

Traffic Flow Theory & Simulation

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Lecture 4
Shockwave theory



Shockwave theory I: Introduction

Applications of the Fundamental Diagram

February 14, 2010

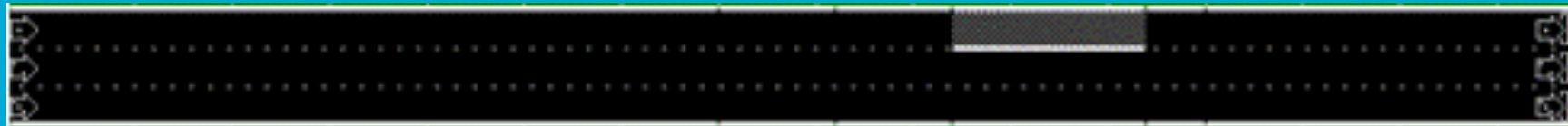
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Intro to shockwave analysis

- Introduce application of fundamental diagram to shockwave analysis with aim to understand importance of field location
- Shockwave analysis:
 - Vehicles are conserved
 - Traffic acts according to the fundamental diagram ($q = Q(k)$)
 - Predicts how inhomogeneous conditions change over time
- FOSIM demonstration
 - Example 3 -> 2 lane drop and emerging shockwaves (roadworks, incident, etc.)

FOSIM example

- Extremely short introduction to FOSIM
 - Build a simple network (8 km road with roadworks at $x = 5$ km to 6 km)
 - Implement traffic demand
 - Assume 10% trucks
- Suppose that upstream traffic flow $>$ capacity of bottleneck
- What will happen?



Questions

- Why does congestion occur
 - Macroscopically?
 - Microscopically?



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Microscopic description

- Congestion at a bottleneck
- Simplest is to compare the system to a (sort of) queuing system
- Drivers arrive at a certain rate (demand) at specific time intervals
- The n 'servers' needs a minimum amount of time to process the drivers (each lane is a server)
- Service time T is a driver-specific (random) variable depending on weather conditions, road and ambient conditions, etc.
(= **minimum time headway of a driver**)
- Note that service time is directly related to car-following behavior
- When another driver arrives when the server is still busy, he / she has to wait a certain amount of time
- Waiting time accumulates -> queuing occurs

Macroscopic description

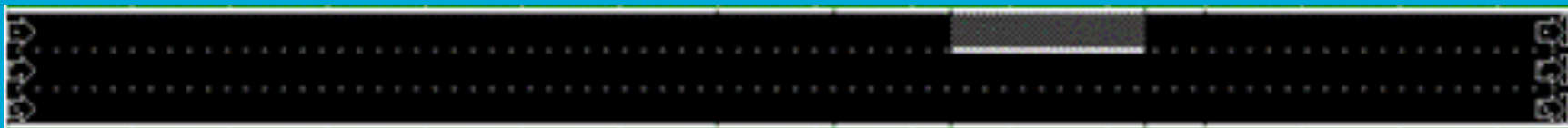
- Compare traffic flow to a fluidic (or better: granular) flow through a narrow bottleneck (hour-glass, funnel)
- If traffic demand at a certain location is larger than the supply (capacity) congestion will occur
- Capacity is determined by number of lanes, weather conditions, driver behavior, etc.
- Excess demand is stored on the motorway to be served in the next time period
- Remainder: focus on **macroscopic** description

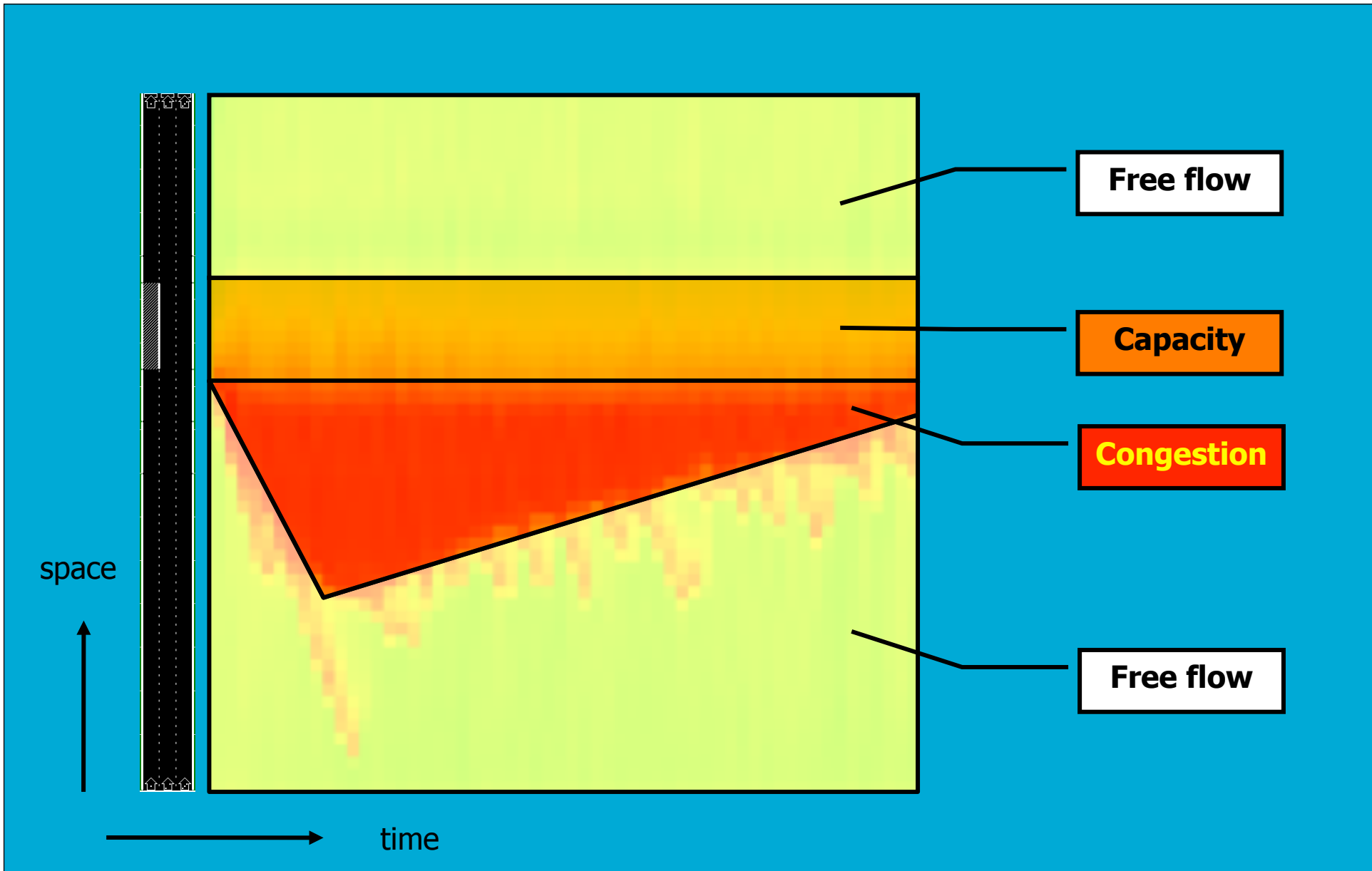


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Questions

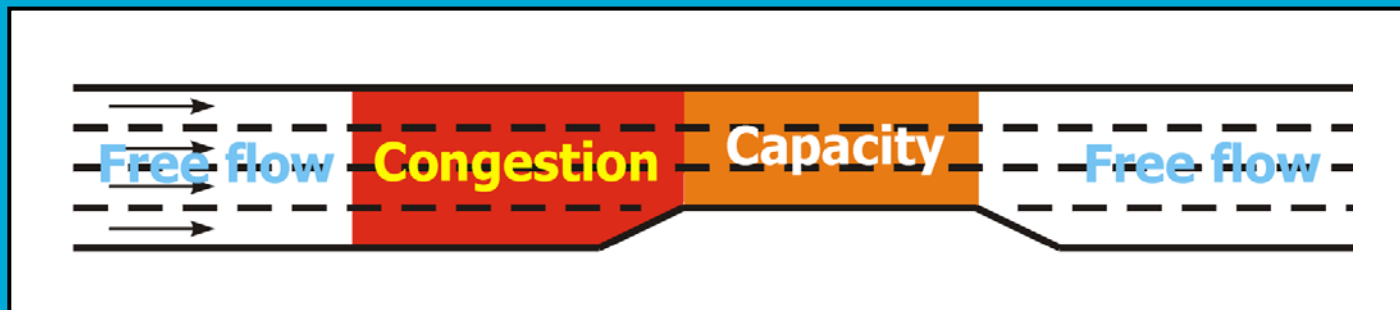
- Why does congestion occur
 - Macroscopically?
 - Microscopically?
- Where does congestion first occur?
- Which traffic conditions (traffic phases) are encountered?
- Where are these conditions encountered?





Definition of a shockwave

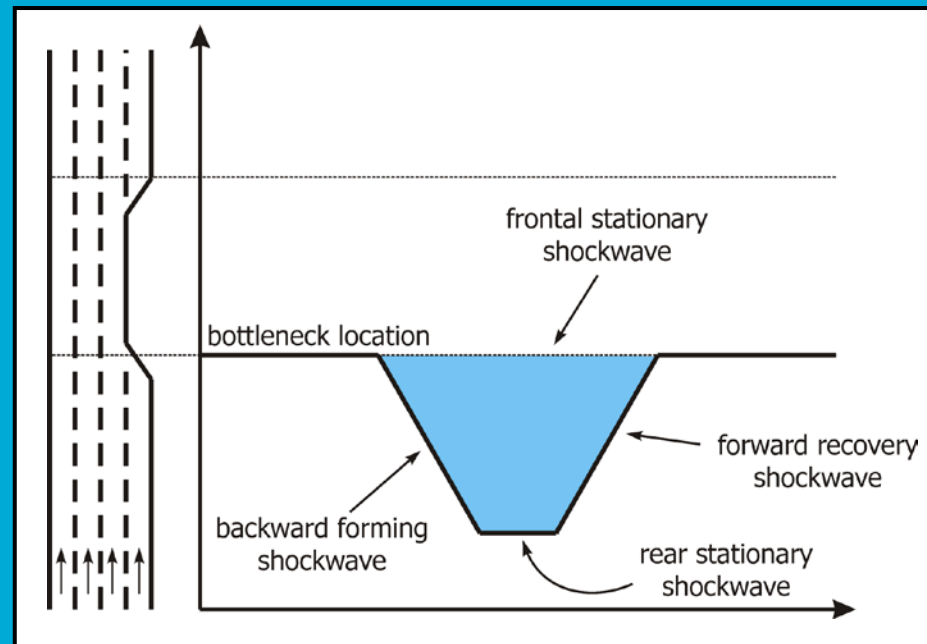
- Consider over-saturated bottleneck
- Traffic conditions change over space and time



- Boundaries between traffic regions are referred to as shockwaves
- Shockwave can be very mild (e.g. platoon of high-speed vehicles catching up to a platoon of slower driver vehicles)
- Very significant shockwave (e.g. free flowing vehicles approaching queue of stopped vehicles)

Definition of a shockwave²

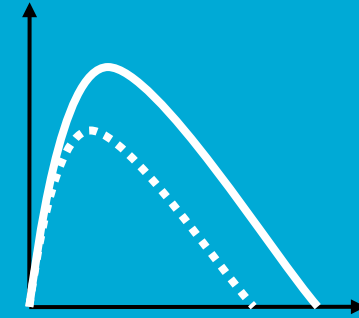
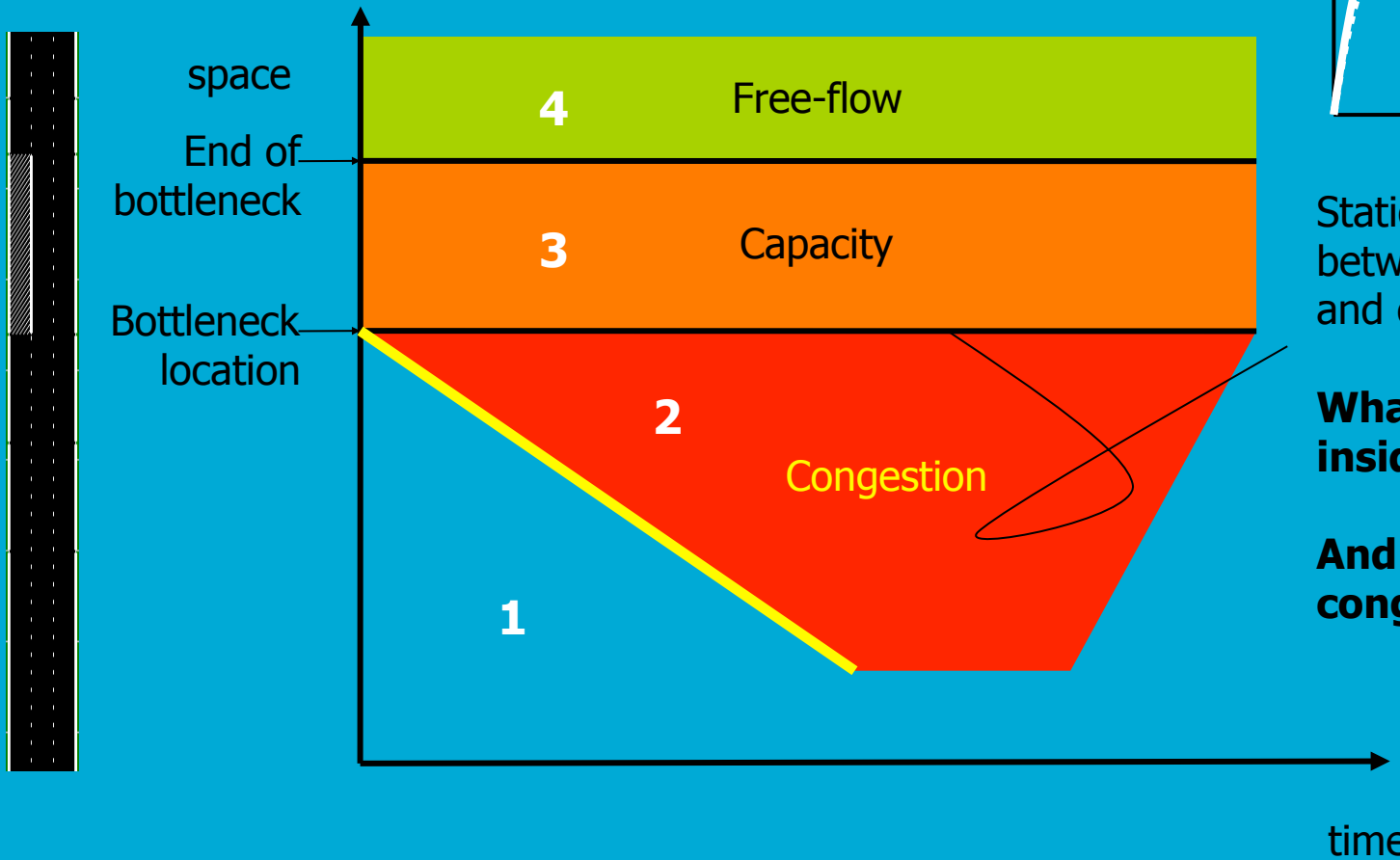
- Shockwave is thus a boundary in the space-time domain that demarks a discontinuity in flow-density conditions
- Example: growing / dissolving queue at a bottleneck



Fundamental diagrams

- What can we say about the FD at the different locations?
- Some 'standard' numbers (for Dutch motorways)
 - Capacity point $\approx n * 2200$ pce/h/lane
 - Critical density $\approx n * 25$ pce/km/lane
 - Jam-density $\approx n * 2200$ pce/km/lane
 - Critical speed ≈ 85 km/h
 - Free speed: to be determined from speed limit
- pce = 'person car equivalent'
- Exact numbers depend on specific characteristics of considered location (traffic composition, road conditions, etc.)

Emerging traffic states

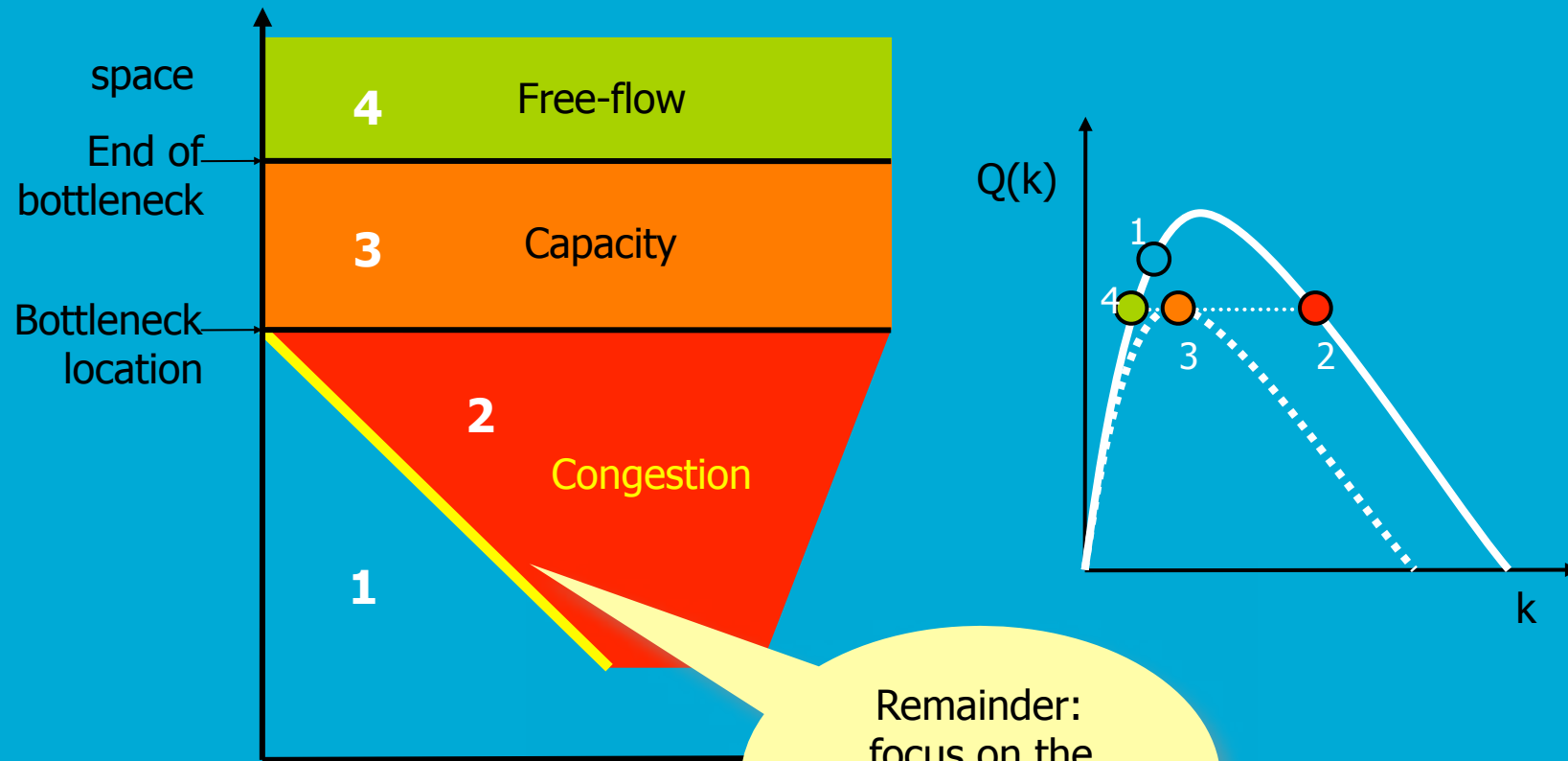


Stationary shockwave between capacity and congested conditions

What is the flow inside congestion?

And upstream of the congestion?

Emerging traffic states

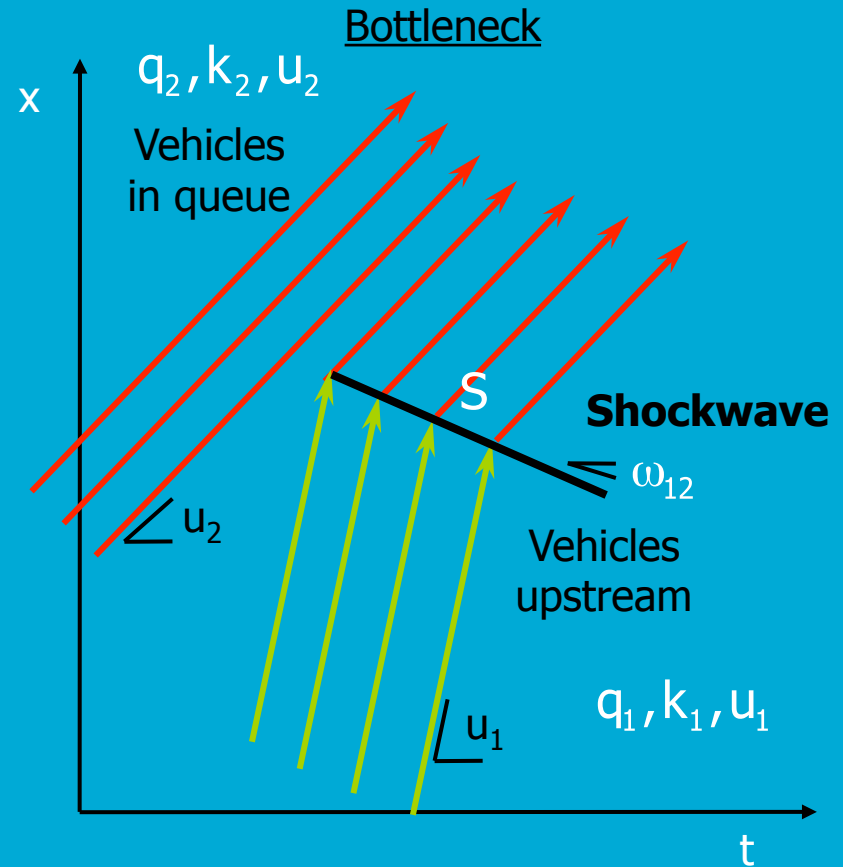
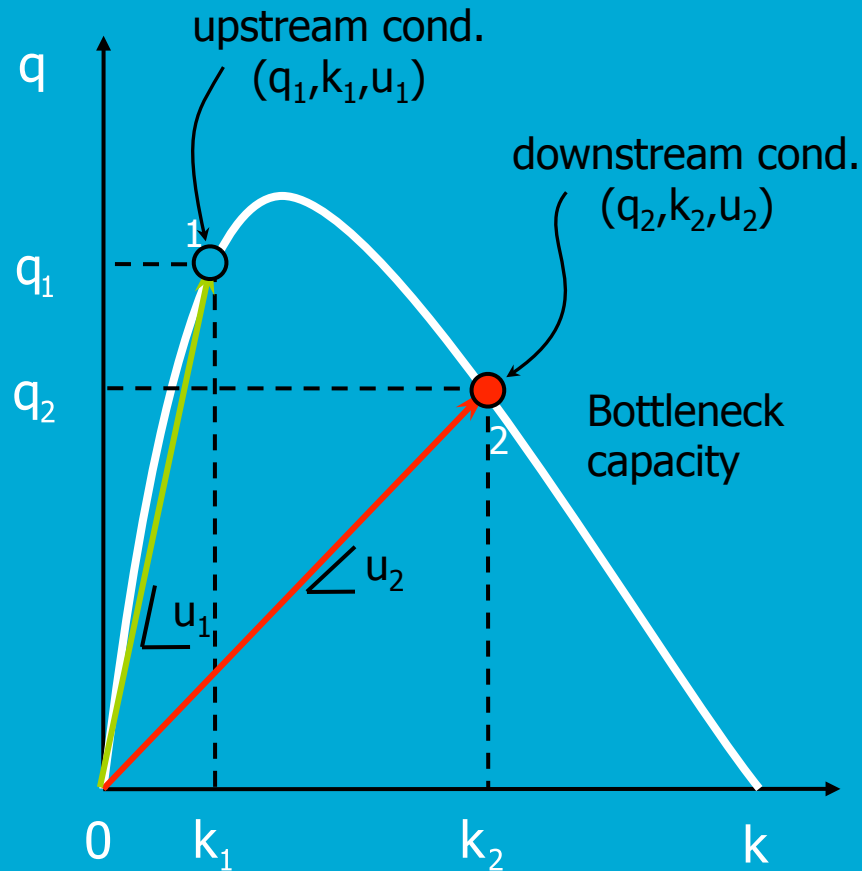


Remainder:
focus on the
dynamics of this
shock!

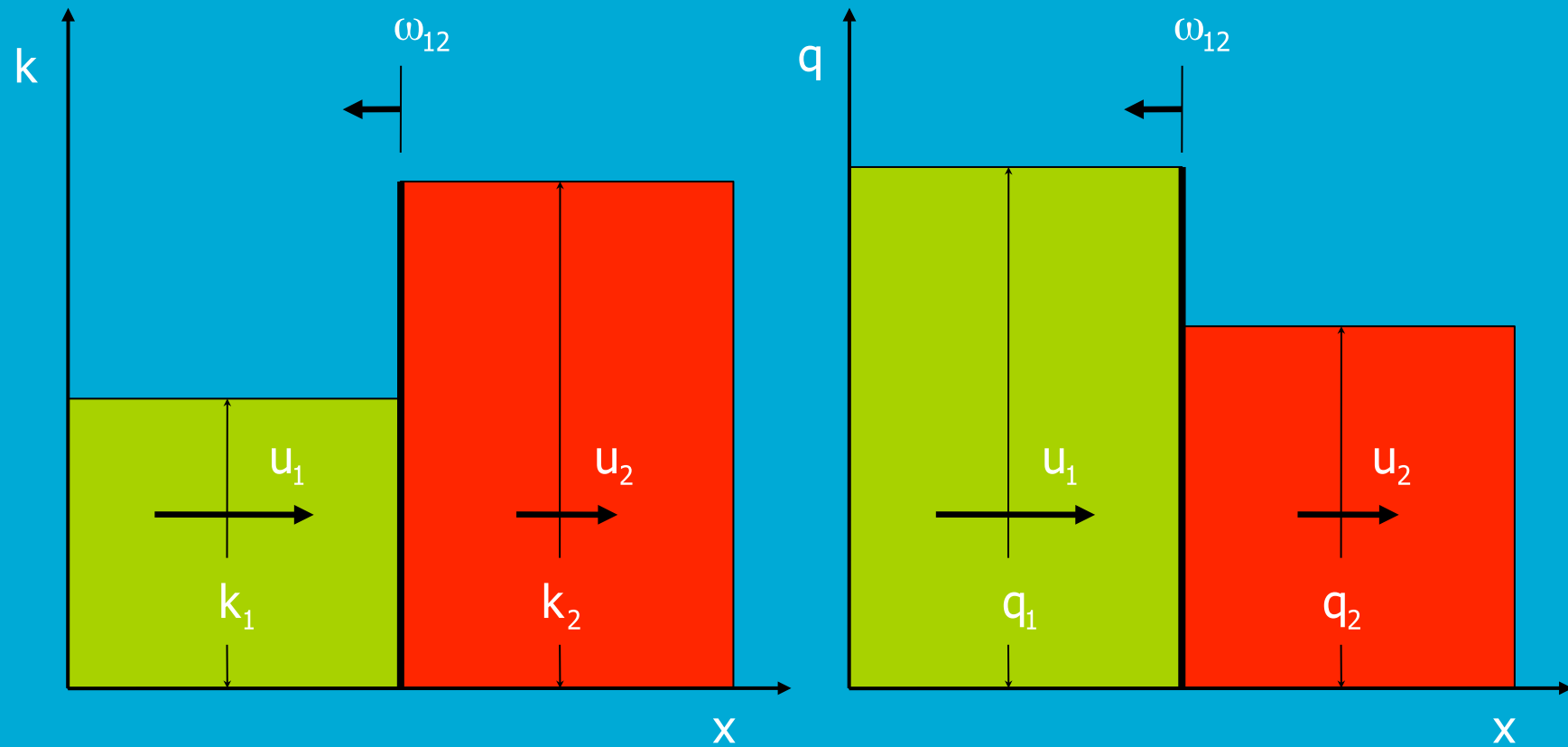
Shockwave equations

- Assume that flow-density relation $Q(k)$ is known for all location x on the road (i.e. different inside and outside bottleneck!)
- Consider queue due to downstream bottleneck
- Consider conditions in the queue
 - Flow $q_2 = C_{b-n}$ (capacity downstream bottleneck)
 - Speed, density follow from q_2 , i.e. $u_2 = U(q_2)$ and $k_2 = q_2/u_2$
- Farther upstream of queue, we have conditions (k_1, u_1, q_1)
- Speed $u_1 > u_2 \rightarrow$ upstream vehicles will catch up with vehicles in queue

Shockwave equations²



Shockwave equations³



Shockwave equations⁴

- Relative speed traffic flow region 1 with respect to S: $u_1 - \omega_{12}$
- Thus flow out of region 1 into shock S equals ([explanation](#))

$$q_S^{\text{in}} = k_1 (u_1 - \omega_{12})$$

- Relative speed traffic flow region 2 with respect to S: $u_2 - \omega_{12}$
- Thus flow into region 2 out of the shock must be

$$q_S^{\text{out}} = k_2 (u_2 - \omega_{12})$$

- Conservation of vehicles over the shock (shock does not destroy or generate vehicles)

$$q_S^{\text{in}} = q_S^{\text{out}} \quad \Leftrightarrow \quad k_1 (u_1 - \omega_{12}) = k_2 (u_2 - \omega_{12})$$

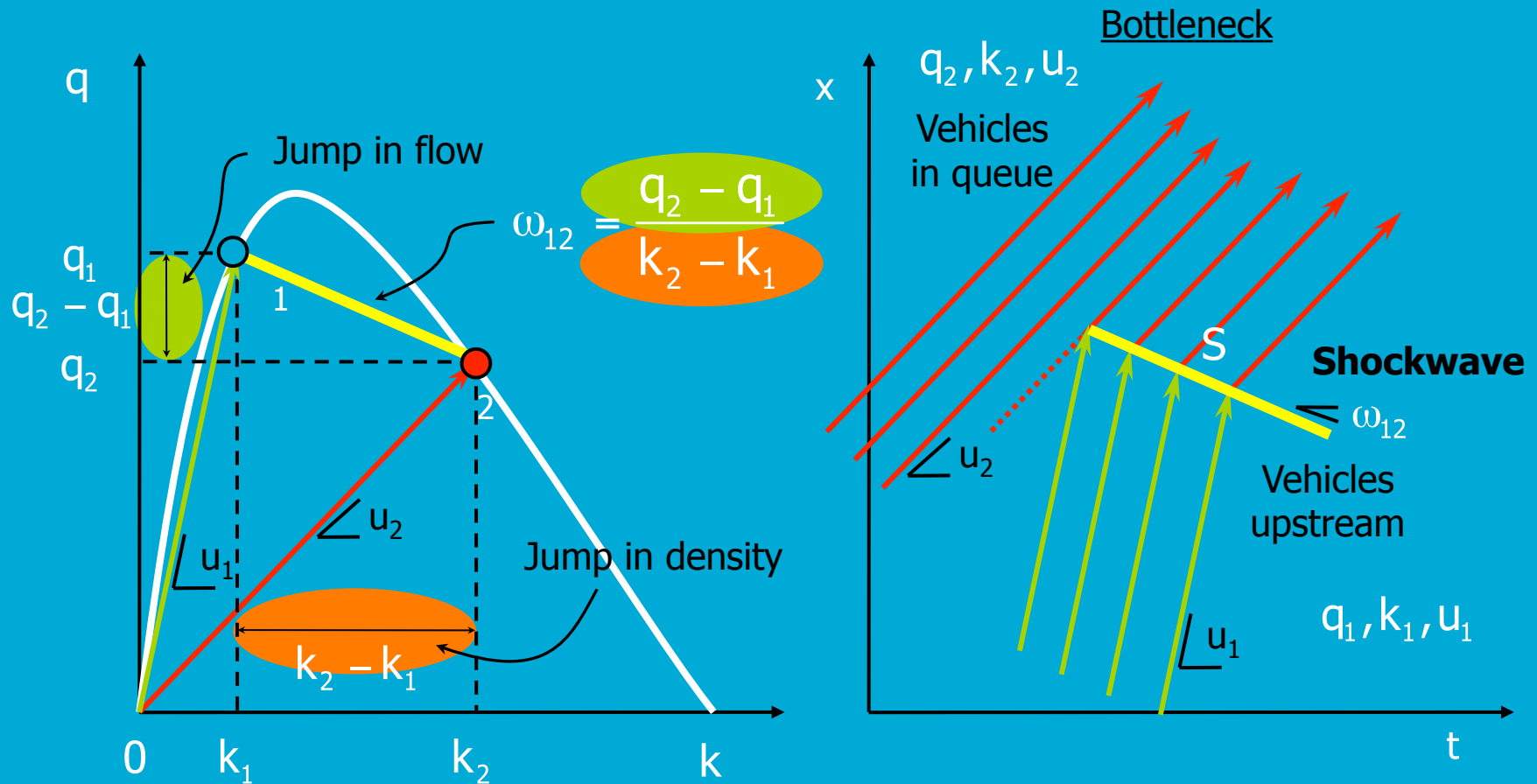
Shockwave equations⁵

- Shockwave speed ω_{12} thus becomes

$$\omega_{12} = \frac{k_2 u_2 - k_1 u_1}{k_2 - k_1} = \frac{q_2 - q_1}{k_2 - k_1}$$

- The speed of the shock equals the ratio of
 - jump of the flow over the shock S and
 - jump in the density over the shock S

Shockwave equations⁶



Shockwave equations⁷

- Remarks:
 - If $k_2 > k_1$ sign shockwave speed negative if $q_1 > q_2$ (backward forming shockwave)
 - If $k_2 > k_1$ sign shockwave speed positive if $q_1 < q_2$ (forward recovery shockwave)
 - If $k_2 > k_1$ sign shockwave speed zero if $q_1 = q_2$ (backward stationary)
- [Classification of shockwaves](#)

Final remarks

- Shockwave theory is applicable when
 - $Q(k)$ is known for all location x
 - Initial conditions are known
 - Boundary conditions (at x_1 AND x_2) are known
- Shockwaves occur when
 - Spatial / temporal discontinuities in speed-flow curve (expressed $Q(k,x)$), e.g. recurrent bottleneck
 - Spatial discontinuities in initial conditions
 - Temporal discontinuities in boundary conditions at x_1 (or x_2)

Applications of shockwave theory

Temporary over-saturation
Traffic lights

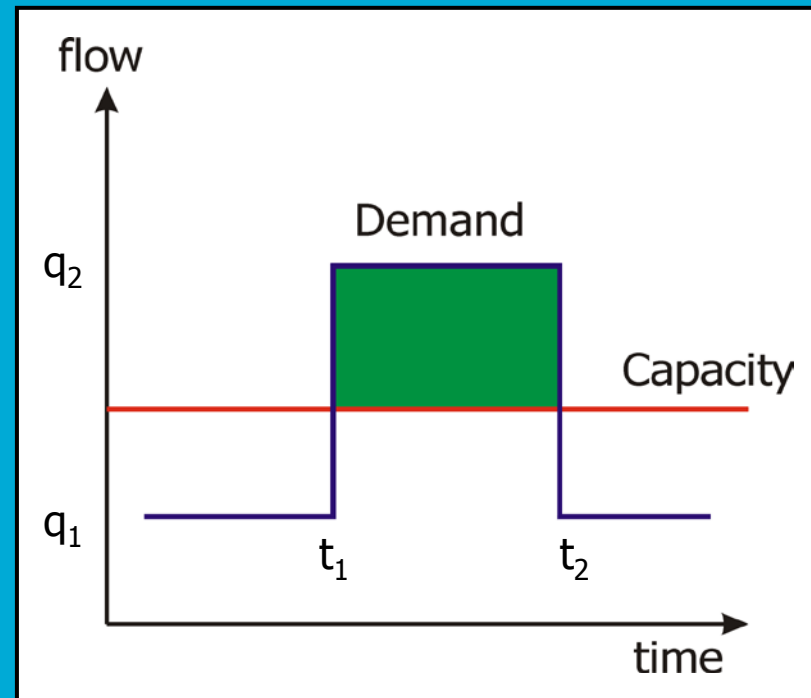
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Shockwaves at a bottleneck

- Temporary over-saturation of a bottleneck
- Traffic demand (upstream)

$$q(t) = \begin{cases} q_1 & t < t_1 \text{ or } t \geq t_2 \\ q_2 & t_1 \leq t < t_2 \end{cases}$$



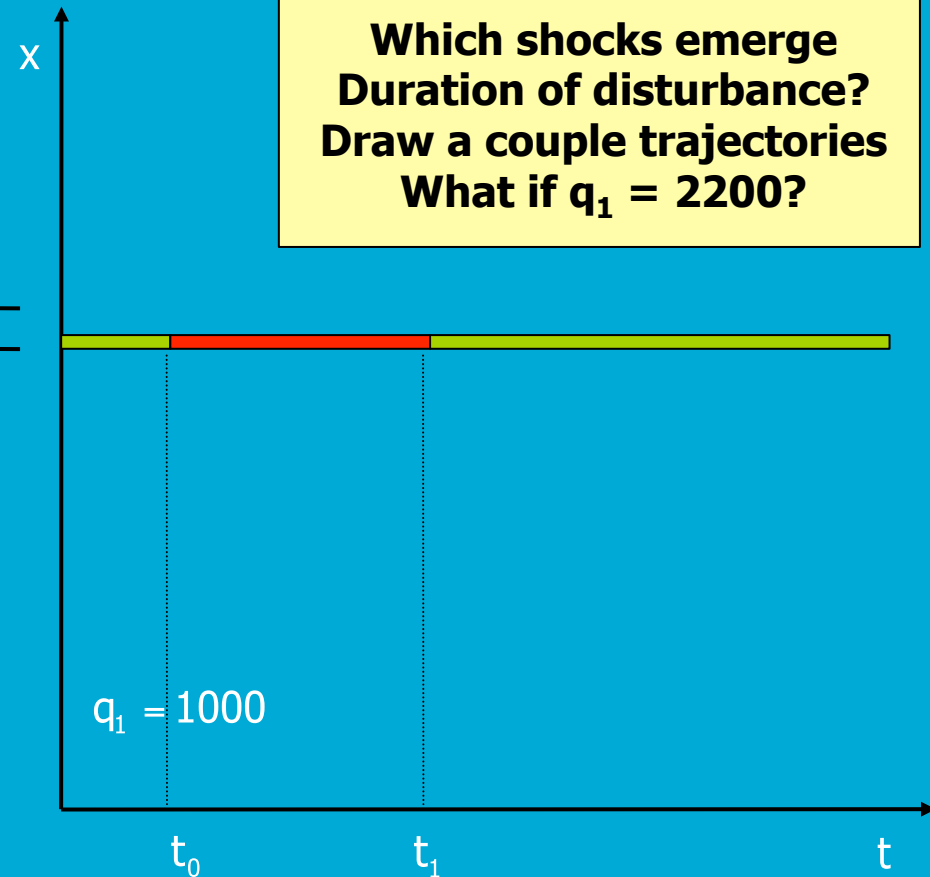
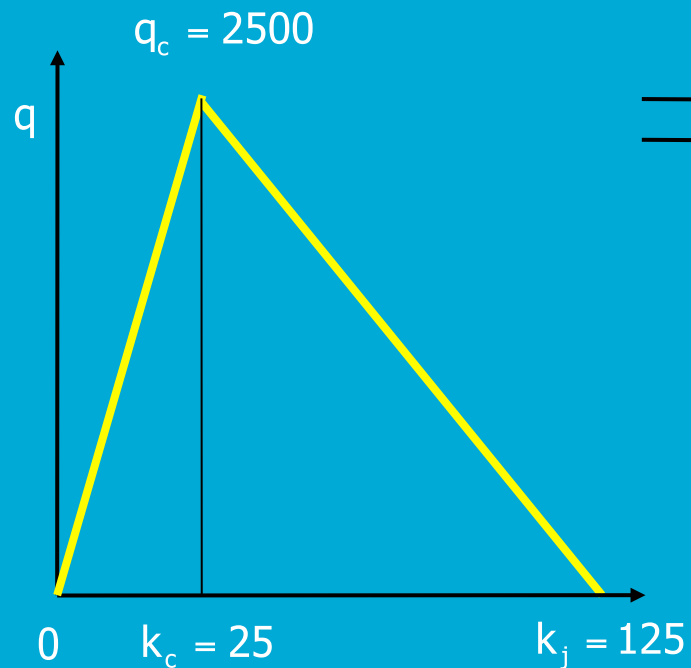
Application of shockwave analysis

Three simple steps to applying shockwave theory:

1. Determine the $Q(k)$ curve for all locations x
2. Determine the following 'external conditions':
 - initial states ($t = t_0$)
 - 'boundary' states (inflow, outflow restrictions, moving bottleneck).present in the x - t plane and the q - k plane
3. Determine the boundaries between the states (=shockwaves) and determine their dynamics
4. Check for any ommisions you may have made (are regions with different states separated by a shockwave?)

Exercise temporary blockade

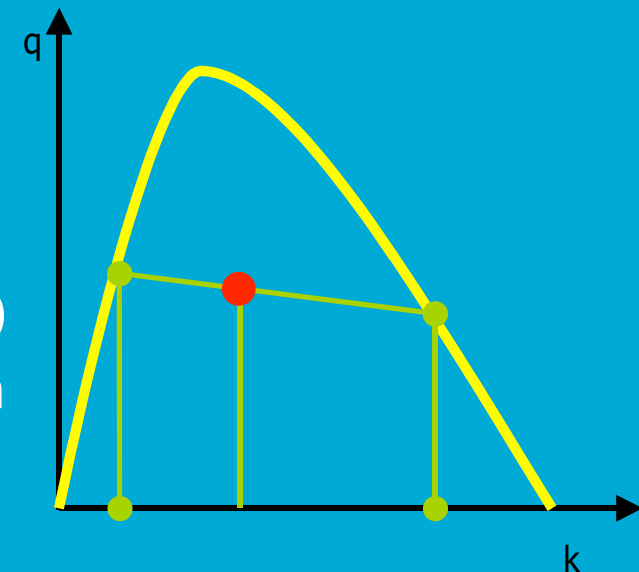
Fundamental diagram
without capacity drop



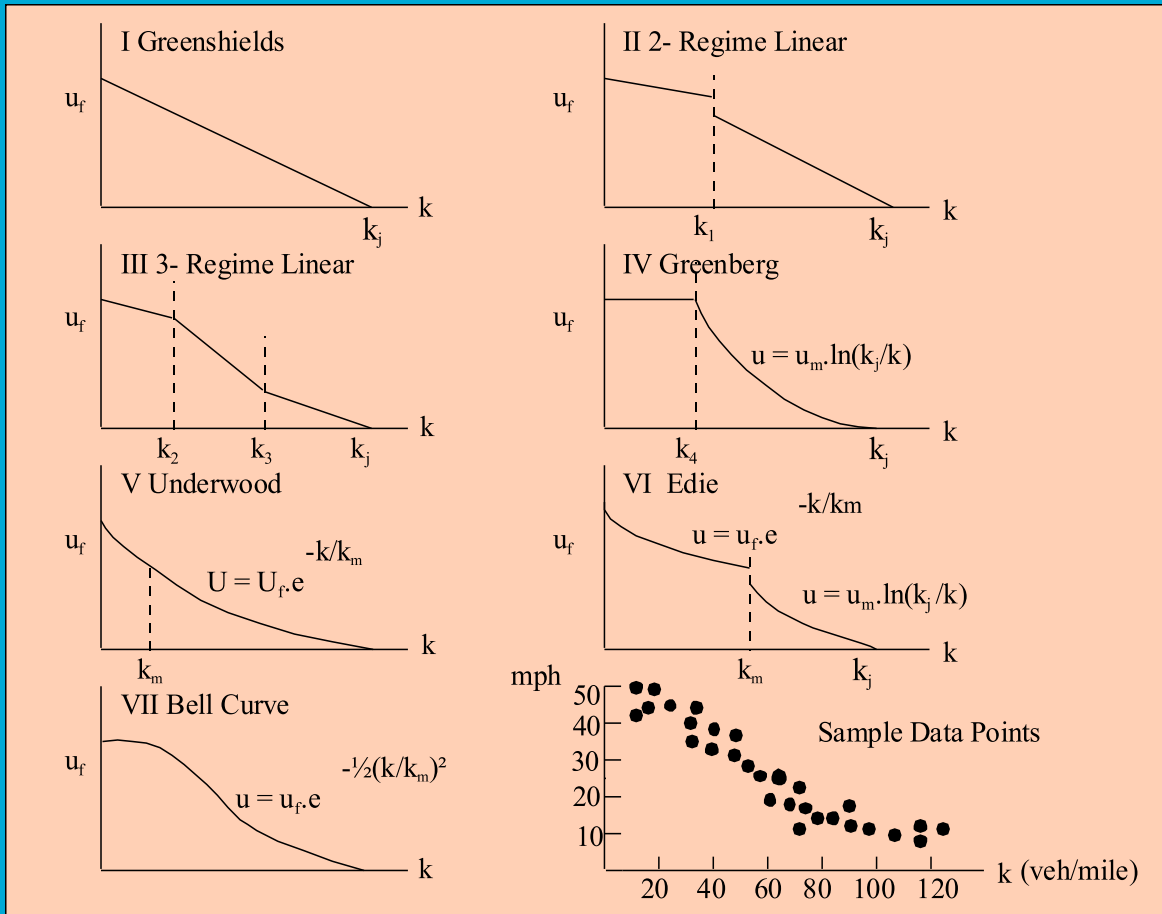
Which shocks emerge
Duration of disturbance?
Draw a couple trajectories
What if $q_1 = 2200$?

Studies of the fundamental diagram

- Need for complete diagram or only a part of it? Will it in general be possible to determine a complete diagram at a cross-section?
- Is the road section homogeneous? Yes: observations at a single cross-section. No: road characteristics are variable over the section and a method such as MO might be suitable
- Mind the period of analysis:
 - too short (1 minute): random fluctuations much influence;
 - too long (1 hour): stationarity cannot be guaranteed (mix different regimes)
- Estimate parameters of the model chosen using (non-linear) regression analysis



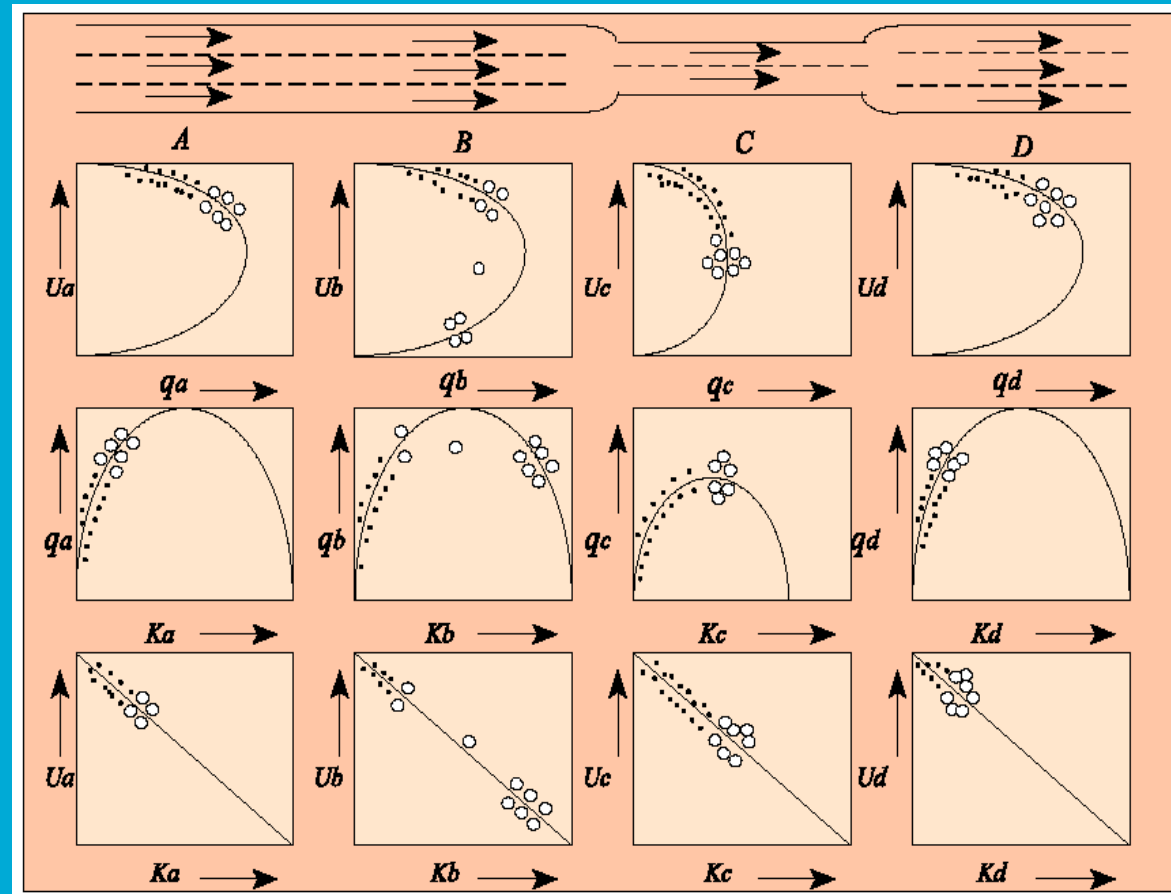
Studies of the fundamental diagram²



- Many models will fit your data
- Hints for choosing models?
- Simplest model possible (parsimony)
- Interpretation of parameters
- Theoretical considerations

Studies of the fundamental diagram³

- Demonstration FOSIM
- Fundamental diagram determined from real-life data, by assuming stationary periods
- Dependent on measurement location
- Flow per lane



Studies of the fundamental diagram⁴

Estimation of free flow capacity using fundamental diagram

- Approach 1:
 - Fit a model $q(k)$ to available data
 - Consider point $dq/dk = 0$
 - Generally not applicable to motorway traffic because $dq/dk = 0$ does not hold at capacity
- Approach 2:
 - Assume fixed value for the critical density k_c
 - Estimate only free-flow branch of the diagram
 - More for comparative analysis

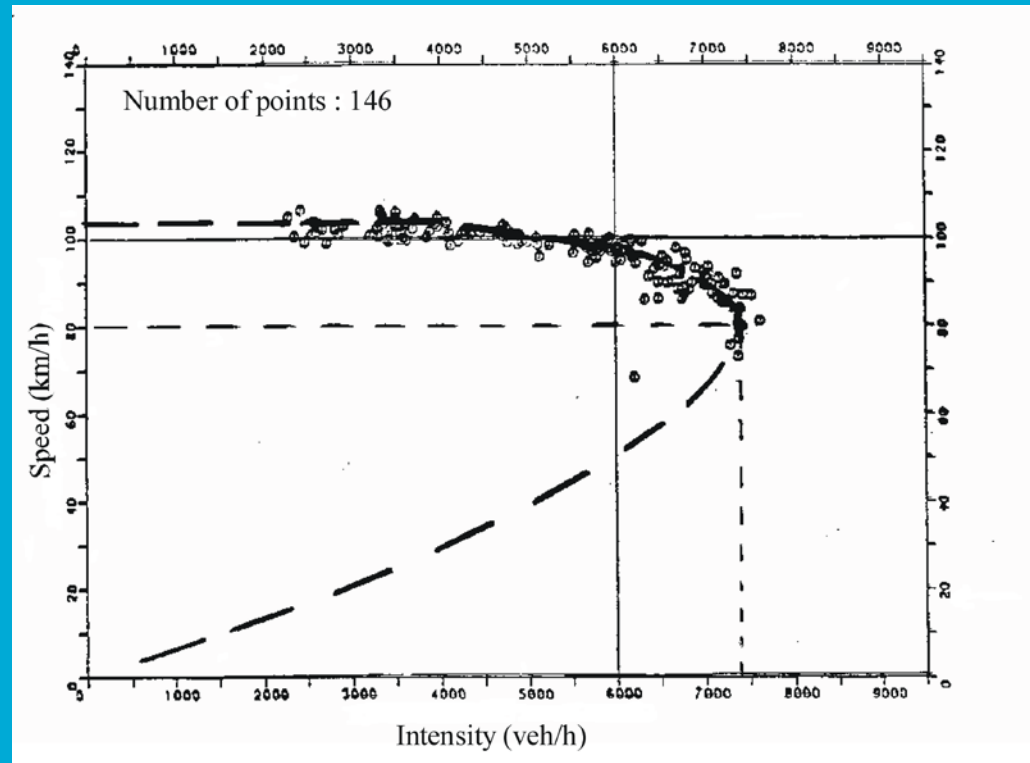
Studies of the fundamental diagram⁵

- Application example: effect of roadway lighting on capacity
- Two and three lane motorway
- Before – after study
 - Difficulties due to different conditions (not only ambient conditions change)
 - See e.g. site SB daylight before – after
 - Effect lighting on capacity approx 2.5% (2 lane) or 1.6% (3 lane)

	Daylight		Darkness	
	Before	After	Before	After
Site I NB Treatment	100	99.1	90.3	92.9
Site I SB Treatment	100	96.8	93.5	95.4
Site II Treatment	100	100.4	95.5	97.1
Site III Comparison	100	98.8	95.1	94.4

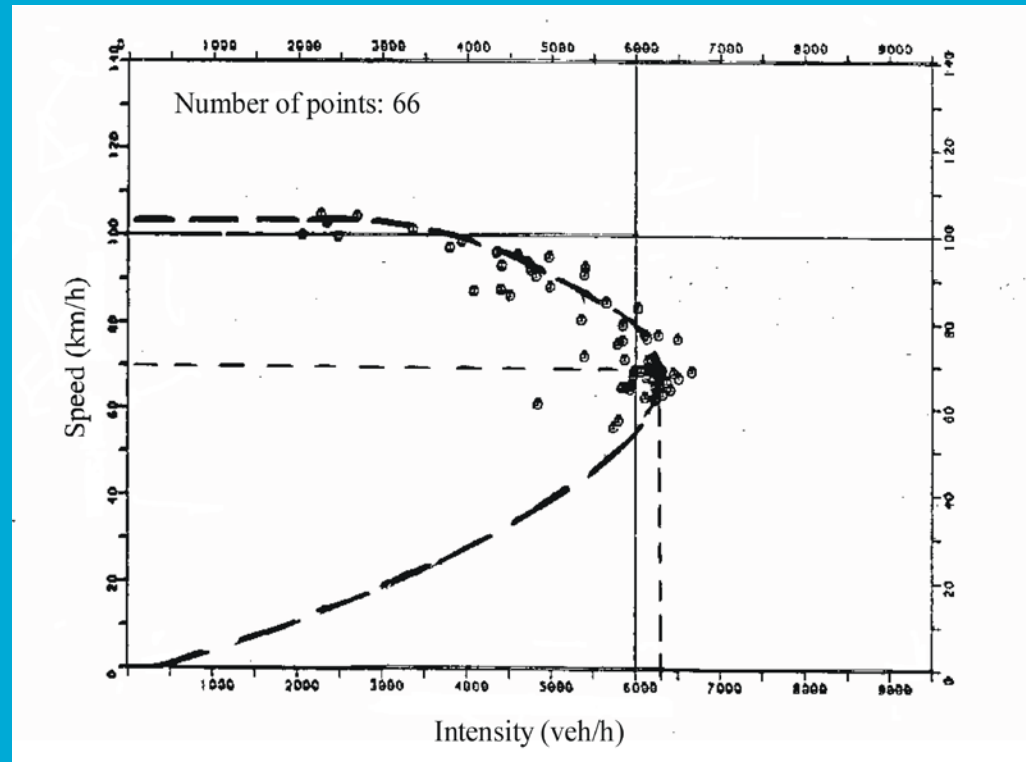
Studies of the fundamental diagram⁶

- Effect on rain on capacity / fundamental diagram



Studies of the fundamental diagram⁷

- Effect on rain on capacity / fundamental diagram



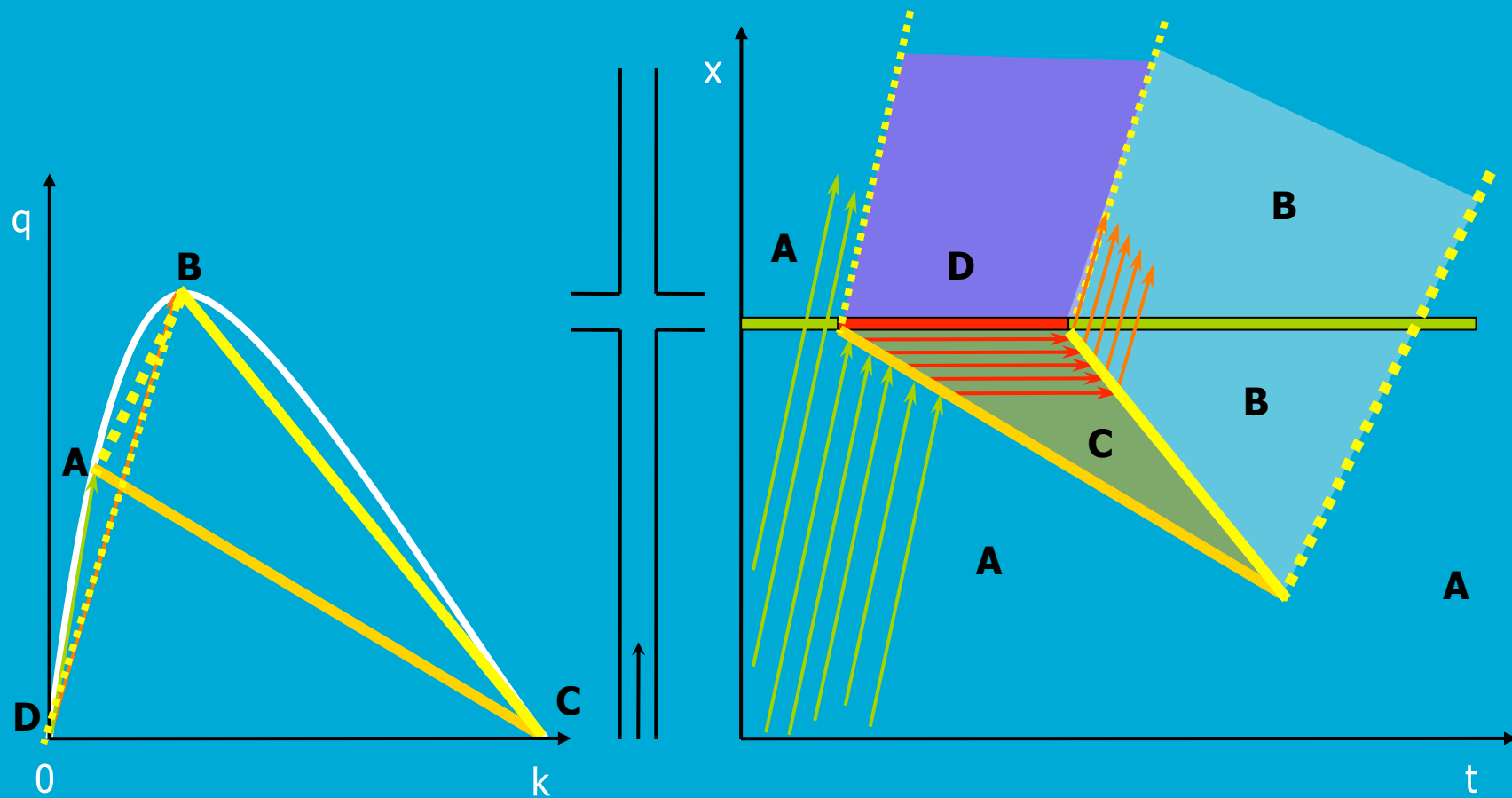
Studies of the fundamental diagram⁸

- Estimating queue discharge rate
- Only in case of observations of oversaturated bottleneck
- Three measurement locations (ideally)
 - Upstream of bottle-neck (does overloading occur?)
 - Downstream of bottle-neck (is traffic flow free?)
 - At the bottle-neck (intensities are capacity measurements if traffic state upstream is congested and the state downstream is free)
- Flow at all three points equal (stationary conditions) and at capacity; use downstream point

Summary of lecture

- Fundamental diagram for a lane and a cross-section
- Shockwave equations
- Application of shockwave analysis
 - shockwave at bottleneck
- Establishing a fundamental diagram from field observations

Shockwaves signalized intersections

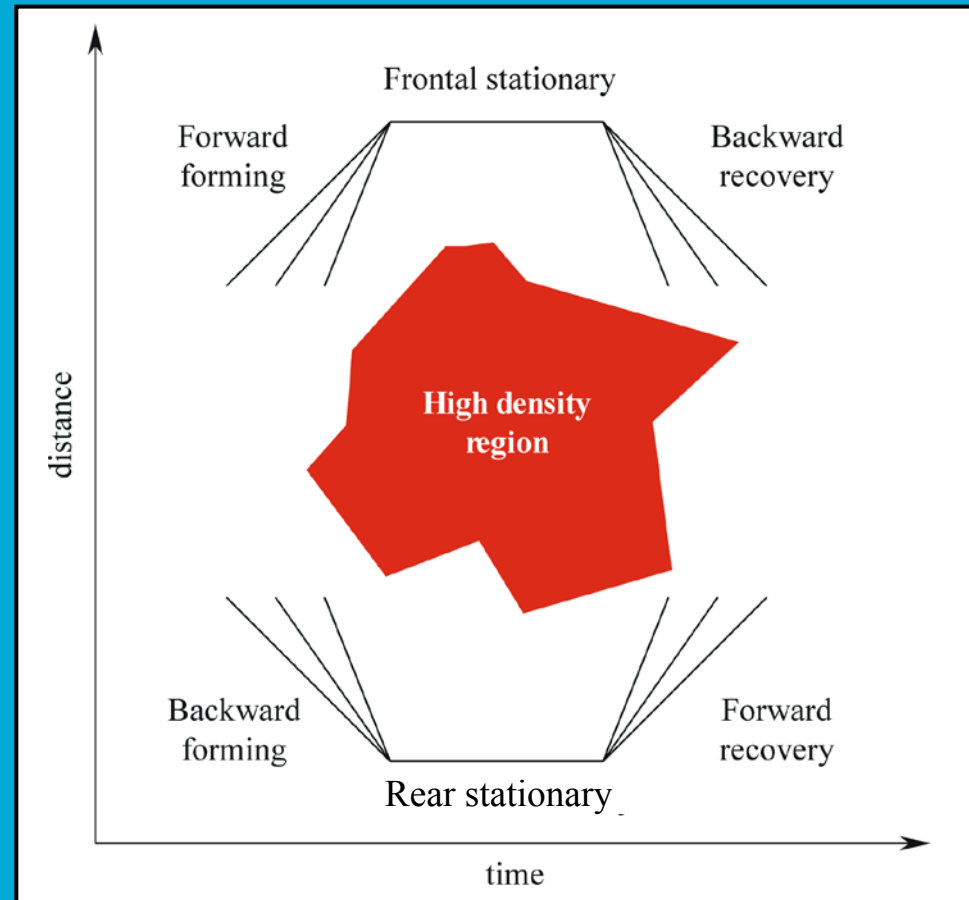


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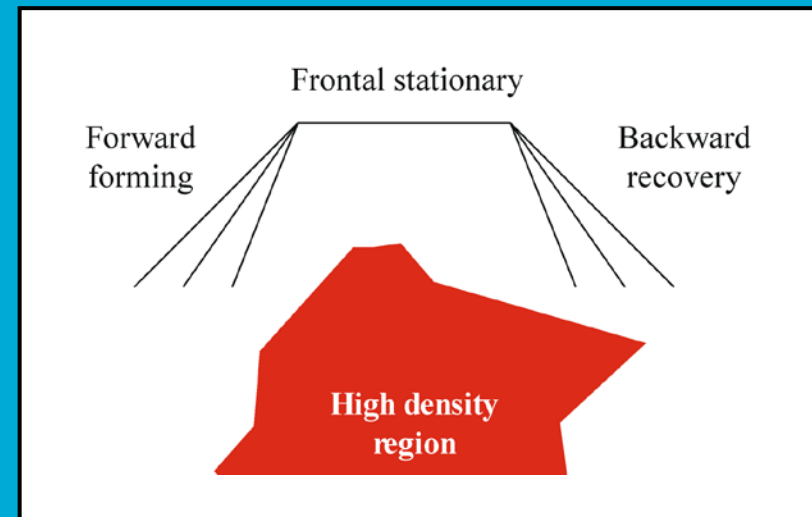
Shockwave classification

- 6 types of shockwaves
- Which situations do they represent?
Examples?



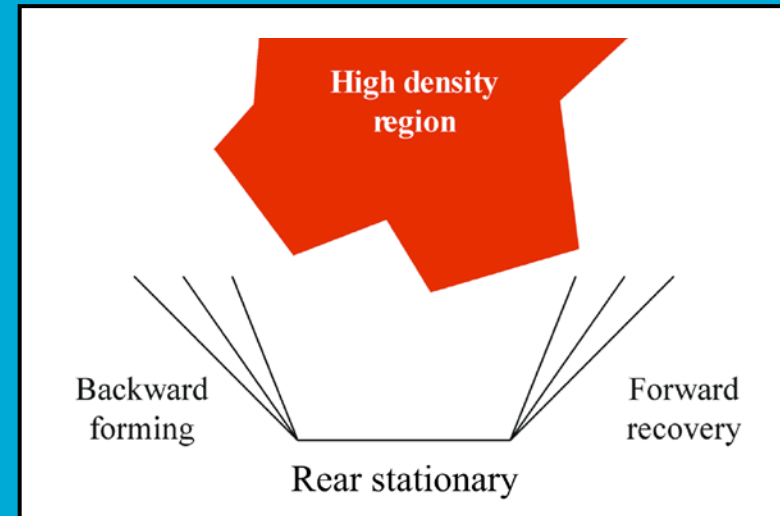
Shockwave classification²

1. Frontal stationary: head of a queue in case of stationary / temporary bottleneck
2. Forward forming: moving bottleneck (slow vehicle moving in direction of the flow given limited passing opportunities)
3. Backward recovery: dissolving queue in case of stationary or temporary bottleneck (demand l.t. supply); forming or dissolving queue for moving bottleneck



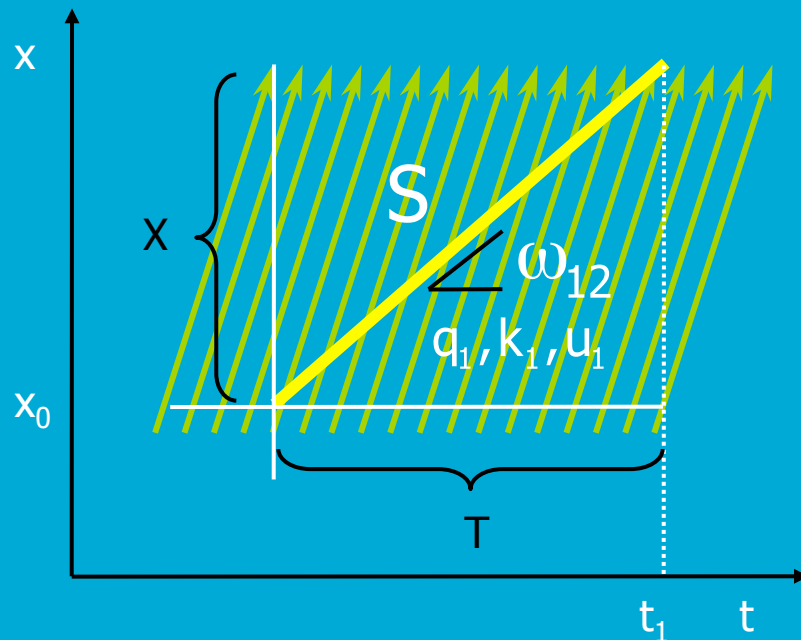
Shockwave classification³

1. Forward recovery: removal of temporary bottleneck (e.g. clearance of incident, opening of bridge, signalized intersection)
2. Backward forming: forming queue in case of stationary, temporary, or moving bottleneck* (demand g.t. supply);
3. Rear stationary: tail of queue in case recurrent congestion when demand is approximately equal to the supply



Flow into shockwave

- Consider a shockwave moving with speed ω_{12}
- Flow into the shockwave = flow observed by moving observer travelling with speed of shockwave



- Number of vehicles observed on S =
 - + Veh. passing x_0 during T
 - Vehicles on X at t_1

$$q_{in}^S T = q_1 T - k_1 X, \quad X = \omega_{12} T$$

$$q_{in}^S T = (k_1 u_1 - k_1 \omega_{12}) T$$

$$q_{in}^S = k_1 (u_1 - \omega_{12})$$