Traffic Flow Theory & Simulation

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Lecture 3
Fundamental diagram

1.

Traffic Flow Variables

Overview of variables

	Local measurements	**Instantaneous measurements**	**Generalized definition (Edie)**
Variable	Cross-section x Period T	Section X Time instant t	Section X Period T
Flow q (veh/h)	$q = \frac{n}{T} = \frac{1}{\bar{h}}$	$q = ku$	$q = \frac{\sum_i d_i}{XT}$
Density k (veh/km)	$k = \frac{q}{u}$	$k = \frac{n}{X} = \frac{1}{\bar{s}}$	$k = \frac{\sum_j r_j}{XT}$
Mean speed u (km/h)	$u_L = \frac{n}{\sum_i (1/v_i)}$	$u = \frac{\sum_j v_j}{n}$	$u = \frac{q}{k}$

2.

Macroscopic flow characteristics

Fundamental Diagram

Overview of lecture

- Fundamental relation (recall)
- Fundamental diagrams
  - Causality
  - Special points
  - Model alternatives
- Studies of the fundamental diagram
  - Effect of weather conditions
- Applications
  - Capacity estimation
  - Effect of rain

Fundamental relation

- Consider traffic flow in stationary + homogeneous state; then

$$q = ku$$

- Which average speeds are to be used? Local (time-mean speed) or instantaneous (space-mean speed)?
- Recall: consider group i with constant speed u_i (per group: $u_L = u_M$)
- Density of group on X equals k_i
- For each group $q_i = k_i u_i$, and thus

$$q = \sum_i q_i = \sum_i k_i u_i \quad \text{and} \quad k = \sum_i k_i = \sum_i \left(q_i / u_i\right)$$

yielding

$$u = \frac{q}{k} = \frac{\sum_i k_i u_i}{\sum_i k_i} = \frac{\sum_i q_i}{\sum_i q_i / u_i} = u_M$$

Fundamental diagram

- Assumptions:
  - On **average** drivers behave the same under similar **stationary conditions**
  - E.g. when drivers have a certain speed *v*, they will on average maintain the same following distance *s*
- Example vehicle *i* follows vehicle *i*–1 at distance headway s_i

$$s_i = s_i^0 + T_i v_i \quad \Leftrightarrow \quad h_i = s_i^0 / v_i + T_i$$

- Distance headway increases as speed increases since driver needs more space to ensure not colliding with the vehicle in front in case of an unexpected situation
- For averages we have

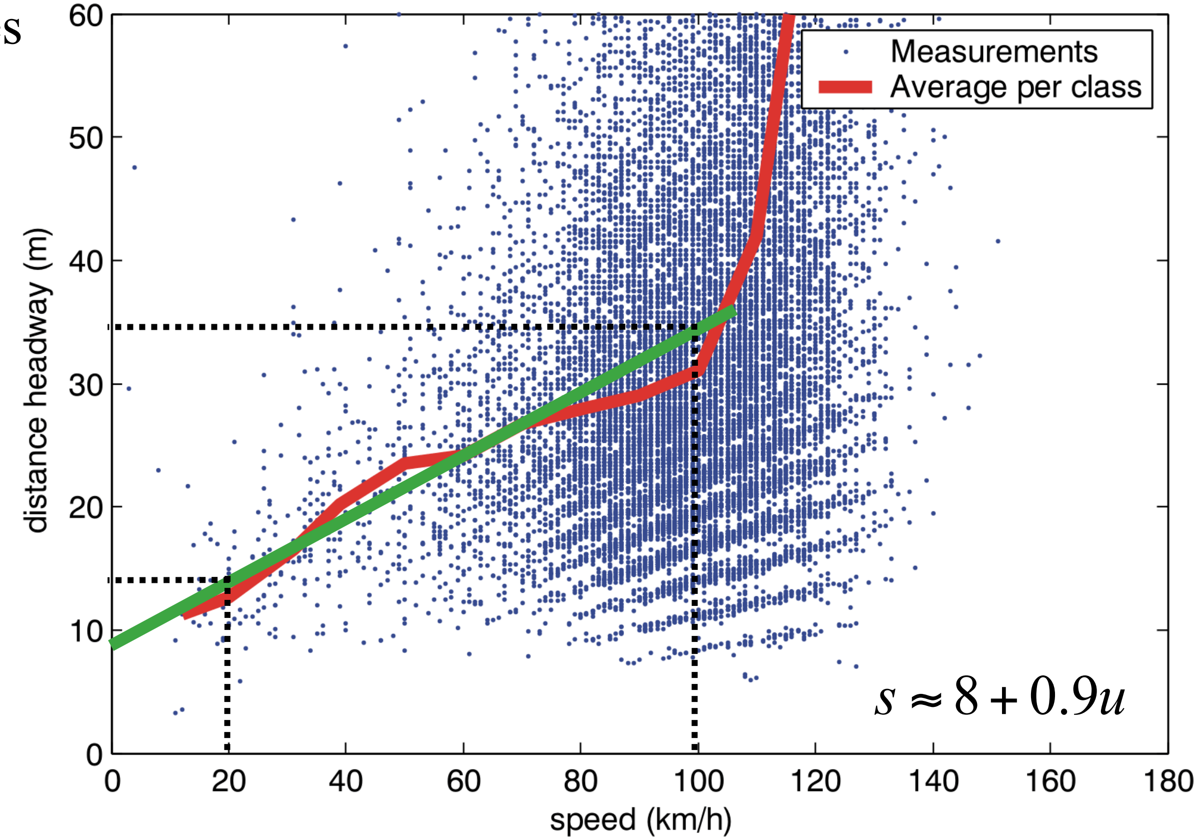
$$\frac{1}{k} = \overline{s} = \tfrac{1}{m}\sum s_i = \tfrac{1}{m}\sum\left(s_i^0 + T_i v_i\right) = \overline{s}_0 + Tu$$

- How about data?

Data collected at A9 motorway (left lane)

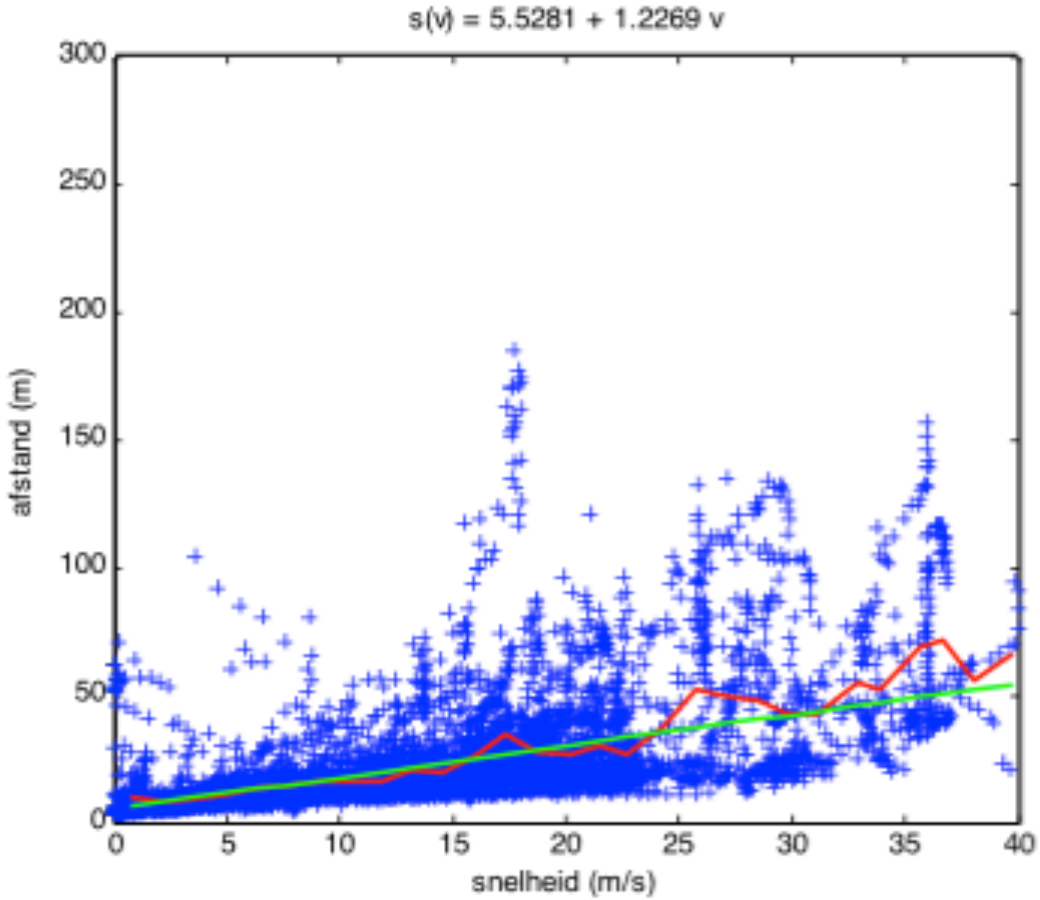
- A9 motorway data
- Green line indicates 'fitted' linear relation between *s* and *u*

- *Much scatter due to differences between vehicles and within driver behaviour, non-stationarity, etc.*



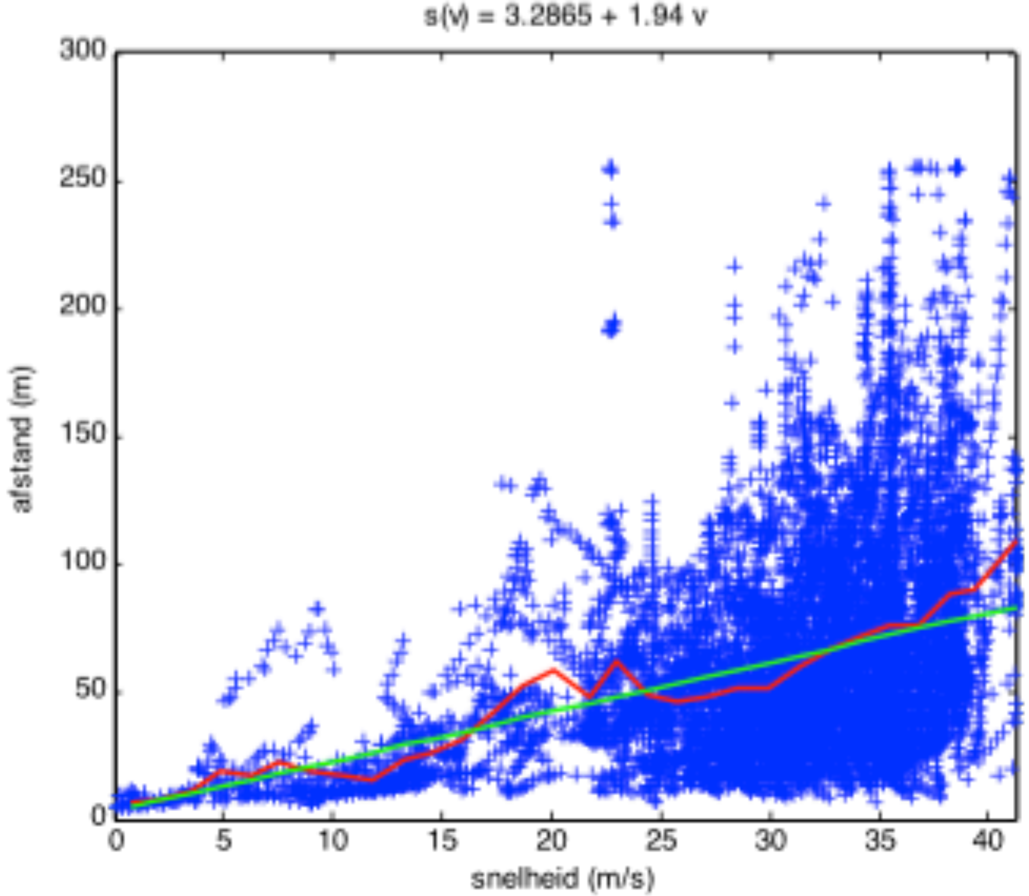
Data from Full Traffic project

- Instrumented vehicles (net distance headways)



Data from Full Traffic project

- Instrumented vehicles (net distance headways)



Fundamental diagram

- For the density we have

$$k = \frac{1}{\overline{s}} = \frac{1}{\overline{s}_0 + Tu} = K(u) \quad \left(u = U(k) = \frac{1}{kT} - \frac{\overline{s}_0}{T} = \frac{\overline{s} - \overline{s}_0}{T} \right)$$

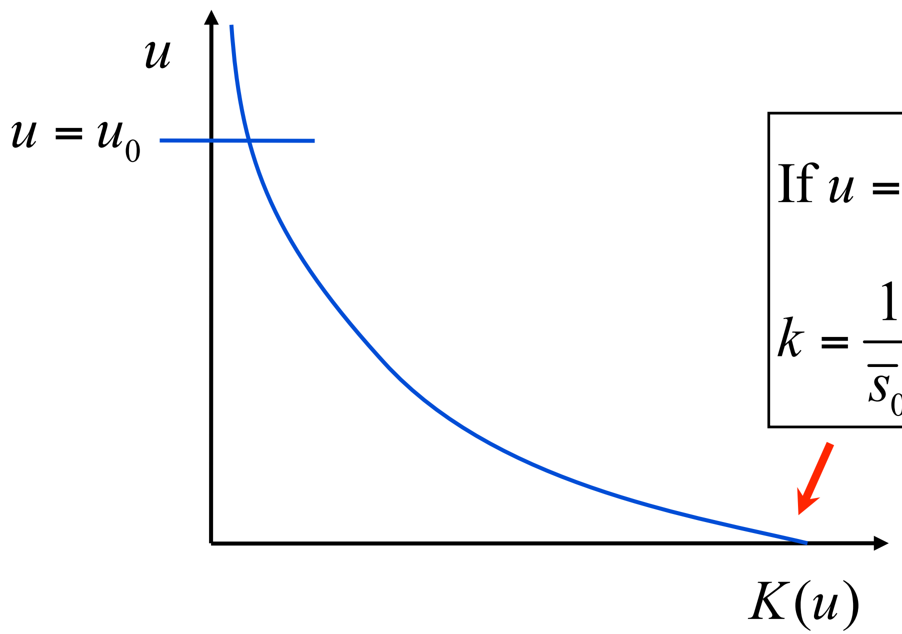
- In other words, the ***density k is a function of the space-mean speed u***
- Relation will be influenced by
  - Road characteristics (grade, width, etc.)
  - Flow composition (% trucks, % commuters)
  - External / environmental conditions (weather, ambient conditions)
  - Traffic regulations
- What happens at $u = 0$? I.e. what density occurs when traffic is stopped?
- Is this relation valid for $k = 0$ (low density traffic)? Why (not)?

Fundamental diagram

- Plot relation between speed and density

For $k \downarrow 0 \rightarrow u = \infty$

$$u = U(k) = \frac{1}{kT} - \frac{\bar{s}_0}{T}$$



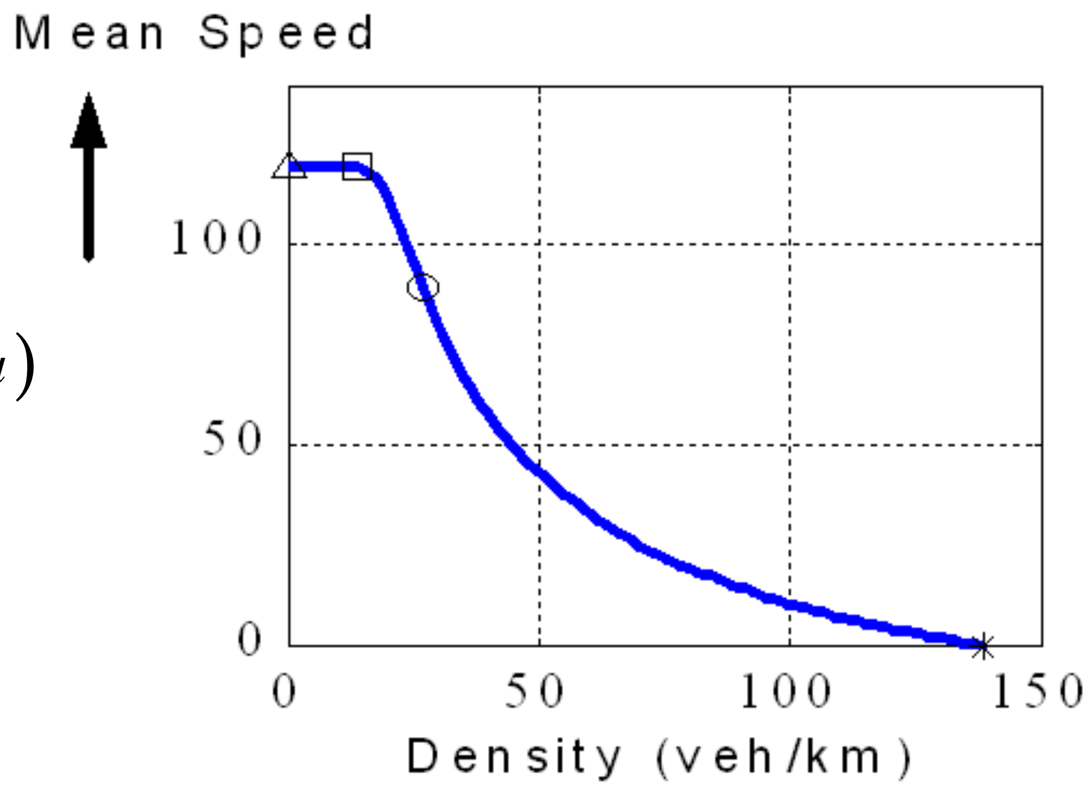
If $u = 0$ then $\frac{1}{kT} - \frac{\bar{s}_0}{T} = 0$

$$k = \frac{1}{\bar{s}_0} = k_{j(am)}$$

Fundamental diagram

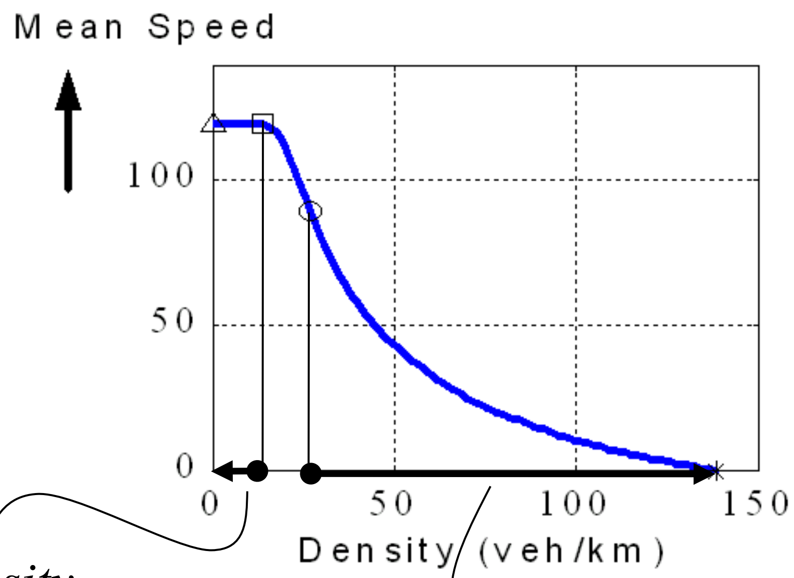
- In sum: considering a stationary / homogeneous flow → reasonable to assume that there exist some relation between density k and instantaneous speed u

$$u = U(k) \text{ or } k = K(u)$$



Fundamental diagram

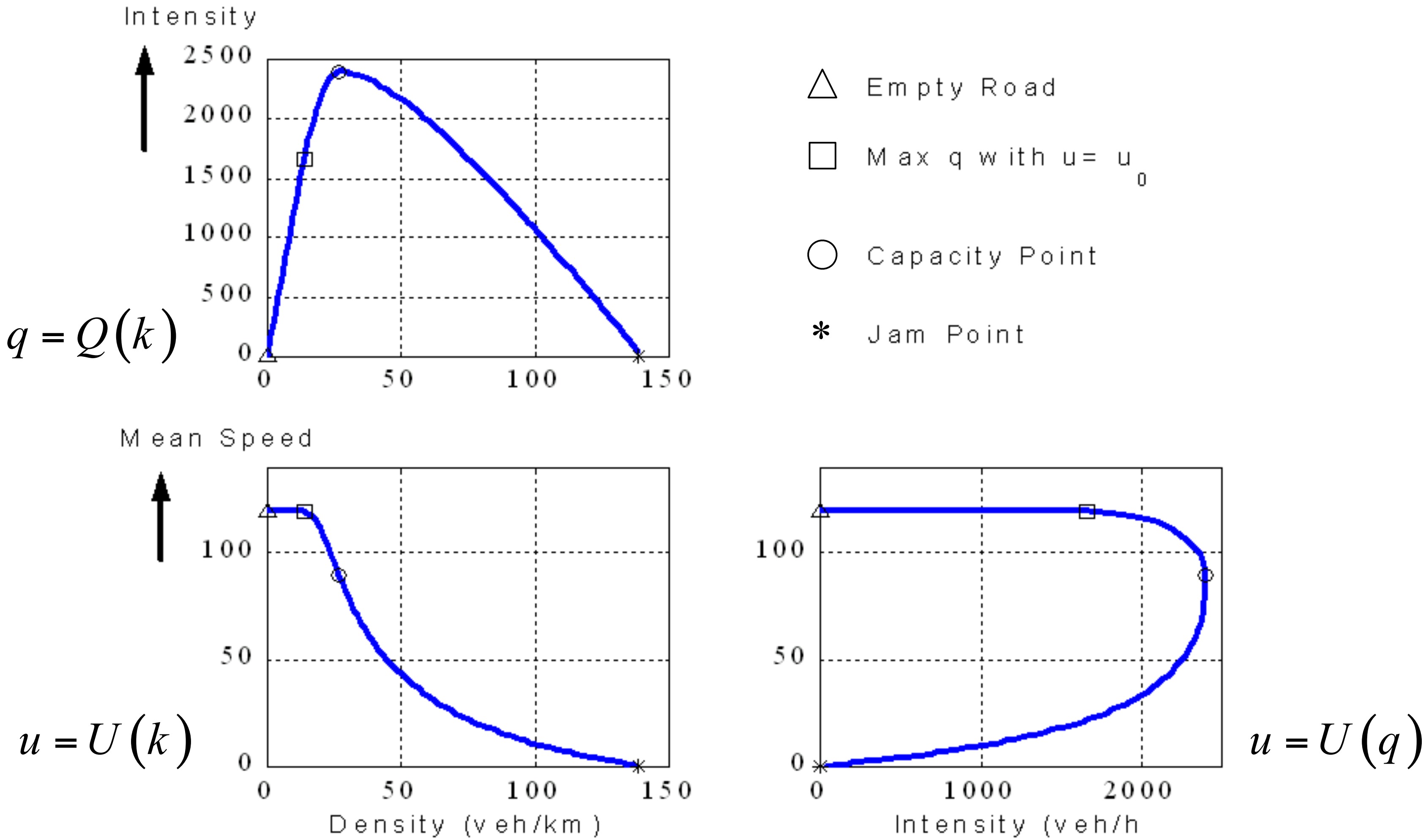
- Three relations are used in traffic flow theory:
  - $q = Q(k)$
  - $u = U(k)$
  - $u = U(q)$
- Relation does not imply causality: in congested conditions density (inverse distance headway) is determined by speed; flow under uncongested conditions is determined by density (via $q = ku_0$)



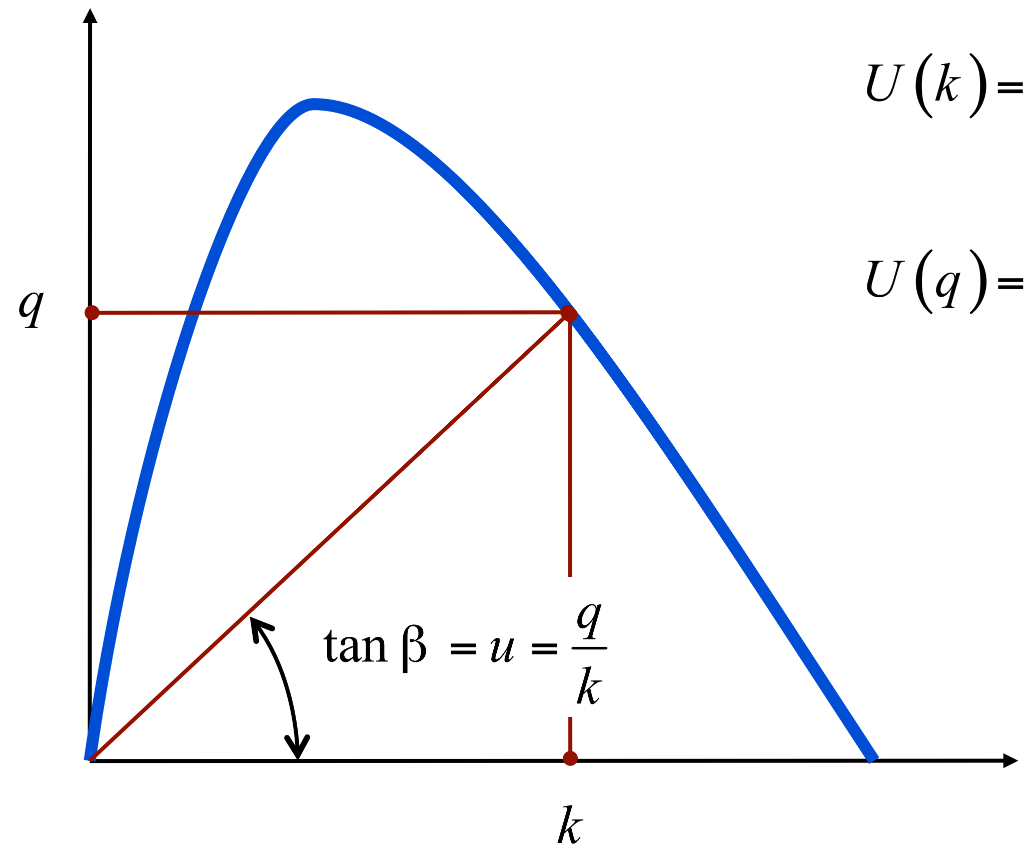
Flow 'determined' by density and free speed

Density determined by speed

Fundamental diagram



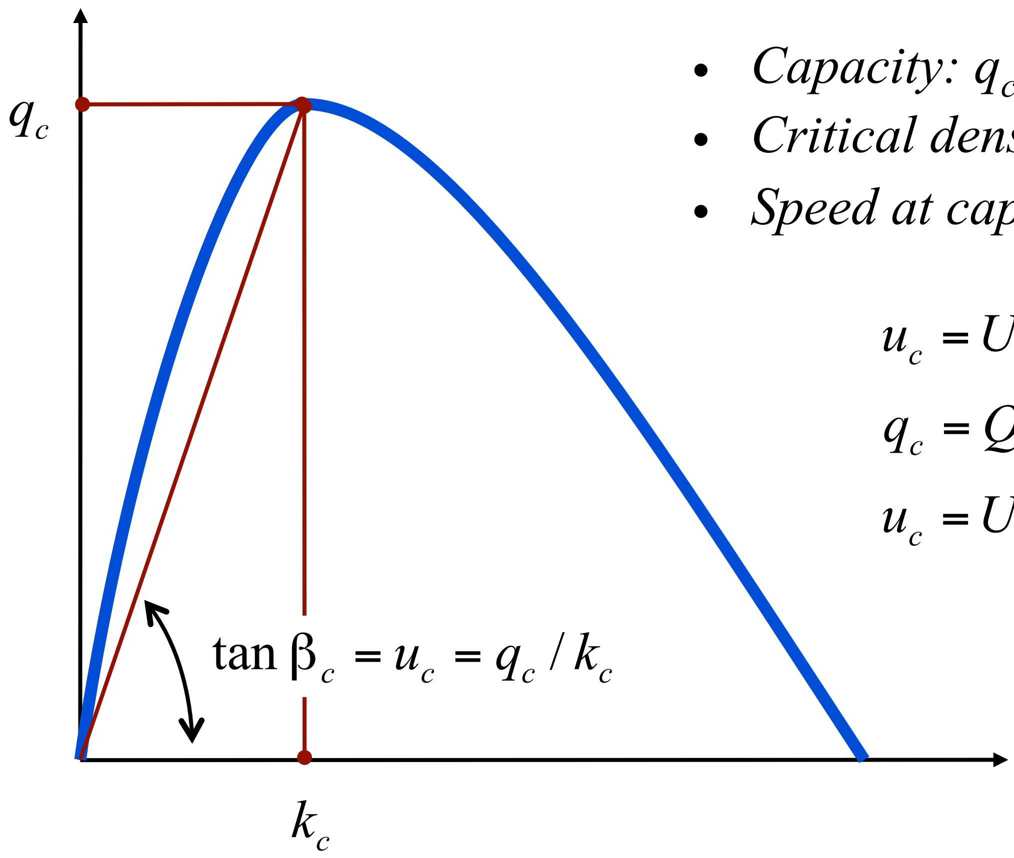
Fundamental diagram



$$U(k) = \frac{q}{k} = \frac{Q(k)}{k}$$

$$U(q) = \frac{q}{k} = \frac{q}{K(q)}$$

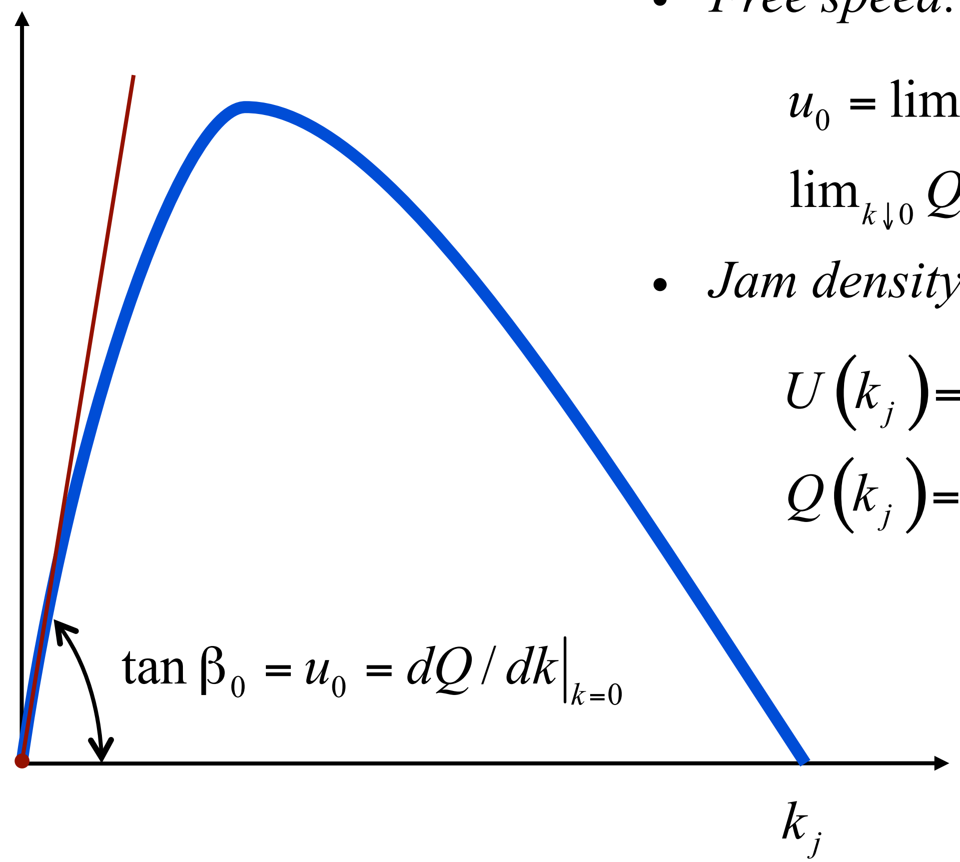
Fundamental diagram



- *Capacity:* q_c
- *Critical density:* k_c
- *Speed at capacity:* u_c

$$u_c = U(q_c)$$
$$q_c = Q(k_c)$$
$$u_c = U(k_c)$$

Fundamental diagram



- *Free speed:* u_0

$$u_0 = \lim_{k \downarrow 0} U(k)$$

$$\lim_{k \downarrow 0} Q(k) = 0$$

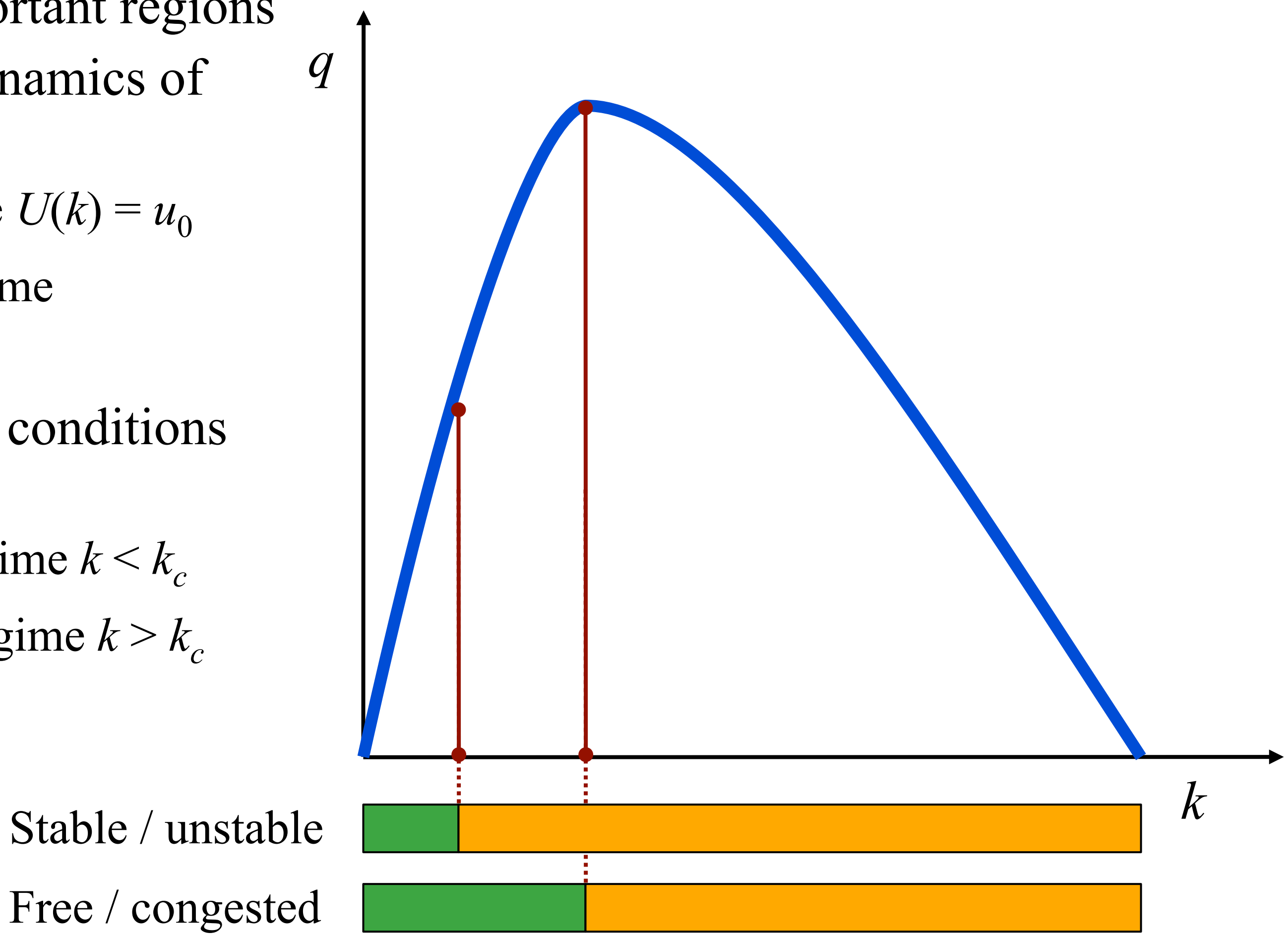
- *Jam density* k_j

$$U(k_j) = 0$$

$$Q(k_j) = 0$$

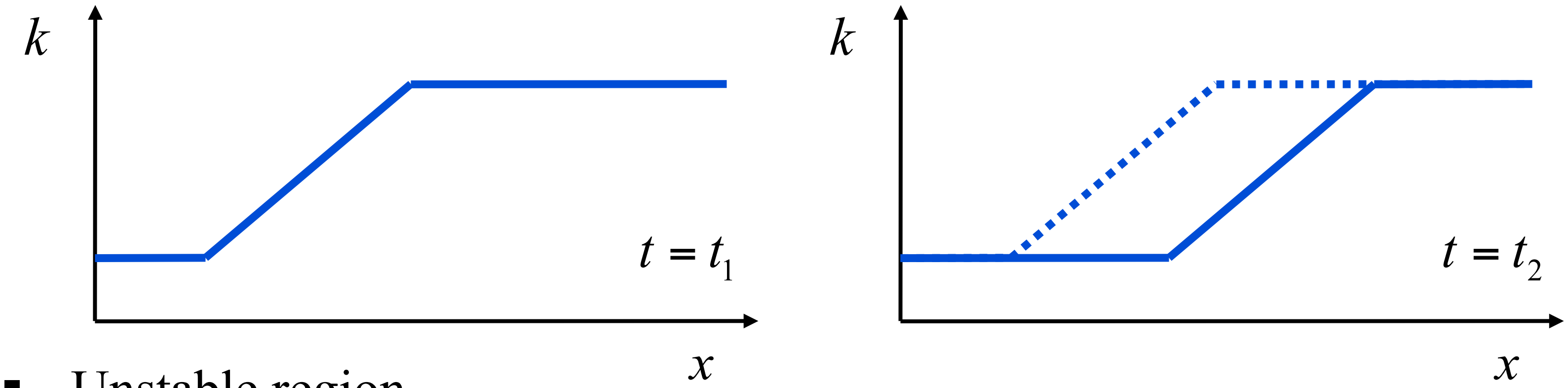
Flow regimes

- Number of important regions
- Based on the dynamics of 'disturbances'
  - Stable regime $U(k) = u_0$
  - Unstable regime
- Based on traffic conditions (LOS)
  - Free flow regime $k < k_c$
  - Congested regime $k > k_c$

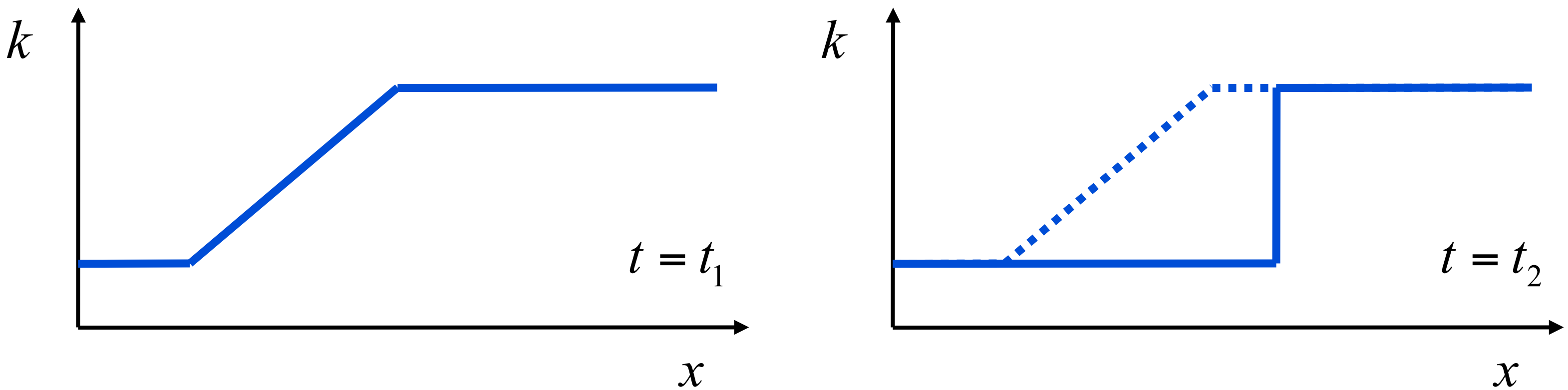


Macroscopic flow models (chapter 8)

- Stable region

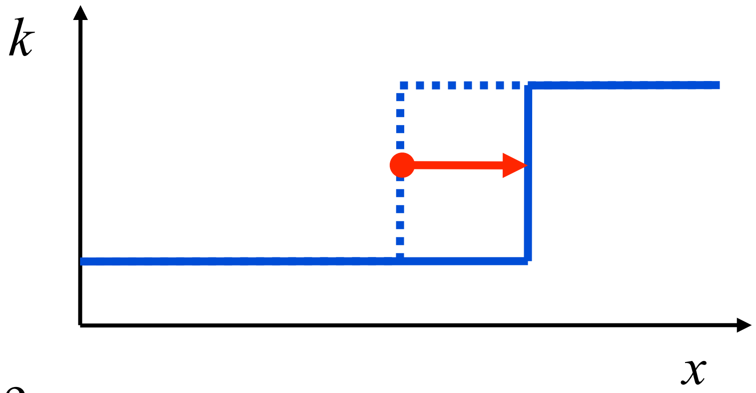


- Unstable region

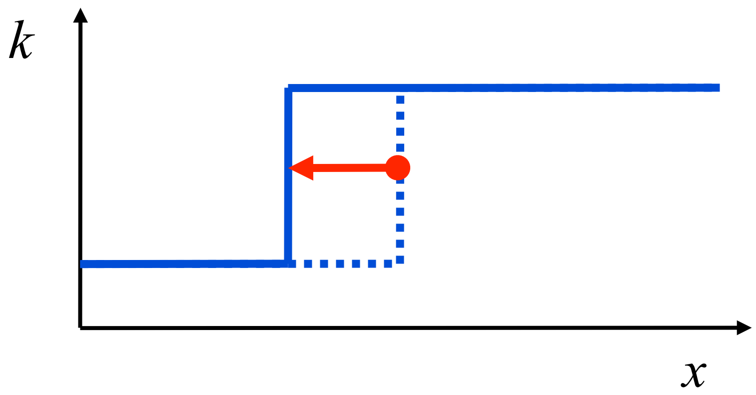


Macroscopic flow models (chapter 8)

- Free flow regime



- Congested regime

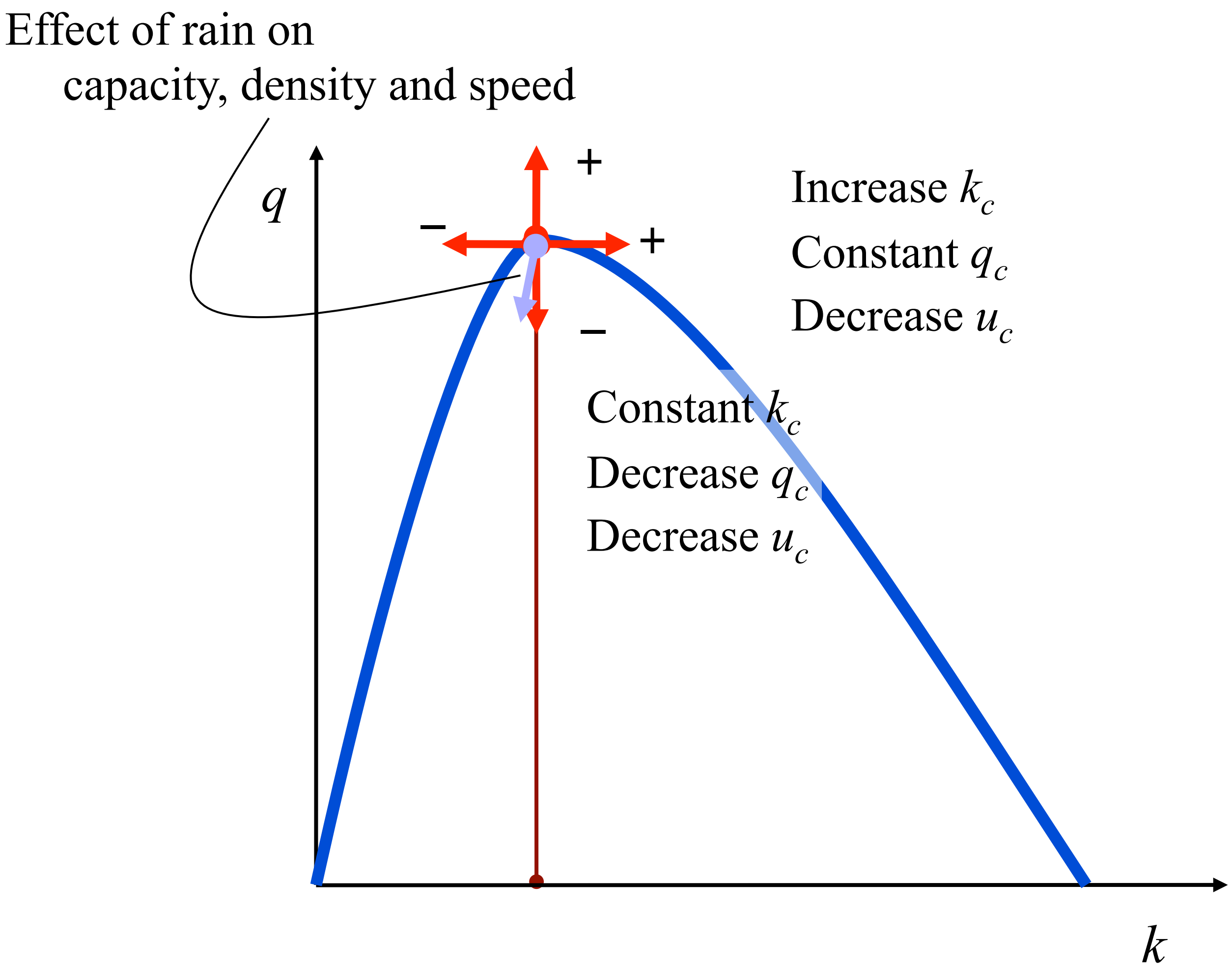


Influence of circumstances on capacity

- Non-ideal conditions

Circumstances	Capacity
Ideal conditions	100%
Darkness (no illumination)	95%
Darkness (with illumination)	97%
Dense Asphalt Concrete (DAC) [2]with rain	91%
Open Asphalt Concrete (OAC) with rain[3]	95%
DAC / rain / darkness	88%
DAC / rain / darkness / illumination	90%
OAC / rain / darkness	91%
OAC / rain / darkness / illumination	92

Effect of circumstances on capacity (2)

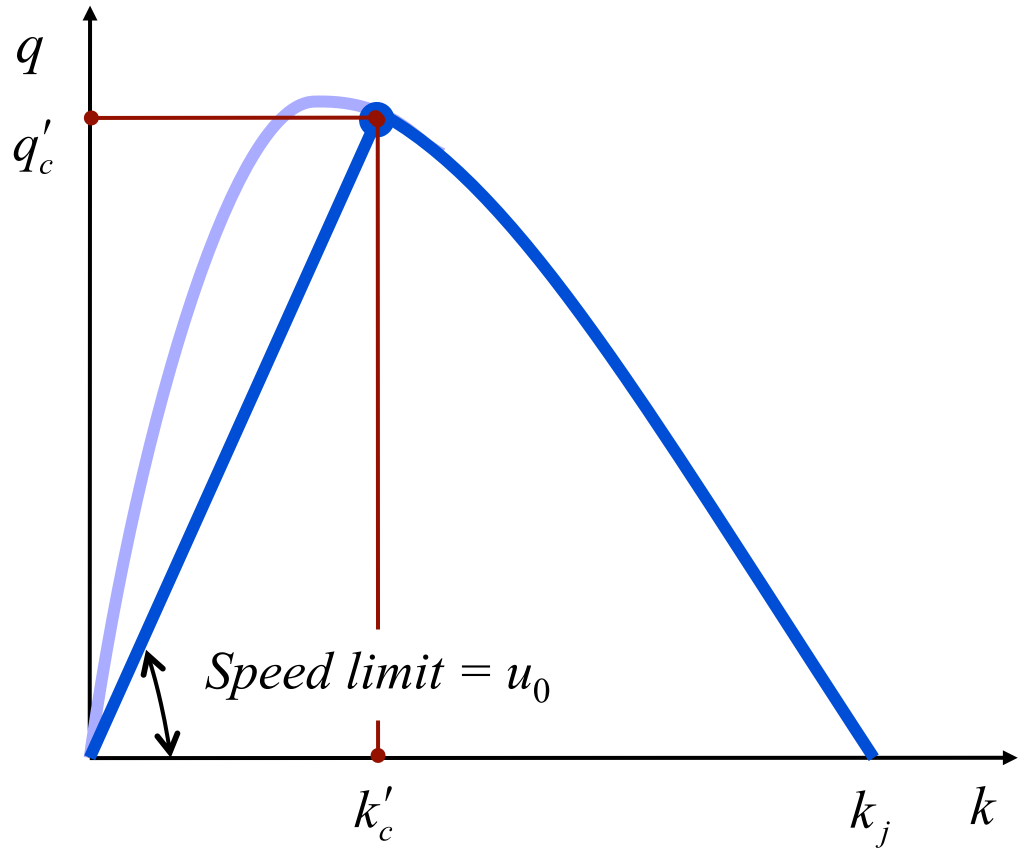


Influence of circumstances on capacity (3)

- Effect of changing speed-limits (e.g. from 120 to 80 km/h)
- Question: how will the fundamental diagram change due to a speed limit?
- Please draw the diagram for a speed limit of 80 km/ h and 50 km/h

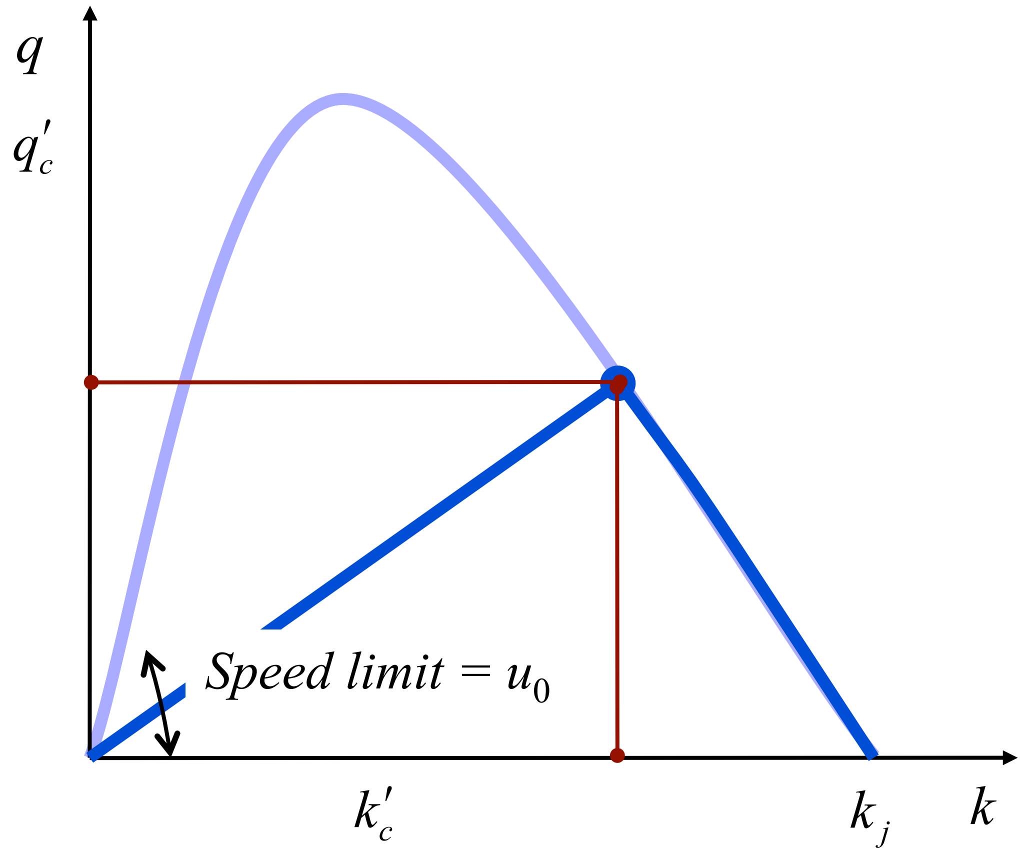
Influence of circumstances on capacity (3)

- Effect of changing speed-limits (e.g. from 120 to 80 km/h)



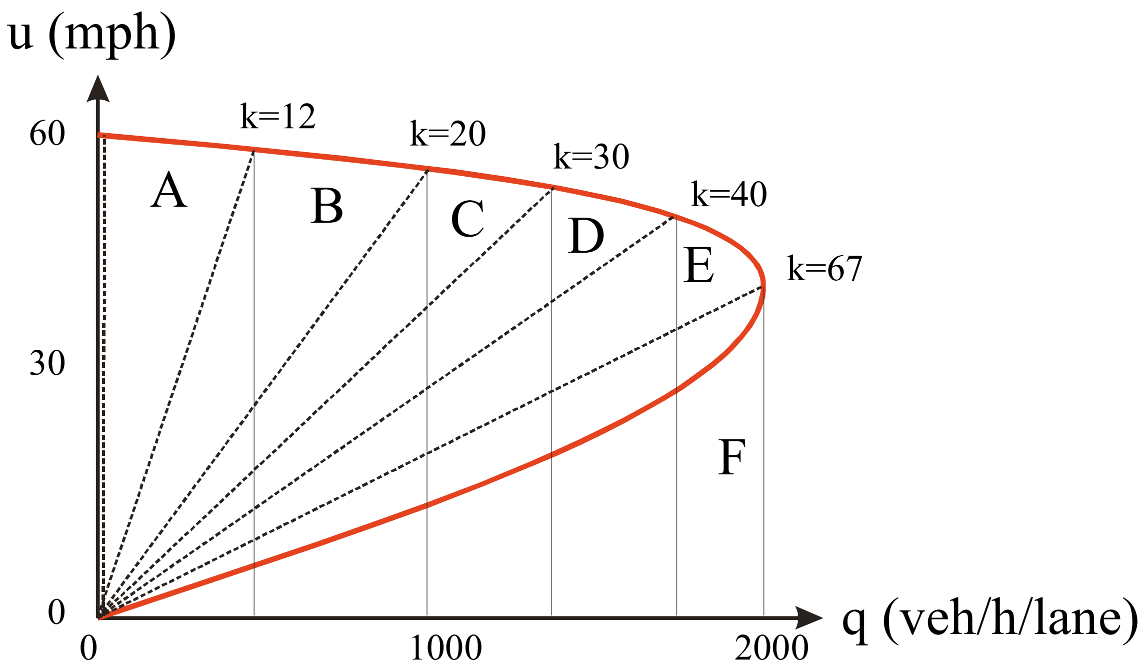
Influence of circumstances on capacity (4)

- Effect of changing speed-limits (e.g. from 120 to 50 km/h)

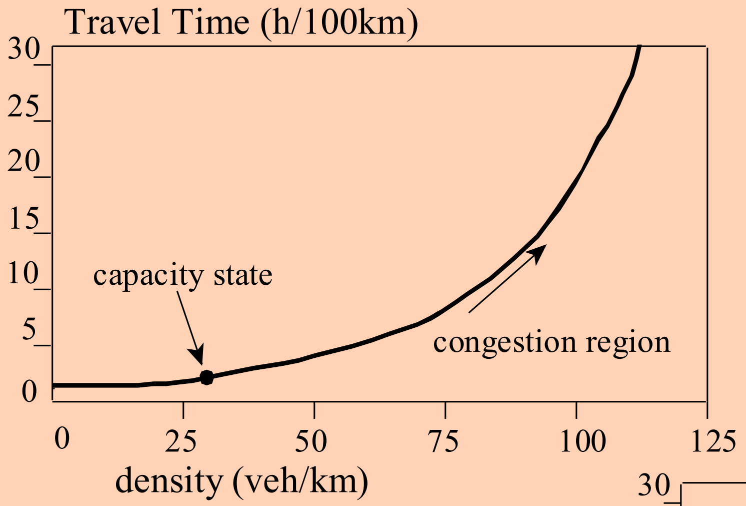


Importance of Fundamental Diagram

- Definition of Level of Service (LOS)
- Relation between intensity and travel time
- Capacity estimation
- Shockwave analysis and macroscopic traffic flow models

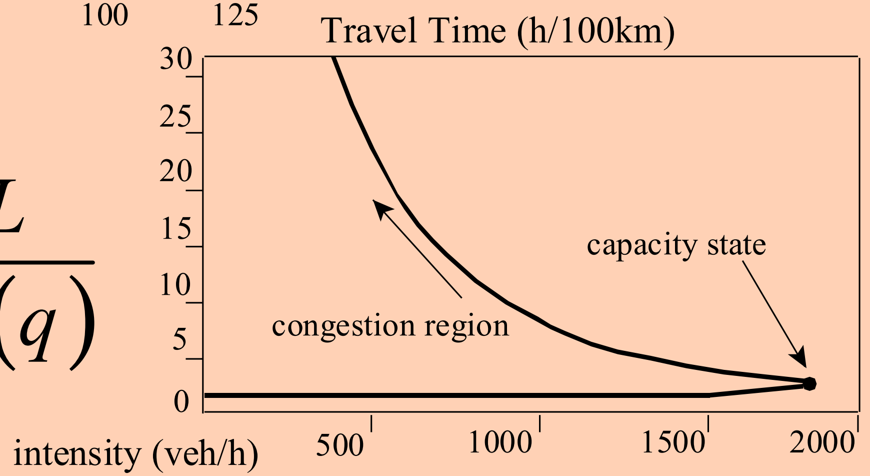


Importance of Fundamental Diagram (2)



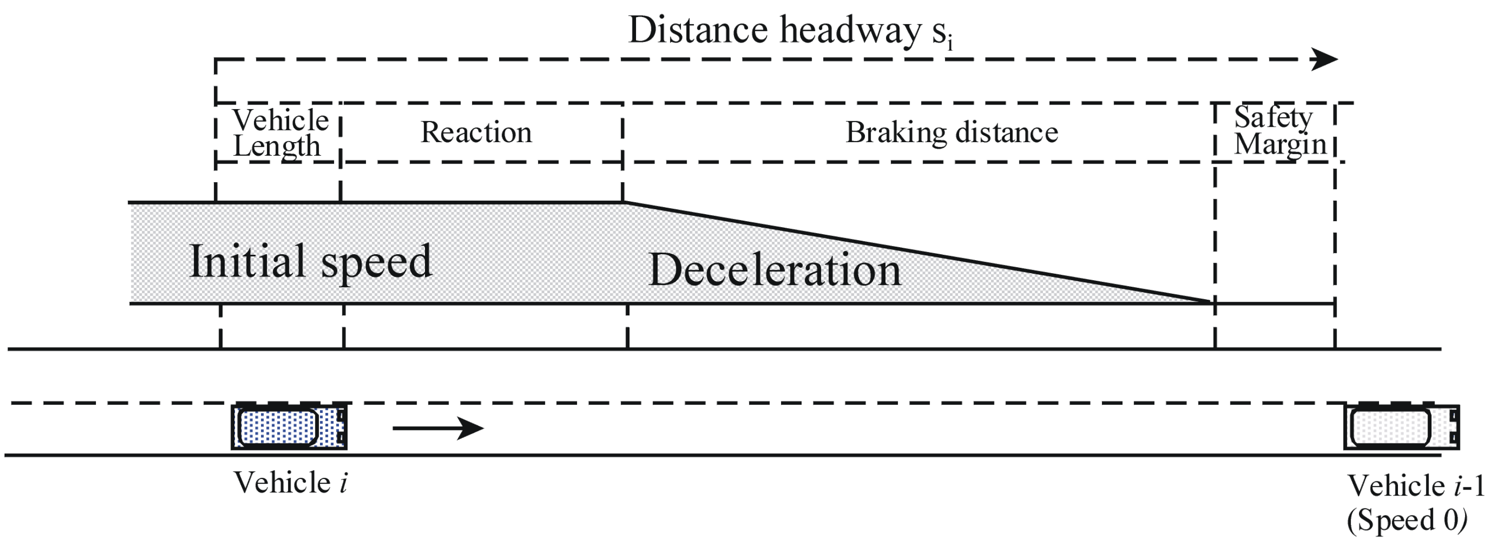
$$TT = \frac{L}{u} = \frac{L}{U(k)}$$

$$TT = \frac{L}{u} = \frac{L}{U(q)}$$



Fundamental diagram and car-following

- Car *i* following car *i*-1
- Behavioral assumptions regarding driver behavior:
  - Keep safe distance between vehicles
  - Vehicle *i* has a max. deceleration rate $a > 0$
  - Vehicle *i*-1 might suddenly brake with deceleration rate $\alpha \cdot a$ to complete stop
  - Safety margin between vehicles when coming to stop = L_m



Exercise

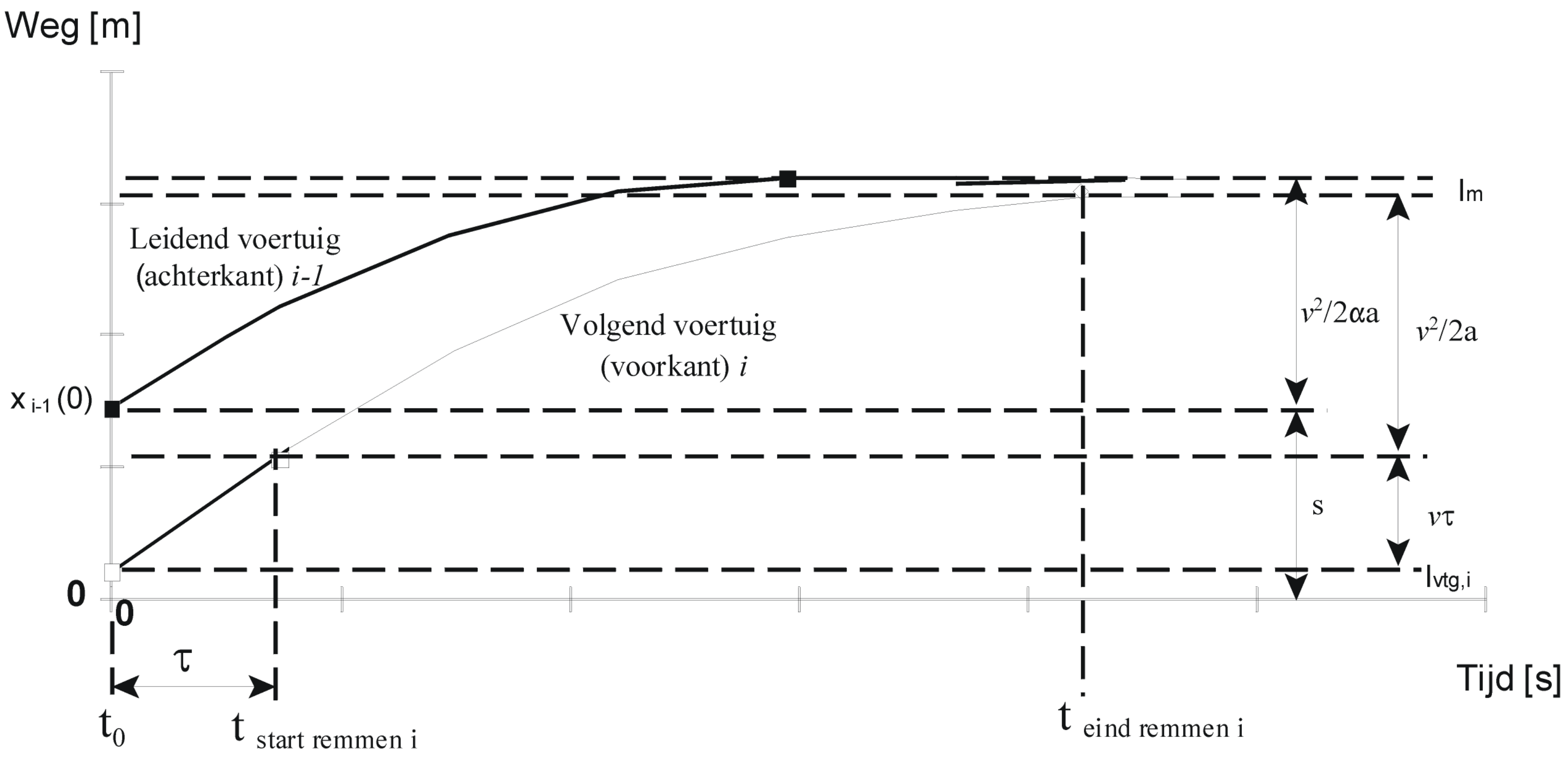
- Assume that driver i has reaction time T_i
- Determine the distance driver i needs to maintain in order to be able to stop at distance L_m when the vehicle in front stops with maximum acceleration
- Initial distance headway? (for $t < t_0$)

$$s_i(t) = \text{distance\_travelled\_to\_stop}_i + L_{veh} + L_m \\ -\text{distance\_travelled\_to\_stop}_{i-1}$$

- This rule results in the fact that after braking, vehicles have net distance L_m

- Based on this driving distance, how can you determine the fundamental diagram from this driving rule?

Fundamental diagram and car-following



Fundamental diagram and car-following (2)

- Factor α expresses attitude towards risk
- For general drivers: $\alpha \geq 1$
- Distance traveled by *i*-1 before coming to standstill: $\frac{v_i^2}{2\alpha a_i}$
- Distance traveled by *i* before coming to standstill: $v_i T_i + \frac{v_i^2}{2a_i}$
- For the safe distance headway we thus find:

$$s_i = \underbrace{L_{veh} + L_m}_{\text{stopping distance between } i \text{ and } i-1} + \underbrace{vT + \frac{v^2}{2a}}_{\text{distance travelled until stopped by vehicle } i} - \underbrace{\frac{v^2}{2\alpha a}}_{\text{distance travelled until stopped by vehicle } i\text{-1}} = s_0 + vT + \frac{v^2}{2a}\left(1 - \frac{1}{\alpha}\right)$$

$$\left(\text{What if } vT + \frac{v^2}{2a} < \frac{v^2}{2\alpha a}?\right)$$

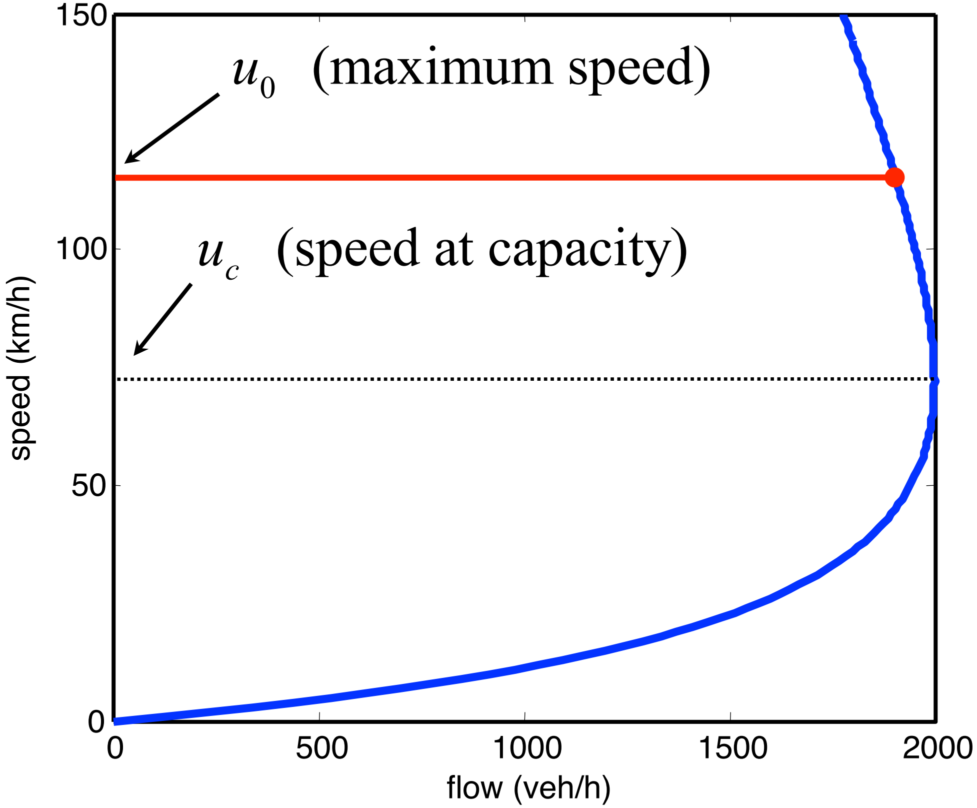
Fundamental diagram and car-following (3)

- Assume that $s_i = s$ is the average distance headway of all vehicles
- $s = 1/k$ and $s_0 = 1/k_j$

$$\frac{1}{k} = \frac{1}{k_j} + uT + \frac{u^2}{2a}\left(1 - \frac{1}{\alpha}\right)$$

$$\Leftrightarrow$$

$$q = \frac{1}{\frac{1}{uk_j} + T + \frac{u}{2a}\left(1 - \frac{1}{\alpha}\right)}$$



Fundamental diagram and car-following (4)

- How can you determine the capacity of this model?
- Determine critical speed u_c for which

$$\frac{dq(u)}{du} = 0$$

®

$$u_c = \sqrt{\frac{2}{k_j}\frac{a\alpha}{\alpha - 1}}, \quad q_c = \frac{u_c k_j}{2 + T u_c k_j} \quad \text{and} \quad k_c = \frac{k_j}{2 + T u_c k_j}$$

- What can we say for $\alpha = 1$?

$$s_i = s_0 + v_i T_i \ \text{®} \ q(u) = \frac{u k_j}{1 + T u k_j}$$

- $q(u)$ is a monotonic increasing function of $u \rightarrow u_c = \infty$

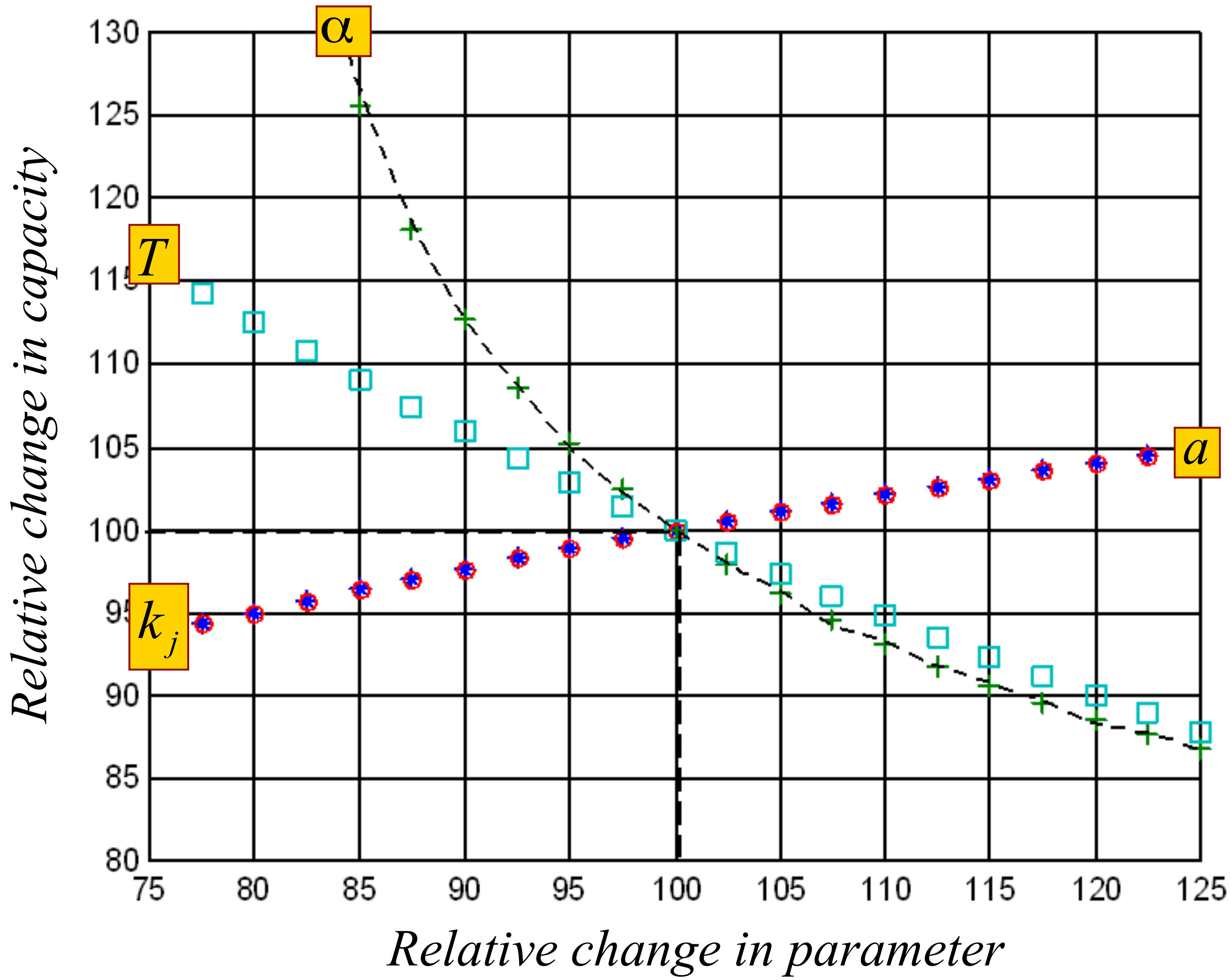
Fundamental diagram and car-following (5)

- Effect of car-following parameters on capacity

$$q_c = \frac{1}{T + \sqrt{2\left(L_{veh} + L_m\right)\frac{1-\alpha^{-1}}{a}}} < \frac{1}{T}$$

- Sensitivity analysis shows that capacity can be increased by:
  - Decrease α (attitude towards risk)
  - Decreasing the (perceived) reaction time
  - Decreasing the vehicle length / safety margin
  - Increasing max. deceleration

Fundamental diagram and car-following (6)



Models of the fundamental diagram

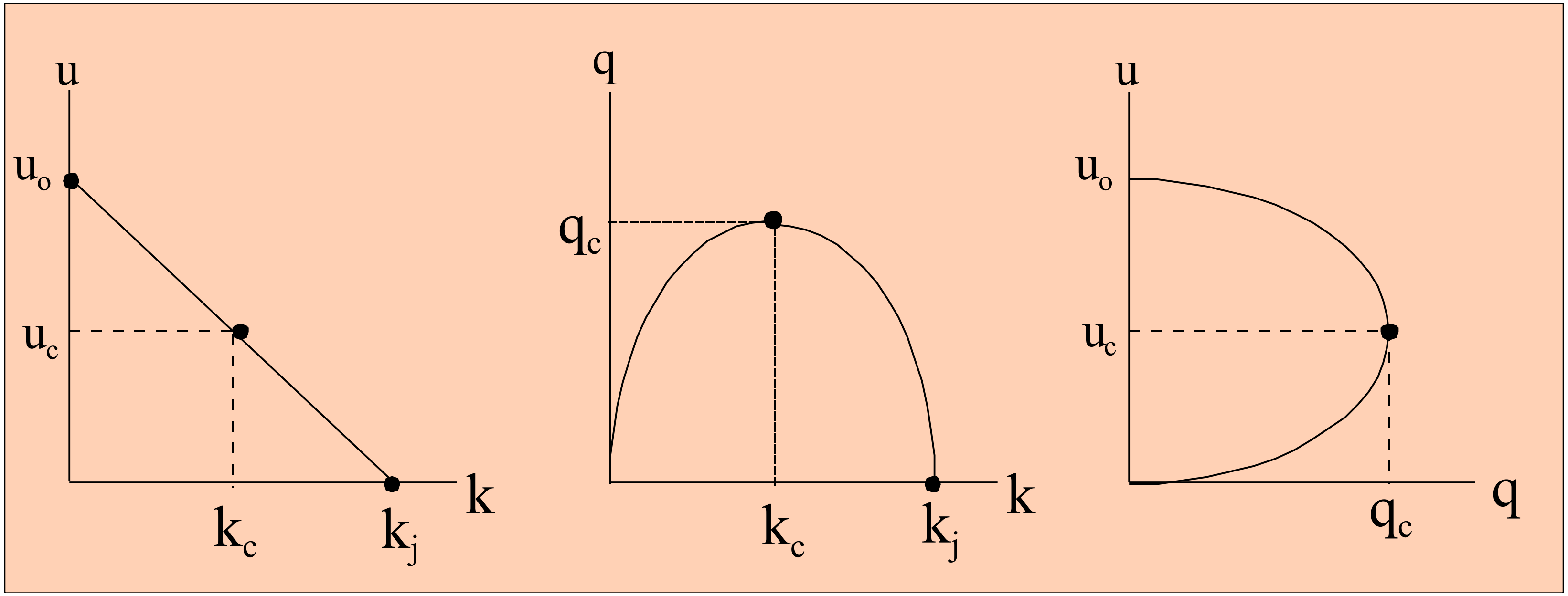
- Approach to model derivation
  1. From microscopic theories about driver behavior (see example, but also car-following models of the stimulus-response type)
  2. Curve fitting to empirical observations
  3. Analogy with flow phenomena in other fields of science (e.g gas-dynamics)

- Remainder will consider some examples of fundamental diagrams that have been / are applied frequently in the past

Models of the fundamental diagram (2)

- Model of Greenshields (based on seven observations!)

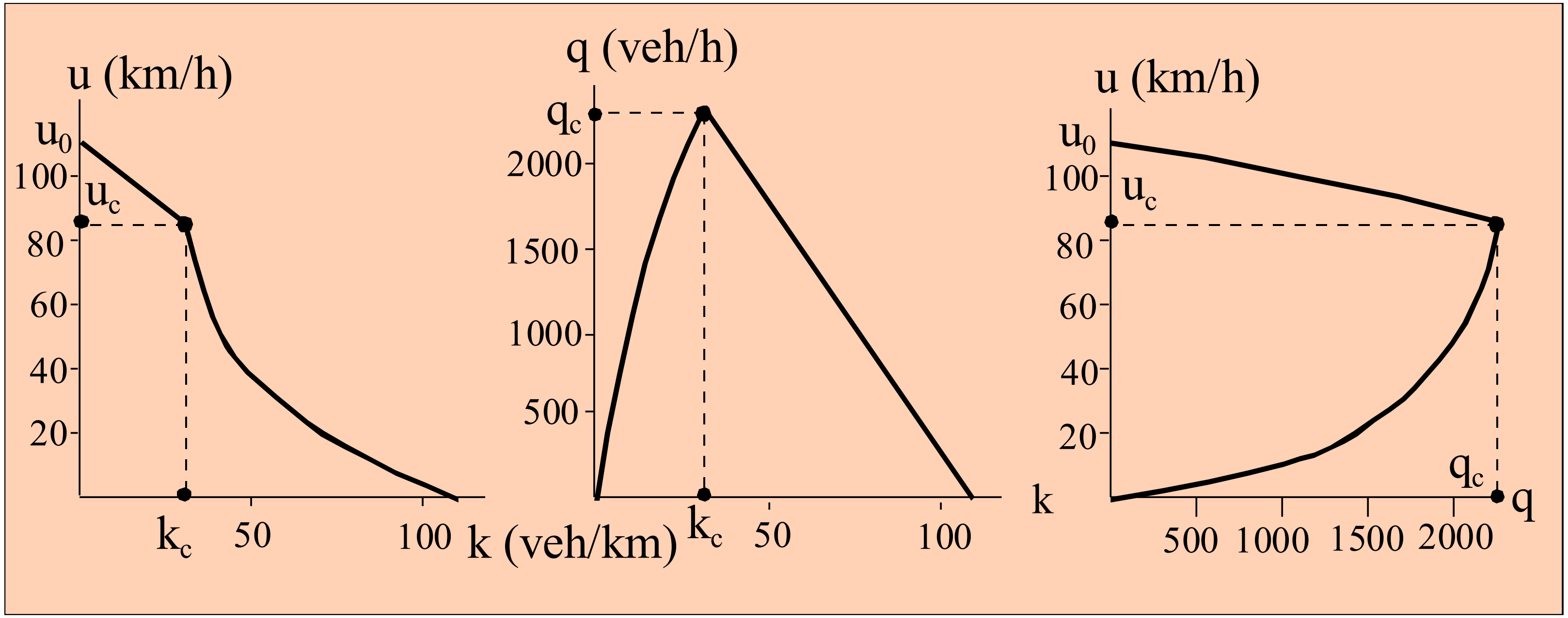
$$u = u_0\left(1 - \frac{k}{k_j}\right) \quad q = u_0 k\left(1 - \frac{k}{k_j}\right) \longrightarrow k_c = \frac{k_j}{2} \quad u_c = \frac{u_0}{2} \quad q_c = \frac{k_j u_0}{4}$$



Models of the fundamental diagram (3)

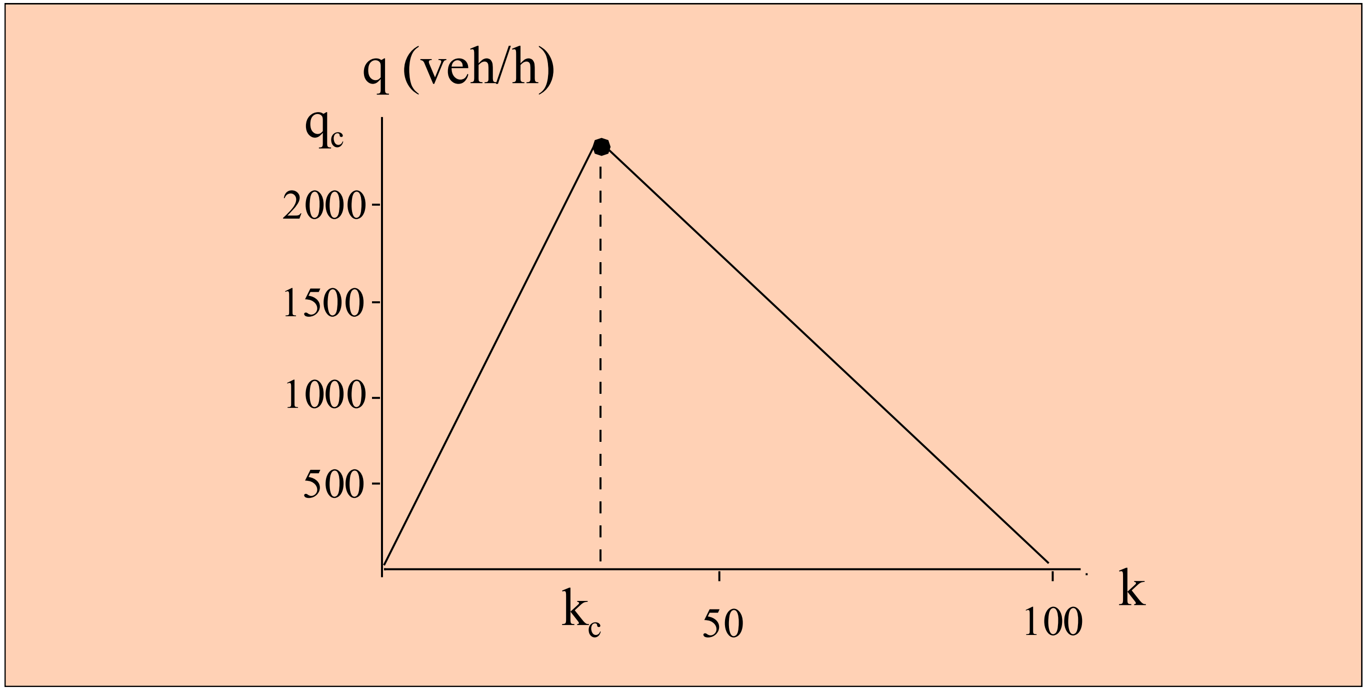
- Fundamental diagram of Smulders

$$u(k) = \begin{cases} u_0\left(1 - k/k_j\right) & k < k_c \\ \beta\left(1/k - 1/k_j\right) & k > k_c \end{cases} \longrightarrow \beta = u_0 k_c$$



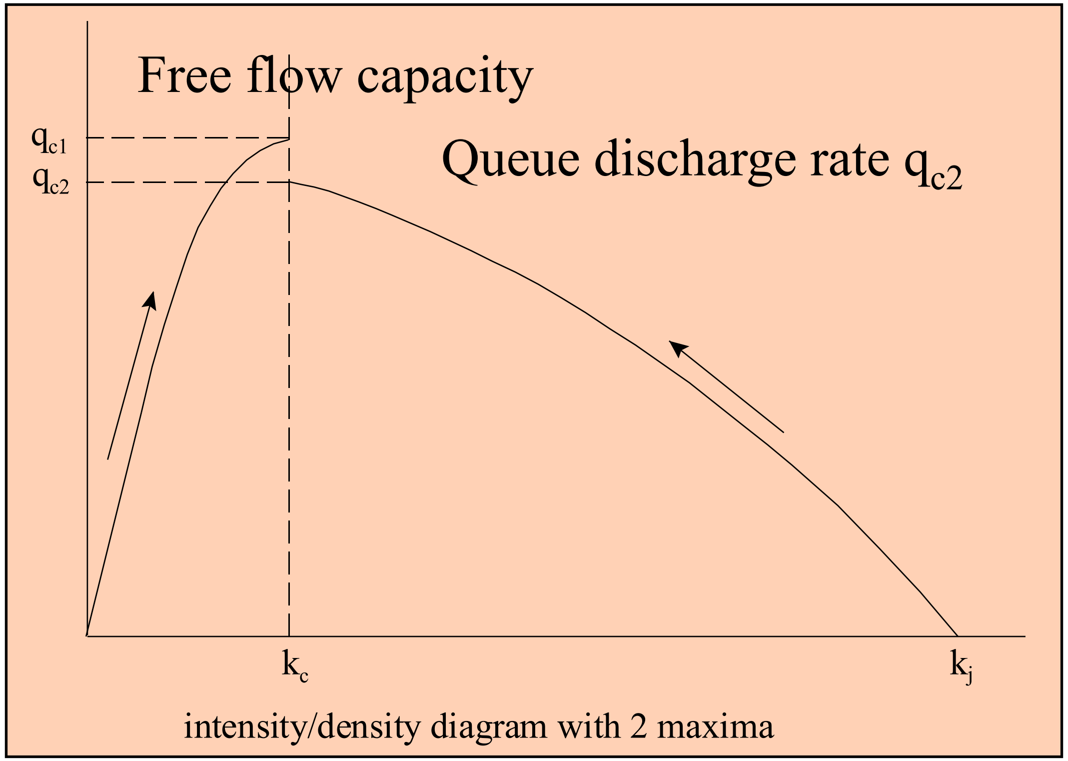
Models of the fundamental diagram (4)

- Schematized model of Daganzo
- Two linear curves
- Free parameters: u_0, q_c, and k_j



Concept of discontinuous diagram

- Discontinuity in the diagram around the capacity point



Wu's model with capacity drop

- Microscopic foundation
- Two regimes + mix
  - Free flow ($0 < k < k_1$)
  - Congested flow ($k_2 < k < k_j$)

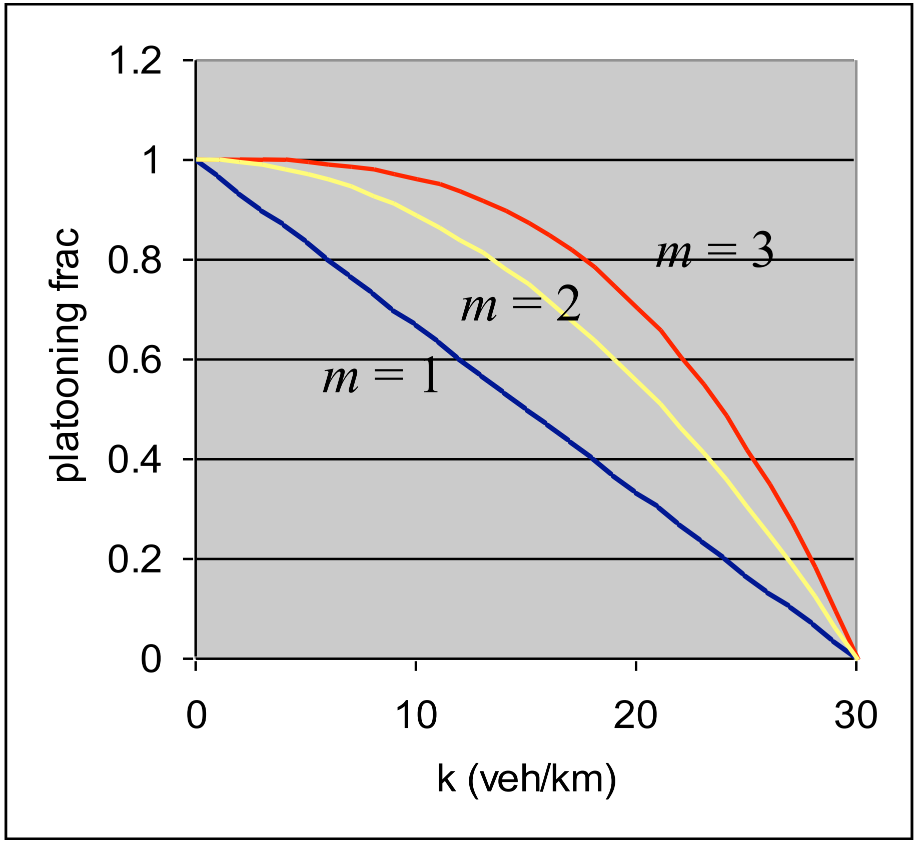
- Model for the free-flow $u(k) = \phi u_0 + (1-\phi) u_p$
  - Free speed u_0
  - Platooning fraction ϕ
  - Platooning speed u_p

Wu's model with capacity drop (2)

- For $k = k_1$ all vehicles are platooning with speed u_p

$$k_1 =$$

- Fraction of platooning vehicles for different roadway types (number of lanes)



- Why h_{net}^f?
  - Observable in practice
  - Constant for all states

Wu's model with capacity drop (3)

- Congested branch ($k > k_2$)
- All vehicles are in car-following state and remain at constant net headway

$$s_{net} = u h_{net}^c \quad \text{where} \quad u = \text{mean speed}$$

$$s_{gross} = s_{net} + 1/k_j = u h_{net}^c + 1/k_j$$

$$k = 1/s_{gross} = \frac{1}{u h_{net}^c + 1/k_j}$$

- Which can be solved for u and leads to

$$u = \frac{1}{h_{net}^c}\left(\frac{1}{k} - \frac{1}{k_j}\right) \quad \text{and} \quad q = \frac{1}{h_{net}^c}\left(1 - \frac{k}{k_j}\right)$$

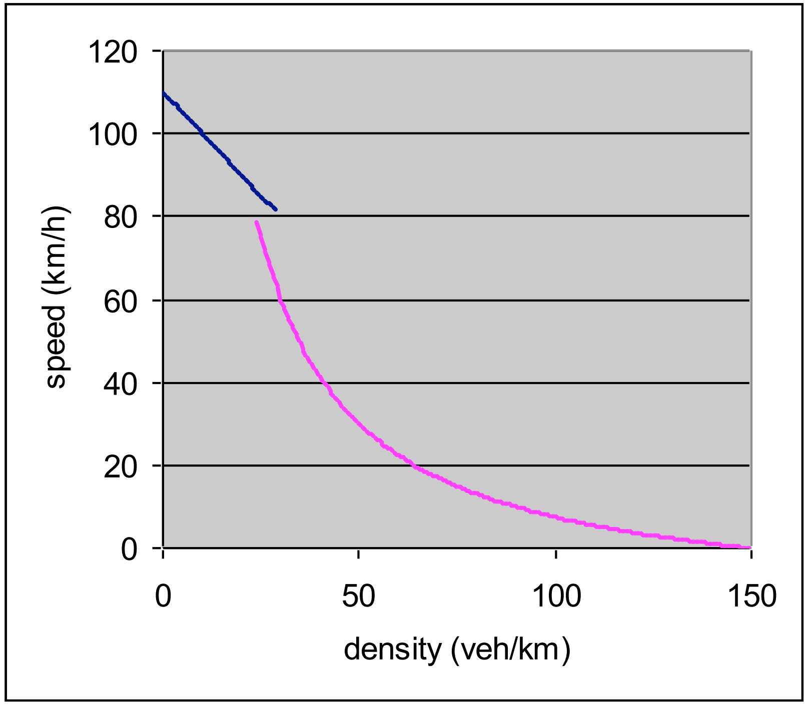
Wu's model with capacity drop (4)

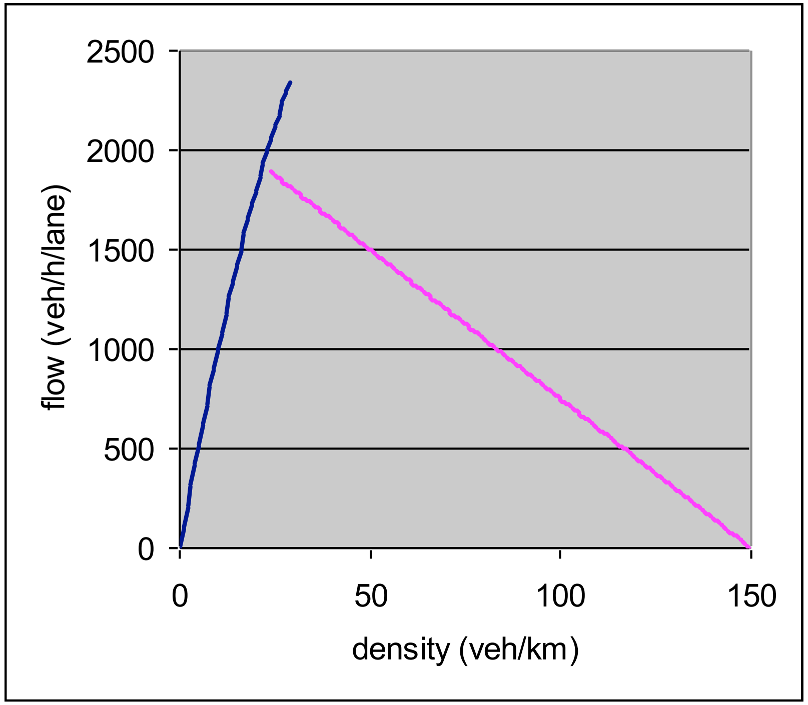
- Determine k_2
- Assumption: maximum speed at congestion is equal to the minimum speed u_p during free flow (at which all vehicles are platooning)

$$u_p = \frac{1}{h_{net}^c}\left(\frac{1}{k_2} - \frac{1}{k_j}\right) \quad \Rightarrow \quad k_2 = \left(u_p h_{net}^c + 1/k_j\right)^{-1}$$

Wu's model with capacity drop (5)

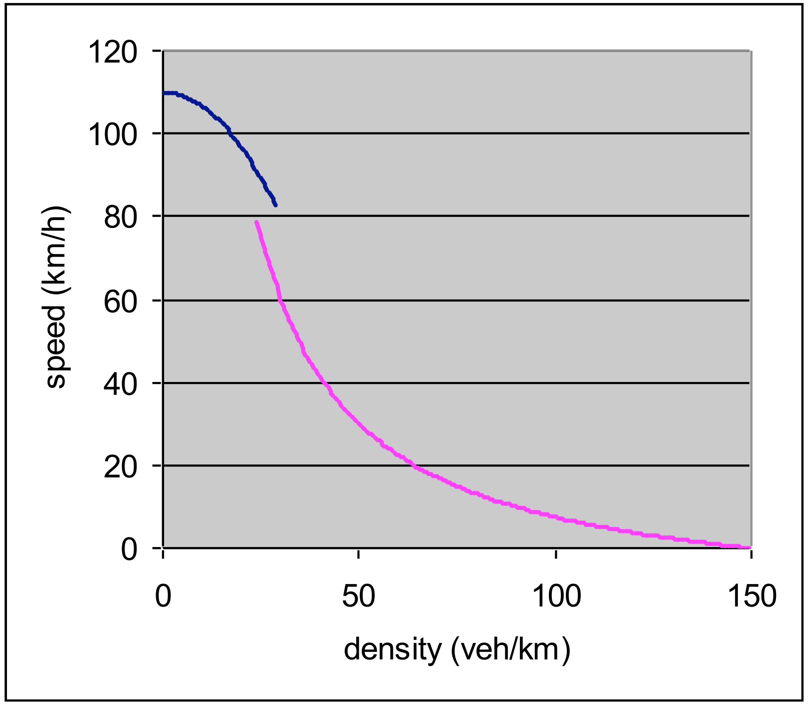
- One-lane road (capacity drop: approx. 21%)

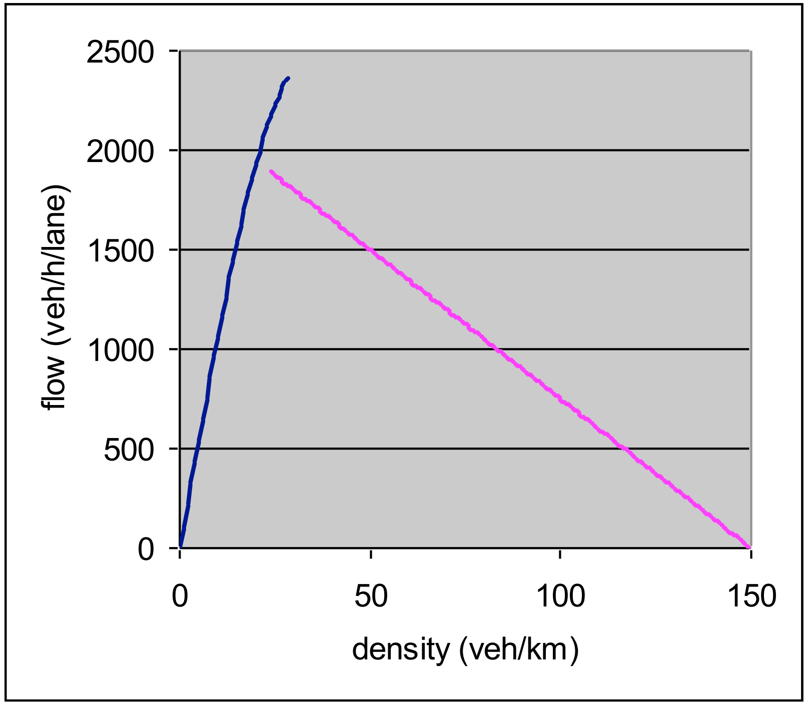




Wu's model with capacity drop (6)

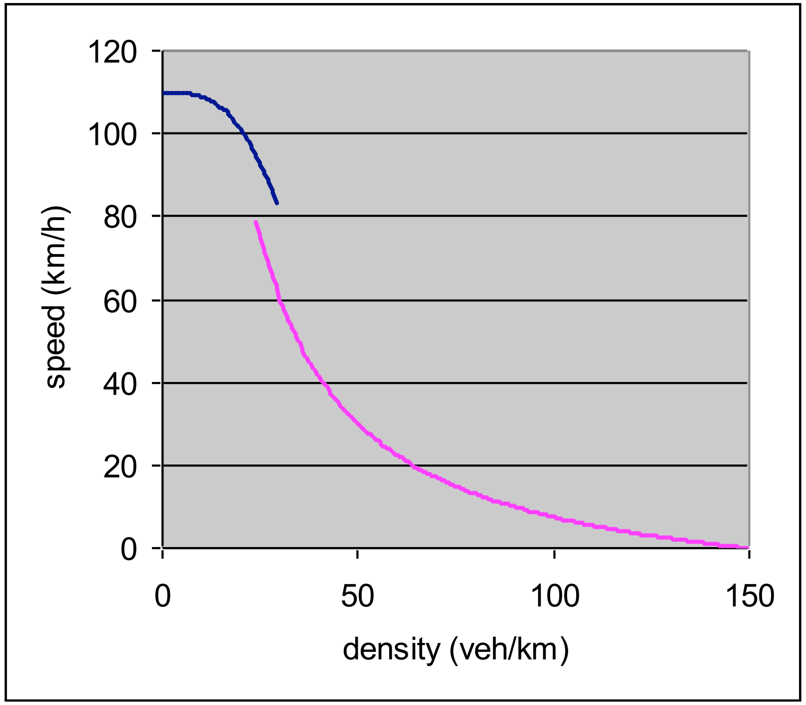
- Two-lane road

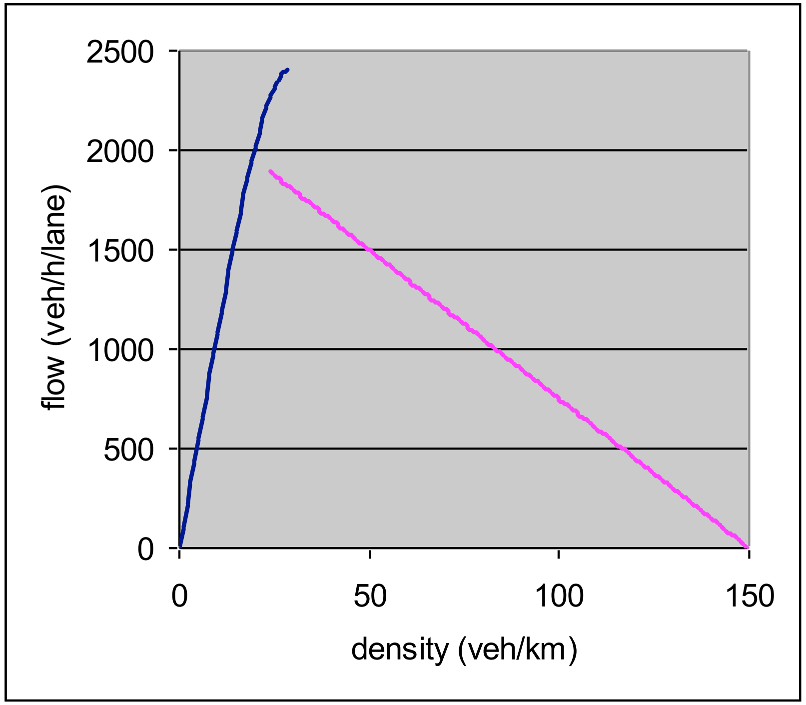




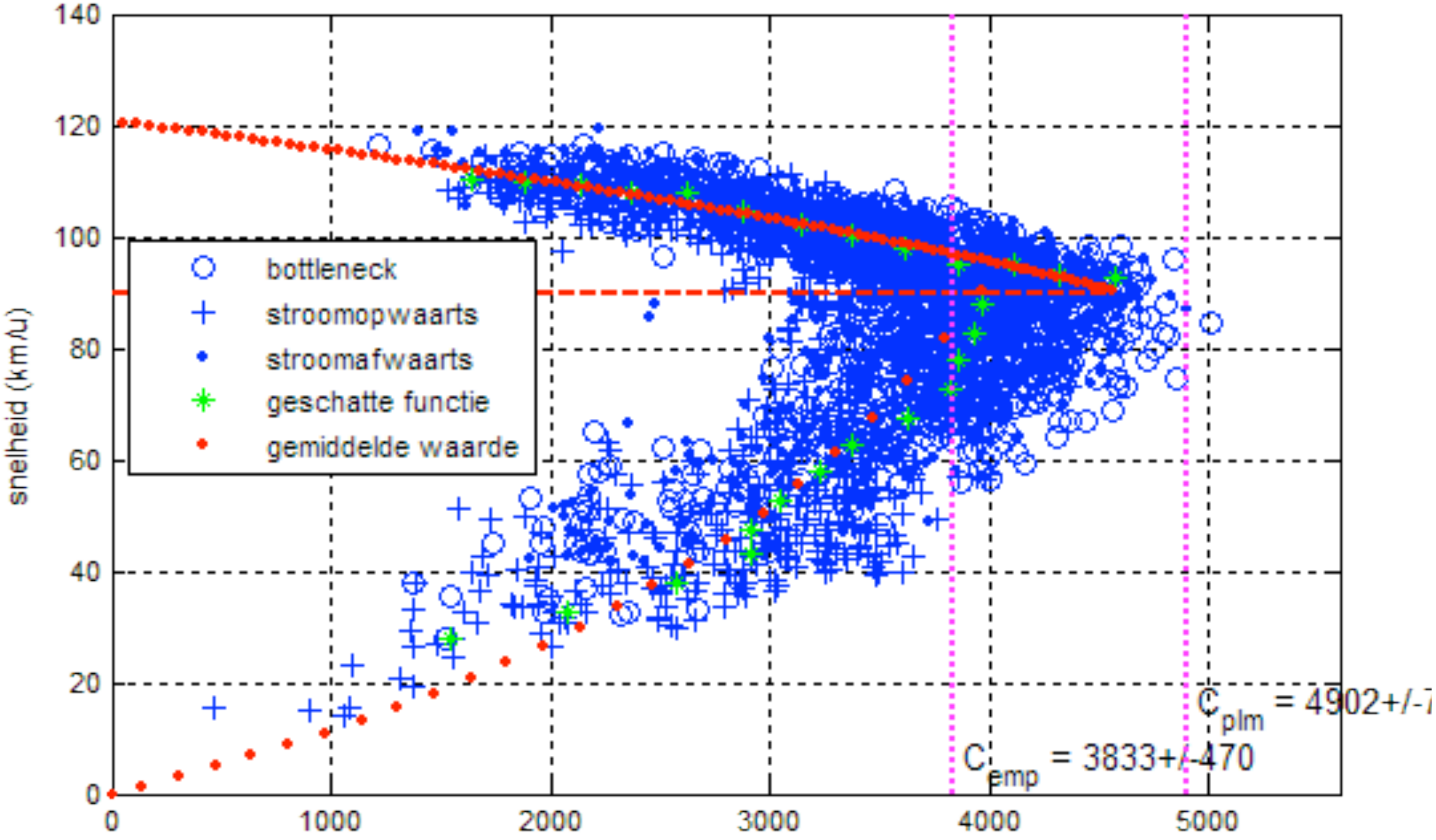
Wu's model with capacity drop (7)

- Three-lane road

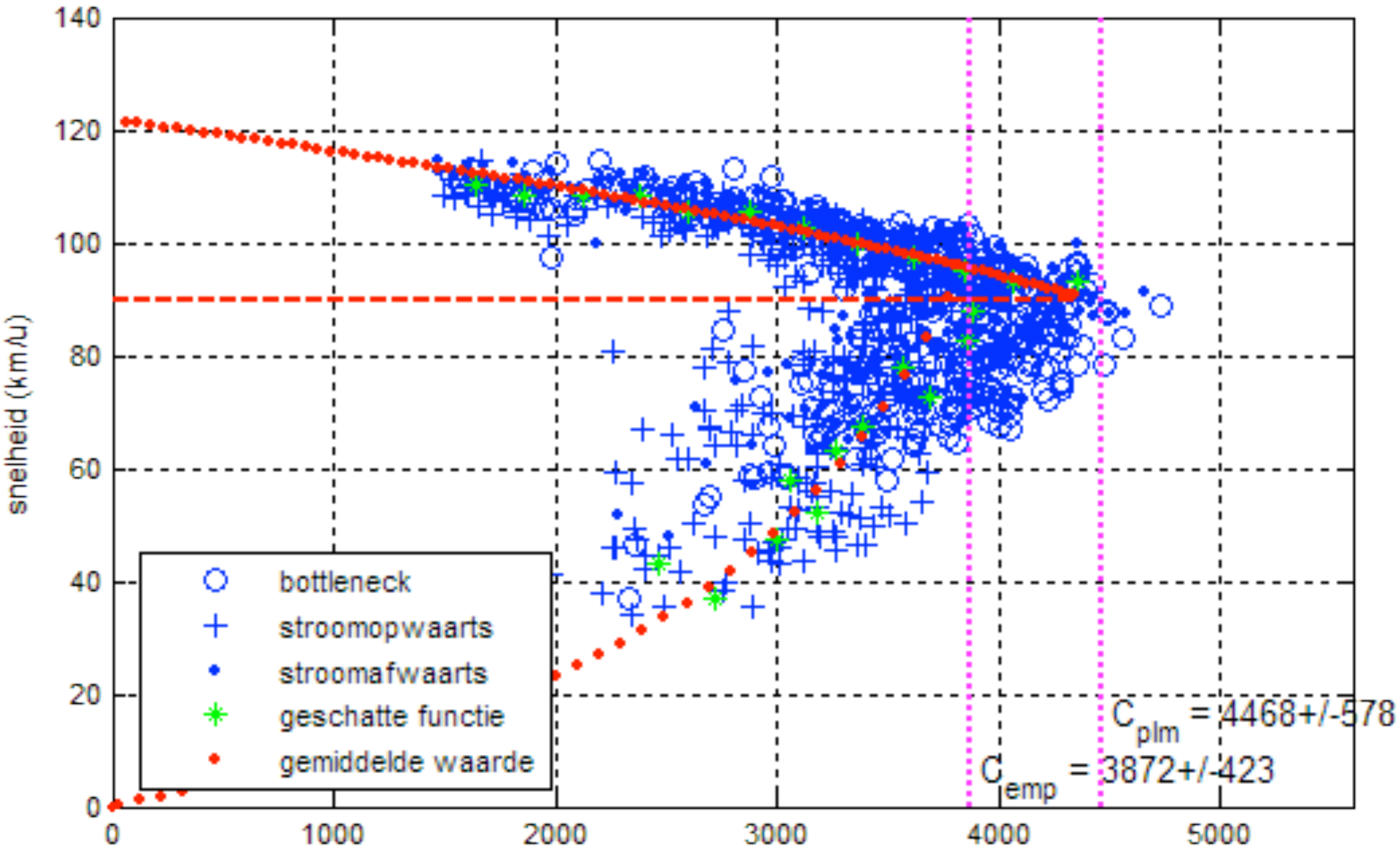




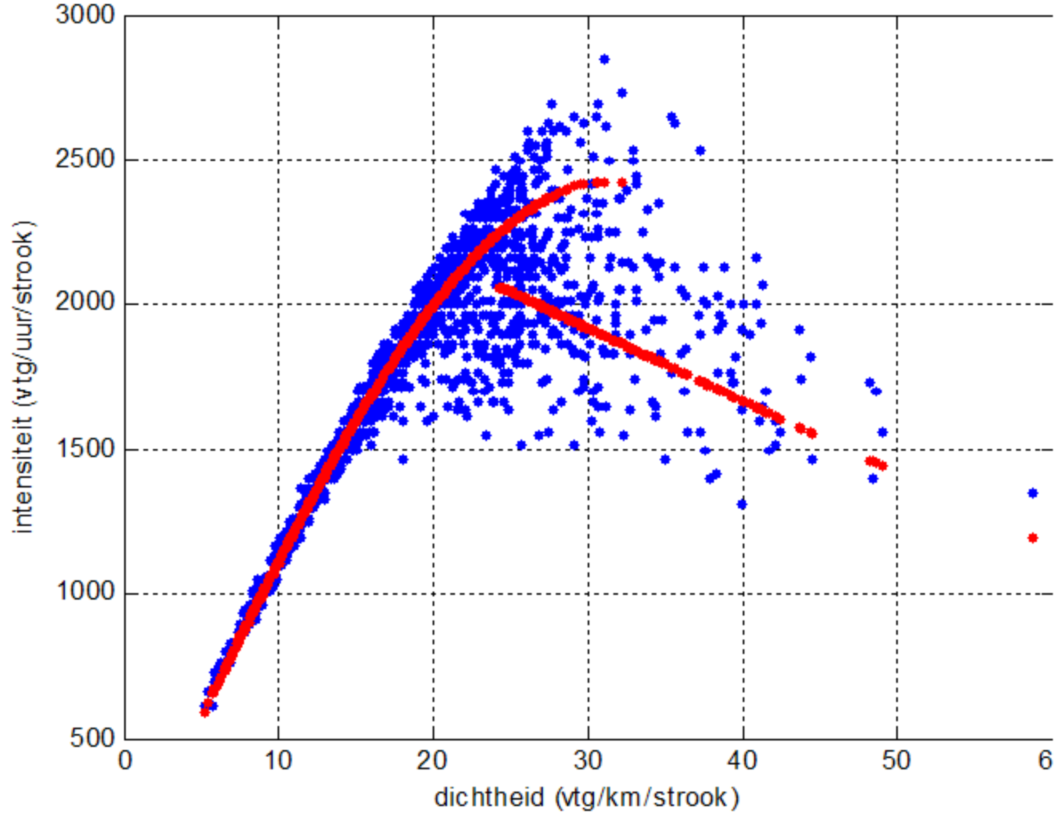
Comparison Wu's diagram and real data



Comparison Wu's diagram and real data



Comparison Wu's diagram and real data



Theory of Kerner

- Kerner: there is no fundamental diagram
- Three fases:
  - Free flow
  - Jam: (near) standstill
  - Synchronized flow
- In the q-k plane, the synchronized flow region is characterized by an 2-D area and **not by a straight line**

Fundamental 'diagram' of Kerner