# Offshore Hydromechanics Part 2

Ir. Peter Naaijen

4. Potential Theory continued

Offshore Hydromechanics, lecture 1


[1]


[2]

Take your laptop, i- or whatever smart-phone and go to:
www.rwpoll.com
Login with session ID



Teacher module II:
- Ir. Peter Naaijen
- p.naaijen@tudelft.nl
- Room 34 B-0-360 (next to towing tank)

Book:
- Offshore Hydromechanics, by J.M.J. Journee & W.W.Massie

Useful weblinks:
- http://www.shipmotions.nl
- Blackboard



## OE4630 module II course content

- +/- 7 Lectures
- Bonus assignments (optional, contributes 20% of your exam grade)
- Laboratory Excercise (starting 30 nov)
  - 1 of the bonus assignments is dedicated to this exercise
  - Groups of 7 students
  - Subscription available soon on BB
- Written exam



Schedule OE4630 D2, Offshore Hydromechanics Pt 2, 2012-2013 **Version 1 (9-11-2012)**
Disclaimer: always track for (last minute) changes in location at huidigeroosters.tudelft.nl!!!

| Date : | Time: | Type: | Teacher: | Location |
|---|---|---|---|---|
| Wed 14 Nov | 13.30 – 16.30 | Lecture | Peter Naaijen | 3mE-CZ D (James Watt) |
| Wed 14 Nov | 16.30 – 17.30 | Assignment assistance /Questions | Peter Naaijen | 3mE-CZ D (James Watt) |
| Fri 16 Nov | 10.30 – 12.30 | Lecture | Peter Naaijen | 3mE-CZ B (Isaac Newton) |
| Mon 19 Nov | 15.30 – 17.30 | Lecture | Peter Naaijen | 3mE-CZ B (Isaac Newton) |
| Tue 20 Nov | 13.30 – 15.30 | Assignment assistance /Questions | Peter Naaijen | 3mE-CZ C (Daniel Bernoulli) |
| Wed 28 Nov | 13.30 – 15.30 | Lecture | Peter Naaijen | 3mE-CZ D (James Watt) |
| Wed 28 Nov | 15.30 – 17.30 | Assignment assistance /Questions | Peter Naaijen | 3mE-CZ D (James Watt) |
| Fri 30 Nov | 10.30 – 13.00 | Lab session | Peter Naaijen | Towing Tank |
| Mon 3 Dec | 15.30 – 17.30 | Lecture | Peter Naaijen | 3mE-CZ B (Isaac Newton) |
| Tue 4 Dec | 13.30 – 16.00 | Lab session | Gideon Hertzberger | Towing Tank |
| Tue 4 Dec | 16.30 – 17.30 | Assignment assistance /Questions | Peter Naaijen | Room Peter Naaijen (34 B 0 360) |
| Mon 10 Dec | 15.30 – 17.30 | Lecture | Peter Naaijen | 3mE-CZ B (Isaac Newton) |
| Mon 17 Dec | 15.30 – 17.30 | Lecture | Peter Naaijen | 3mE-CZ B (Isaac Newton) |
| Mon 7 Jan | 15.30 – 17.30 | Lecture | Peter Naaijen | 3mE-CZ B (Isaac Newton) |

Lecture notes:

- Disclaimer: Not everything you (should) learn is in the lecture notes (lees: niet alles wat op het tentamen gevraagd kan worden staat in diktaat…) -7

Make personal notes during lectures!!

- Don't save your questions 'till the break -7

Ask if anything is unclear



# Learning goals Module II, behavior of floating bodies in waves

| | | |
|---|---|---|
| • Definition of ship motions<br>Motion Response in regular waves:<br>• How to use RAO's<br>• Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces<br>• How to solve RAO's from the equation of motion<br>Motion Response in irregular waves:<br>•How to determine response in irregular waves from RAO's and wave spectrum without forward speed | | |
| 3D linear Potential Theory<br>•How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential<br>•How to determine Velocity Potential | | |
| Motion Response in irregular waves:<br>• How to determine response in irregular waves from RAO's and wave spectrum with forward speed<br>• Make down time analysis using wave spectra, scatter diagram and RAO's | Ch. 8 | |
| Structural aspects:<br>• Calculate internal forces and bending moments due to waves | | |
| Nonlinear behavior:<br>• Calculate mean horizontal wave force on wall<br>• Use of time domain motion equation | Ch.6 | |



# Introduction

[3]



# Introduction

Offshore oil resources have to be explored in deeper water floating structures instead of bottom founded

[4]

# Introduction

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Inertia forces on sea fastening due to accelerations:




# Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Direct wave induced structural loads


[5]

Minimum required air gap to avoid wave damage



# Introduction

Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations


[5]


[5]

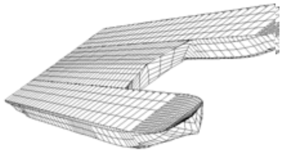

[6]

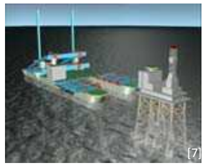


# Introduction

Decommissioning / Installation / Pipe laying -7 Excalibur / Allseas 'Pieter Schelte'

- Motion Analysis

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# Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Forces on mooring system, motion envelopes loading arms


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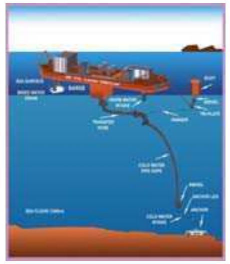
# Introduction

Floating Offshore: More than just oil


Floating wind farm [9]


OTEC [10]



# Introduction

Floating Offshore: More than just oil


[11]


[12]

Wave energy conversion



# Introduction

Floating Offshore: More than just oil

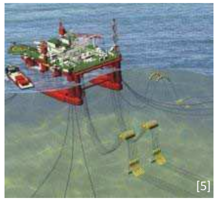

Mega Floaters

# Introduction

## Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations
- Downtime analysis

| Winter Data of Areas 8, 9, 15 and 16 of the North Atlantic (Global Wave Statistics) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T_2$ (s) | | | | | | | | | | | |
| Hs (m) | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 | 12.5 | 13.5 | Total |
| 14 | 0 | 0 | 0 | 0 | 2 | 30 | 154 | 362 | 466 | 370 | 202 | 1586 |
| 13 | 0 | 0 | 0 | 0 | 3 | 33 | 145 | 293 | 322 | 219 | 101 | 1116 |
| 12 | 0 | 0 | 0 | 0 | 7 | 72 | 289 | 539 | 548 | 345 | 149 | 1949 |
| 11 | 0 | 0 | 0 | 0 | 17 | 160 | 585 | 996 | 931 | 543 | 217 | 3449 |
| 10 | 0 | 0 | 0 | 1 | 41 | 363 | 1200 | 1852 | 1579 | 843 | 310 | 6189 |
| 9 | 0 | 0 | 0 | 4 | 109 | 845 | 2485 | 3443 | 2648 | 1283 | 432 | 11249 |
| 8 | 0 | 0 | 0 | 12 | 295 | 1996 | 5157 | 6323 | 4333 | 1882 | 572 | 20570 |
| 7 | 0 | 0 | 0 | 41 | 818 | 4723 | 10537 | 11242 | 6755 | 2594 | 703 | 37413 |
| 6 | 0 | 0 | 1 | 138 | 2273 | 10967 | 20620 | 18718 | 9665 | 3222 | 767 | 66371 |
| 5 | 0 | 0 | 7 | 471 | 6187 | 24075 | 36940 | 27702 | 11969 | 3387 | 694 | 111432 |
| 4 | 0 | 0 | 31 | 1586 | 15757 | 47072 | 56347 | 33539 | 11710 | 2731 | 471 | 169244 |
| 3 | 0 | 0 | 148 | 5017 | 34720 | 74007 | 64809 | 28964 | 7804 | 1444 | 202 | 217115 |
| 2 | 0 | 4 | 681 | 13441 | 56847 | 77259 | 45013 | 13962 | 2725 | 381 | 41 | 210354 |
| 1 | 0 | 40 | 2699 | 23284 | 47839 | 34532 | 11554 | 2208 | 282 | 27 | 2 | 122467 |
| 0 | 5 | 350 | 3314 | 8131 | 5858 | 1598 | 216 | 18 | 1 | 0 | 0 | 19491 |
| Total | 5 | 394 | 6881 | 52126 | 170773 | 277732 | 256051 | 150161 | 61738 | 19271 | 4863 | 999995 |



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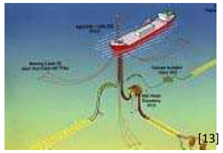

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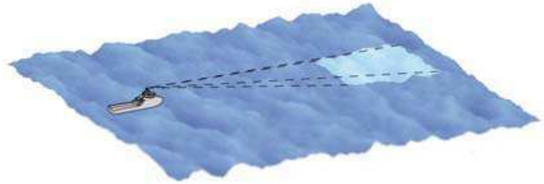
Real-time motion prediction

Using X-band radar remote wave observation





# Definitions & Conventions

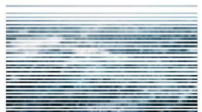
Regular waves

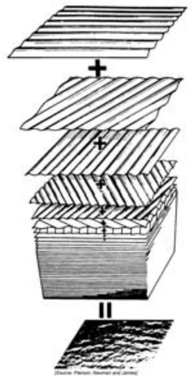
Ship motions

[5]



apparently irregular but can be considered as a superposition of a finite number of regular waves, each having own frequency, amplitude and propagation direction





[5]

Regular waves



## Regular waves

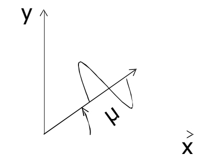
regular wave propagating in direction μ:

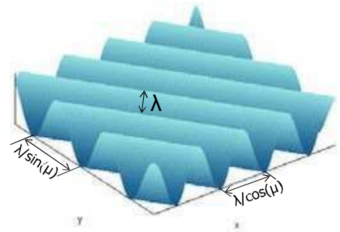
$$(t,x)\quad {}_a\cos\quad t\quad kx\cos\quad ky\sin$$

$$k\quad 2\quad /$$

$$2\quad /T$$

Linear solution Laplace equation

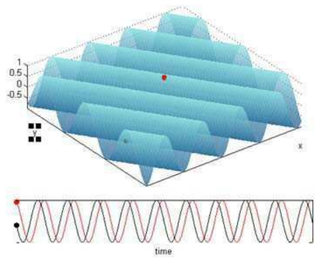





- Regular waves
- regular wave propagating in direction μ

$$(t,x)\quad {}_a\cos\quad t\quad kx\cos\quad ky\sin$$




Phase angle wave at black dot with respect to wave at red dot:

$$k\ \ x\ \ x\ \cos\quad k\ \ y\ \ y\ \sin$$

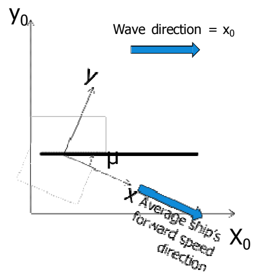


## Co-ordinate systems

Definition of systems of axes

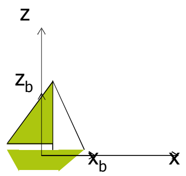
Earth fixed: ($x_0$, $y_0$, $z_0$)

wave direction with respect to ship's exes system:



## Behavior of structures in waves

Ship's body bound axes system ($x_b$,$y_b$,$z_b$) follows all ship motions

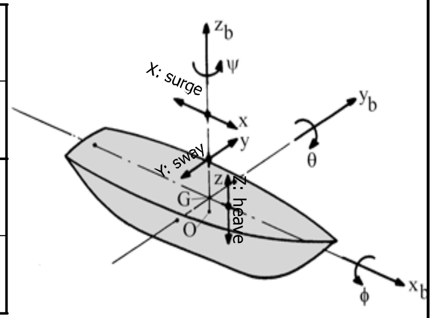





## Behavior of structures in waves

Definition of translations

| | | NE | EN |
|---|---|---|---|
| 1 | x | Schrikken | Surge |
| 2 | y | Verzetten | Sway |
| 3 | z | Dompen | Heave |

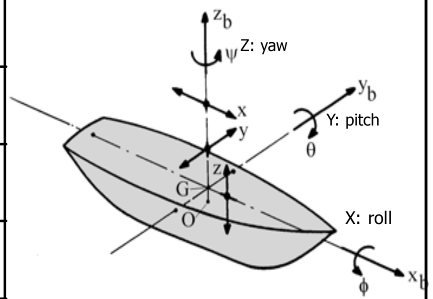





## Behavior of structures in waves

Definition of rotations

| | | | |
|---|---|---|---|
| | | | |
| 5 | y | Stampen | Pitch |
| 6 | z | Gieren | Yaw |






## How do we describe ship motion response?

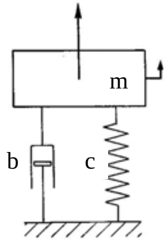
Rao's

Phase angles

## Mass-Spring system:

$m\ddot{z} + b\dot{z} + cz = F_a \cos(\omega t)$ Motion equation

$z(t) = z_a \cos(\omega t + \varepsilon)$ Steady state solution






## Motions of and about COG

Amplitude / Phase angle

*Surge(schrikken)*: $x = x_a \cos(\omega t + \varepsilon_{x\zeta})$

*Sway(verzetten)*: $y = y_a \cos(\omega t + \varepsilon_{y\zeta})$

*Heave(dompen)*: $z = z_a \cos(\omega t + \varepsilon_{z\zeta})$

*Roll(rollen)*: $\langle \text{phi} \rangle \phi = \phi_a \cos(\omega t + \varepsilon_{\phi\zeta})$

*Pitch(stampen)*: $\langle \text{theta} \rangle \theta = \theta_a \cos(\omega t + \varepsilon_{\theta\zeta})$

*Yaw(gieren)*: $\langle \text{psi} \rangle \psi = \psi_a \cos(\omega t + \varepsilon_{\psi\zeta})$

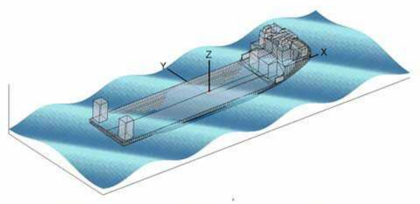
Phase angles $\varepsilon$ are related to undisturbed wave at origin of steadily translating ship-bound system of axes (→COG)



## Motions of and about COG

Phase angles $\varepsilon$ are related to undisturbed wave at origin of steadily

translating ship-bound system of axes (→COG)






## Motions of and about COG

*Surge(schrikken)*: $x = x_a \cos(\omega t + \varepsilon_{x\zeta})$ RAOSurge: $\frac{x_a}{\zeta_a}(\omega, \mu)$

*Sway(verzetten)*: $y = y_a \cos(\omega t + \varepsilon_{y\zeta})$ RAOSway: $\frac{y_a}{\zeta_a}(\omega, \mu)$

*Heave(dompen)*: $z = z_a \cos(\omega t + \varepsilon_{z\zeta})$ RAOHeave: $\frac{z_a}{\zeta_a}(\omega, \mu)$

*Roll(rollen)*: $\langle \text{phi} \rangle \phi = \phi_a \cos(\omega t + \varepsilon_{\phi\zeta})$ RAORoll: $\frac{\phi_a}{\zeta_a}(\omega, \mu)$

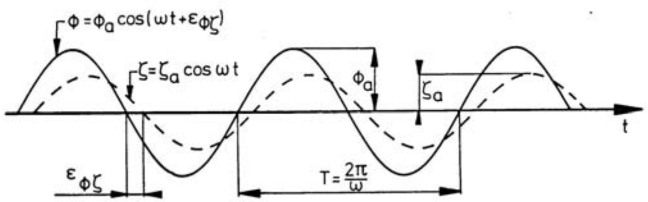
*Pitch(stampen)*: $\langle \text{theta} \rangle \theta = \theta_a \cos(\omega t + \varepsilon_{\theta\zeta})$ RAOPitch: $\frac{\theta_a}{\zeta_a}(\omega, \mu)$

*Yaw(gieren)*: $\langle \text{psi} \rangle \psi = \psi_a \cos(\omega t + \varepsilon_{\psi\zeta})$ RAOYaw: $\frac{\psi_a}{\zeta_a}(\omega, \mu)$

RAO and phase depend on:

- Wave frequency
- Wave direction

## Example: roll signal




Displacement $\phi = \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right)$

Velocity... $\dot{\phi} = -\omega_e \phi_a \sin\left(\omega_e t + \varepsilon_{\phi\zeta}\right) = \omega_e \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta} + \pi / 2\right)$

Acceleration... $\ddot{\phi} = -\omega_e^2 \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right) = \omega_e^2 \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta} + \pi\right)$



## Motions of and about COG

1 *Surge(schrikken)*: $x = x_a \cos\left(\omega_e t + \varepsilon_{x\zeta}\right)$

2 *Sway(verzetten)*: $y = y_a \cos\left(\omega_e t + \varepsilon_{y\zeta}\right)$

3 *Heave(dompen)*: $z = z_a \cos\left(\omega_e t + \varepsilon_{z\zeta}\right)$

4 *Roll(rollen)*: $\langle \text{phi} \rangle \phi = \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right)$

5 *Pitch(stampen)*: $\langle \text{theta} \rangle \theta = \theta_a \cos\left(\omega_e t + \varepsilon_{\theta\zeta}\right)$

6 *Yaw(gieren)*: $\langle \text{psi} \rangle \psi = \psi_a \cos\left(\omega_e t + \varepsilon_{\psi\zeta}\right)$

- Frequency of input (regular wave) and output (motion) is ALWAYS THE SAME !!
- Phase can be positive ! (shipmotion ahead of wave elevation at COG)
- Due to symmetry: some of the motions will be zero
- Ratio of motion amplitude / wave amplitude = RAO (Response Amplitude Operator)
- RAO's and phase angles depend on wave frequency and wave direction
- RAO's and phase angles must be calculated by dedicated software or measured by experiments
- Only some special cases in which 'common sense' is enough:



## Consider Long waves relative to ship dimensions

What is the RAO of pitch in head waves ?

- Phase angle heave in head waves ?...
- RAO pitch in head waves ?...
- Phase angle pitch in head waves ?...
- Phase angle pitch in following waves ?...



## Consider very long waves compared to ship dimensions

60

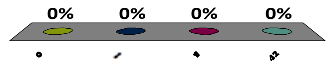
What is the RAO for heave in head waves ?

A. 0

B. ∞

C. 1

D. 42

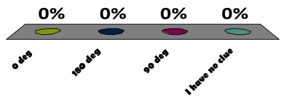


**Consider very long waves compared to ship dimensions**

60

What is the phase for heave in head waves ?

A. 0 deg
B. 180 deg
C. 90 deg
D. I have no clue






**Consider very long waves compared to ship dimensions**

60

What is the phase for heave in head waves ?

A. 0 deg
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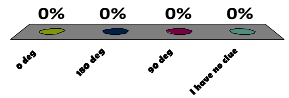


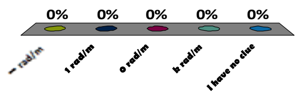



**Consider very long waves compared to ship dimensions**

90

What is the RAO for pitch in head waves ?

A. ∞ rad/m
B. 1 rad/m
C. 0 rad/m
D. k rad/m
E. I have no clue



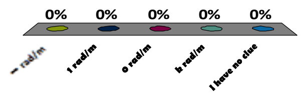



**Consider very long waves compared to ship dimensions**

90

What is the RAO for pitch in head waves ?

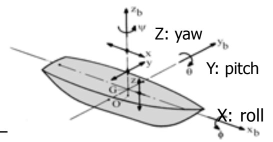
A. ∞ rad/m
B. 1 rad/m
C. 0 rad/m
D. k rad/m
E. I have no clue

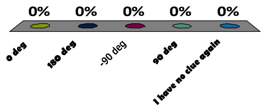


## Consider very long waves compared to ship dimensions

What is the phase for pitch in head waves ? **120**

A. 0 deg
B. 180 deg
C. -90 deg
D. 90 deg
E. I have no clue again



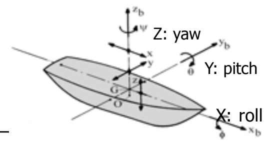



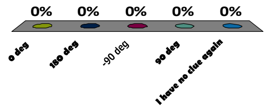



## Consider very long waves compared to ship dimensions

What is the phase for pitch in head waves ? **120**

A. 0 deg
B. 180 deg
C. -90 deg
D. 90 deg
E. I have no clue again





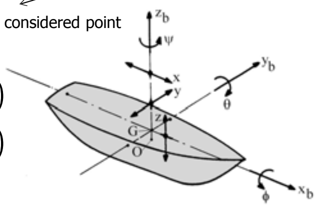

## Local motions (in steadily translating axes system)

- **Only variations!!**
- **Linearized!!**

$$\begin{pmatrix} x_P(t) \\ y_P(t) \\ z_P(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} + \begin{pmatrix} 0 & -\psi(t) & \theta(t) \\ \psi(t) & 0 & -\phi(t) \\ -\theta(t) & \phi(t) & 0 \end{pmatrix} \cdot \begin{pmatrix} x_{bP} \\ y_{bP} \\ z_{bP} \end{pmatrix}$$

6 DOF Ship motions — Location considered point

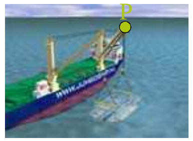
$$x_P(t) = x(t) - y_{bP}\psi(t) + z_{bP}\theta(t)$$
$$y_P(t) = y(t) + x_{bP}\psi(t) - z_{bP}\phi(t)$$
$$z_P(t) = z(t) - x_{bP}\theta(t) + y_{bP}\phi(t)$$





## Local Motions

The tip of an onboard crane, location:


For a frequency ω=0.6 the RAO's and phase angles of the ship motions are:

| SURGE | | SWAY | | HEAVE | | ROLL | | PITCH | | YAW | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| - | degr | - | degr | - | degr | deg/m | degr | deg/m | degr | deg/m | degr |
| 1.014E-03 | 3.421E+02 | 5.992E-01 | 2.811E+02 | 9.991E-01 | 3.580E+02 | 2.590E+00 | 1.002E+02 | 2.424E-03 | 1.922E+02 | 2.102E-04 | 5.686E+01 |

Calculate the RAO and phase angle of the transverse horizontal motion (y-direction)

**Complex notation of harmonic functions**

$$1\ Surge(schrikken):\quad x = x_a \cos\left(\omega_e t + \varepsilon_{x\zeta}\right)$$

$$= \mathrm{Re}\left(x_a e^{i(\omega t + \varepsilon_{x\zeta})}\right)$$

$$= \mathrm{Re}\left(x_a e^{i\varepsilon_{x\zeta}} \cdot e^{i\omega t}\right)$$

Complex motion amplitude

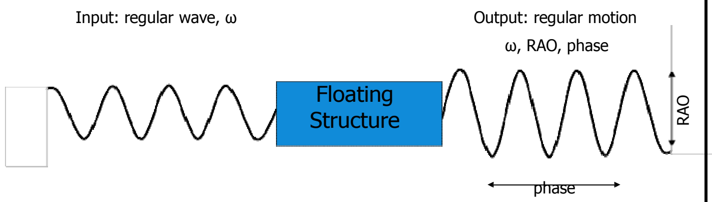
$$= \mathrm{Re}\left(\hat{x}_a \cdot e^{i\omega t}\right)$$

- :



# Relation between Motions and Waves

## How to calculate RAO's and phases ?

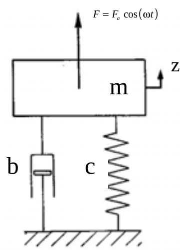





# Mass-Spring system:

Forces acting on body:

...?





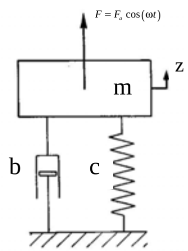

# Mass-Spring system:

$$m\ddot{z} + b\dot{z} + cz = F_a \cos(\omega t)$$

Transient solution

$$z(t) = A_t e^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\,\omega_0 t + \varphi_t\right)$$

$$\left(\zeta = \frac{b}{2\sqrt{mc}}\right)$$ Damping ratio



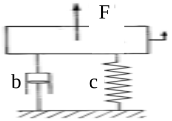

Steady state solution:

$$z(t) = z_a \cos(\omega t + \varepsilon)$$

$$\varepsilon = a\tan\left(\frac{(-b\omega)}{\left(-\omega^2(m) + c\right)}\right)$$

$$z_a = \sqrt{\frac{F_a^{\,2}}{\left(\left(-(m)\omega^2 + c\right)^2 + (b\omega)^2\right)}}$$

# Moving ship in waves:


$m_3\ddot{z} + b_3\dot{z} + c_3 z = F_{a3}\cos(\omega t)$

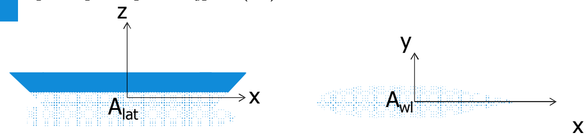
Restoring coefficient for **heave** ?

m for **roll** ?

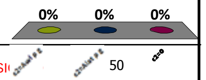


# What is the hydrostatic spring coefficient for the sway motion ?

$m_2\ddot{y} + b_2\dot{y} + c_2 \cdot y = F_{a2}\cos(\omega t)$

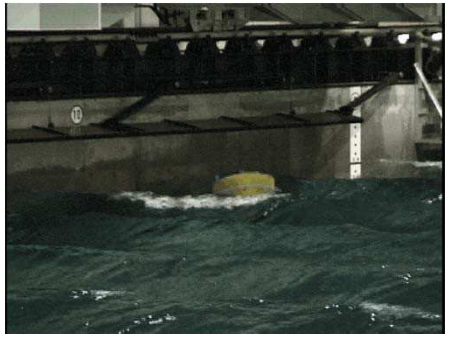



A. $c_2 = A_{wl}\ \rho\ g$
B. $c_2 = A_{lat}\ \rho\ g$
C. $c_2 = 0$






Non linear stability issue...





# Roll restoring

Roll restoring coefficient:

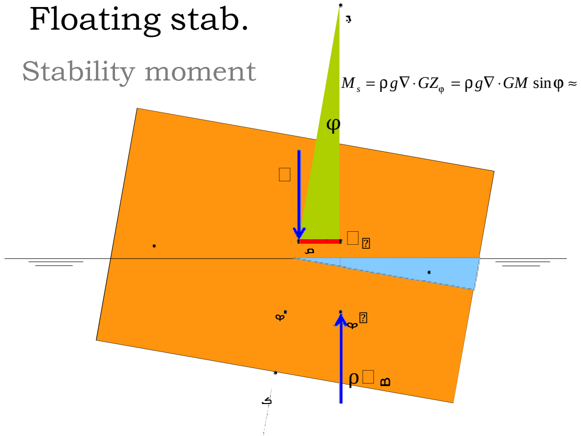
$$c_4 = \rho g \nabla \cdot GM$$

What is the point the ship rotates around statically speaking ? (Ch 2)

# Floating stab.

## Stability moment

$$M_s = \rho g \nabla \cdot GZ_\varphi = \rho g \nabla \cdot GM \sin\varphi \approx \rho g \nabla \cdot GM \cdot \varphi$$

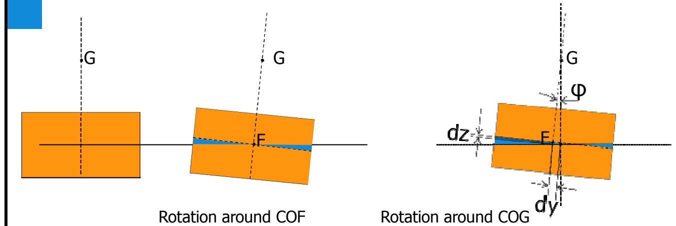




# Moving ship in waves:

$m_4\ddot{\varphi} + b_4\dot{\varphi} + c_4\varphi = F_{a4}\cos(\omega t)$

Restoring coefficient for **roll** ?




Rotation around COF

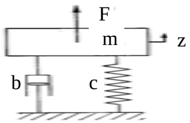
Rotation around COG
=Rotation around COF
+vertical translation
+horizontal translation

$dz = FG - FG\cos\varphi \approx 0$

$dy = FG\sin\varphi \approx FG\varphi$



# Moving ship in waves:
# Not in air but in water!




$F = m\ddot{z}$

SHIP MOTION : HEAVE

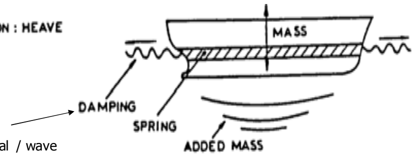
$\longrightarrow F_w$

$\longrightarrow -c \cdot z$

$\longrightarrow -b \cdot \dot{z}$

$\longrightarrow -a \cdot \ddot{z}$

(Only potential / wave damping)




$$(m+a)\ddot{z} + b\dot{z} + c \cdot z = F_w$$



# Moving ship in waves:

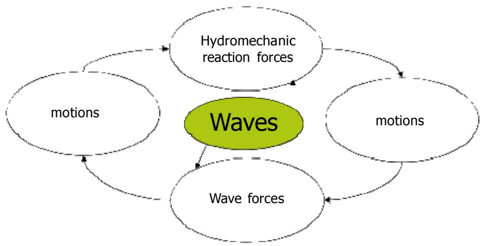
Analogy / differences with mass-spring system:

| |  |  |
|---|---|---|
| External force | F(t) | Wave exciting force<br>Has a phase angle w r t undisturbed wave at COG |
| restoring force | c*z | Archimedes: bouyancy |
| Damping force | b*dz/dt | Hydrodynamic damping |
| Inertia force | M*d ²z/dt ² | Mass + Hydrodynamic Mass |

Depend on frequency !

## Moving ship in waves:

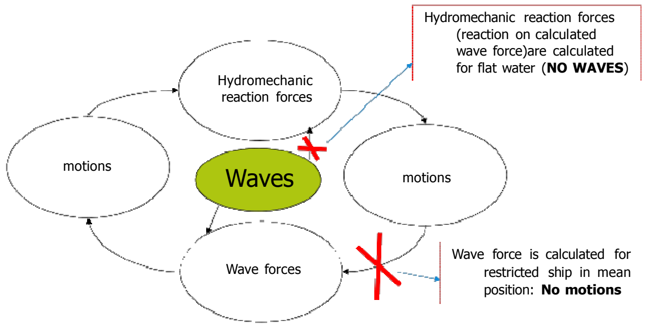
$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$






## Moving ship in waves:

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$






## Right hand side of m.e.: Wave Exciting Forces

- Incoming: regular wave with given frequency and propagation direction
- Assuming the vessel is not moving



## Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x)=\zeta_a\cos(\omega t-kx\cos\mu-ky\sin\mu)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

## Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x) = \zeta_a \cos(\omega t - kx\cos\mu - ky\sin\mu)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?



## Potential Theory

What is potential theory ?:
way to give a mathematical description of flowfield

Most complete mathematical description of flow is viscous Navier-Stokes equation:



## Navier-Stokes vergelijkingen:

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\lambda\nabla\cdot V+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left(\lambda\nabla\cdot V+2\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]$$

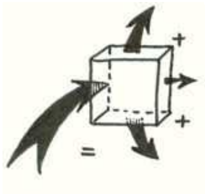
$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left(\lambda\nabla\cdot V+2\mu\frac{\partial w}{\partial z}\right)$$

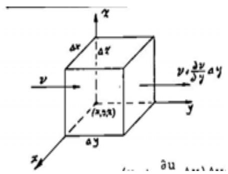
(not relaxed)



→

Apply principle of continuity on control volume:





Continuity: what comes in,
must go out

This results in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$



If in addition the flow is considered to be irrotational and non viscous →

Velocity potential function can be used to describe water motions
Main property of velocity potential function:

for potential flow, a function Φ(x,y,z,t) exists whose derivative in a certain arbitrary direction equals the flow velocity in that direction. This function is called the velocity potential



From definition of velocity potential:

$$u=\frac{\partial \Phi}{\partial x}, v=\frac{\partial \Phi}{\partial v}, w=\frac{\partial \Phi}{\partial z}$$

Substituting in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Results in Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2}+\frac{\partial^2 \Phi}{\partial y^2}+\frac{\partial^2 \Phi}{\partial z^2}=0$$



## Summary

- Potential theory is mathematical way to describe flow

Important facts about velocity potential function Φ:

- definition: Φ is a function whose derivative in any direction equals the flow velocity in that direction
- Φ describes non-viscous flow
- Φ is a scalar function of space and time (NOT a vector!)

## Summary

- Velocity potential for regular wave is obtained by
  - Solving Laplace equation satisfying:
    1. Seabed boundary condition
    2. Dynamic free surface condition

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh(k(h+z))}{\cosh(kh)} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$

3. Kinematic free surface boundary condition results in:
   Dispersion relation = relation between wave frequency and wave length

$$\omega^2 = kg\tanh(kh)$$

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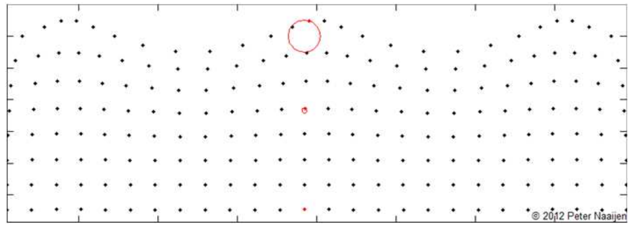
## Water Particle Kinematics

trajectories of water particles in infinite water depth

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$

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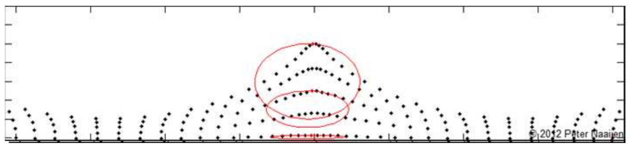
## Water Particle Kinematics

trajectories of water particles in finite water depth

$$\Phi(x,y,z,t) = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh(k(h+z))}{\cosh(kh)} \cdot \sin(kx\cos\mu + ky\sin\mu - \omega t)$$

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## Pressure

Pressure in the fluid can be found using Bernouilli equation for unsteady flow:

$$\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}(u^2 + w^2) + \frac{p}{\rho} + gz = 0$$

$$p = -\rho\frac{\partial\Phi}{\partial t} - \tfrac{1}{2}\rho(u^2 + w^2) - \rho g z$$

1st order fluctuating pressure

2nd order (small quantity squared=small enough to neglect)

Hydrostatic pressure (Archimedes)

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## Potential Theory

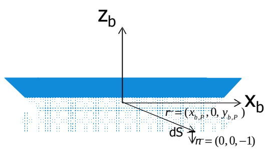
From all these velocity potentials we can derive:

- Pressure
- Forces and moments can be derived from pressures:

$$\bar{F} = -\iint_S (p \cdot \bar{n})\, dS$$

$$\bar{M} = -\iint_S p \cdot (\bar{r} \times \bar{n})\, dS$$

Verify these formulae (incl the signs!) yourself in order to understand them. Just check e.g. the force in heave direction ($F_3$) and the pitch moment ($M_5$) induced by a pressure on an infinite piece of hull surface $dS$ at location $\bar{r}$ :

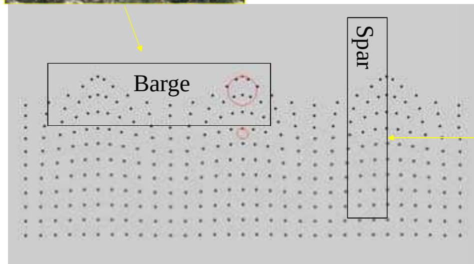





## Wave Force

Determination $F_w$

- Froude Krilov
- Diffraction





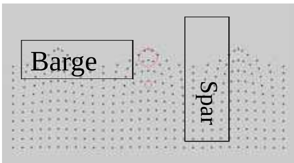


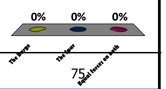
## Which structure experiences the highest vertical wave load acc. to potential theory ?

A. The Barge

B. The Spar

C. Equal forces on both





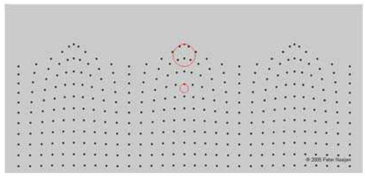

## Flow superposition

$$(m + a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = F_w$$

Considering a fixed structure (ignoring the motions) we will try to find a description of the disturbance of the flow by the presence of the structure in the form of a velocity potential. We will call this one the diffraction potential and added to the undisturbed wave potential (for which we have an analytical expression) it will describe the total flow due to the waves.

1. Flow due to Undisturbed wave

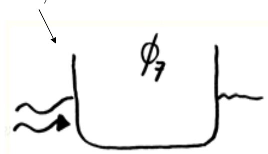
$$\Phi_0 = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(\omega t - kx \cos \mu - ky \sin \mu)$$



2. Flow due to Diffraction

Has to be solved. What is boundary condition at body surface ?

$\Phi_7$



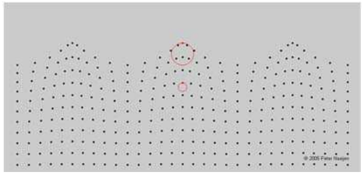
## Exciting force due to waves

$$(m+a)\ddot{z} + b\dot{z} + c\cdot z = F_w$$

$$= F_{FK} + F_D$$

1. Undisturbed wave force (Froude-Krilov)

$$\Phi_0 = -\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot \sin\left(\omega t - kx\cos\mu - ky\sin\mu + \varepsilon\right)$$



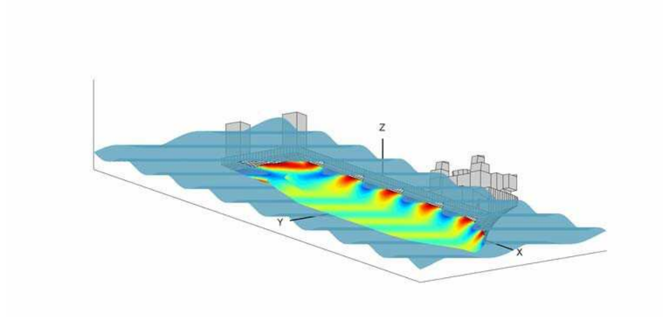
2. Diffraction force

Has to be solved. What is boundary condition at body surface ?

$\Phi_7$

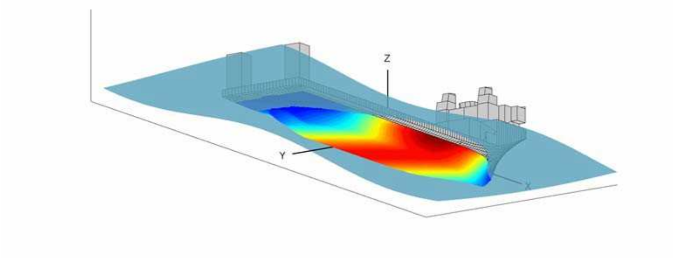



## Pressure due to undisturbed incoming wave
T=4 s



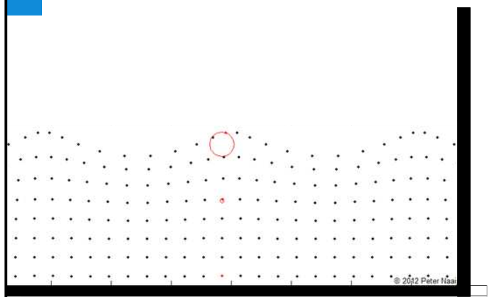



## Pressure due to undisturbed incoming wave
T=10 s



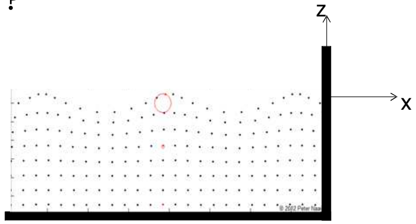



## Wave Forces
## Wave force acting on vertical wall



## What is the formulation of diffraction potential $\Phi_7$ ?

$$\Phi_0(x,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)$$




A. $\Phi_7(x,z,t) = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx + \omega t)$

B. $\Phi_7(x,z,t) = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)$

C. $\Phi_7(x,z,t) = 0$

0%

120

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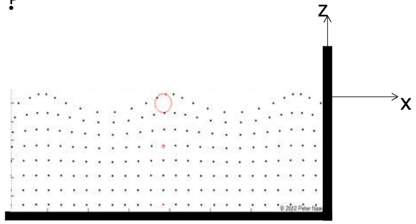
## What is the formulation of diffraction potential $\Phi_7$ ?

$$\Phi_0(x,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)$$




A. $\Phi_7(x,z,t) = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx + \omega t)$

B. $\Phi_7(x,z,t) = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)$

C. $\Phi_7(x,z,t) = 0$

0%

120

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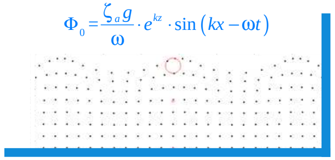
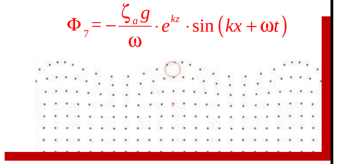
## Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z} + b\cdot\dot{z} + c\cdot z = F_W = F_{FK} + F_D$$

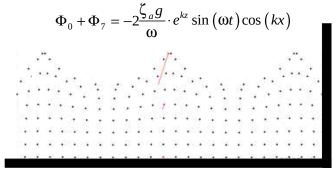
$$\Phi_0 = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)$$



$$\Phi_7 = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx + \omega t)$$



$$\Phi_0 + \Phi_7 = -2\frac{\zeta_a g}{\omega} \cdot e^{kz} \sin(\omega t)\cos(kx)$$



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## Force on the wall

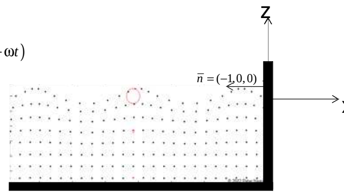
$$\overline{F} = -\int_{-\infty}^{0} p \cdot \overline{n} dz$$

$$\Phi_0 = \frac{\zeta_a g}{\omega} e^{kz} \sin(kx - \omega t), \Phi_7 = -\frac{\zeta_a g}{\omega} e^{kz} \sin(kx + \omega t)$$

$$p = -\rho \frac{\partial \Phi}{\partial t} = -\rho \frac{\partial(\Phi_0 + \Phi_7)}{\partial t} =$$

$$-\rho \frac{\partial\left(-2\frac{\zeta_a g}{\omega} \cdot e^{kz} \sin(\omega t)\cos(kx)\right)}{\partial t} =$$

$$2\rho\zeta_a g \cdot e^{kz} \cos(kx)\cos(\omega t)$$




$$\overline{n} = (-1, 0, 0)$$

$$x = 0$$

$$F_x = \int_{-\infty}^{0} 2\rho\zeta_a g \cdot e^{kz} \cos(\omega t) dz = \left[2\rho \frac{\zeta_a g}{k} \cdot e^{kz} \cos(\omega t)\right]_{-\infty}^{0} =$$

$$2\rho \frac{\zeta_a g}{k} \cdot \cos(\omega t) - 0$$

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## Left hand side of m.e.: Hydromechanic reaction forces

- NO incoming waves:
- Vessel moves with given frequency



## Recap: Motion equation

$$(m+a)\cdot\ddot{z} + b\cdot\dot{z} + c\cdot z = +F_{FK} + F_D = F_W$$

Hydromechanic force depends on motion

Wave Force independent of motion



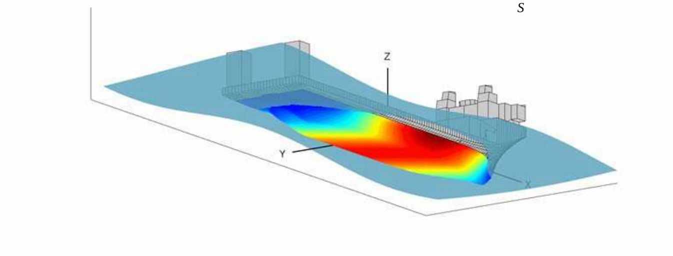
## Pressure / force due to undisturbed incoming wave

T=10 s

$$p = -\rho\frac{\partial\Phi}{\partial t}$$

$$\vec{F} = -\iint_S (p\cdot\vec{n})\,dS$$

$$\vec{M} = -\iint_S p\cdot(\vec{r}\times\vec{n})\,dS$$

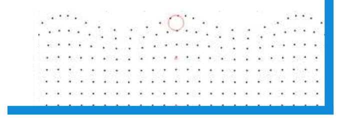




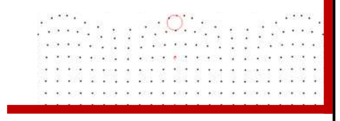

## Calculating diffraction force

$$(m+a)\cdot\ddot{z} + b\cdot\dot{z} + c\cdot z = F_W = F_{FK} + F_D$$

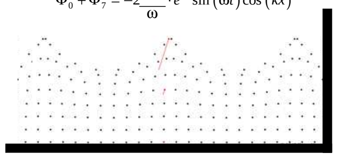
$$\Phi_0 = \frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx-\omega t)$$



$$\Phi_7 = -\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx+\omega t)$$



$$\Phi_0 + \Phi_7 = -2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)$$



## left hand side: reaction forces

$$(m+a)\cdot\ddot z + b\cdot\dot z + c\cdot z = +F_{FK} + F_D = F_W$$

Hydromechanic force depends on motion

Wave Force independent of motion



## Hydrodynamic coefficients

Determination of a and b:

- Forced oscillation with known frequency and amplitude
- Measure Force needed to oscillate the model


6 Degree of Freedom Forced Oscillation tests

July-August 2004

TUDelft



## Determine added mass and damping

Experimental procedure:

Oscillate model (i.e. impose known harmonic motion, z) → Measure required force, $F_{oscillation}$ → Subtract known reaction forces from measured $F_{oscillation}$ → Split remainder into damping and added mass coefficient

$$(m+a)\cdot\ddot z + b\cdot\dot z + c\cdot z = F_{oscillation}$$

$$z = z_a\cos(\omega t),\ \dot z = -\omega z_a\sin(\omega t),\ \ddot z = -\omega^2 z_a\cos(\omega t)$$

$$\left(-\omega^2(m+a)+c\right) z_a\cos\omega t - \omega b z_a\sin\omega t = F_{a,osc}\cdot\cos(\omega t+\varepsilon_{F,z})$$

$$-\omega^2 a z_a\cos\omega t - \omega b z_a\sin\omega t = F_{a,osc}\cdot\cos(\omega t+\varepsilon_{F,z}) + (\omega^2 m - c) z_a\cos\omega t$$



## Equation of motion

$$(m+a)\cdot\ddot z + b\cdot\dot z + c\cdot z = F_W$$

Hydrodynamic coefficients:

a=added mass coefficient= force on ship per 1 m/s² acceleration →
a * acceleration = hydrodynamic inertia force

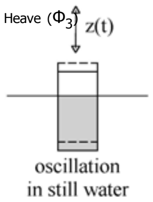
b=damping coefficient= force on ship per 1 m/s velocity →
b * velocity = hydrodynamic damping force

## Calculating hydrodynamic coeffiecients
## added mass and damping

For each of the 6 possible motions, the flow is described by a radiation potential function. The incoming waves are ignored for this. By finding a description of the flow, the pressures and consequently the forces can be determined later

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

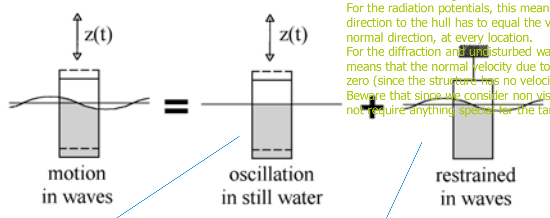
- Oscillation in desired direction **in still water**
- To prevent water from penetrating through the hull: we need the radiation velocity potentials : $\Phi_1 - \Phi_6$
- From potentials, we can calculate forces on body and the corresponding coefficients




## Solving the Laplace equation
Summary

The boundary conditions are the same as those used for the undisturbed wave (Ch 5) however, we have an additional boundary now which is the hull of the structure: it has to be water tight!
For the radiation potentials, this means: the flow in normal direction to the hull has to equal the velocity of the hull in normal direction, at every location.
For the diffraction an[redacted]isturbed wave potential it means that the normal velocity due to their sum must be zero (since the structure has no velocity itself).
Beware that since we consider non viscous flow, we do not require anything special for the tangential velocity!




Radiation potential $\Phi_{1.....6}$

Boundary Condition: $\frac{\partial \Phi_{1.....6}}{\partial n} = v_n$

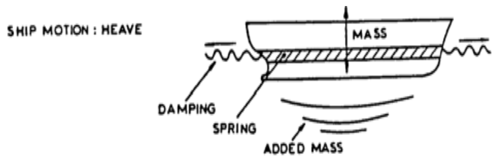
Undisturbed wave potential $\Phi_0$

Diffraction potential $\Phi_7$

Boundary Condition: $\frac{\partial \Phi_0}{\partial n} + \frac{\partial \Phi_7}{\partial n} = 0$



## Moving ship in waves:
## Not in air but in water!




$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$

$$\bar{F} = -\iint_S (p\cdot\bar{n})\,dS$$

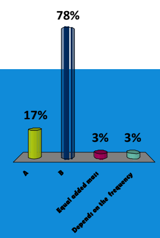
$$\bar{M} = -\iint_S p\cdot(\bar{r}\times\bar{n})\,dS$$

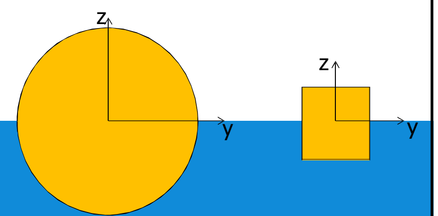
$$p = -\rho\frac{\partial \Phi}{\partial t}$$



## Which body will have the largest added mass for roll ?

A. A
B. B
C. Equal added mass
D. Depends on the frequency



## Equation of motion

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=+F_{FK}+F_D=F_W$$

**To solve equation of motion for certain frequency:**

- Determine spring coefficient:
  - c → follows from geometry of vessel
- Determine required hydrodynamic coefficients for desired frequency:
  - a, b → computer / experiment
- Determine amplitude and phase of $F_w$ of regular wave with amplitude =1:
  - Computer / experiment: $F_w = F_{wa}\cos(\omega t+\varepsilon_{Fw,\zeta})$
- As we consider the response to a regular wave with frequency ω:
  Assume steady state response: $z=z_a\cos(\omega t+\varepsilon_{z,\zeta})$
  and substitute in equation of motion:



## Equation of motion

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

$$z=z_a\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$$

$$\dot{z}=-z_a\omega\sin\left(\omega t+\varepsilon_{z,\zeta}\right)$$

$$\ddot{z}=-z_a\omega^2\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$$

$$\left(c-\omega^2(m+a)\right)\cdot z_a\cos\left(\omega t+\varepsilon_{z,\zeta}\right)+b\cdot -z_a\omega\sin\left(\omega t+\varepsilon_{z,\zeta}\right)=F_{Wa}\cos\left(\omega t+\varepsilon_{F_W,\zeta}\right)$$

Now solve the equation for the unknown motion amplitude $z_a$ and phase angle $\varepsilon_{z,\zeta}$



## Equation of motion

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

Now solve the equation for the unknown motion amplitude $z_a$ and phase angle $\varepsilon_{z,\zeta}$ for 1 frequency

System is linear

If wave amplitude doubles → wave force doubles → motion doubles

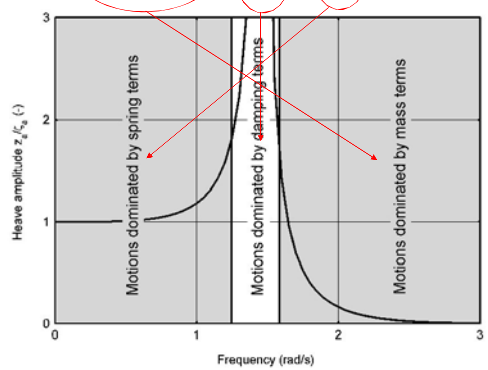
$$(m+a)\cdot\frac{\ddot{z}}{\zeta_a}+b\cdot\frac{\dot{z}}{\zeta_a}+c\cdot\frac{z}{\zeta_a}=\frac{F_W}{\zeta_a}$$

Substitue solution $\frac{z}{\zeta_a}=\frac{z_a}{\zeta_a}\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$ and solve RAO and phase

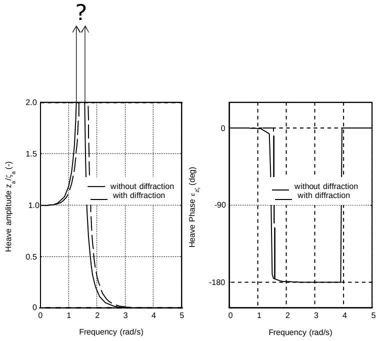


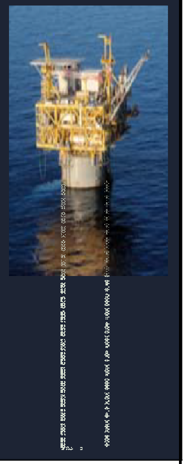
## RAO

$$(m+a)\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$



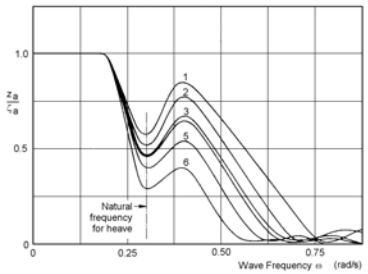
## Calculated RAO spar with potential theory

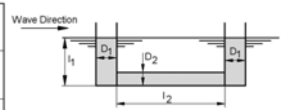




## Frequency Response of semi-submersible




| Platform | Dimensions (m) | | | |
|---|---|---|---|---|
| | $D_1$ | $l_1$ | $D_2$ | $l_2$ |
| 1 | 11.11 | 16.52 | 10.06 | 111.10 |
| 2 | 11.11 | 20.71 | 8.91 | 133.50 |
| 3 | 11.11 | 25.82 | 8.02 | 155.80 |
| 4 | 25.00 | 31.61 | 17.33 | 150.00 |
| 5 | 25.00 | 38.12 | 15.30 | 175.00 |
| 6 | 25.00 | 55.06 | 11.62 | 225.00 |

*[Source: Hooft, 1970]*



## What is 'linear' ???

*R*

1. Linear waves:
   - 'nice' regular harmonic (<u>cosine</u> shaped) waves
   - Wave steepness small: free surface boundary condition satisfied at <u>mean still water</u> level
     - Pressures and fluid velocities are proportional to wave elevation and have same frequency as elevation
2. linearised wave exciting force:
   - Wave force independent of motions
   - Wave force only on <u>mean</u> wetted surface

*L*

3. Motion amplitudes are small
   - Restoring force proportional to motion amplitude
   - Hydrodynamic reaction forces proportional to motion amplitude

**Motions are proportional to wave height !**

**Motions have same frequency as waves**



## Learning goals Module II, behavior of floating bodies in waves

| | |
|---|---|
| • Definition of ship motions<br>**Motion Response in regular waves:**<br>• How to use RAO's<br>• Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces<br>• How to solve RAO's from the equation of motion<br>**Motion Response in irregular waves:**<br>• How to determine response in irregular waves from RAO's and wave spectrum without forward speed | Ch.6 |
| **3D linear Potential Theory**<br>• How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential<br>• How to determine Velocity Potential | Ch. 7 |
| **Motion Response in irregular waves:**<br>• How to determine response in irregular waves from RAO's and wave spectrum with forward speed<br>• Determine probability of exceedence<br>• Make down time analysis using wave spectra, scatter diagram and RAO's | Ch. 8 |
| **Structural aspects:**<br>• Calculate internal forces and bending moments due to waves | Ch. 8 |
| **Nonlinear behavior:**<br>• Calculate mean horizontal wave force on wall<br>• Use of time domain motion equation | Ch.6 |

## Learning goals Module II, behavior of floating bodies in waves

| | |
|---|---|
| • Definition of ship motions<br>**Motion Response in regular waves:**<br>• How to use RAO's<br>• Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces<br>• How to solve RAO's from the equation of motion<br>**Motion Response in irregular waves:**<br>• ~~How to determine response in irregular waves from RAO's and wave spectrum without forward speed~~ | Ch.6 |
| **3D linear Potential Theory**<br>• How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential → Today<br>• How to determine Velocity Potential → Next week | Ch. 7 |
| **Motion Response in irregular waves:**<br>• How to determine response in irregular waves from RAO's and wave spectrum with forward speed<br>• Determine probability of exceedence<br>• Make down time analysis using wave spectra, scatter diagram and RAO's | Ch. 8 |
| **Structural aspects:**<br>• Calculate internal forces and bending moments due to waves | Ch. 8 |
| **Nonlinear behavior:**<br>• Calculate mean horizontal wave force on wall<br>• Use of time domain motion equation | Ch.6 |



2D Potential theory (strip theory) p. 7-12 until p. 7-35 SKIP THIS PART



## Calculating hydrodynamic coeffiecients and diffraction force

$$(m+a)\ddot{z}+b\dot{z}+c\cdot z=F_w=F_{FK}+F_D$$

m and c = piece of cake

$F_{FK}$ = almost easy

a, b, and $F_D$ = kind of difficult → Ch. 7



## Calculating hydrodynamic coeffiecients and diffraction force

p7-4 course notes

- Radiation: $-a_3\ddot{z}-b_3\dot{z}$
- Incoming wave
- Diffraction
- Hydrostatic buoyancy: $-c_3\cdot z$

$$m\ddot{z}=\sum F=F_{r3}+F_{w3}+F_{d3}+F_{s3}$$

$$(m+a)\ddot{z}+b\dot{z}+cz=F_{w3}+F_{d3}$$

Next slides we'll consider the left hand side of this motion equation: we will try to write the hydrodynamic reaction force F, that the structure feels as a result of its motions in such a way that we can incorporate them in the well known motion equation of a damped mass-spring system.

## Calculating hydrodynamic coeffiecients and diffraction force

$$m\cdot\ddot{z} = \sum F = F_{r3} + F_{w3} + F_{d3} + F_{s3}$$

Radiation Force: $F_{r3} = -a_3\cdot\ddot{z} - b_3\cdot\dot{z}$

To calculate force: first describe fluid motions due to given heave motion by means of radiation potential:



## Potential theory
### Radiation potential

$$(m+a)\cdot\ddot{z} + b\cdot\dot{z} + c\cdot z = F_w + F_d$$

Radiation potential heave $\Phi_3(x, y, z, t)$

= flow due to heave motion

Knowing the potential, calculating resulting force is straight forward:

$$\bar{F} = -\iint_S (p\cdot\bar{n})\,dS$$

$$\bar{M} = -\iint_S p\cdot(\bar{r}\times\bar{n})\,dS$$

$$p = -\rho\frac{\partial\Phi}{\partial t}$$

$$\bar{F} = \iint_S \left(\rho\frac{\partial\Phi}{\partial t}\cdot\bar{n}\right)dS$$

$$\bar{M} = \iint_S \rho\frac{\partial\Phi}{\partial t}\cdot(\bar{r}\times\bar{n})\,dS$$



## Potential theory
### Radiation potential

$$(m+a)\cdot\ddot{z} + b\cdot\dot{z} + c\cdot z = F_w + F_d$$

Radiation potential heave $\Phi_3(x, y, z, t)$

= flow due to motions, larger motions → 'more' flow

**Problem:** But we don't know the motions !! (we need the flow to calculate the motions…and we need the motions to calculate the flow…)

**Solution:** radiation potential is written as function of motion:



## Potential theory
### Radiation potential

**Solution:** radiation potential is written as function of velocity of the motion

$$\Phi_3(\underline{x}, t) = \Re\{\phi_3(\underline{x})\cdot v_3(t)\}$$

P7-5 eq. 7.17

**Only <u>space</u> dependent** **Only <u>time</u> dependent**

Suppose we would know the velocity potential due to heave motion: $\Phi_3$
Assuming linearity this will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Let's define the amplitude and the phase angle of this potential function to be related to the velocity of the heave motion ($\dot{z}$ or in complex notation: $v_3$).
So we write the potential function $\Phi_3$ as a complex product of:
$\phi_3$ (which can be considered as a complex transfer function between potential and heave velocity) and the heave velocity $v_3$

## Potential theory

Radiation potential

$$\Phi_3(\underline{x},t) = \Re\{\phi_3(\underline{x})\cdot v_3(t)\}$$

**Only space dependent** — **Only time dependent**

Complex notation:

$$s_3(t) = s_{a3}\cdot e^{-i\omega t}$$

$$v_3(t) = \dot{s}(t) = -i\omega s_{a3}\cdot e^{-i\omega t}$$

$$z(t) = z_a\cos(\omega t+\varepsilon_{z,\zeta}) = \Re\{z_a e^{-i\varepsilon_{z,\zeta}} e^{-i\omega t}\} = \Re\{s_{a3}e^{-i\omega t}\}$$

$$v_3(t) = -i\omega z_a e^{-i\varepsilon_{z,\zeta}} e^{-i\omega t} = -i\omega s_{a3}e^{-i\omega t}$$

$s_{a3}$: Complex heave motion amplitude

$v_{a3}$: Complex heave velocity amplitude



## Potential theory

Let's consider Heave motion:

$$\Phi_3(x,t) = \Re\{\phi_3\cdot v_{a3}\cdot e^{-i\omega t}\} = \Re\{\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}\}$$

(the circled term $-i\omega\cdot s_{a3}\cdot e^{-i\omega t}$ points to $v_3(t)$)

$v_{a3}$ = complex amplitude of heave velocity

$s_{a3}$ = complex amplitude of heave displacement

Potential not necessarely in phase with heave velocity $v_3$ →

$\phi_3$ = <u>complex</u> amplitude of heave radiation potential (devided by $-i\omega s_{a3}$)

Remember:
$\Phi_3$ will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Suppose that at a certain location, this function has a phase angle ε related to the heave velocity and the ratio between its amplitude and the amplitude of the heave velocity is a.

**Verify** that in that case:
$\phi_3 = ae^{-i\varepsilon}$



## Potential theory

Let's consider forces and moments <u>due to heave motion</u>

$$\bar{F}_{r3} = \iint_S \left(\rho\frac{\partial\Phi_3}{\partial t}\cdot\bar{n}\right)dS$$

$$\bar{M}_{r3} = \iint_S \rho\frac{\partial\Phi_3}{\partial t}\cdot(\bar{r}\times\bar{n})\,dS$$

$$\Phi_3(\underline{x},t) = \phi_3\cdot v_{a3}\cdot e^{-i\omega t} = \phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}$$

$$\bar{F}_{r3} = \iint_S\left(\rho\frac{\partial(\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t})}{\partial t}\cdot\bar{n}\right)dS$$

$$\bar{M}_{r3} = \iint_S \rho\frac{\partial(\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t})}{\partial t}\cdot(\bar{r}\times\bar{n})\,dS$$



## Potential theory

some re writing, considering only heave force due to heave motion:

$$F_{r33} = \Re\left\{\iint_S \rho\frac{\partial(\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t})}{\partial t}\cdot n_3\,dS\right\}$$

Only <u>space</u> dependent — Only <u>time</u> dependent

$$= \Re\left\{-\rho\cdot i\omega\cdot s_{a3}\iint_S \phi_3\cdot\frac{\partial e^{-i\omega t}}{\partial t}\cdot n_3\cdot dS\right\}$$

This 3 component force vector $\bar{F}$ is what we call the hydro**dynamic** reaction force that the structure experiences due to its heave motion.

## Potential theory

Radiation Force due to heave motion is 3 component vector:

$$F_{r13} = \Re\left\{-\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_1 \cdot dS \cdot e^{-i\omega t}\right\}$$ Surge force due to heave motion

$$F_{r23} = \Re\left\{-\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_2 \cdot dS \cdot e^{-i\omega t}\right\}$$ Sway force due to heave motion

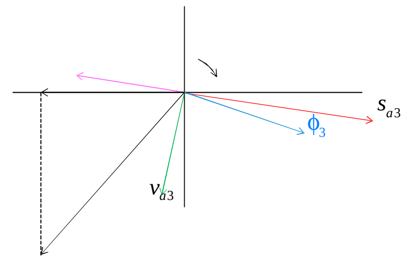
$$F_{r33} = \Re\left\{-\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t}\right\}$$ Heave force due to heave motion

In the following, only heave force due to heave motion is considered: $F_{r33}$



## Potential theory

Radiation potential




= Radiation force in heave direction,

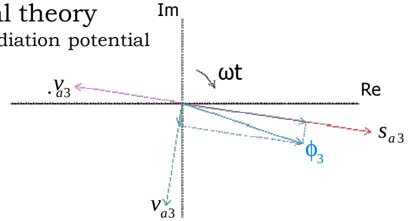
$$m \cdot \ddot{z} + c \cdot z + F_{r33} = F_{w3} + F_{d3} =$$

$$m \cdot \dot{v}_{a3} \cdot e^{-i\omega t} + c \cdot s_{a3} \cdot e^{-i\omega t} + F_{r33}$$



## Potential theory

Radiation potential




$$F_{r33} = \Re\left\{-\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t}\right\}$$ = Radiation force in heave direction, due to heave motion

$$m \cdot \ddot{z} + c \cdot z + F_{r33} = F_{w3} + F_{d3}$$

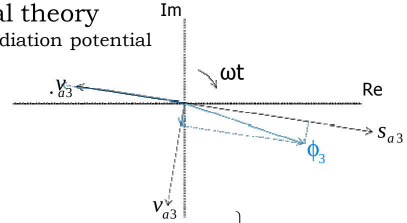
$$\Re\left\{m \cdot \dot{v}_{a3} \cdot e^{-i\omega t} + c \cdot s_{a3} \cdot e^{-i\omega t} + F_{r33}\right\} = F_{w3} + F_{d3}$$

$$-a \cdot \ddot{z} = \Re\left\{-a \cdot \dot{v}_{a3} \cdot e^{-i\omega t}\right\} \qquad -b \cdot \dot{z} = \Re\left\{-b \cdot v_{a3} \cdot e^{-i\omega t}\right\}$$



## Potential theory

Radiation potential




$$F_{r33} = \Re\left\{-\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t}\right\} = \Re\left\{-a \cdot \dot{v}_{a3} \cdot e^{-i\omega t} - b \cdot v_{a3} \cdot e^{-i\omega t}\right\}$$

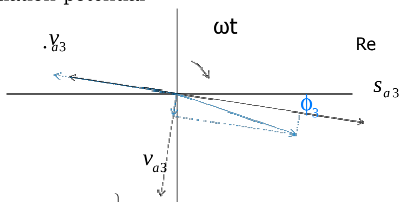
$$= \Re\left\{a \cdot \omega^2 \cdot s_{a3} \cdot e^{-i\omega t} + b \cdot i\omega \cdot s_{a3} \cdot e^{-i\omega t}\right\}$$

$$= \Re\left\{a\omega^2 \cdot s_{a3} \cdot e^{-i\omega t} + ib\omega \cdot s_{a3} \cdot e^{-i\omega t}\right\}$$

$$\Re\left\{-\rho \cdot \omega^2 \cdot s_{a3} \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t}\right\} = \Re\left\{a\omega^2 \cdot s_{a3} \cdot e^{-i\omega t} + ib\omega \cdot s_{a3} \cdot e^{-i\omega t}\right\}$$

## Potential theory

Radiation potential




$$\Re\left\{ -\rho \cdot \omega^2 \cdot s_{a3} \cdot \iint_S \phi_3 \cdot n_3 \cdot dS \cdot e^{-i\omega t} \right\} = \Re\left\{ a\omega^2 \cdot s_{a3} \cdot e^{-i\omega t} + ib\omega \cdot s_{a3} \cdot e^{-i\omega t} \right\}$$

After dividing by $s_{a3} \cdot e^{-i\omega t}$
Both Im and Re part have to be equalled!

$$-\rho\omega^2 \iint_S \phi_3 \cdot n_3 \cdot dS = a\omega^2 + ib\omega$$

$$a = -\rho\Re\left\{ \iint_S \phi_3 \cdot n_3 \cdot dS \right\}$$

$$b = -\rho\omega\Im\left\{ \iint_S \phi_3 \cdot n_3 \cdot dS \right\}$$



## Potential theory

resulting from heave motions, $\Phi_3$

### Forces

$$a_{33} = -\Re\left\{ \rho \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0 \right\}$$

$$b_{33} = -\Im\left\{ \rho\omega \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0 \right\}$$

$$a_{13} = -\Re\left\{ \rho \iint_{S_0} \phi_3 \cdot n_1 \cdot dS_0 \right\}$$

$$b_{13} = -\Im\left\{ \rho\omega \iint_{S_0} \phi_3 \cdot n_1 \cdot dS_0 \right\}$$

$$a_{23} = -\Re\left\{ \rho \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0 \right\}$$

$$b_{23} = -\Im\left\{ \rho\omega \iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0 \right\}$$

### Moments

$$a_{43} = -\Re\left\{ \rho \iint_{S_0} \phi_3 \cdot (\vec{r} \times \vec{n})_1 \, dS_0 \right\}$$

$$b_{53} = -\Im\left\{ \rho\omega \iint_{S_0} \phi_3 \cdot (\vec{r} \times \vec{n})_2 \cdot dS_0 \right\}$$

$$a_{53} = -\Re\left\{ \rho \iint_{S_0} \phi_3 \cdot (\vec{r} \times \vec{n})_2 \cdot dS_0 \right\}$$

$$b_{53} = -\Im\left\{ \rho\omega \iint_{S_0} \phi_3 \cdot (\vec{r} \times \vec{n})_2 \cdot dS_0 \right\}$$

$$a_{63} = -\Re\left\{ \rho \iint_{S_0} \phi_3 \cdot (\vec{r} \times \vec{n})_3 \cdot dS_0 \right\}$$

$$b_{63} = -\Im\left\{ \rho\omega \iint_{S_0} \phi_3 \cdot (\vec{r} \times \vec{n})_3 \cdot dS_0 \right\}$$



## Solving the Laplace equation

coupled equation of motion:

$$\begin{bmatrix} M+a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & M+a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & M+a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & I_{xx}+a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & I_{yy}+a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & I_{zz}+a_{66} \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ \varphi \\ \theta \\ \psi \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{pmatrix}$$



## Potential theory

Now let's consider the right hand side of the motion equation: the excitation forces that the structure feels due to the waves.


Radiation potential $\Phi_{1.....6}$

Boundary Condition: $\frac{\partial \Phi_{1.....6}}{\partial n} = v_n$

Undisturbed wave potential $\Phi_0$

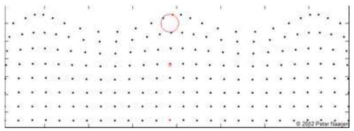
Diffraction potential $\Phi_7$

Boundary Condition: $\frac{\partial \Phi_0}{\partial n} + \frac{\partial \Phi_7}{\partial n} = 0$

## Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

Potential is known from Ch. 5:



Has to be solved. What is boundary condition at body surface ?

$\Phi_7$




## Calculating hydrodynamic coeffiecient and diffraction force

$$F_{FK}+F_D$$

Linear relation between undisturbed wave and diffraction potential →

$$\Phi_7=\phi_7\cdot\dot{\zeta}=\phi_7\cdot -i\omega\cdot\zeta_a\cdot e^{-i\omega t}=\phi_7\cdot -i\omega\cdot\zeta_0\cdot e^{-i\omega t}$$

Notation p 7-39, 7-40:

$\zeta_0=\zeta_7$ = amplitude undisturbed wave (at origin, so real)

$\zeta_{1\ldots6}$ = amplitude motions (complex)



## Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

Undisturbed wave force (Froude-Krilov)

$$\zeta=\zeta_a\cos\left(kx\cos(\mu)+ky\sin(\mu)-\omega t\right)$$

$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin\left(kx\cos(\mu)+ky\sin(\mu)-\omega t\right)$$

$$=\Re\left\{-i\frac{\zeta_0 g}{\omega}\cdot e^{kz}\cdot e^{ik(x\cdot\cos(\mu)+y\cdot\sin(\mu))}\cdot e^{-i\omega t}\right\}$$

Analogue to the radiation potential we write the known undisturbed wave potential function as a transfer function $\Phi_0$ multiplied with the velocity of the undisturbed wave elevation at the origin of the axes system.

$$=\Re\left\{-i\omega\cdot\frac{g}{\omega^2}\cdot e^{kz}\cdot e^{ik(x\cdot\cos(\mu)+y\cdot\sin(\mu))}\cdot\zeta_0\cdot e^{-i\omega t}\right\}$$

$$=\Re\left\{\phi_0\cdot -i\omega\cdot\zeta_0\cdot e\right\}$$

axes system (COG) in complex notation

Error p.7-39 eq. 7.151!!

$$\phi_0(x,y,z)=\frac{g}{\omega^2}e^{kz}\cdot e^{i(kx\cos(\mu)+ky\sin(\mu))}$$



## Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F=F+F$$

we write the known undisturbed wave potential function as a transfer function Φ multiplied with the velocity of the undisturbed wave elevation at the origin of the axes system.
We also do this for the unknown function we call $\Phi_7$

$$\Phi_0+\Phi_7=-i\omega\cdot(\phi_0+\phi_7)\cdot\zeta_0\cdot e^{-i\omega t}$$

Pressure:

$$p_w=-\rho\frac{\partial(\Phi_0+\Phi_7)}{\partial t}=\rho\omega^2\cdot(\phi_0+\phi_7)\cdot\zeta_0\cdot e^{-i\omega t}$$

= pressure due to incoming and diffracted wave

## Potential Theory

Forces and moments can be derived from pressures:

$$\overline{F} = -\iint_S \left( p \cdot \overline{n} \right) dS$$

$$\overline{M} = -\iint_S p \cdot \left( \overline{r} \times \overline{n} \right) dS$$

Knowing the potentials, pressures due to incoming and diffracted wave can be determined. Integrating these acc to the equations here finally gives the wave exciting forces.



## Learning goals Module II, behavior of floating bodies in waves

| | |
|---|---|
| • Definition of ship motions<br>**Motion Response in regular waves:**<br>• How to use RAO's<br>• Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces<br>• How to solve RAO's from the equation of motion<br>**Motion Response in irregular waves:**<br>• ~~How to determine response in irregular waves from RAO's and wave spectrum without forward speed~~ | |
| **3D linear Potential Theory**<br>• How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential<br>• How to determine Velocity Potential → Next week! | Ch. 7 |
| **Motion Response in irregular waves:**<br>• How to determine response in irregular waves from RAO's and wave spectrum with forward speed<br>• Determine probability of exceedence<br>• Make down time analysis using wave spectra, scatter diagram and RAO's | Ch. 8 |
| **Structural aspects:**<br>• Calculate internal forces and bending moments due to waves | Ch. 8 |
| **Nonlinear behavior:**<br>• Calculate mean horizontal wave force on wall<br>• Use of time domain motion equation | Ch.6 |

# Sources images

[1] Towage of SSDR Transocean Amirante, source: Transocean
[2] Tower Mooring, source: unknown
[3] Rogue waves, source: unknown
[4] Bluewater Rig No. 1, source:  Friede & Goldman, LTD/GNU General Public License
[5] Source: unknown
[6] Rig Neptune, source: Seafarer Media
[7] Pieter Schelte vessel, source: Excalibur
[8] FPSO design basis, source: Statoil
[9] Floating wind turbines, source: Principle Power Inc.
[10] Ocean Thermal Energy Conversion (OTEC), source: Institute of Ocean Energy/Saga University
[11] ABB generator, source: ABB
[12] A Pelamis installed at the Agucadoura Wave Park off Portugal, source:  S.Portland/Wikipedia
[13] Schematic of Curlew Field, United Kingdom, source: offshore-technology.com
[14] Ocean Quest Brave Sea, source: Zamakona Yards
[15] Medusa, A Floating SPAR Production Platform, source: Murphy USA