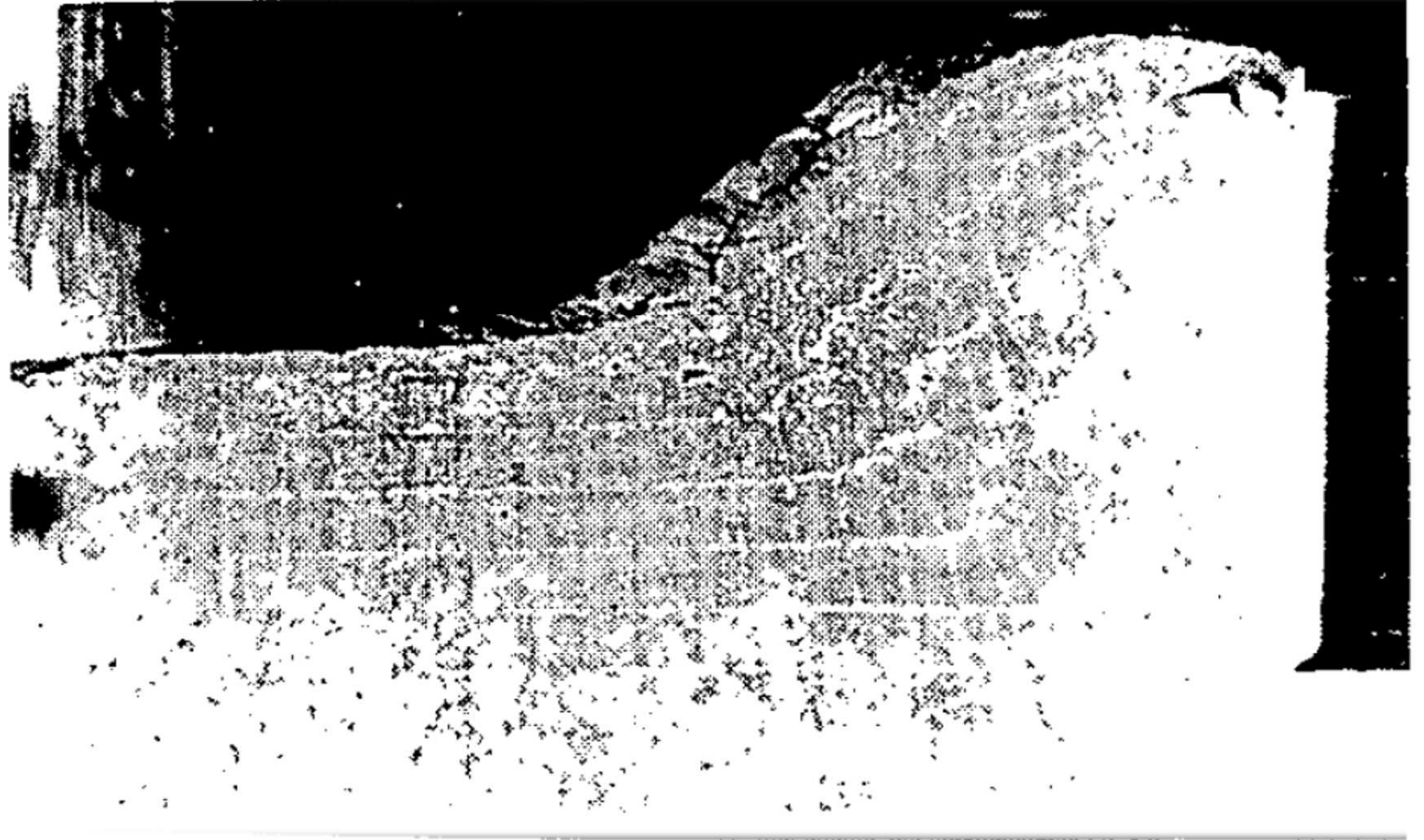
# Hatamura Chijiwa Plastic Loam 75 deg.



(d) Cutting angle  $\alpha = 75^\circ$

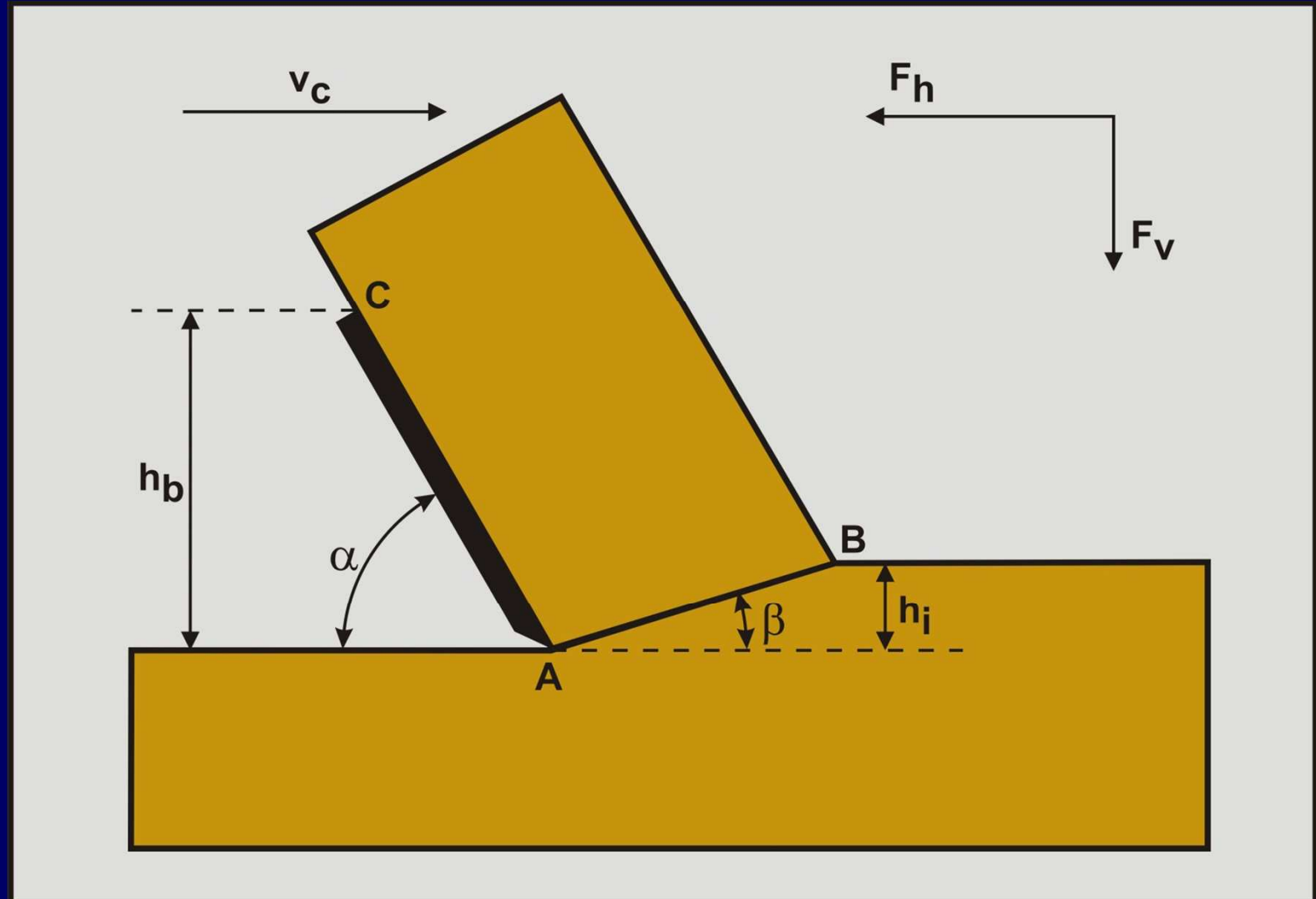


# Hatamura Chijiwa Plastic Loam 90 deg.



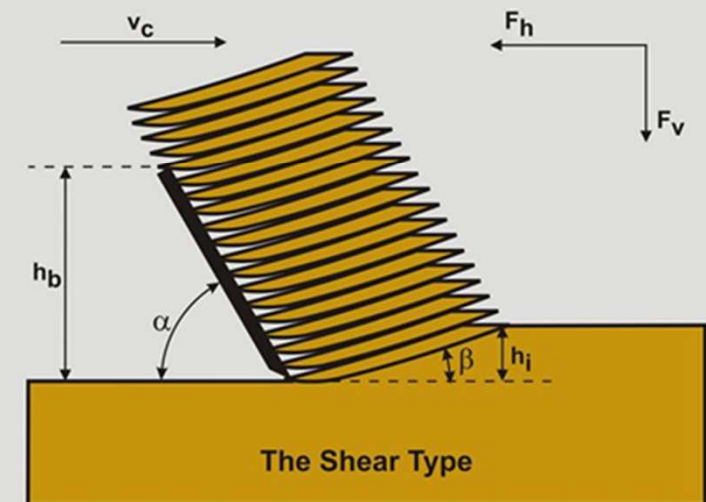
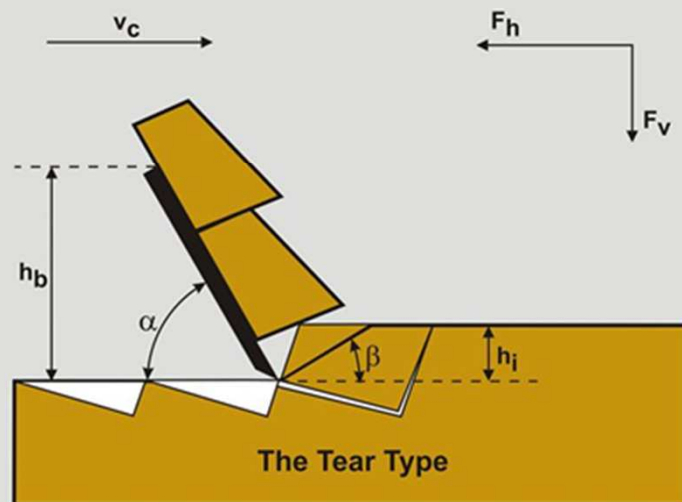
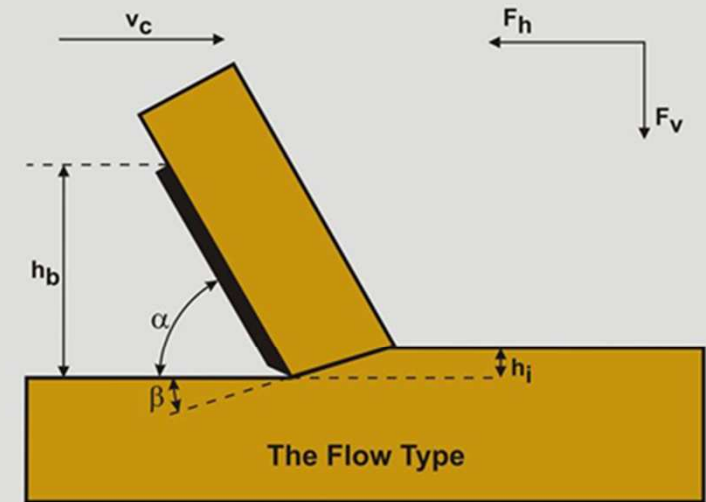
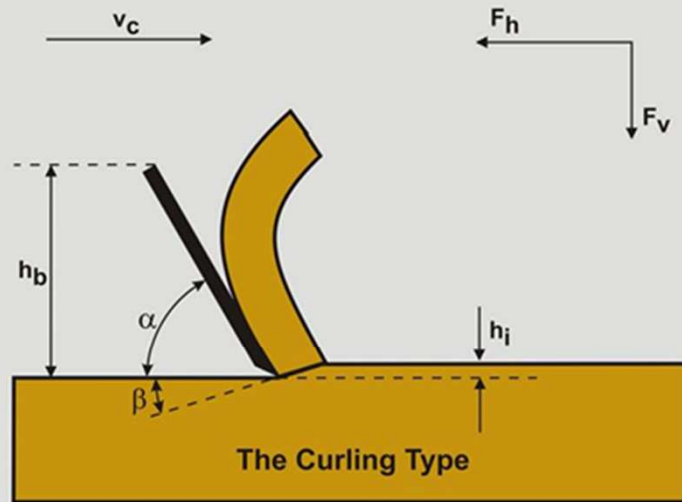
(e) Cutting angle  $\alpha = 90^\circ$

# Definitions





# Cutting Mechanisms

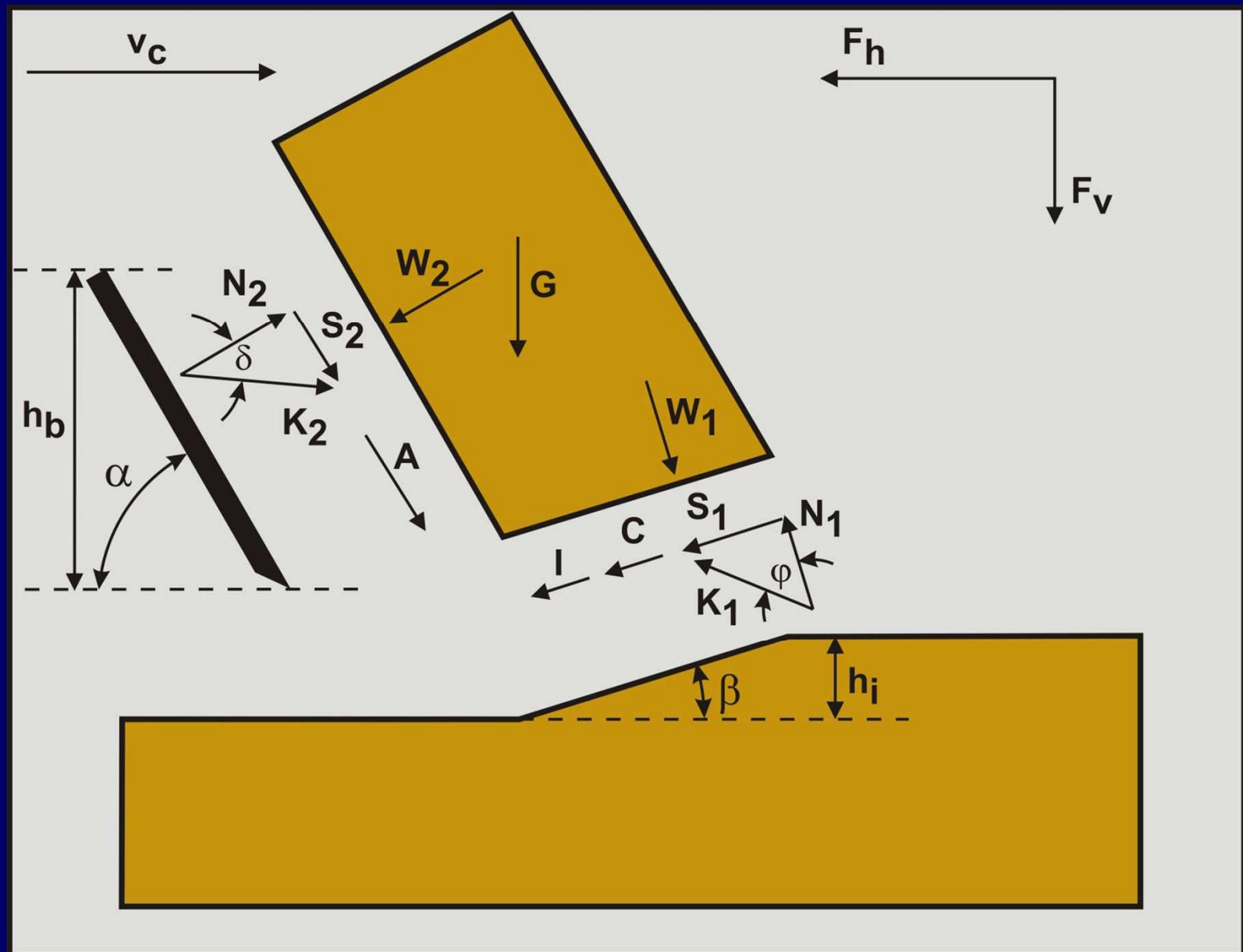




# Cutting Forces

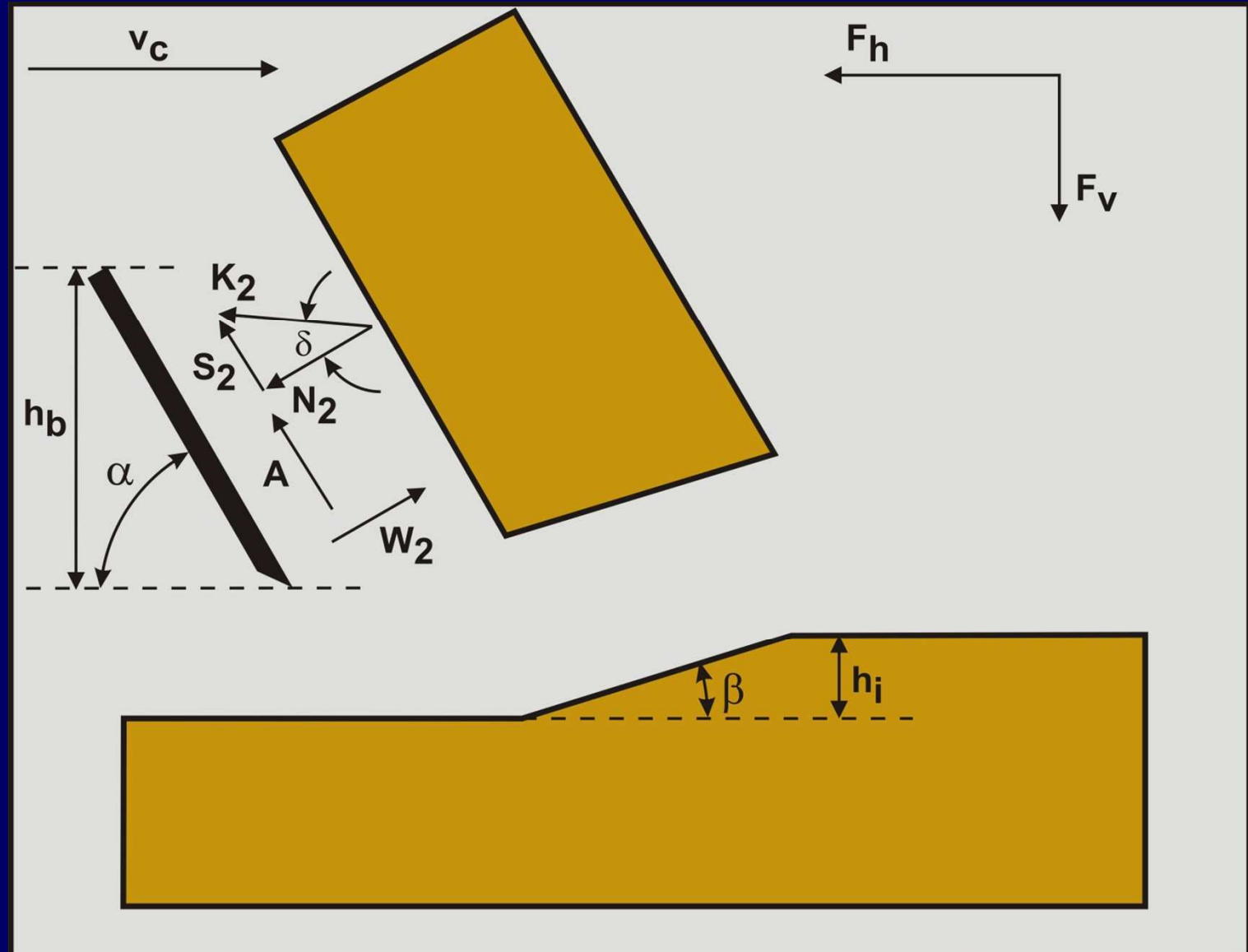


# Forces on the Layer Cut



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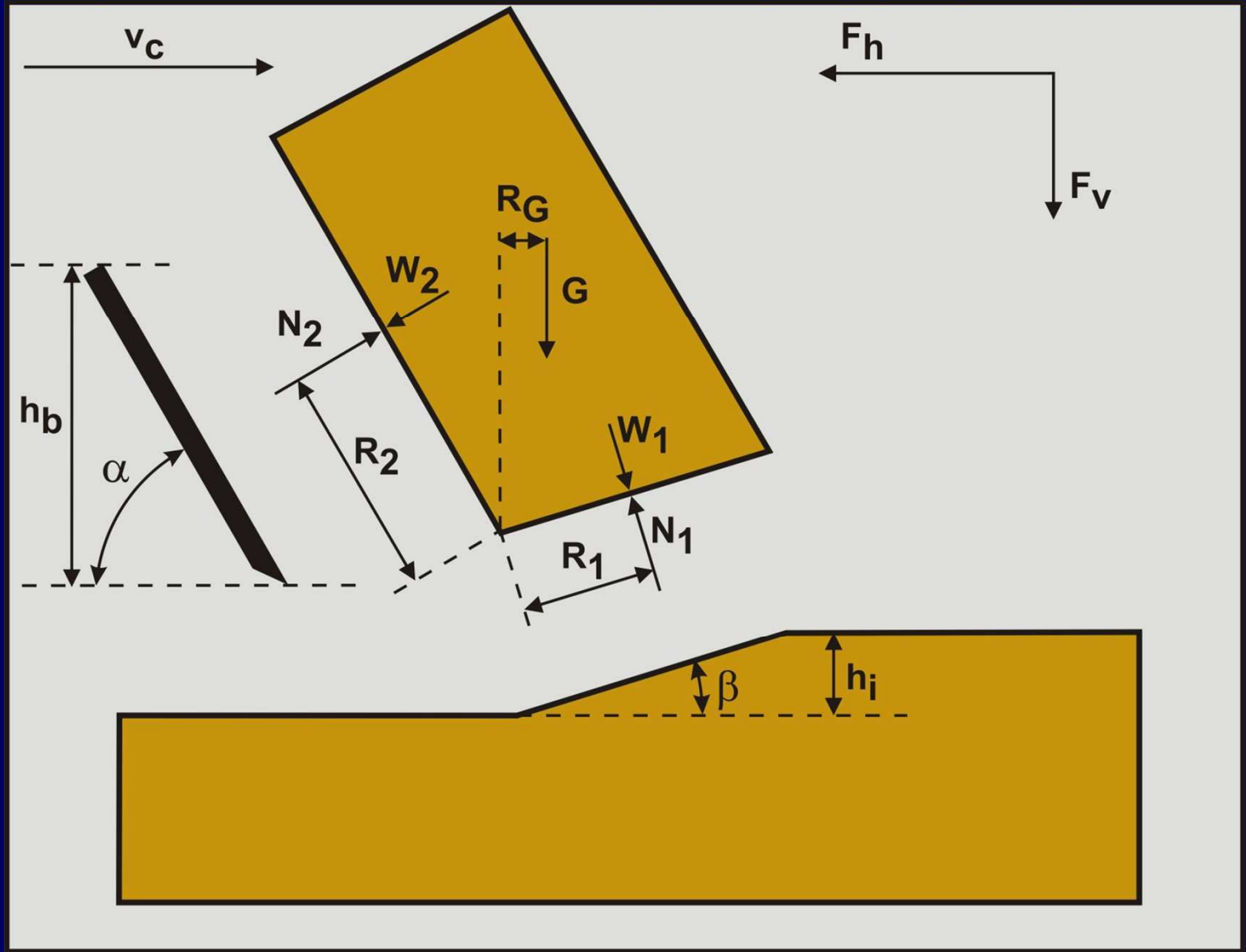
# Forces on the Blade



Faculty of 3mE - Dredging Engineering



# Moments



## Resulting Equations

$$K_2 = \frac{W_2 \cdot \sin(\alpha + \beta + \varphi) + W_1 \cdot \sin(\varphi) + G \cdot \sin(\beta + \varphi)}{\sin(\alpha + \beta + \delta + \varphi)}$$

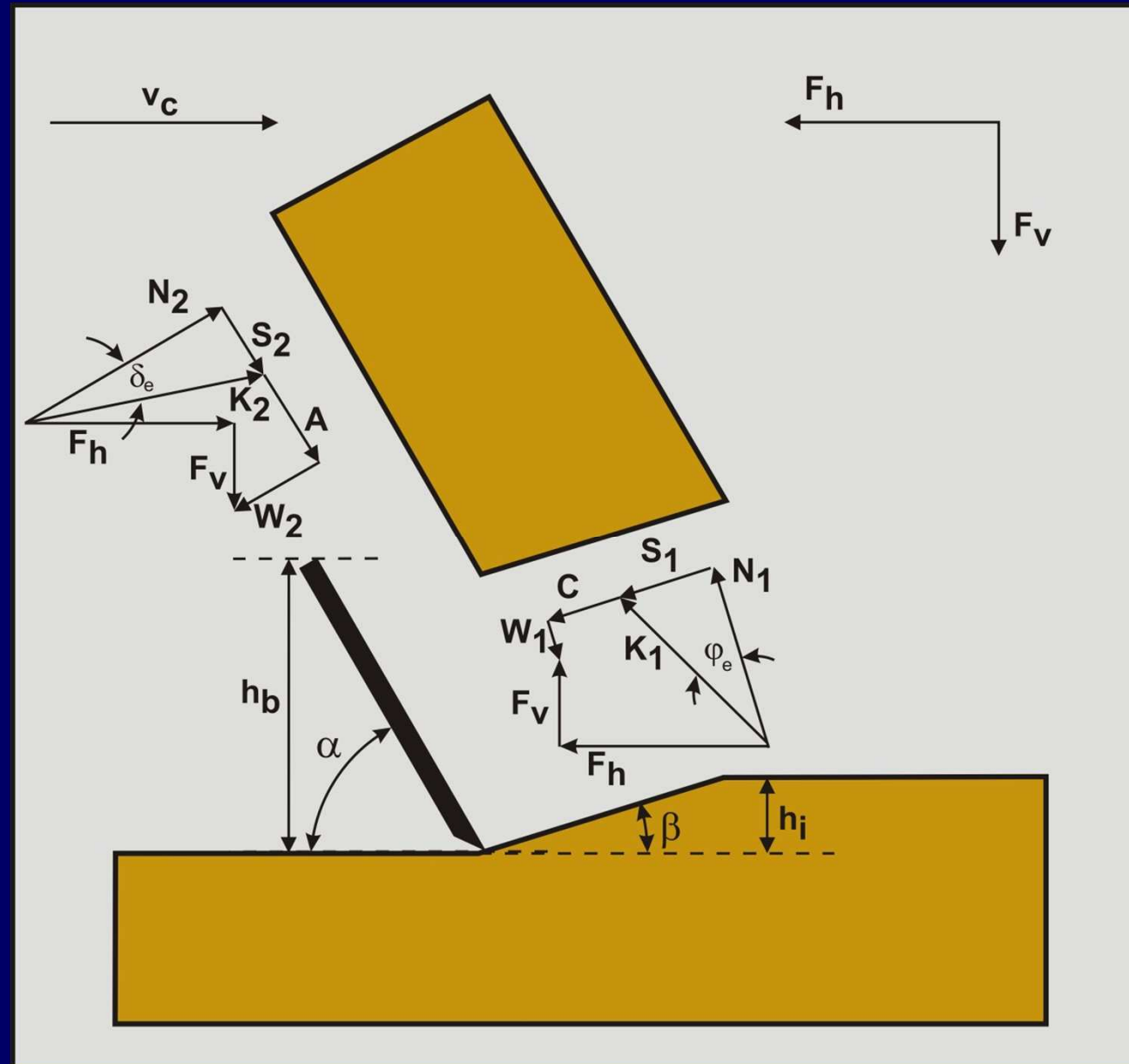
$$\frac{+I \cdot \cos(\varphi) + C \cdot \cos(\varphi) - A \cdot \cos(\alpha + \beta + \varphi)}{\sin(\alpha + \beta + \delta + \varphi)}$$

$$F_h = -W_2 \cdot \sin(\alpha) + K_2 \cdot \sin(\alpha + \delta) + A \cdot \cos(\alpha)$$

$$F_v = -W_2 \cdot \cos(\alpha) + K_2 \cdot \cos(\alpha + \delta) - A \cdot \sin(\alpha)$$



# Vector Diagram



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# Which Terms in Which Soil

	Gravity	Inertia	Pore Pressure	Cohesion	Adhesion	Friction
Dry sand						
Saturated sand						
Clay						
Atmospheric rock						
Hyperbaric rock						

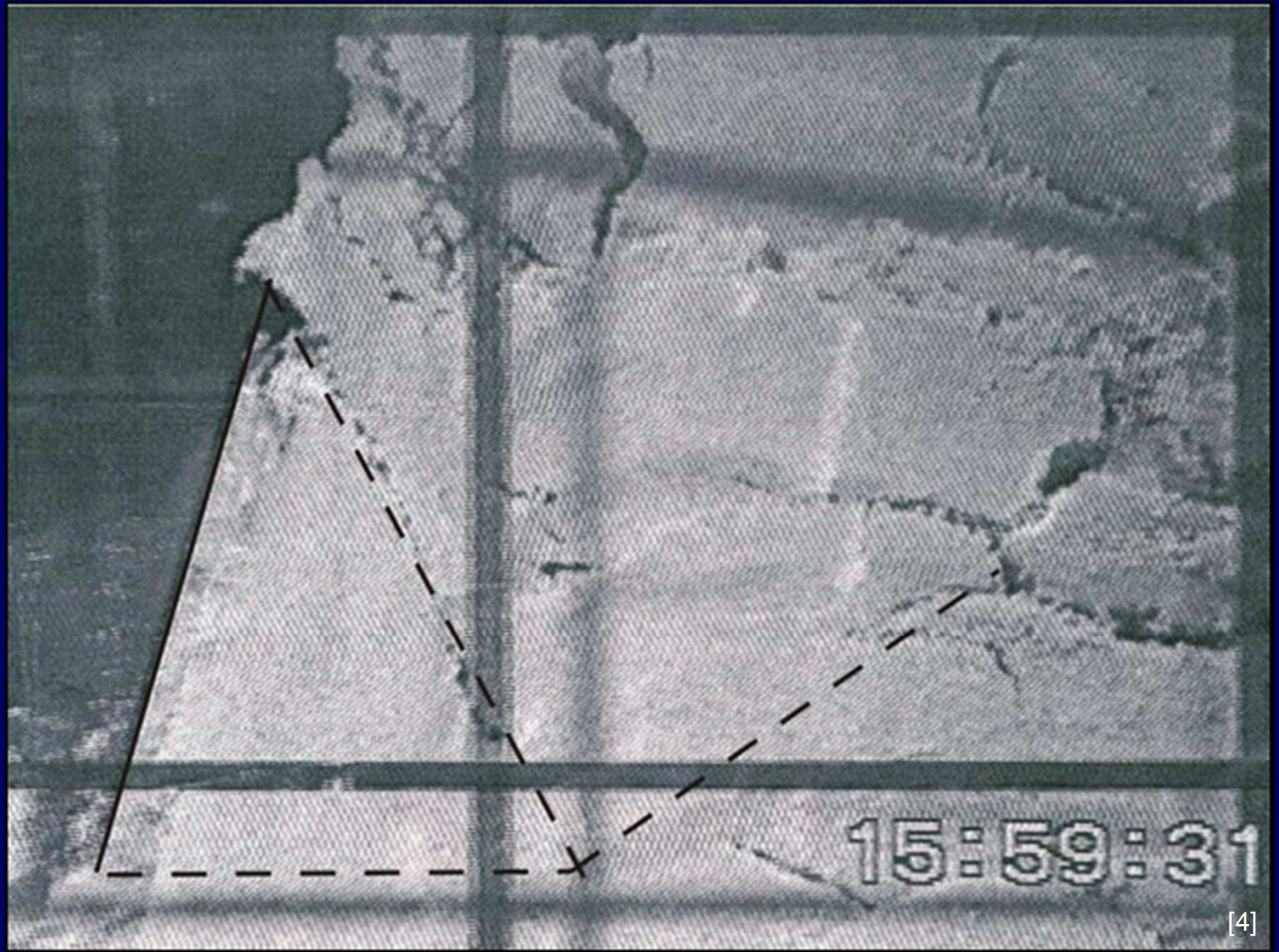






# Cutting Forces with Wedge

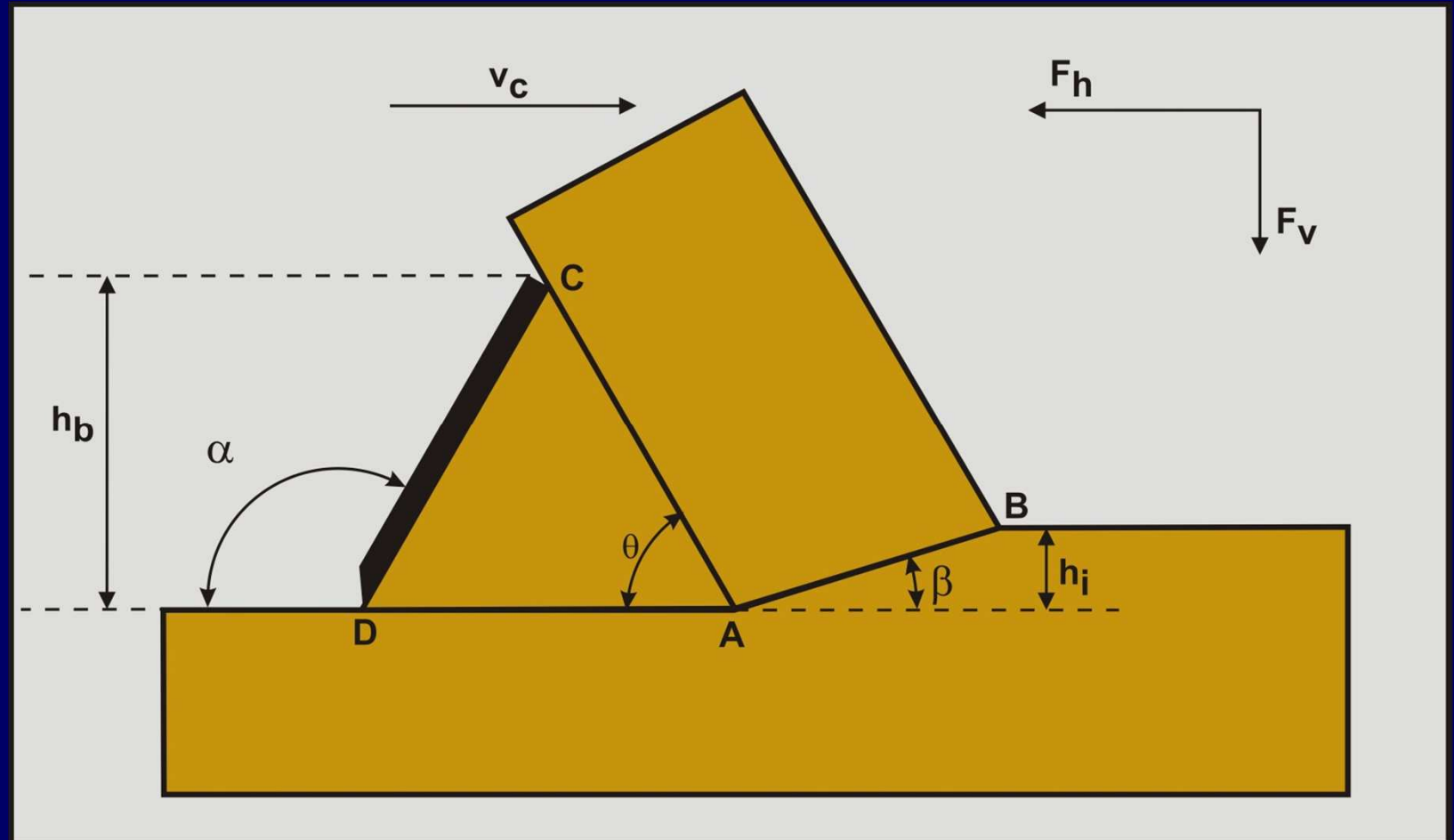
# A Wedge in Dry Sand



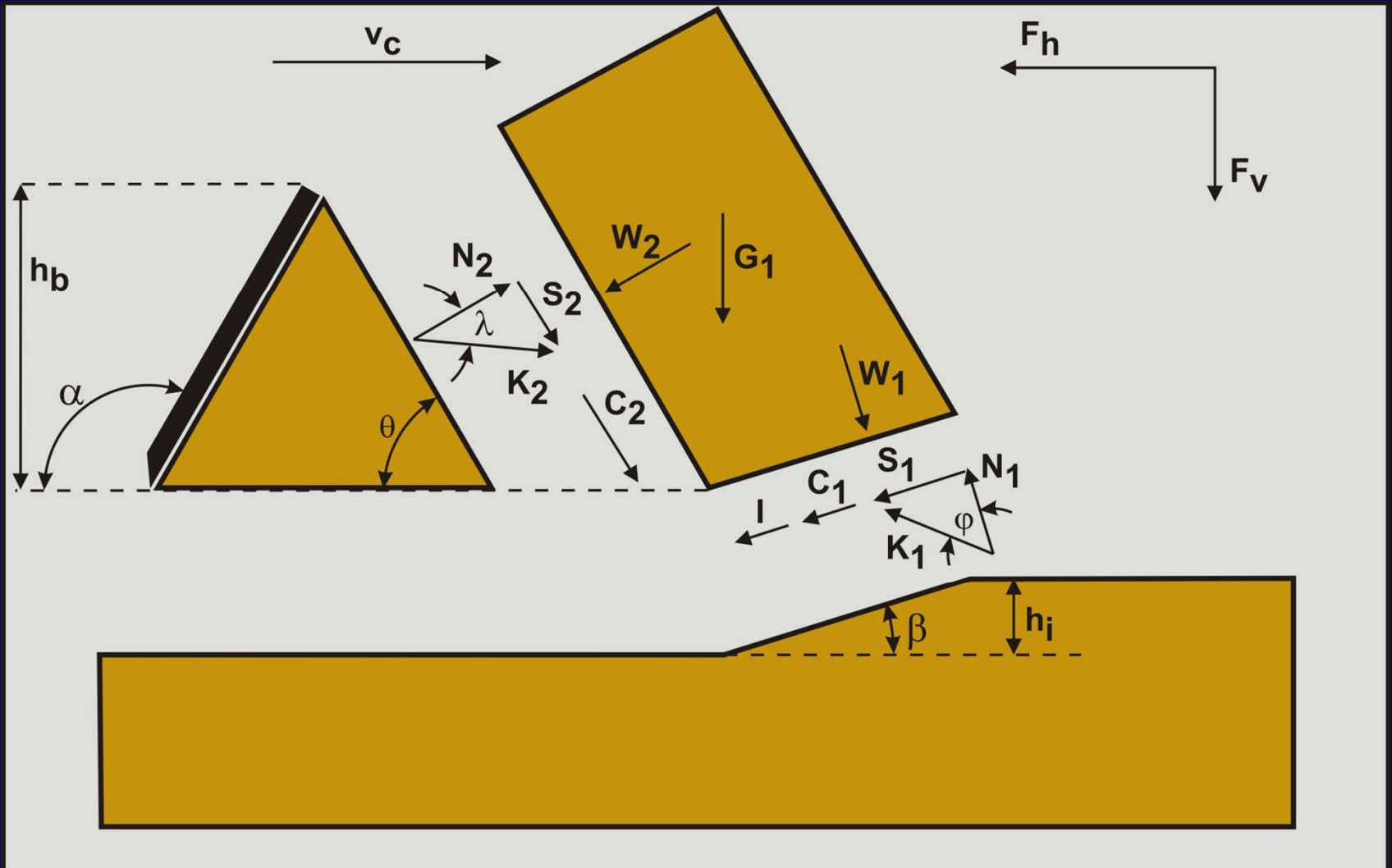
[4]



# Wedge Definitions

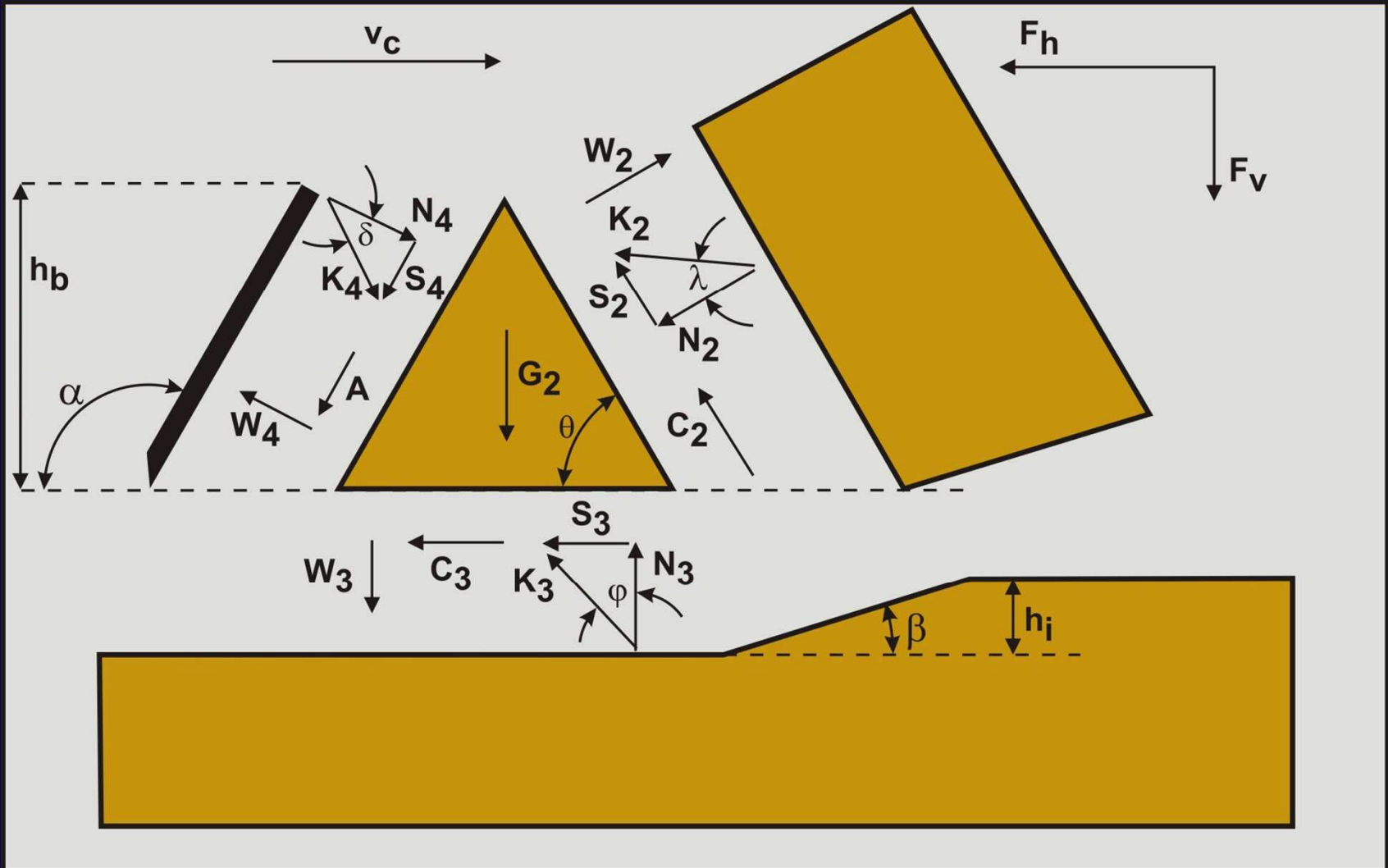


# Forces on Layer Cut

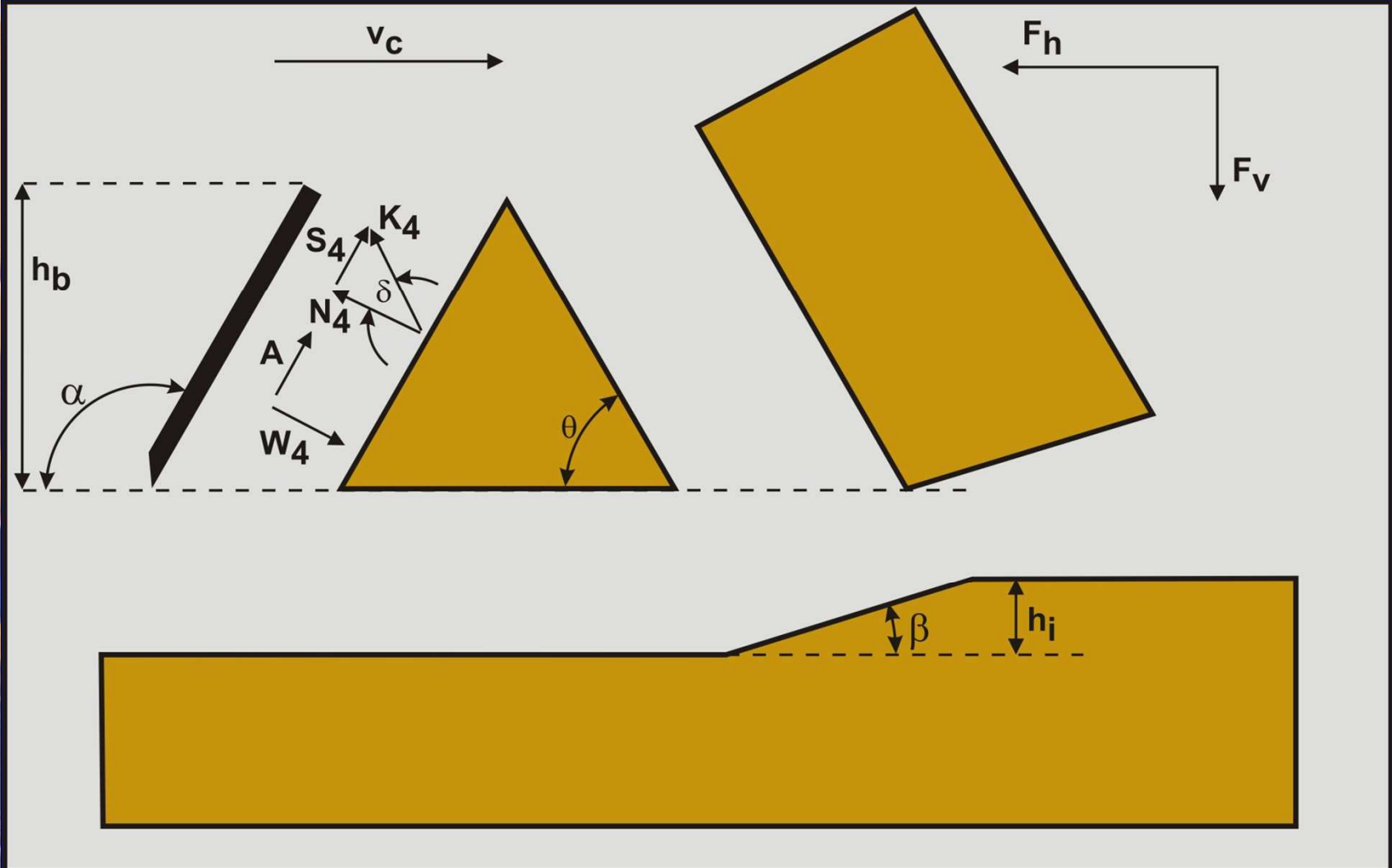




# Forces on the Wedge

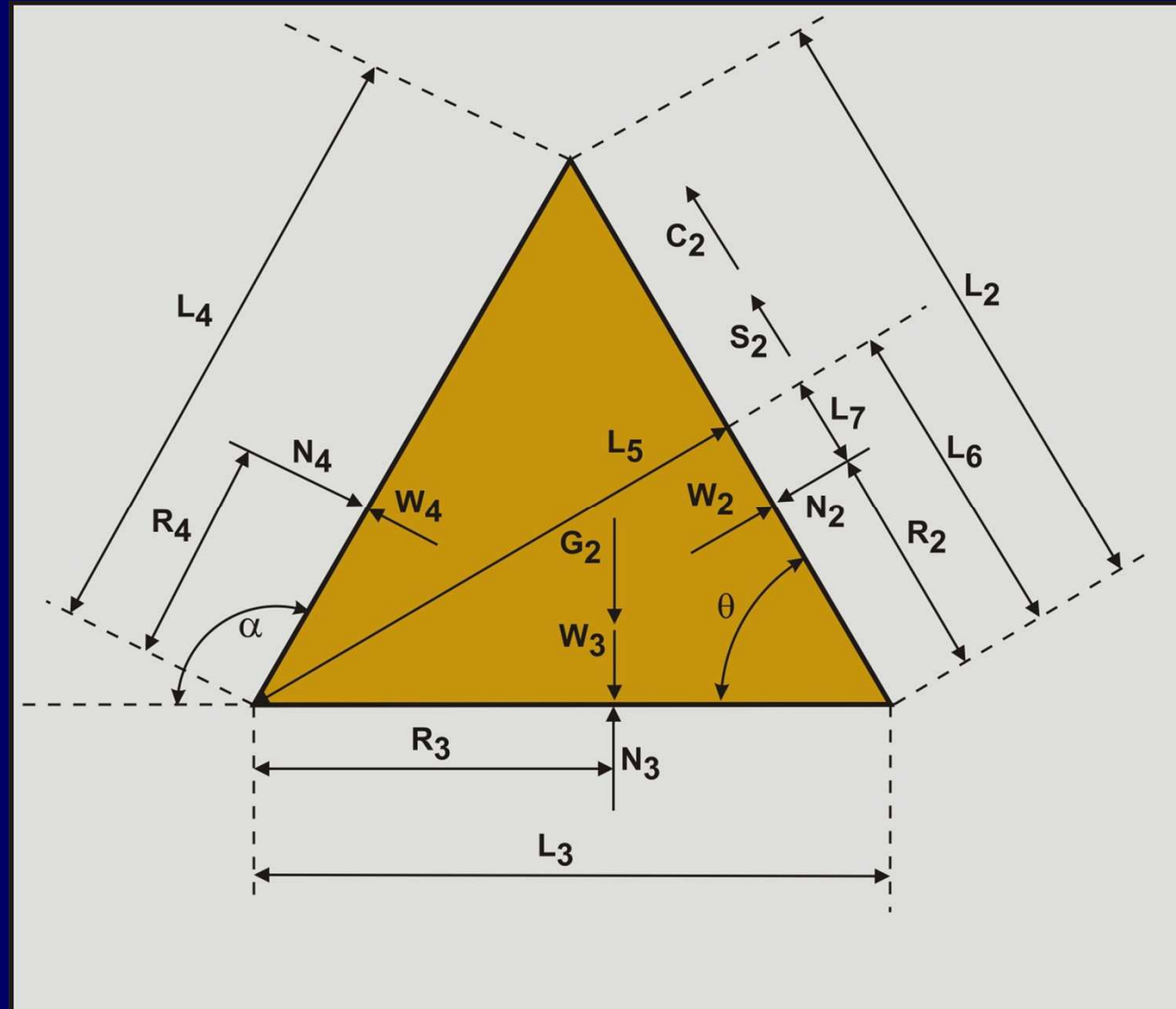


# Forces on the Blade





# Moments on the Wedge



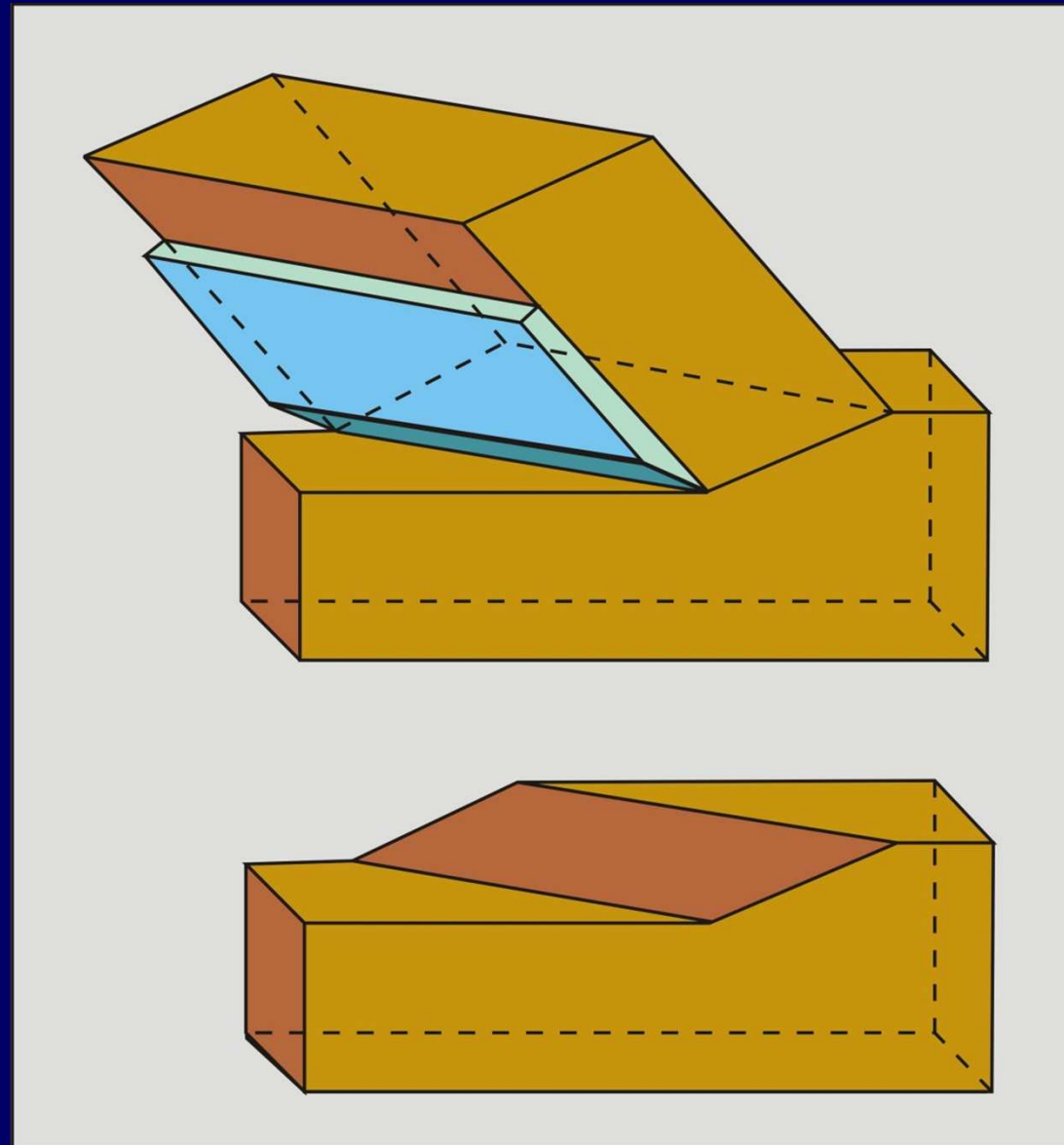
Faculty of 3mE - Dredging Engineering



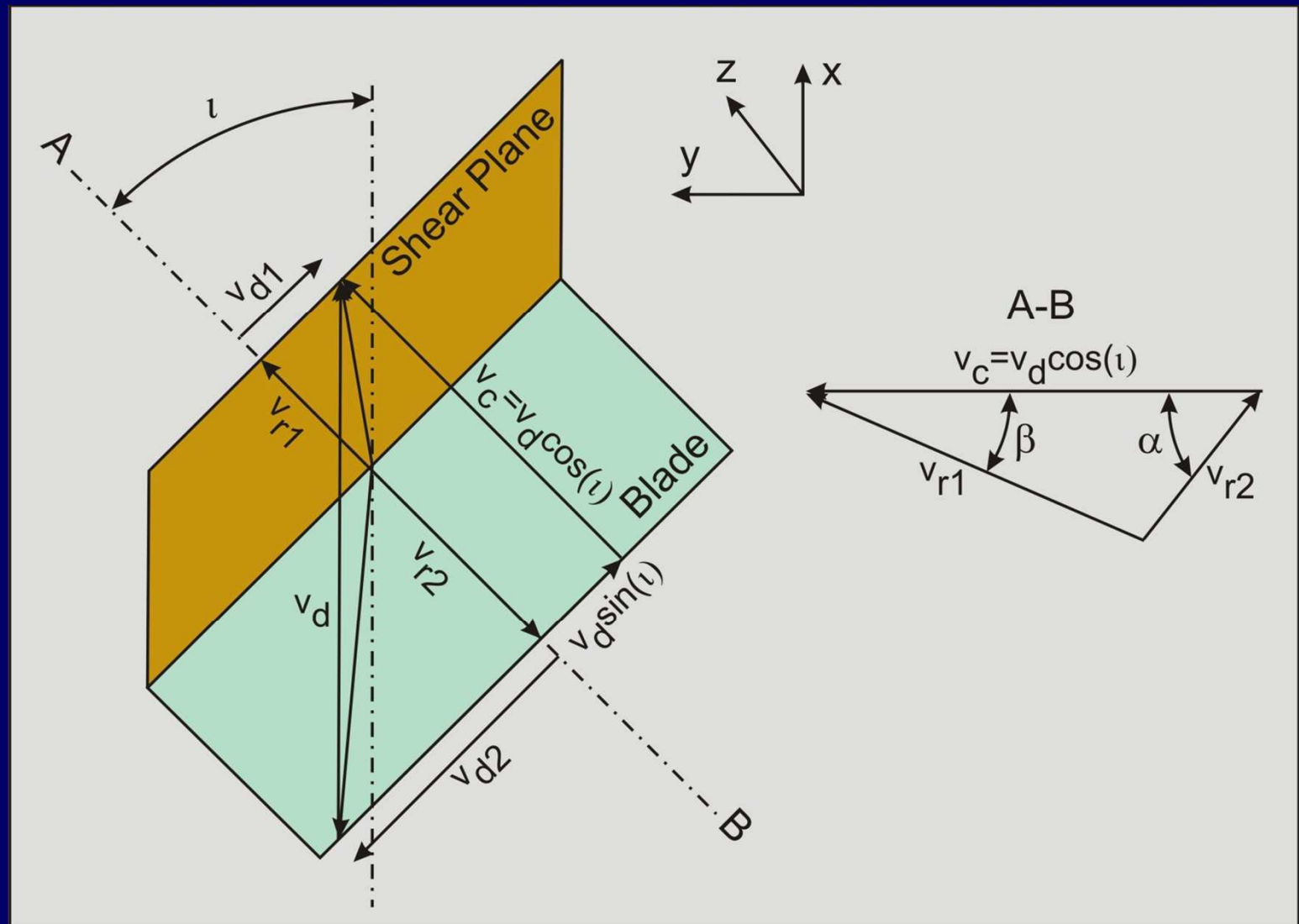
# Snow Plough Effect



# Snow Plough Effect

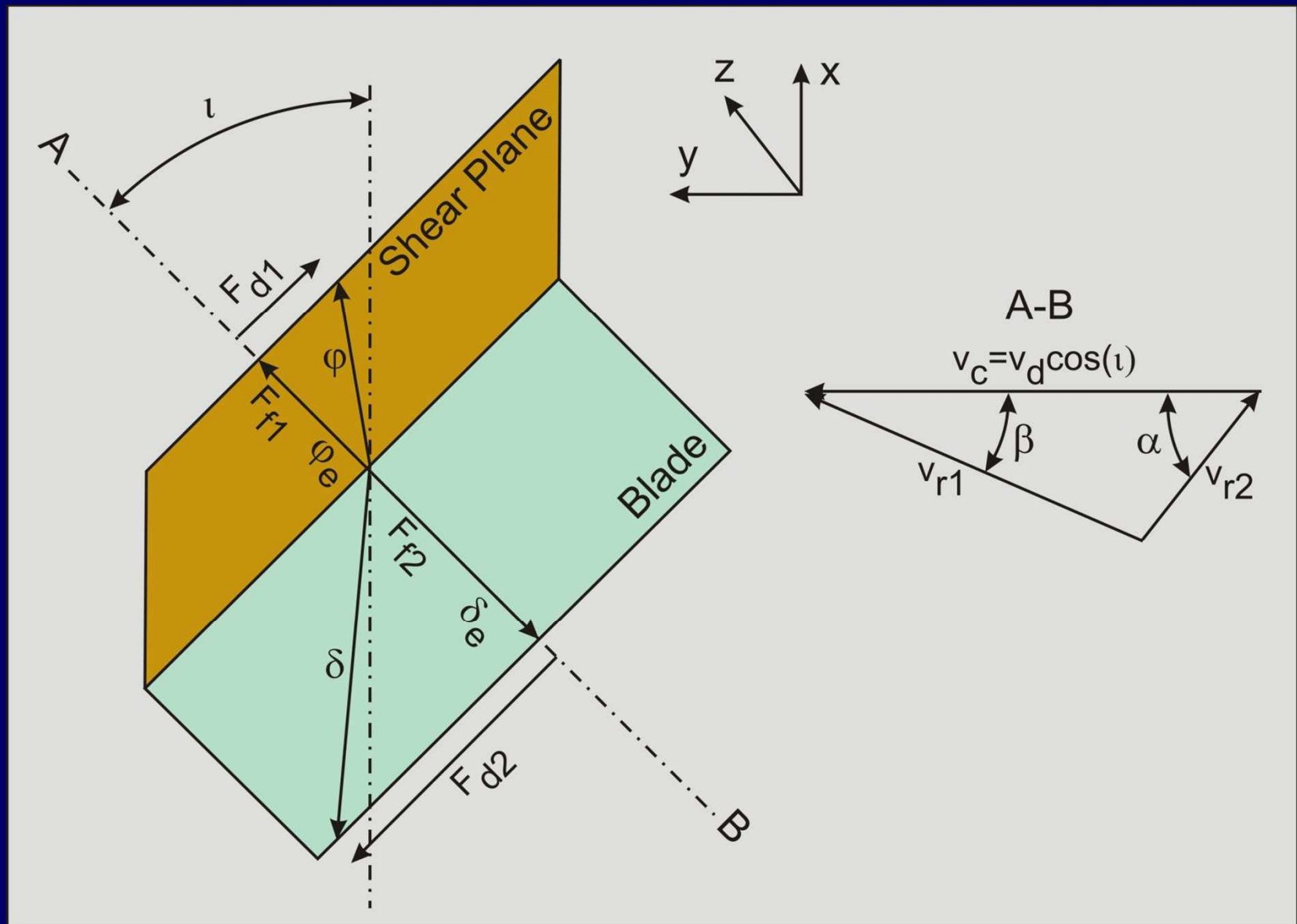


# Snow Plough Velocities





# Snow Plough Forces



# Effective Friction & Shear Stress

$$\tan(\varphi_e) = \tan(\varphi) \cdot \cos\left(\operatorname{atan}\left(\frac{v_{d1}}{v_{r1}}\right)\right)$$

$$\tan(\delta_e) = \tan(\delta) \cdot \cos\left(\operatorname{atan}\left(\frac{v_{d2}}{v_{r2}}\right)\right)$$

$$c_e = c \cdot \cos\left(\operatorname{atan}\left(\frac{v_{d1}}{v_{r1}}\right)\right)$$

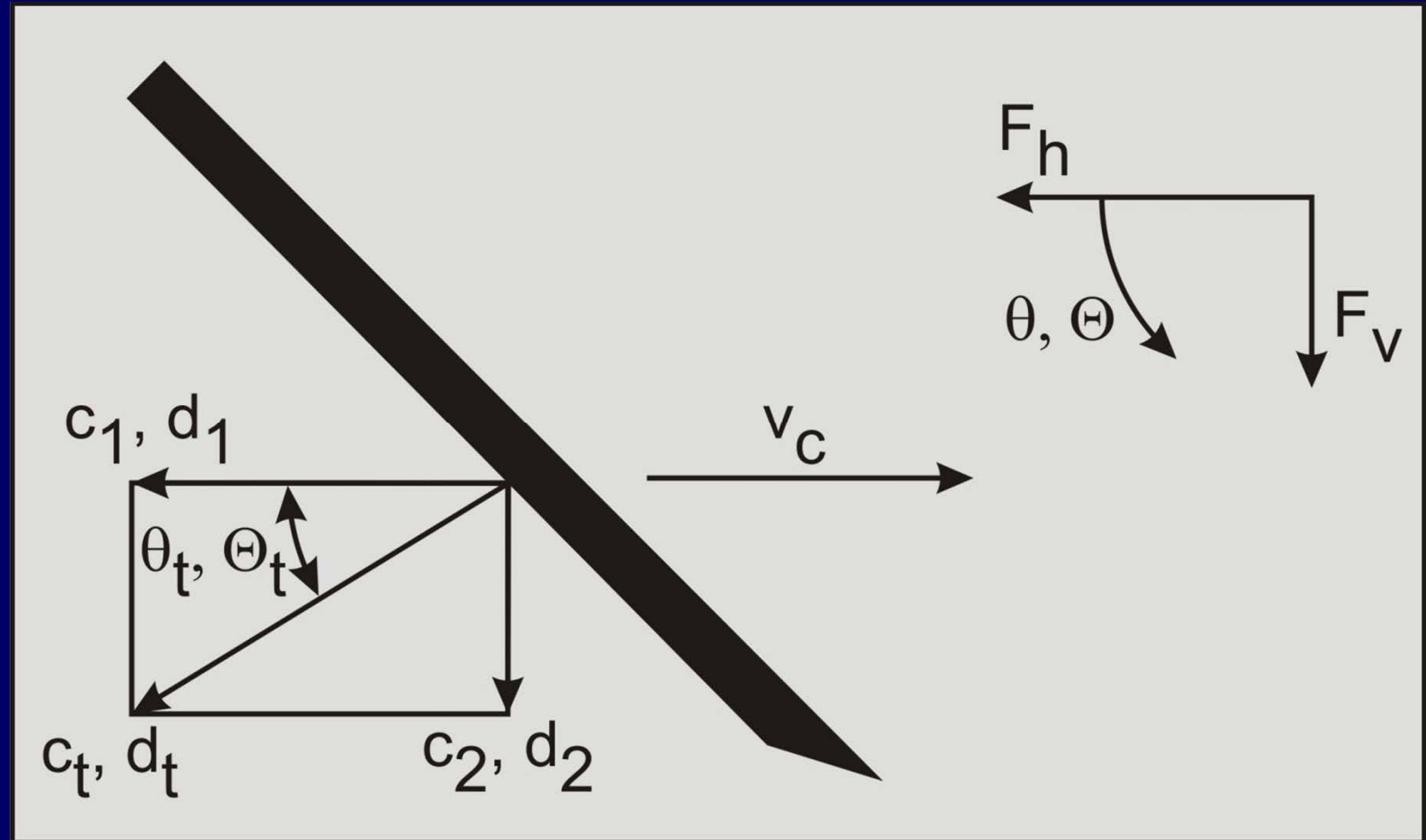
$$a_e = a \cdot \cos\left(\operatorname{atan}\left(\frac{v_{d2}}{v_{r2}}\right)\right)$$





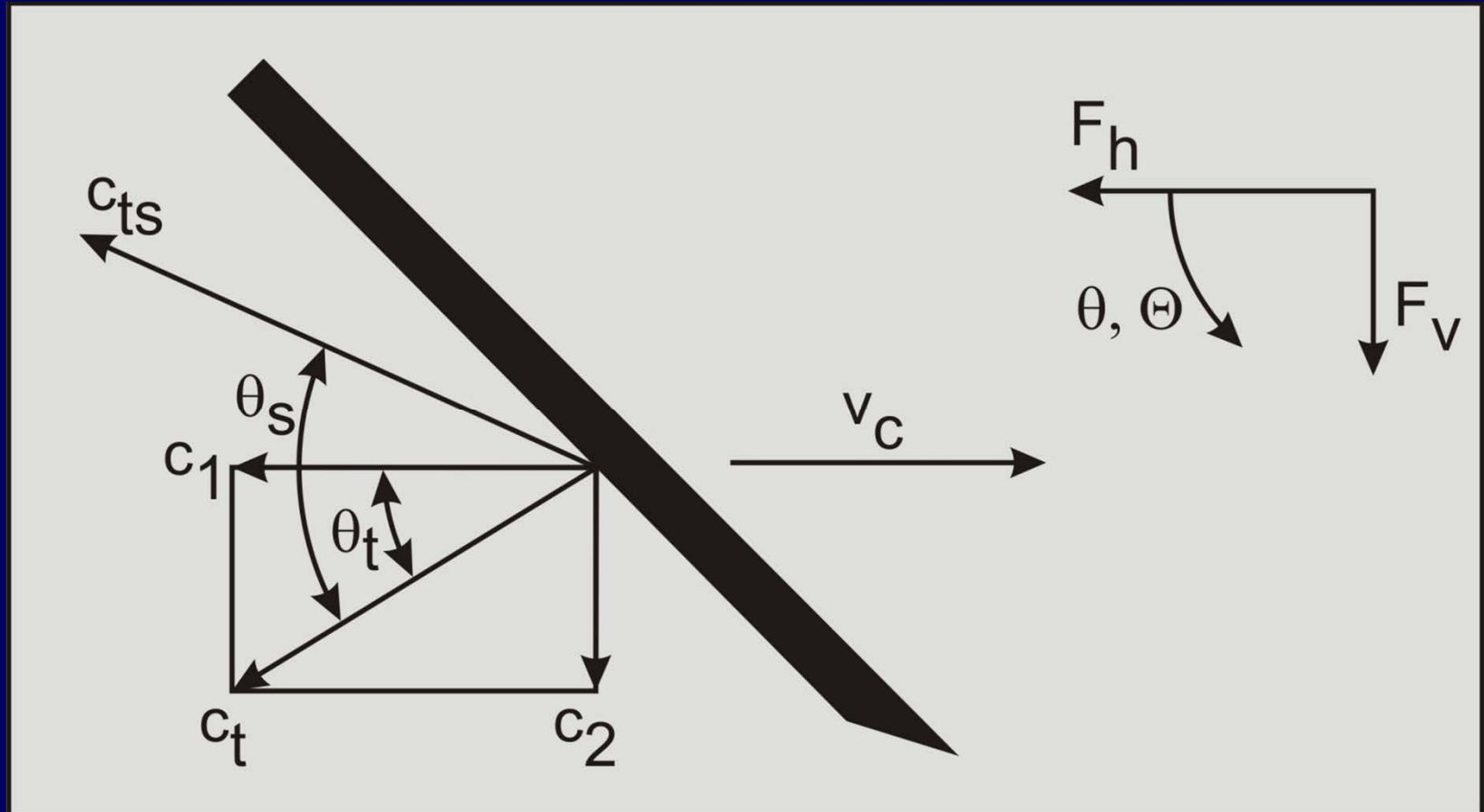
# Wear and 3D Effects

# Forces on the Blade

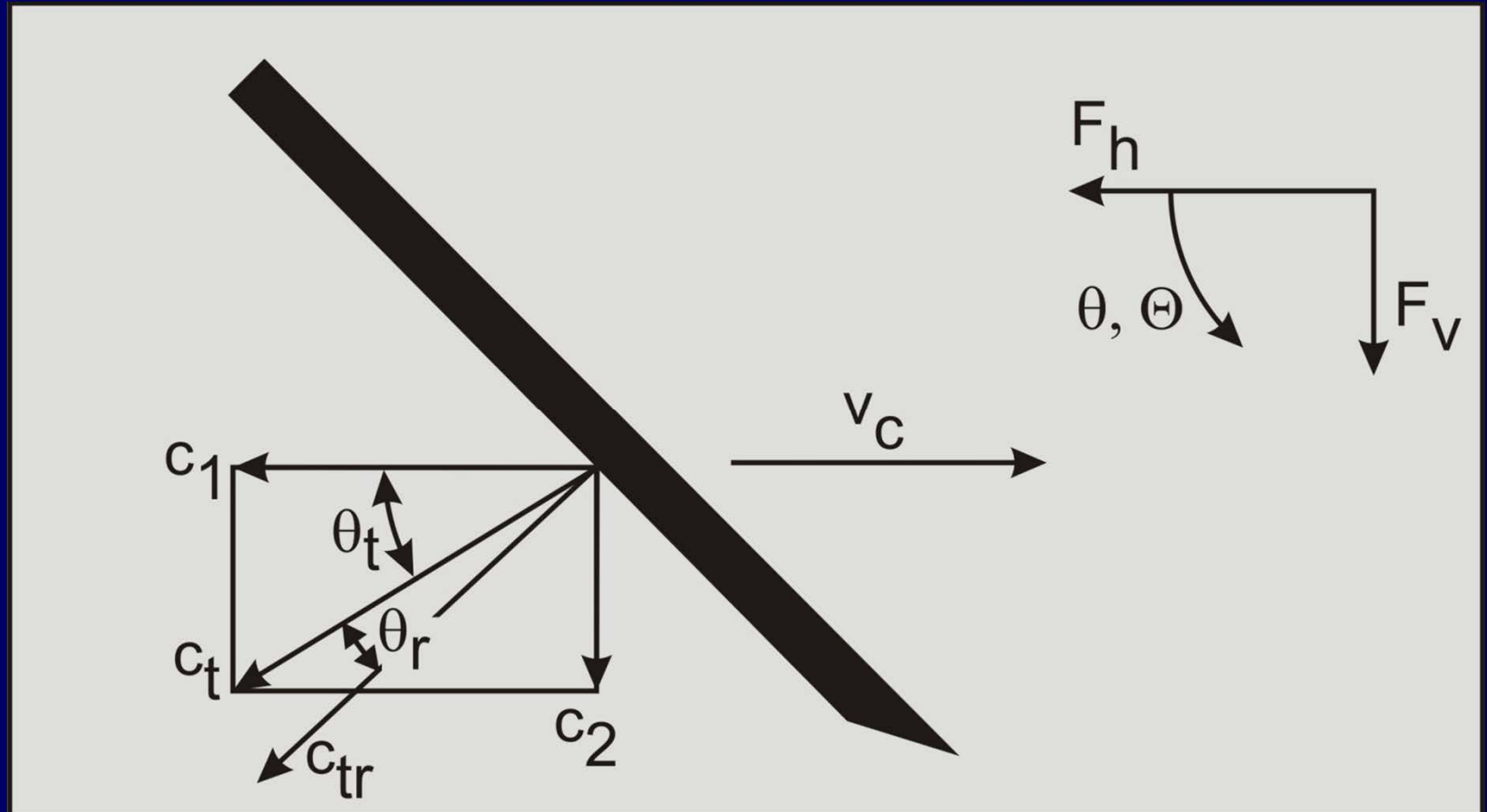




# Wear



# 3D Effects







# Questions?

# Sources images

1. A model cutter head, source: Delft University of Technology.
2. Off shore platform, source: Castrol (Switzerland) AG
3. Off shore platform, source: <http://www.wireropetraining.com>
4. Diagram of the failure pattern with Rake angle 120, source: TUDelft/S.A.Miedema