Traffic Flow Theory & Simulation

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Lecture 4
Shockwave theory
Shockwave theory I: Introduction

Applications of the Fundamental Diagram
Intro to shockwave analysis

• Introduce application of fundamental diagram to shockwave analysis with aim to understand importance of field location

• Shockwave analysis:
  • Vehicles are conserved
  • Traffic acts according to the fundamental diagram \(q = Q(k)\)
  • Predicts how inhomogeneous conditions change over time

• FOSIM demonstration
  • Example 3 -> 2 lane drop and emerging shockwaves (roadworks, incident, etc.)
FOSIM example

- Extremely short introduction to FOSIM
  - Build a simple network (8 km road with roadworks at $x = 5$ km to 6 km)
  - Implement traffic demand
  - Assume 10% trucks
- Suppose that upstream traffic flow > capacity of bottleneck
- What will happen?
Questions

• Why does congestion occur
  • Macroscopically?
  • Microscopically?

Photo by My Europe

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Microscopic description

- Congestion at a bottleneck
- Simplest is to compare the system to a (sort of) queuing system
- Drivers arrive at a certain rate (demand) at specific time intervals
- The n ‘servers’ needs a minimum amount of time to process the drivers (each lane is a server)
- Service time T is a driver-specific (random) variable depending on weather conditions, road and ambient conditions, etc. (= minimum time headway of a driver)
- Note that service time is directly related to car-following behavior
- When another driver arrives when the server is still busy, he / she has to wait a certain amount of time
- Waiting time accumulates -> queuing occurs
Macroscopic description

- Compare traffic flow to a fluidic (or better: granular) flow though a narrow bottleneck (hour-glass, funnel)

- If traffic demand at a certain location is larger than the supply (capacity) congestion will occur

- Capacity is determined by number of lanes, weather conditions, driver behavior, etc.

- Excess demand is stored on the motorway to be served in the next time period

- Remainder: focus on macroscopic description

Photo by My Europe
Questions

- Why does congestion occur
  - Macroscopically?
  - Microscopically?

- Where does congestion first occur?
- Which traffic conditions (traffic phases) are encountered?
- Where are these conditions encountered?
Definition of a shockwave

- Consider over-saturated bottleneck
- Traffic conditions change over space and time

- Boundaries between traffic regions are referred to as shockwaves
- Shockwave can be very mild (e.g. platoon of high-speed vehicles catching up to a platoon of slower driver vehicles)
- Very significant shockwave (e.g. free flowing vehicles approaching queue of stopped vehicles)
Definition of a shockwave

- Shockwave is thus a boundary in the space-time domain that demarks a discontinuity in flow-density conditions
- Example: growing / dissolving queue at a bottleneck

![Diagram of shockwave concepts](image)
Fundamental diagrams

• What can we say about the FD at the different locations?
• Some ‘standard’ numbers (for Dutch motorways)
  • Capacity point $\approx n \times 2200 \text{ pce/h/lane}$
  • Critical density $\approx n \times 25 \text{ pce/km/lane}$
  • Jam-density $\approx n \times 2200 \text{ pce/km/lane}$
  • Critical speed $\approx 85 \text{ km/h}$
  • Free speed: to be determined from speed limit
• pce = ‘person car equivalent’

• Exact numbers depend on specific characteristics of considered location (traffic composition, road conditions, etc.)
Emerging traffic states

- Stationary shockwave between capacity and congested conditions
- What is the flow inside congestion?
- And upstream of the congestion?
Emerging traffic states

- Free-flow
- Capacity
- Congestion
- End of bottleneck
- Bottleneck location

Remainder: focus on the dynamics of this shock!
Shockwave equations

- Assume that flow-density relation $Q(k)$ is known for all location $x$ on the road (i.e. different inside and outside bottleneck!)
- Consider queue due to downstream bottleneck
- Consider conditions in the queue
  - Flow $q_2 = C_{b-n}$ (capacity downstream bottleneck)
  - Speed, density follow from $q_2$, i.e. $u_2 = U(q_2)$ and $k_2 = q_2/u_2$

- Farther upstream of queue, we have conditions $(k_1, u_1, q_1)$
- Speed $u_1 > u_2 \rightarrow$ upstream vehicles will catch up with vehicles in queue
Shockwave equations

upstream cond. \((q_1,k_1,u_1)\)

downstream cond. \((q_2,k_2,u_2)\)

Bottleneck capacity

Vehicles in queue

Shockwave

Vehicles upstream
Shockwave equations

\[ \omega_{12} \]

\[ u_1 \quad k_1 \]

\[ u_2 \quad k_2 \]

\[ q_1 \quad q_2 \]

\[ u_1 \quad u_2 \]
Shockwave equations

- Relative speed traffic flow region 1 with respect to S: \( u_1 - \omega_{12} \)
- Thus flow out of region 1 into shock S equals (explanation)
  \[
  q_{S}^{in} = k_1 \left( u_1 - \omega_{12} \right)
  \]
- Relative speed traffic flow region 2 with respect to S: \( u_2 - \omega_{12} \)
- Thus flow into region 2 out of the shock must be
  \[
  q_{S}^{out} = k_2 \left( u_2 - \omega_{12} \right)
  \]
- Conservation of vehicles over the shock (shock does not destroy or generate vehicles)
  \[
  q_{S}^{in} = q_{S}^{out} \iff k_1 \left( u_1 - \omega_{12} \right) = k_2 \left( u_2 - \omega_{12} \right)
  \]
Shockwave equations

- Shockwave speed \( \omega_{12} \) thus becomes

\[
\omega_{12} = \frac{k_2 u_2 - k_1 u_1}{k_2 - k_1} = \frac{q_2 - q_1}{k_2 - k_1}
\]

- The speed of the shock equals the ratio of:
  - jump of the flow over the shock S and
  - jump in the density over the shock S
Shockwave equations

\[ q_2, k_2, u_2 \]

Vehicles in queue

Bottleneck

Shockwave

Jump in flow

\[ \omega_{12} = \frac{q_2 - q_1}{k_2 - k_1} \]

Jump in density

0 \quad k_1 \quad k_2 \quad k

\[ q_1, k_1, u_1 \]

Vehicles upstream

\[ u_1, u_2 \]
Shockwave equations

- Remarks:
  - If $k_2 > k_1$ sign shockwave speed negative if $q_1 > q_2$ (backward forming shockwave)
  - If $k_2 > k_1$ sign shockwave speed positive if $q_1 < q_2$ (forward recovery shockwave)
  - If $k_2 > k_1$ sign shockwave speed zero if $q_1 = q_2$ (backward stationary)

- Classification of shockwaves
Final remarks

- Shockwave theory is applicable when
  - $Q(k)$ is known for all location $x$
  - Initial conditions are known
  - Boundary conditions (at $x_1$ AND $x_2$) are known

- Shockwaves occur when
  - Spatial / temporal discontinuities in speed-flow curve (expressed $Q(k,x)$), e.g. recurrent bottleneck
  - Spatial discontinuities in initial conditions
  - Temporal discontinuities in boundary conditions at $x_1$ (or $x_2$)
Applications of shockwave theory

Temporary over-saturation
Traffic lights
Shockwaves at a bottleneck

- Temporary over-saturation of a bottleneck
- Traffic demand (upstream)

\[
q(t) = \begin{cases} 
q_1 & t < t_1 \text{ or } t \geq t_2 \\
q_2 & t_1 \leq t < t_2 
\end{cases}
\]
Application of shockwave analysis

Three simple steps to applying shockwave theory:
1. Determine the Q(k) curve for all locations x
2. Determine the following ‘external conditions’:
   - initial states ($t = t_0$)
   - ‘boundary’ states (inflow, outflow restrictions, moving bottleneck).
   present in the x-t plane and the q-k plane
3. Determine the boundaries between the states (=shockwaves) and determine their dynamics
4. Check for any omissions you may have made (are regions with different states separated by a shockwave?)
Shockwaves at bottleneck

capacity of bottleneck

0 \ k_1 \ k_2 \ k

q

q_1

q_2

x

t_1 \ t_2

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Exercise temporary blockade

Fundamental diagram without capacity drop

Which shocks emerge
Duration of disturbance?
Draw a couple trajectories
What if $q_1 = 2200$?

$q_c = 2500$

$q_1 = 1000$

$t_0$

$t_1$

$t$
Studies of the fundamental diagram

- Need for complete diagram or only a part of it? Will it in general be possible to determine a complete diagram at a cross-section?
- Is the road section homogeneous? Yes: observations at a single cross-section. No: road characteristics are variable over the section and a method such as MO might be suitable
- Mind the period of analysis:
  - too short (1 minute): random fluctuations much influence;
  - too long (1 hour): stationarity cannot be guaranteed (mix different regimes)
- Estimate parameters of the model chosen using (non-linear) regression analysis
Studies of the fundamental diagram$^2$

- Many models will fit your data
- Hints for choosing models?
- Simplest model possible (parsimony)
- Interpretation of parameters
- Theoretical considerations
Studies of the fundamental diagram

- **Demonstration FOSIM**
- Fundamental diagram determined from real-life data, by assuming stationary periods
- Dependent on measurement location
- Flow per lane
Studies of the fundamental diagram

Estimation of free flow capacity using fundamental diagram

• Approach 1:
  • Fit a model $q(k)$ to available data
  • Consider point $dq/dk = 0$
  • Generally not applicable to motorway traffic because $dq/dk = 0$ does not hold at capacity

• Approach 2:
  • Assume fixed value for the critical density $k_c$
  • Estimate only free-flow branch of the diagram
  • More for comparative analysis
Studies of the fundamental diagram

- **Application example**: effect of roadway lighting on capacity
- Two and three lane motorway
- Before – after study
  - Difficulties due to different conditions (not only ambient conditions change)
  - See e.g. site SB daylight before – after
  - Effect lighting on capacity approx 2.5% (2 lane) or 1.6% (3 lane)

<table>
<thead>
<tr>
<th></th>
<th>Daylight</th>
<th>Darkness</th>
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<tbody>
<tr>
<td></td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td>Site I NB Treatment</td>
<td>100</td>
<td>99.1</td>
</tr>
<tr>
<td>Site I SB Treatment</td>
<td>100</td>
<td>96.8</td>
</tr>
<tr>
<td>Site II Treatment</td>
<td>100</td>
<td>100.4</td>
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<tr>
<td>Site III Comparison</td>
<td>100</td>
<td>98.8</td>
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Studies of the fundamental diagram\textsuperscript{6}

- Effect on rain on capacity / fundamental diagram

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fundamental_diagram.png}
\caption{Number of points: 146}
\end{figure}
Studies of the fundamental diagram

- Effect on rain on capacity / fundamental diagram
Studies of the fundamental diagram

- Estimating queue discharge rate
- Only in case of observations of oversaturated bottleneck
- Three measurement locations (ideally)
  - Upstream of bottle-neck (does overloading occur?)
  - Downstream of bottle-neck (is traffic flow free?)
  - At the bottle-neck (intensities are capacity measurements if traffic state upstream is congested and the state downstream is free)
- Flow at all three points equal (stationary conditions) and at capacity; use downstream point
Summary of lecture

- Fundamental diagram for a lane and a cross-section
- Shockwave equations
- Application of shockwave analysis
  - shockwave at bottleneck
- Establishing a fundamental diagram from field observations
Shockwaves signalized intersections
Shockwave classification

- 6 types of shockwaves
- Which situations do they represent? Examples?
Shockwave classification

1. **Frontal stationary**: head of a queue in case of stationary / temporary bottleneck
2. **Forward forming**: moving bottleneck (slow vehicle moving in direction of the flow given limited passing opportunities)
3. **Backward recovery**: dissolving queue in case of stationary or temporary bottleneck (demand l.t. supply); forming or dissolving queue for moving bottleneck
Shockwave classification

1. **Forward recovery**: removal of temporary bottleneck (e.g. clearance of incident, opening of bridge, signalized intersection)
2. **Backward forming**: forming queue in case of stationary, temporary, or moving bottleneck* (demand g.t. supply);
3. **Rear stationary**: tail of queue in case recurrent congestion when demand is approximately equal to the supply

*Note: The diagram at the bottom right of the page illustrates the concepts of forward recovery, backward forming, and rear stationary.
Flow into shockwave

- Consider a shockwave moving with speed $\omega_{12}$
- Flow into the shockwave = flow observed by moving observer travelling with speed of shockwave

- Number of vehicles observed on $S =$
  + Veh. passing $x_0$ during $T$
  - Vehcles on $X$ at $t_1$

\[
q^{s}_T = q_1 T - k_1 X, \quad X = \omega_{12} T
\]
\[
q^{s}_T = (k_1 u_1 - k_1 \omega_{12}) T
\]
\[
q^{s}_T = k_1 (u_1 - \omega_{12})
\]