Electronic Power Conversion

Line-Frequency Diode Rectifiers

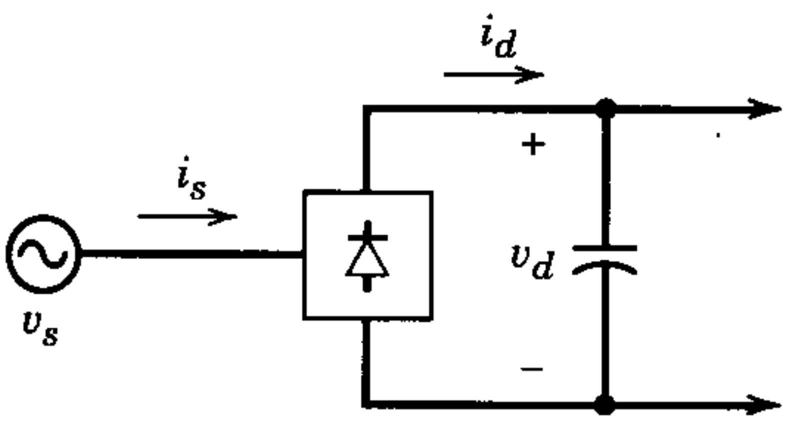
5. Line-frequency diode rectifiers

- Conversion: line frequency ac → uncontrolled dc (control often in next converter stage)
- Applications: power supplies, ac/dc/ac drives, dc servo drives, ...

☺Low cost, popular

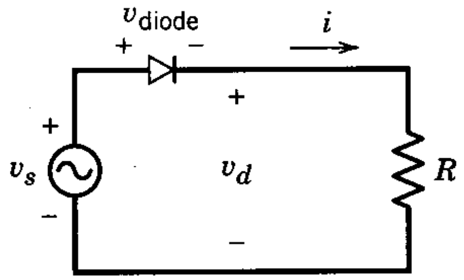
☹Large capacitor needed for ripple-free dc voltage → distorted line current

☹Filters required to comply with standards on allowable line-current harmonic

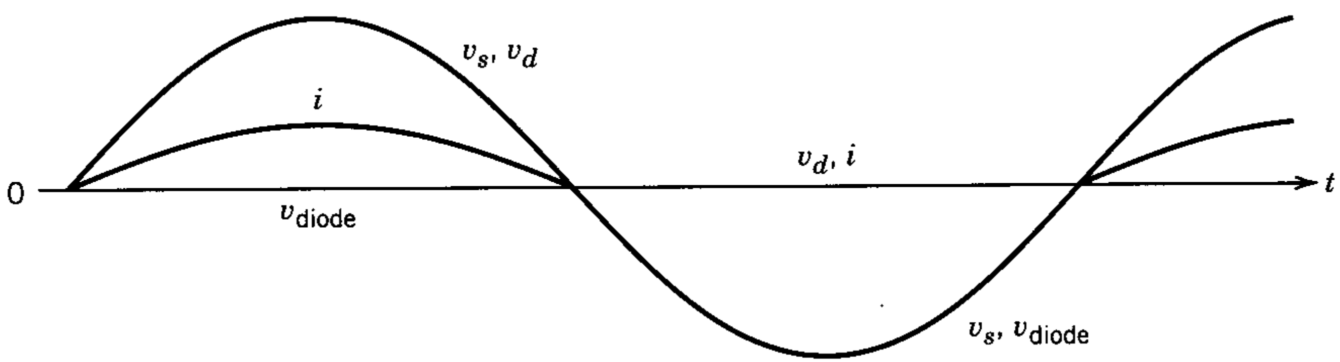


Single-phase diode rectifiers

- Basic rectifier, single diode, purely resistive load
- Large ripple in v_d



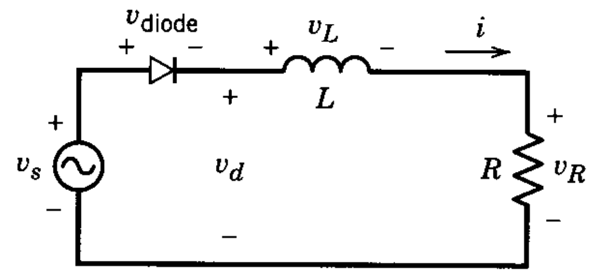
(a)



(b)

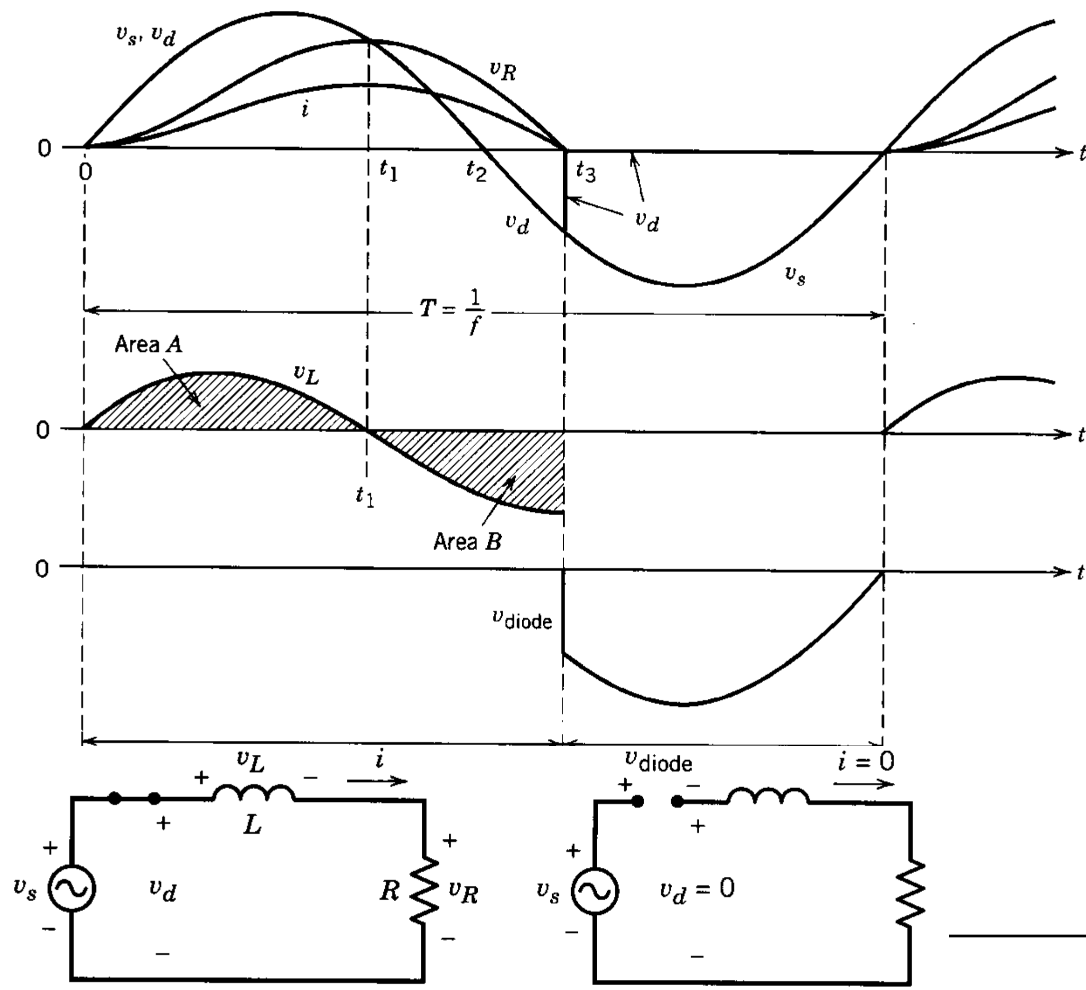
Single-phase diode rectifiers

- Basic rectifier, single diode, inductive load



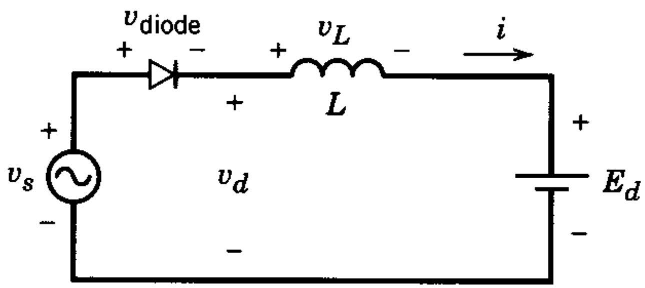
$$v_L = L \cdot \frac{di}{dt} = v_s - Ri$$

$$V_{L,av} = \int_0^{t_3} v_L \, dt = 0$$

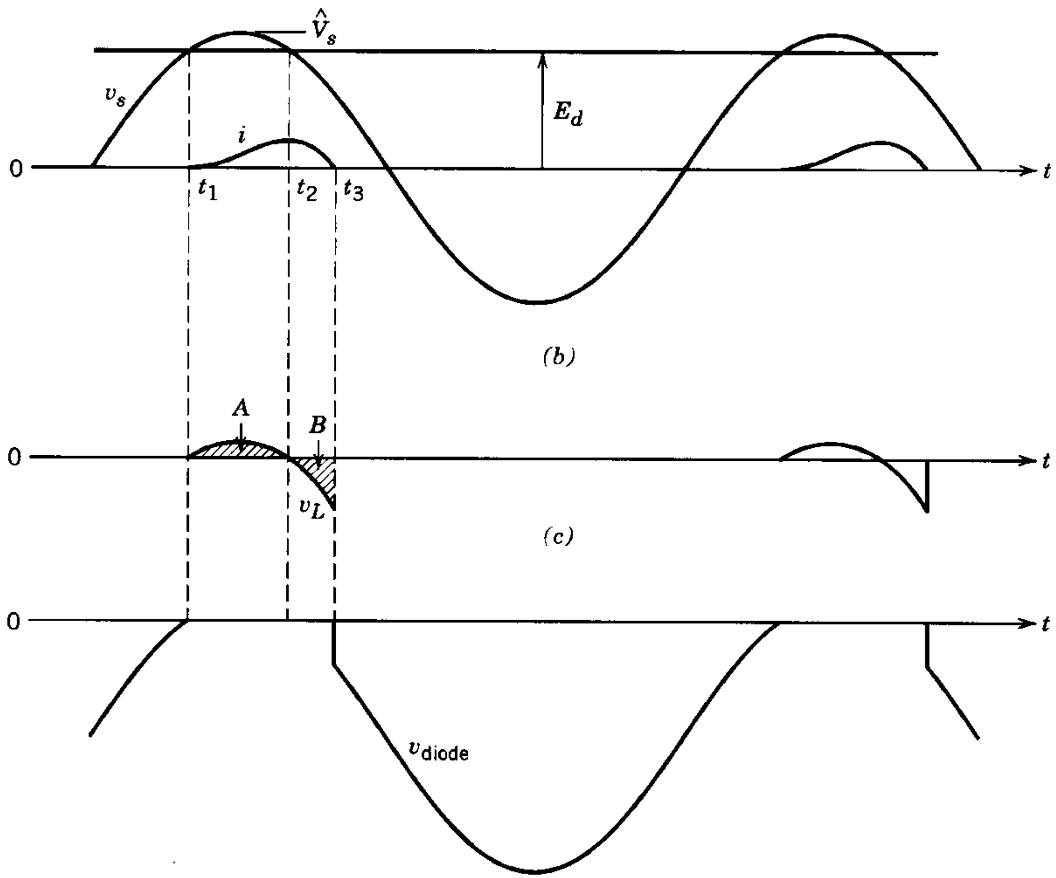


Single-phase diode rectifiers

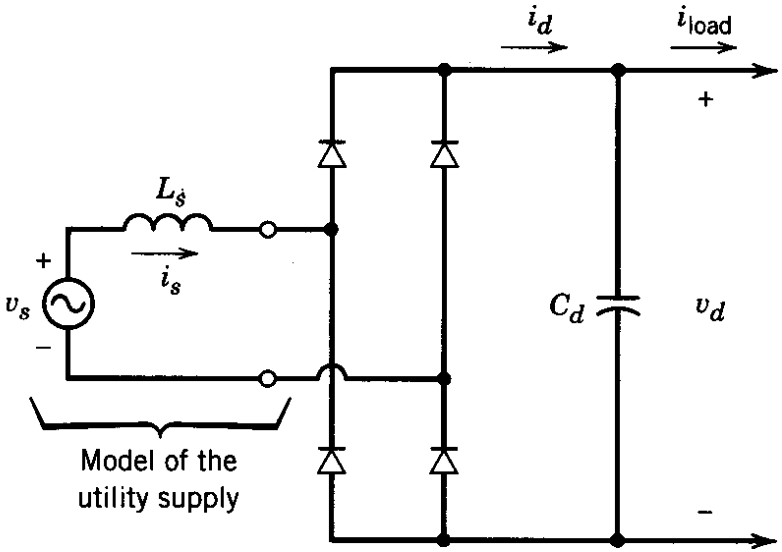
- Single diode, load with internal dc voltage, load with large C

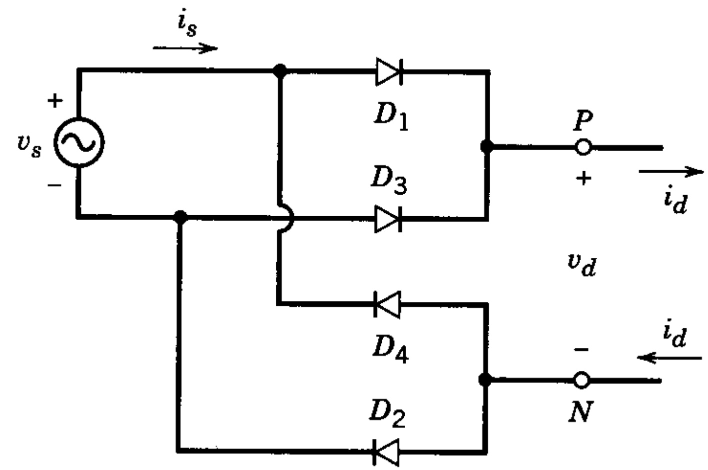


$$V_{L,av} = \int_{t_1}^{t_3} v_L dt = 0$$



Single-phase bridge rectifier



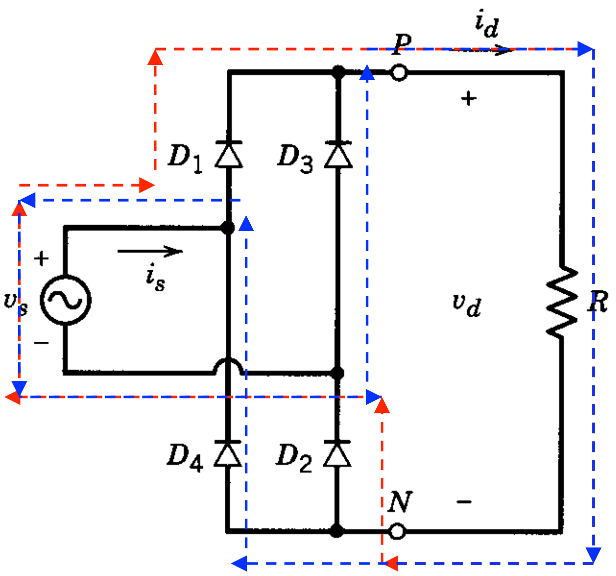


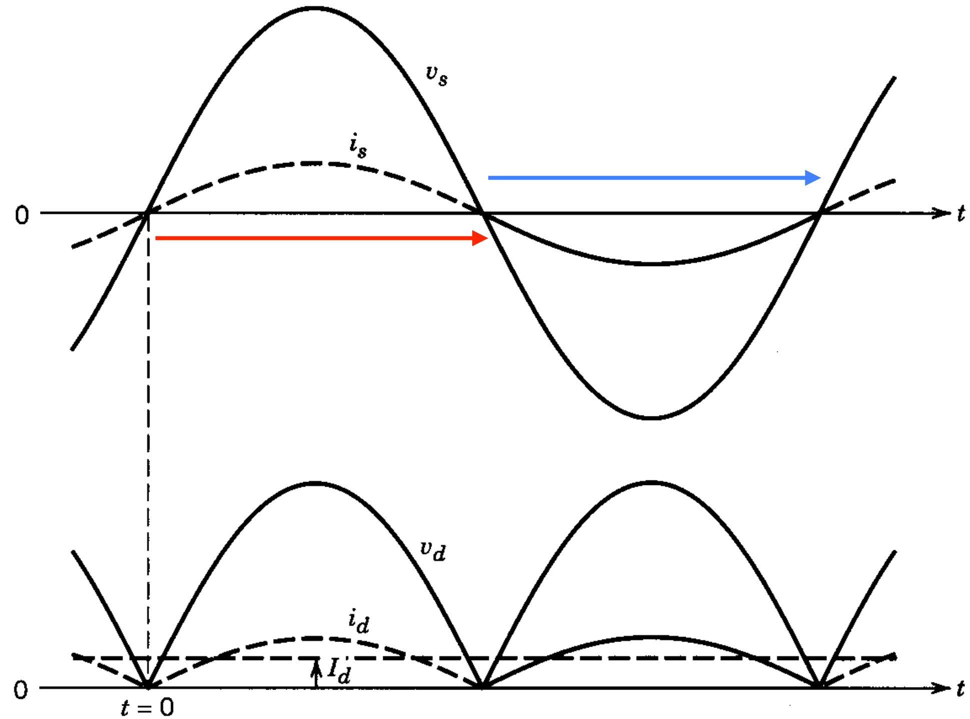
+ rail:
Diode with highest anode potential is conducting)

- rail:
Diode with lowest cathode potential is conducting

Single-phase bridge rectifier

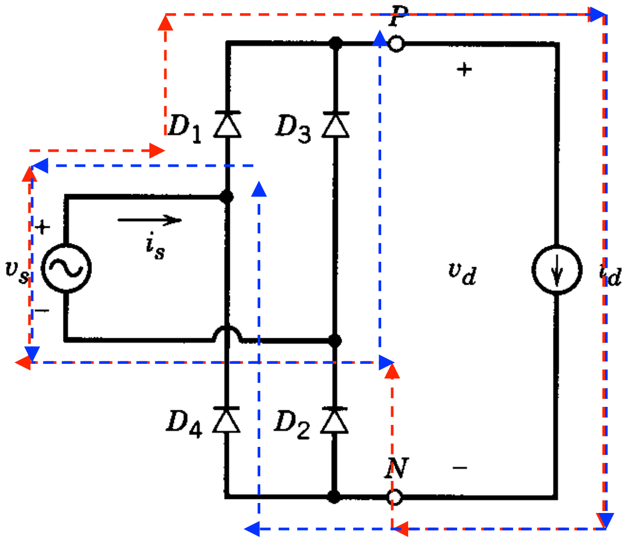
- Resistive load





Single-phase bridge rectifier

- Resistive load



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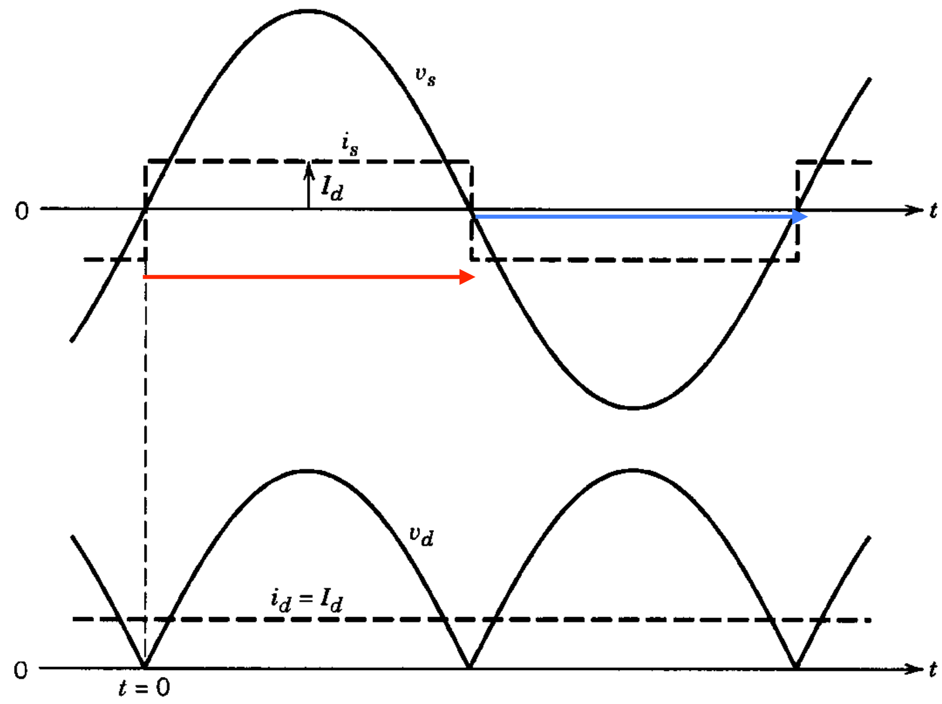
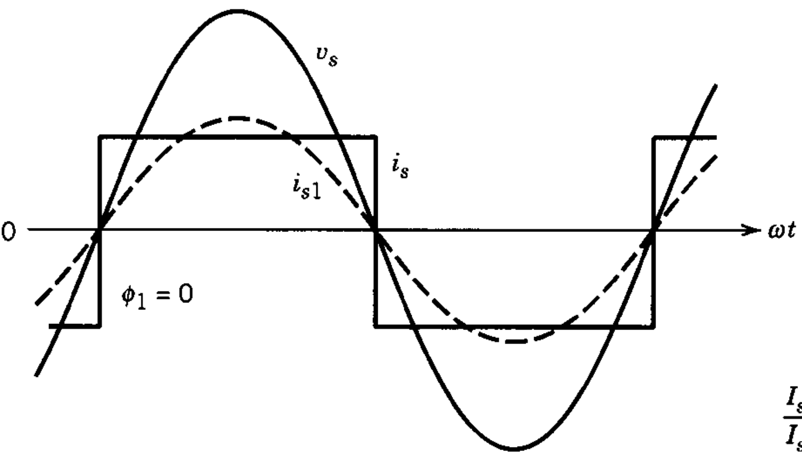


- Output voltage

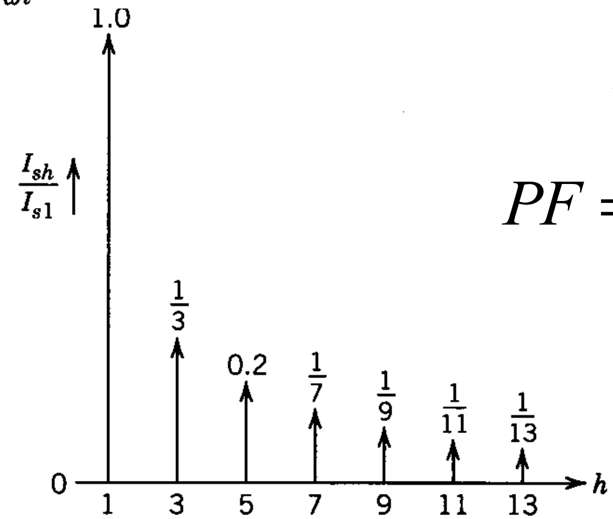
$$V_{d0} = \frac{1}{T/2}\int_0^{T/2} \sqrt{2}V_s \sin\omega t\,dt = \frac{2}{\pi}\sqrt{2}V_s = 0.9V_s$$

- For inductive load



Fourier analysis of i_s:

$$I_{s1} = \frac{2}{\pi}\sqrt{2}I_d = 0.9I_d \qquad I_{sh} = I_{s1}/h$$

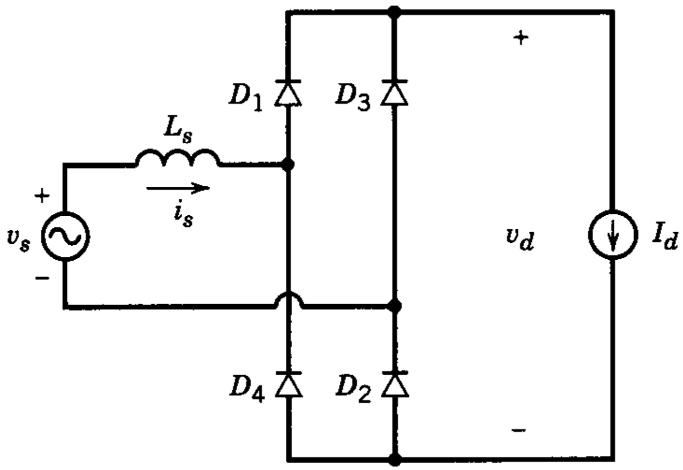


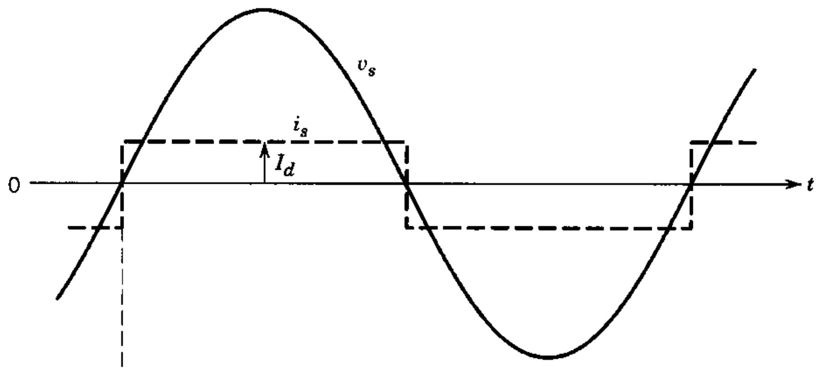
$$DF = I_{s1}/I_d = 0.9$$

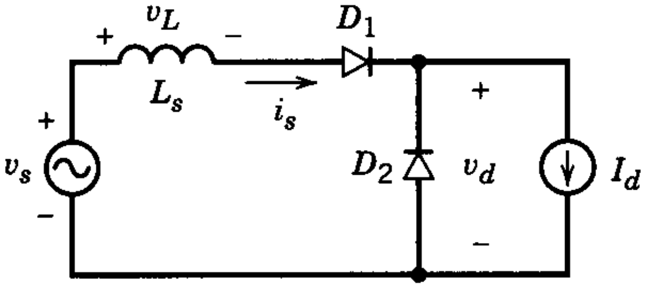
$$PF = (I_{s1}/I_d)\cdot\cos\varphi = 0.9$$

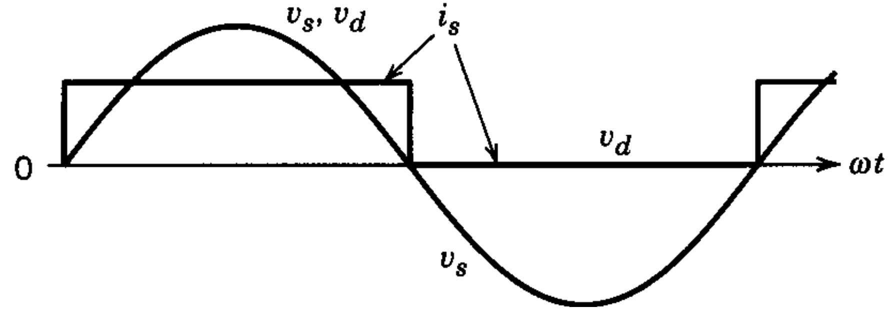
Single-phase bridge rectifier

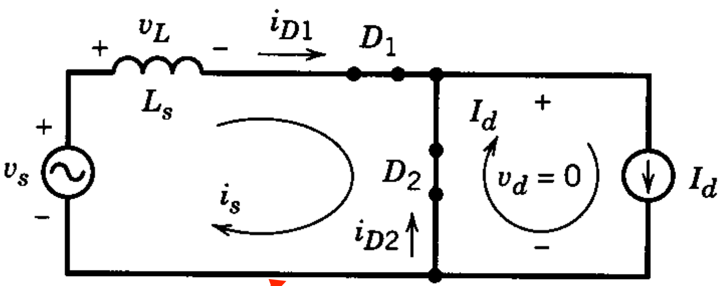
- Large inductive load - Effect of Ls on current commutation
- i_s (inductor current) can not change from I_d to $-I_d$ instantaneously

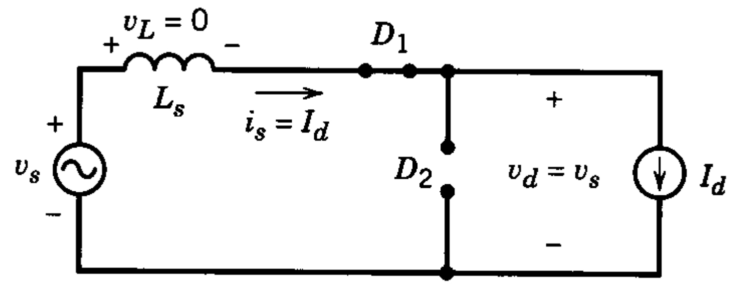


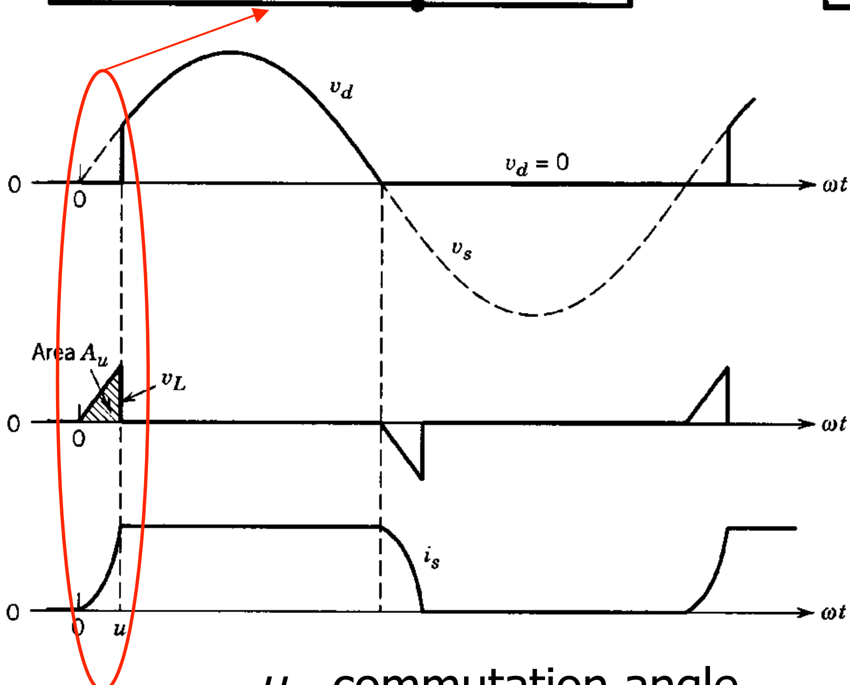












$$v_L = \sqrt{2}\ V_s\ \sin\omega t = L_s \frac{di_s}{dt} \qquad 0 < \omega t < u$$

$$i_s(t) = i_s(0) + \frac{1}{L_s}\int_0^t \sqrt{2}\ V_s\ \sin\omega t\ d(t)$$

$$i_s(0) = 0 \qquad i_s(u/\omega) = I_d$$

$$\omega L_s\ I_d = \sqrt{2}\ V_s\ (\ 1 - \cos u)$$

$$\cos u = 1 - \frac{\omega L_s I_d}{\sqrt{2}\ V_s} \qquad A_u = \int_0^u \sqrt{2}\, V_s\ \sin\omega t\ d(\omega t)$$

u - commutation angle

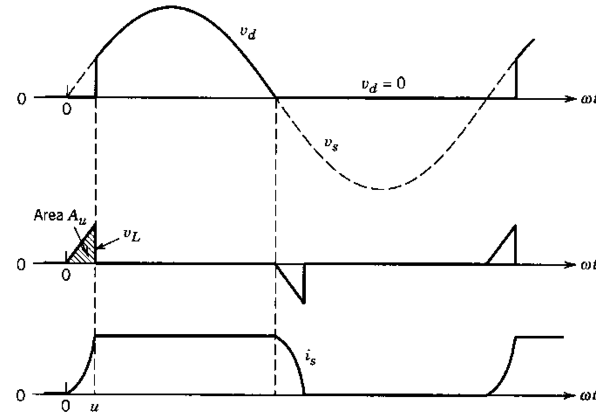
Voltage drop due to current commutation

For $L_s = 0$: $V_d = \frac{1}{2\pi}\int_0^{\pi}\sqrt{2}\,V_s\,\sin\omega t\,d(\omega t) = \frac{2\sqrt{2}\,V_s}{2\pi} = 0.45\,V_s$

For $L_s \neq 0$: $V_d = \frac{1}{2\pi}\int_u^{\pi}\sqrt{2}\,V_s\,\sin\omega t\,d(\omega t)$

$$V_d = 0.45V_s - \frac{1}{2\pi}\int_0^{u}\sqrt{2}\,V_s\sin\omega t\,d(\omega t)$$

$$= 0.45V_s - \frac{area\ A_u}{2\pi} = 0.45V_s - \frac{\omega L_s}{2\pi}\,I_d$$

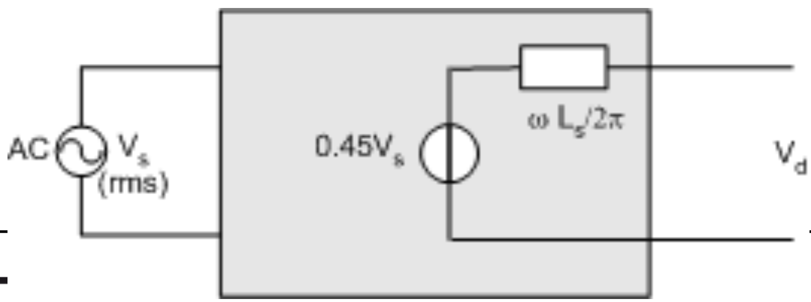


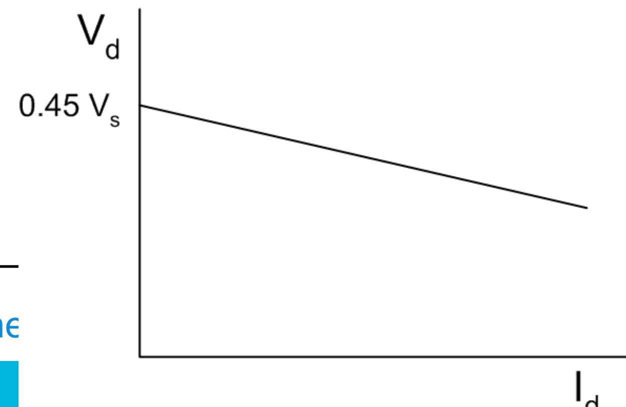
Every cycle the area A_u is lost once

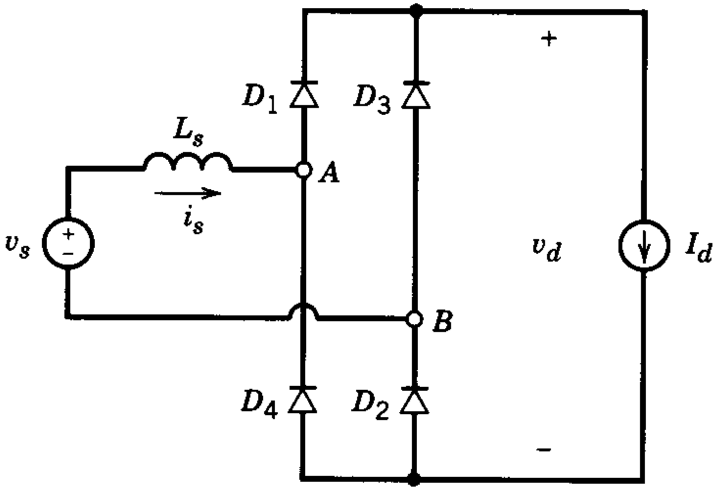
$$\Delta V_d = \frac{area\ A_u}{2\pi} = \frac{\omega L_s}{2\pi}\,I_d$$

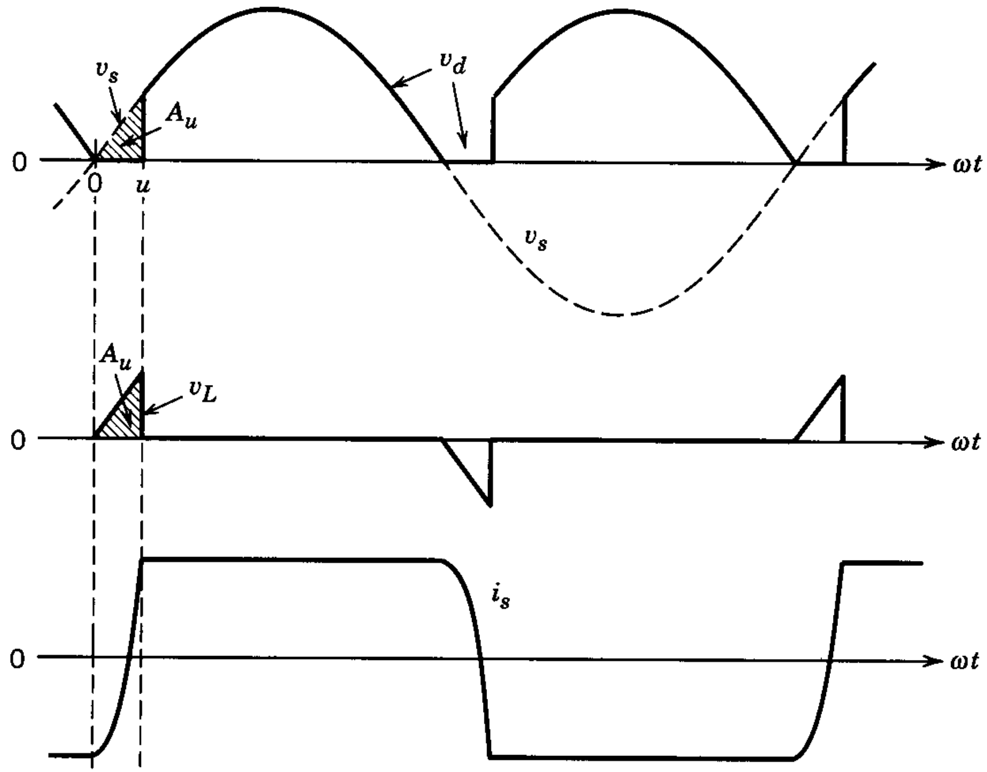
= apparent resistor (no losses)

Static model of dc-side of rectifier





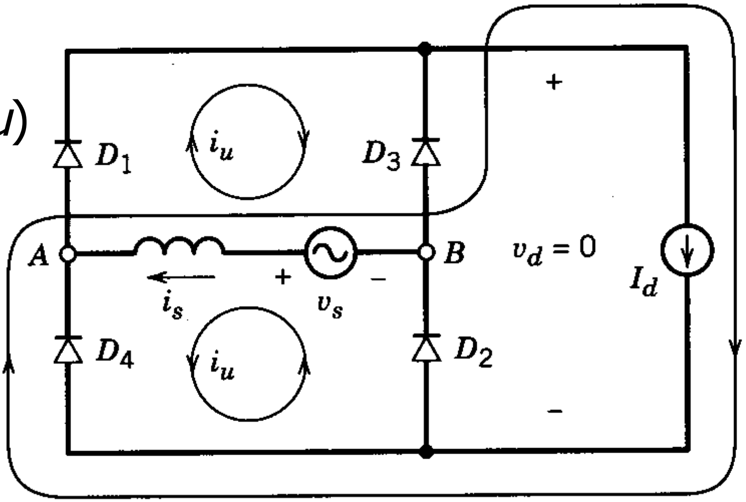




commutation: 3+4 ==> 1+2

1. describe problem as superposition of I_d and commutation current i_u
2. identify commutation meshes and driving voltages
3. note that I_d is constant

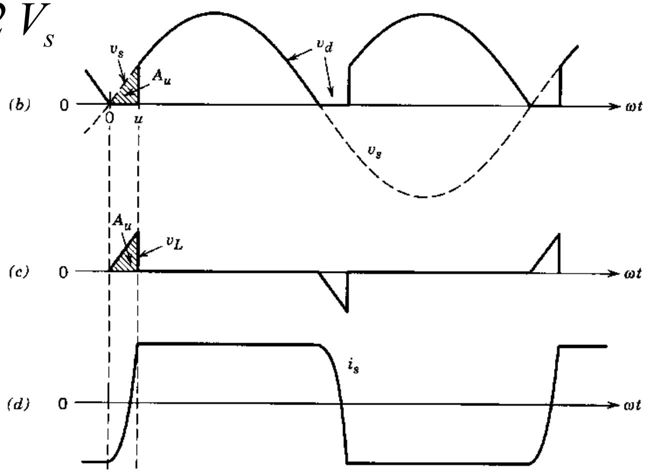
Current $i_s(t)$ goes from $-I_d$ to I_d in interval $(0,u)$



$$i_s(\omega t) = i_s(0) + \frac{1}{\omega L_s}\int_0^{\omega t} \sqrt{2}\, V_s \sin \omega t \; d(\omega t)$$

$$\omega L_s \Delta i_s = \int_0^{u} \sqrt{2}\, V_s \sin \omega t \; d(\omega t) \quad with \quad \Delta i_s = 2\, I_d$$

or $\quad 2\omega L_s I_d = \sqrt{2}\, V_s\,(1 - \cos u) \quad \rightarrow \quad \cos u = 1 - \dfrac{2\omega L_s I_d}{\sqrt{2}\, V_s}$



Shaded area forces the current from $-I_d$ to I_d :

$$A_u = \int_0^{u} \sqrt{2}\; V_s \sin \omega t \;\; d(\omega t)$$

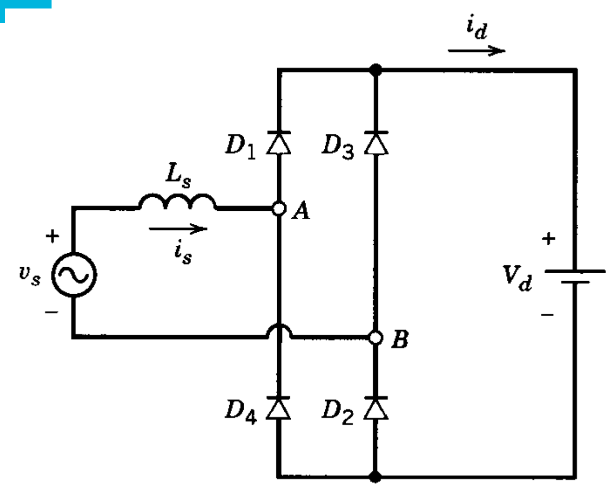
$$2\, \omega L_s I_d = \int_0^{u} \sqrt{2}\, V_s \sin \omega t \; d(\omega t)$$

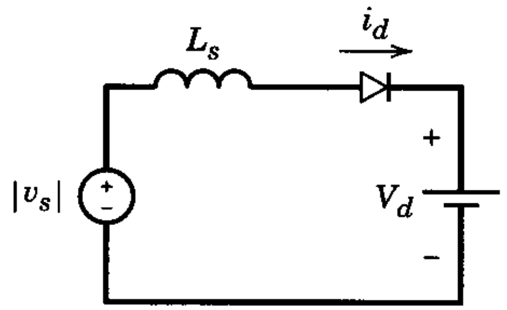
Output voltage: $\quad V_d = 0.9\, V_s - \dfrac{2\, Area\ A_u}{2\pi} = 0.9\, V_s - \dfrac{2\, \omega L_s}{\pi} I_d$

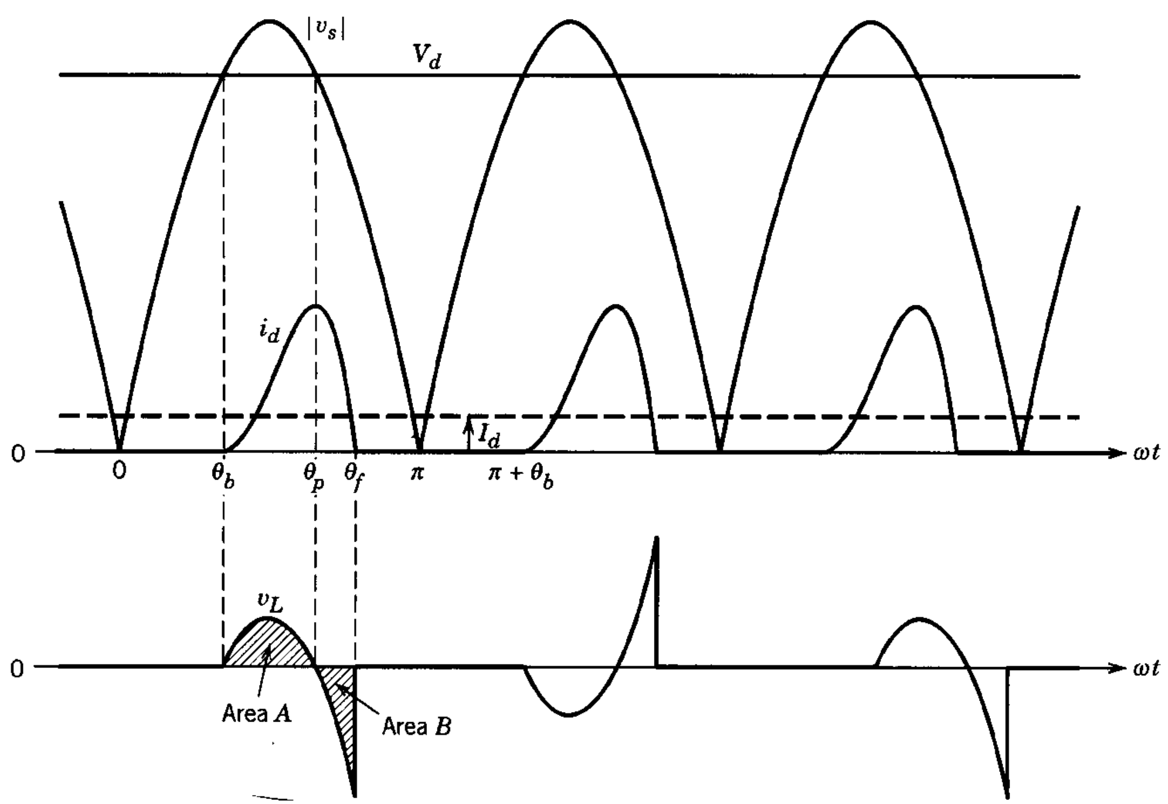
Every cycle the area A_u is lost twice

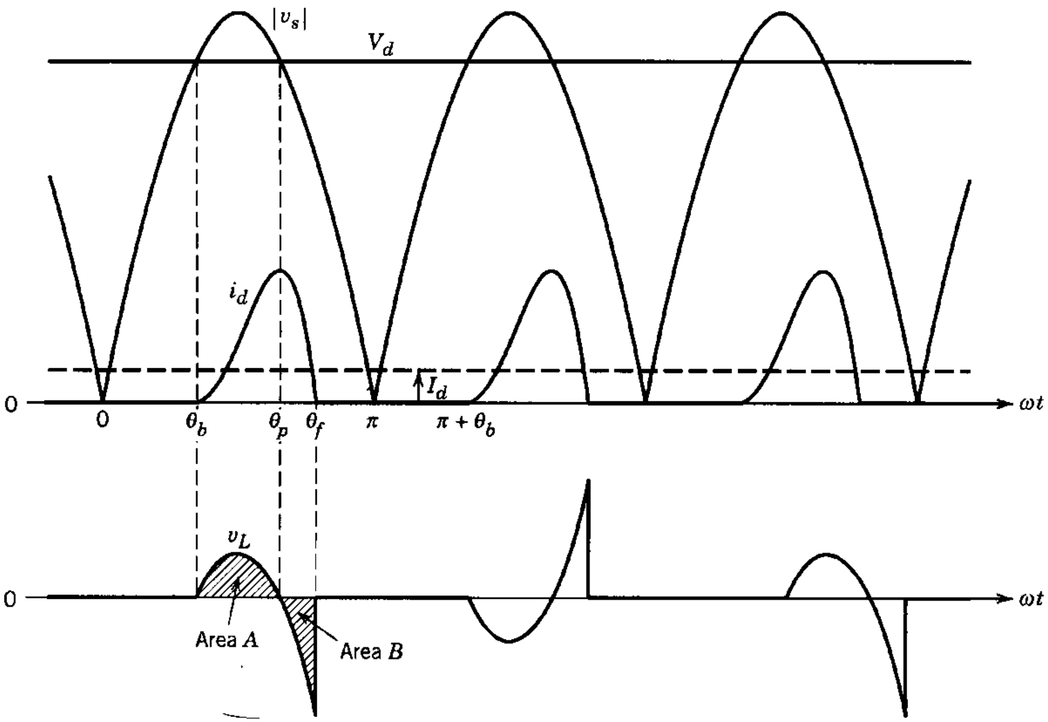
Single-phase bridge rectifier

- Constant dc-side voltage
  (Approximation of RC-load with RC>> 10 ms)









$$V_d = \sqrt{2}\,V_s \sin\theta_b$$

- i_d has maximum when v_s crosses V_d again ↓

$$i_d(\theta) = \frac{1}{\omega L_s} \int_{\theta_b}^{\theta} (\sqrt{2}\;V_s \sin\omega t\; - V_d)\; d(\omega t)$$

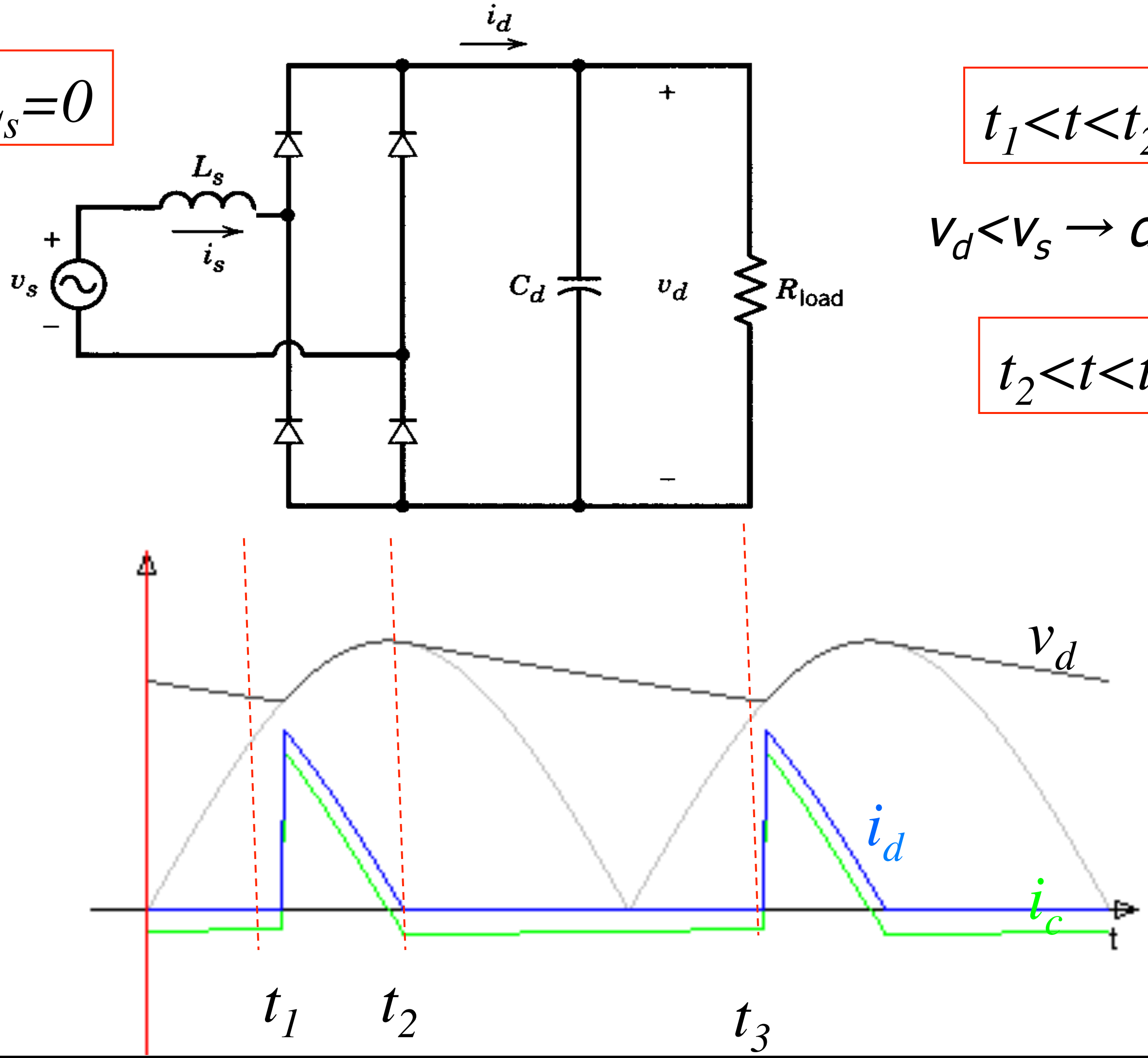
- θ_f follows from:

$$0 = \frac{1}{\omega L_s} \int_{\theta_b}^{\theta_f} (\sqrt{2}\;V_s \sin\omega t\; - V_d)\; d(\omega t)$$

- high peak line current

Practical bridge rectifier

$L_s=0$



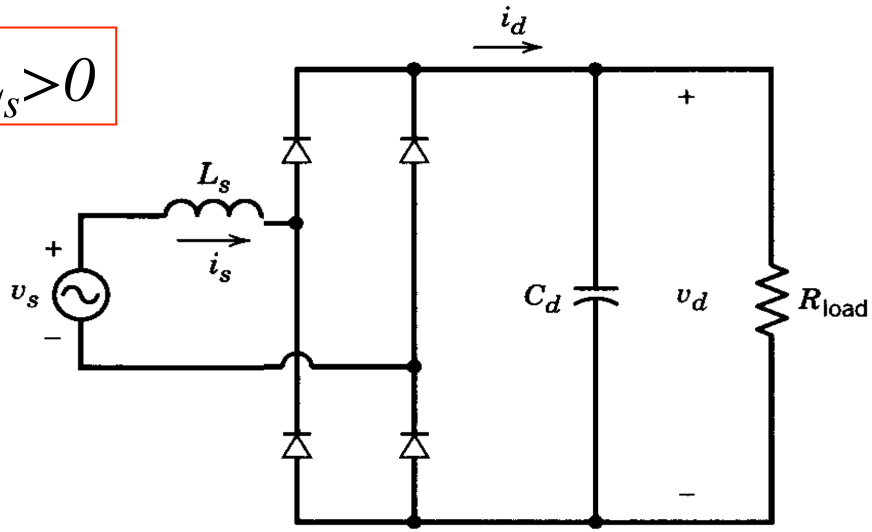
$t_1<t<t_2$

$v_d<v_s \rightarrow$ *diodes conducting*

$t_2<t<t_3$

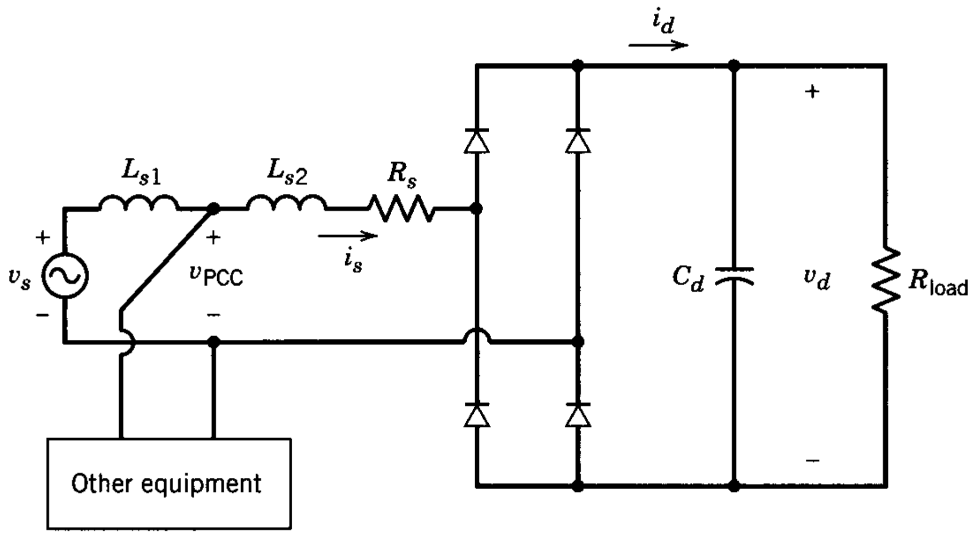
Practical bridge rectifier

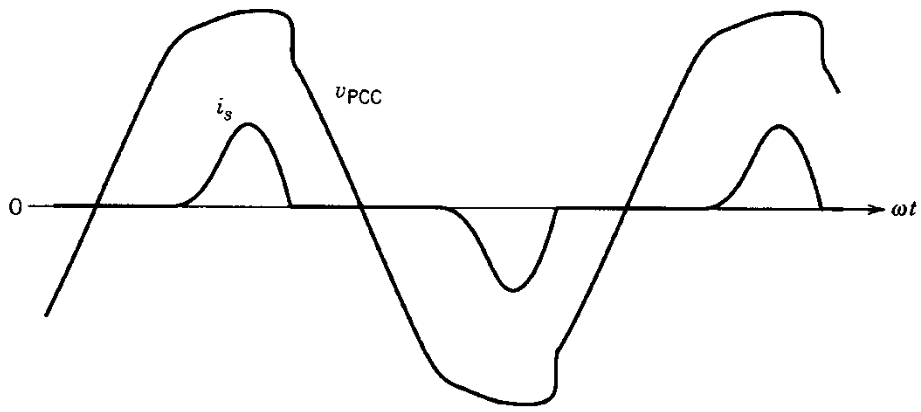
$L_s > 0$



$v_d < v_s \rightarrow$ *diodes conducting*

Line voltage distortion





$$v_{pcc} = v_s - L_{s1}\frac{di_s}{dt}$$

i_s contains harmonics:

$$i_s = i_{s1} + \sum_{h=2}^{\infty} i_{sh}$$

$$v_{pcc} = v_s - L_{s1}\frac{di_1}{dt} - L_{s1}\sum_{h=2}^{\infty}\frac{di_{sh}}{dt}$$

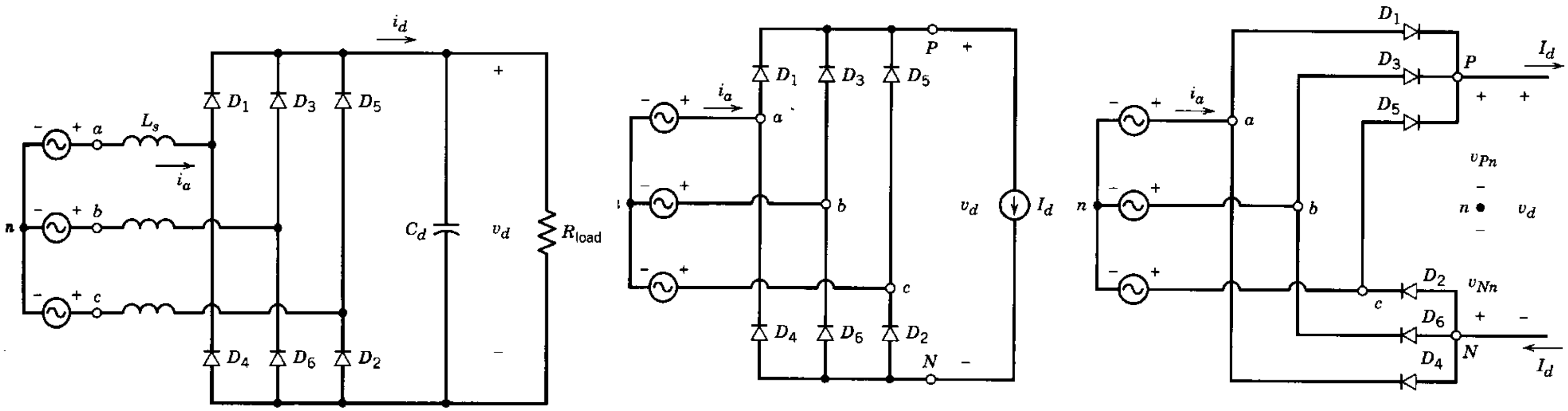
Fundamental component of v_{pcc}

$$v_{pcc,1} = v_s - L_{s1}\frac{di_{s1}}{dt}$$

Voltage distortion component

$$v_{pcc,dis} = -L_{s1}\sum_{h=2}^{\infty}\frac{di_{sh}}{dt}$$

Three phase full-bridge rectifiers



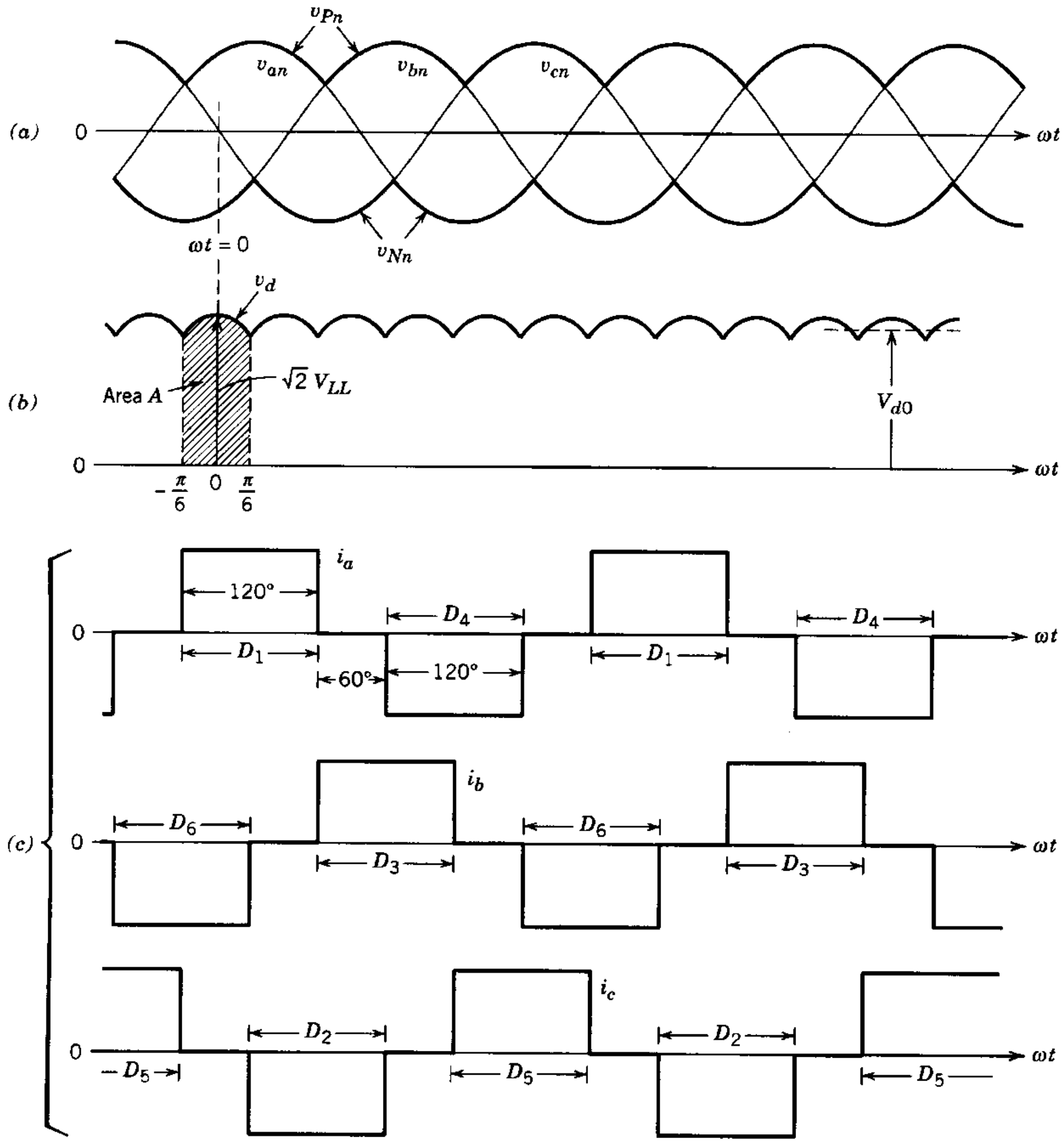
(a) (b)

Assume: i_d constant and equal to I_d *and* $L_s = 0$

How many diodes are conducting?
Which diodes are conducting?
Diode with highest anode potential is conducting (+rail)
Diode with lowest cathode potential is conducting (-rail)

Fig. (b): $$v_d = v_{Pn} - v_{Nn}$$

i_a = $+I_d$ when diode 1 is conducting
= $-I_d$ when diode 4 is conducting
= 0 when neither diode 1 or 4 is conducting

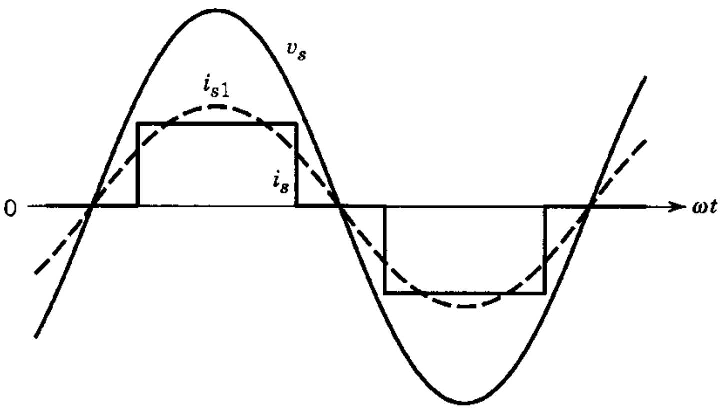


For: $-\frac{1}{6}\pi < \omega t < \frac{1}{6}\pi$

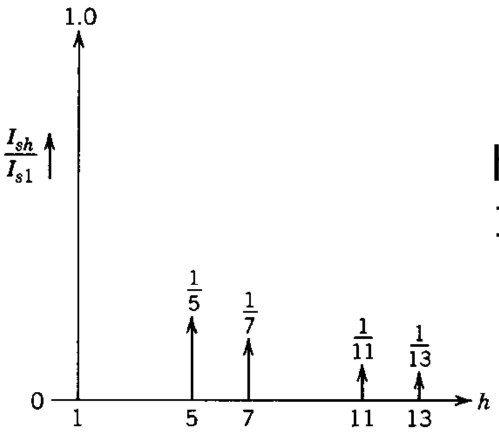
$$v_d(t) = v_{ab}(t) = \sqrt{2}\; V_{LL}\;\; \cos \omega t$$

$$V_{do} = \frac{1}{\pi/3} \int_{-\pi/6}^{\pi/6} \sqrt{2}\, V_{LL} \cos \omega t$$

$$= \frac{3}{\pi} \sqrt{2}\, V_{LL} = 1.35\, V_{LL}$$



(a)



(b)

h = 5, 7, 11, 13, 17, ...

$$I_s = \sqrt{\frac{2}{3}}\ I_d = 0.816\ I_d$$

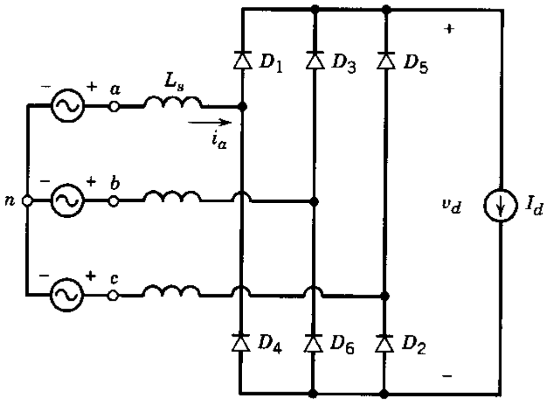
From Fourier analysis: $I_{sh} = \frac{I_{s1}}{h}$ with $I_{s1} = \frac{\sqrt{6}}{\pi}\ I_d = 0.78\ I_d$

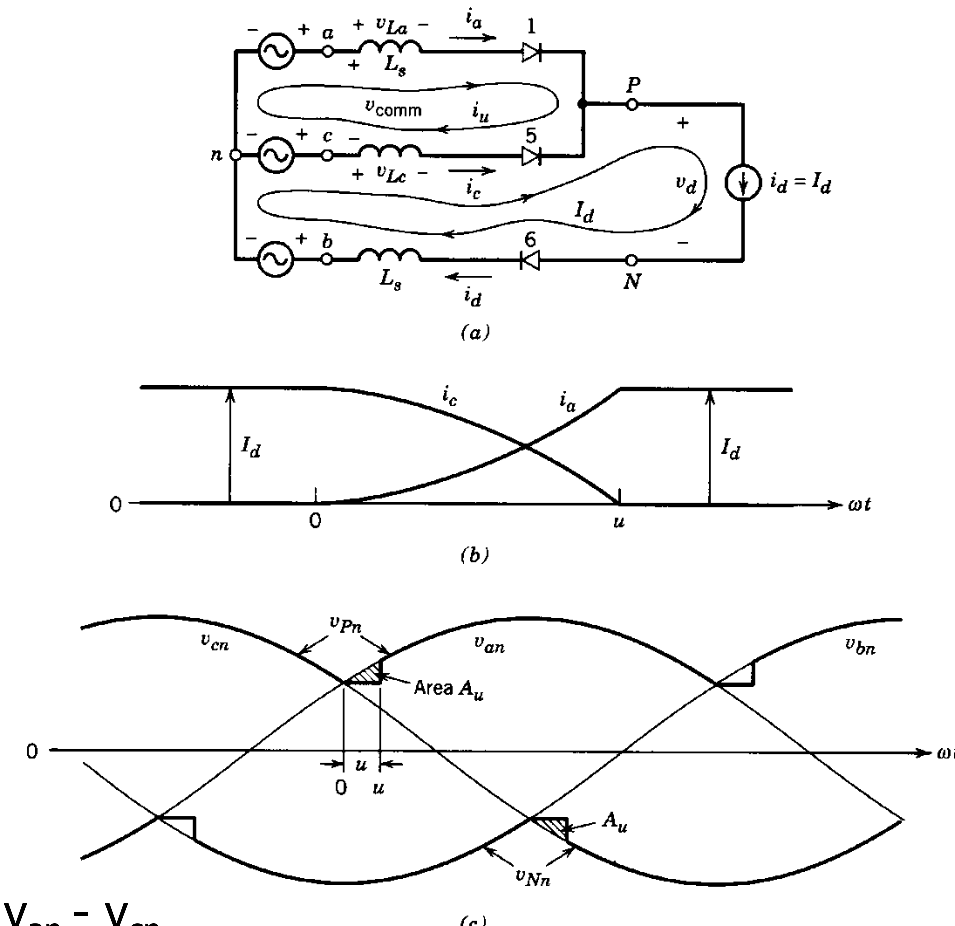
$$DF = \frac{I_{s1}}{I_s} = \frac{3}{\pi} = 0.955 \quad and \quad DPF = \cos\alpha$$

$$PF = DF \cdot DPF = \frac{I_{s1}}{I_s} \cdot 1 = \frac{3}{\pi} = 0.955$$

Three phase full-bridge rectifiers

Commutation





Commutation starts when $v_{ac} = v_{cn}$

Driving voltage in a mesh: $v_{comm} = v_{an} - v_{cn}$

$$v_{comm} = v_{an} - v_{cn} = v_{La} - v_{Lc}$$

so: $$v_{comm} = 2L_s \frac{di_u}{dt}$$

$$i_u(t) = i_u(0) + \frac{1}{L_s}\int_0^t \frac{v_{an} - v_{cn}}{2}\, d(t) \quad \rightarrow \quad \omega L_s \Delta i_d = \int_0^u \frac{\sqrt{2}\, V_{LL} \sin \omega t}{2}\, dt$$

$$\omega 2 L_s I_d = \sqrt{2}\, V_{LL}(1 - \cos u) \quad \rightarrow \quad \cos u = 1 - \frac{2\omega L_s I_d}{\sqrt{2}\, V_s}$$

During commutation: $$v_{Pn} = v_{an} - L_s \frac{di_u}{dt} = \frac{v_{an} + v_{cn}}{2}$$

Shaded area forces the current from 0 to I_d in one inductor:

$$A_u = \frac{1}{2}\int_0^u \sqrt{2}\, V_{LL} \sin \omega t \, d(\omega t)$$ or: $$A_u = \omega L_s I_d$$

Every period the area A_u is lost six times

Voltage drop: $$\Delta V_{do} = \frac{6 \cdot area\ A_u}{2\pi} = \frac{3\,\omega L_s}{\pi} I_d \qquad V_d = V_{do} - \Delta V_d = 1.35 V_{LL} - \frac{3\omega L_s}{\pi} I_d$$

Image credits

- All uncredited diagrams are from the book “Power Electronics: Converters, Applications, and Design” by N. Mohan, T.M. Undeland and W.P. Robbins.
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