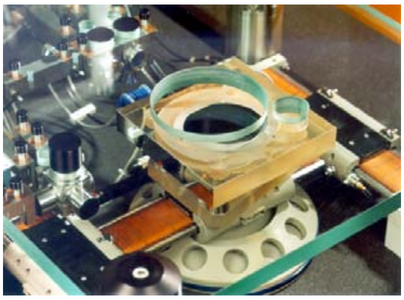
Mechatronic system design

Mechatronic system design wb2414-2012/2013
Course part 3

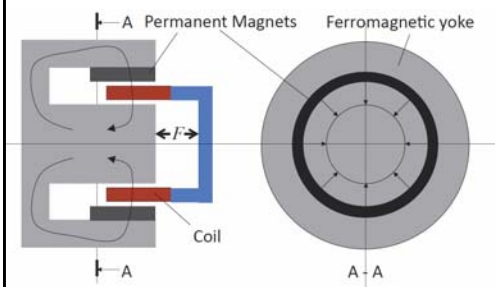


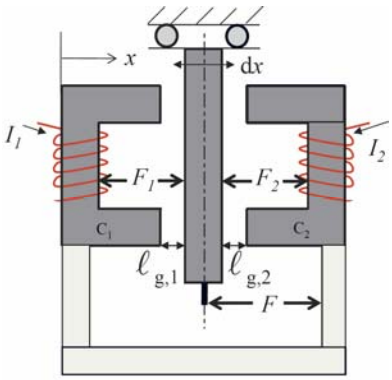
Electromagnetic actuators

Prof.ir. R.H.Munnig Schmidt
Mechatronic System Design

Learning goals. The student:

- Can select and calculate a single axis functional electromagnetic actuator for a given specification, working according to the Lorentz or reluctance force generation principle.

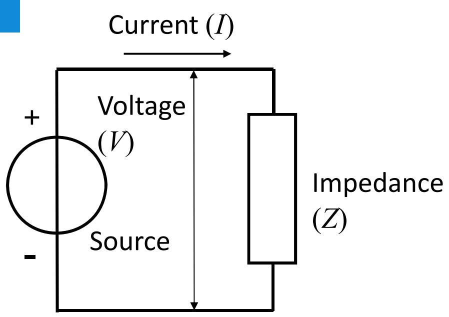




Contents

- Electricity and electromagnetism
- Electromagnetic actuators
  - Lorentz actuator
  - Variable reluctance actuator
  - Hybrid actuator

Ohm's law, the definition of impedance



$$V = IR \qquad I = \frac{V}{R}$$

Power in Watt (W):

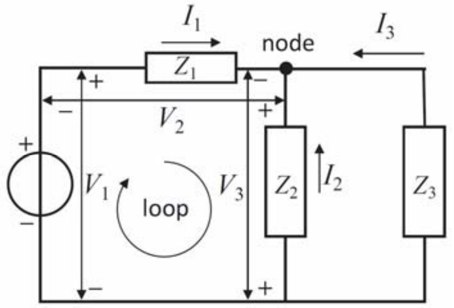
$$P = IV = I^2 R = \frac{V^2}{R}$$

Complex impedance $Z(f)$

$$V(f) = I(f)Z(f) \qquad I(f) = \frac{V(f)}{Z(f)}$$

Rules of Kirchhoff (network theory)

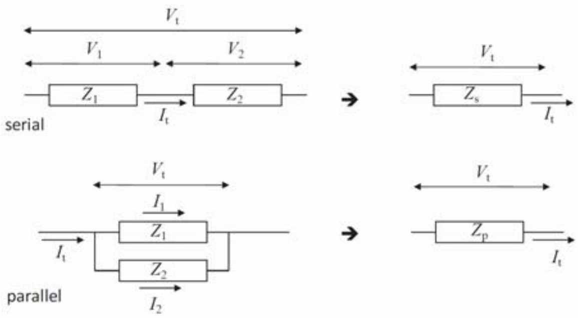
1. At any node of an electronic circuit all currents add to zero.
   - No charge storage in a node
2. Following any loop in an electronic circuit all voltages add to zero.



$$I_1 + I_2 + I_3 = 0$$
$$V_1 + V_2 + V_3 = 0$$

Combination of Impedances

Serial
- Current is shared,
- Voltage is divided



$$Z_s = \frac{V_t}{I_t} = \frac{V_1 + V_2}{I_t} =$$
$$= \frac{I_t Z_1 + I_t Z_2}{I_t} = Z_1 + Z_2$$

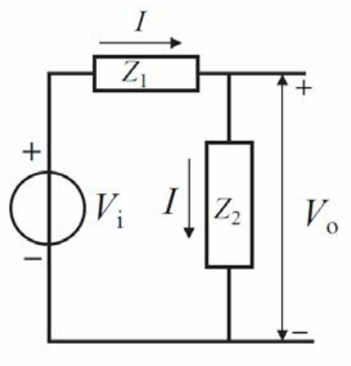
Parallel
- Voltage is shared
- current is divided

$$Z_p = \frac{V_t}{I_t} = \frac{V_t}{I_1 + I_2} = \frac{V_t}{\frac{V_t}{Z_1} + \frac{V_t}{Z_2}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}}$$

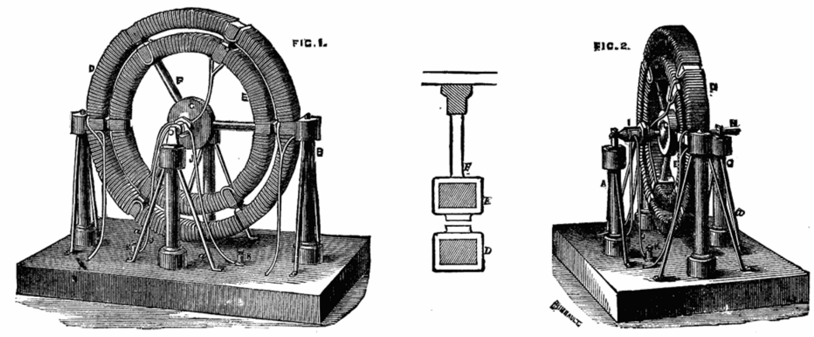
A voltage divider

$$I = \frac{V_i}{Z_1 + Z_2}$$

$$V_o = I \cdot Z_2 = \frac{V_i Z_2}{Z_1 + Z_2}$$



One of the oldest electromotors The Elias motor of 1842



An actuator is an electromotor for limited movements

Definition of terms in Maxwell's equations

$\mathbf{E}$ =	Electric field	[V/m]
$\mathbf{J}$ =	Electric current density	[A/m^2]
q =	Electric charge	[C]
ϵ_0 =	electric permittivity in vacuum	[As/Vm]
ρ_q =	Electric charge density	[C/m^3]
$\mathbf{B}$ =	Magnetic field	[T]
B =	Magnetic flux density	[T]
Φ =	Magnetic flux	[W]
μ =	Magnetic permeability	[Vs/Am]
$\mathbf{H}$ =	Magnetizing field	[A/m]
H =	Magnetic field strength	[A/m]

Maxwell equations for magnetics

$\mathbf{E}$ =	Electric field	[V/m]
$\mathbf{J}$ =	Electric current density	[A/m^2]
q =	Electric charge	[C]
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H =	Magnetic field strength	[A/m]

Gauss law (magnetic):

$$\oiint_S (\mathbf{B}\cdot\hat{\mathbf{n}})\,\mathrm{d}S = 0$$

$$\mathrm{div}\mathbf{B} = \nabla\cdot\mathbf{B} = 0$$

Faraday's law:

$$\oint_L \mathbf{E}\cdot\mathrm{d}l = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S (\mathbf{B}\cdot\mathbf{n})\,\mathrm{d}S$$

$$\mathrm{rot}\mathbf{E} = \nabla\times\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$$

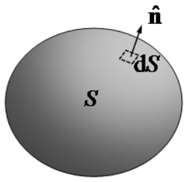
Ampère's law:

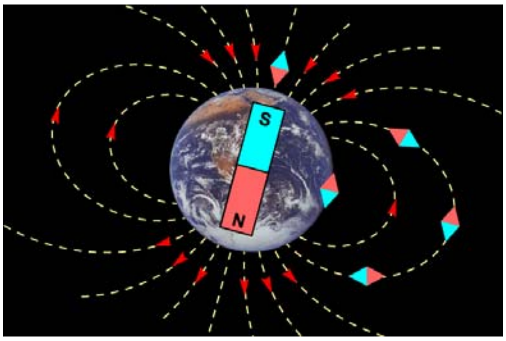
$$\oint_L \mathbf{B}\cdot\mathrm{d}l = \mu_0 I + \epsilon_0\mu_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_S (\mathbf{E}\cdot\mathbf{n})\,\mathrm{d}S$$

$$\mathrm{rot}\mathbf{B} = \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \epsilon_0\mu_0\frac{\partial}{\partial t}\mathbf{E}$$

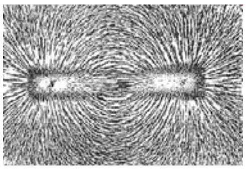
Gauss Law, magnetic fieldlines are closed loops

$$\oint\!\!\oint_S (\mathbf{B} \cdot \hat{\mathbf{n}})\, \mathrm{d}S = 0$$





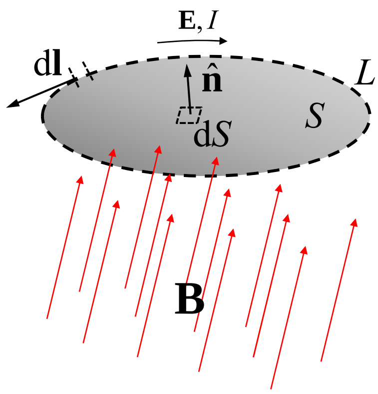
- A magnetic field has its origin in a **dipole**, North and South pole
- Density of fieldlines is proportional to flux density



Faraday's law, a changing magnetic field causes an electric field over a wire

$$\oint_L \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_S (\mathbf{B} \cdot \mathbf{n})\, \mathrm{d}S$$

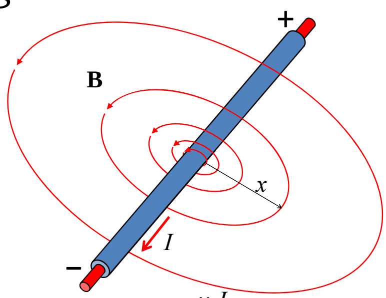
The line-integral of the electrical field over a closed loop L equals the change of the flux through the open surface S bounded by the loop L. This is a voltage source (EMF), where the current is driven in the direction of the electric field.



Ampere's law, Current through a wire gives a magnetic field

Not relevant for electromagnetic actuators

$$\oint_L \mathbf{B}\cdot \mathrm{d}l = \mu_0 I + \epsilon_0\mu_0 \frac{\mathrm{d}}{\mathrm{d}t}\iint_S (\mathbf{E}\cdot\mathbf{n})\,\mathrm{d}S$$



The line-integral of the magnetic field over a closed loop L is proportional to the current through the surface S enclosed by the loop L (plus a surface-integral term related to electric fields)

$$B(x) = \frac{\mu_0 I}{2\pi x}$$

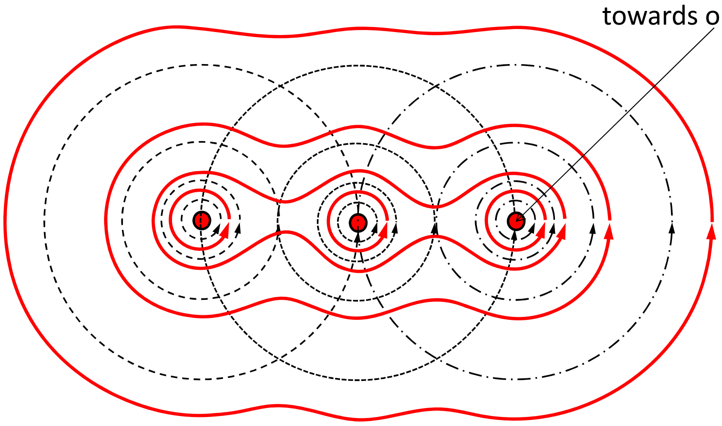
$$H(x) = \frac{B(x)}{\mu_0} = \frac{I}{2\pi x}$$

$$\mu_0 = 4\pi \cdot 10^{-7}\ [\mathrm{Vs/Am}]$$

in vacuum/air

The magnetic field of more current carrying wires

Current directed towards observer



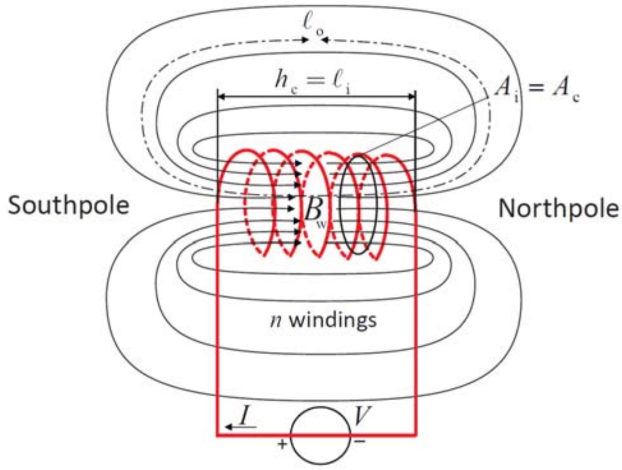
The total magnetic field is a linear vectorial combination of the magnetic field from each separate winding.

Average magnetic field from a coil

$$B_{\mathrm{w,av}} = \kappa \frac{\mu_0 n I}{h_\mathrm{c}}$$

$$H_{\mathrm{w,av}} = \frac{B_\mathrm{w}}{\mu_0} = \kappa \frac{nI}{h_\mathrm{c}}$$

$$\approx 0.3 < \kappa < 1$$

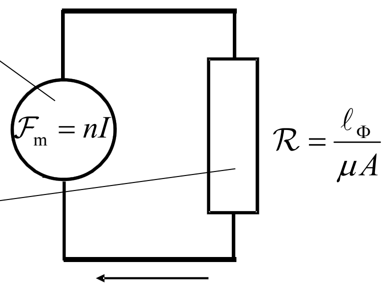


Infinitely long coil: $\kappa = 1$

Hopkinson's law of magnetics vs Ohm's law on electricity

Magnetomotive force ($\mathcal{F}_\mathrm{m}$) vs Electromotive force = Voltage ($\mathcal{F}_\mathrm{e} = V$)
Magnetic flux (Φ) vs electric current (I)
Magnetic reluctance ($\mathcal{R}$) vs electrical resistance (R)

- The Magnetomotive force is the amount of windings times the current
- The reluctance is proportional to the average length of the flux-path and inversely proportional to cross-section times the magnetic permeability.
- The magnetic flux follows Hopkinson's law of magnetics



$$\Phi_w = \frac{\mathcal{F}_\mathrm{m}}{\mathcal{R}} = \frac{nI}{\mathcal{R}}$$

The magnetic flux generated inside a coil

$$B_{w,av} = \frac{\Phi_{\mathrm{w}}}{A_{\mathrm{c}}} = \frac{\mathcal{F}_{\mathrm{m}}}{A_{\mathrm{c}}\mathcal{R}} = \frac{nI}{A_{\mathrm{c}}\mathcal{R}}$$

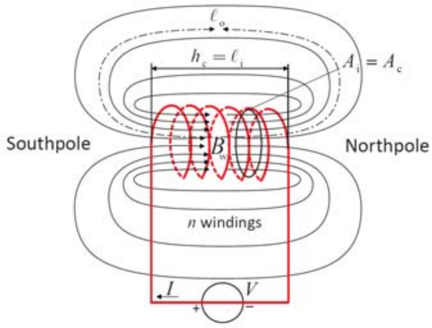
$$\mathcal{R} = \mathcal{R}_{\mathrm{i}} + \mathcal{R}_{\mathrm{o}} = \frac{\ell_{\mathrm{i}}}{A_{\mathrm{i}}\mu_0} + \frac{\ell_{\mathrm{o}}}{A_{\mathrm{o}}\mu_0} = \frac{\ell_{\mathrm{i}}}{\kappa A_{\mathrm{i}}\mu_0} \quad (\approx 0.3 < \kappa < 1)$$

This approximates to:

$$B_{\mathrm{w,av}} = \mu_0 H_{\mathrm{w,av}} = \kappa \frac{\mu_0 nI}{\ell_{\mathrm{i}}}$$

$$H_{\mathrm{w,av}} = \kappa \frac{nI}{\ell_{\mathrm{i}}}$$

Righthand rule: North and Southpole



Magnetic energy

The energy equals the integral of $B{*}H$, which are connected by μ_0

$$E_{\mathrm{m}} = \iiint_{V_\Phi} \int_0^{B_{\mathrm{xyz}}} H \mathrm{d}B \mathrm{d}V \quad [J]$$

$$H = \frac{B}{\mu_0} \quad \Rightarrow \quad E_{\mathrm{m}} = \int_0^{V_{\mathrm{i}}} \int_0^{B_{\mathrm{xyz}}} \frac{B}{\mu_0} \mathrm{d}B \mathrm{d}V \quad [J]$$

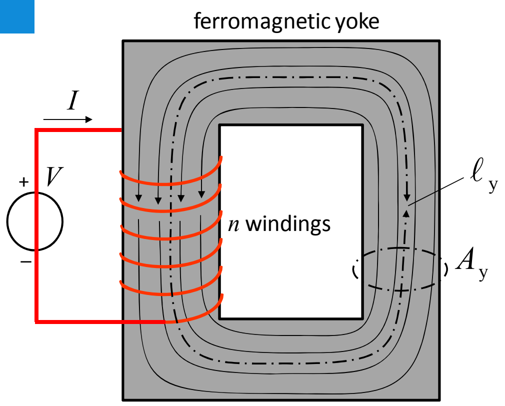
Applying some relations like $V_\Phi = A_\Phi \ell_{\mathrm{y}}$ and Hopkinson's law:

$$\int_0^{V_\Phi} \mathrm{d}V = \int_0^{\ell_{\mathrm{y}}} \iint_{A_\Phi} \mathrm{d}\ell \mathrm{d}A, \qquad \int_0^{\ell_{\mathrm{y}}} H_{\mathrm{w}} \mathrm{d}\ell = \mathcal{F}_{\mathrm{m}} = \Phi\mathcal{R}, \qquad \iint_{A_\Phi} \int_0^{B_{\mathrm{xyz}}} \mathrm{d}A \mathrm{d}B = \int_0^{\Phi_{\mathrm{w}}} \mathrm{d}\Phi$$

Results in :

$$E_{\mathrm{m}} = \iiint_{V_\Phi} \int_0^{B_{\mathrm{xyz}}} H \mathrm{d}B \mathrm{d}V = \int_0^{\Phi_{\mathrm{w}}} \mathcal{F}_{\mathrm{m}} \mathrm{d}\Phi = \int_0^{\Phi_{\mathrm{w}}} \Phi\mathcal{R} \mathrm{d}\Phi = \frac{1}{2}\Phi_{\mathrm{w}}^2 \mathcal{R} \triangleq \frac{1}{2}\frac{\mathcal{F}^2}{\mathcal{R}}$$

Adding a ferromagnetic material reduces the reluctance



$$\mathcal{F}_m = nI$$

$$\mathcal{R} = \frac{\ell_y}{\mu_0 \mu_r A_y}$$

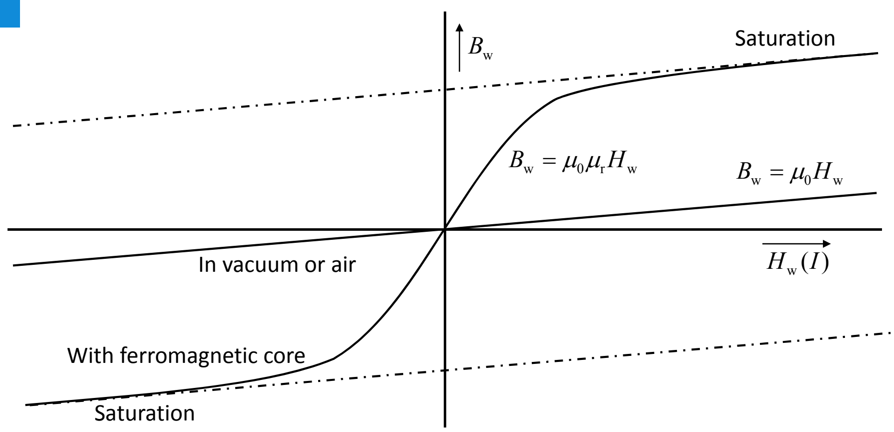
$$\Phi_y = \Phi_w = \frac{\mathcal{F}_m}{\Re} = \frac{A_y \mu_0 \mu_r nI}{\ell_y}$$

$$B_w = \frac{\Phi_y}{A_y} = \frac{\mu_0 \mu_r nI}{\ell_y} \quad \Rightarrow$$

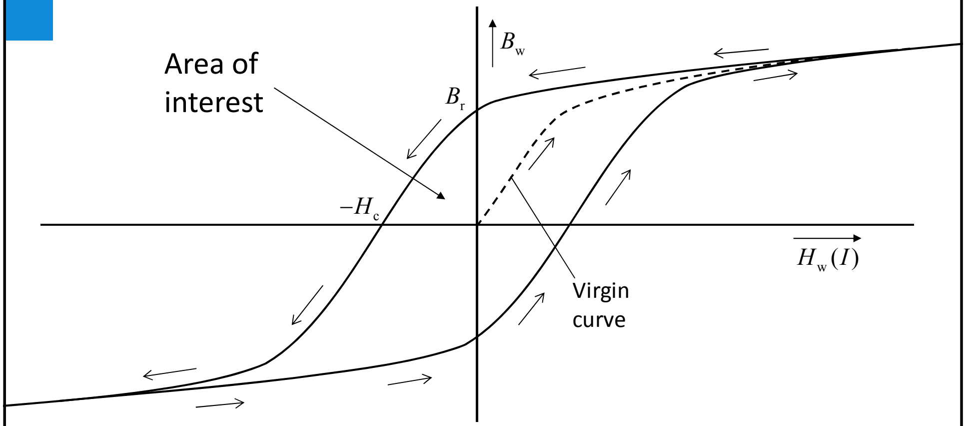
$$H_w = \frac{B_w}{\mu_0 \mu_r} = \frac{nI}{\ell_y}$$

With a large μ_r the flux increases

Magnetisation curve of ferromagnetic material (μ_r > 1)



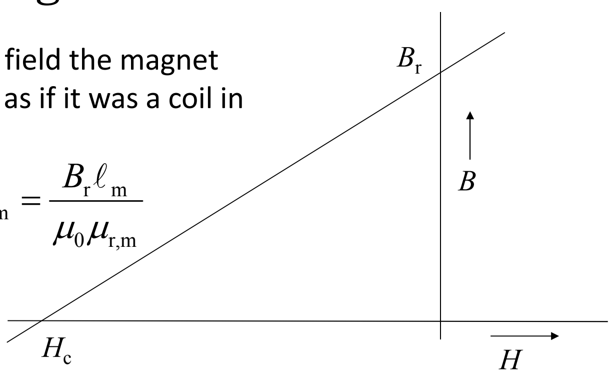
A permanent magnet is a ferromagnetic material with a high hysteresis



PM materials act like a current carrying coil

Without an external field the magnet creates its own field as if it was a coil in air

$$\mathcal{F}_m = nI_{equivalent} = H_c \ell_m = \frac{B_r \ell_m}{\mu_0 \mu_{r,m}}$$

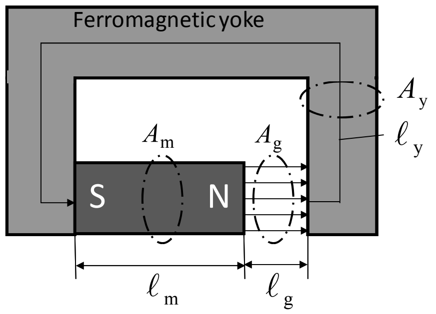


Modern PM $\mu_{r,m} = 1$

$$\mathcal{F}_m = nI_{equivalent} = H_c \ell_m = \frac{B_r \ell_m}{\mu_0}$$

$B_r = 1$ T is equivalent to 800000 Amp turns per meter length of magnet

Use a permanent magnet to create a magnetic field in an airgap.



A_y, ℓ_y, A_m, A_g, ℓ_m, ℓ_g

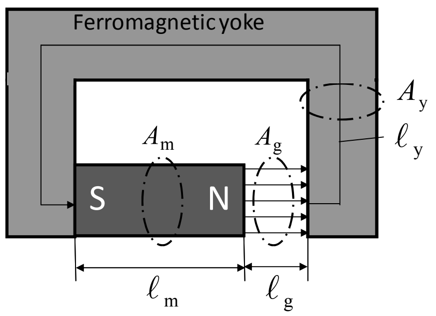
The magnetic flux in the magnet equals:

$$\Phi_\mathrm{m} = \frac{\mathcal{F}_\mathrm{m}}{\mathcal{R}_\mathrm{t}} = \frac{B_\mathrm{r}}{\mu_0}\frac{\ell_\mathrm{m}}{\mathcal{R}_\mathrm{t}}$$

$$\mathfrak{R}_\mathrm{t} = \frac{\ell_\mathrm{m}}{A_\mathrm{m}\mu_0} + \frac{\ell_\mathrm{y}}{A_\mathrm{y}\mu_0\mu_\mathrm{r}} + \frac{\ell_\mathrm{g}}{A_\mathrm{g}\mu_0} \Rightarrow \Phi_\mathrm{m} = \frac{B_\mathrm{r}A_\mathrm{m}}{1 + \frac{A_\mathrm{m}\ell_\mathrm{y}}{A_\mathrm{y}\ell_\mathrm{m}\mu_\mathrm{r}} + \frac{A_\mathrm{m}\ell_\mathrm{g}}{A_\mathrm{g}\ell_\mathrm{m}}}$$

Practical approximation

$$\Phi_\mathrm{m} = \frac{B_\mathrm{r}A_\mathrm{m}}{1 + \frac{A_\mathrm{m}\ell_\mathrm{y}}{A_\mathrm{y}\ell_\mathrm{m}\mu_\mathrm{r}} + \frac{A_\mathrm{m}\ell_\mathrm{g}}{A_\mathrm{g}\ell_\mathrm{m}}}$$



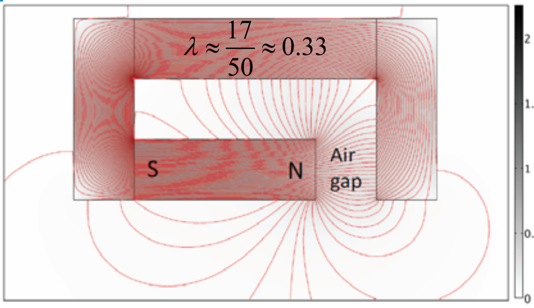
A_y, ℓ_y, A_m, A_g, ℓ_m, ℓ_g

In practice $\mu_\mathrm{r} \gg A_\mathrm{m}\ell_\mathrm{y} / A_\mathrm{y}\ell_\mathrm{m}$

$$\Phi_\mathrm{m} = \frac{B_\mathrm{r}A_\mathrm{m}}{1 + \frac{A_\mathrm{m}\ell_\mathrm{g}}{A_\mathrm{g}\ell_\mathrm{m}}}$$

But the flux in the air gap is smaller because magnetic flux is lost outside the air gap by “fringe/stray flux”.
Air is “conductive” for magnetic fields.

The fringe flux takes in practice between 25 to 75% of the total flux!



$\lambda = 1 - \text{loss}$

$\Phi_g = B_g A_g = \lambda \cdot \Phi_m$

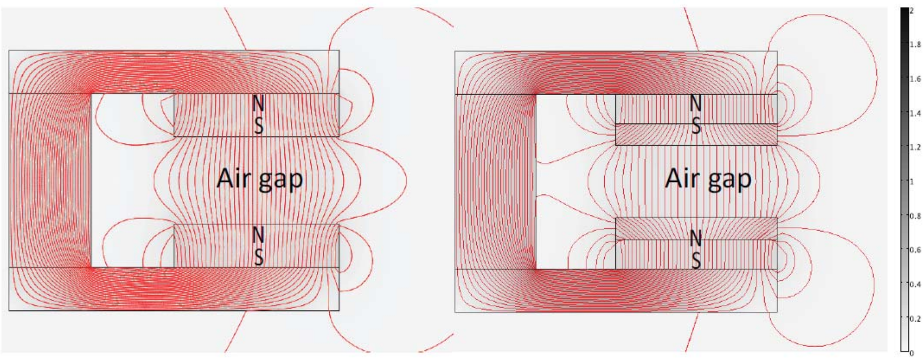
$\sim 0.25 < \lambda < \sim 0.75$

If $A_g = A_m$ and $\ell_g = \ell_m$

$$B_g = \frac{\lambda \cdot \Phi_m}{A_g} = \frac{A_m}{A_g} \cdot \frac{\lambda B_r}{1 + \frac{A_m \ell_g}{A_g \ell_m}} = \frac{\lambda B_r}{\frac{A_g}{A_m} + \frac{\ell_g}{\ell_m}}$$

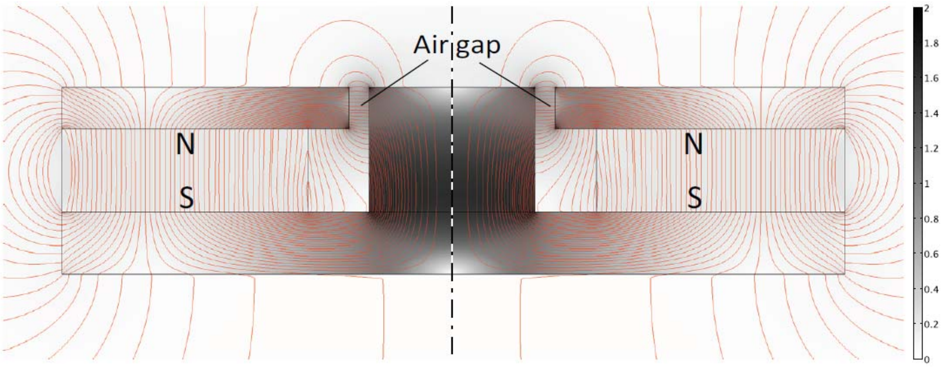
$$B_g = \frac{\lambda B_r}{2}$$

Flat magnets have less fringe flux and pole pieces are not efficient!



Concentration of flux by iron parts

$A_m >> A_g$
B_g (~0,7T) > B_r (~0,4T)
Much fringe flux !



Contents

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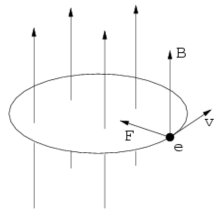
  - Lorentz actuator
  - Variable reluctance actuator
  - Hybrid actuator

With the field in the airgap an actuator can be made with a current wire inserted in the gap

Lorentz Force on a charged particle



$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

where
- **F** is the vectorial force (in Newton)
- **E** is the electric field (in Volts per meter)
- **B** is the magnetic field (in Tesla)
- *q* is the electric charge of the particle (in coulombs)
- **v** is the instantaneous velocity of the particle (in meters per second)
- and × is the cross product.

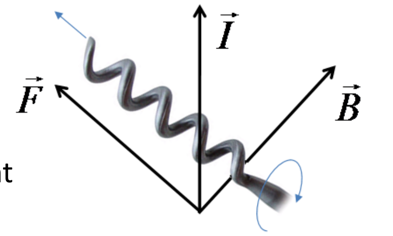
For the magnetic force on a current this relation leads to

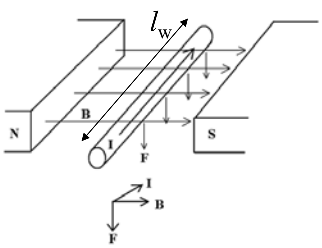
The Lorentz Force with $q \cdot v = I$

$$F = BI\ell_w \sin\alpha$$

α = the angle between the current and the magnetic field

ℓ_w = length of the wire in the field





Corkscrew rule due to cross product

Formulate the Lorentz force differently to avoid mistakes

The Lorentz Force

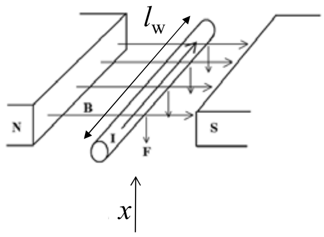
$$F = BI\ell_w \sin\alpha$$

Can also be written as : $(\alpha = \pi/2)$

$$F = I\frac{d\Phi_w}{dx}$$

Because

$$\frac{d\Phi_w}{dx} = B\ell_w$$



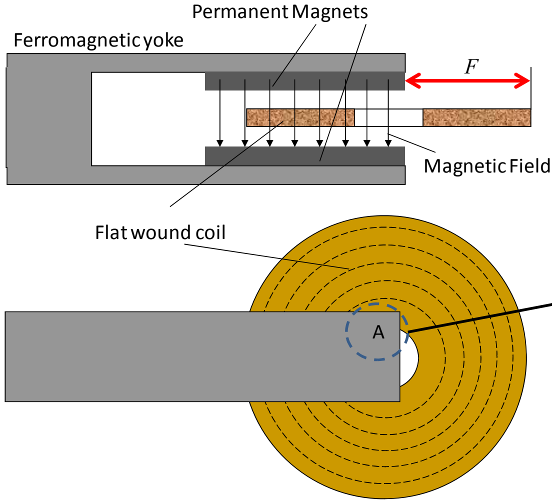
For multiple windings this becomes

$$F = nI\frac{d\Phi_w}{dx}$$

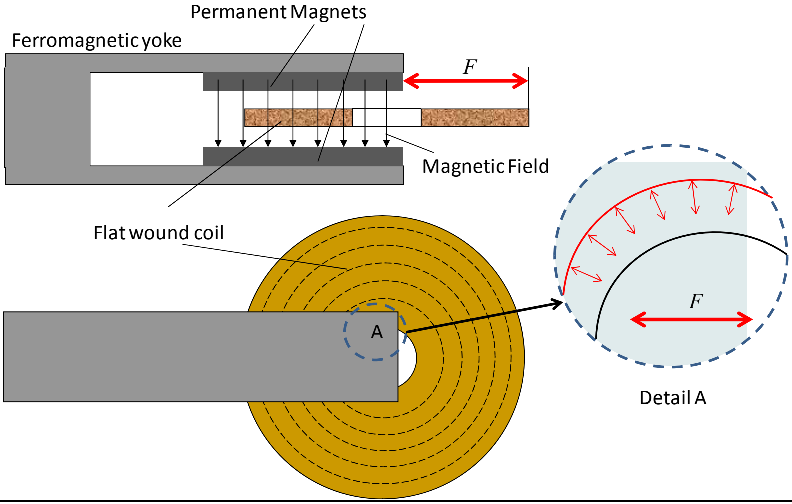
n = number of windings

Φ_w = flux per winding

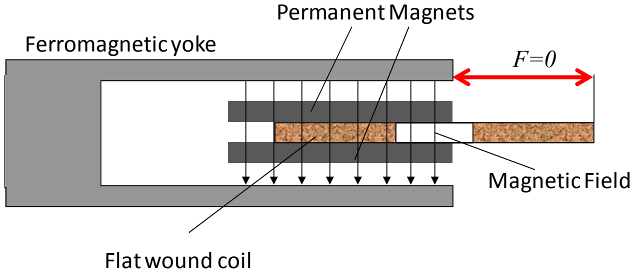
A first example, a flat Lorentz actuator



Efficiency of a flat Lorentz actuator

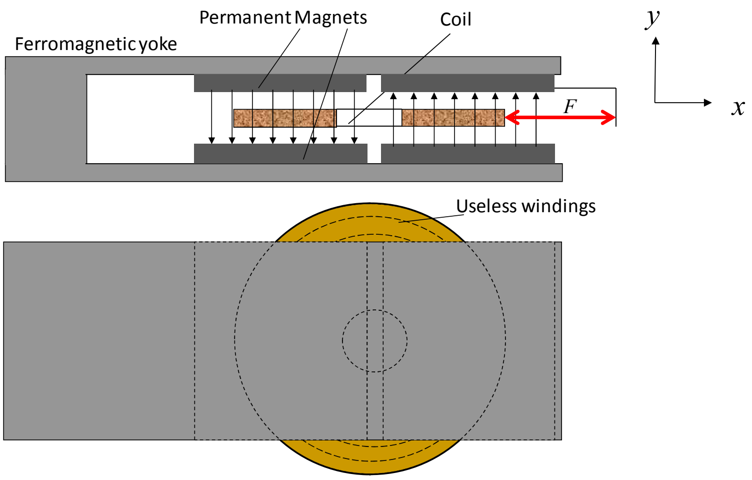


Importance of dΦ_w/dx, risk of non functional actuator

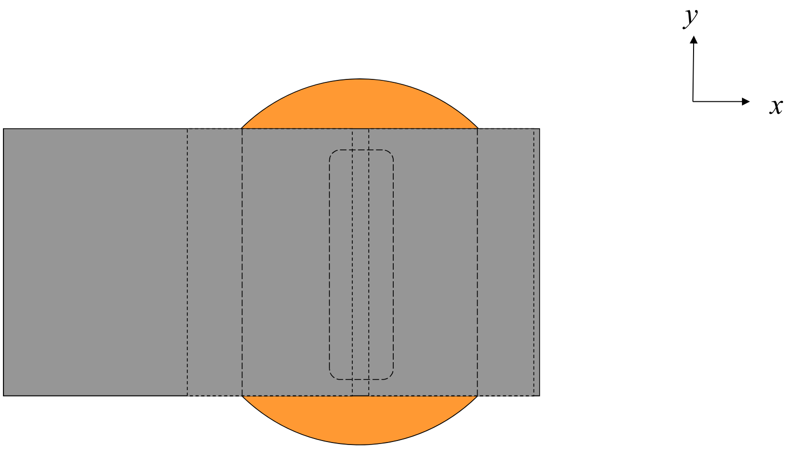


There should be motion between coil and permanent magnetic field.
The force acts between the current and the source of the magnetic field.

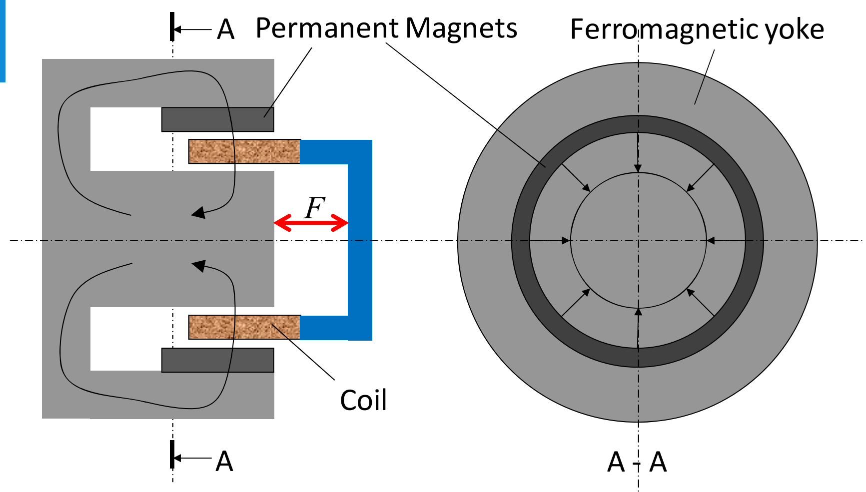
Extending in the x direction



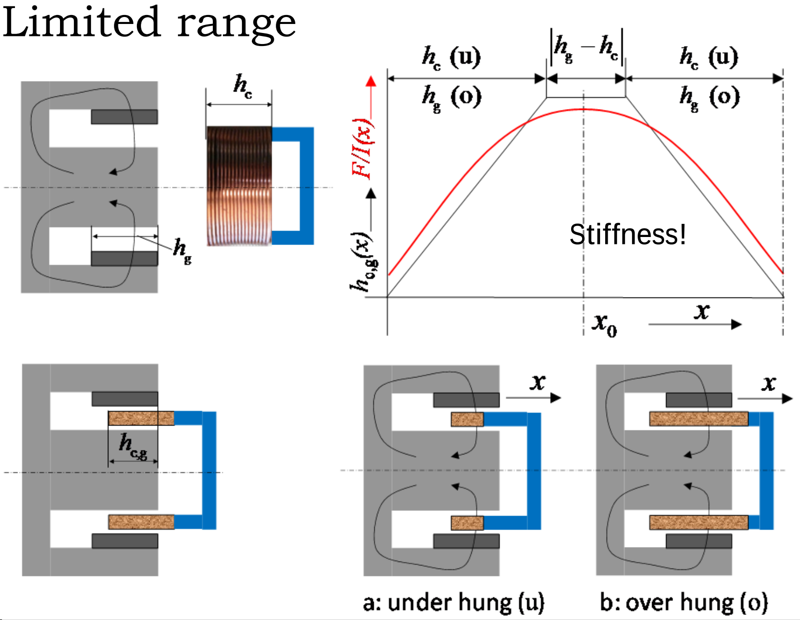
Extending in the y direction



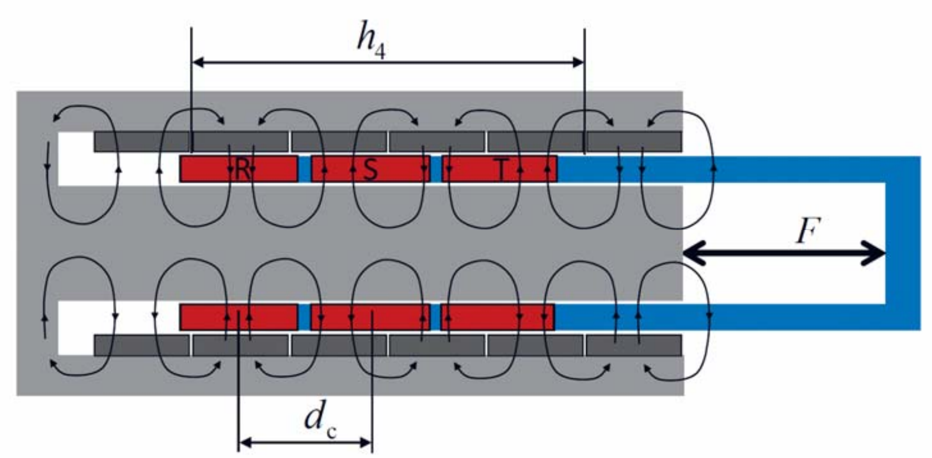
The loudspeaker (moving coil) motor



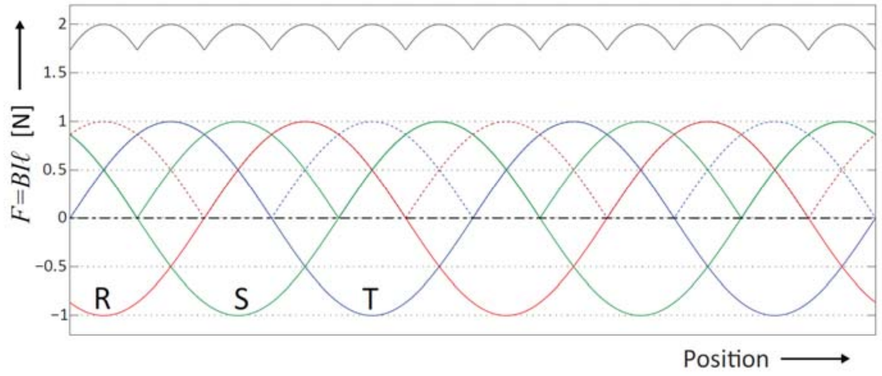
Limited range



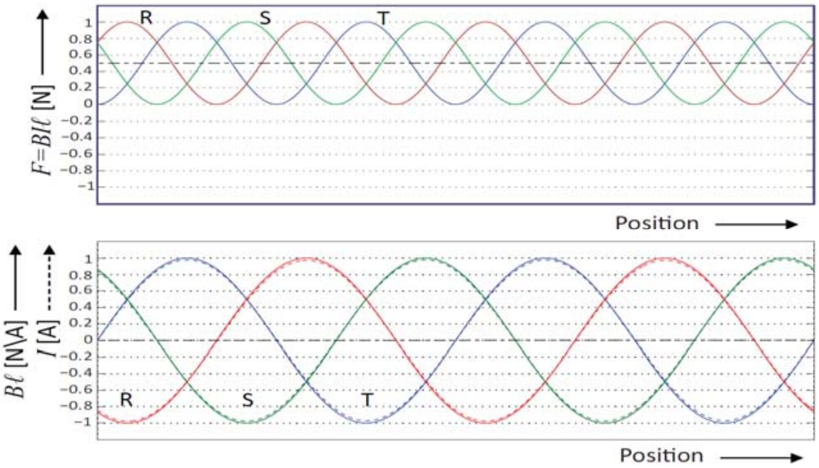
Commutation of the coils



Three phase force with switching at zero-crossings (mechanical commutation)



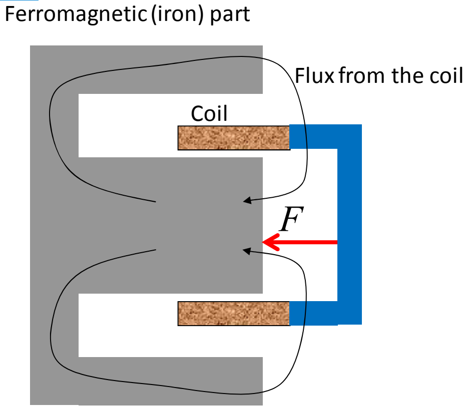
Three phase force with sinusoidal currents



Contents

- Basic electromagnetism
- Electromagnetic actuators
  - Lorentz actuator
  - Variable reluctance actuator
  - Hybrid actuator

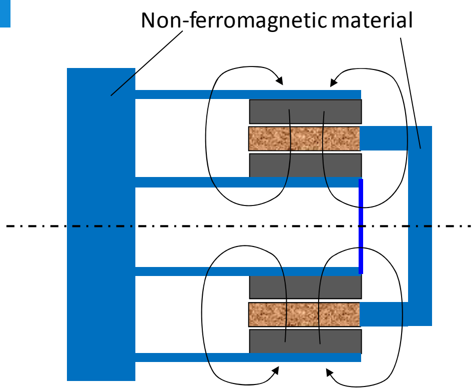
A Lorentz actuator also has some non-linear reluctance force



- Reluctance force is force on the coil without the permanent magnets
- The flux by the current in the coil will pull the coil in the iron
- This force is unidirectional and depending on the position

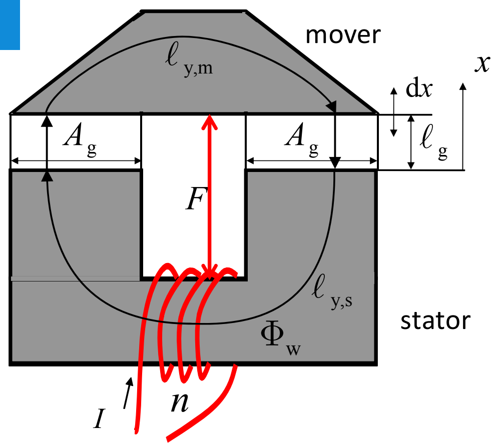
➔ **Stiffness!**

Ultimate solution. Avoid iron!



- Remove the iron and use more magnet material. (Higher cost!)
- Reluctance of outer path is same order of magnitude as the reluctance of the gap between the magnets (Larger flux path and larger cross-section)

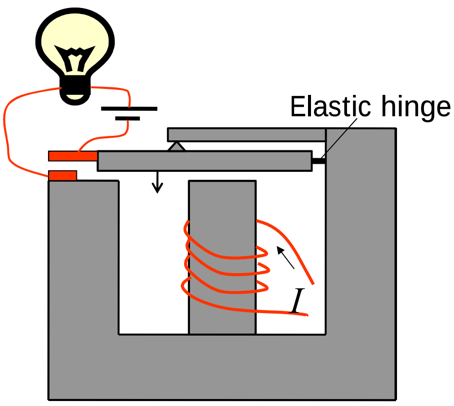
Variable reluctance actuator



Force based on energy balance
Energy stored in magnetic field

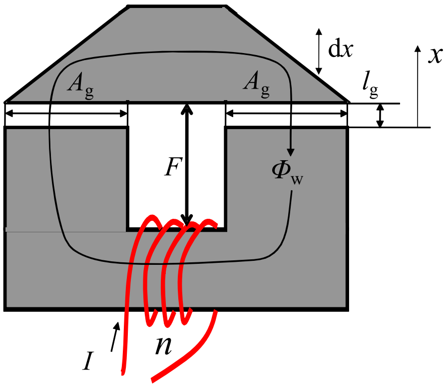
$$F = -\left(\frac{nI}{\ell_g}\right)^2 \frac{A_g \mu_0}{4}$$

Application of variable reluctance actuator, the current relay switch



- Avalanche effect
  - Elastic hinge is pre-strained
  - Current rises until mover starts to move
  - Resulting higher force speeds up movement
  - Strong and fast connection of electric contacts

Force of magnetic field to ferromagnetic material



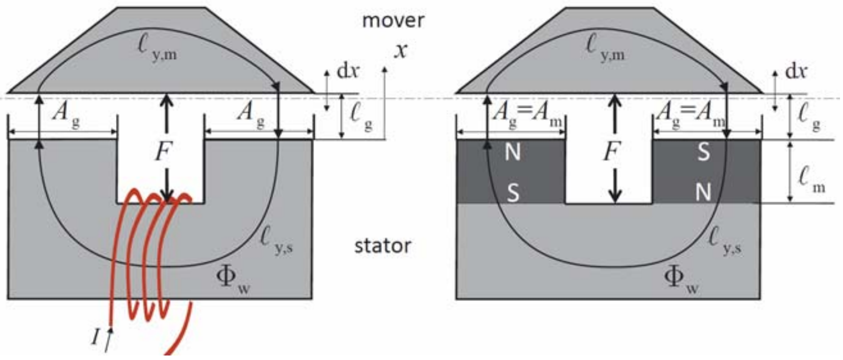
$$B_g = \frac{\Phi_w}{A_g} = \frac{nI}{\mathcal{R}A_g} \approx \frac{nI\mu_0}{2\ell_g}$$

$$\Rightarrow nI \approx \frac{2B_g\ell_g}{\mu_0}$$

$$F = -\left(\frac{nI}{\ell_g}\right)^2 \frac{A_g\mu_0}{4} \approx -\frac{B_g^2 A_g}{\mu_0}$$

"Magnetic pressure" $P_m \approx \frac{F}{2A_g} \approx \frac{B_g^2}{2\mu_0}$ 1T ➔ 0.4 Mpa (4bar)

The relation for attraction force is only valid for reluctance force



$$F \approx -\frac{B_g^2 A_g}{\mu_0}$$

Two permanent magnets will attract or repel each other.
This is linear force related to the Lorentz force