Electronic Power Conversion

Review of Basic Electrical and Magnetic Circuit Concepts

3. Review of Basic Electrical and Magnetic Circuit Concepts

- Notation
- Electric circuits
  - Steady state – sinusoidal, non-sinusoidal
  - Power – apparent, real, reactive
  - Power factor, THD, harmonics
- Magnetic circuits
  - Ampere's law, Faraday's law
  - Magnetic reluctance and magnetic circuits
  - Inductor
  - Transformer

Notation

- Notation:
  - Lowercase – instantaneous value e.g. v_o ($=v_0(t)$)
  - Uppercase – average or rms value e.g. V_o
  - Bold – phasor, vector

- Steady state:
  - State of operation where waveforms repeat with a period T that is specific for the system.

Instantaneous, Average Power and RMS current

- Instantaneous power

$$p = v\,i$$

- Average power

$$P_{av} = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T v\,i\,dt$$

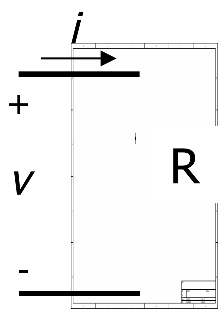
- Power to resistive load

$$P_{av,R} = R\,\frac{1}{T}\int_0^T i^2 dt$$

or

$$P_{av,R} = R\,I^2$$

$$P_{av,R} = \frac{V^2}{R}$$



- I – RMS (Root-mean-square)

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

- Example

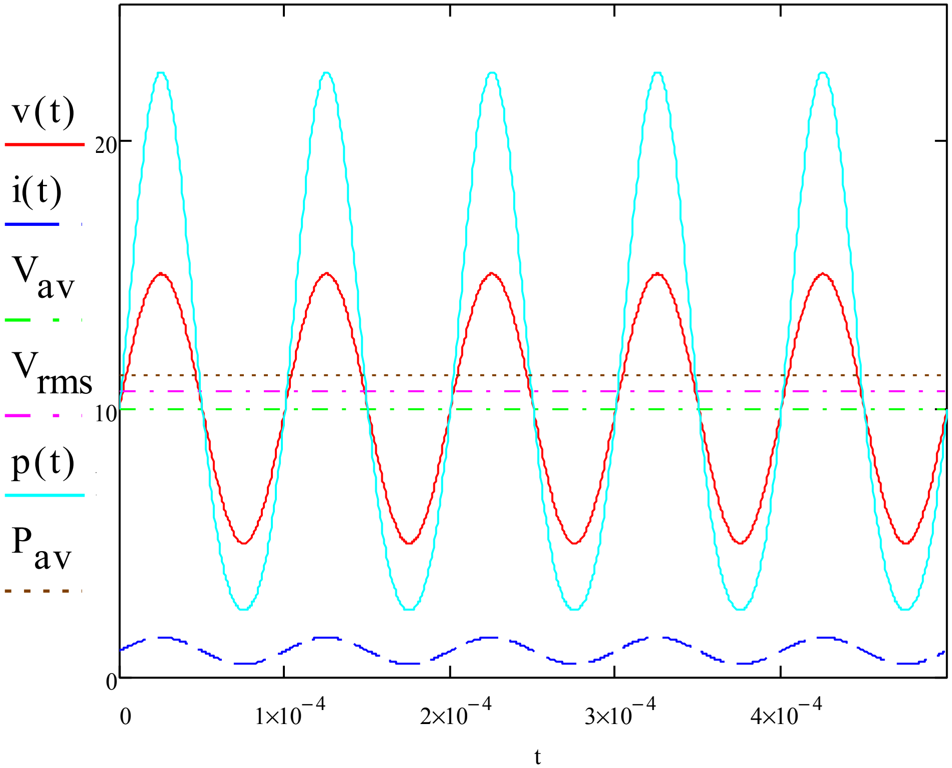
$$v = 10V + 5V\sin(2\cdot\pi\cdot 10kHz\cdot t)$$

$$V_{av} = 10V$$

$$V_{rms} = 10.607V$$

$$R = 10\Omega$$

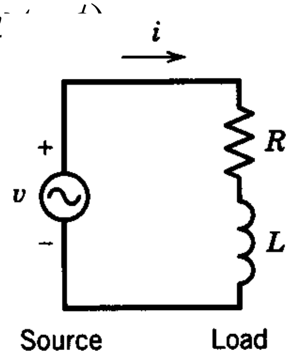
$$P_{av} = 11.25W$$

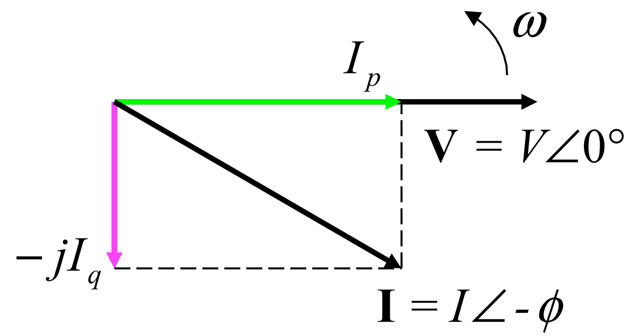


Steady-State ac Sinusoidal Waveforms

- Resistive/inductive load

$v(t) = \sqrt{2}\, V\ \cos\omega_1 t \qquad i(t) = \sqrt{2}\, I\ \cos(\omega$





- Phasor representation

$\mathbf{V} = V\, e^{j0} \qquad \mathbf{I} = I\, e^{-j\phi}$

Complex impedance

$$\mathbf{Z} = R + j\omega L = Ze^{j\phi}$$

$$\mathbf{V} = \mathbf{I}\cdot\mathbf{Z}$$

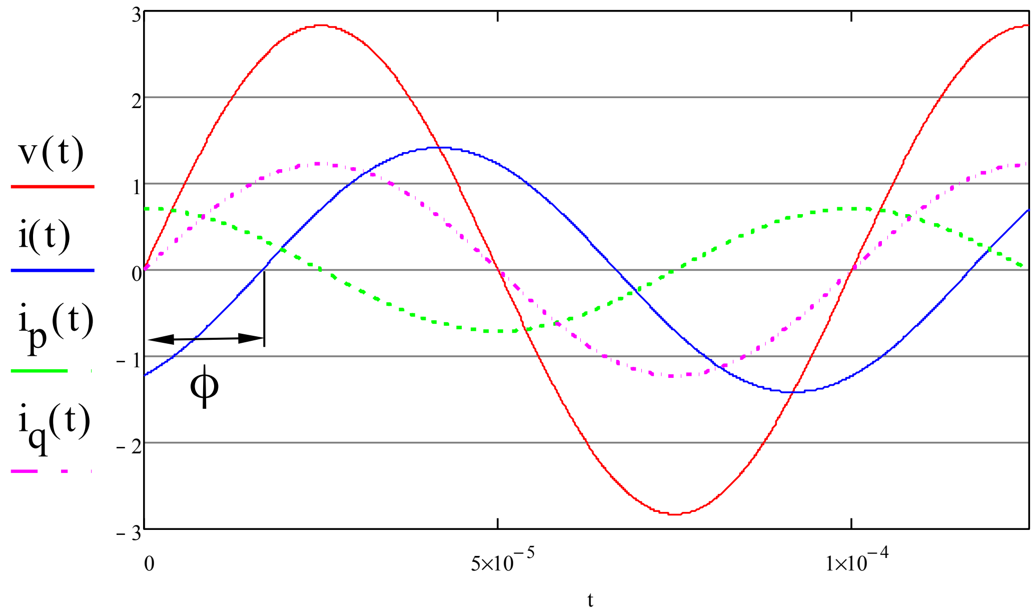
$$i\ (t) = i_p(t) + i_q(t)$$

$$i_p(t) = \sqrt{2} I_p \cos\omega t$$

$$= (\sqrt{2} I\cos\phi)\cos\omega t$$

$$i_q(t) = \sqrt{2} I_q \sin\omega t$$

$$= (\sqrt{2} I\sin\phi)\sin\omega t$$



Complex, Real and Reactive Power

- Complex power [VA]

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = Se^{j\phi}$$

- Apparent power

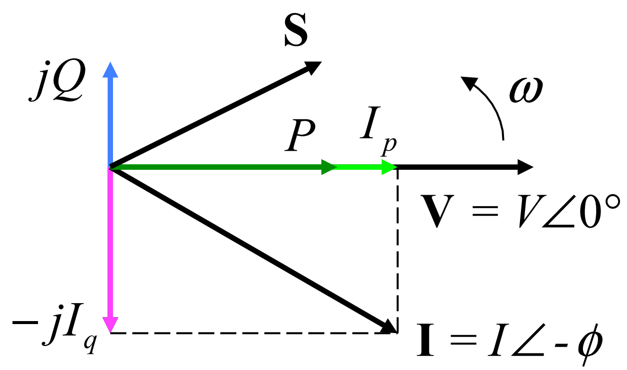
$$S = V\,I$$

- Real average power [W]

$$P = \mathrm{Re}[\mathbf{S}] = \mathrm{VI}\cos\phi$$

- Reactive power [VAr]

$$Q = \mathrm{Im}[\mathbf{S}] = \mathrm{VI}\sin\phi$$



$$\mathbf{S} = P + jQ$$

- Only I_p is responsible for power transfer from source to load!

Complex, Real and Reactive Power

- Physical meaning
  - Apparent power *S* influences cost and size:
    - Insulation level and magnetic core size depend on V
    - Conductor size depends on I
  - Real power *P* – useful work and losses;
  - Reactive power *Q* – preferably zero.

Power Factor *(arbeidsfactor)*

- Power factor
  - How effectively is energy transferred between source and load

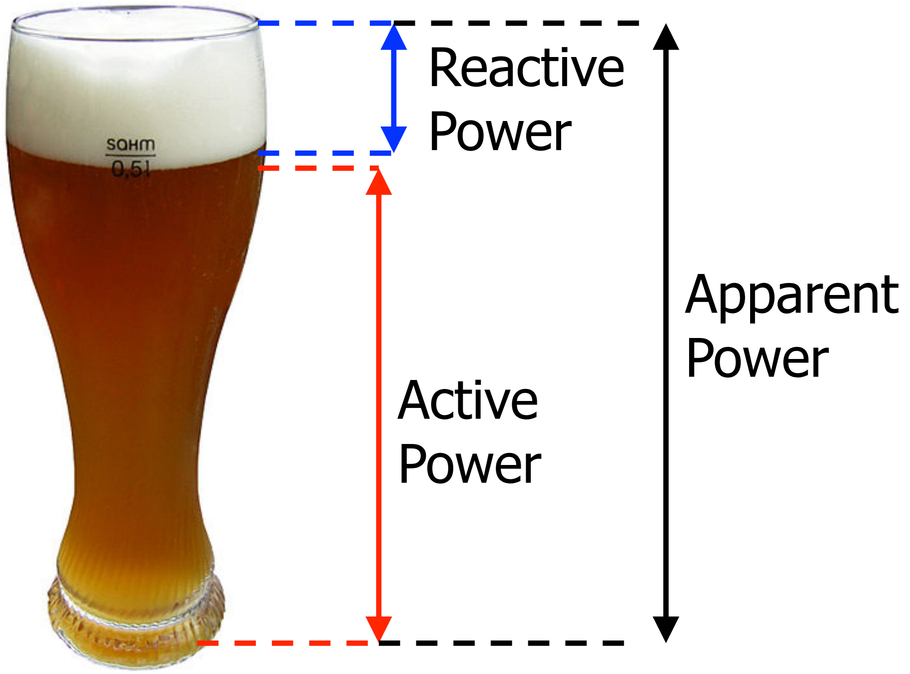
$$PF \, @ \frac{P}{S}$$

  - For sinusoidal systems:

$$PF = \frac{P}{S} = \frac{V \cdot I \cdot \cos\phi}{V \cdot I} = \cos\phi$$

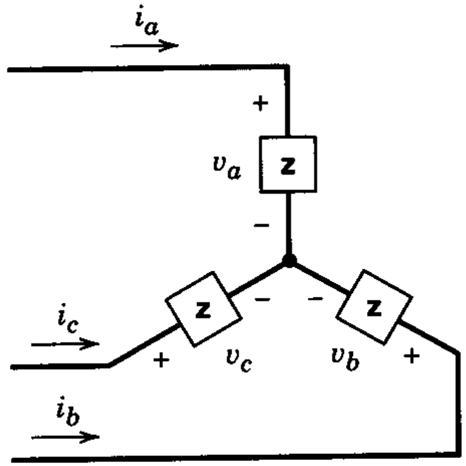
Power Factor – Beer Mug Analogy

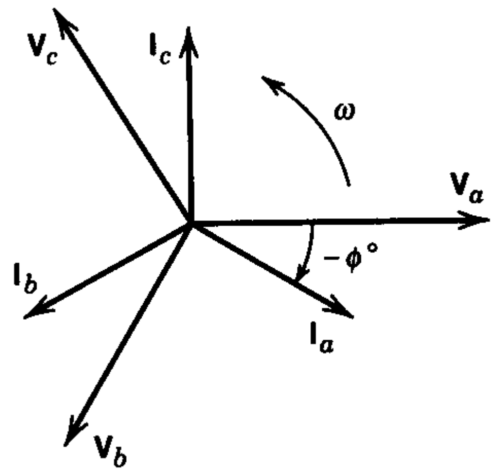
- Glass (over)sized to hold beer *and* foam
- Power wires (over)sized for Watts *and* VArs



Source: Wikimedia Commons CC-BY-SA

Three Phase Circuits





$$V_{LL} = \sqrt{3}V$$

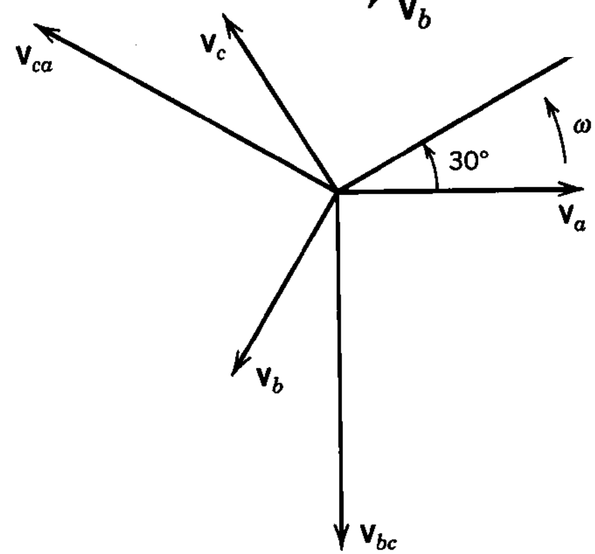
$$P_{ph} = VI\cos\varphi$$

$$S_{ph} = VI$$

$$S_{3ph} = 3VI = \sqrt{3}V_{LL}I$$

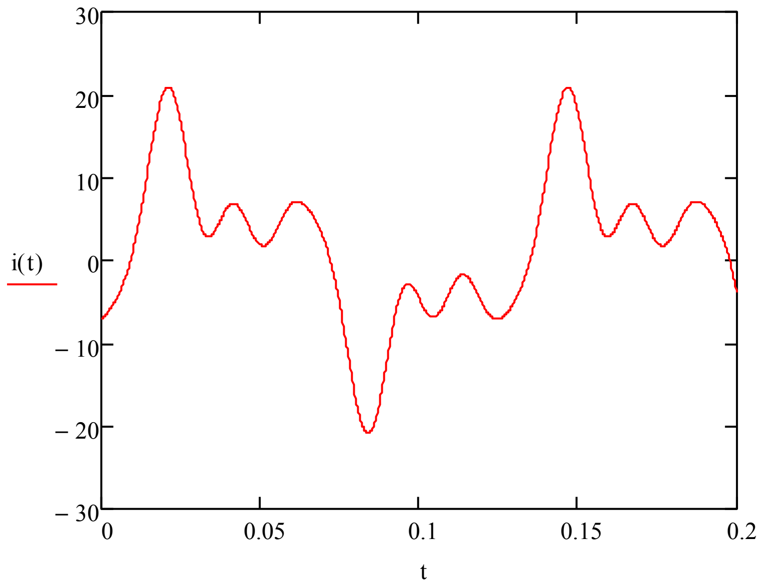
$$P_{3ph} = 3VI\cos\varphi = \sqrt{3}V_{LL}I\cos\varphi$$

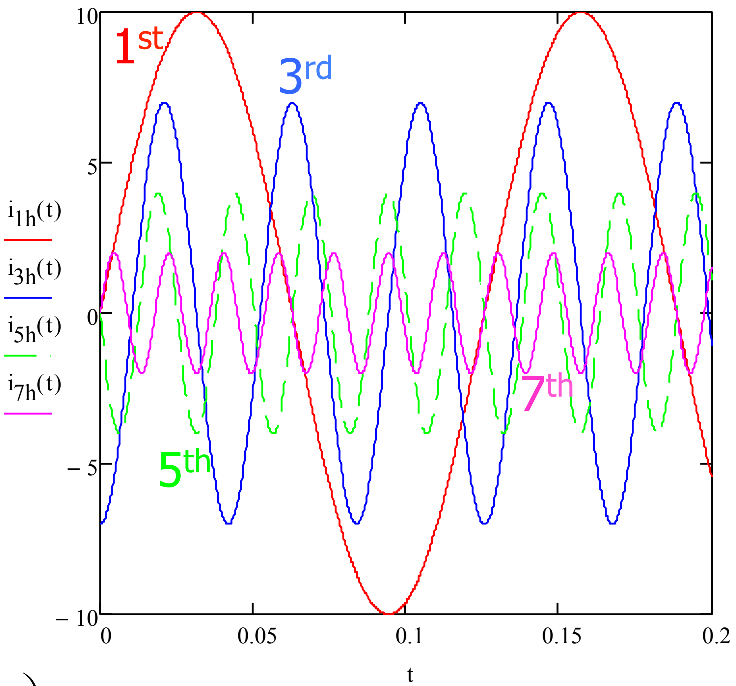
$$PF = \cos\phi$$



Steady-State Non-Sinusoidal Waveforms

- Periodic non-sinusoidal waveform
  - Represented as sum of its harmonics





$$i(t) := 10A\,\sin(\omega_1 \cdot t) + 7 \cdot A \cdot \sin\left(3\omega_1 \cdot t - \frac{\pi}{2}\right) - 4 \cdot A \cdot \sin(5\omega_1 \cdot t) + 2 \cdot A\,\sin(7\omega_1 \cdot t)$$

Fourier Analysis of Non-Sinusoidal Waveforms

- Non-sinusoidal periodic waveform *f(t)*

$$f(t) = F_0 + \sum_{h=1}^{\infty} f_h(t) = \frac{1}{2} \cdot a_0 + \sum_{h=1}^{\infty} \{a_h \cdot \cos(h\omega t) + b_h \cdot \sin(h\omega t)\}$$

where

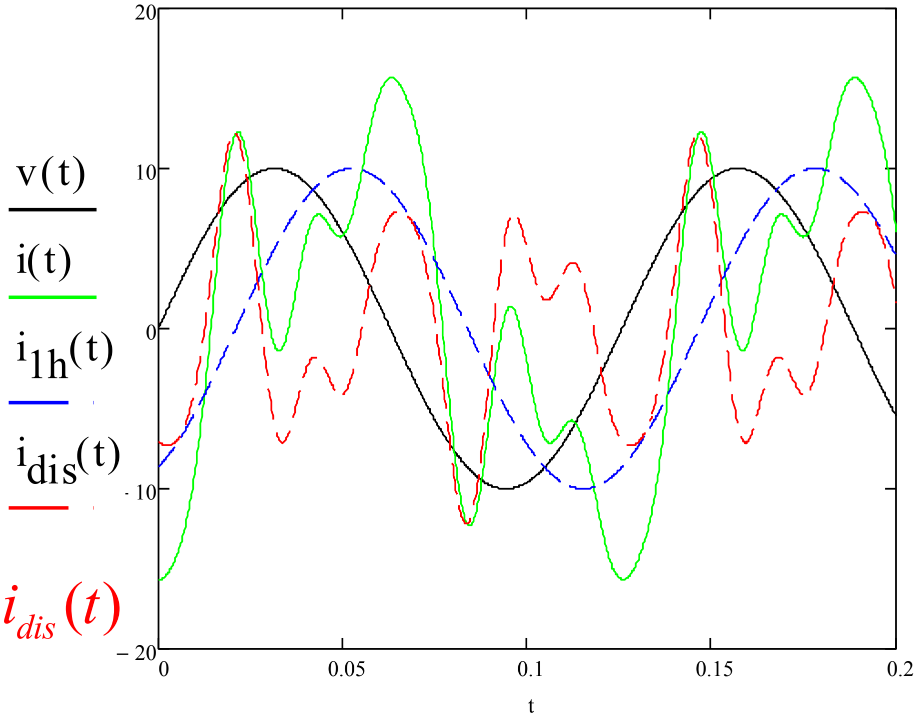
$$F_0 = \frac{1}{2} a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(t) d(\omega t) = \frac{1}{T} \int_0^{T} f(t) d(t)$$

$$a_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(h\omega t) d(\omega t) \qquad h = 1, \ldots, \infty$$

$$b_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(h\omega t) d(\omega t) \qquad h = 1, \ldots, \infty$$

Line Current Distortion

$$v_s(t) = \sqrt{2}\, V_s \sin \omega_1 t$$



$$i_s(t) = i_{s1}(t) + \sum_{h=2}^{\infty} i_{sh}(t) = i_{s1}(t) + i_{dis}(t)$$

$$i_s(t) = \sqrt{2}\, I_{s1} \sin(\omega_1 t - \phi_1) + \sum_{h=2}^{\infty} \sqrt{2}\, I_{sh} \sin(\omega_h t - \phi_h)$$

current i = fundamental harmonic i_{s1} + distortion current i_{dis}

Total Harmonic Distortion (THD)

- A measure of how much a composite current deviates from an ideal sine wave
- Caused by the way that electronic loads draw current

$$i_s(t) = \sqrt{2}\, I_{s1} \sin(\omega_1 t - \phi_1) + \sum_{h=2}^{\infty} \sqrt{2}\, I_{sh} \sin(\omega_h t - \phi_h)$$

$$I_s = \sqrt{\frac{1}{T_1} \int_0^{T_1} i_s^2(t)\, dt} \quad \text{or} \quad I_s = \sqrt{I_{s1}^2 + \sum_{h=2}^{\infty} I_{sh}^2}$$

and: $$I_{dis} = \sqrt{I_s^2 - I_{s1}^2} = \sqrt{\sum_{h=2}^{\infty} I_{sh}^2}$$

Total harmonic distortion: $$THD = \sqrt{\sum_{h=2}^{\infty} \left(\frac{I_{sh}}{I_{s1}} \right)^2}$$

Harmonics

- Negative effects of harmonics
  - Conductor overheating
  - Capacitors can be affected by heat rise increases due to power loss and reduced life time
  - Distorted voltage waveform
  - Increased losses (e.g. transformers overheating)
  - Fuses and circuit breakers fault operation
- Harmonic standards
  - International Electrotechnical Commission Standard IEC-555
  - IEEE/ANSI Standard 519

Power Factor

$$PF \triangleq \frac{P}{S}$$

with $P = \frac{1}{T_1}\int_0^{T_1} p(t)\,dt = \frac{1}{T_1}\int_0^{T_1} v_s(t)\, i_s(t)\,dt$ Real power (*vermogen*)

and $S = V_s\ I_s$ Apparent power (*schijnbaar vermogen*)

$$P = \frac{1}{T_1}\int_0^{T_1} \underbrace{\sqrt{2}\, V_s \sin(\omega_1 t)}_{v_s(t)} \cdot \underbrace{\sum_{h=1}^{\infty} \sqrt{2}\, I_{sh} \sin(\omega_h t - \phi_h)}_{i_s(t)}\, dt$$

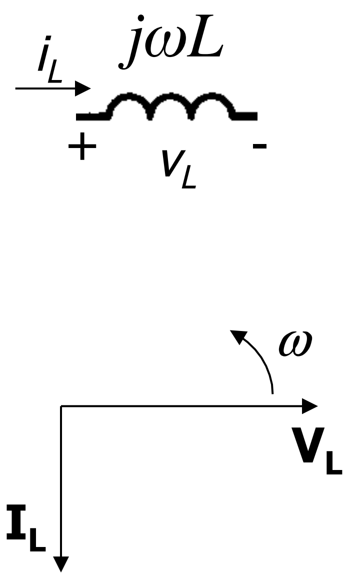
$$P = \frac{1}{T_1}\int_0^{T_1} \sqrt{2}\, V_s \sin(\omega_1 t) \cdot \sqrt{2}\, I_{s1} \sin(\omega_1 t - \phi_1)\, dt = V_s I_{s1}\ \cos\phi_1$$

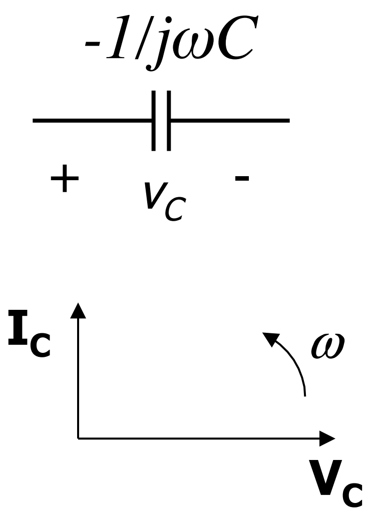
$$PF = \frac{V_s I_1 \cos\phi_1}{V_s I_s} = \frac{I_{s1}}{I_s} \cos\phi_1$$

DPF

Inductor and Capacitor Phasor Diagrams

- Phasors are <u>only</u> applicable to sinusoidal steady state waveforms.





Inductor and Capacitor Response

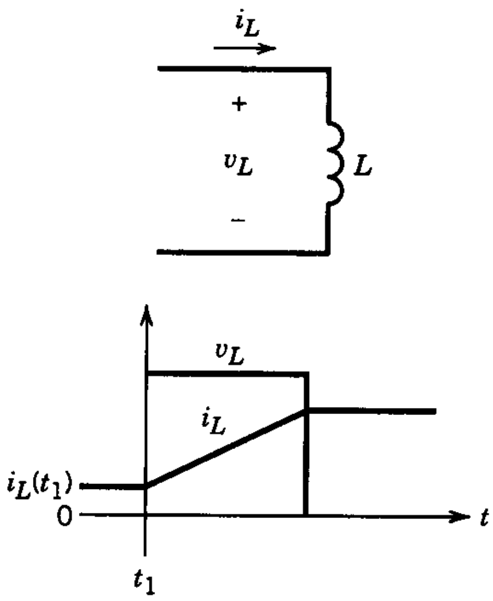
- Time domain:

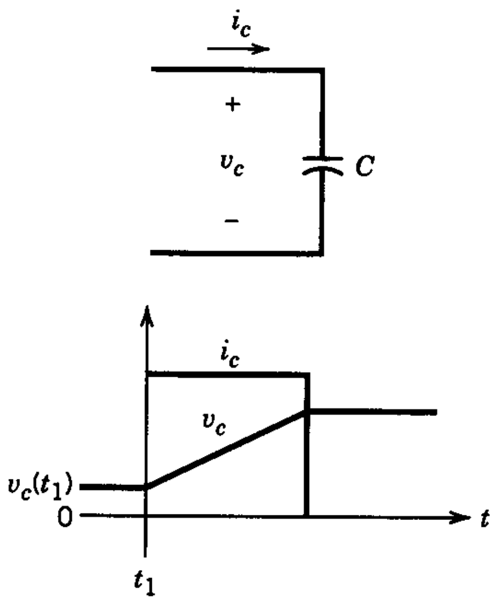
$$v_L(t) = L \cdot \frac{di_L}{dt}$$

$$i_C(t) = C \cdot \frac{dv_C}{dt}$$

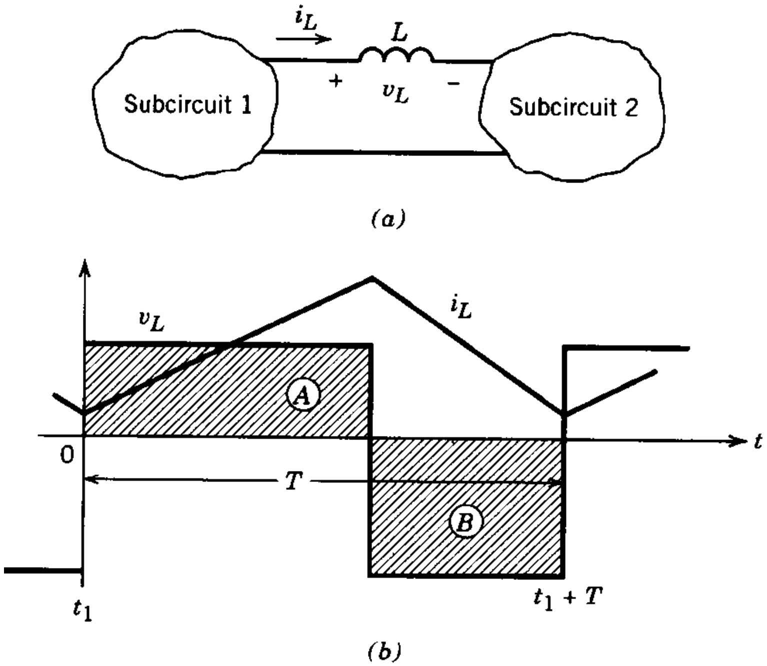
$$i_L(t) = i_L(t_1) + \frac{1}{L}\int_{t_1}^{t} v_L(t)\, dt$$

$$v_c(t) = v_c(t_1) + \frac{1}{C}\int_{t_1}^{t} i_c(t)\, dt$$





Inductor in Steady State
Volt-second Balance



Steady state implies:

$$v(t+T)=v(t)$$

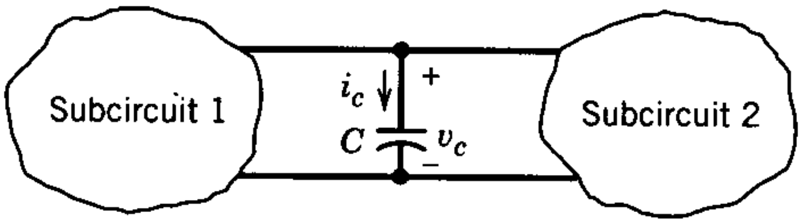
$$i(t+T)=i(t)$$

$$i_L(t)=i_L(t_1)+\frac{1}{L}\int_{t_1}^{t} v_L(t)\,dt \quad \Rightarrow \quad \int_{t_1}^{t_1+T} v_L(t)\,dt = 0$$

$$\boxed{v_{L,av} = 0}$$

Net change in flux is zero

Capacitor in Steady State
Amp-second balance



$$v_c(t) = v_c(t_1) + \frac{1}{C}\int_{t_1}^{t} i_c(t)\,dt$$

$$\int_{t_1}^{t_1+T} i_C(t)\,dt = 0 \qquad \boxed{i_{C,av} = 0}$$

Net change in charge is zero

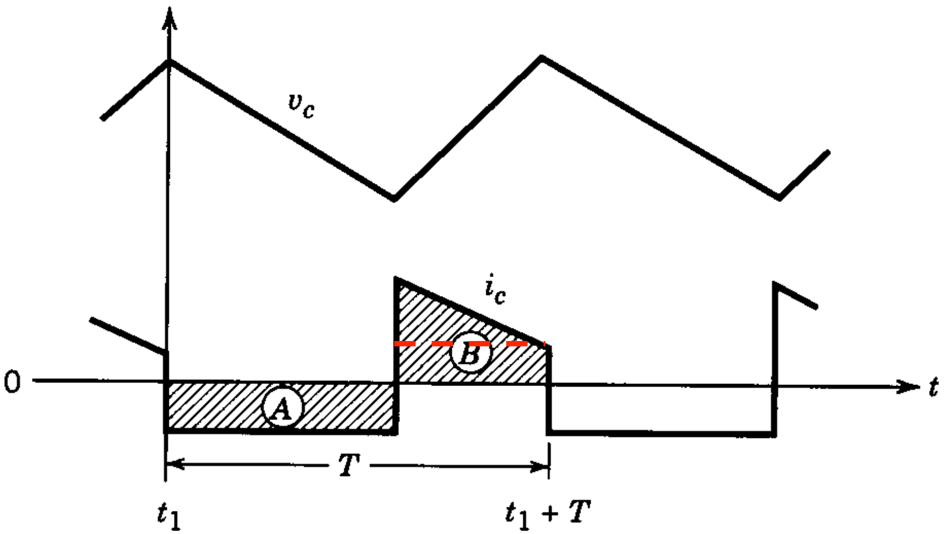
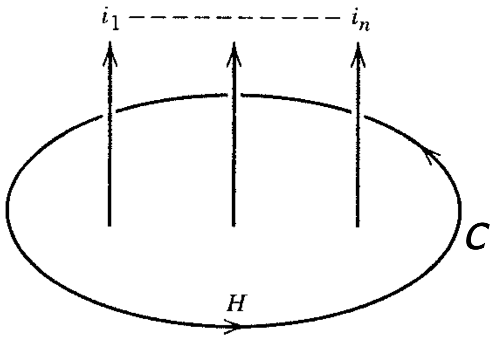


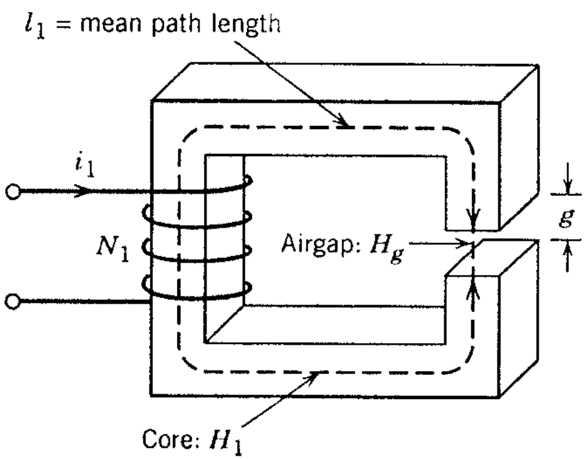
Fig. 3-9
Note: error in fig 3-9 from textbook i_C in the second interval should be constant,
-----
because v_c has a constant derivative in that interval

Basic Magnetics Theory

Ampere's law
(4th Maxwell's equation)



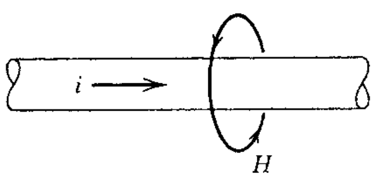
$$\oint_C \mathbf{H}\, d\mathbf{l} = \sum i$$

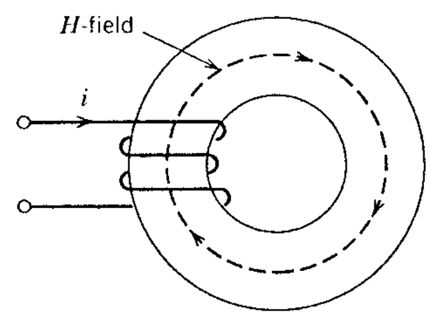


$$\sum_{K\ \text{core sections}} H_k l_k = \sum_{M\ \text{windings}} N_m I_m$$

$$H_i l + H_g l_g = N_1 i$$

Right-hand rule:

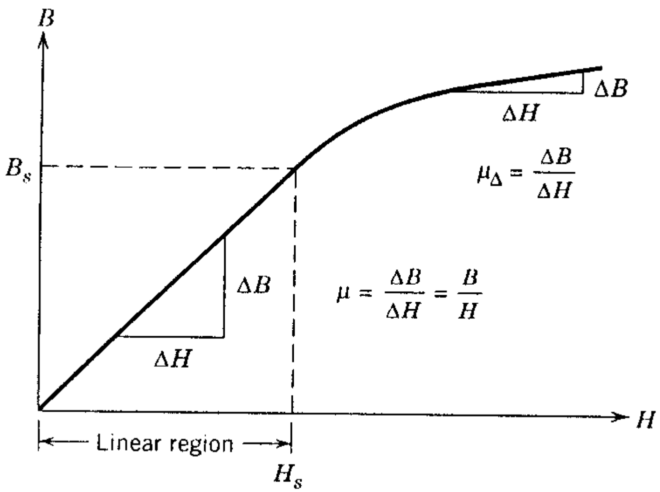




H-field, B-field and Material Properties

- Relationship between **B** and **H** given by:
  - material properties
  - often approximated by:

$$\mathbf{B} = \mu \mathbf{H}$$

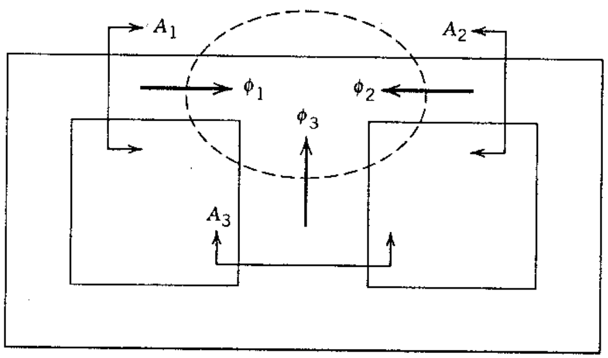


- Continuity of flux

Flux (by def.): $\phi = \iint_A \mathbf{B} \cdot d\mathbf{A}$

$$\phi_{closedArea} = \oint\!\!\oint_A \mathbf{B} \cdot d\mathbf{A} = 0$$

Continuity of flux:
Gauss's law of magnetism
(2^{nd} Maxwell's equation)



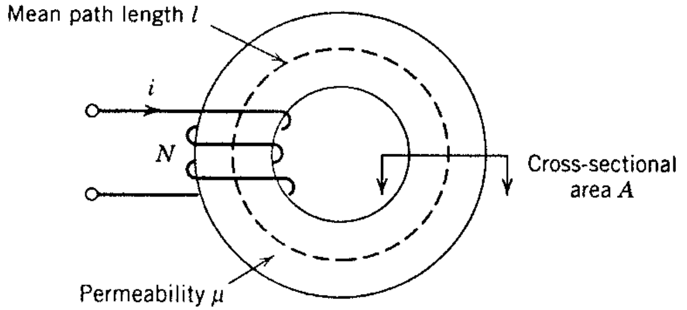
Magnetic Reluctance

Ampere's law:

$$\sum_m N_m I_m = \sum_k H_k l_k = \sum_k (B_k A_k) \frac{l_k}{\mu_k A_k} = \sum_k \phi_k \frac{l_k}{\mu_k A_k} = \phi \sum_k \frac{l_k}{\mu_k A_k} = \phi \cdot \Re_k$$

$$\mathbf{B} = \mu \, \mathbf{H}$$

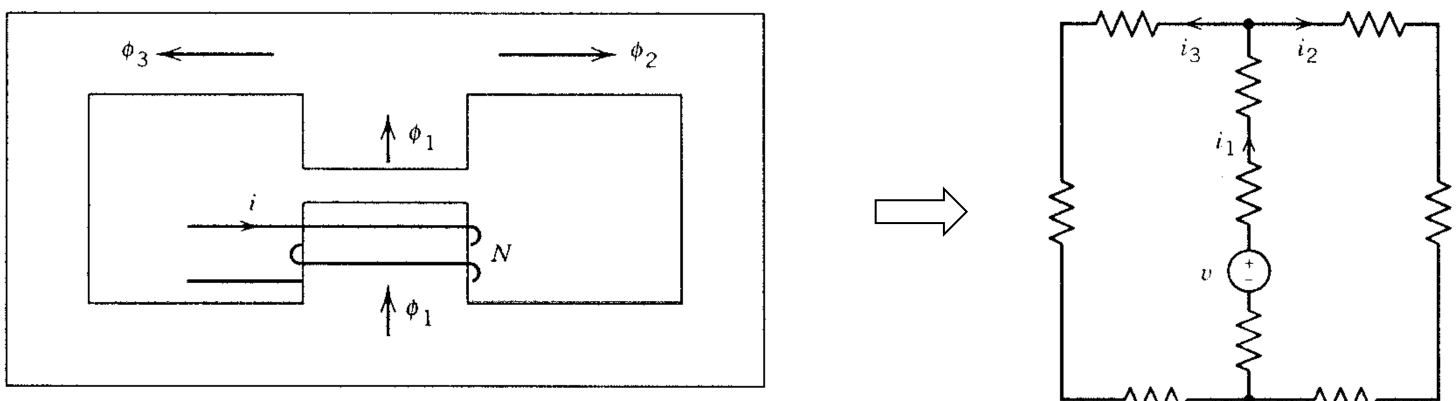
Magnetic reluctance: $\Re_k = \frac{l_k}{\mu_k A_k}$



Magnetic	Electrical
$\Re_k$	R
ϕ	i
Ni	v

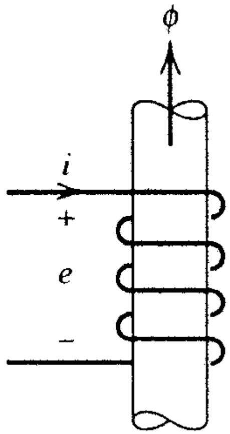
Magnetic Circuit Analysis

Magnetic	Electrical	
$Ni = \phi \cdot \Re_k$	$v = i \cdot R$	Ohm's law:
$\sum_m N_m I_m = \phi \sum_k \Re_k$	$\sum_m v_m = i \sum_k R_k$	Kirchhoff's voltage law
$\sum_k \phi_k = 0$	$\sum_k i_k = 0$	Kirchhoff's current law

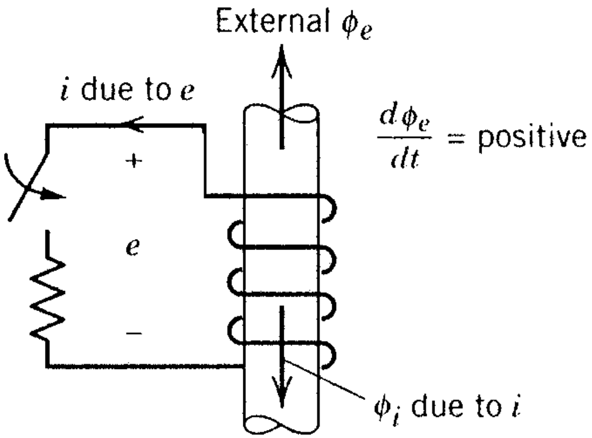


Faraday's law

$$e = \frac{d\Psi}{dt} = \frac{d(N\phi)}{dt} = NA\frac{dB}{dt}$$



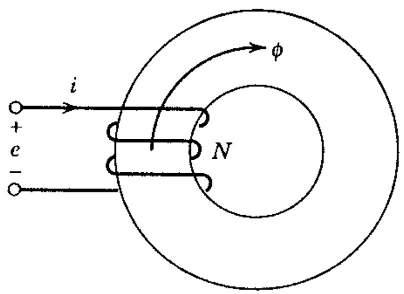
(a)



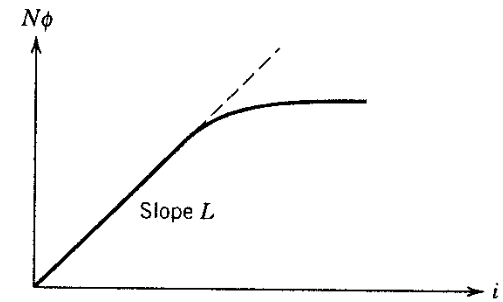
(b)

Inductor

- Coil self-inductance



(a)



(b)

$$\left.\begin{array}{l} L \,@\, \dfrac{N\phi}{i} \\ e = \dfrac{d(N\phi)}{dt} \end{array}\right\} \rightarrow \quad e = L\frac{di}{dt}$$

$$\left.\begin{array}{l} L = \dfrac{N\phi}{i} \\ \phi\,\Re = N\,i \end{array}\right\} \rightarrow \quad L = \frac{N^2}{\Re} = \frac{\mu\, N^2 A}{l}$$

$$\uparrow$$

$$\Re = \frac{l}{\mu A}$$

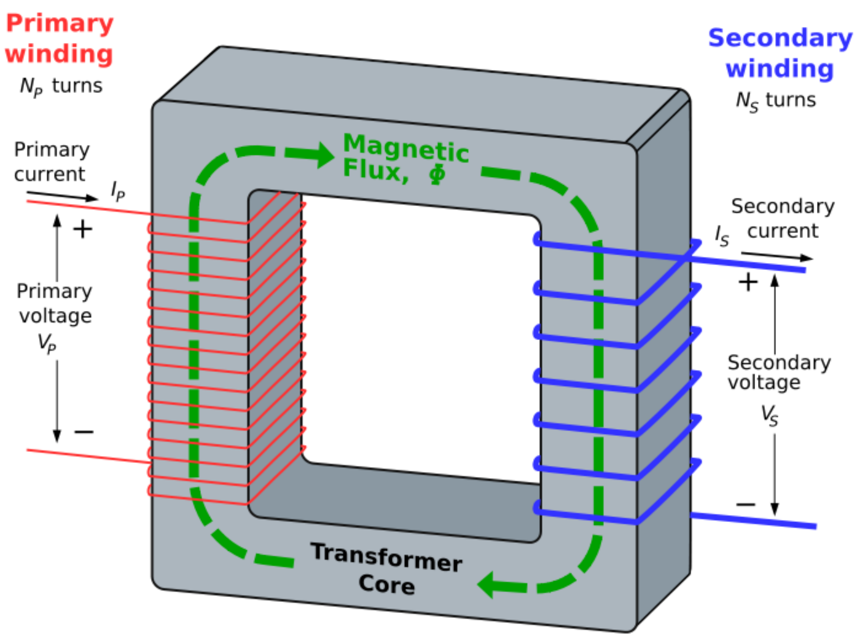
Transformer

- Ideal transformer

Faraday's law

$$v_1 = N_1 \cdot \frac{d\phi}{dt} \qquad v_2 = N_2 \cdot \frac{d\phi}{dt}$$

$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2}}$$

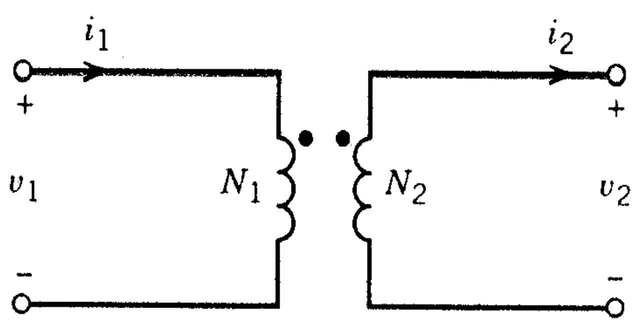


Ampere's law

$$\sum_{i=1,2} N_i i_i = \phi \cdot \Re \qquad N_1 i_1 - N_2 i_2 = \phi \cdot \Re$$

$$\boxed{i_1 N_1 = i_2 N_2}$$

Core reluctance $\Re = \frac{l}{\mu A} \approx 0$

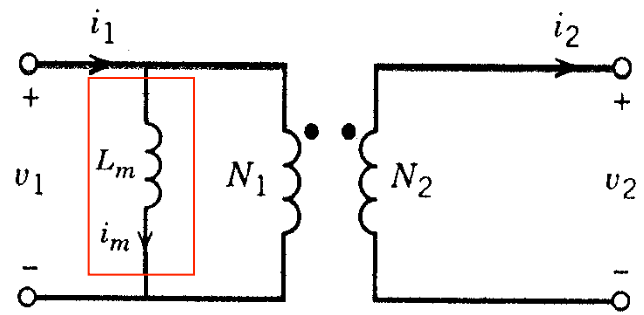


Transformer

- Magnetising inductance

$$\Re = \frac{l}{\mu A} \neq 0$$

Non-zero
core reluctance



$$\left.\begin{aligned} N_1 i_1 - N_2 i_2 &= \phi \cdot \Re \\ v_1 &= N_1 \cdot \frac{d\phi}{dt} \end{aligned}\right\} \quad v_1 = \frac{N_1}{\Re} \cdot \frac{d}{dt}\left(i_1 - \frac{N_2}{N_1} i_2\right) = L_m \cdot \frac{d}{dt} i_m$$

$$L_m = \frac{N_1^2}{\Re}$$

$$i_m = i_1 - \frac{N_2}{N_1} i_2$$

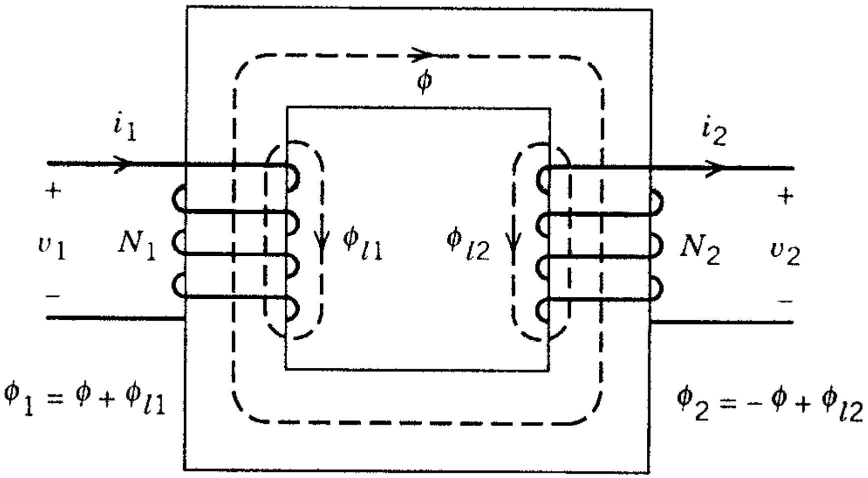
Transformer

- Leakage flux

$\phi_{l1,2}$ – leakage flux

$\phi_1 = \phi + \phi_{l1}$

$\phi_2 = -\phi + \phi_{l2}$



$R_{1,2}$ – ohmic resistances of the windings

$$v_1 = R_1 i_1 + N_1 \frac{d\phi_1}{dt} = R_1 i_1 + N_1 \left(\frac{d\phi_{l1}}{dt} + \frac{d\phi}{dt} \right)$$

$$v_2 = -R_2 i_2 - N_2 \frac{d\phi_2}{dt} = -R_2 i_2 - N_2 \left(\frac{d\phi_{l2}}{dt} - \frac{d\phi}{dt} \right)$$

Equivalent Transformer Circuit

$$v_1 = R_1 i_1 + N_1 \frac{d\phi_1}{dt} = R_1 i_1 + N_1 \left(\frac{d\phi_{l1}}{dt} + \frac{d\phi}{dt} \right) \quad L = N \cdot \phi / i \quad v_1 = R_1 i_1 + L_{l1} \frac{di_1}{dt} + L_m \frac{di_m}{dt}$$

$$v_2 = -R_2 i_2 - N_2 \frac{d\phi_2}{dt} = -R_2 i_2 - N_2 \left(\frac{d\phi_{l2}}{dt} - \frac{d\phi}{dt} \right) \quad \Rightarrow \quad v_2 = -R_2 i_2 - L_{l2} \frac{di_2}{dt} + \frac{N_2}{N_1} L_m \frac{di_m}{dt}$$

$$\Rightarrow \quad v_1 = R_1 i_1 + L_{l1} \frac{di_1}{dt} + e_1$$

$$i_1 = i_m + \frac{N_2}{N_1} i_2 \qquad v_2 = -R_2 i_2 - L_{l2} \frac{di_2}{dt} + e_2$$

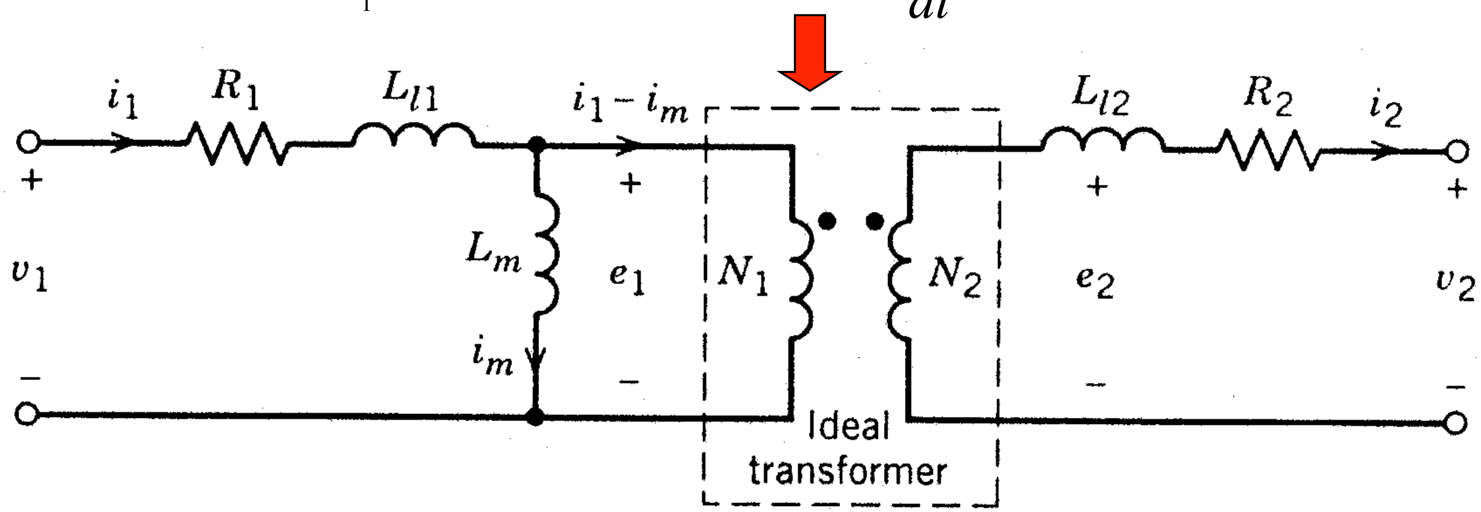


Image credits

- All uncredited diagrams are from the book “Power Electronics: Converters, Applications, and Design” by N. Mohan, T.M. Undeland and W.P. Robbins.
- All other uncredited images are from research done at the EWI faculty.