Offshore Hydromechanics Module 1

Dr. ir. Pepijn de Jong

3. Potential Flows part 1

Introduction

Overview

	Tutorial				Lecture				Online Assignments	
Week	**date**	**time**	**location**	**topic**	**date**	**time**	**location**	**topic**	**deadline**	**topic**
2					11-Sep	8:45-10:30	3mE-CZ B	Intro, Hydrostatics, Stability		
3					18-Sep	8:45-10:30	DW-Room 2	Hydrostatics, Stability		
4	23-Sep	8:45-10:30	TN-TZ4.25	Hydrostatics, Stability	25-Sep	8:45-10:30	3mE-CZ B	Potential Flows	27-Sep	Hydrostatics, Stability
5					02-Oct	8:45-10:30	3mE-CZ B	Potential Flows		
6	07-Oct	8:45-10:30	TN-TZ4.25	Potential Flows	09-Oct	8:45-10:30	3mE-CZ B	Real Flows	11-Oct	Potential Flows
7	14-Oct	8:45-10:30	TN-TZ4.25	Real Flows	16-Oct	8:45-10:30	3mE-CZ B	Real Flows, Waves	18-Oct	Real Flows
8					23-Oct	8:45-10:30	3mE-CZ B	Waves	25-Oct	Waves
Exam	30-Oct	9:00-12:00	TN-TZ4.25	Exam						

Introduction

E-Assessment

- Grade counted as follows: exam 80%, bonus assignments 20%
  - If it improves your final grade…
  - Only bonus assignments count

- E-Assessment Potential Flows:
  - Formative Exercises (set of 5, 4 tries, minimum 3/5 score)
  - Bonus Assignment

Introduction

Topics of Module 1

• Problems of interest	Chapter 1
• Hydrostatics	Chapter 2
• Floating stability	Chapter 2
• **Constant potential flows**	**Chapter 3**
• Constant real flows	Chapter 4
• Waves	Chapter 5

Learning Objectives

Chapter 3

- Understand the basic principles behind potential flow
- To schematically model flows applying basic potential flow elements and the superposition principle
- To perform basic flow computations applying potential flow theory

Fluid Mechanics Laws

Basic assumptions (Euler flow, potential flow)

- Homogeneous — properties are evenly spread over fluid
- Continuous — no bubbles, holes, particles, shocks etc
- Incompressible — $\rho = constant$
- Non-viscous — $\mu = 0$

- Without the latter 2 assumptions you end up with the Navier-Stokes eqs.

Fluid Mechanics Laws

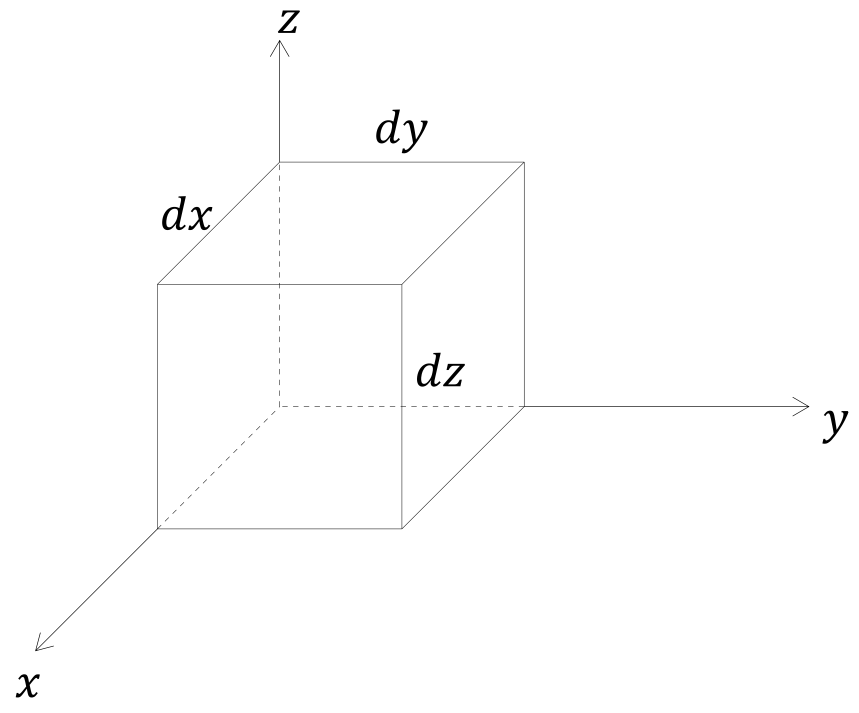
Basic assumptions

- Assumptions:
  - Are restrictive: they limit applicability of calculations
  - Are (often) necessary: to obtain a solution within a reasonable amount of effort
  - Discrepancies often addressed in a semi-empirical manner:
    - Very simplified models or coefficients based on experimental data

Fluid Mechanics Laws

Continuity (Conservation of mass)

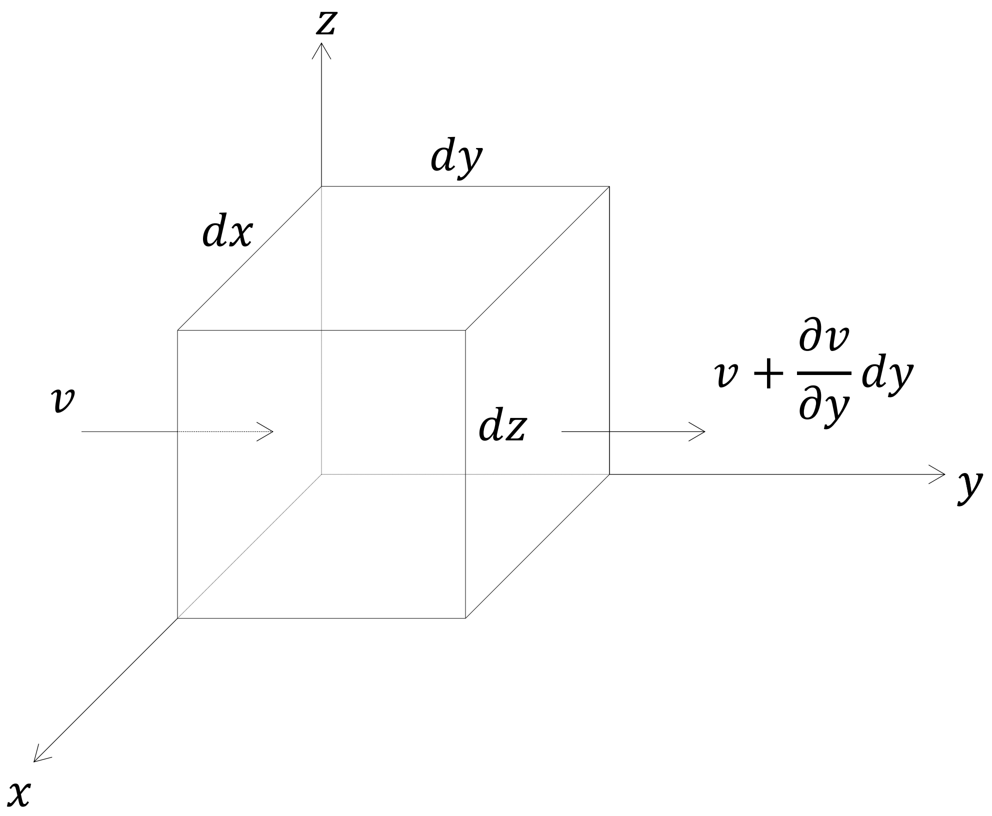
- Physical principle:
  - Mass can be neither created nor destroyed



Fluid Mechanics Laws

Continuity

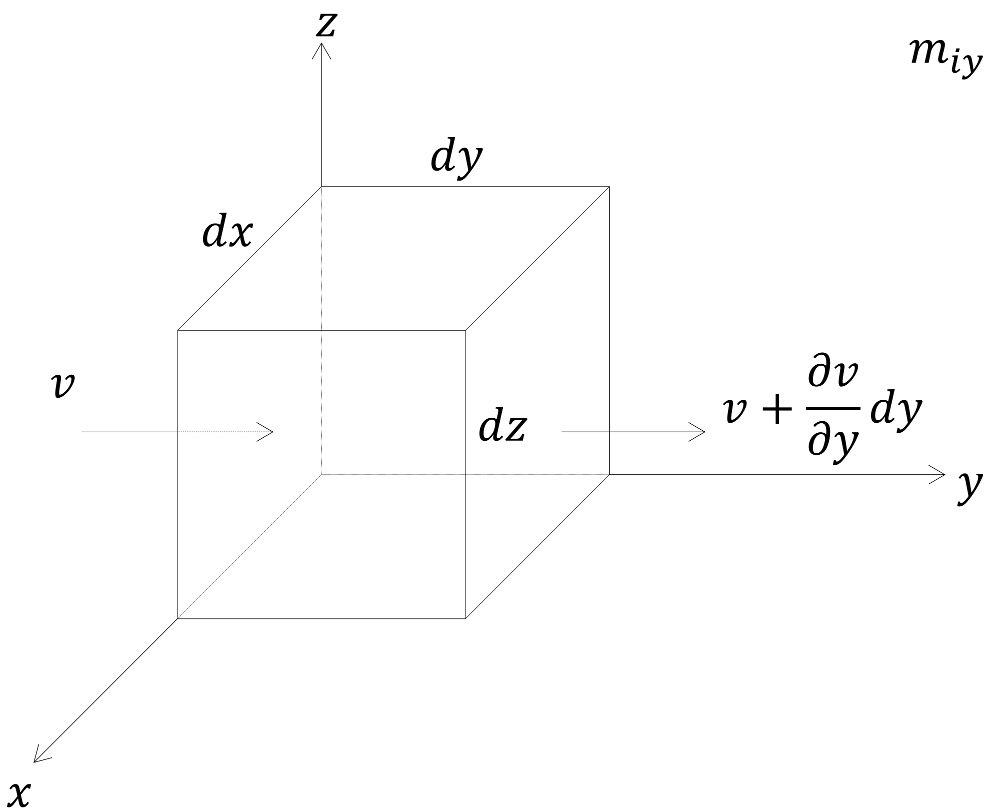
Net mass flow out of control volume
=
Time rate of decrease of mass within control volume



Fluid Mechanics Laws

Continuity

Net mass flow out of control volume
=
Time rate of decrease of mass within control volume



$$m_{iy} = \rho \cdot v dx dz \cdot dt = \rho v \cdot dx dz dt$$

$$m_{oy} = \left[\rho v + \frac{\partial \rho v}{\partial y} dy\right] \cdot dx dz dt$$

Net mass flow out of control volume:

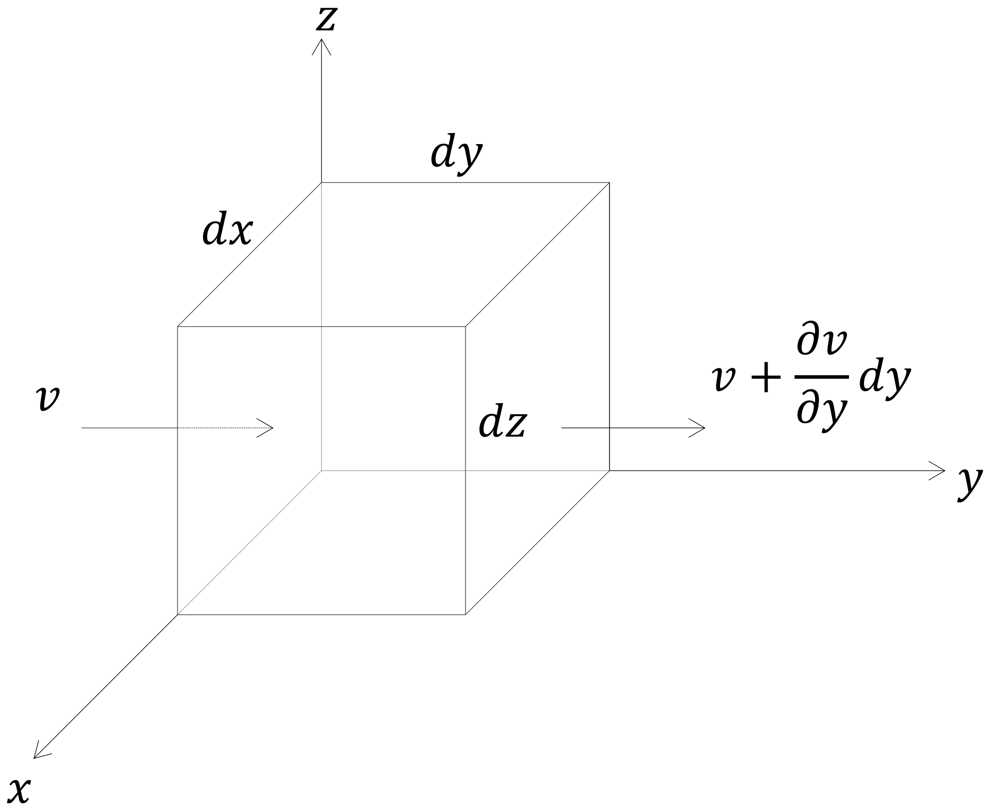
$$\frac{dm_y}{dt} = \frac{dm_{oy}}{dt} - \frac{dm_{iy}}{dt}$$

$$\frac{dm_y}{dt} = \frac{\partial \rho v}{\partial y} \cdot dx dy dz$$

Fluid Mechanics Laws

Continuity

Net mass flow out of control volume
=
Time rate of decrease of mass within control volume



Net mass flow out of CV:

$$\frac{dm_x}{dt} = \frac{\partial \rho u}{\partial x} \cdot dxdydz$$

$$\frac{dm_y}{dt} = \frac{\partial \rho v}{\partial y} \cdot dxdydz$$

$$\frac{dm_z}{dt} = \frac{\partial \rho w}{\partial z} \cdot dxdydz$$

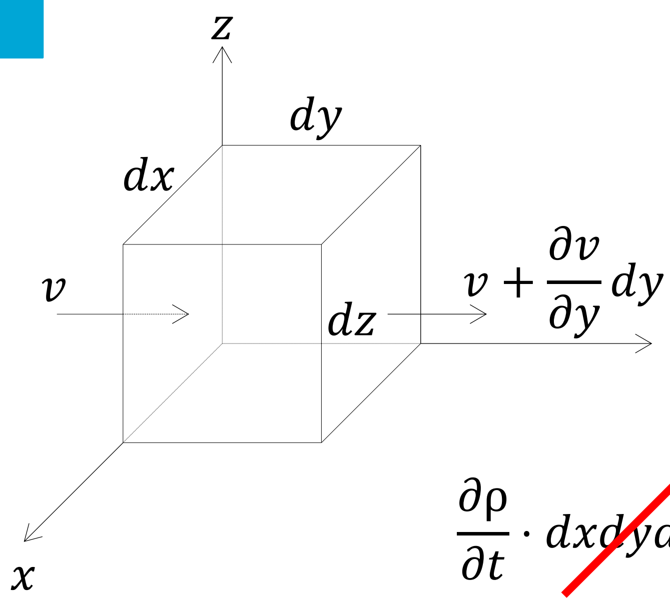
Time rate of mass decrease within CV:

$$-\frac{\partial \rho}{\partial t} \cdot dxdydz$$

Fluid Mechanics Laws

Continuity

Net mass flow out of control volume
=
Time rate of decrease of mass within control volume



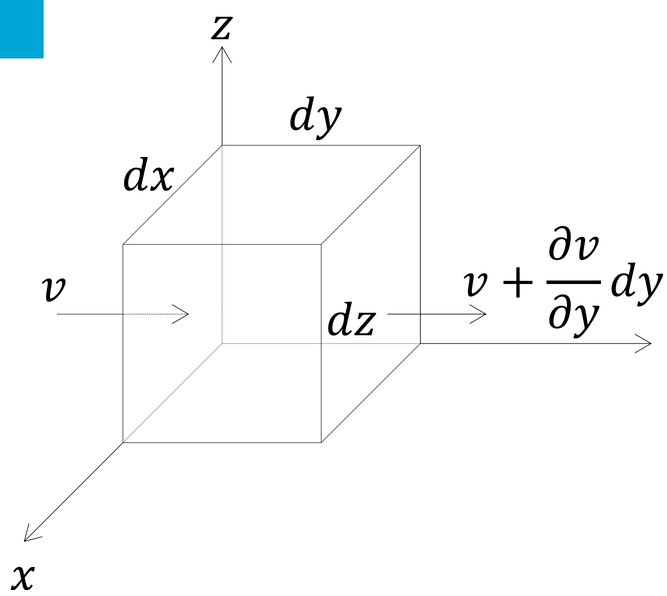
$$\frac{\partial \rho}{\partial t} \cdot dxdydz + \frac{\partial \rho u}{\partial x} \cdot dxdydz + \frac{\partial \rho v}{\partial y} \cdot dxdydz + \frac{\partial \rho w}{\partial z} \cdot dxdydz = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

Fluid Mechanics Laws

Continuity

Net mass flow out of control volume
=
Time rate of decrease of mass within control volume



$$v + \frac{\partial v}{\partial y} dy$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

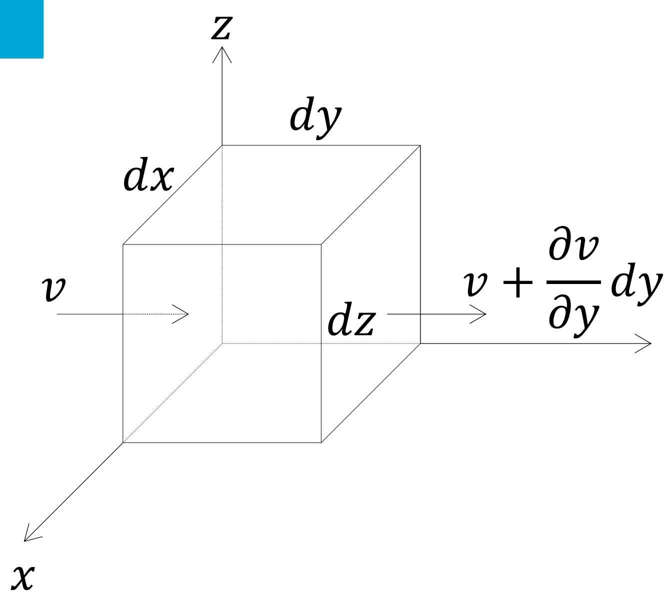
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Fluid Mechanics Laws

Continuity

Net mass flow out of control volume = Time rate of decrease of mass within control volume



Incompressible flow:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\nabla \cdot \vec{V} = 0$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Fluid Mechanics Laws

Conservation of momentum

- Apply Newton's second law for:
  - Incompressible fluid → constant density
  - Inviscid fluid → no tangential stresses in fluid

- Then the conservation of momentum yields the 'Euler Equations'

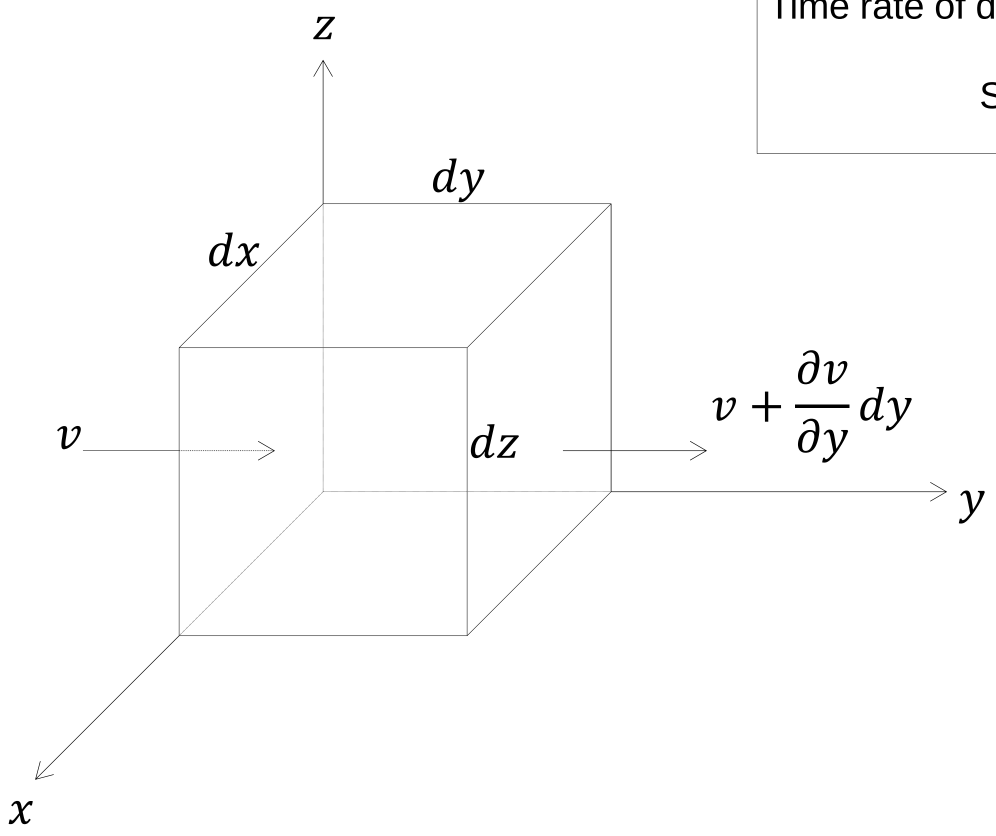
- (Linear) Momentum:

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}\left(m\vec{V}\right)$$

Fluid Mechanics Laws

Con. of momentum

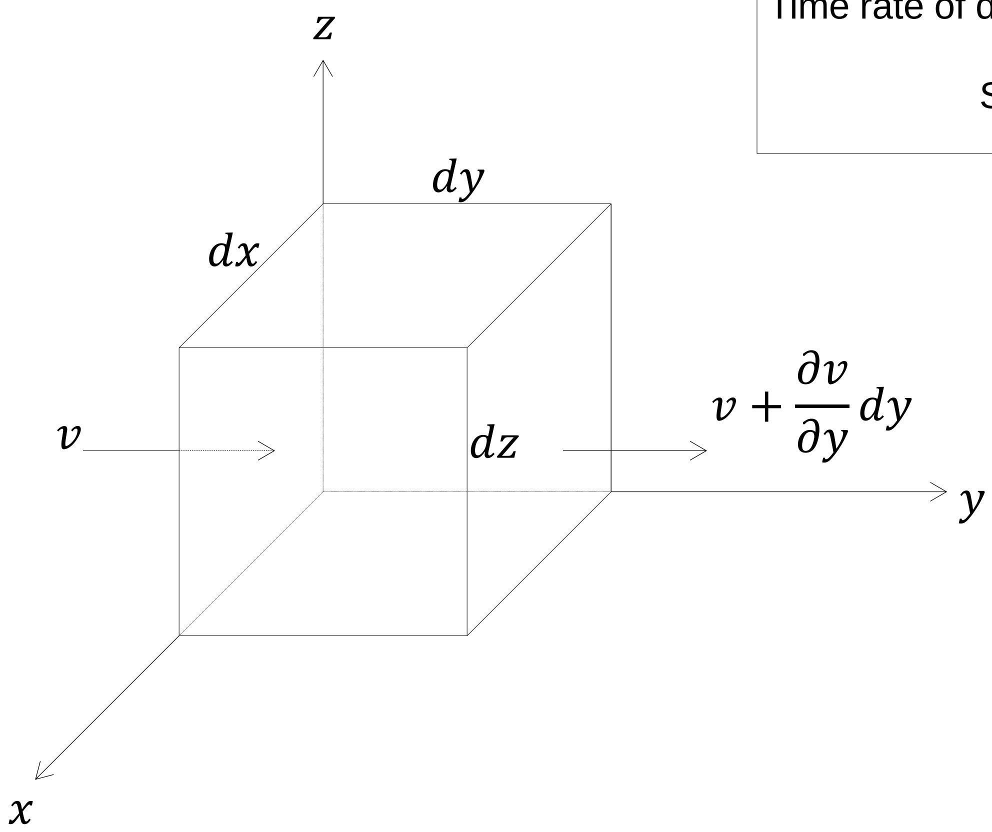
Net momentum flux out of control volume
=
Time rate of decrease of momentum within control volume
+
Sum of forces on control volume



Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume
-
Time rate of decrease of momentum within control volume
=
Sum of forces on control volume



Momentum flux:

$$\frac{d}{dt}\left(m\vec{V}\right) \rightarrow \frac{dm}{dt}\vec{V} = \rho v dxdz \cdot \vec{V}$$

Inlet momentum flux:

$$\rho v dxdz \cdot \vec{V} = \rho v \vec{V} \cdot dxdz$$

Outlet momentum flux:

$$\left[\rho v \vec{V} + \frac{\partial}{\partial x}\left(\rho v \vec{V}\right) dy\right] dxdz$$

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Fluid Mechanics Laws

Con. of momentum

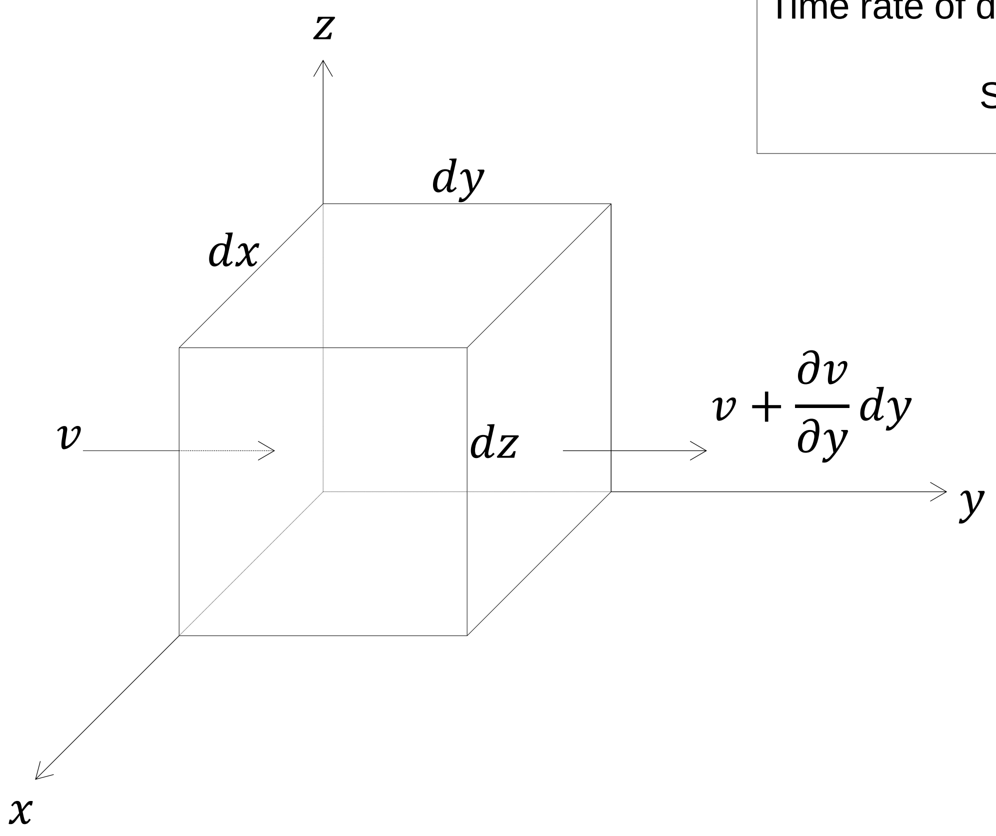
Net momentum flux out of control volume

\-

Time rate of decrease of momentum within control volume

=

Sum of forces on control volume



Inlet momentum flux:

$$\rho v \vec{V} \cdot dxdz$$

Outlet momentum flux:

$$\left[\rho v \vec{V} + \frac{\partial}{\partial y}\left(\rho v \vec{V}\right) dy\right] dxdz$$

Net momentum flux (out):

$$\frac{\partial}{\partial y}\left(\rho v \vec{V}\right) dxdydz$$

Fluid Mechanics Laws

Con. of momentum

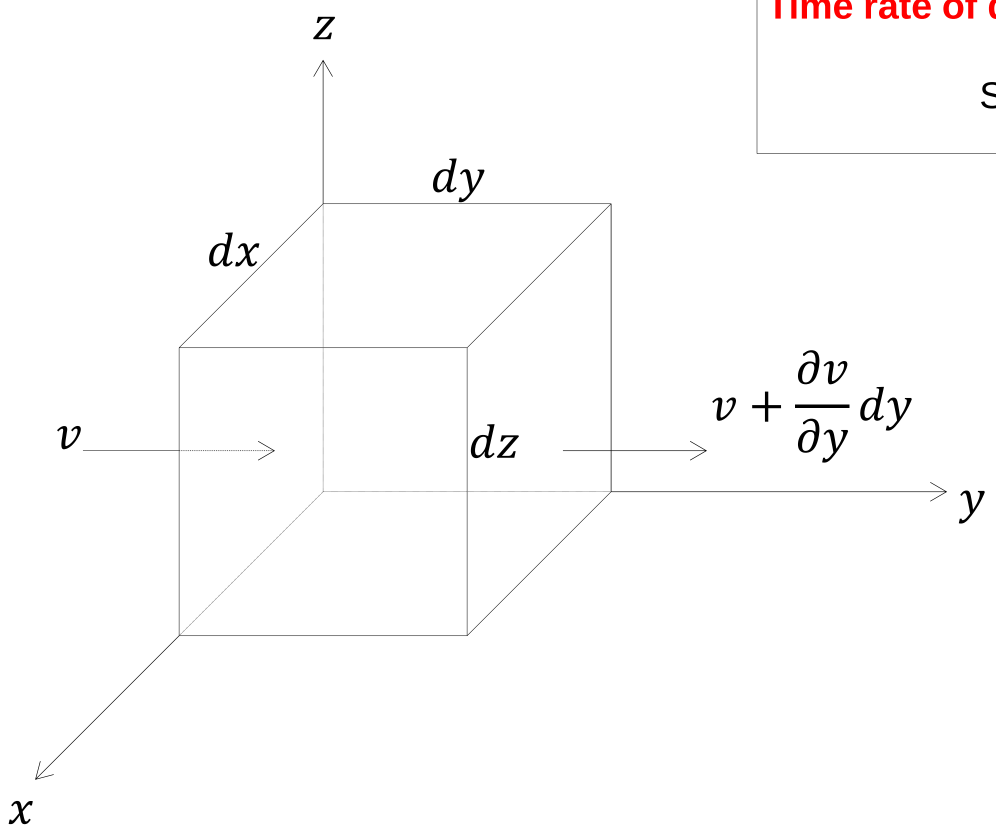
Net momentum flux out of control volume

-

Time rate of decrease of momentum in control volume

=

Sum of forces on control volume



Time rate of momentum in CV:

$$\frac{d}{dt}\left(m\vec{V}\right) \rightarrow \frac{\partial}{\partial t}\left(\rho dV\vec{V}\right)$$

Time rate of decrease of momentum in CV:

$$-\frac{d}{dt}\left(\rho\vec{V}\right)dxdydz$$

Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume

-

Time rate of decrease of momentum in control volume

=

Sum of forces on control volume

Net momentum flux (out):

$$\frac{\partial}{\partial x}\left(\rho u\vec{V}\right)dxdydz$$

$$\frac{\partial}{\partial y}\left(\rho v\vec{V}\right)dxdydz$$

$$\frac{\partial}{\partial z}\left(\rho w\vec{V}\right)dxdydz$$

Time rate of decrease of momentum in CV:

$$-\frac{\partial}{\partial t}\left(\rho\vec{V}\right)dxdydz$$

Result:

Sum of forces

$$\left[\frac{\partial}{\partial t}\left(\rho\vec{V}\right)+\frac{\partial}{\partial x}\left(\rho u\vec{V}\right)+\frac{\partial}{\partial y}\left(\rho v\vec{V}\right)+\frac{\partial}{\partial z}\left(\rho w\vec{V}\right)\right]dxdydz=\sum F$$

Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume

-

Time rate of decrease of momentum in control volume

=

Sum of forces on control volume

Further reduction is possible:

$$\left[\frac{\partial}{\partial t}(\rho\vec{V})+\frac{\partial}{\partial x}(\rho u\vec{V})+\frac{\partial}{\partial y}(\rho v\vec{V})+\frac{\partial}{\partial z}(\rho w\vec{V})\right]dxdydz=\sum F$$

$$\left[\vec{V}\left(\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{V})\right)+\rho\left(\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}\right)\right]dxdydz=\sum F$$

Conservation of mass!

$$\rho\left(\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}\right)dxdydz=\sum F$$

Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume
-
Time rate of decrease of momentum in control volume
=
Sum of forces on control volume

$$\rho\left(\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}\right)dxdydz=\sum F$$

Can be written as:

$$\rho\frac{D\vec{V}}{Dt}dxdydz=\sum F$$

Using the 'substantial derivative':
The total derivative of a particle that moves with the fluid through the control volume:

$$\frac{D\vec{V}}{Dt}=\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}$$

Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume
-
Time rate of decrease of momentum in control volume
=
Sum of forces on control volume

Gravity forces:

$$d\vec{F}_{grav} = \rho\vec{g}dxdydz$$

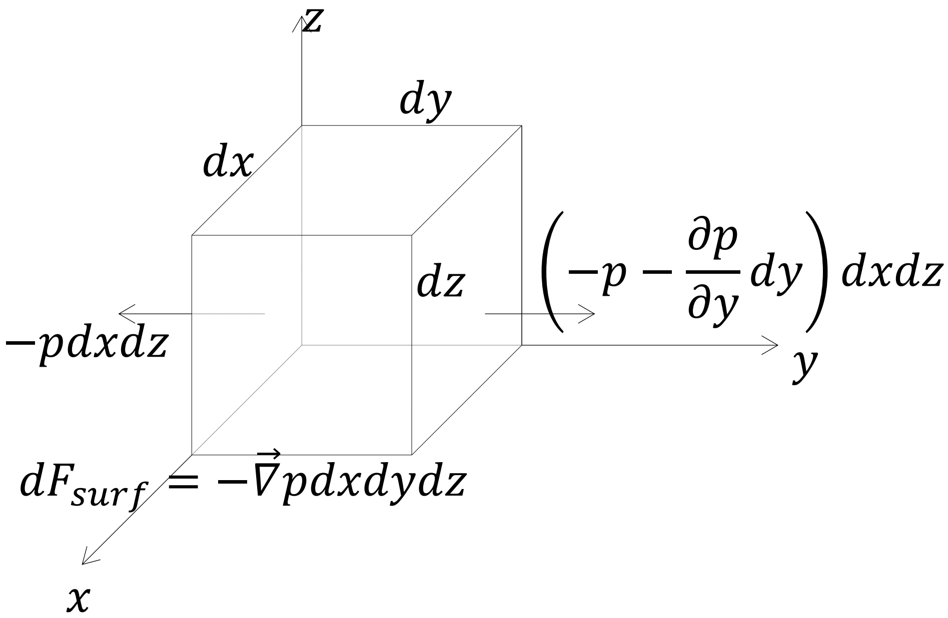
$$\vec{g} = [0,0,-g]$$

Surface forces (neglecting viscous stresses):

$$dF_{xsurf} = \frac{-\partial p}{\partial x}dxdydz$$

$$dF_{ysurf} = \frac{-\partial p}{\partial y}dydxdz$$

$$dF_{zsurf} = \frac{-\partial p}{\partial z}dzdxdy$$



$$dF_{surf} = -\vec{\nabla}pdxdydz$$

Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume
-
Time rate of decrease of momentum in control volume
=
Sum of forces on control volume

Gravity forces:

$$d\vec{F}_{grav} = \rho\vec{g}dxdydz$$

Surface forces (neglecting viscous stresses):

$$dF_{surf} = -\vec{\nabla}pdxdydz$$

Conservation of momentum:

$$\rho\frac{D\vec{V}}{Dt}dxdydz = \rho\vec{g}dxdydz - \vec{\nabla}pdxdydz$$

Fluid Mechanics Laws

Con. of momentum

Net momentum flux out of control volume
-
Time rate of decrease of momentum in control volume
=
Sum of forces on control volume

Gravity forces:

$$d\vec{F}_{grav} = \rho\vec{g}dxdydz$$

Surface forces (neglecting viscous stresses):

$$dF_{surf} = -\vec{\nabla}pdxdydz$$

Conservation of momentum:

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \vec{\nabla}p$$

Fluid Mechanics Laws

Con. of momentum

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Gravity forces:

$$d\vec{F}_{grav} = \rho\vec{g}dxdydz$$

Surface forces (neglecting viscous stresses):

$$dF_{surf} = -\vec{\nabla}pdxdydz$$

Conservation of momentum:

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \vec{\nabla}p$$

Euler Equation for inviscid flow

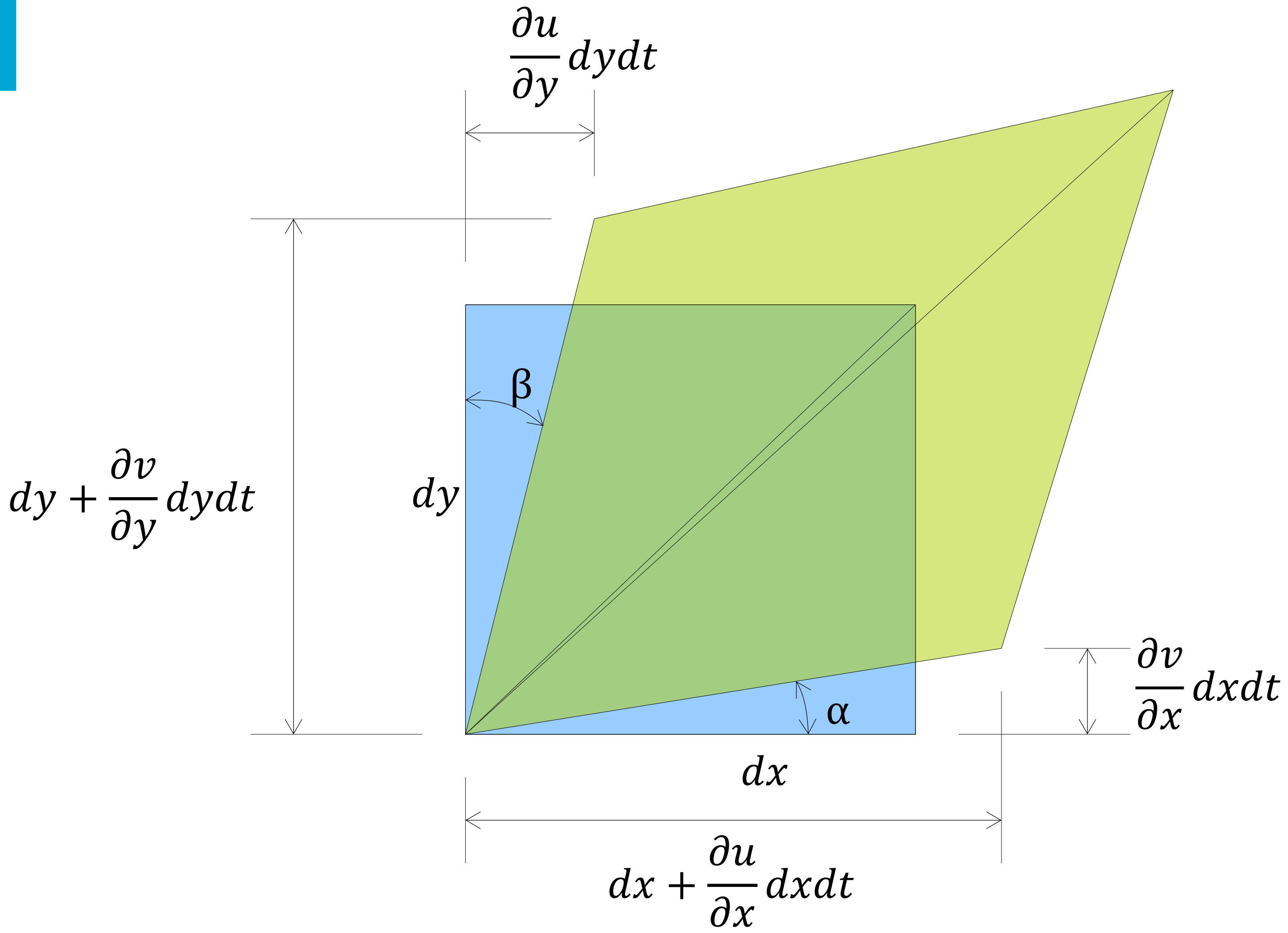
Fluid Mechanics Laws

Deformation and rotation (2D)

- Stresses within fluid will deform the cube considered before
- Explanation in book (p. 3-4 and 3-5) very shady/shaky
- We will consider a 2D slice of the cube

Fluid Mechanics Laws

Deformation and rotation (2D)



Fluid Mechanics Laws

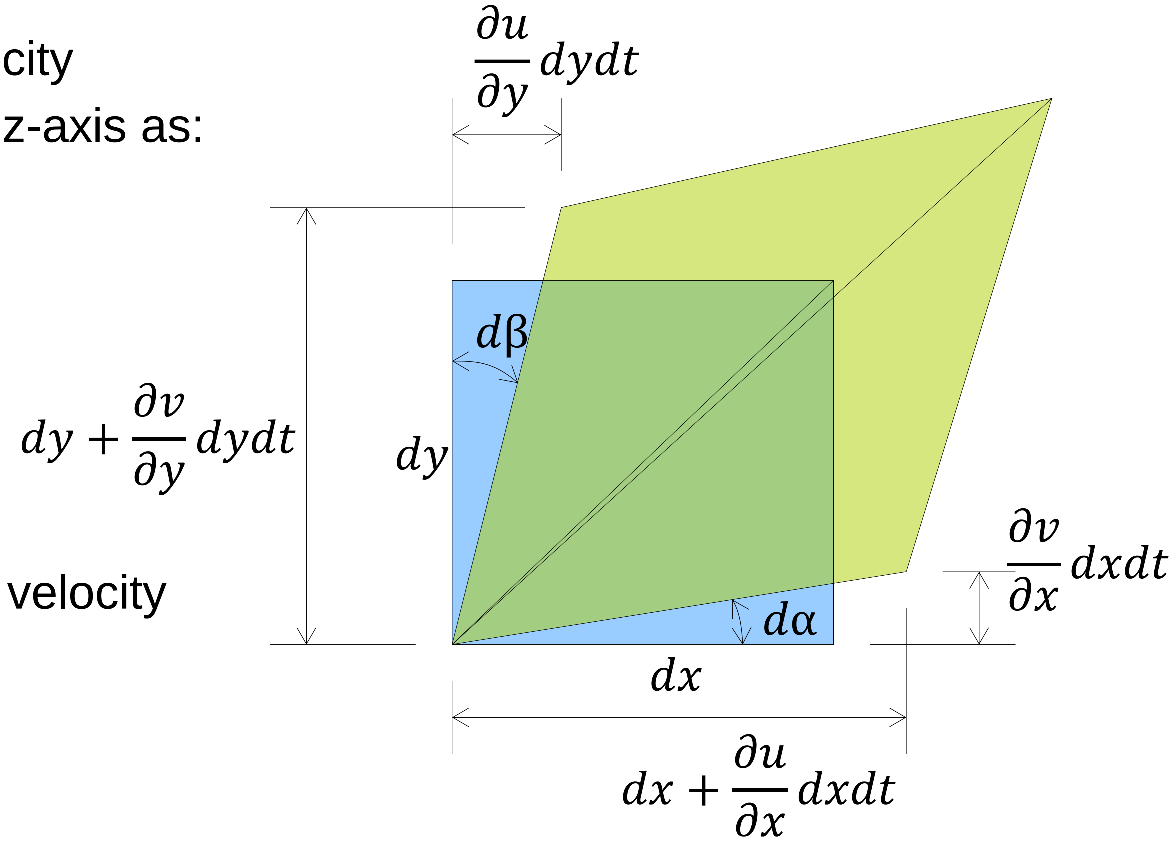
Deformation and rotation (2D)

- Define angular velocity (or **rotation**) about z-axis as:

$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right)$$

- Define deformation velocity (or **dilatation)** as:

$$\frac{1}{2}\left(\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right)$$



Fluid Mechanics Laws

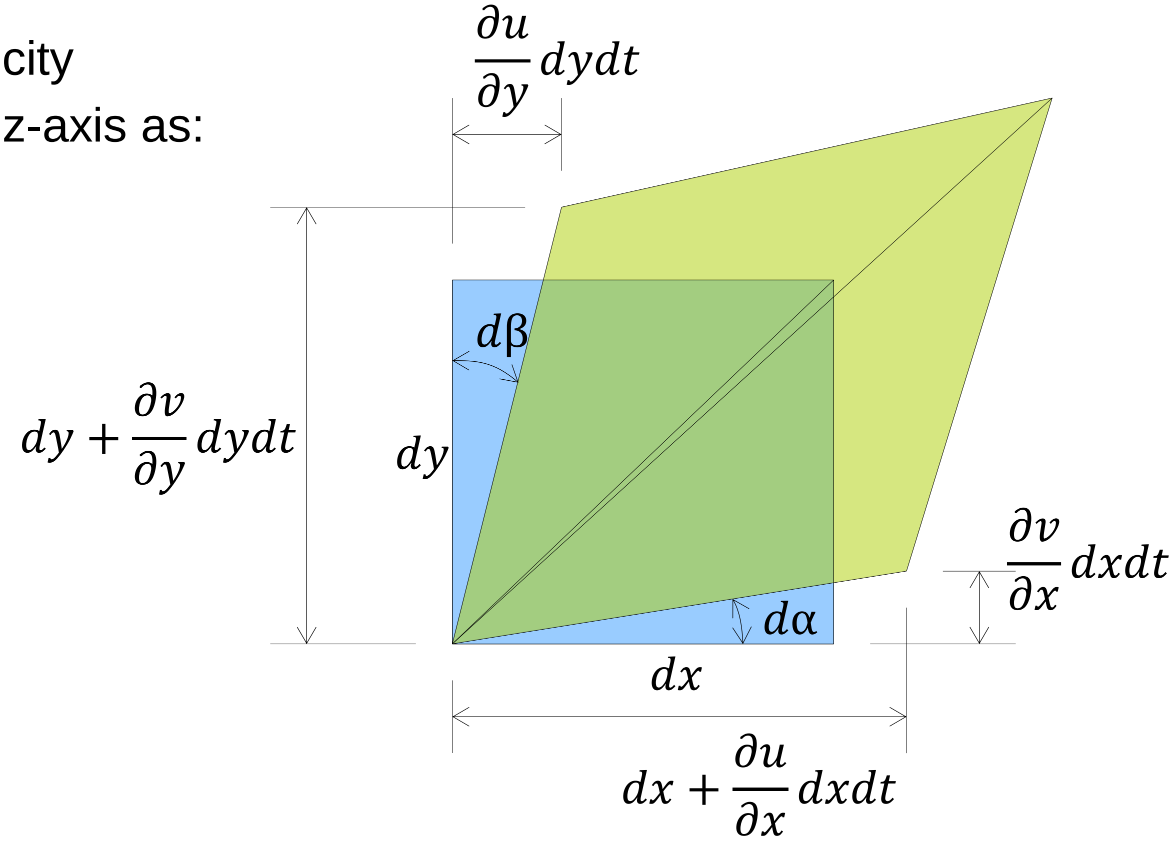
Deformation and rotation (2D)

- Define angular velocity (or **rotation**) about z-axis as:

$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right)$$

$$d\alpha = \lim_{dt\to 0}\left[\tan^{-1}\frac{\frac{\partial v}{\partial x}dxdt}{dx + \frac{\partial u}{\partial x}dxdt}\right]$$

$$d\beta = \lim_{dt\to 0}\left[\tan^{-1}\frac{\frac{\partial u}{\partial y}dydt}{dy + \frac{\partial v}{\partial y}dydt}\right]$$



Fluid Mechanics Laws

Deformation and rotation (2D)

- Define angular velocity
  (or **rotation**) about z-axis as:

$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\frac{\partial v}{\partial x}dt}{dt} - \frac{\frac{\partial u}{\partial y}dt}{dt}\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$d\alpha = \lim_{dt\to 0}\left[\tan^{-1}\frac{\frac{\partial v}{\partial x}dxdt}{dx + \frac{\partial u}{\partial x}dxdt}\right] = \frac{\partial v}{\partial x}dt \qquad d\beta = \lim_{dt\to 0}\left[\tan^{-1}\frac{\frac{\partial u}{\partial y}dydt}{dy + \frac{\partial v}{\partial y}dydt}\right] = \frac{\partial u}{\partial y}dt$$

Fluid Mechanics Laws

Rotation in 3D and vorticity

- Rotation in 3D

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

- Rotation equals the 'curl' of the velocity vector:

$$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{V}$$

- Vorticity of a fluid is defined as twice the rotation:

$$\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V}$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Fluid Mechanics Laws

Summarizing:

- Conservation of mass (continuity):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0$$

Incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \nabla \cdot \vec{V} = 0$$

- Conservation of momentum (inviscid flow):

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \vec{\nabla} p$$

- Rotation of a fluid element:

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

Vorticity:

$$\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V}$$

Fluid Mechanics Laws

Velocity Potential

- Assumptions
  - Homogeneous
  - Continuous
  - Incompressible
  - Non-viscous (inviscid)
- Extra assumption:
  - **Irrotational flow** $\rightarrow$ $\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V} = 0$

Fluid Mechanics Laws

Velocity Potential

- Theorem in vector calculus:

  In case the curl of a vector is zero, then the vector must be the gradient of a *scalar* function

$$\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V} = 0$$

- Thus:

$$\vec{V} = \nabla \Phi$$

- where Φ is a scalar function

- Φ is known as the *Velocity Potential*

Fluid Mechanics Laws

Velocity Potential

- The velocity potential is a function of time and position:

$$\Phi(x, y, z, t)$$

- The spatial derivatives of the velocity potential equal the velocity components at a time and position:

$$\frac{\partial \Phi}{\partial x} = u \qquad \frac{\partial \Phi}{\partial y} = v \qquad \frac{\partial \Phi}{\partial z} = w$$

- Potential lines are defined as:

$$\Phi(x, y, z, t) = constant$$

Fluid Mechanics Laws

Velocity Potential

- In 2D polar coordinates:

$$\Phi(r, \theta, t)$$

$$v_r = \frac{\partial \Phi}{\partial r}$$

$$v_\theta = \frac{1}{r} \cdot \frac{\partial \Phi}{\partial \theta}$$

Fluid Mechanics Laws

Velocity Potential

- Continuity equation for incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- Velocity components:

$$u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z}$$

- Continuity equation for potential flow:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

$\nabla^2 \Phi = 0$ Laplace equation

Fluid Mechanics Laws

Velocity Potential

- Irrotational flow (in 2D):

$$u = \frac{\partial \Phi}{\partial x} \quad \text{thus:} \quad \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\frac{\partial \Phi}{\partial x} = \frac{\partial^2 \Phi}{\partial y \partial x}$$

$$v = \frac{\partial \Phi}{\partial y} \quad \text{thus:} \quad \frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\frac{\partial \Phi}{\partial y} = \frac{\partial^2 \Phi}{\partial x \partial y}$$

$$\frac{\partial^2 \Phi}{\partial y \partial x} = \frac{\partial^2 \Phi}{\partial x \partial y} \quad \text{thus:} \quad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

Fluid Mechanics Laws

Velocity Potential

- Irrotational flow (in 3D):

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \quad \text{in the (x,y) plane}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \quad \text{in the (y,z) plane}$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \quad \text{in the (x,z) plane}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Fluid Mechanics Laws

Bernoulli Equation

- Recall the Euler Equations (sheet 29): $\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \vec{\nabla}p$

Fluid Mechanics Laws

Bernoulli Equation

- Recall the Euler Equations (sheet 29): $$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \vec{\nabla}p$$

- Note that:

$$\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$$

$$(\vec{V} \cdot \nabla)\vec{V} = \nabla\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V}$$

Fluid Mechanics Laws

Bernoulli Equation

- Recall the Euler Equations (sheet 29):

$$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \vec{\nabla}p$$

- Note that:

$$\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$$

$$(\vec{V} \cdot \nabla)\vec{V} = \nabla\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V}$$

- Then:

$$\rho\left(\frac{\partial\vec{V}}{\partial t} + \nabla\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V}\right) = \rho\vec{g} - \vec{\nabla}p$$

Fluid Mechanics Laws

Bernoulli Equation

- Recall the Euler Equations (sheet 29):

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \vec{\nabla} p$$

- Note that:

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$$

$$(\vec{V} \cdot \nabla)\vec{V} = \nabla\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V}$$

- Then:

$$\frac{\partial \vec{V}}{\partial t} + \nabla\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V} - \vec{g} + \vec{\nabla}\frac{p}{\rho} = 0$$

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Fluid Mechanics Laws

Bernoulli Equation

- Dot with small displacement along streamline *d**r*** = *(dx, dy, dz)*:

$$\left[\frac{\partial \vec{V}}{\partial t} + \nabla\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V} - \vec{g} + \nabla\frac{p}{\rho}\right] \cdot d\vec{r} = 0$$

- Then ‘work done’ by fluid along *d**r***:

$$\frac{\partial \vec{V}}{\partial t} \cdot d\vec{r} + d\left(\frac{1}{2}V^2\right) + \vec{\zeta} \times \vec{V} \cdot d\vec{r} - g dz + \frac{dp}{\rho} = 0$$

- This can be integrated along any two points along a streamline, however:

$\vec{\zeta} \times \vec{V} \cdot d\vec{r}$ is a difficult to evaluate term

Fluid Mechanics Laws

Bernoulli Equation

- Possibilities to deal with $\vec{\zeta} \times \vec{V} \cdot d\vec{r}$:

    - $\vec{V}$ is zero; no flow only hydrostatics
    - $\vec{\zeta}$ is zero; irrotational flow
    - $d\vec{r}$ is perpendicular to $\vec{\zeta} \times \vec{V}$; very rare solution
    - $d\vec{r}$ is parallel to $\vec{\zeta} \times \vec{V}$; integrate along *streamline*

Fluid Mechanics Laws

Bernoulli Equation

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  - $\vec{V}$ is zero; no flow only hydrostatics
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Fluid Mechanics Laws

Bernoulli Equation

- Possibilities to deal with $\vec{\zeta} \times \vec{V} \cdot d\vec{r}$:
  - $\vec{V}$ is zero; no flow only hydrostatics
  - $\vec{\zeta}$ **is zero; irrotational flow → POTENTIAL FLOW**
  - $d\vec{r}$ is perpendicular to $\vec{\zeta} \times \vec{V}$; very rare solution
  - $d\vec{r}$ **is parallel to** $\vec{\zeta} \times \vec{V}$ **; integrate along *streamline***

Fluid Mechanics Laws

Bernoulli Equation

- Potential flow: $\vec{V} = \nabla\Phi$

$$\frac{\partial \vec{V}}{\partial t} \cdot d\vec{r} + d\left(\frac{1}{2}V^2\right) - g dz + \frac{dp}{\rho} = 0$$

$$\frac{\partial \nabla\Phi}{\partial t} \cdot d\vec{r} + d\frac{1}{2}(\nabla\Phi)^2 - g dz + \frac{dp}{\rho} = 0$$

$$d\frac{\partial \Phi}{\partial t} + d\frac{1}{2}(\nabla\Phi)^2 - g dz + \frac{dp}{\rho} = 0$$

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Fluid Mechanics Laws

Bernoulli Equation

- Potential flow: $\vec{V} = \nabla\Phi$

$$d\frac{\partial\Phi}{\partial t} + d\frac{1}{2}(\nabla\Phi)^2 - gdz + \frac{dp}{\rho} = 0$$

- Finally simple integration yields (between any two points):

$$\int_1^2 d\frac{\partial\Phi}{\partial t} + \int_1^2 d\left(\frac{1}{2}V^2\right) - \int_1^2 gdz + \int_1^2 \frac{dp}{\rho} = 0$$

$$\frac{\partial\Phi}{\partial t}_2 - \frac{\partial\Phi}{\partial t}_1 + \frac{1}{2}(V_2^2 - V_1^2) - g(z_2 - z_1) + \frac{p_2 - p_1}{\rho} = 0$$

Fluid Mechanics Laws

Bernoulli Equation

- Potential flow: $\vec{V} = \nabla\Phi$

$$d\frac{\partial\Phi}{\partial t} + d\frac{1}{2}(\nabla\Phi)^2 - g dz + \frac{dp}{\rho} = 0$$

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$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 - gz + \frac{p}{\rho} = constant$$

Bernoulli equation

Fluid Mechanics Laws

Steady and Unsteady Flow

- Steady flow: at any point in flow the velocity is independent of time

$$\frac{d\vec{V}}{dt} = 0 \qquad \frac{\partial \Phi}{\partial t} + \frac{1}{2}\cancel{(\nabla\Phi)^2} - gz + \frac{p}{\rho} = constant$$

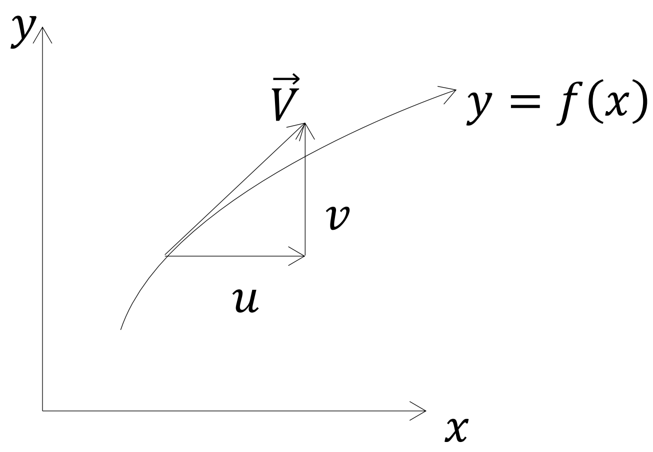
- Unsteady flow: any other flow
  - E.g. waves
  - Motions of floating objects in a flow
  - etc.

Potential Flow

Stream lines (2D)

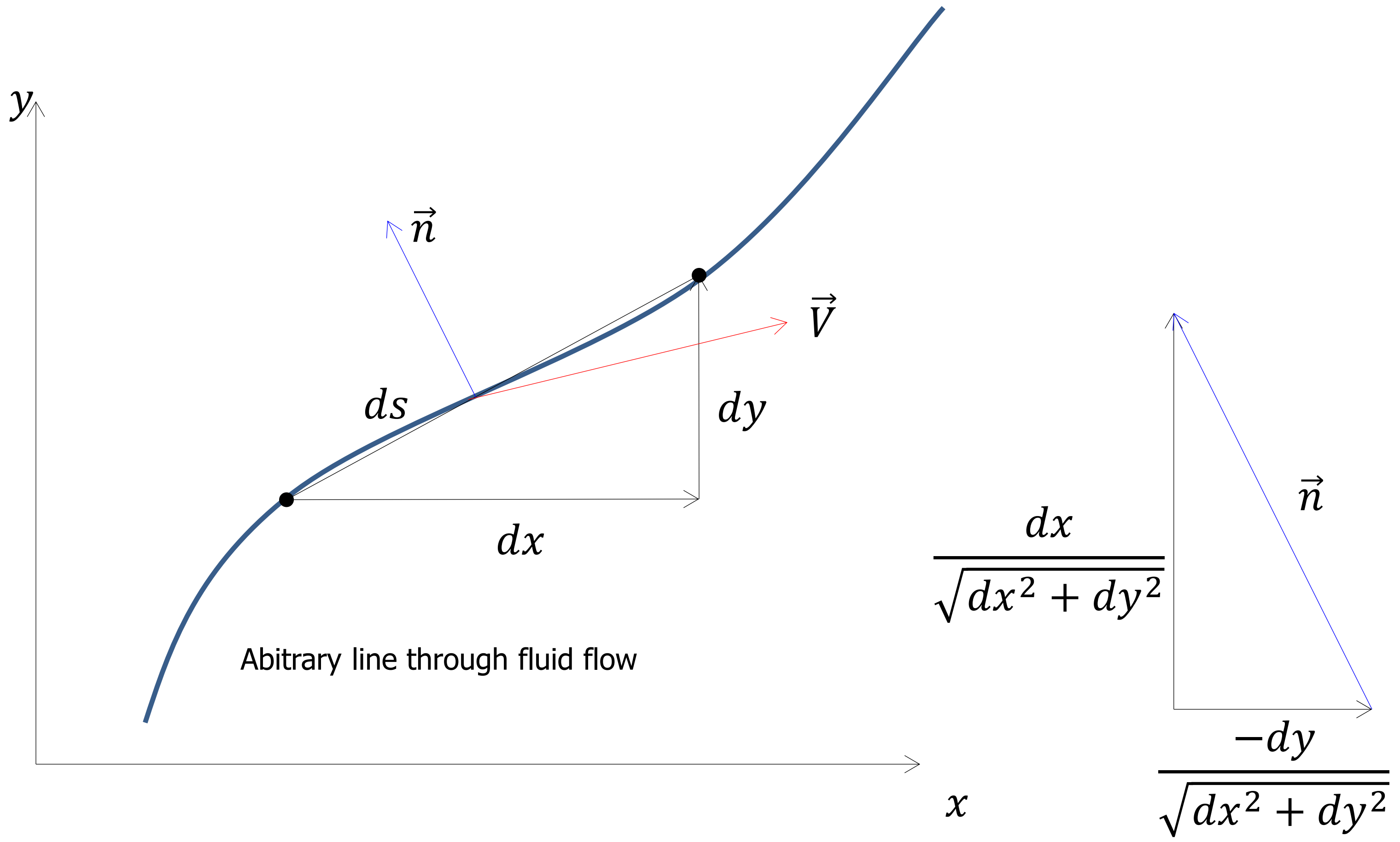
- Definition:
  - A line that follows the flow (as if you would have injected dye into the flow)

- Stream line: curve tangent to flow velocity vectors at a time instant:



Potential Flow

Stream function (2D)



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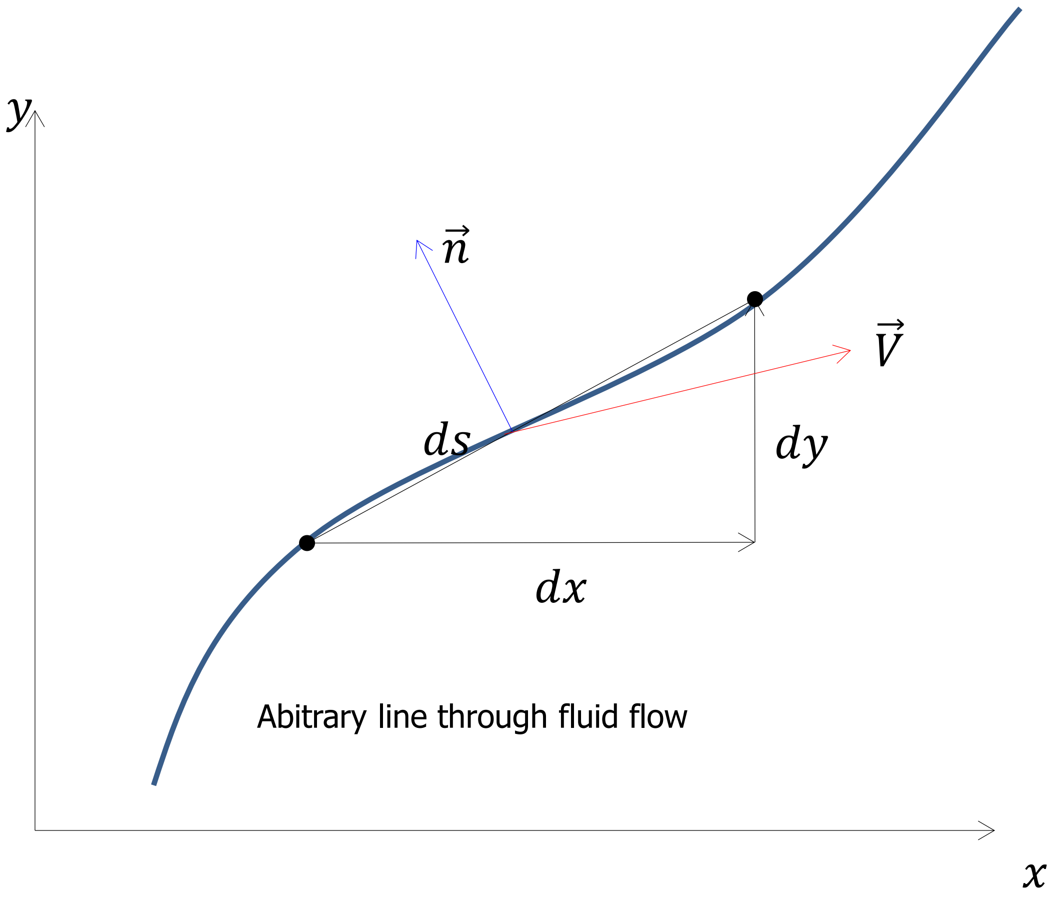
Potential Flow

Stream function (2D)



Rate of flow through ds:

$$d\Psi = -(\vec{V} \cdot \vec{n})ds$$

And:

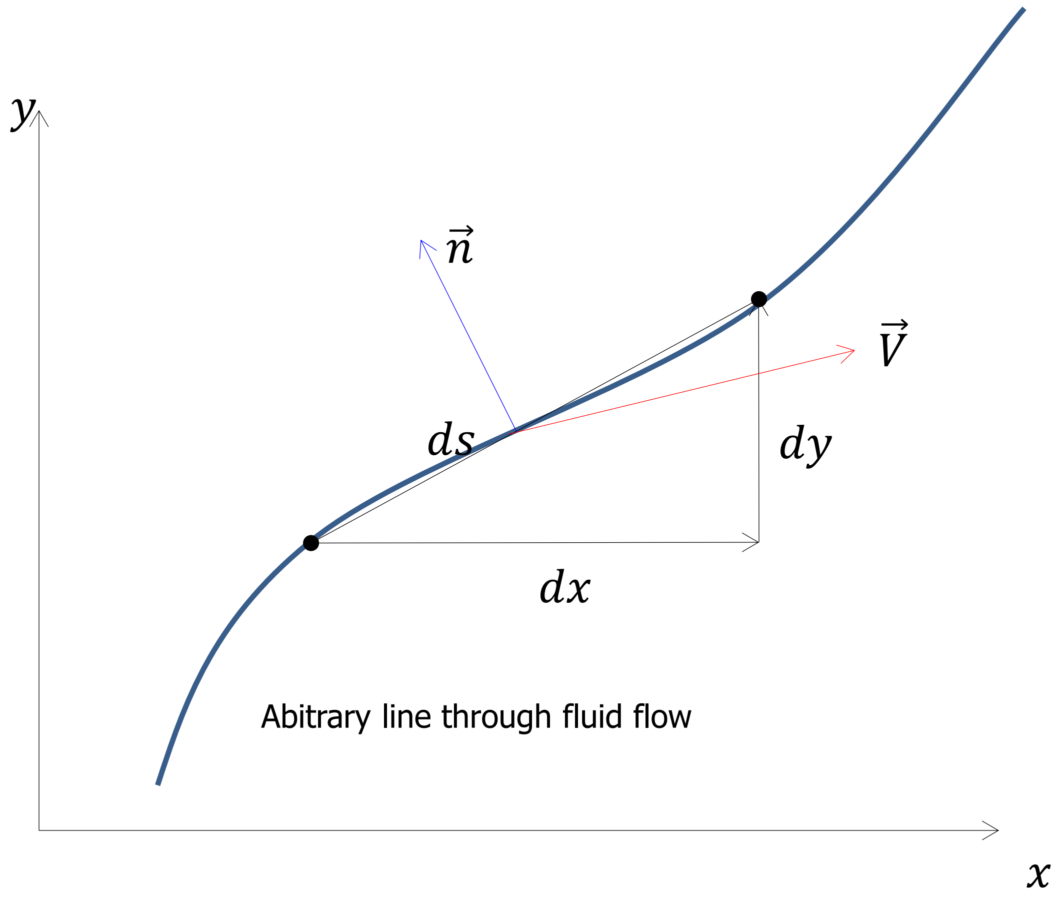
$$\vec{V} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\vec{n} = \frac{1}{\sqrt{dx^2 + dy^2}} \begin{bmatrix} -dy \\ dx \end{bmatrix}$$

$$ds = \sqrt{dx^2 + dy^2}$$

Potential Flow

Stream function (2D)



Rate of flow through ds:

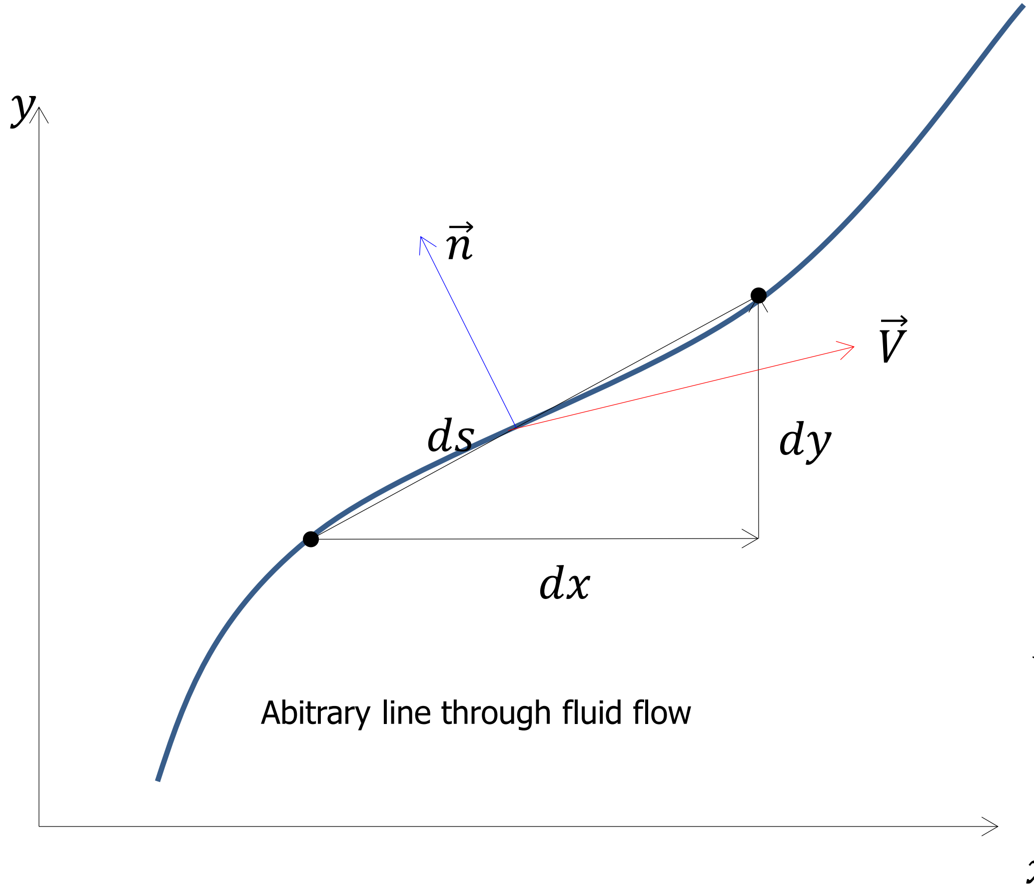
$$d\Psi = -\begin{bmatrix} u \\ v \end{bmatrix} \cdot \begin{bmatrix} -dy \\ dx \end{bmatrix}$$

$$d\Psi = udy - vdx$$

Potential Flow

Stream function (2D)

$$d\Psi = udy - vdx$$



Flow in y-direction:

$$d\Psi = -vdx$$

$$\frac{d\Psi}{dx} = -v$$

Flow in x-direction:

$$d\Psi = udy$$

$$\frac{d\Psi}{dy} = \mathrm{u}$$

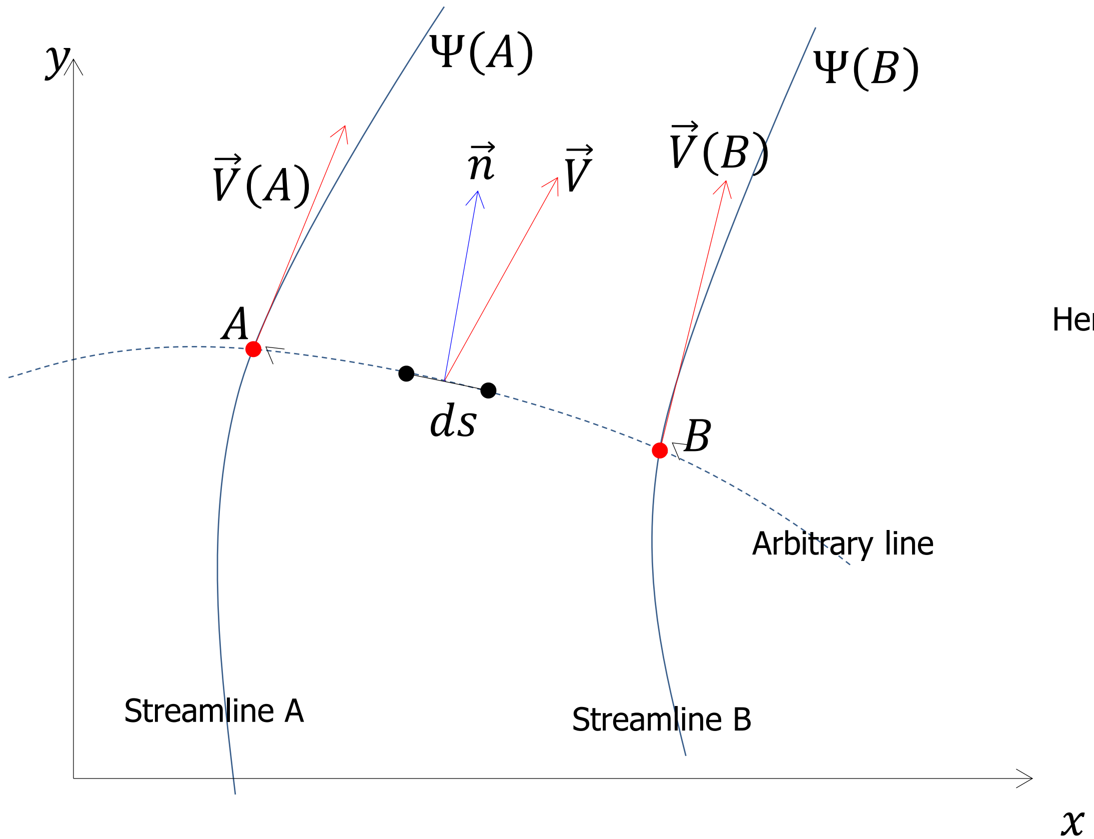
Ψ is the stream function

Its value is constant on streamline:

$$d\Psi = udy - vdx = 0$$

Potential Flow

Stream function (2D)



Definition stream funtion:

$$d\Psi = udy - vdx$$

$$\frac{d\Psi}{dx} = -v \qquad \frac{d\Psi}{dy} = u$$

Hence:

$$\Delta\Psi_{A\rightarrow B} = \int_A^B -(\vec{V}\cdot\vec{n})ds$$
$$= \int_A^B (udy - vdx)$$
$$= \int_A^B d\Psi$$
$$= \Psi(B) - \Psi(A)$$

Potential Flow

Stream funtion (2D)

- Ψ is the (2D) stream function, with:

$$\frac{d\Psi}{dy} = u \qquad \frac{d\Psi}{dx} = -v$$

- Difference of Ψ between neighboring stream lines: rate of flow between streamlines

Potential flow properties

Summary

- Orthogonality: $$\frac{d\Psi}{dy} = \frac{d\Phi}{dx} = u \qquad \frac{d\Psi}{dx} = -\frac{d\Phi}{dy} = -v$$
- Impervious boundaries equals streamline: $$\frac{d\Phi}{dn} = 0 \qquad \Psi = constant$$
- Conditions far away from disturbance: $$R \to \infty \Rightarrow \Phi \to \Phi_\infty \wedge \Psi \to \Psi_\infty$$
- Steady and unsteady flow: $$\frac{d\vec{V}}{dt} = 0, \qquad \frac{d\Phi}{dt} = 0$$
- Uniform flow (*s* coordinate along streamline): $$\frac{d\vec{V}}{ds} = 0$$

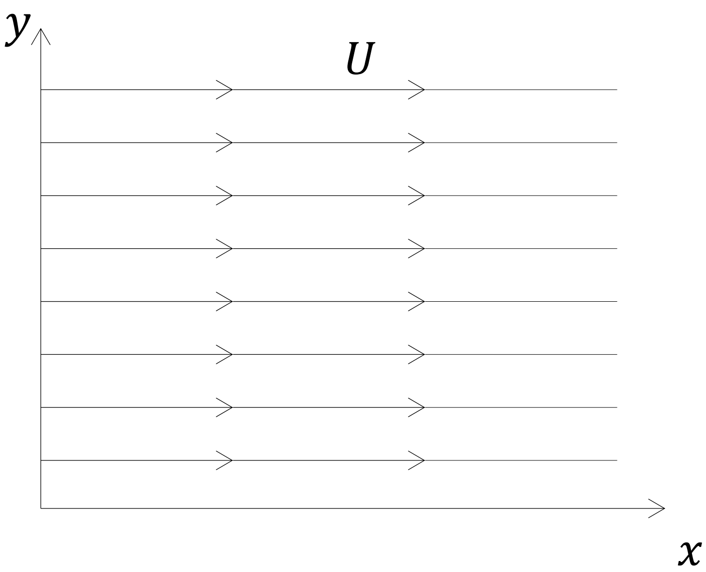
Potential flow elements

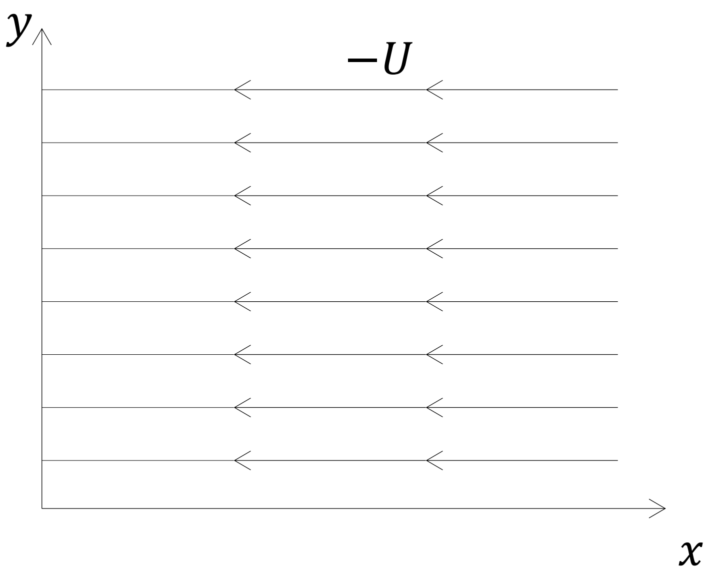
Introduction

- Using the previous we can define 'flow elements'
- Building blocks that respect the assumptions of potential flow:
  - Homogeneous
  - Continuous
  - Inviscid
  - Incompressible
  - Irrotational
- We can add these elements up to construct realistic flow patterns
- Modeling of submerged bodies by matching streamlines to body outline
- Using the velocity potential, stream function and Bernoulli equation to find velocities, pressures and eventually fluid forces on bodies

Potential flow elements

Uniform flow





$$\Phi = U \cdot x$$

$$\Psi = U \cdot y$$

$$u = \frac{d\Phi}{dx} = \frac{d\Psi}{dy} = U$$

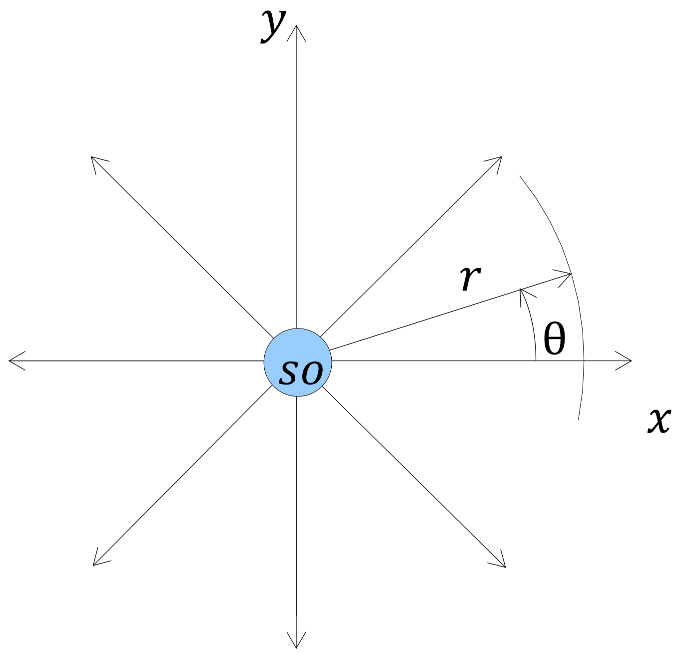
$$\Phi = -U \cdot x$$

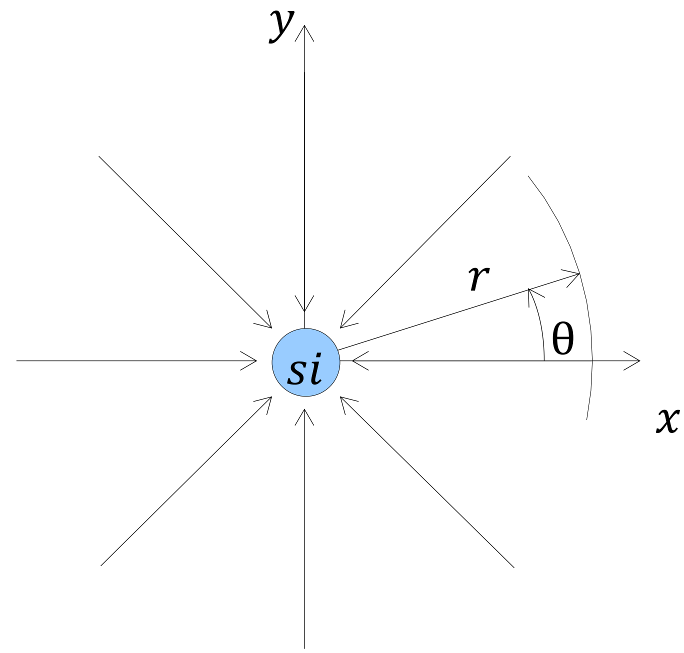
$$\Psi = -U \cdot y$$

$$u = \frac{d\Phi}{dx} = \frac{d\Psi}{dy} = -U$$

Potential flow elements

Source and sink flow





$$\Phi = +\frac{Q}{2\pi} \cdot \ln r = +\frac{Q}{2\pi} \cdot \ln\sqrt{x^2 + y^2}$$

$$\Psi = +\frac{Q}{2\pi} \cdot \theta = +\frac{Q}{2\pi} \cdot \arctan\frac{y}{x}$$

$$\Phi = -\frac{Q}{2\pi} \cdot \ln r = -\frac{Q}{2\pi} \cdot \ln\sqrt{x^2 + y^2}$$

$$\Psi = -\frac{Q}{2\pi} \cdot \theta = -\frac{Q}{2\pi} \cdot \arctan\frac{y}{x}$$

Potential flow elements

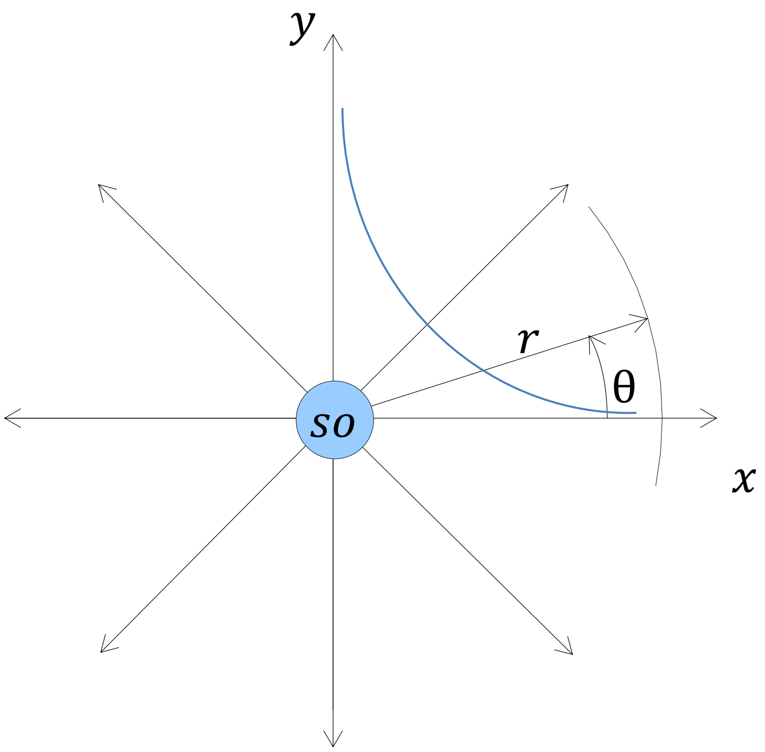
Source and sink flow

$$\Phi = +\frac{Q}{2\pi} \cdot \ln r$$

$$\Psi = +\frac{Q}{2\pi} \cdot \theta$$

$$v_r = \frac{\partial \Phi}{\partial r} = \frac{1}{r} \cdot \frac{\partial \Psi}{\partial \theta} = \frac{Q}{2\pi r}$$

$$v_\theta = \frac{1}{r} \cdot \frac{\partial \Phi}{\partial \theta} = -\frac{\partial \Psi}{\partial r} = 0$$



Potential flow elements

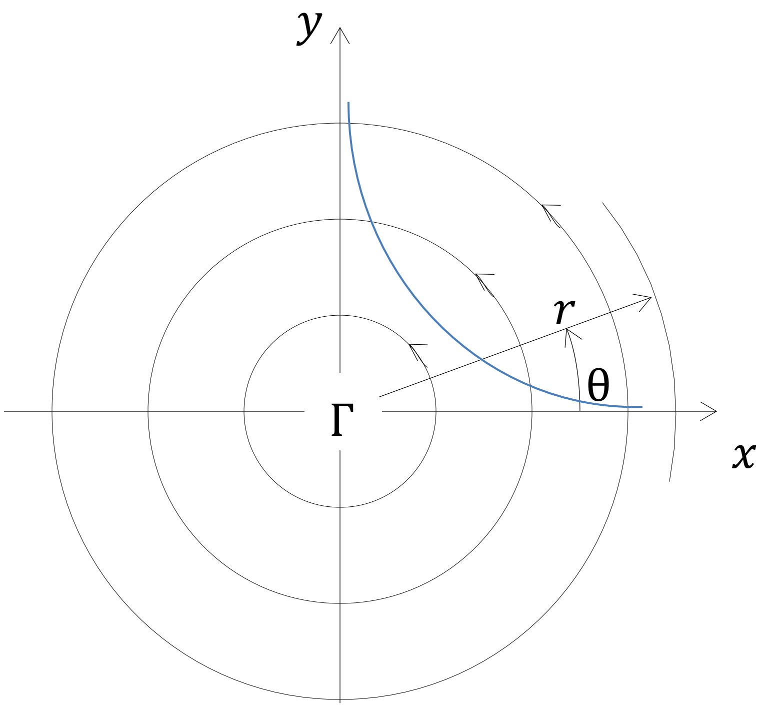
Circulation or vortex elements

$$\Phi = +\frac{\Gamma}{2\pi} \cdot \theta$$

$$\Psi = +\frac{\Gamma}{2\pi} \cdot \ln r$$

$$v_r = \frac{\partial \Phi}{\partial r} = \frac{1}{r} \cdot \frac{\partial \Psi}{\partial \theta} = 0$$

$$v_\theta = \frac{1}{r} \cdot \frac{\partial \Phi}{\partial \theta} = -\frac{\partial \Psi}{\partial r} = \frac{\Gamma}{2\pi r}$$



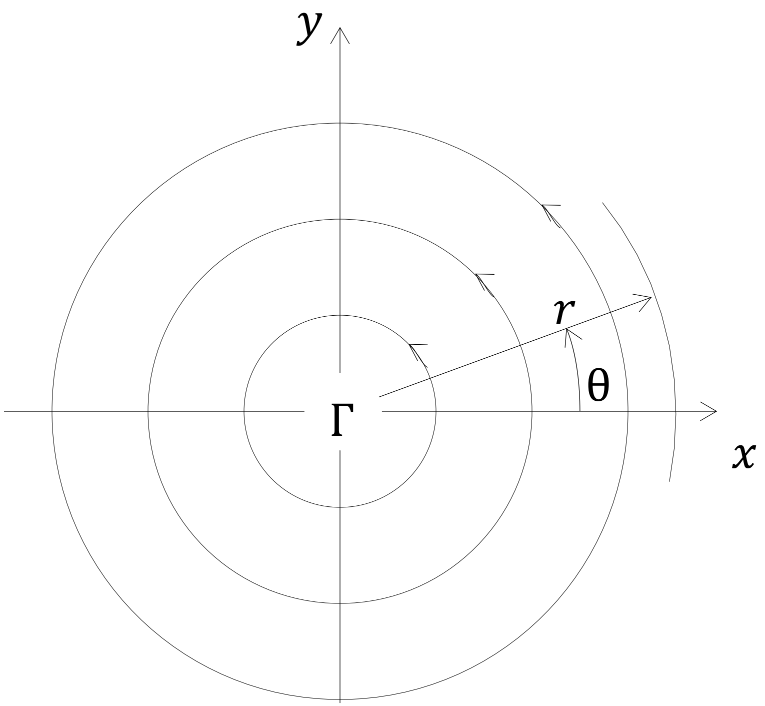
Potential flow elements

Circulation or vortex elements

$$\Phi = +\frac{\Gamma}{2\pi}\cdot\theta \qquad \Psi = +\frac{\Gamma}{2\pi}\cdot\ln r$$

$$v_r = \frac{\partial\Phi}{\partial r} = \frac{1}{r}\cdot\frac{\partial\Psi}{\partial\theta} = 0$$

$$v_\theta = \frac{1}{r}\cdot\frac{\partial\Phi}{\partial\theta} = -\frac{\partial\Psi}{\partial r} = \frac{\Gamma}{2\pi r}$$



Circulation strenght constant:

$$\Gamma = \oint v_\theta\cdot ds = 2\pi r\cdot v_\theta = constant$$

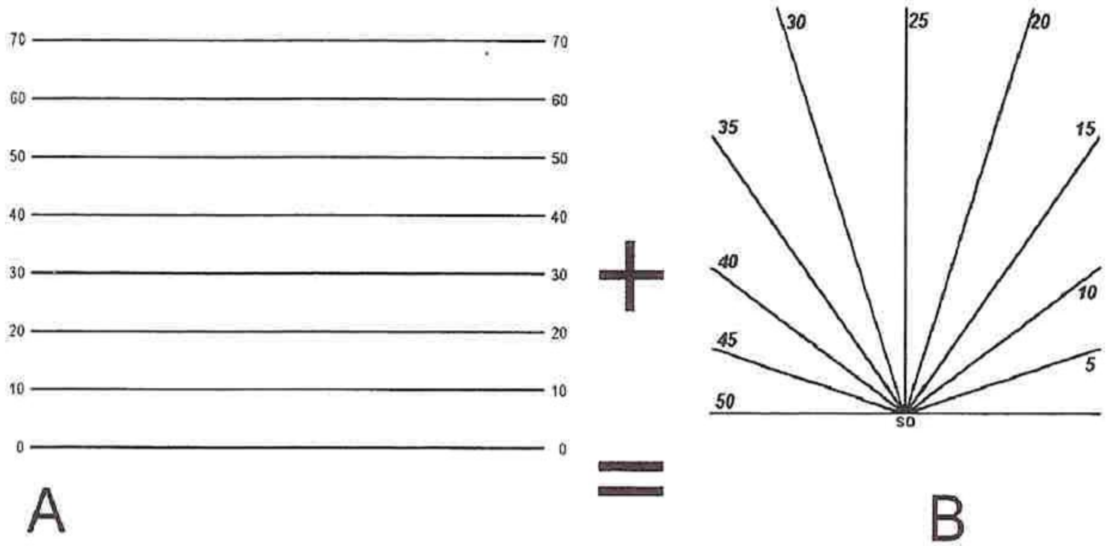
Therefore: no rotation, origin singular point: velocity infinite

Superposition of potential flow elements

Methodology (source in positive uniform flow)

- The resulting velocity fields, potential fields or stream function fields may be simply superposed to find the combined flow patterns

(Using stream function values)



Uniform flow field

Source flow field

Superposition of potential flow elements

Methodology (source in positive uniform flow)

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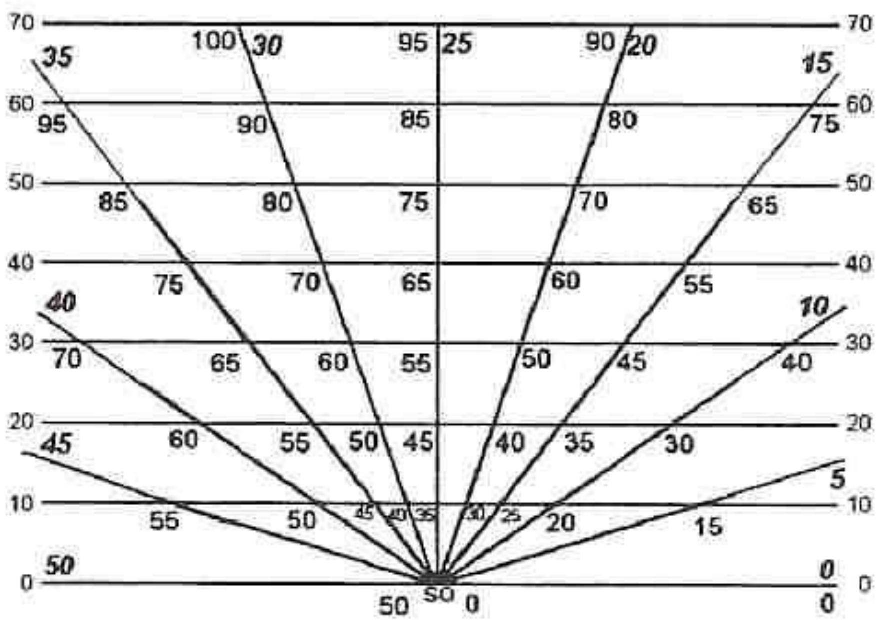
(Using stream function values)



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Superposition of potential flow elements

Methodology (source in positive uniform flow)

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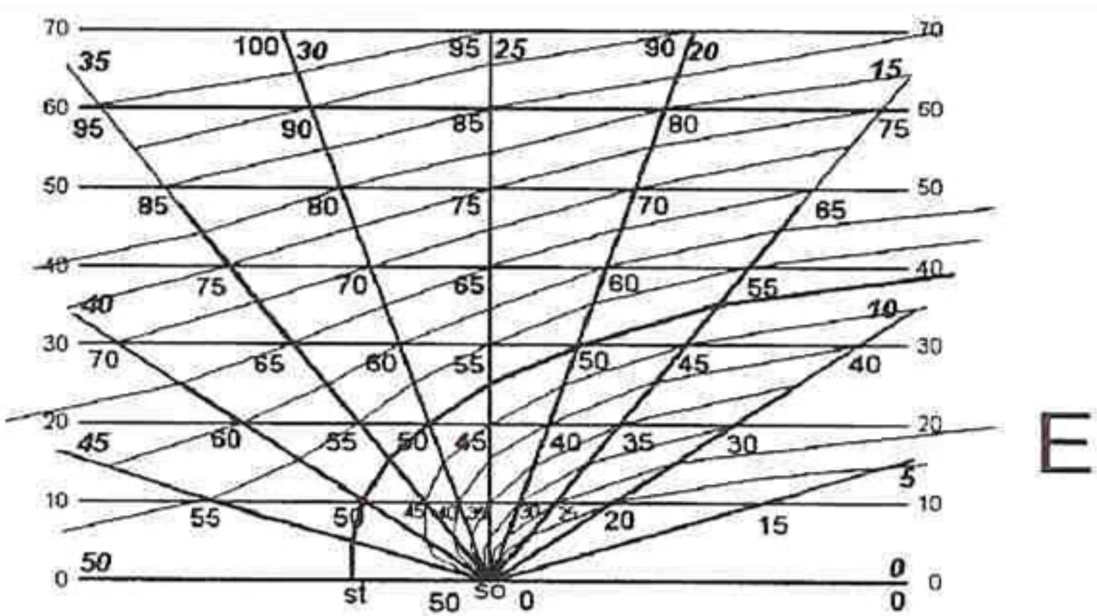
(Using stream function values)



Superposition of potential flow elements

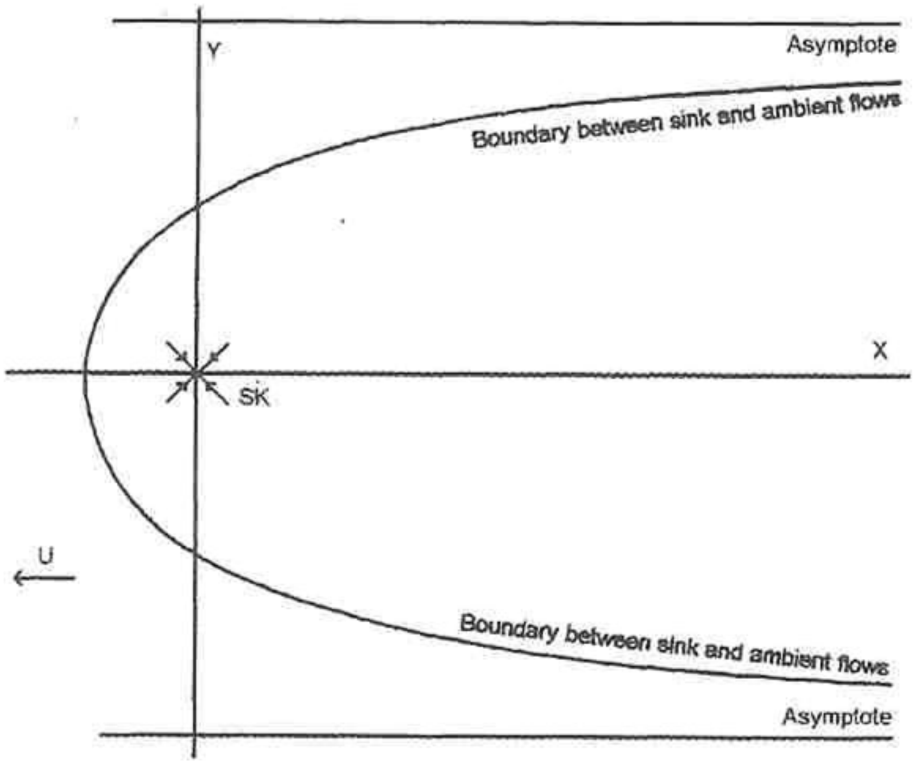
Sink in negative uniform flow

- Besides graphically this works also with formulas:

$$\Psi = -\frac{Q}{2\pi} \cdot \arctan\frac{y}{x} - U_\infty \cdot y$$

$$\Phi = -\frac{Q}{2\pi} \cdot \ln\sqrt{x^2 + y^2} - U_\infty \cdot x$$

For instance:
Find location stagnation point (Blackboard...)



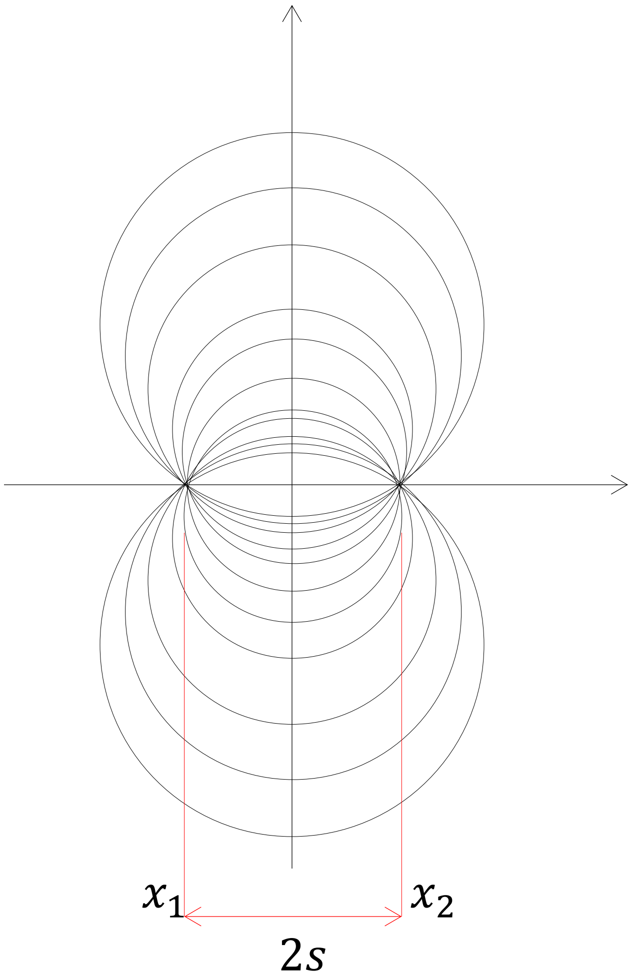
Superposition of potential flow elements

Separated source and sink

$$\Psi_{source} = +\frac{Q}{2\pi} \cdot \theta_1 = +\frac{Q}{2\pi} \cdot \arctan\frac{y}{x_1}$$

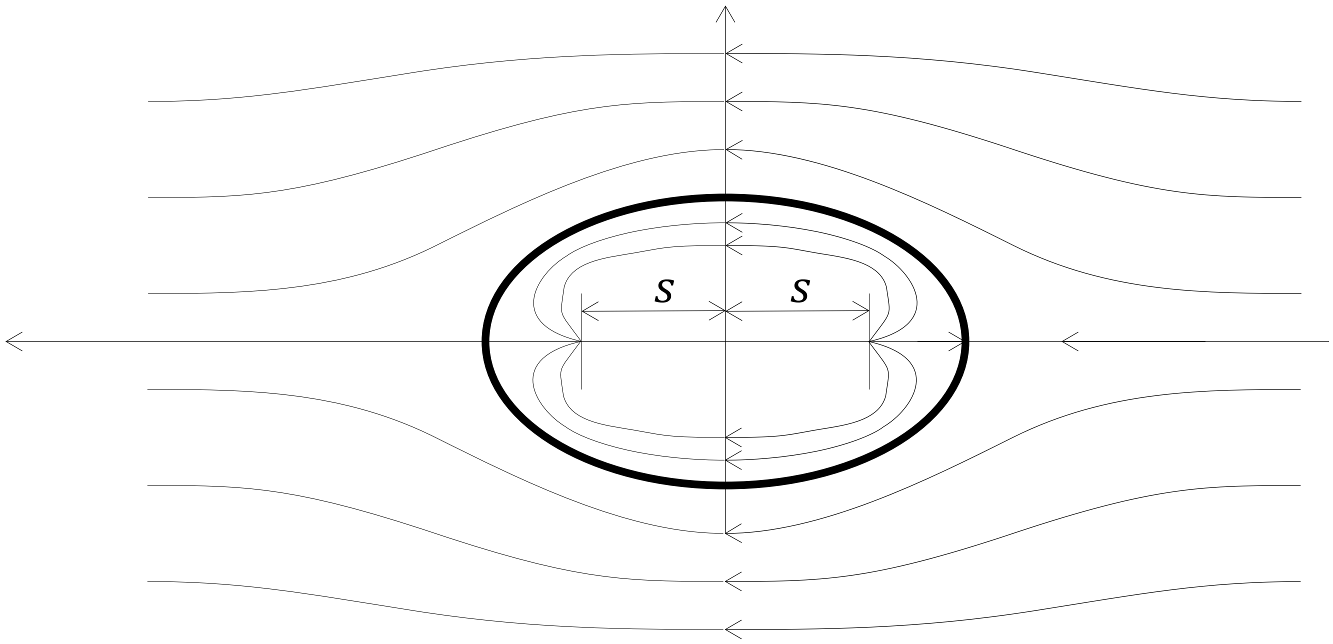
$$\Psi_{sink} = -\frac{Q}{2\pi} \cdot \theta_2 = -\frac{Q}{2\pi} \cdot \arctan\frac{y}{x_2}$$

$$\Psi = \frac{Q}{2\pi} \cdot \arctan\frac{2ys}{x^2 + y^2 - s^2}$$



Superposition of potential flow elements

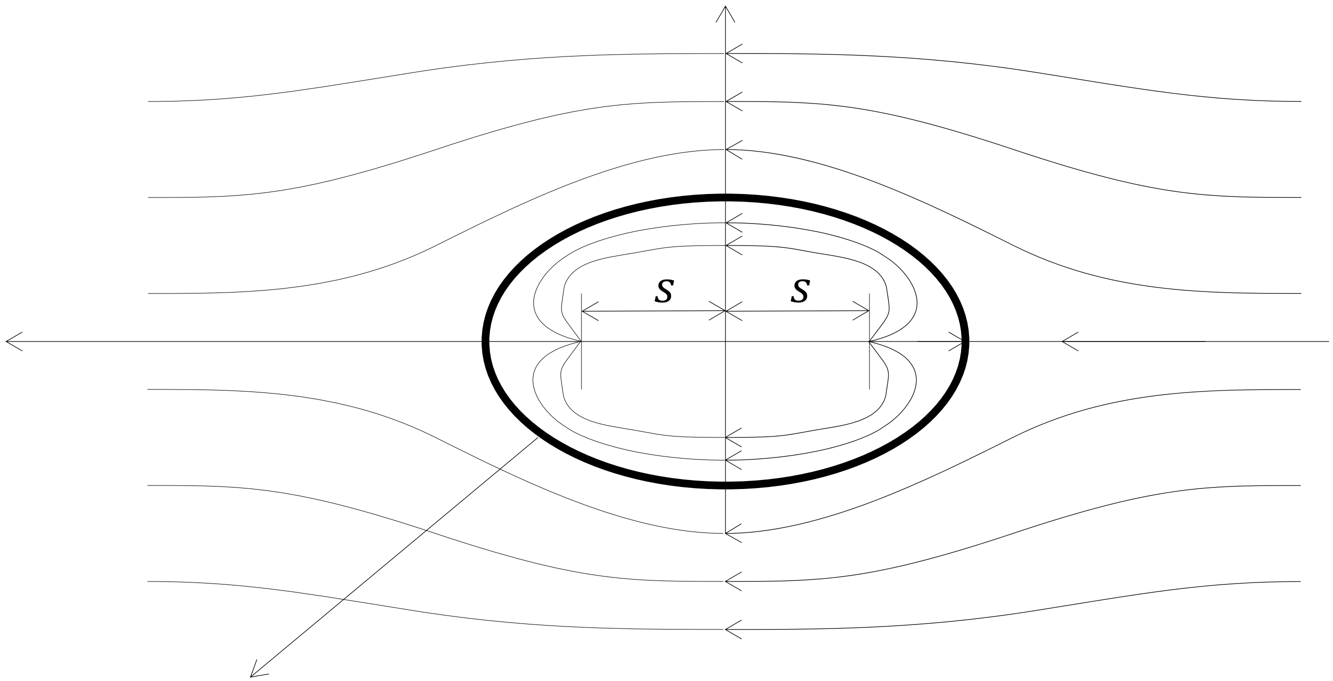
Separated source and sink in uniform flow



$$\Psi = \frac{Q}{2\pi} \cdot \arctan \frac{2ys}{x^2 + y^2 - s^2} + U_\infty y$$

Superposition of potential flow elements

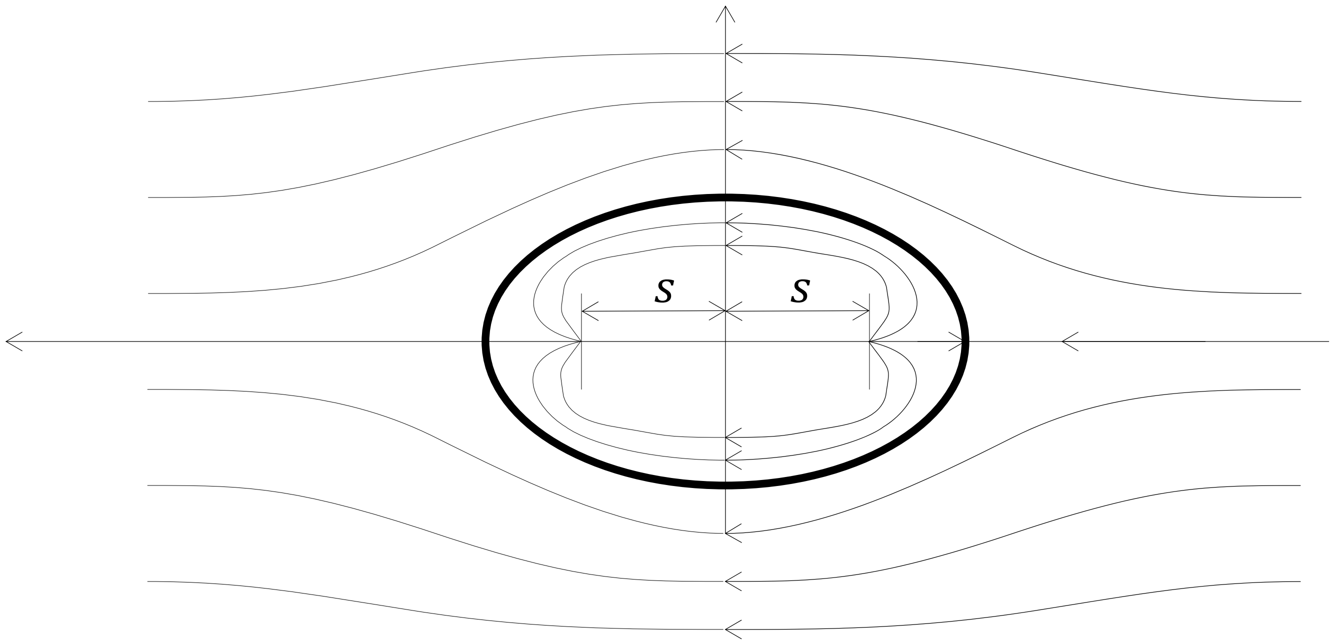
Separated source and sink in uniform flow



Streamline resembles fixed boundary (Rankine oval)
The flow outside this streamline resembles flow around solid boundary with this shape
Shape can be changed by using more source-sinks along x-axis with different strenghts

Superposition of potential flow elements

Separated source and sink in uniform flow



This approach can be extended to form ship forms in 2D or 3D:

Rankine ship forms

Useful for simple flow computations

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Superposition of potential flow elements

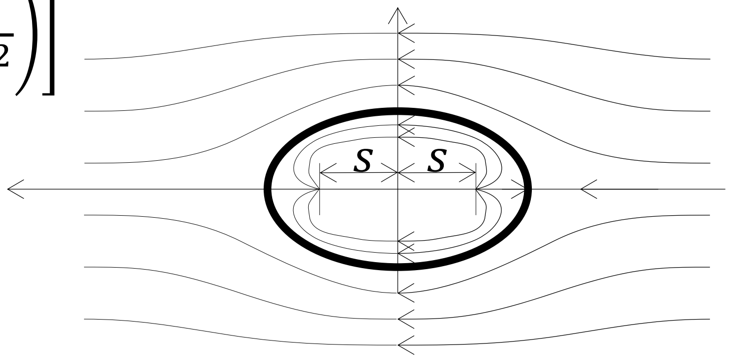
Doublet or dipole

When distance 2s becomes zero a new basic flow element is produced:

Doublet or dipole producing flow in positive x-direction

$$\Psi = \lim_{s\to 0}\left[\frac{Q}{2\pi}\cdot \arctan\left(\frac{2ys}{x^2+y^2-s^2}\right)\right]$$

$$\Psi = \lim_{s\to 0}\left[\frac{Q}{\pi}s\cdot\left(\frac{y}{x^2+y^2-s^2}\right)\right]$$



Note: in book errors w.r.t. to doublet and its orientation!

Superposition of potential flow elements

Doublet or dipole

When distance 2s becomes zero a new basic flow element is produced:

Doublet or dipole producing flow in positive x-direction

$$\Psi = \lim_{s \to 0} \left[\frac{Q}{2\pi} \cdot \arctan\left(\frac{2ys}{x^2 + y^2 - s^2} \right) \right]$$

$$\Psi = \lim_{s \to 0} \left[\frac{Q}{\pi} s \cdot \left(\frac{y}{x^2 + y^2 - s^2} \right) \right]$$

Set constant: $\mu = \frac{Q}{\pi} s$

Superposition of potential flow elements

Doublet or dipole

When distance 2s becomes zero a new basic flow element is produced:

Doublet or dipole producing flow in positive x-direction

$$\Psi = \lim_{s \to 0} \left[\frac{Q}{2\pi} \cdot \arctan\left(\frac{2ys}{x^2 + y^2 - s^2} \right) \right]$$

$$\Psi = \lim_{s \to 0} \left[\frac{Q}{\pi} s \cdot \left(\frac{y}{x^2 + y^2 - s^2} \right) \right]$$

Disappears when: $s \to 0$

Set constant: $\mu = \frac{Q}{\pi} s$

Superposition of potential flow elements

Doublet or dipole

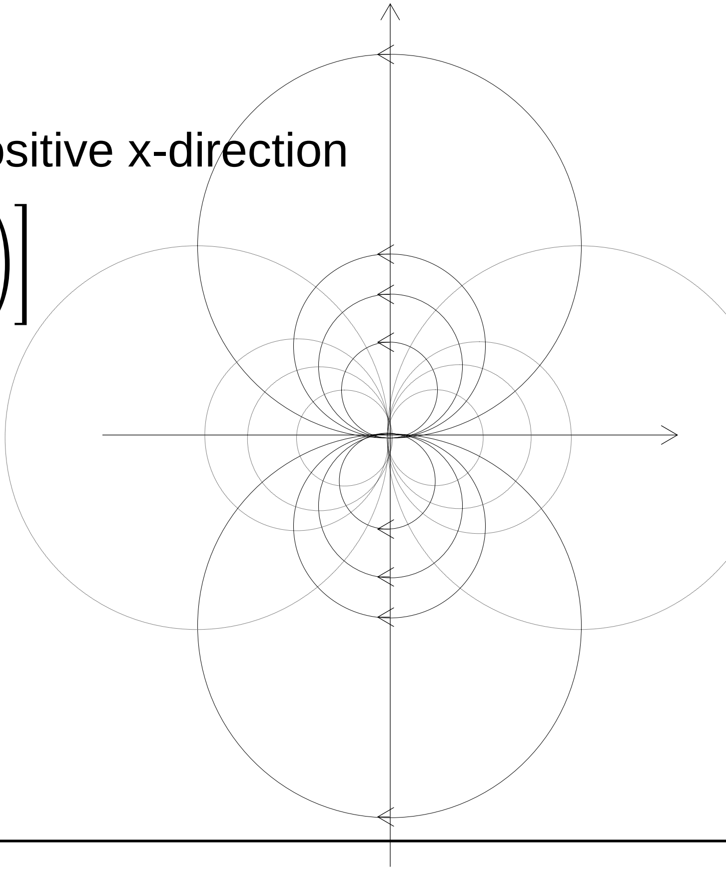
When distance 2s becomes zero a new basic flow element is produced:

Doublet or dipole producing flow in positive x-direction

$$\Psi = \lim_{s \to 0} \left[\frac{Q}{2\pi} \cdot \arctan\left(\frac{2ys}{x^2 + y^2 - s^2} \right) \right]$$

$$\Psi = \mu \cdot \frac{y}{x^2 + y^2} = \mu \cdot \frac{\sin\theta}{r}$$

$$\Phi = -\mu \cdot \frac{x}{x^2 + y^2} = -\mu \cdot \frac{\cos\theta}{r}$$



Superposition of potential flow elements

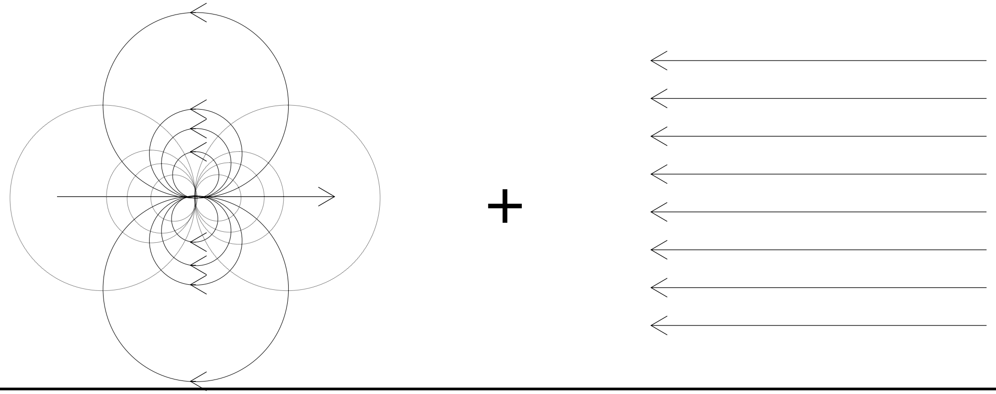
Doublet in a uniform flow

$$\Phi = -\mu \cdot \frac{x}{x^2 + y^2} - U_\infty x \qquad \Psi = \mu \cdot \frac{y}{x^2 + y^2} - U_\infty y$$

$$\Phi = -\mu \cdot \frac{\cos\theta}{r} - U_\infty r\cos\theta \qquad \Psi = \mu \cdot \frac{\sin\theta}{r} - U_\infty r\sin\theta$$

Wrong in book!

Doublet pointing in positive x-direction, uniform flow in negative x-direction:



Superposition of potential flow elements

Doublet in a uniform flow

$$\Phi = -\mu \cdot \frac{x}{x^2 + y^2} - U_\infty x \qquad \Psi = \mu \cdot \frac{y}{x^2 + y^2} - U_\infty y$$

$$\Phi = -\mu \cdot \frac{\cos\theta}{r} - U_\infty r\cos\theta \qquad \Psi = \mu \cdot \frac{\sin\theta}{r} - U_\infty r\sin\theta$$

Set $\Psi = 0$ then:

$$\Psi = y\left[\frac{\mu}{x^2 + y^2} - U_\infty\right] = 0$$

True when:

$$y = 0$$

$$\frac{\mu}{x^2 + y^2} - U_\infty = 0 \rightarrow x^2 + y^2 = \frac{\mu}{U_\infty}$$

Superposition of potential flow elements

Doublet in a uniform flow: flow around a circle

- The radius of the circle:

$$R = \sqrt{\frac{\mu}{U_\infty}}$$

- Doublet strength needed for radius R:

$$\mu = U_\infty R^2$$

$$\Phi = -\mu \cdot \frac{\cos\theta}{r} - U_\infty r\cos\theta$$

$$\Psi = \mu \cdot \frac{\sin\theta}{r} - U_\infty r\sin\theta$$

- This yields the following:

$$\Phi = -\frac{U_\infty R^2 \cos\theta}{r} - U_\infty r\cos\theta = RU_\infty \left[\frac{R}{r} - \frac{r}{R}\right]\cos\theta$$

$$\Psi = \frac{U_\infty R^2 \sin\theta}{r} - U_\infty r\sin\theta = RU_\infty \left[\frac{R}{r} - \frac{r}{R}\right]\sin\theta$$

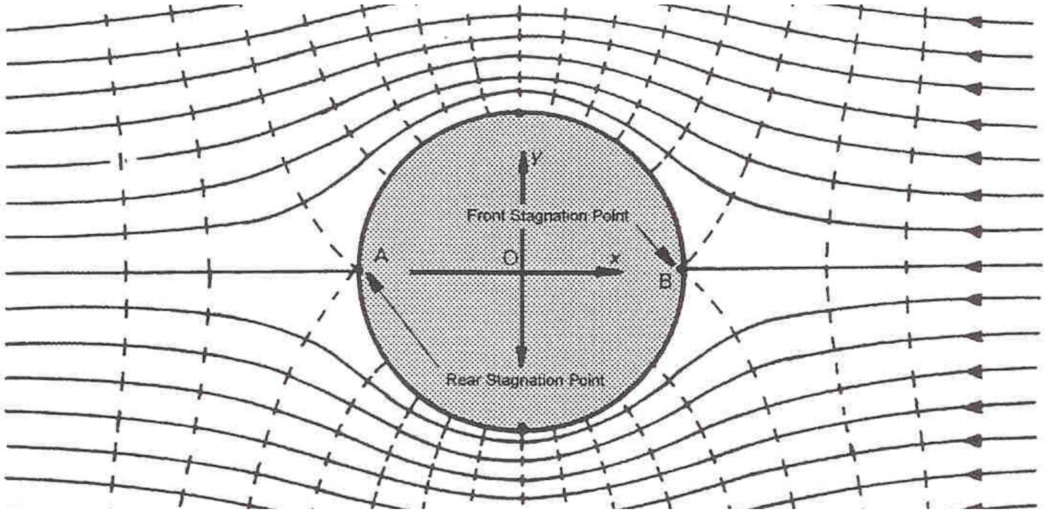
Superposition of potential flow elements

Doublet in a uniform flow: flow around a circle

$$\Phi = -RU_\infty\left[\frac{R}{r} - \frac{r}{R}\right]\cos\theta$$

$$\Phi = -U_\infty R^2 \cdot \frac{x}{x^2+y^2} - U_\infty x$$

$$u = \frac{d\Phi}{dx} = U_\infty R^2 \frac{x^2-y^2}{\left(x^2+y^2\right)^2} - U_\infty$$



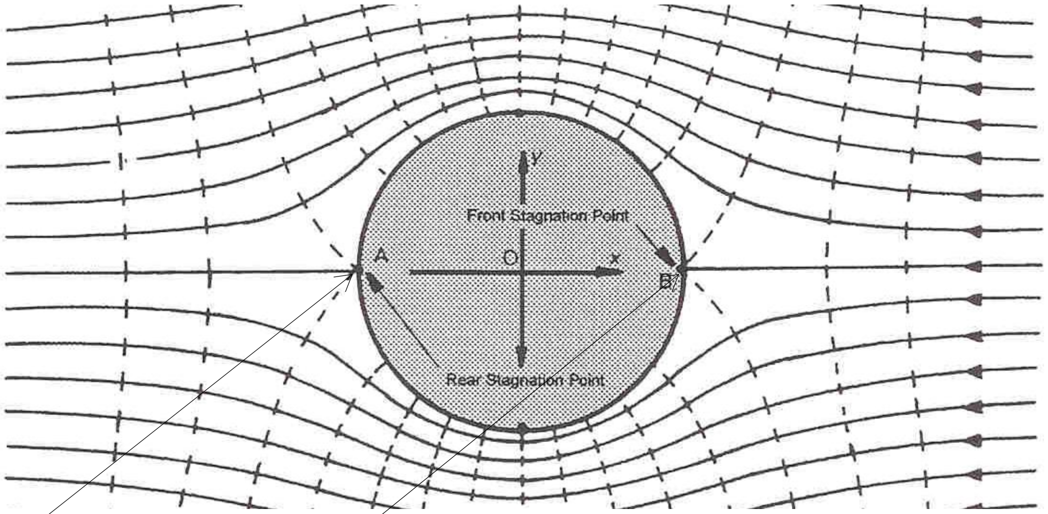
Superposition of potential flow elements

Doublet in a uniform flow: flow around a circle

$$\Phi = -RU_\infty \left[\frac{R}{r} - \frac{r}{R}\right] \cos\theta$$

$$\Phi = -U_\infty R^2 \cdot \frac{x}{x^2 + y^2} - U_\infty x$$

$$u = \frac{d\Phi}{dx} = U_\infty R^2 \frac{x^2 - y^2}{(x^2 + y^2)^2} - U_\infty$$



$$x = \pm R, \qquad y = 0$$

$$u = \frac{d\Phi}{dx} = U_\infty R^2 \frac{R^2 - 0^2}{(R^2 + 0^2)^2} - U_\infty = U_\infty R^2 \frac{R^2}{R^4} - U_\infty = 0$$

Stagnation points!

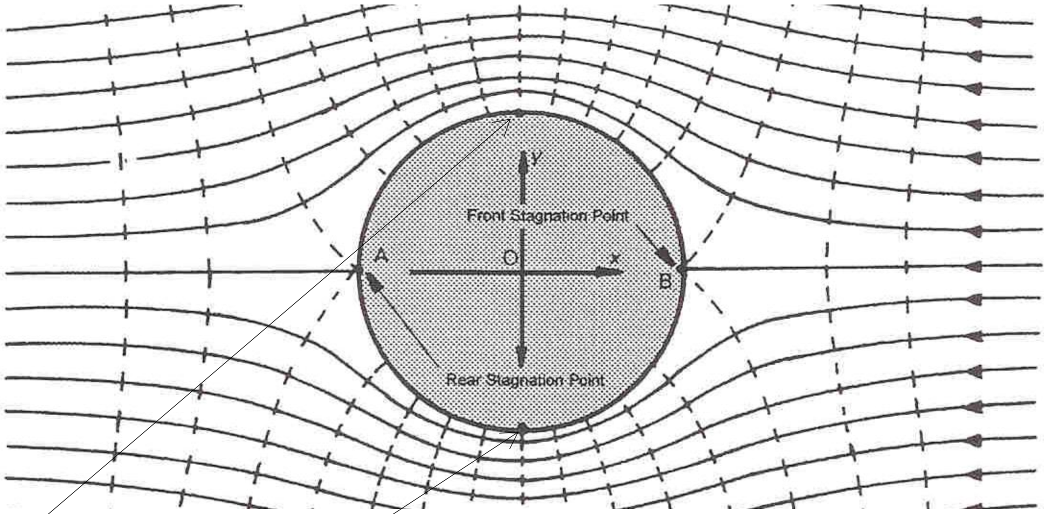
Superposition of potential flow elements

Doublet in a uniform flow: flow around a circle

$$\Phi = -RU_\infty \left[\frac{R}{r} - \frac{r}{R}\right] \cos\theta$$

$$\Phi = -U_\infty R^2 \cdot \frac{x}{x^2 + y^2} - U_\infty x$$

$$u = \frac{d\Phi}{dx} = U_\infty R^2 \frac{x^2 - y^2}{(x^2 + y^2)^2} - U_\infty$$



$$x = 0, \qquad y = \pm R$$

$$u = \frac{d\Phi}{dx} = U_\infty R^2 \frac{0^2 - R^2}{(0^2 + R^2)^2} - U_\infty = -U_\infty R^2 \frac{R^2}{R^4} - U_\infty = -2U_\infty$$

Superposition of potential flow elements

Evaluate velocities on cylinder wall

$$\Psi = \mu \cdot \frac{\sin\theta}{r} - U_\infty r \sin\theta$$

- Generally, velocity on cylinder wall:

$$v_\theta(r = R) = -\left[\frac{\partial \Psi}{\partial r}\right]_{r=R} = -\frac{\partial}{\partial r}\left[\frac{U_\infty R^2 \sin\theta}{r} - U_\infty r \sin\theta\right] = \ldots = -2U_\infty \sin\theta$$

Superposition of potential flow elements

Evaluate pressures on cylinder wall

- Use the Bernoulli equation:

$$\frac{1}{2}\rho U_\infty^2 + 0 = p + \frac{1}{2}\rho v_\theta^2$$

$$v_\theta = -2U_\infty \sin\theta$$

Pressure at stagnation points:

$$v = 0$$

Pressure at cylinder boundary:

$$v_r = 0$$

Assuming constant elevation

- Result:

$$p = \frac{1}{2}\rho U_\infty^2 [1 - 4\sin^2\theta]$$

Superposition of potential flow elements

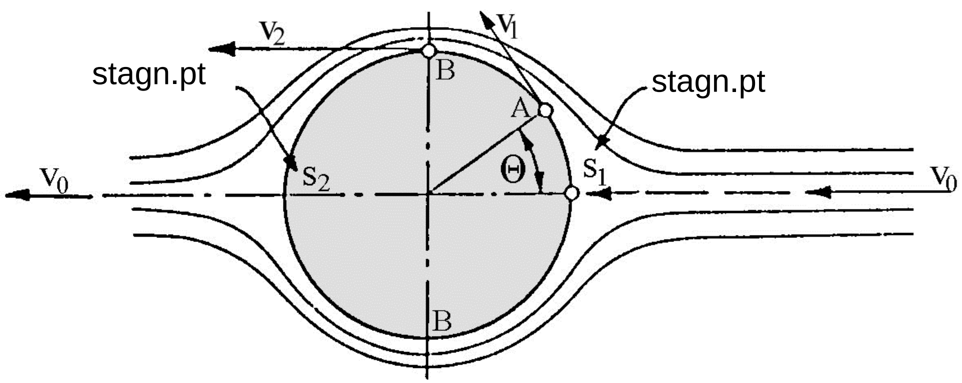
Evaluate pressures on cylinder wall

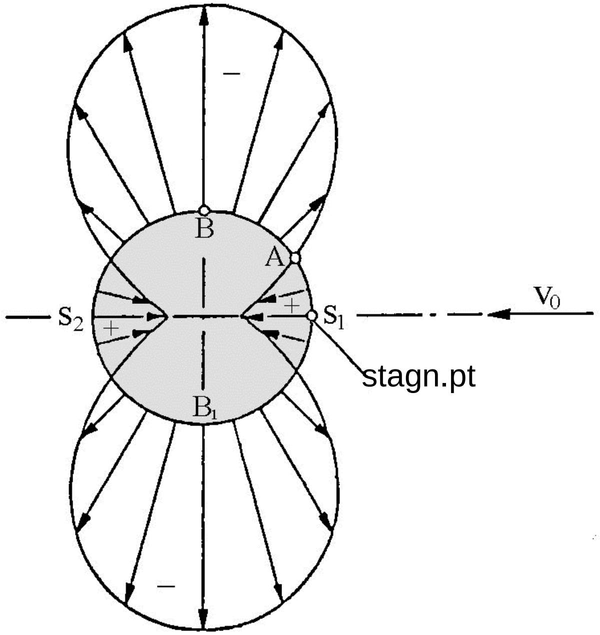
- Velocity profile:

$$v_\theta = -2U_\infty \sin\theta$$

- Pressure profile:

$$p = \frac{1}{2}\rho U_\infty^2[1 - 4\sin^2\theta]$$



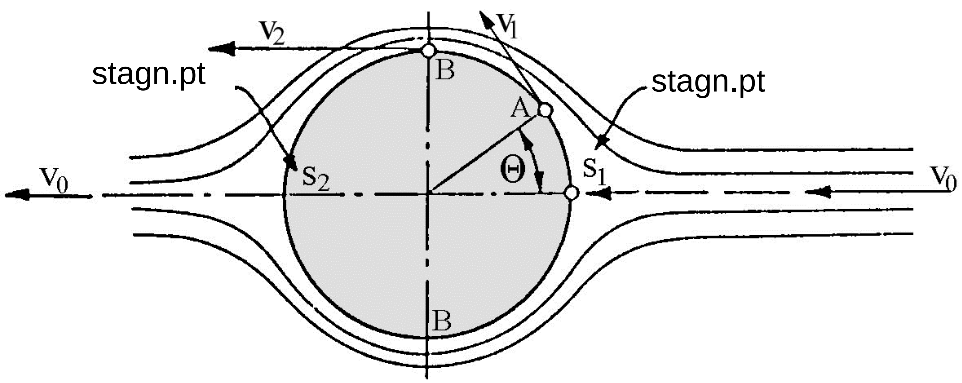


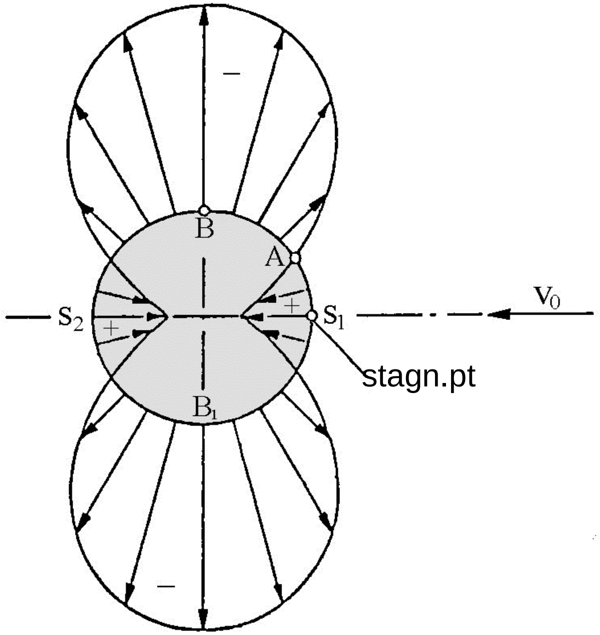
Superposition of potential flow elements

Evaluate pressures on cylinder wall

- Velocity profile: $$v_\theta = -2U_\infty \sin\theta$$
- Pressure profile: $$p = \frac{1}{2}\rho U_\infty^2[1 - 4\sin^2\theta]$$

•No net resulting force!!!
•D'Alembert's Paradox

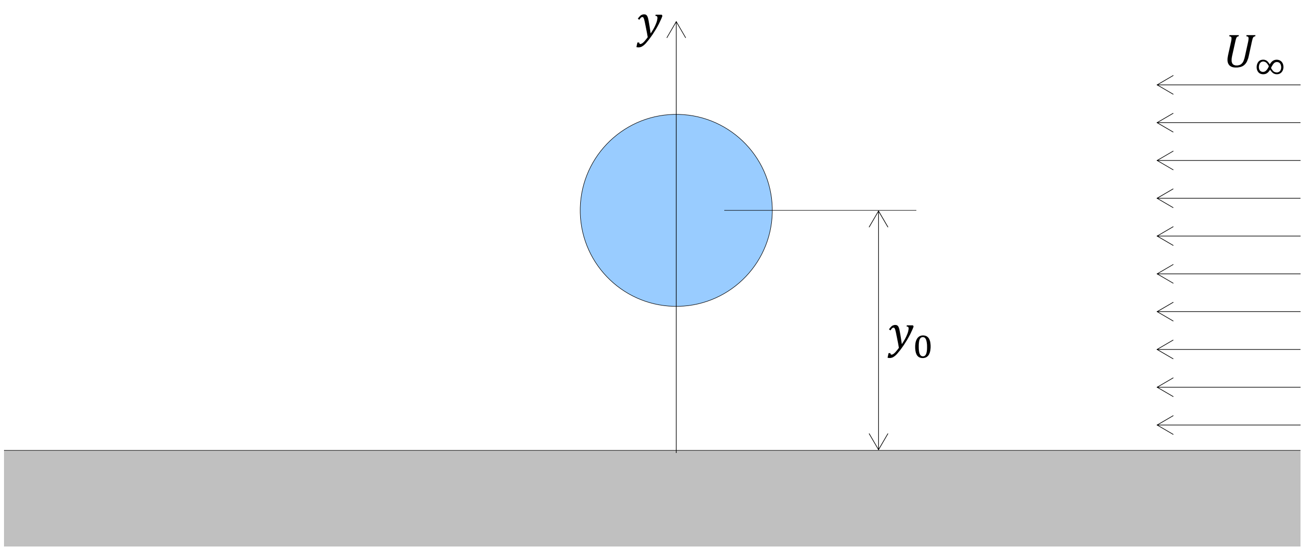




Superposition of potential flow elements

Pipeline near seabed

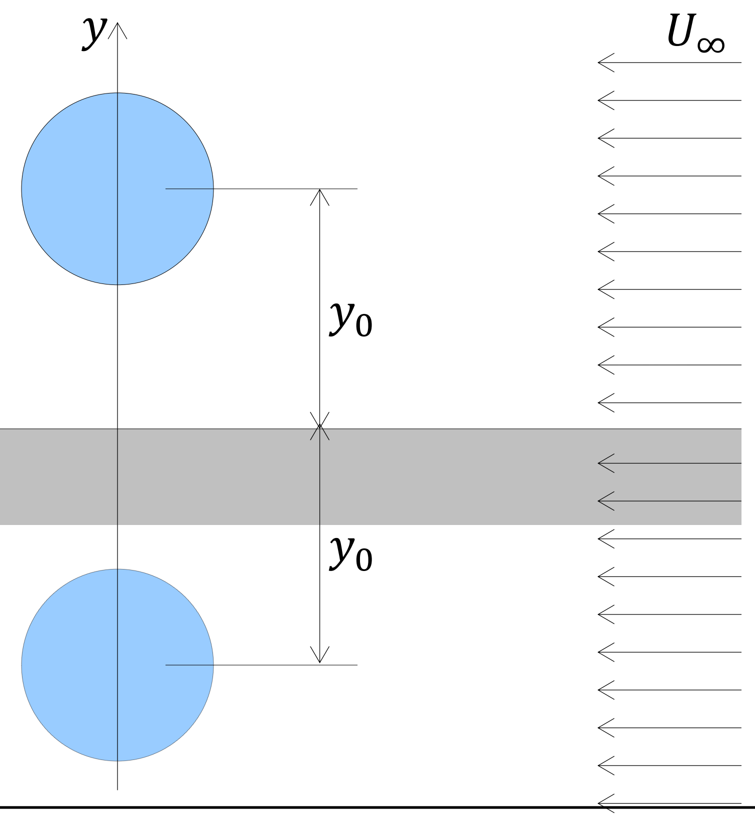
- How to calculate the flow around a pipeline near the seabed?



Superposition of potential flow elements

Pipeline near seabed

- Mirror flow in seabed!
- Superpose flows:
  - undisturbed
  - cylinder
  - mirrored cylinder

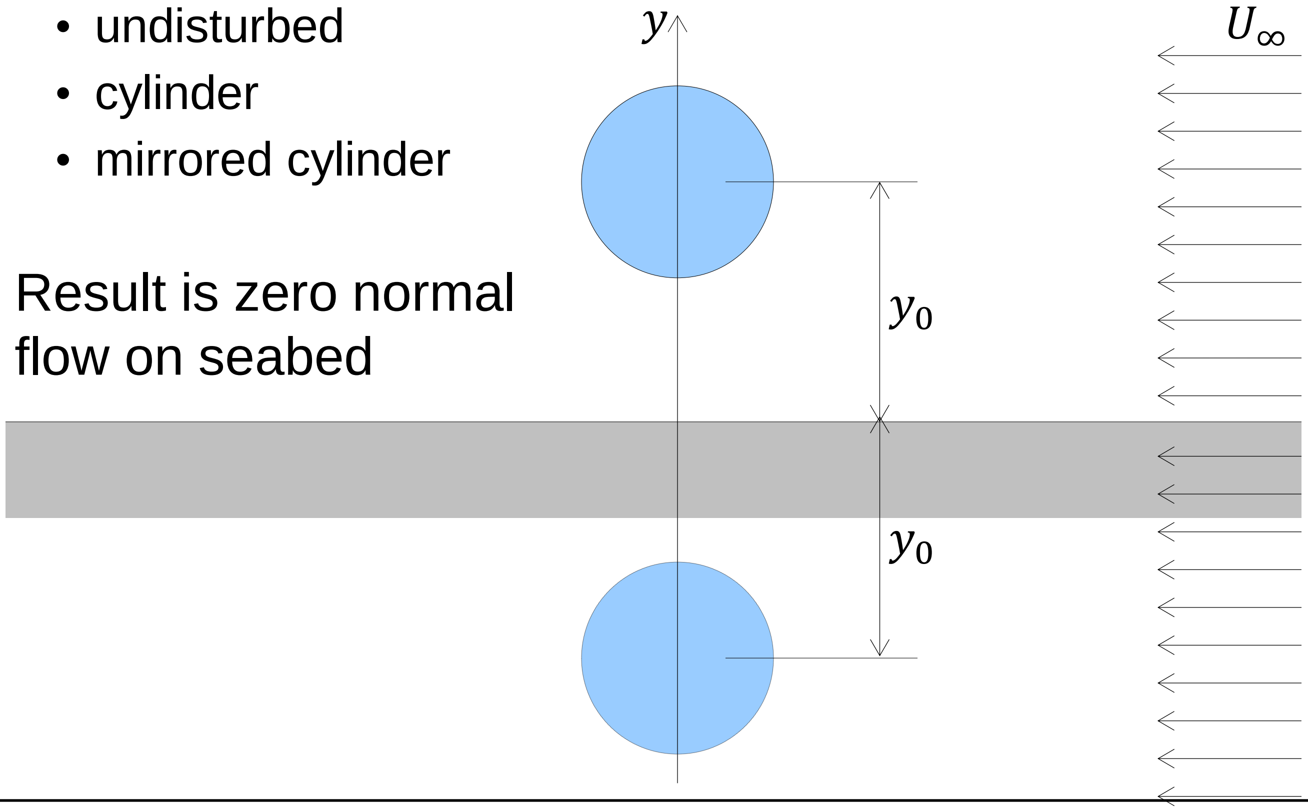


Superposition of potential flow elements

Pipeline near seabed

- Mirror flow in seabed!
- Superpose flows:
  - undisturbed
  - cylinder
  - mirrored cylinder

- Result is zero normal flow on seabed



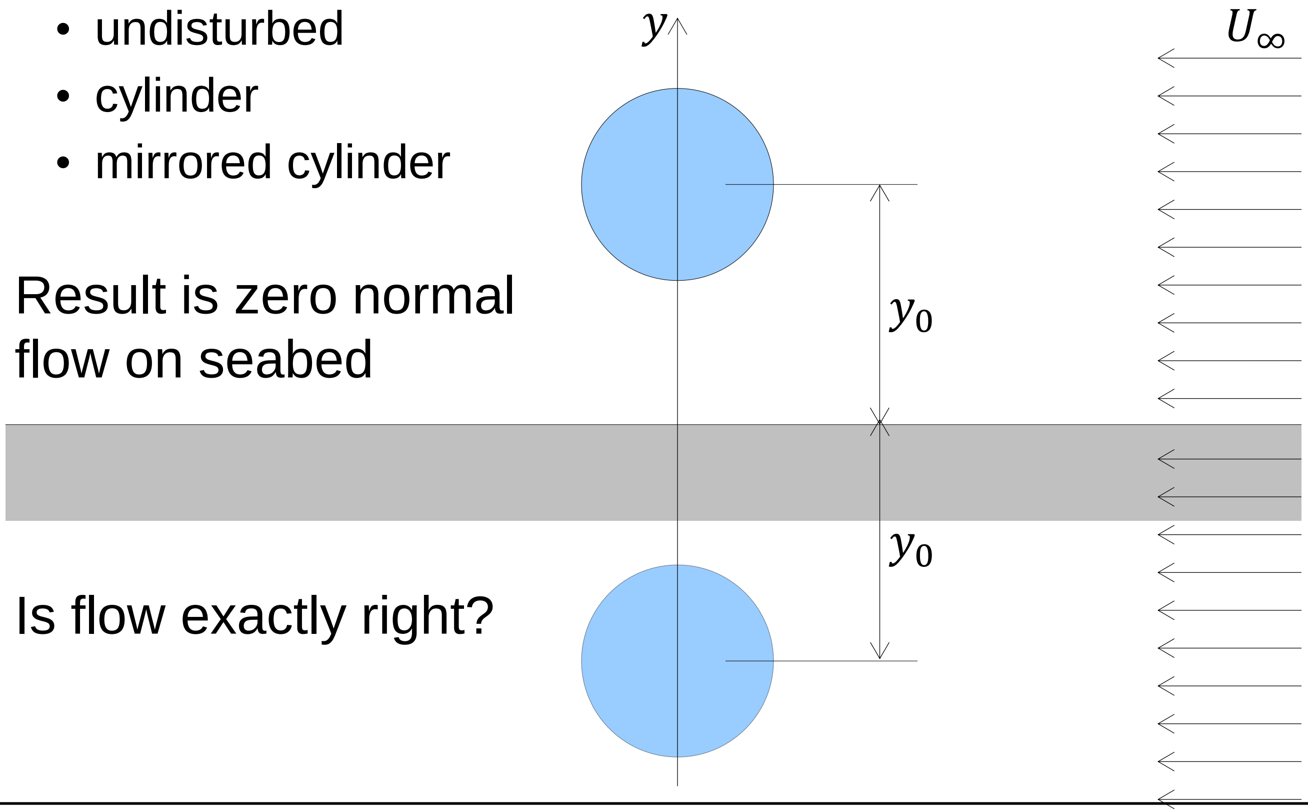
Superposition of potential flow elements

Pipeline near seabed

- Mirror flow in seabed!
- Superpose flows:
  - undisturbed
  - cylinder
  - mirrored cylinder

- Result is zero normal flow on seabed

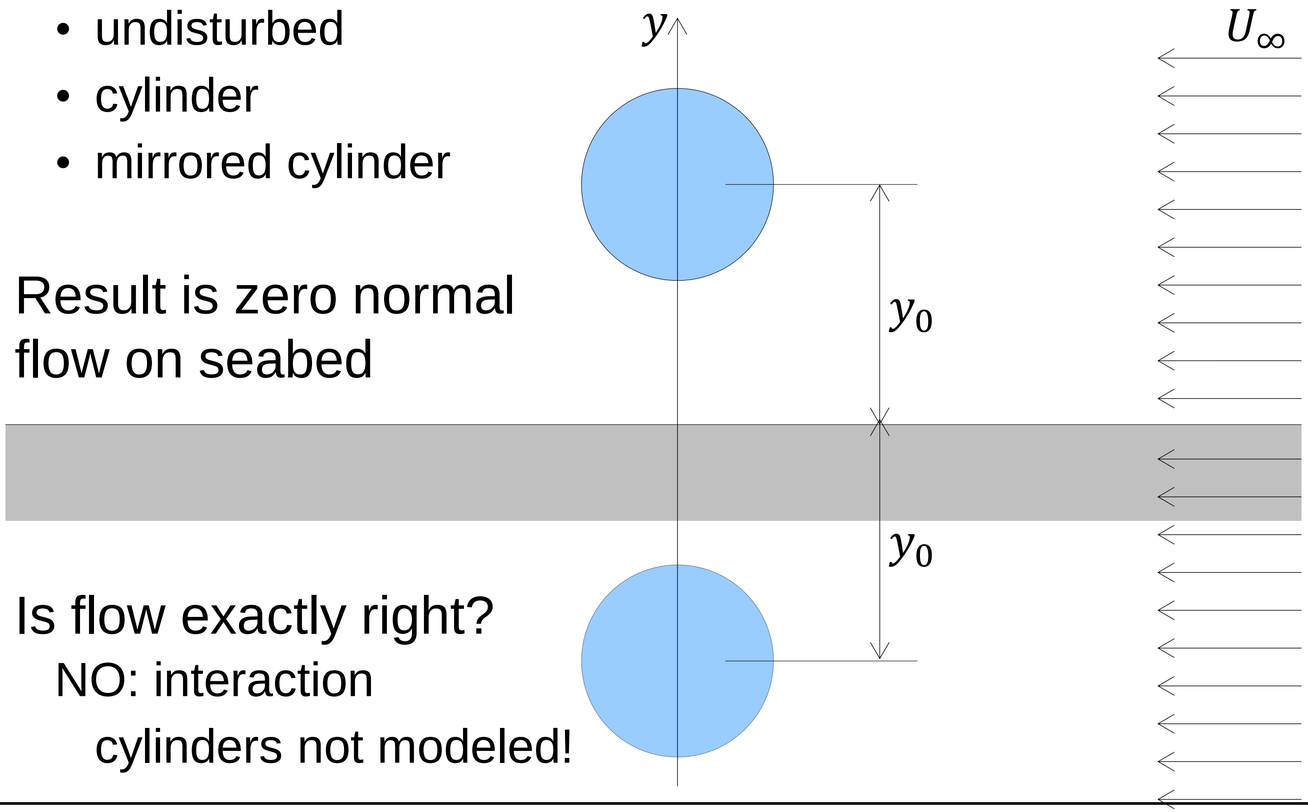
- Is flow exactly right?



Superposition of potential flow elements

Pipeline near seabed

- Mirror flow in seabed!
- Superpose flows:
  - undisturbed
  - cylinder
  - mirrored cylinder



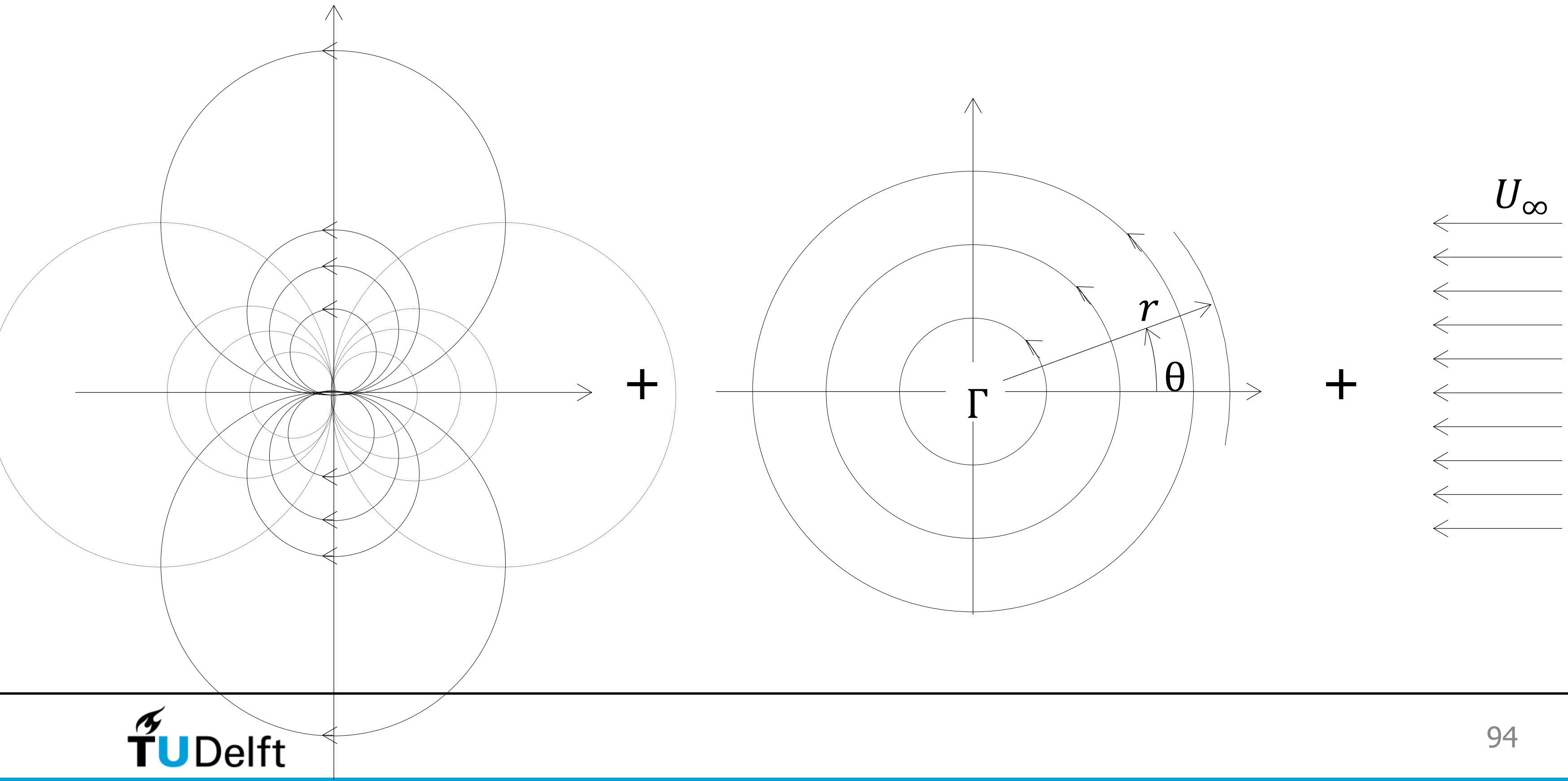
- Result is zero normal flow on seabed

- Is flow exactly right?
  NO: interaction
    cylinders not modeled!

Superposition of potential flow elements

Circulation

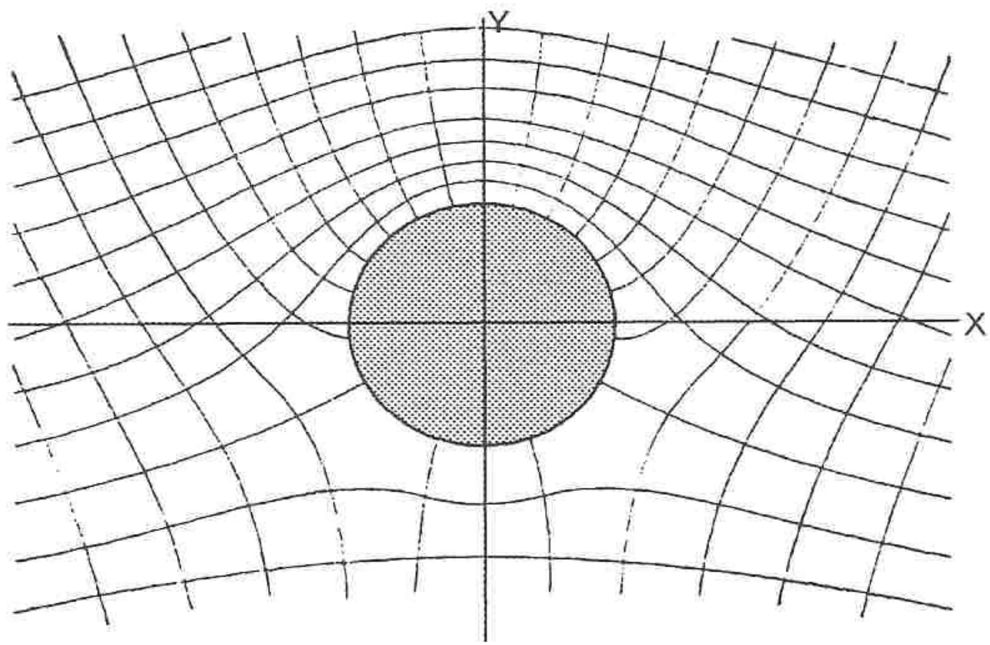
- Add circulation (or vortex flow element)
- Resulting velocity field:



Superposition of potential flow elements

Circulation

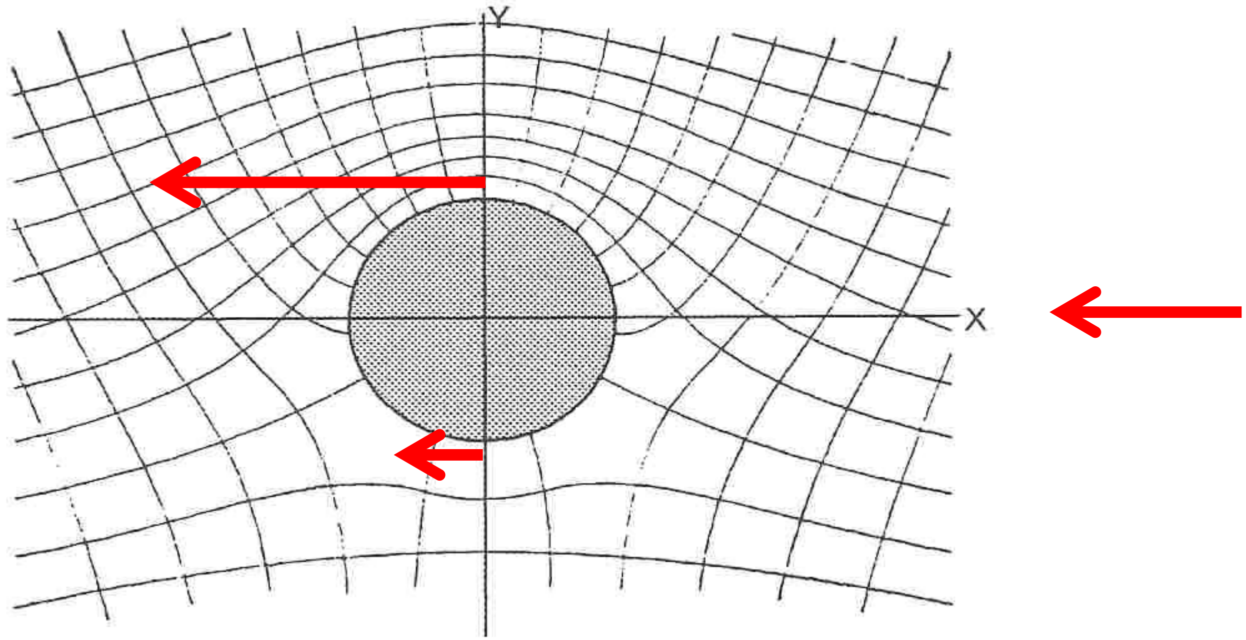
- Add circulation (or vortex flow element)
- Resulting velocity field:



Superposition of potential flow elements

Circulation

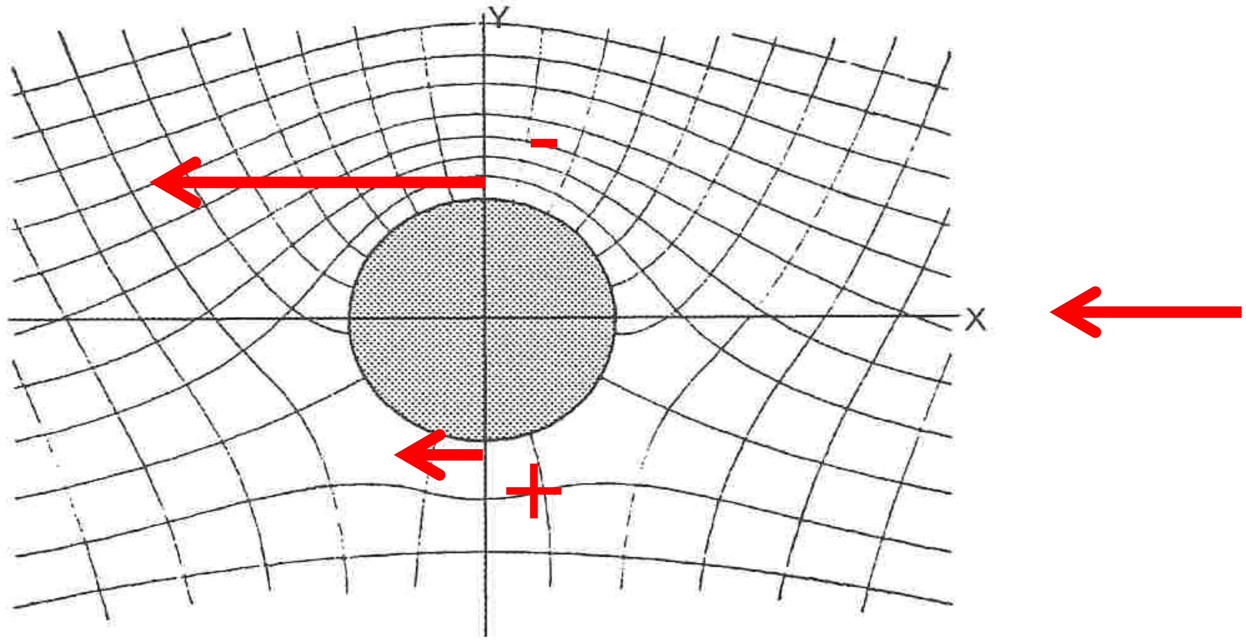
- Add circulation (or vortex flow element)
- Resulting velocity field:



Superposition of potential flow elements

Circulation

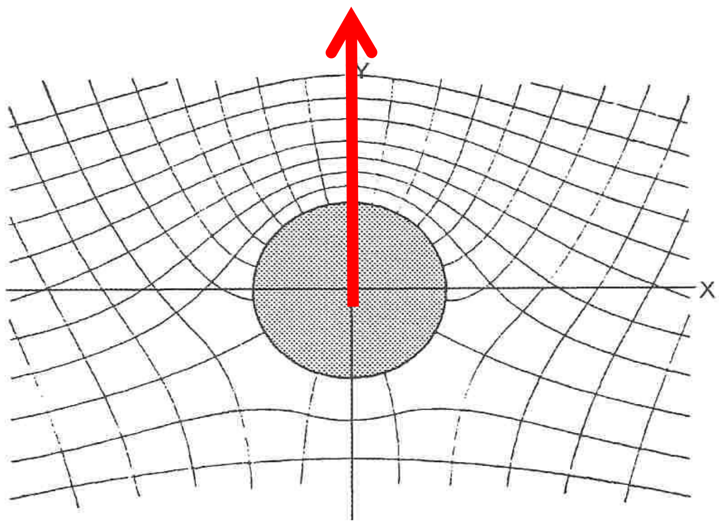
- Add circulation (or vortex flow element)
- Resulting velocity field:



Superposition of potential flow elements

Circulation

- Now integration of pressure yields a net force perpendicular to the undisturbed flow direction: the **lift** force
- However: still no net force in the flow direction: no **drag**



Sources images

All images are from the book *Offshore Hydromechanics* by Journée and Massie.