Offshore Hydromechanics Part 2

Ir. Peter Naaijen

3. Linear Potential Theory

Offshore Hydromechanics, lecture 1

Take your laptop, i- or whatever smart-phone and go to:
www.rwpoll.com
Login with session ID

Teacher module II:
- Ir. Peter Naaijen
- p.naaijen@tudelft.nl
- Room 34 B-0-360 (next to towing tank)

Book:
- Offshore Hydromechanics, by J.M.J. Journee & W.W.Massie

Useful weblinks:
- http://www.shipmotions.nl
- Blackboard

OE4630 module II course content

- +/- 7 Lectures
- Bonus assignments (optional, contributes 20% of your exam grade)
- Laboratory Excercise (starting 30 nov)
  - 1 of the bonus assignments is dedicated to this exercise
  - Groups of 7 students
  - Subscription available soon on BB
- Written exam

Schedule OE4630 D2, Offshore Hydromechanics Pt 2, 2012-2013 **Version 1 (9-11-2012)**
Disclaimer: always track for (last minute) changes in location at huidigeroosters.tudelft.nl!!!

Date :	Time:	Type:	Teacher:	Location
Wed 14 Nov	13.30 – 16.30	Lecture	Peter Naaijen	3mE-CZ D (James Watt)
Wed 14 Nov	16.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ D (James Watt)
Fri 16 Nov	10.30 – 12.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 19 Nov	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Tue 20 Nov	13.30 – 15.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ C (Daniel Bernoulli)
Wed 28 Nov	13.30 – 15.30	Lecture	Peter Naaijen	3mE-CZ D (James Watt)
Wed 28 Nov	15.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ D (James Watt)
Fri 30 Nov	10.30 – 13.00	Lab session	Peter Naaijen	Towing Tank
Mon 3 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Tue 4 Dec	13.30 – 16.00	Lab session	Gideon Hertzberger	Towing Tank
Tue 4 Dec	16.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	Room Peter Naaijen (34 B 0 360)
Mon 10 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 17 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 7 Jan	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)

Lecture notes:

- Disclaimer: Not everything you (should) learn is in the lecture notes (lees: niet alles wat op het tentamen gevraagd kan worden staat in diktaat…) -7

Make personal notes during lectures!!

- Don't save your questions 'till the break -7

Ask if anything is unclear

Learning goals Module II, behavior of floating bodies in waves

• Definition of ship motions Motion Response in regular waves: • How to use RAO's • Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces • How to solve RAO's from the equation of motion Motion Response in irregular waves: •How to determine response in irregular waves from RAO's and wave spectrum without forward speed		
3D linear Potential Theory •How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential •How to determine Velocity Potential		
Motion Response in irregular waves: • How to determine response in irregular waves from RAO's and wave spectrum with forward speed • Make down time analysis using wave spectra, scatter diagram and RAO's	Ch. 8	
Structural aspects: • Calculate internal forces and bending moments due to waves		
Nonlinear behavior: • Calculate mean horizontal wave force on wall • Use of time domain motion equation	Ch.6	

Introduction


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Introduction

Offshore oil resources have to be explored in deeper water floating structures instead of bottom founded


[4]

Introduction

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Inertia forces on sea fastening due to accelerations:

Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Direct wave induced structural loads

Minimum required air gap to avoid wave damage

Introduction

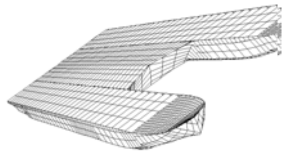
Reasons to study waves and ship behavior in waves:

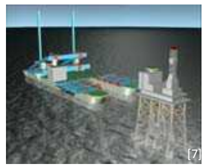
- Determine allowable / survival conditions for offshore operations

Introduction

Decommissioning / Installation / Pipe laying -7 Excalibur / Allseas 'Pieter Schelte'

- Motion Analysis





Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Forces on mooring system, motion envelopes loading arms


[8]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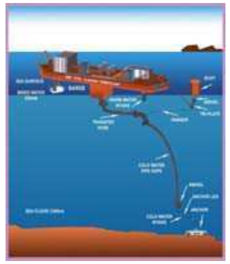
Introduction

Floating Offshore: More than just oil


Floating wind farm [9]


OTEC [10]

Introduction

Floating Offshore: More than just oil


[11]


[12]

Wave energy conversion

Introduction

Floating Offshore: More than just oil

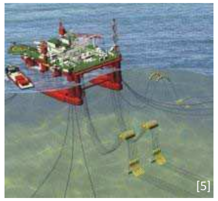
Mega Floaters

Introduction

Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations
- Downtime analysis

Winter Data of Areas 8, 9, 15 and 16 of the North Atlantic (Global Wave Statistics)												
	T_2 (s)											
H_s (m)	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	Total
14	0	0	0	0	2	30	154	362	466	370	202	1586
13	0	0	0	0	3	33	145	293	322	219	101	1116
12	0	0	0	0	7	72	289	539	548	345	149	1949
11	0	0	0	0	17	160	585	996	931	543	217	3449
10	0	0	0	1	41	363	1200	1852	1579	843	310	6189
9	0	0	0	4	109	845	2485	3443	2648	1283	432	11249
8	0	0	0	12	295	1996	5157	6323	4333	1882	572	20570
7	0	0	0	41	818	4723	10537	11242	6755	2594	703	37413
6	0	0	1	138	2273	10967	20620	18718	9665	3222	767	66371
5	0	0	7	471	6187	24075	36940	27702	11969	3387	694	111432
4	0	0	31	1586	15757	47072	56347	33539	11710	2731	471	169244
3	0	0	148	5017	34720	74007	64809	28964	7804	1444	202	217115
2	0	4	681	13441	56847	77259	45013	13962	2725	381	41	210354
1	0	40	2699	23284	47839	34532	11554	2208	282	27	2	122467
0	5	350	3314	8131	5858	1598	216	18	1	0	0	19491
Total	5	394	6881	52126	170773	277732	256051	150161	61738	19271	4863	999995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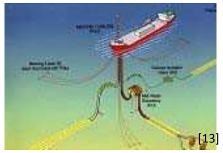
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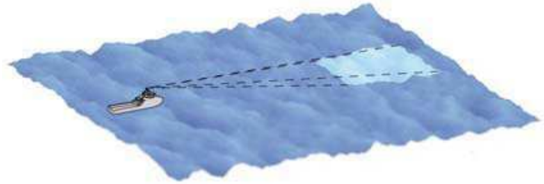
Real-time motion prediction
Using X-band radar remote wave observation



Definitions & Conventions
Regular waves
Ship motions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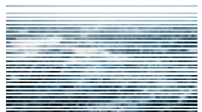
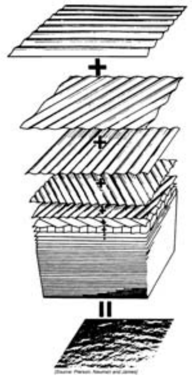
apparently irregular but can be considered as a superposition of a finite number of regular waves, each having own frequency, amplitude and propagation direction

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Regular waves

Regular waves

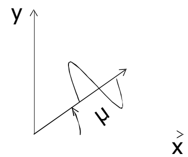
regular wave propagating in direction μ:

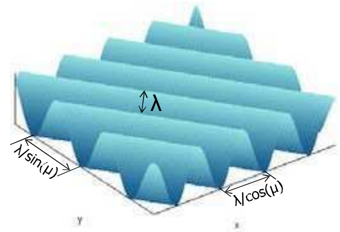
$$(t, x) \quad {}_a \cos \quad t \quad kx\cos \quad ky\sin$$

$$k \quad 2 \quad /$$

$$2 \quad / T$$

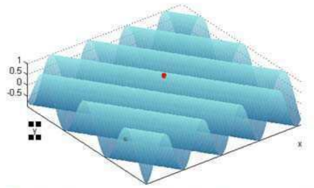
Linear solution Laplace equation

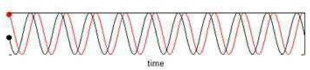




- Regular waves
- regular wave propagating in direction μ

$$(t, x) \quad {}_a \cos \quad t \quad kx\cos \quad ky\sin$$





Phase angle wave at black dot with respect to wave at red dot:

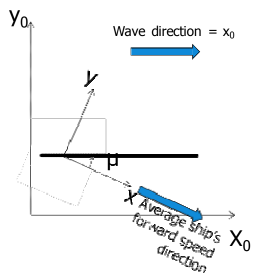
$$k \quad x \quad x \cos \quad k \quad y \quad y \sin$$

Co-ordinate systems

Definition of systems of axes

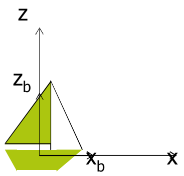
Earth fixed: (x_0, y_0, z_0)

wave direction with respect to ship's exes system:



Behavior of structures in waves

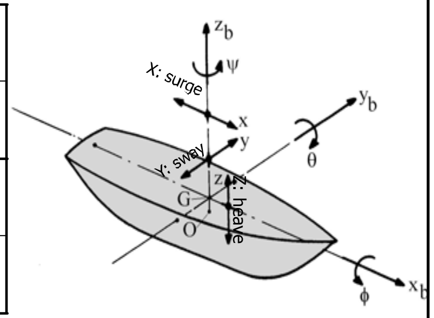
Ship's body bound axes system (x_b,y_b,z_b) follows all ship motions



Behavior of structures in waves

Definition of translations

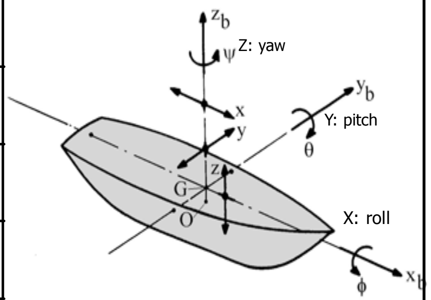
		NE	EN
1	x	Schrikken	Surge
2	y	Verzetten	Sway
3	z	Dompen	Heave



Behavior of structures in waves

Definition of rotations

5	y	Stampen	Pitch
6	z	Gieren	Yaw



How do we describe ship motion response?

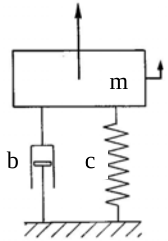
Rao's

Phase angles

Mass-Spring system:

$m\ddot{z} \quad b\dot{z} \quad cz \quad F_a \cos \quad t$ Motion equation

$z \; t \quad z_a \cos \quad t$ Steady state solution



Motions of and about COG

Amplitude Phase angle

$Surge(schrikken): \; x \quad x_a \cos \quad t \quad _x$

$Sway(verzetten): \; y \quad y_a \cos \quad t \quad _y$

$Heave(dompen): \; z \quad z_a \cos \quad t \quad _z$

$Roll(rollen):$ ⟨phi⟩ $_a \cos \quad t$

$Pitch(stampen):$ ⟨theta⟩ $_a \cos \quad t$

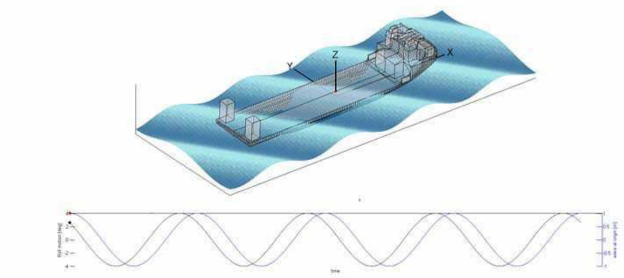
$Yaw(gieren):$ ⟨psi⟩ $_a \cos \quad t$

Phase angles are related to undisturbed wave at origin of steadily translating ship-bound system of axes (COG)

Motions of and about COG

Phase angles are related to undisturbed wave at origin of steadily

translating ship-bound system of axes (COG)



Motions of and about COG

$Surge(schrikken): \; x \quad x_a \cos \quad t \quad _x$ — $RAOSurge: \frac{x}{_a}$,

$Sway(verzetten): \; y \quad y_a \cos \quad t \quad _y$ — $RAOSway: \frac{y_a}{_a}$,

$Heave(dompen): \; z \quad z_a \cos \quad t \quad _z$ — $RAOHeave: \frac{z_a}{_a}$,

$Roll(rollen):$ ⟨phi⟩ $_a \cos \quad t$ — $RAORoll: \frac{_a}{_a}$,

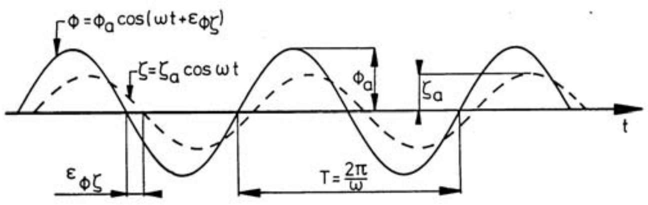
$Pitch(stampen):$ ⟨theta⟩ $_a \cos \quad t$ — $RAOPitch: \frac{_a}{_a}$,

$Yaw(gieren):$ ⟨psi⟩ $_a \cos \quad t$ — $RAOYaw: \frac{_a}{_a}$,

RAO and phase depend on:

- Wave frequency
- Wave direction

Example: roll signal



Displacement $_a \cos \quad _e t$

Velocity… $_e \; _a \sin \quad t_e \qquad _e \; _a \cos \quad t_e \qquad / 2$

Acceleration… $^2_e \; _a \cos \quad _e t \qquad ^2_e \; _a \cos \quad _e t$

Motions of and about COG

1 Surge(schrikken): $x \quad x_a \cos \quad _e t \quad _x$

2 Sway(verzetten): $y \quad y_a \cos \quad _e t \quad _y$

3 Heave(dompen): $z \quad z_a \cos \quad _e t \quad _z$

4 Roll(rollen): ⟨phi⟩ $_a \cos \quad _e t$

5 Pitch(stampen): ⟨theta⟩ $_a \cos \quad _e t$

6 Yaw(gieren): ⟨psi⟩ $_a \cos \quad _e t$

- Frequency of input (regular wave) and output (motion) is ALWAYS THE SAME !!
- Phase can be positive ! (shipmotion ahead of wave elevation at COG)
- Due to symmetry: some of the motions will be zero
- Ratio of motion amplitude / wave amplitude = RAO (Response Amplitude Operator)
- RAO's and phase angles depend on wave frequency and wave direction
- RAO's and phase angles must be calculated by dedicated software or measured by experiments
- Only some special cases in which 'common sense' is enough:

Consider Long waves relative to ship dimensions

What is the RAO of pitch in head waves ?

- Phase angle heave in head waves ?...
- RAO pitch in head waves ?...
- Phase angle pitch in head waves ?...
- Phase angle pitch in following waves ?...

Local motions (in steadily translating axes system)

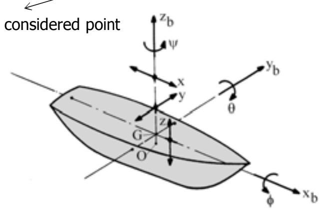
- Onlyvariations!!
- Linearized!!

$$\begin{Bmatrix} y_P & t \\ z_P & t \end{Bmatrix} \quad \begin{Bmatrix} y & t \\ z & t \end{Bmatrix} \quad \begin{Bmatrix} & t & 0 & & t \\ & & t & t & 0 \end{Bmatrix} \begin{Bmatrix} y_{bP} \\ z \end{Bmatrix}$$

6 DOF Ship motions

Location considered point

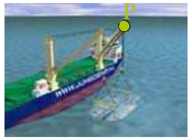
$$x_P \; t \quad x \; t \quad y_{bP} \quad t \quad z_{bP} \quad t$$
$$y_P \; t \quad y \; t \quad x_{bP} \quad t \quad z_{bP} \quad t$$
$$z_P \; t \quad z \; t \quad x_{bP} \quad t \quad y_{bP} \quad t$$



Local Motions

Example 3: horizontal crane tip motions

The tip of an onboard crane, location:
xb,yb,zb = -40, -9.8, 25.0



For a frequency ω=0.6 the RAO's and phase angles of the ship motions are:

SURGE		SWAY		HEAVE		ROLL		PITCH		YAW	
RAO	phase	RAO	phase	RAO	phase	RAO	phase	RAO	phase	RAO	phase
-	degr	-	degr	-	degr	deg/m	degr	deg/m	degr	deg/m	degr
1.014E-03	3.421E+02	5.992E-01	2.811E+02	9.991E-01	3.580E+02	2.590E+00	1.002E+02	2.424E-03	1.922E+02	2.102E-04	5.686E+01

Calculate the RAO and phase angle of the transverse horizontal motion (y-direction) of the crane tip.

Complex notation of harmonic functions

1 $Surge(schrikken):\ x \quad x_a \cos \quad {}_e t \quad {}_x$

$\mathrm{Re}\ x_a e^{i \quad t \quad x}$

$\mathrm{Re}\ x_a e^{\ x} \ e^{i \quad t}$

Complex motion amplitude

$\mathrm{Re}\ \hat{x}_a \ e^{i \quad t}$

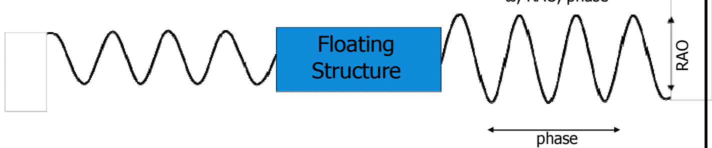
- :

Relation between Motions and Waves

How to calculate RAO's and phases ?

Input: regular wave, ω

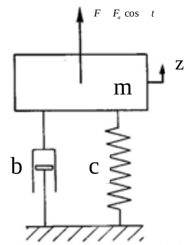
Output: regular motion
ω, RAO, phase



Mass-Spring system:

Forces acting on body:

…?



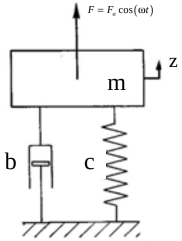
Mass-Spring system:

$m\ddot{z} + b\dot{z} + cz = F_a\cos(\omega t)$

Transient solution

$z(t) = A_t e^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\,\omega_0 t + \varphi_t\right)$

$\left(\zeta = \frac{b}{2\sqrt{mc}}\right)$ Damping ratio



Steady state solution:

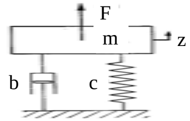
$z(t) = z_a\cos(\omega t + \varepsilon)$

$\varepsilon = a\tan\left(\frac{(-b\omega)}{(-\omega^2(m)+c)}\right)$

$z_a = \sqrt{\frac{F_a^2}{\left((-(m)\omega^2+c)^2+(b\omega)^2\right)}}$

Moving ship in waves:

[14]



$m_3\ddot{z} + b_3\dot{z} + c_3 z = F_{a3}\cos(\omega t)$

Restoring coefficient for **heave** ?

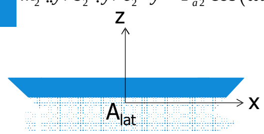
$m_4\ddot{\varphi} + b_4\dot{\varphi} + c_4\varphi = F_{a4}\cos(\omega t)$

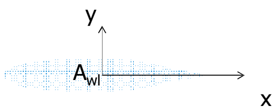
Restoring coefficient for **roll** ?

m for **roll** ?

What is the hydrostatic spring coefficient for the sway motion ?

$m_2\ddot{y} + b_2\dot{y} + c_2\cdot y = F_{a2}\cos(\omega t)$

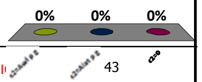




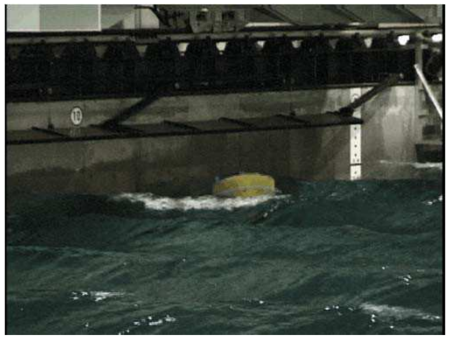
A. $c_2 = A_{wl}\,\rho\,g$

B. $c_2 = A_{lat}\,\rho\,g$

C. $c_2 = 0$



Non linear stability issue...



Roll restoring

Roll restoring coefficient:

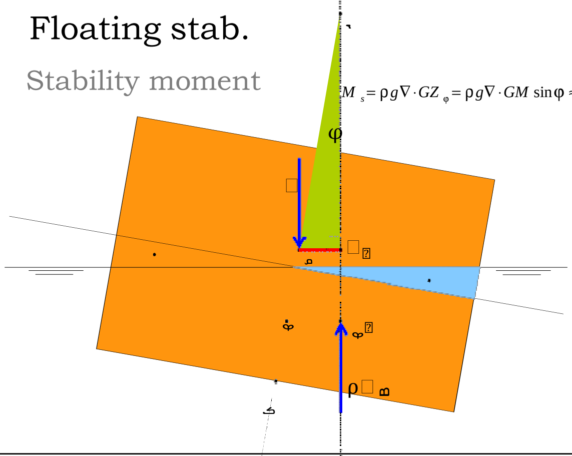
$$c_4 = \rho g \nabla \cdot GM$$

What is the point the ship rotates around statically speaking ? (Ch 2)

Floating stab.

Stability moment

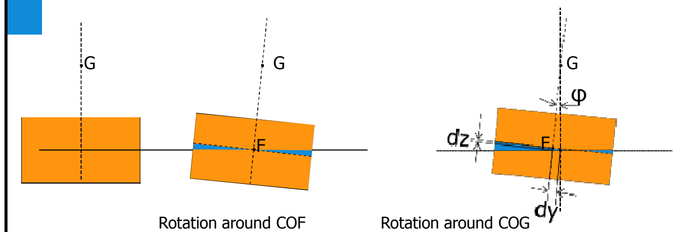
$$M_s = \rho g \nabla \cdot GZ_\varphi = \rho g \nabla \cdot GM \sin\varphi \approx \rho g \nabla \cdot GM \cdot \varphi$$



Moving ship in waves:

$$m_4\ddot{\varphi} + b_4\dot{\varphi} + c_4\varphi = F_{a4}\cos(\omega t)$$

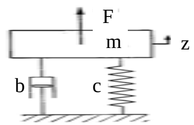
Restoring coefficient for **roll** ?



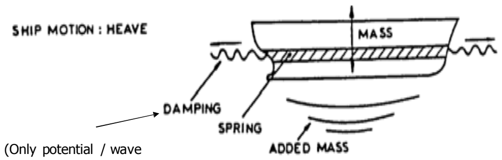
Rotation around COF

Rotation around COG
=Rotation around COF
+vertical translation $dz = FG - FG\cos\varphi \approx 0$
+horizontal translation $dy = FG\sin\varphi \approx FG\varphi$

Moving ship in waves:
Not in air but in water!



$F = m\ddot{z}$



→ F_w
→ $-c \cdot z$
→ $-b\dot{z}$ (Only potential / wave damping)
→ $-a\ddot{z}$

$$(m + a)\ddot{z} + b\dot{z} + c \cdot z = F_w$$

Moving ship in waves:

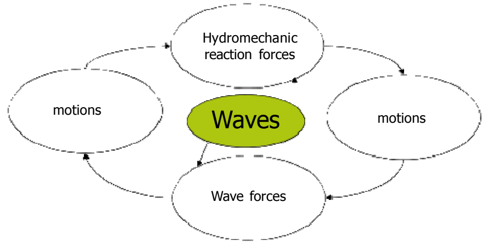
Analogy / differences with mass-spring system:

		
External force	F(t)	Wave exciting force Has a phase angle w r t undisturbed wave at COG
restoring force	c*z	Archimedes: bouyancy
Damping force	b*dz/dt	Hydrodynamic damping
Inertia force	$M*d^2z/dt^2$	Mass + Hydrodynamic Mass

Depend on frequency !

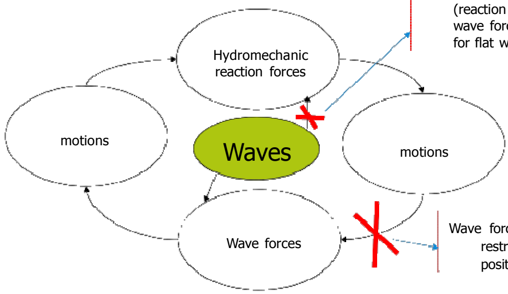
Moving ship in waves:

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$



Moving ship in waves:

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$



Hydromechanic reaction forces (reaction on calculated wave force)are calculated for flat water (**NO WAVES**)

Wave force is calculated for restricted ship in mean position: **No motions**

Right hand side of m.e.: Wave Exciting Forces

- Incoming: regular wave with given frequency and propagation direction
- Assuming the vessel is not moving

Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x) = \zeta_a \cos(\omega t - kx\cos\mu - ky\sin\mu)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x) = \zeta_a \cos(\omega t - kx\cos\mu - ky\sin\mu)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

Potential Theory

What is potential theory ?:
way to give a mathematical description of flowfield

Most complete mathematical description of flow is viscous Navier-Stokes equation:

Navier-Stokes vergelijkingen:

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\lambda\nabla\cdot V+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]$$

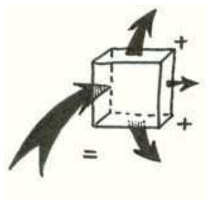
$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left(\lambda\nabla\cdot V+2\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]$$

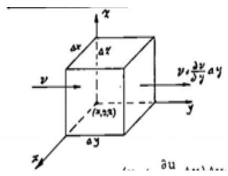
$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left(\lambda\nabla\cdot V+2\mu\frac{\partial w}{\partial z}\right)$$

(not relaxed)

Water is hard to compress, we will assume this is impossible →

Apply principle of continuity on control volume:





Continuity: what comes in,
must go out

This results in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

If in addition the flow is considered to be irrotational and non viscous →

Velocity potential function can be used to describe water motions
Main property of velocity potential function:

for potential flow, a function Φ(x,y,z,t) exists whose derivative in a certain arbitrary direction equals the flow velocity in that direction. This function is called the velocity potential.

From definition of velocity potential:

$$u=\frac{\partial \Phi}{\partial x}, v=\frac{\partial \Phi}{\partial v}, w=\frac{\partial \Phi}{\partial z}$$

Substituting in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Results in Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2}+\frac{\partial^2 \Phi}{\partial y^2}+\frac{\partial^2 \Phi}{\partial z^2}=0$$

Summary

- Potential theory is mathematical way to describe flow

Important facts about velocity potential function Φ:

- definition: Φ is a function whose derivative in any direction equals the flow velocity in that direction
- Φ describes non-viscous flow
- Φ is a scalar function of space and time (NOT a vector!)

Summary

- Velocity potential for regular wave is obtained by
  - Solving Laplace equation satisfying:
    1. Seabed boundary condition
    2. Dynamic free surface condition

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh(k(h+z))}{\cosh(kh)} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$

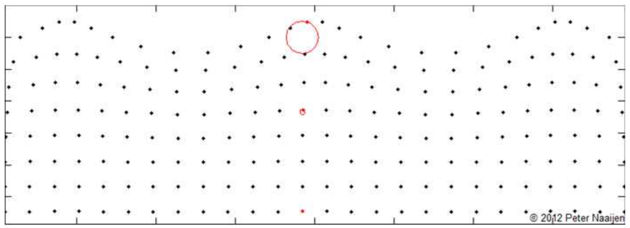
3. Kinematic free surface boundary condition results in:
   Dispersion relation = relation between wave frequency and wave length

$$\omega^2 = kg\tanh(kh)$$

Water Particle Kinematics

trajectories of water particles in infinite water depth

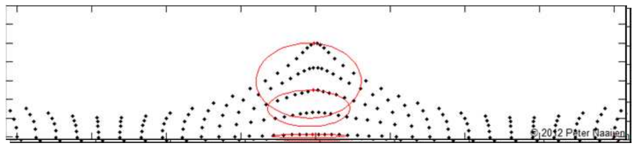
$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$



Water Particle Kinematics

trajectories of water particles in finite water depth

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh(k(h+z))}{\cosh(kh)} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$



Pressure

Pressure in the fluid can be found using Bernouilli equation for unsteady flow:

$$\frac{\partial \Phi}{\partial t} + \tfrac{1}{2}(u^2 + w^2) + \frac{p}{\rho} + gz = 0$$

$$p = -\rho \frac{\partial \Phi}{\partial t} - \tfrac{1}{2}\rho(u^2 + w^2) - \rho g z$$

1^{st} order fluctuating pressure

2^{nd} order (small quantity squared=small enough to neglect)

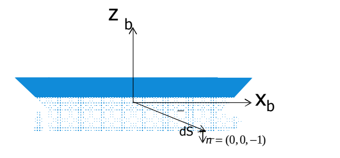
Hydrostatic pressure (Archimedes)

Potential Theory

- Pressure
- Forces and moments can be derived from pressures:

$$\bar{M} = -\iint_S p \cdot (\bar{r} \times \bar{n})\, dS$$

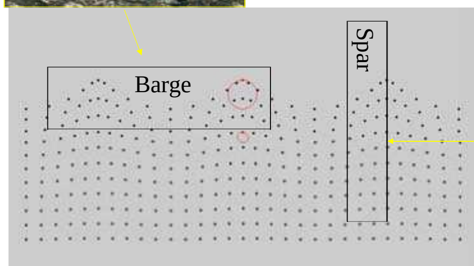
Verify these formulae (incl the signs!) yourself in order to understand them. Just check e.g. the force in heave direction (F_3) and the pitch moment (M_2) induced by a pressure on an infinite piece of hull surface dS at location $\bar{r}$:



Wave Force

Determination F_w

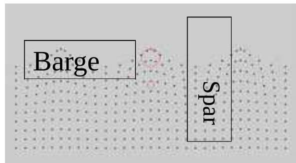
- Froude Krilov
- Diffraction

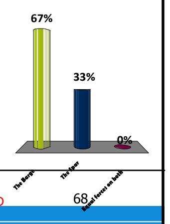


[15]

Which structure experiences the highest vertical wave load acc. to potential theory ?

A. The Barge
B. The Spar
C. Equal forces on both





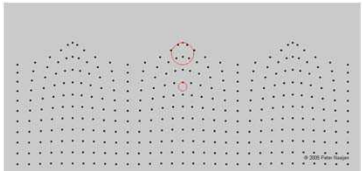
Flow superposition

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w$$

Considering a fixed structure (ignoring the motions) we will try to find a description of the disturbance of the flow by the presence of the structure in the form of a velocity potential. We will call this one the diffraction potential and added to the undisturbed wave potential (for which we have an analytical expression) it will describe the total flow due to the waves.

1. Flow due to Undisturbed wave

$$\Phi_0=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(\omega t-kx\cos\mu-ky\sin\mu)$$



2. Flow due to Diffraction

Has to be solved. What is boundary condition at body surface ?

Φ_7

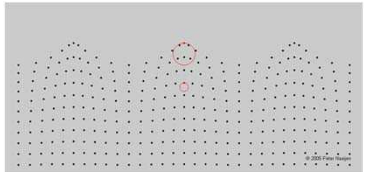
Exciting force due to waves

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w$$

$$=F_{FK}+F_D$$

1. Undisturbed wave force (Froude-Krilov)

$$\Phi_0=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(\omega t-kx\cos\mu-ky\sin\mu+\varepsilon)$$

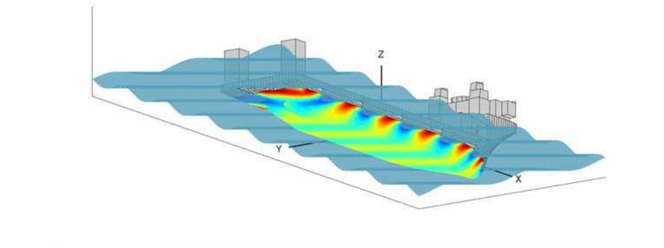


2. Diffraction force

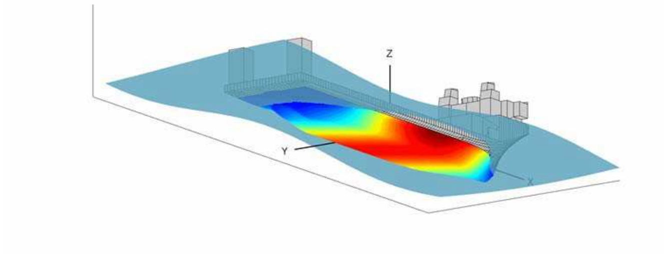
Has to be solved. What is boundary condition at body surface ?

Φ_7

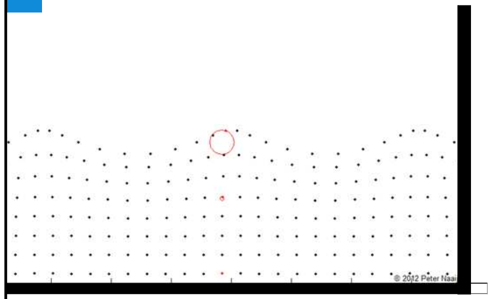
Pressure due to undisturbed incoming wave T=4 s



Pressure due to undisturbed incoming wave T=10 s



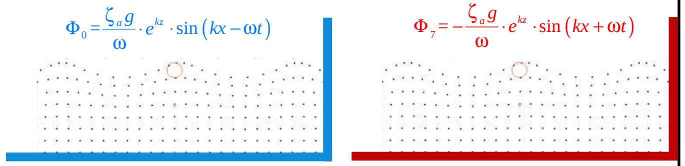
Wave Forces
Wave force acting on vertical wall



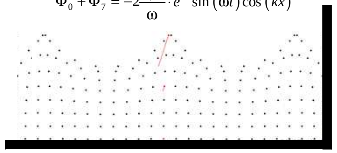
Calculating hydrodynamic coeffecient and diffraction force

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx-\omega t) \qquad \Phi_7=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx+\omega t)$$



$$\Phi_0+\Phi_7=-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)$$



Force on the wall

$$\bar{F}=-\int_{-\infty}^{0} p\cdot\bar{n}dz$$

$$\Phi_0=\frac{\zeta_a g}{\omega}e^{kz}\sin(kx-\omega t),\Phi_7=-\frac{\zeta_a g}{\omega}e^{kz}\sin(kx+\omega t)$$

$$p=-\rho\frac{\partial\Phi}{\partial t}=-\rho\frac{\partial(\Phi_0+\Phi_7)}{\partial t}=$$

$$-\rho\frac{\partial\left(-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)\right)}{\partial t}=$$

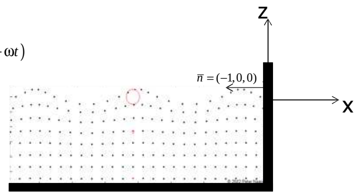
$$2\rho\zeta_a g\cdot e^{kz}\cos(kx)\cos(\omega t)$$

$$\bar{n}=(-1,0,0)$$

$$x=0$$

$$F_x=\int_{-\infty}^{0}2\rho\zeta_a g\cdot e^{kz}\cos(\omega t)dz=\left[2\rho\frac{\zeta_a g}{k}\cdot e^{kz}\cos(\omega t)\right]_{-\infty}^{0}=$$

$$2\rho\frac{\zeta_a g}{k}\cdot\cos(\omega t)-0$$



Left hand side of m.e.: Hydromechanic reaction forces

- NO incoming waves:
- Vessel moves with given frequency

Recap: Motion equation

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=+F_{FK}+F_D=F_W$$

Hydromechanic force depends on motion

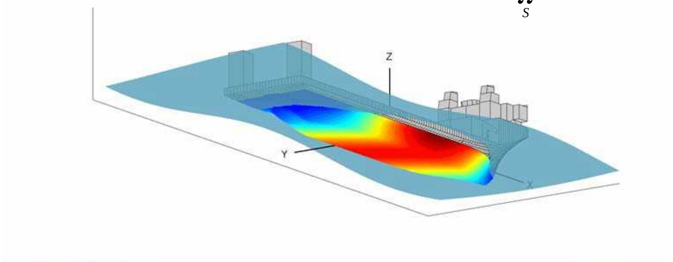
Wave Force independent of motion

Pressure / force due to undisturbed incoming wave
T=10 s

$$p=-\rho\frac{\partial\Phi}{\partial t}$$

$$\overline{F}=-\iint_S(p\cdot\overline{n})\,dS$$

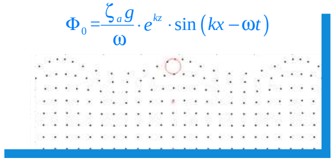
$$\overline{M}=-\iint_S p\cdot(\overline{r}\times\overline{n})\,dS$$



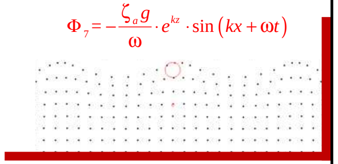
Calculating diffraction force

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

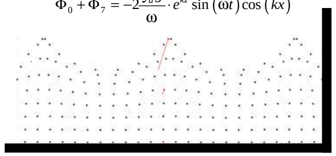
$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx-\omega t)$$



$$\Phi_7=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx+\omega t)$$



$$\Phi_0+\Phi_7=-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)$$



left hand side: reaction forces

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=+F_{FK}+F_D=F_W$$

Hydromechanic force depends on motion

Wave Force independent of motion

Hydrodynamic coefficients

Determination of a and b:

- Forced oscillation with known frequency and amplitude
- Measure Force needed to oscillate the model


6 Degree of Freedom Forced Oscillation tests

July-August 2004

TUDelft

Determine added mass and damping

Experimental procedure:

Oscillate model (i.e. impose known harmonic motion, z) → Measure required force, $F_{oscillation}$ → Subtract known reaction forces from measured $F_{oscillation}$ → Split remainder into damping and added mass coefficient

$$(m+a)\ddot{z} + b\dot{z} + c\cdot z = F_{oscillation}$$

$$z = z_a\cos(\omega t),\quad \dot{z} = -\omega z_a\sin(\omega t),\quad \ddot{z} = -\omega^2 z_a\cos(\omega t)$$

$$\left(-\omega^2(m+a)+c\right)z_a\cos\omega t - \omega b z_a\sin\omega t = F_{a,osc}\cdot\cos(\omega t+\varepsilon_{F,z})$$

$$-\omega^2 a z_a\cos\omega t - \omega b z_a\sin\omega t = F_{a,osc}\cdot\cos(\omega t+\varepsilon_{F,z}) + (\omega^2 m - c)z_a\cos\omega t$$

Equation of motion

$$(m+a)\ddot{z} + b\dot{z} + c\cdot z = F_W$$

Hydrodynamic coefficients:

a=added mass coefficient= force on ship per 1 m/s² acceleration →
a * acceleration = hydrodynamic inertia force

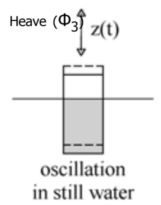
b=damping coefficient= force on ship per 1 m/s velocity →
b * velocity = hydrodynamic damping force

Calculating hydrodynamic coefficients
added mass and damping

For each of the 6 possible motions, the flow is described by a radiation potential function. The incoming waves are ignored for this. By finding a description of the flow, the pressures and consequently the forces can be determined later

$$(m+a)\ddot{z} + b\dot{z} + c\cdot z = F_W$$

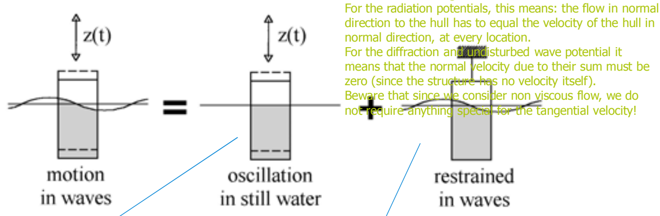
- Oscillation in desired direction **in still water**
- To prevent water from penetrating through the hull: we need the radiation velocity potentials : $\Phi_1 - \Phi_6$
- From potentials, we can calculate forces on body and the corresponding coefficients



Solving the Laplace equation

Summary

The boundary conditions are the same as those used for the undisturbed wave (Ch 5) however, we have an additional boundary now which is the hull of the structure: it has to be water tight!
For the radiation potentials, this means: the flow in normal direction to the hull has to equal the velocity of the hull in normal direction, at every location.
For the diffraction and undisturbed wave potential it means that the normal velocity due to their sum must be zero (since the structure has no velocity itself).
Beware that since we consider non viscous flow, we do not require anything special for the tangential velocity!



Radiation potential $\Phi_{1.....6}$

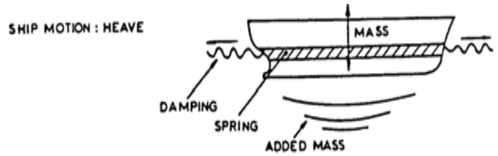
Boundary Condition: $\frac{\partial \Phi_{1.....6}}{\partial n} = v_n$

Undisturbed wave potential Φ_0

Diffraction potential Φ_7

Boundary Condition: $\frac{\partial \Phi_0}{\partial n} + \frac{\partial \Phi_7}{\partial n} = 0$

Moving ship in waves: Not in air but in water!



$$(m+a)\ddot{z} + b \cdot \dot{z} + c \cdot z = F_w$$

$$\bar{F} = -\iint_S (p \cdot \bar{n})\, dS$$

$$\bar{M} = -\iint_S p \cdot (\bar{r} \times \bar{n})\, dS$$

$$p = -\rho \frac{\partial \Phi}{\partial t}$$

Equation of motion

$$(m+a)\ddot{z} + b \cdot \dot{z} + c \cdot z = +F_{FK} + F_D = F_W$$

To solve equation of motion for certain frequency:

- Determine spring coefficient:
  - c → follows from geometry of vessel
- Determine required hydrodynamic coefficients for desired frequency:
  - a, b → computer / experiment
- Determine amplitude and phase of F_w of regular wave with amplitude =1:
  - Computer / experiment: $F_w = F_{wa}\cos(\omega t + \varepsilon_{Fw,\zeta})$
- As we consider the response to a regular wave with frequency ω:
  Assume steady state response: $z = z_a \cos(\omega t + \varepsilon_{z,\zeta})$
  and substitute in equation of motion:

Equation of motion

$$(m+a)\ddot{z} + b \cdot \dot{z} + c \cdot z = F_w$$

$$z = z_a \cos(\omega t + \varepsilon_{z,\zeta})$$

$$\dot{z} = -z_a \omega \sin(\omega t + \varepsilon_{z,\zeta})$$

$$\ddot{z} = -z_a \omega^2 \cos(\omega t + \varepsilon_{z,\zeta})$$

$$\left(c - \omega^2 (m+a)\right) \cdot z_a \cos(\omega t + \varepsilon_{z,\zeta}) + b \cdot -z_a \omega \sin(\omega t + \varepsilon_{z,\zeta}) = F_{wa} \cos(\omega t + \varepsilon_{F_w,\zeta})$$

Now solve the equation for the unknown motion amplitude z_a and phase angle $\varepsilon_{z,\zeta}$

Equation of motion

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

Now solve the equation for the unknown motion amplitude z_a and phase angle $\varepsilon_{z,\zeta}$ for 1 frequency

System is linear

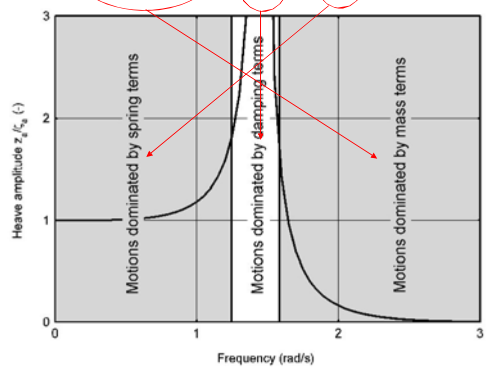
If wave amplitude doubles → wave force doubles → motion doubles

$$(m+a)\cdot\frac{\ddot{z}}{\zeta_a}+b\cdot\frac{\dot{z}}{\zeta_a}+c\cdot\frac{z}{\zeta_a}=\frac{F_W}{\zeta_a}$$

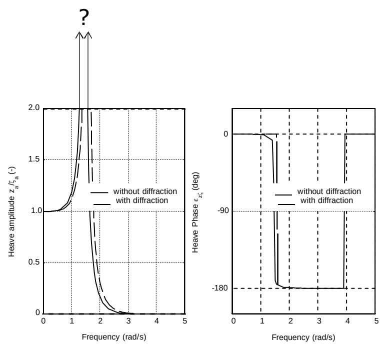
Substitue solution $\frac{z}{\zeta_a}=\frac{z_a}{\zeta_a}\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$ and solve RAO and phase

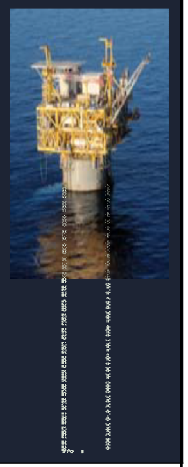
RAO

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

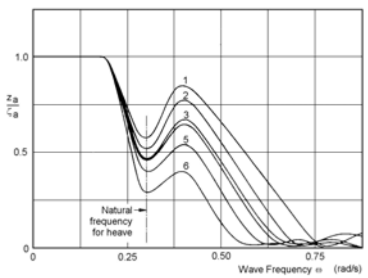


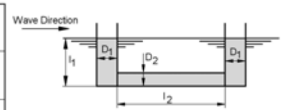
Calculated RAO spar with potential theory





Frequency Response of semi-submersible





Platform	Dimensions (m)			
	D_1	l_1	D_2	l_2
1	11.11	16.52	10.06	111.10
2	11.11	20.71	8.91	133.50
3	11.11	25.82	8.02	155.80
4	25.00	31.61	17.33	150.00
5	25.00	38.12	15.30	175.00
6	25.00	55.06	11.62	225.00

[Source: Hooft, 1970]

What is 'linear' ???

R

1. Linear waves:
   - 'nice' regular harmonic (cosine shaped) waves
   - Wave steepness small: free surface boundary condition satisfied at mean still water level
     - Pressures and fluid velocities are proportional to wave elevation and have same frequency as elevation
2. linearised wave exciting force:
   - Wave force independent of motions
   - Wave force only on mean wetted surface

L

3. Motion amplitudes are small
   - Restoring force proportional to motion amplitude
   - Hydrodynamic reaction forces proportional to motion amplitude

Motions are proportional to wave height !

Motions have same frequency as waves

Learning goals Module II, behavior of floating bodies in waves

• Definition of ship motions **Motion Response in regular waves:** • How to use RAO's • Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces • How to solve RAO's from the equation of motion **Motion Response in irregular waves:** • ~~How to determine response in irregular waves from RAO's and wave spectrum without forward speed~~	Ch.6
3D linear Potential Theory •How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential •How to determine Velocity Potential	Ch. 7
Motion Response in irregular waves: • How to determine response in irregular waves from RAO's and wave spectrum with forward speed • Determine probability of exceedence • Make down time analysis using wave spectra, scatter diagram and RAO's	Ch. 8
Structural aspects: • Calculate internal forces and bending moments due to waves	Ch. 8
Nonlinear behavior: • Calculate mean horizontal wave force on wall • Use of time domain motion equation	Ch.6

Learning goals Module II, behavior of floating bodies in waves

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2D Potential theory (strip theory) p. 7-12 until p. 7-35 SKIP THIS PART

Calculating hydrodynamic coeffiecients and diffraction force

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w=F_{FK}+F_D$$

m and c = piece of cake

F_{FK} = almost easy

a, b, and F_D = kind of difficult ⟶ Ch. 7

Calculating hydrodynamic coeffiecients and diffraction force

p7-4 course notes

- Radiation: $-a_3\ddot{z}-b_3\dot{z}$
- Incoming wave
- Diffraction
- Hydrostatic buoyancy: $-c_3\cdot z$

$$m\ddot{z}=\sum F=F_{r3}+F_{w3}+F_{d3}+F_{s3}$$

$$(m+a)\ddot{z}+b\dot{z}+cz=F_{w3}+F_{d3}$$

Next slides we'll consider the left hand side of this motion equation: we will try to write the hydrodynamic reaction force F, that the structure feels as a result of its motions in such a way that we can incorporate them in the well known motion equation of a damped mass-spring system.

Calculating hydrodynamic coeffiecients and diffraction force

$$m\ddot{z}=\sum F=F_{r3}+F_{w3}+F_{d3}+F_{s3}$$

Radiation Force: $F_{r3}=-a_3\ddot{z}-b_3\dot{z}$

To calculate force: first describe fluid motions due to given heave motion by means of radiation potential:

Potential theory

Radiation potential

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w+F_d$$

Radiation potential heave $\Phi_3(x,y,z,t)$

= flow due to heave motion

Knowing the potential, calculating resulting force is straight forward:

$$\bar{F}=-\iint_S (p\cdot\bar{n})\,dS$$

$$\bar{M}=-\iint_S p\cdot(\bar{r}\times\bar{n})\,dS$$

$$p=-\rho\frac{\partial\Phi}{\partial t}$$

$$\bar{F}=\iint_S\left(\rho\frac{\partial\Phi}{\partial t}\cdot\bar{n}\right)dS$$

$$\bar{M}=\iint_S\rho\frac{\partial\Phi}{\partial t}\cdot(\bar{r}\times\bar{n})\,dS$$

Potential theory

Radiation potential

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w+F_d$$

Radiation potential heave $\Phi_3\left(x,y,z,t\right)$

= flow due to motions, larger motions → 'more' flow

Problem: But we don't know the motions !! (we need the flow to calculate the motions…and we need the motions to calculate the flow…)

Solution: radiation potential is written as function of motion:

Potential theory

Radiation potential

Solution: radiation potential is written as function of velocity of the motion

$$\Phi_3(\underline{x},t)=\Re\{\phi_3(\underline{x})\cdot v_3(t)\}$$

P7-5 eq. 7.17

Only space dependent — **Only time dependent**

Suppose we would know the velocity potential due to heave motion: Φ_3
Assuming linearity this will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Let's define the amplitude and the phase angle of this potential function to be related to the velocity of the heave motion ($\dot{z}$ or in complex notation: v_3).
So we write the potential function Φ_3 as a complex product of:
ϕ_3 (which can be considered as a complex transfer function between potential and heave velocity) and the heave velocity v_3

Potential theory

Radiation potential

$$\Phi_3(\underline{x},t)=\Re\{\phi_3(\underline{x})\cdot v_3(t)\}$$

Only space dependent — **Only time dependent**

Complex notation:

$$s_3\left(t\right)=s_{a3}\cdot e^{-i\omega t}$$

$$v_3\left(t\right)=\dot{s}\left(t\right)=-i\omega s_{a3}\cdot e^{-i\omega t}$$

$$z\left(t\right)=z_a\cos\left(\omega t+\varepsilon_{z,\zeta}\right)=\Re\left\{z_a e^{-i\varepsilon_{z,\zeta}}e^{-i\omega t}\right\}=\Re\left\{s_{a3}e^{-i\omega t}\right\}$$

$$v_3\left(t\right)=-i\omega z_a e^{-i\varepsilon_{z,\zeta}}e^{-i\omega t}=-i\omega s_{a3}e^{-i\omega t}$$

s_{a3} Complex heave motion amplitude

v_{a3} Complex heave velocity amplitude

Potential theory

Let's consider Heave motion:

$$\Phi_3(\underline{x},t)=\Re\{\phi_3\cdot v_{a3}\cdot e^{-i\omega t}\}=\Re\{\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}\}$$

($-i\omega\cdot s_{a3}\cdot e^{-i\omega t}$ = $v_3(t)$)

v_{a3} = complex amplitude of heave velocity

s_{a3} = complex amplitude of heave displacement

Potential not necessarely in phase with heave velocity v_3 →

ϕ_3 = <u>complex</u> amplitude of heave radiation potential (devided by $-i\omega s_{a3}$)

Remember:
Φ_3 will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Suppose that at a certain location, this function has a phase angle ε related to the heave velocity and the ratio between its amplitude and the amplitude of the heave velocity is a.

Verify that in that case:
$\phi_3=ae^{-i\varepsilon}$

Potential theory

Let's consider forces and moments due to heave motion

$$\vec{F}_{r3} = \iint_S \left(\rho \frac{\partial \Phi_3}{\partial t} \cdot \vec{n} \right) dS$$

$$\Phi_3(x,t) = \phi_3 \cdot v_{a3} \cdot e^{-i\omega t} = \phi_3 \cdot -i\omega \cdot s_{a3} \cdot e^{-i\omega t}$$

$$\vec{M}_{r3} = \iint_S \rho \frac{\partial \Phi_3}{\partial t} \cdot (\vec{r} \times \vec{n})\, dS$$

$$\vec{F}_{r3} = \iint_S \left(\rho \frac{\partial \left(\phi_3 \cdot -i\omega \cdot s_{a3} \cdot e^{-i\omega t}\right)}{\partial t} \cdot \vec{n} \right) dS$$

$$\vec{M}_{r3} = \iint_S \rho \frac{\partial \left(\phi_3 \cdot -i\omega \cdot s_{a3} \cdot e^{-i\omega t}\right)}{\partial t} \cdot (\vec{r} \times \vec{n})\, dS$$

Potential theory

some re writing, considering only heave force due to heave motion:

$$F_{r33} = \Re\left\{ \iint_S \left(\rho \frac{\partial \left(\phi_3 \cdot -i\omega \cdot s_{a3} \cdot e^{-i\omega t}\right)}{\partial t} \cdot n_3 \right) dS \right\}$$

Only space dependent

Only time dependent

This 3 component force vector $\vec{F}$ is what we call the hydrodynamic reaction force that the structure experiences due to its heave motion.

$$= \Re\left\{ -\rho \cdot i\omega \cdot s_{a3} \iint_S \phi_3 \cdot \frac{\partial e^{-i\omega t}}{\partial t} \cdot n_3 \cdot dS \right\}$$

$$= \Re\left\{ -\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t} \right\}$$

Potential theory

Radiation Force due to heave motion is 3 component vector:

$$F_{r13} = \Re\left\{ -\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_1 \cdot dS \cdot e^{-i\omega t} \right\}$$ Surge force due to heave motion

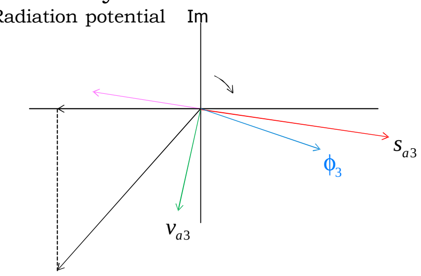
$$F_{r23} = \Re\left\{ -\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_2 \cdot dS \cdot e^{-i\omega t} \right\}$$ Sway force due to heave motion

$$F_{r33} = \Re\left\{ -\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t} \right\}$$ Heave force due to heave motion

In the following, only heave force due to heave motion is considered: F_{r33}

Potential theory

Radiation potential



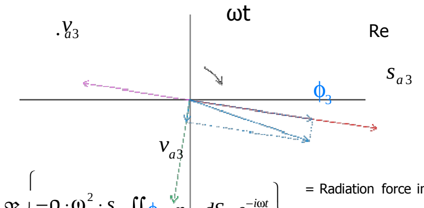
= Radiation force in heave direction,

$$m \cdot \ddot{z} + c \cdot z + F_{r33} = F_{w3} + F_{d3} =$$

$$m \cdot \ddot{v}_{3} \cdot e^{-i\omega t} + c \cdot s_{a3} \cdot e^{-i\omega t} + F_{r33}$$

Potential theory

Radiation potential



$F_{r33} = \Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S \phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}$ = Radiation force in heave direction, due to heave motion

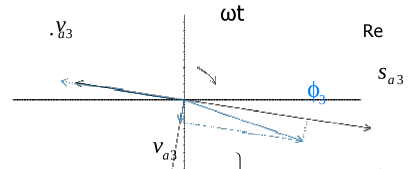
$m\ddot z + c\cdot z + F_{r33} = F_{w3} + F_{d3}$

$\Re\left\{m\cdot\dot v_{a3}\cdot e^{-i\omega t} + c\cdot s_{a3}\cdot e^{-i\omega t} + F_{r33}\right\} = F_{w3} + F_{d3}$

$-a\cdot\ddot z = \Re\left\{-a\cdot\dot v_{a3}\cdot e^{-i\omega t}\right\}$ | $-b\cdot\dot z = \Re\left\{-b\cdot v_{a3}\cdot e^{-i\omega t}\right\}$

Potential theory

Radiation potential



$F_{r33} = \Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S \phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\} = \Re\left\{-a\cdot\dot v_{a3}\cdot e^{-i\omega t} - b\cdot v_{a3}\cdot e^{-i\omega t}\right\}$

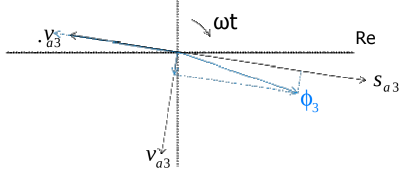
$= \Re\left\{a\cdot\omega^2\cdot s_{a3}\cdot e^{-i\omega t} + b\cdot i\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$

$= \Re\left\{a\omega^2\cdot s_{a3}\cdot e^{-i\omega t} + ib\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$

$\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S \phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\} = \Re\left\{a\omega^2\cdot s_{a3}\cdot e^{-i\omega t} + ib\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$

Potential theory

Radiation potential



$= \Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S \phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\} = \Re\left\{a\omega^2\cdot s_{a3}\cdot e^{-i\omega t} + ib\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$

After dividing by $s_{a3}\cdot e^{-i\omega t}$

Both Im and Re part have to be equalled!

$-\rho\omega^2\iint_S \phi_3\cdot n_3\cdot dS = a\omega^2 + ib\omega$

$a = -\rho\Re\left\{\iint_S \phi_3\cdot n_3\cdot dS\right\}$

$b = -\rho\omega\Im\left\{\iint_S \phi_3\cdot n_3\cdot dS\right\}$

Potential theory

Since the hydrodynamic reaction force depends on the motions (which is why we wrote the radiation potential as a function of the velocity of the motion in the first place) we want it to appear at the left hand side of the motion equation.

Remember that the motion equation for an ordinary damped mass spring system is:
$m\ddot z + b\dot z + cz = F$

Let's consider the hydrodynamic heave reaction force due to heave motion and write it as:
$F_3 = -a\ddot z - b\dot z$ That way we end up with a motion equation almost identical to what we already know from 'ordinary' dynamics.

heave:

$F_3 = \Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S \phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}$ **Heave force due to heave motion**

This force has a certain phase angle with respect to motion:

part in phase with motion acceleration is: $-\Re\left\{\frac{\partial^2\left(s_{a3}\cdot e^{-i\omega t}\right)}{\partial t^2}\right\}\cdot a_{33} = -\ddot z\, a_{33}$

part in phase with motion velocity is $-\Re\left\{\frac{\partial\left(s_{a3}\cdot e^{-i\omega t}\right)}{\partial t}\right\}\cdot b_{33} = -\dot z\, b_{33}$

+

F_3

$(m + a_{33})\ddot z + b_{33}\dot z + c\cdot z = F_W$

Potential theory

heave:

$$\frac{\partial\; s_{a3}\cdot e}{\partial t^2}\cdot a_{33} = \ddot{z}\, a_{33} + \frac{\partial\left(s_{a3}\cdot e^{-i\omega t}\right)}{\partial t}\cdot b_{33} = \dot{z}\, b_{33} = F_3$$

Verify this!

$$a_{33} = -\Re\left\{\rho \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$$

$$b_{33} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$$

p. 7-41

otential theory

resulting from heave motions, Φ_3

Forces	Moments
$a_{33} = -\Re\left\{\rho \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$	$a_{43} = -\Re\left\{\rho \iint_{S_0} \phi_3 \cdot (r\times n)_1\, dS_0\right\}$
$b_{33} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$	$b_{43} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot (r\times n)_1 \cdot dS_0\right\}$
$a_{13} = -\Re\left\{ \iint_{S} {}_{3} \; {}_{1} \; {}_{0}\right\}$	$a_{53} = -\Re\left\{ \iint_{S_0} {}_{3} \; - \; - \; {}_{2} \; {}_{0}\right\}$
$b_{13} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot n_1 \cdot dS_0\right\}$	$b_{53} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot (r\times n)_2 \cdot dS_0\right\}$
$a_{23} = -\Re\left\{\rho \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$	$a_{63} = -\Re\left\{\rho \iint_{S_0} \phi_3 \cdot (r\times n)_3 \cdot dS_0\right\}$
$b_{23} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$	$b_{63} = -\Im\left\{\rho\omega \iint_{S_0} \phi_3 \cdot (r\times n)_3 \cdot dS_0\right\}$

Solving the Laplace equation

coupled equation of motion:

$$\begin{bmatrix} M+a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & M+a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & M+a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & I_{xx}+a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & I_{yy}+a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & I_{zz}+a_{66} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \\ \varphi \\ \theta \\ \psi \end{Bmatrix} = \begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$$

Sources images

[1] Towage of SSDR Transocean Amirante, source: Transocean
[2] Tower Mooring, source: unknown
[3] Rogue waves, source: unknown
[4] Bluewater Rig No. 1, source: Friede & Goldman, LTD/GNU General Public License
[5] Source: unknown
[6] Rig Neptune, source: Seafarer Media
[7] Pieter Schelte vessel, source: Excalibur
[8] FPSO design basis, source: Statoil
[9] Floating wind turbines, source: Principle Power Inc.
[10] Ocean Thermal Energy Conversion (OTEC), source: Institute of Ocean Energy/Saga University
[11] ABB generator, source: ABB
[12] A Pelamis installed at the Agucadoura Wave Park off Portugal, source: S.Portland/Wikipedia
[13] Schematic of Curlew Field, United Kingdom, source: offshore-technology.com
[14] Ocean Quest Brave Sea, source: Zamakona Yards
[15] Medusa, A Floating SPAR Production Platform, source: Murphy USA