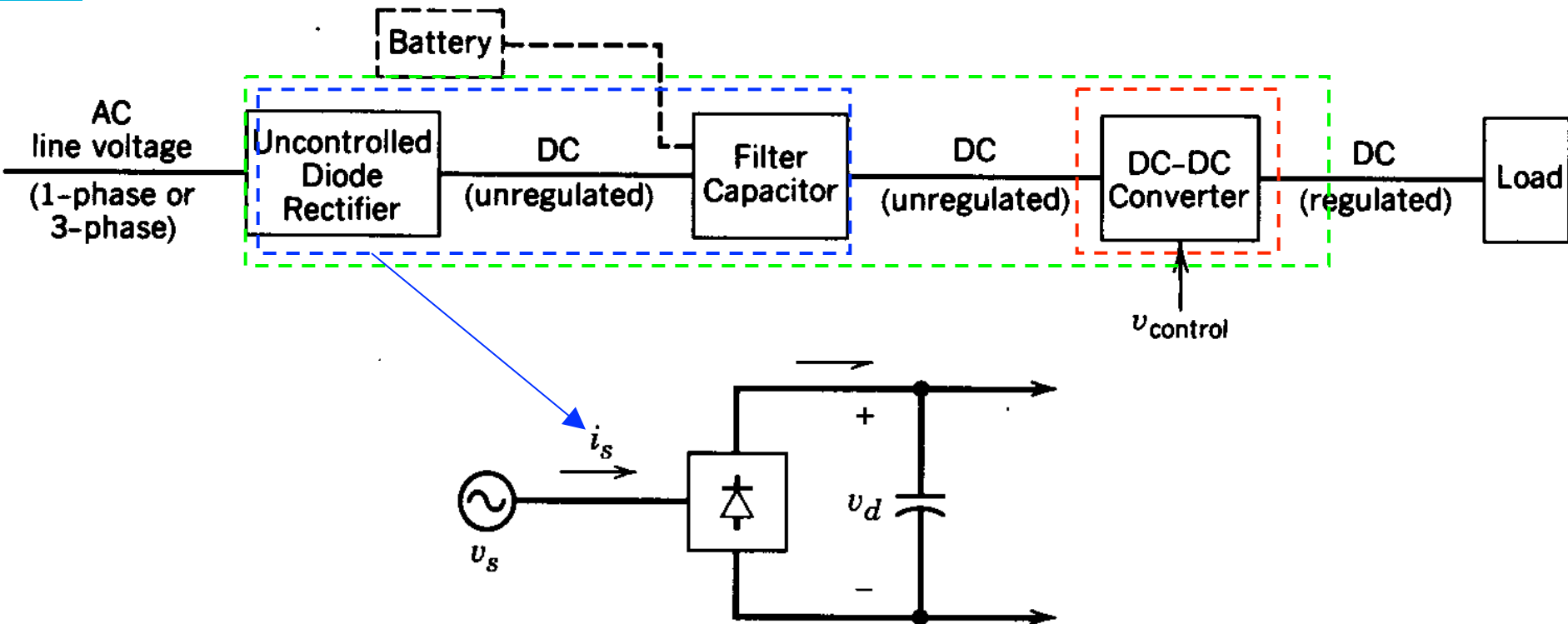


Electronic Power Conversion

DC-DC switch mode converters

7. DC-DC switch mode converters



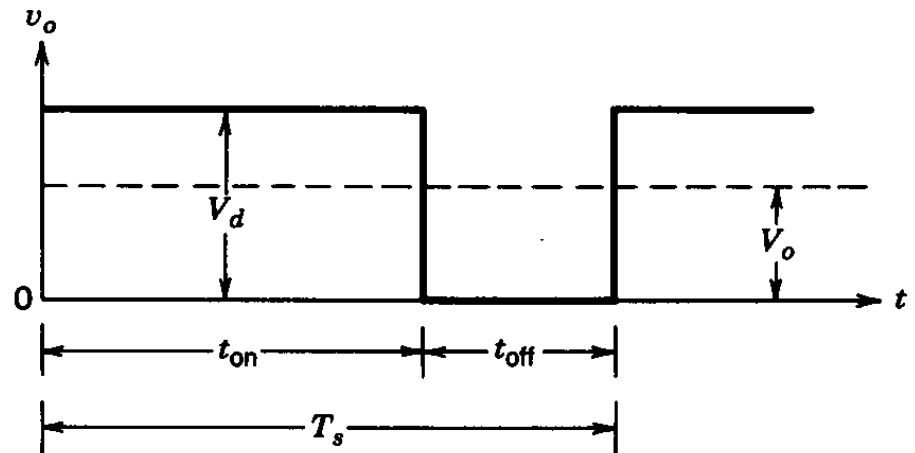
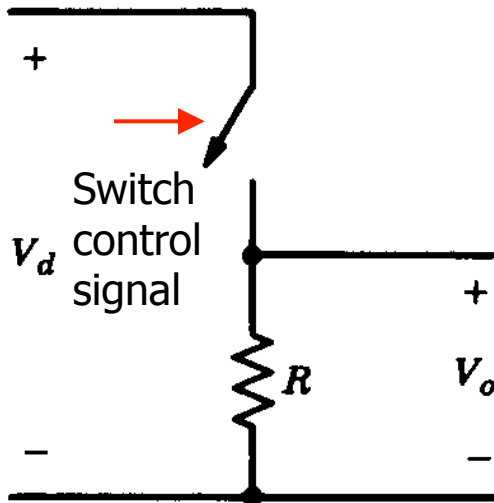
- Basic DC-DC converters
 - Step-down converter (*neerchopper*)
 - Step-up converter (*opchopper*)
- Derived circuits
 - Step-down/step-up converter (flyback)
 - (Ćuk-converter)
 - Full-bridge converter (*volle brug omzetter*)

Applications

- DC-motor drives
- SMPS

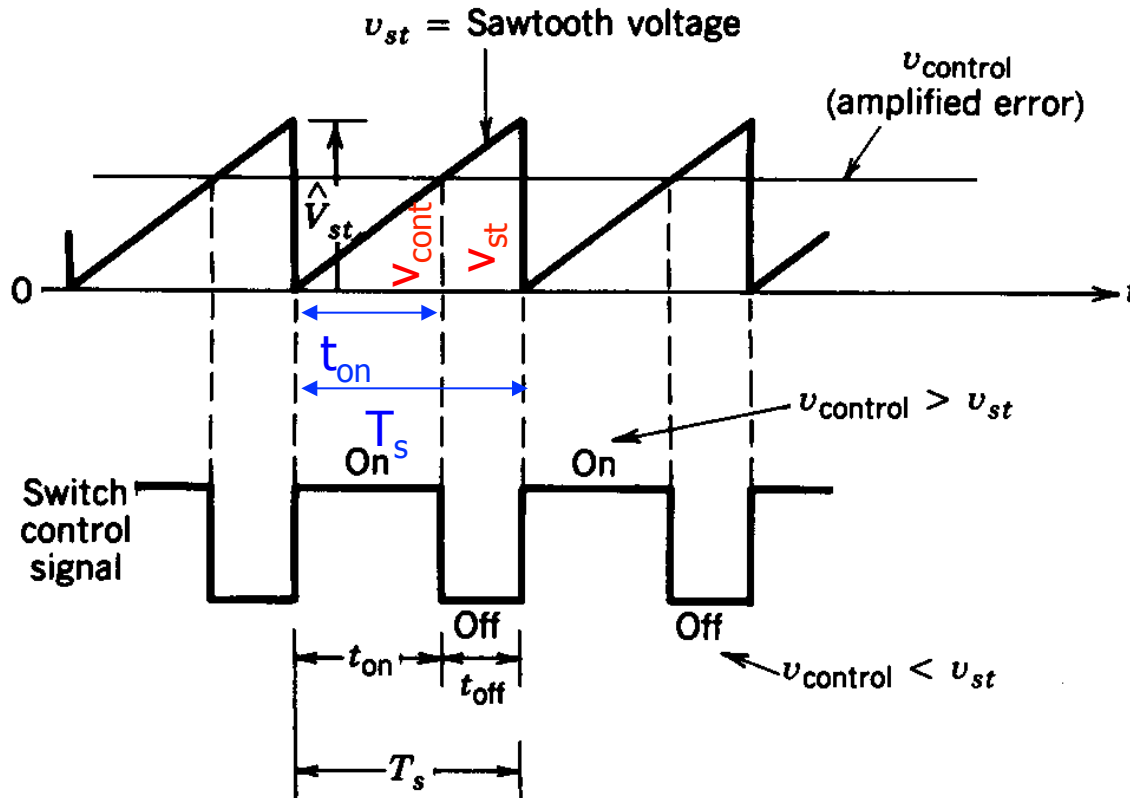
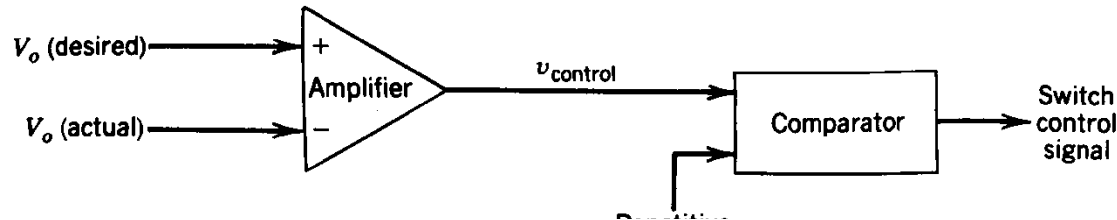
Control of dc-dc converters

- Output voltage has to be kept constant even though input voltage and output load may fluctuate
 - Controlled by switch on and off durations



$$V_o = \frac{1}{T_s} \int_0^{T_s} v_o(t) dt = \frac{1}{T_s} (t_{on} \cdot V_d + t_{off} \cdot 0) = \frac{t_{on}}{T_s} V_d$$

Control of dc-dc converters

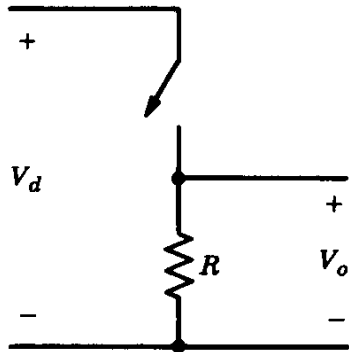


$$\frac{t_{on}}{T_s} = \frac{v_{control}}{\hat{V}_{st}}$$

$$D = \frac{t_{on}}{T_s}$$

(switching frequency $f_s = \frac{1}{T_s}$)

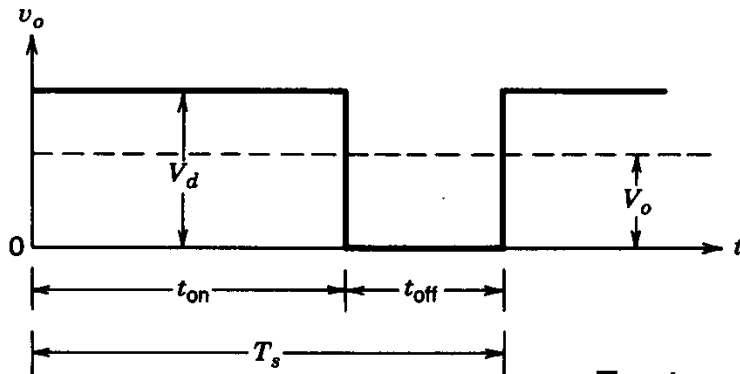
Step-down (Buck) converter



$$V_0 = \frac{1}{T_s} \int_0^{T_s} v_0(t) dt = \frac{1}{T_s} \left(\int_0^{DT_s} v_d(t) dt + \int_{DT_s}^{T_s} 0 dt \right)$$

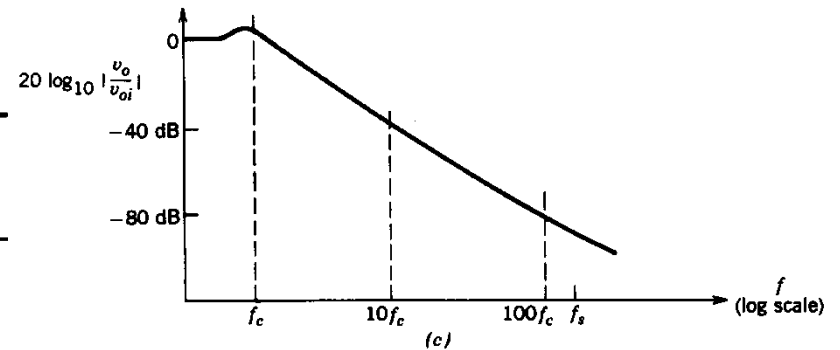
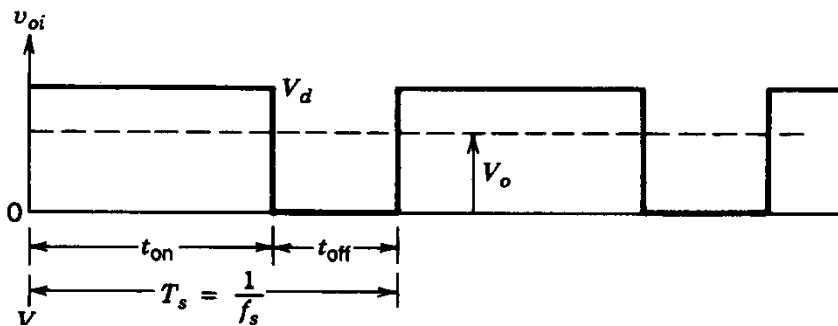
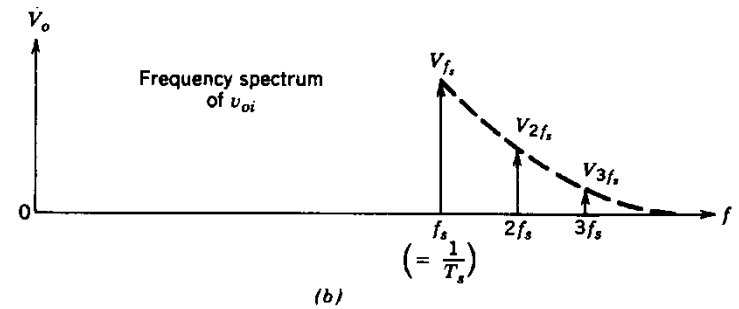
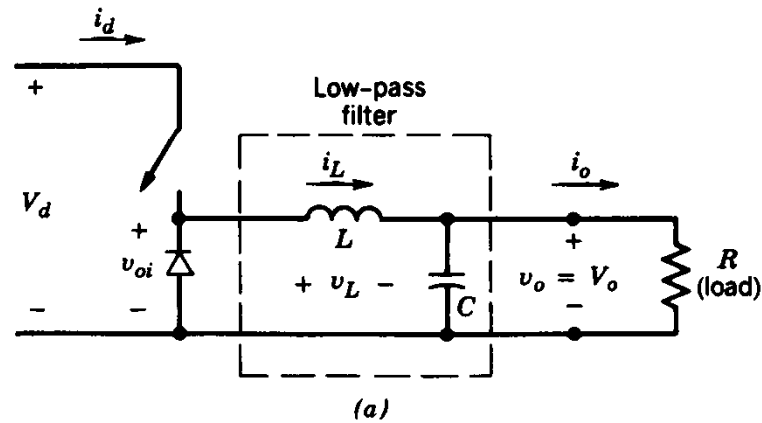
$$= \frac{t_{on}}{T_s} V_d = D V_d$$

$$V_0 = k v_{control}$$



Features of basic circuit:

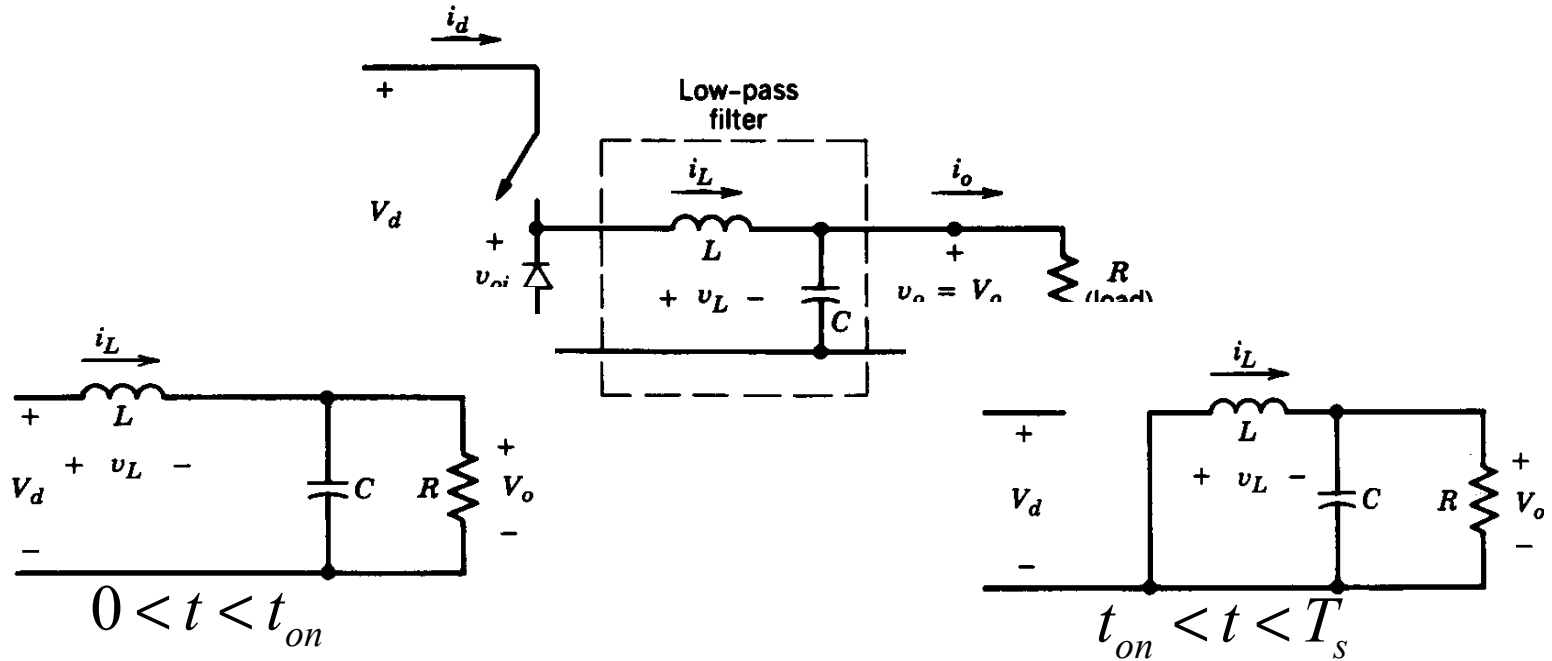
- switch stress when load is inductive
- fluctuating output voltage



Features of basic circuit:

- switch stress when load is inductive → diode
- fluctuating output voltage → filter used

Continuous conduction mode



$$v_L = V_d - V_o$$

$$i_L(t) = i_L(0) + \frac{1}{L} \int_0^t v_L(t) dt =$$

$$= i_L(0) + \frac{V_d - V_o}{L} t$$

$$v_L = V_o$$

$$i_L(t) = i_L(t_{on}) + \frac{1}{L} \int_{t_{on}}^t v_L(t) dt =$$

$$= i_L(t_{on}) + \frac{-V_o}{L} (t - t_{on})$$

$$0 < t < t_{on}$$

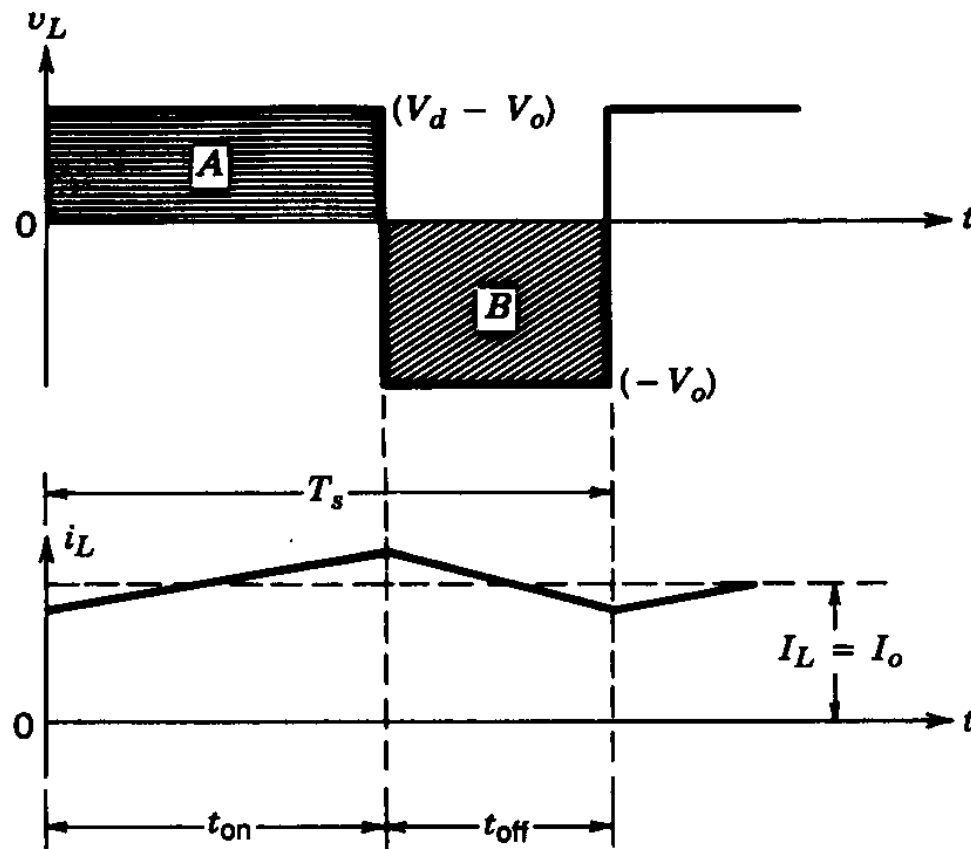
$$v_L = V_d - V_o$$

$$i_L(t) = i_L(0) + \frac{V_d - V_o}{L} t$$

$$t_{on} < t < T_s$$

$$v_L = -V_o$$

$$i_L(t) = i_L(t_{on}) + \frac{-V_o}{L} (t - t_{on})$$



$$0 < t < t_{on}$$

$$v_L = V_d - V_o$$

$$t_{on} < t < T_s$$

$$v_L = -V_o$$

$$v_{L,av} = 0 \quad \rightarrow \quad (V_d - V_o)t_{on} = V_o(T_s - T_{on})$$

$$(V_d - V_o) \cdot t_{on} + (-V_o) \cdot t_{off} = 0 \quad / T_s$$

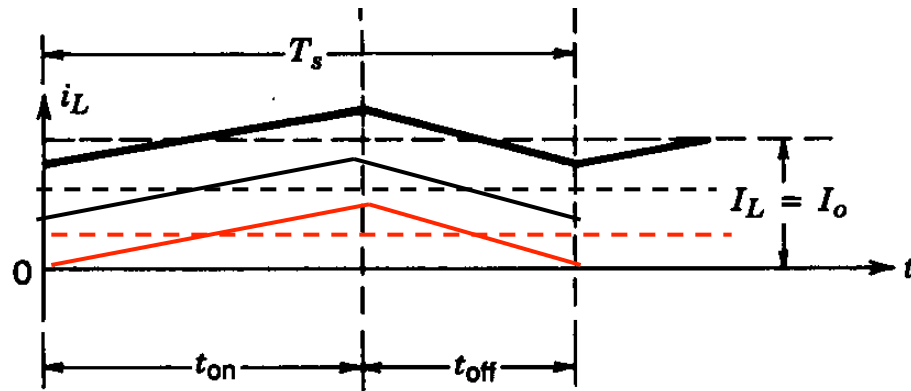
$$V_o = D \cdot V_d$$

$$P_{out} = P_{in}$$

$$V_d I_d = V_o I_o$$

$$\frac{I_o}{I_d} = \frac{V_d}{V_o} = \frac{1}{D}$$

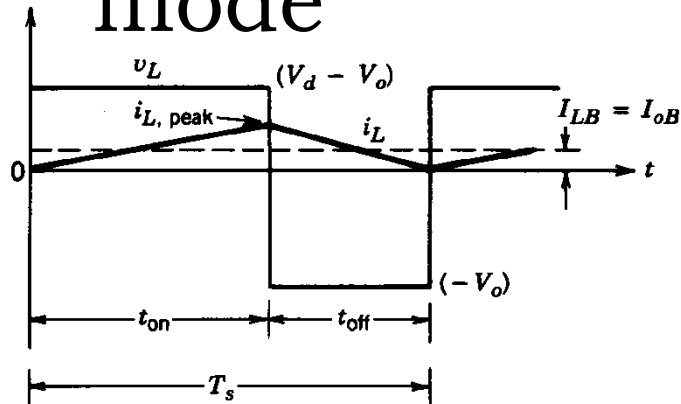
Continuous and discontinuous mode



$$R \uparrow \rightarrow I_o \downarrow$$

Boundary between continuous and discontinuous mode

Boundary continuous- discontinuous mode



On boundary B:

$$I_{LB} = \frac{1}{2} i_{L, peak} \quad \text{so} \quad I_{OB} = \frac{t_{on}}{2L} (V_d - V_o) \quad (a)$$

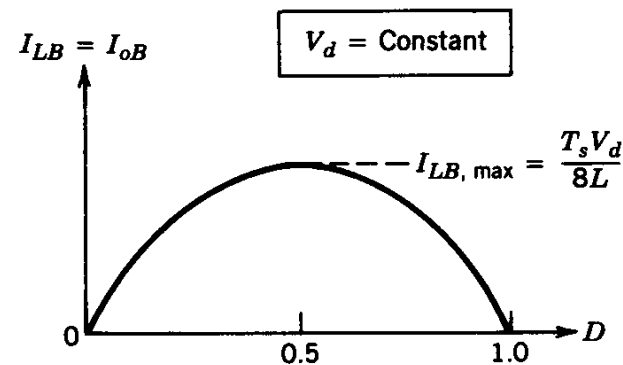
$$= \frac{DT_s}{2L} (V_d - V_o)$$

$$(b) \quad \frac{V_o}{V_d} = D$$

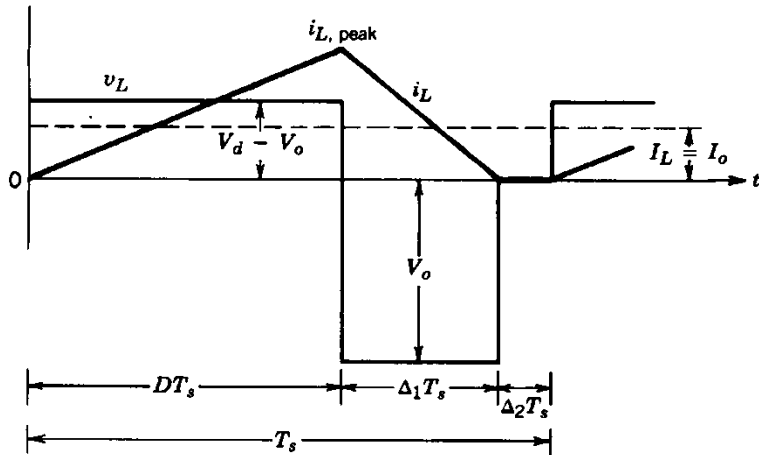
Eliminate either V_o or V_d from (a) and (b) depending on what is constant.

Assume V_d constant:

$$\text{Boundary: } I_{OB} = \frac{T_s V_d}{2L} D(1 - D)$$



Discontinuous conduction mode



CCM: $V_0 = D V_d$

DCM: $V_0 = f(V_d, D, I_0)$

$$V_{L,av} = 0 \quad \Rightarrow \quad (V_d - V_0)DT_s - V_0 \Delta_1 T_s = 0 \quad \Rightarrow \quad \frac{V_0}{V_d} = \frac{D}{D + \Delta_1} \quad (1)$$

Δ_1 ? There is a relationship between Δ_1 and I_0 :

$$I_0 = i_{L,peak} \frac{D + \Delta_1}{2} \quad (2) \quad \text{with} \quad i_{L,peak} = \frac{V_0 \Delta_1 T_s}{L} \quad (3) \quad \Rightarrow \quad I_0 = \frac{V_0 \Delta_1 T_s}{L} \frac{(D + \Delta_1)}{2} \quad (4)$$

Eliminate Δ_1 from (1) & (4) \Rightarrow

$$\frac{V_0}{V_d} = \frac{D^2}{D^2 + \frac{2L}{T_s V_d} I_0}$$

or

$$\frac{V_0}{V_d} = \frac{D^2}{D^2 + \frac{1}{4} \left(I_0 / I_{LB,max} \right)}$$

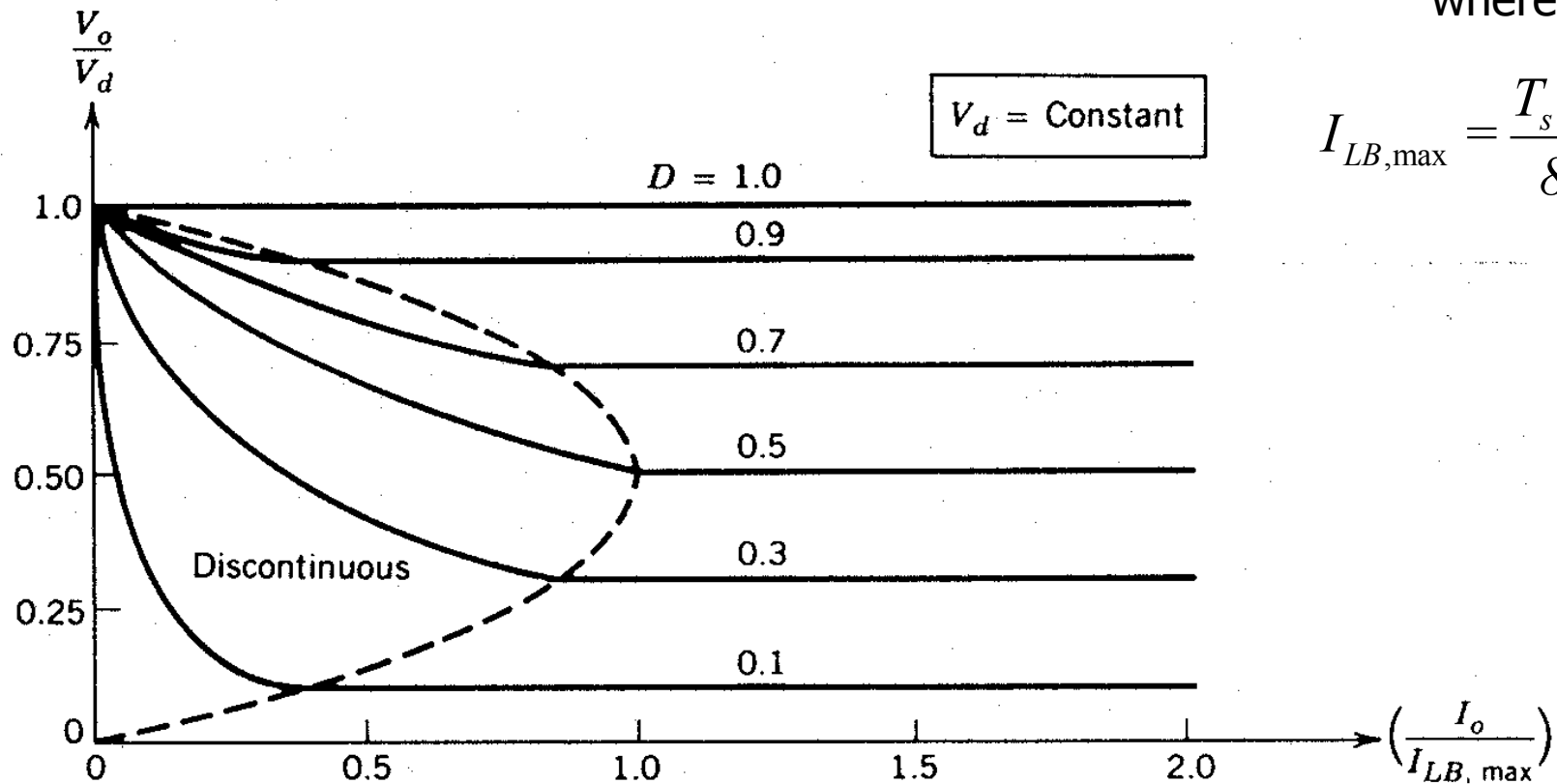
$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{2L}{T_s V_d} I_o}$$

or

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I}{4} (I_o / I_{LB,max})}$$

where:

$$I_{LB,max} = \frac{T_s V_d}{8L}$$



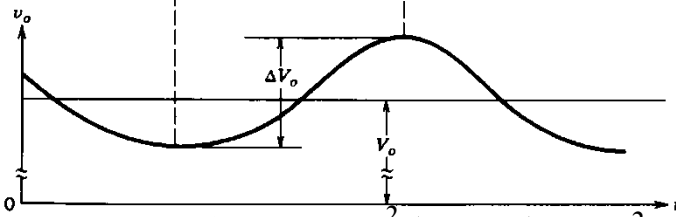
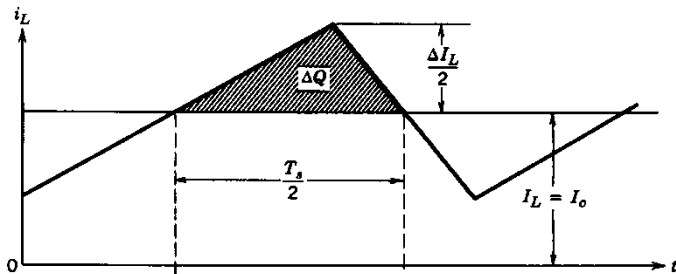
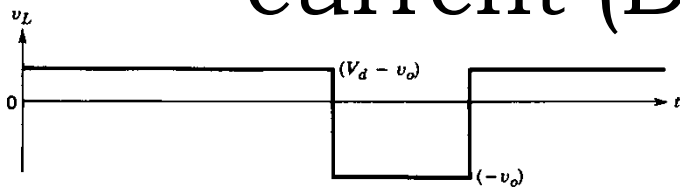
$$I_{LB,max} = \frac{T_s V_d}{8L}$$

$D(V_d, I_o)$ for V_o constant

Boundary: $I_{LB} = \frac{T_s V_o}{2L} (1 - D)$ (5) $I_{LB, \max} = \frac{T_s V_o}{2L}$

(1) - (4) still hold $D = \frac{V_o}{V_d} \left(\frac{I_o / I_{LB, \max}}{1 - V_o / V_d} \right)^{1/2}$

Output voltage ripple and capacitor current (Down conv.)



or:
$$\frac{\Delta V_0}{V_0} = \frac{T_s^2 (1-D)}{8LC} = \frac{\pi^2}{2} (1-D) \left(\frac{f_c}{f_s} \right)^2$$

$$\Delta V_0 = \frac{\Delta Q}{C} = \frac{I}{C} \cdot \frac{1}{2} \text{ base} \cdot \text{height}$$

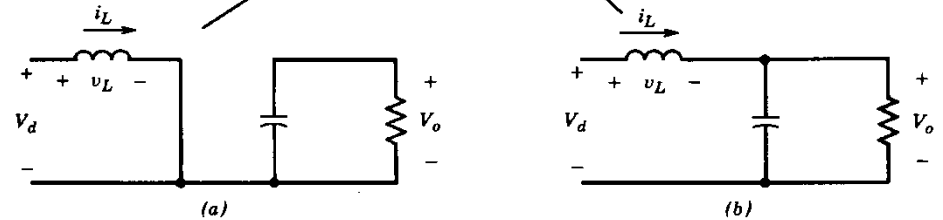
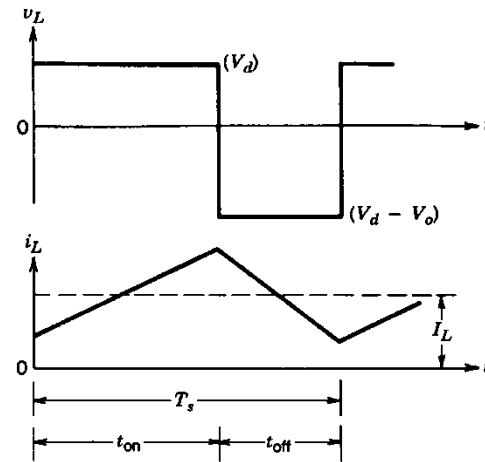
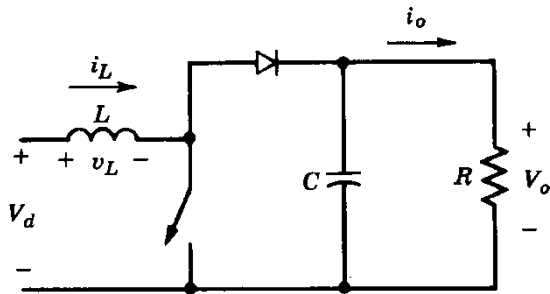
2.Height:
$$\Delta I_L = \frac{V_0}{L} (1-D) T_s$$

→
$$\Delta V_0 = \frac{1}{C} \cdot \frac{1}{2} \cdot \frac{T_s}{2} \cdot \frac{V_0}{2L} (1-D) T_s$$

with:
$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

- Only valid for continuous conduction mode (CCM)

Step-up (or boost) converter

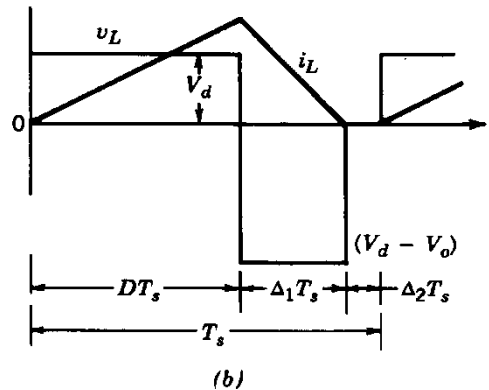
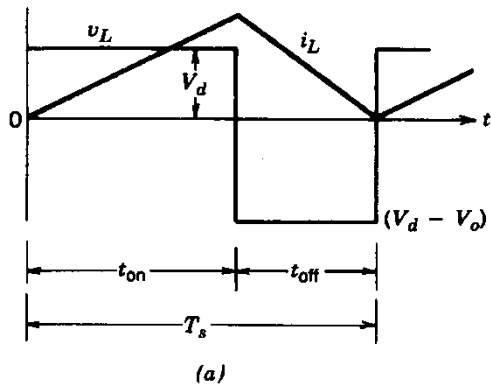


$$V_{L,av} = 0 \quad \rightarrow \quad V_d D T_s + (V_d - V_o) (1 - D) T_s = 0$$

$$\frac{V_o}{V_d} = \frac{1}{1 - D}$$

Boost converter in DCM

same procedure as down converter



CCM: $V_0 = \frac{V_d}{1-D}$

DCM: $V_0 = f(V_d, D, I_d)$

From the time waveforms:

$$V_{L,av} = 0 \quad \rightarrow \quad V_d D T_s + (V_d - V_0) \Delta_1 T_s = 0 \quad \rightarrow \quad \frac{V_0}{V_d} = \frac{\Delta_1 + D}{\Delta_1} \quad (1)$$

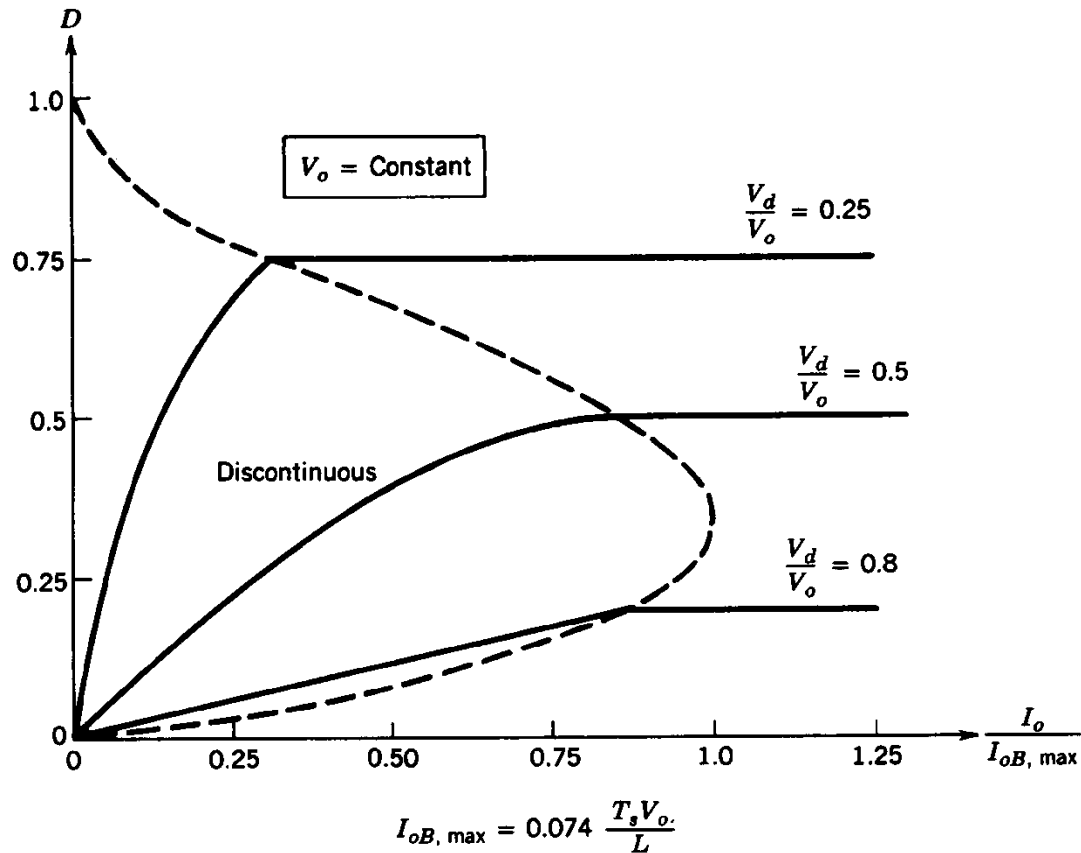
$$\hat{I}_L = f(V_d, D) \quad (2)$$

$$I_{0,av} = f(\hat{I}_L, D, \Delta_1) \quad (3)$$

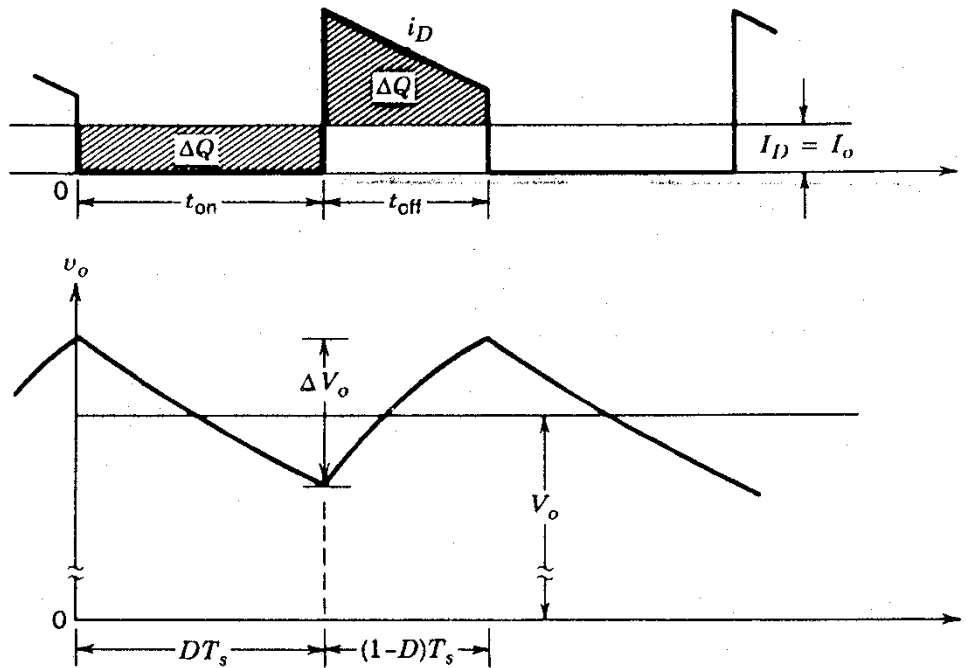
Eliminate \hat{I}_L , D and Δ_1 from (1), (2) and (3)

$V_o = \text{constant}$

Solution not important, however here is the plot:

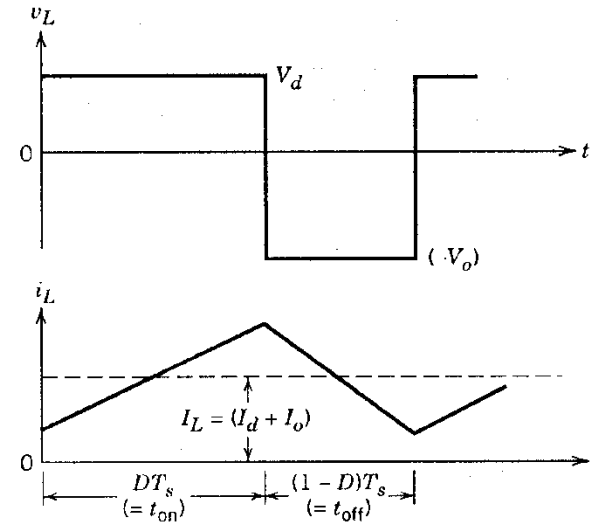
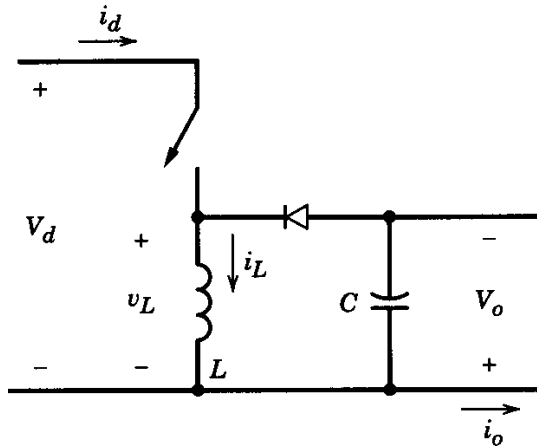


Boost converter - output voltage ripple and capacitor currents



$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o DT_s}{C} = \frac{V_o}{R} \frac{DT_s}{C}$$

Buck-boost converter



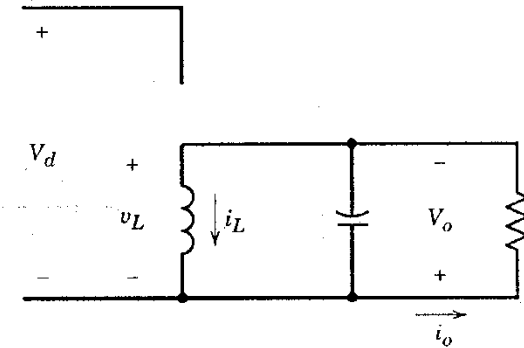
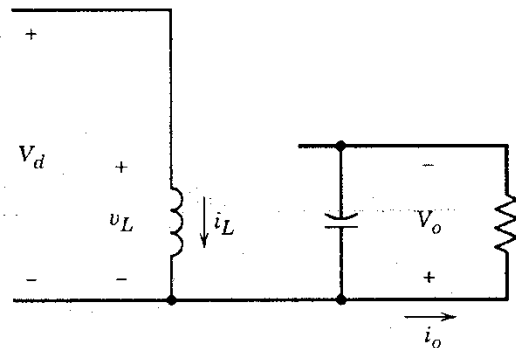
$$V_{L,av} = 0$$

$$V_d D T_s + (-V_o) (1-D) T_s = 0$$

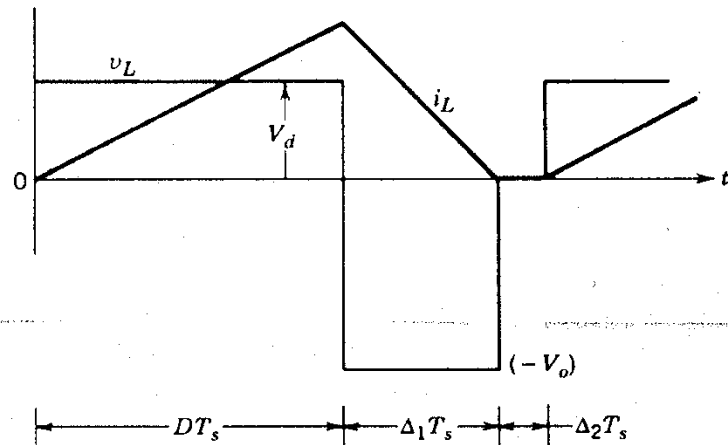
$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

and

$$\frac{I_o}{I_d} = \frac{1-D}{D}$$



Discontinuous conduction mode (DCM)



Method: find several relations:

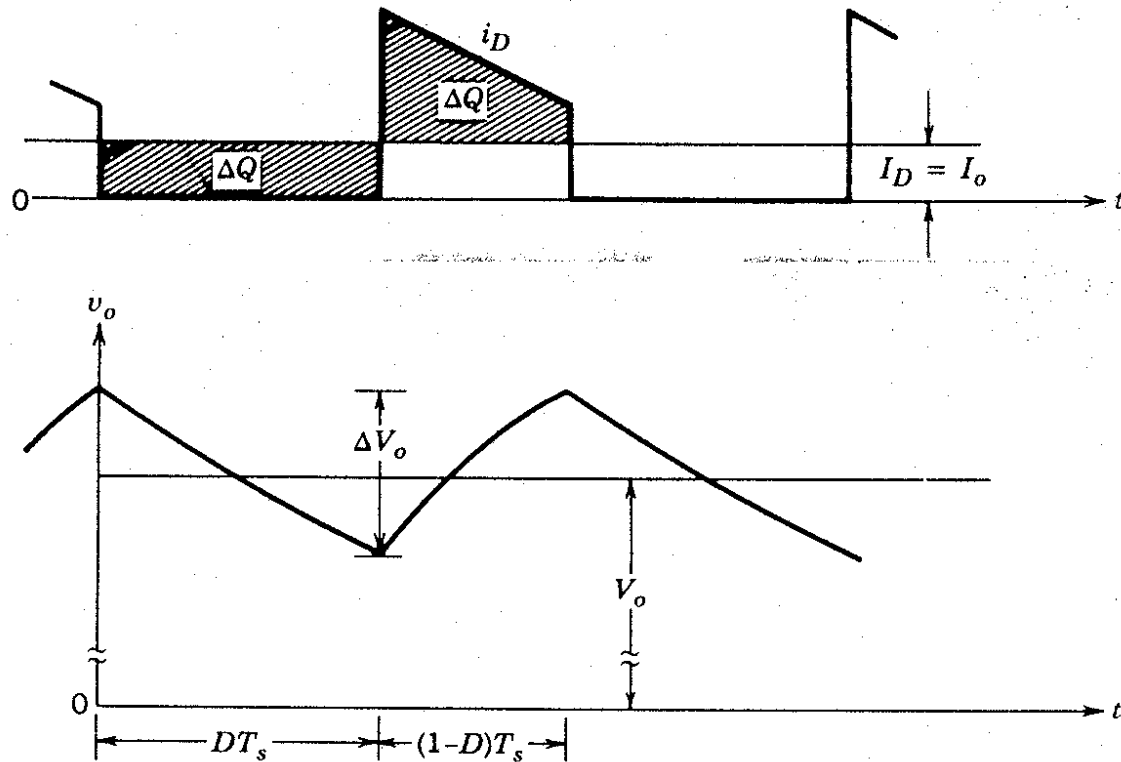
$$V_{L,av} = 0 \quad \rightarrow \quad V_d D T_s + (-V_o) \Delta_1 T_s = 0 \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D}{\Delta_1} \quad (1)$$

$$\hat{I}_L = f(V_d, D) \quad (2)$$

$$I_{0,av} = f(\hat{I}_L, D, \Delta_1) \quad (3)$$

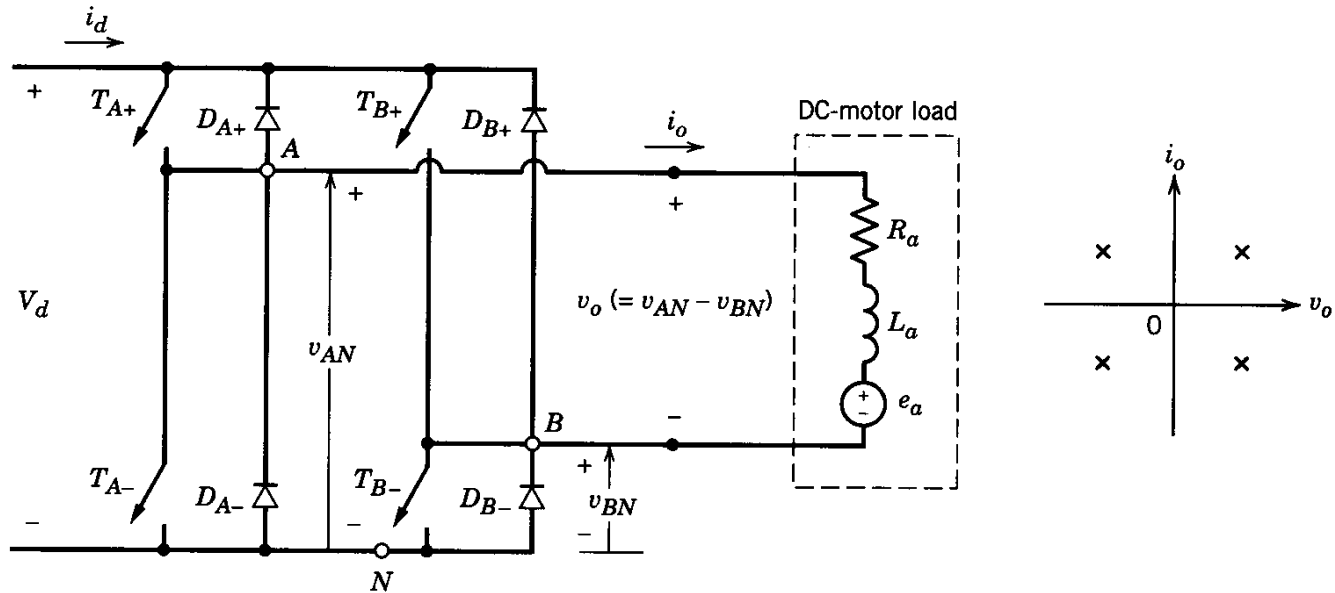
Eliminate \hat{I}_L , D and Δ_1 from (1), (2) and (3)

Voltage ripple and capacitor current

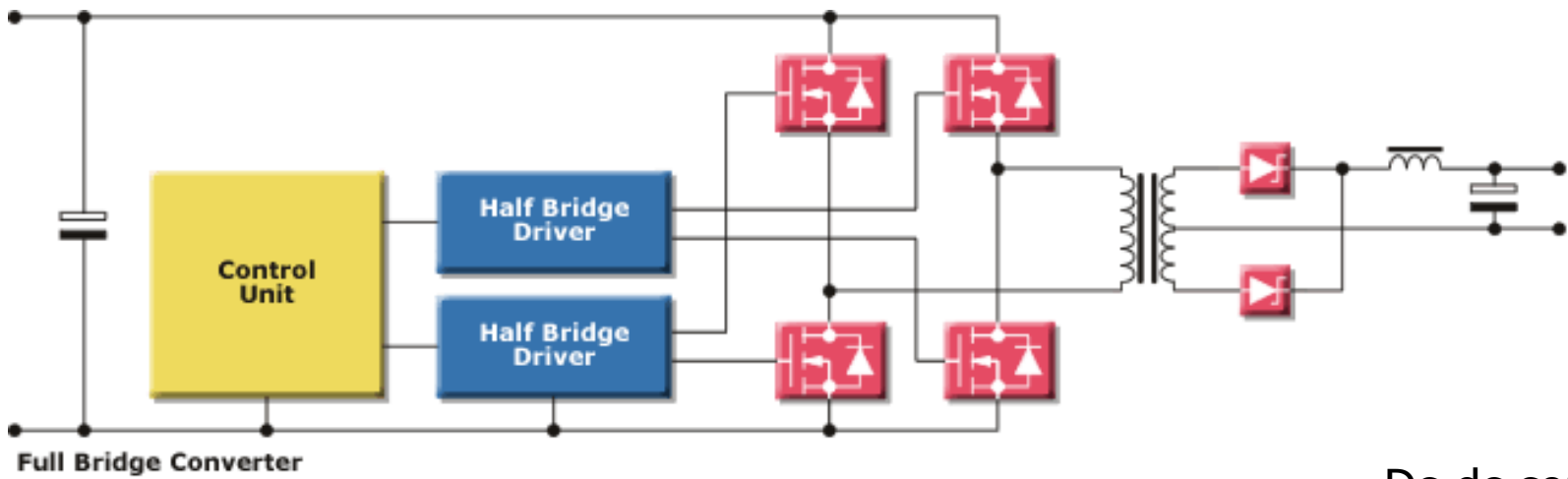


Calculate voltage ripple $\Delta V_o = ?$

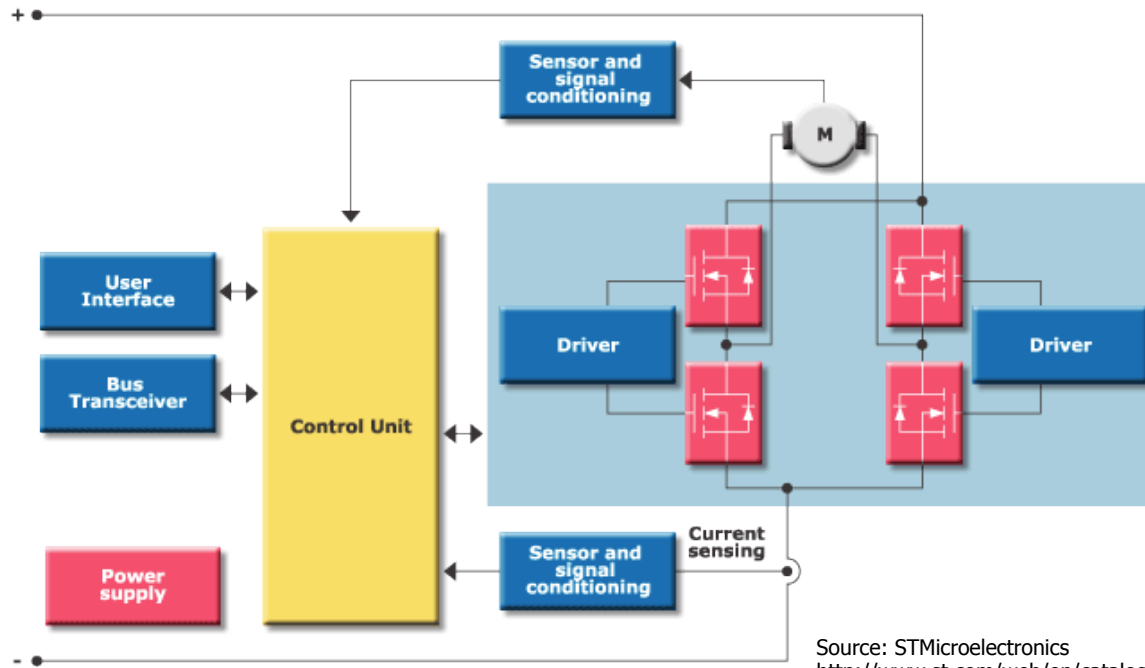
Full-bridge dc-dc converter



- Either + or - switch is on so output voltage is solely determined by status of the switch-gate signal (not by the current polarity)
- Current polarity determines whether T or D is conducting
- Applications:
 - 4-quadrant dc motor drives
 - dc-ac sine wave conversion (Chapter 8)

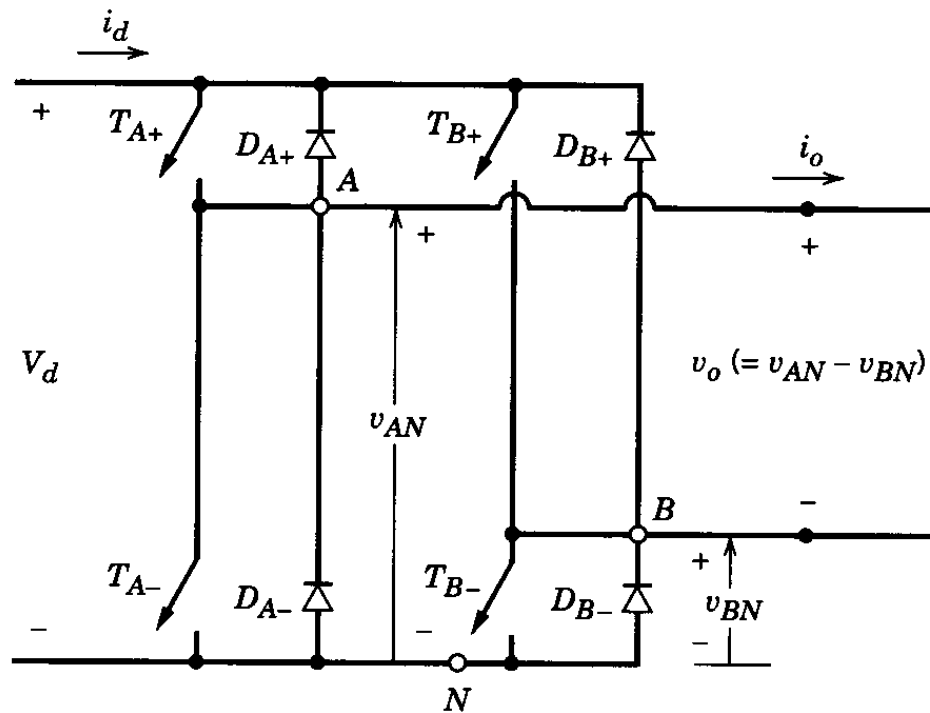


Dc-dc converter



Dc motor drives

Source: STMicroelectronics
<http://www.st.com/web/en/catalog/apps/SE413/AS214/AC899>



$$V_{AN} = V_d D_{T_{A+}} \quad D_{T_{A-}} = 1 - D_{T_{A+}} \quad \rightarrow \quad V_{BN} = V_d D_{T_{B+}}$$

Two PWM control switching strategies

- PWM bipolar switching: simultaneous synchronized operation of the phase legs
- PWM unipolar switching: independent operation of the phase legs

Bipolar voltage switching

- single control signal $v_{control}$

$$v_{control} > v_{tri} \quad T_{A+}, T_{B-}$$

$$v_{control} < v_{tri} \quad T_{A-}, T_{B+}$$

$$V_{AN} = D_1 V_d \quad V_{BN} = (1 - D_1) V_d$$

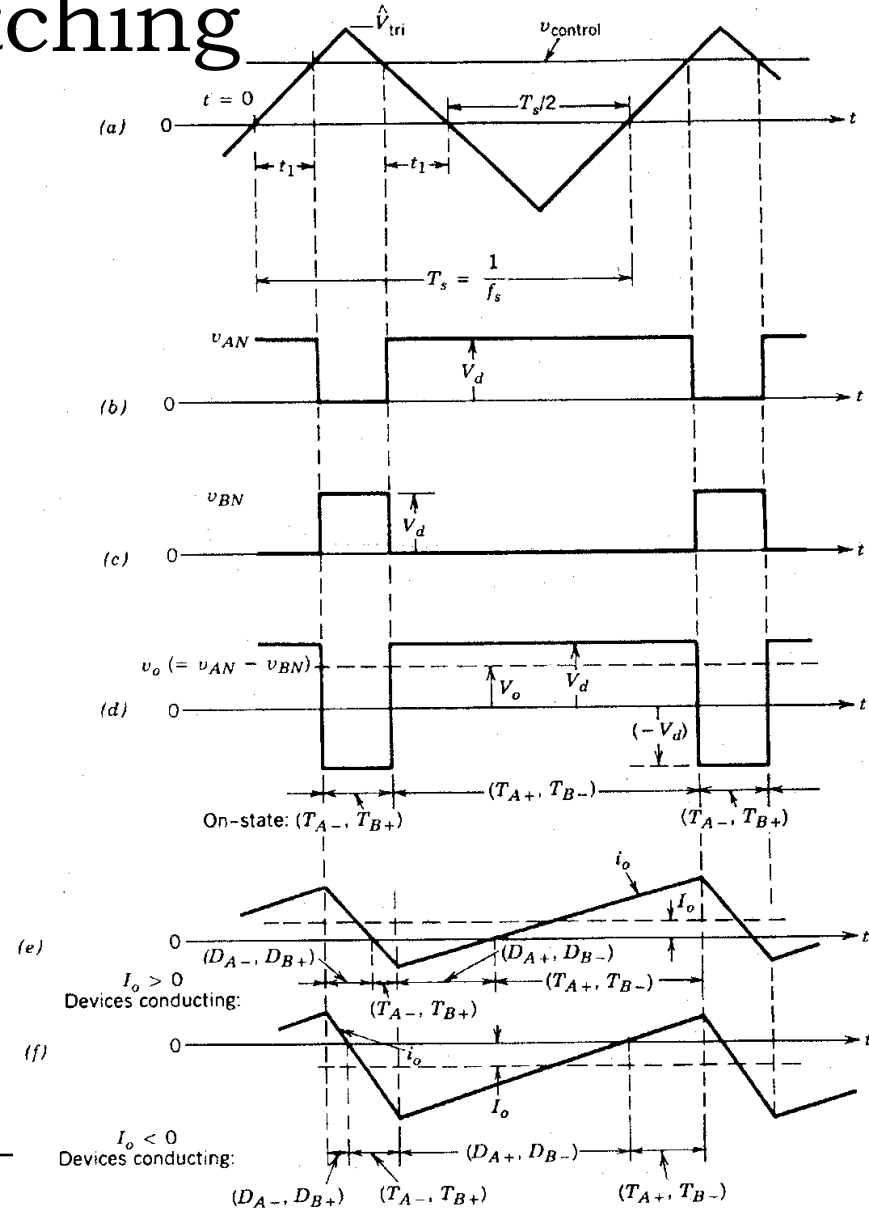
$$V_0 = V_{AN} - V_{BN} = (2D_1 - 1) V_d$$

$$V_0 = \frac{v_{control}}{\hat{V}_{tri}} V_d = k v_{control}$$

with $k = \frac{V_d}{\hat{V}_{tri}}$

4 quadrant operation:

All combinations of polarities of (average) current and voltage possible



Unipolar voltage switching

- double control signal $+v_{control}$ and $-v_{control}$

$$v_{control} > v_{tri} \quad T_{A+}$$

$$-v_{control} > v_{tri} \quad T_{B+}$$

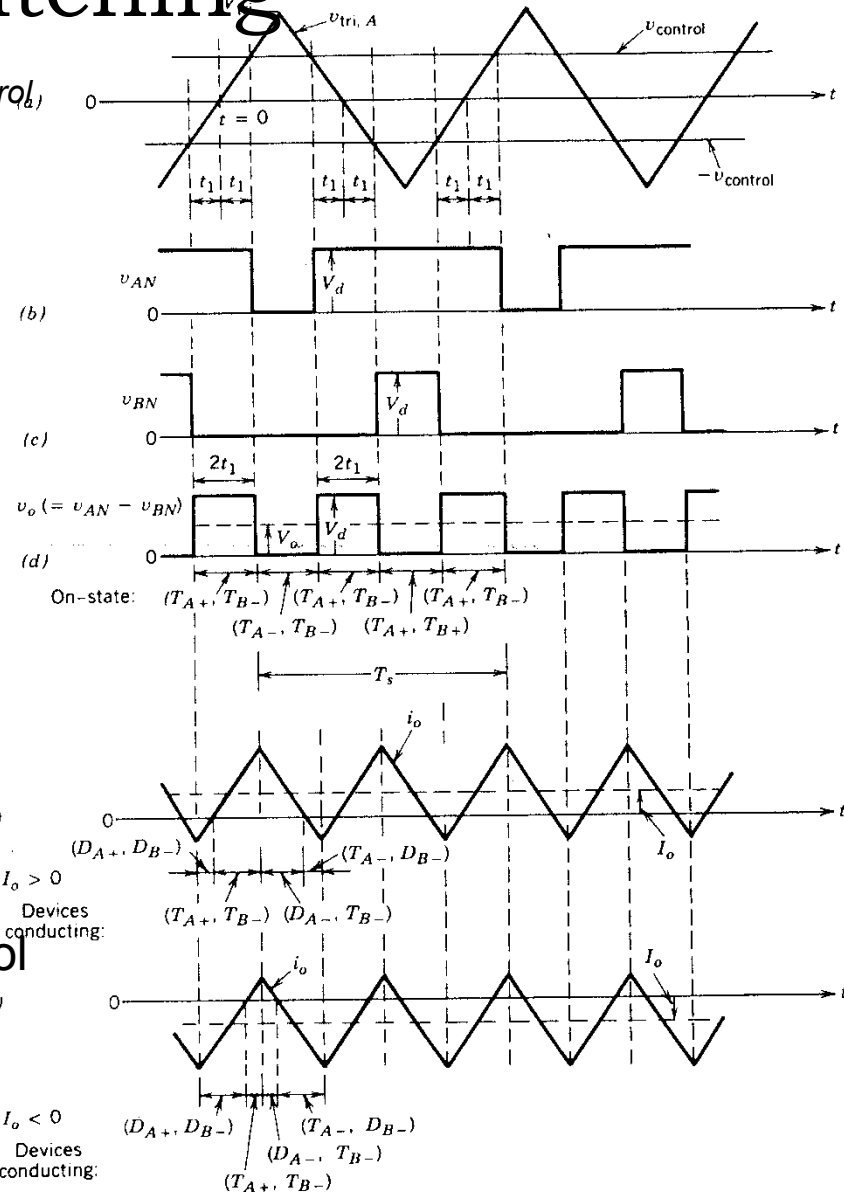
$$V_0 = V_{AN} - V_{BN} = (2D_1 - 1) V_d$$

$$V_0 = \frac{v_{control}}{\hat{V}_{tri}} V_d = k v_{control}$$

with $k = \frac{V_d}{\hat{V}_{tri}} = \text{constant}$

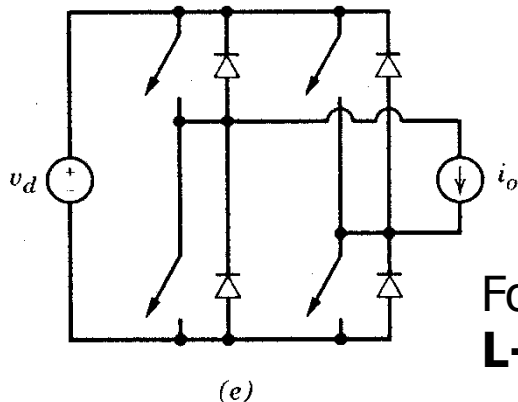
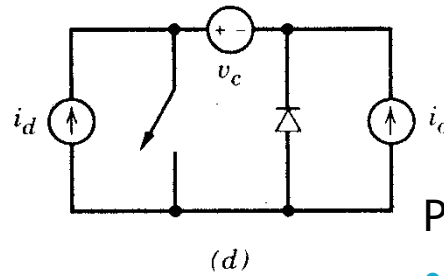
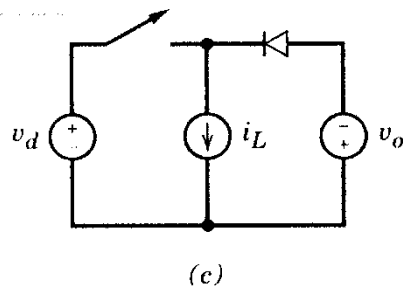
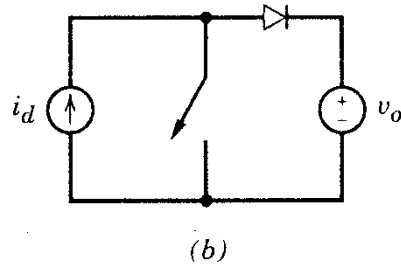
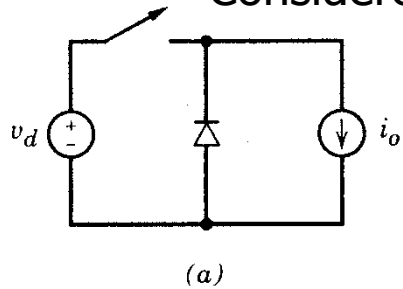
Comparison unipolar/bipolar:

- ripple in i_o at $2f_s$
- smaller ripple current for the same L
- slightly more complex control; 2 control signals required



Summary

Considered circuits



Assumptions for analysis:

- all components ideal (no voltage drop on switches, infinitely fast)
- output capacitor voltage assumed to be constant → voltage source
- output inductor current assumed to be constant → current source

Phase arm:

- uni- and bipolar switching
- control signal uniquely determines $v_{AN}(t)$ and $v_{BN}(t)$
- current depends on $V_{AB}(t)$ and load
- 1, 2 and 4 quadrant operation

For all these circuits:

L-side voltage always smaller than C-side voltage

Comparison of dc-converters

- switch utilization
- transformer core utilization
- size of passives

Switch Utilisation Ratio $SUR = \frac{P_0}{P_T}$ with $P_T = \hat{V}_T \cdot \hat{I}_T$ and $P_0 = V_0 \cdot I_0$

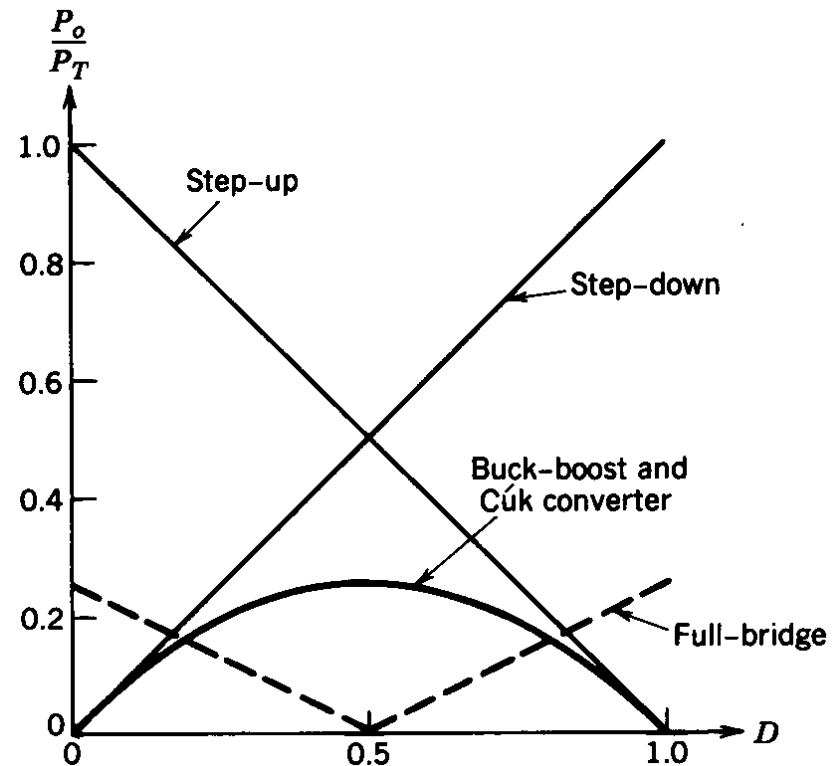
Problem:

Sketch $\frac{\hat{V}_T}{V_0}$ and $\frac{\hat{I}_T}{I_0}$

as a function of D

- for:
- step down converter
 - step-up converter
 - buck-boost converter

Assume for simplicity L to be infinitely large (flat current waveform)



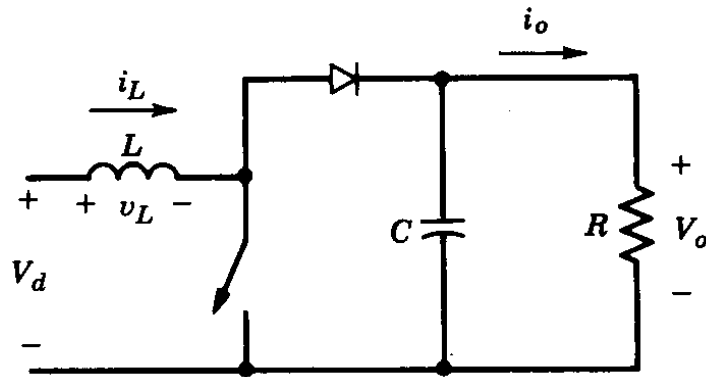
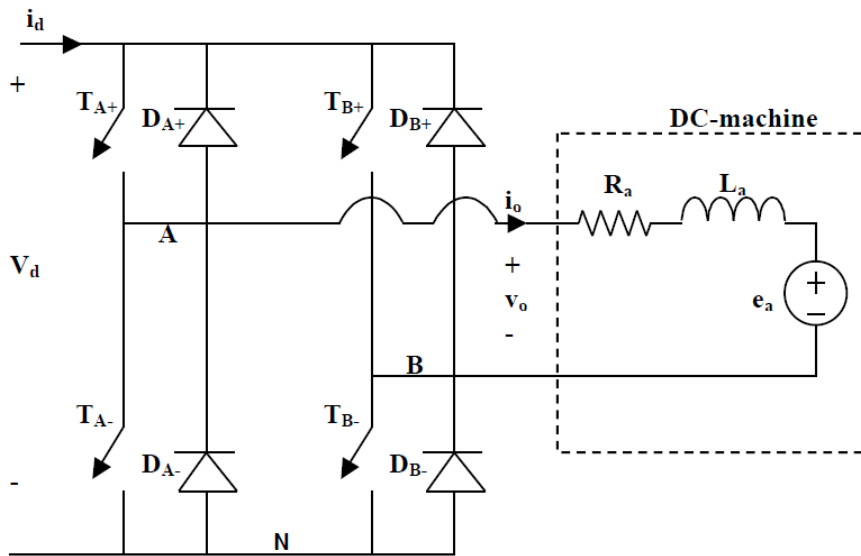


Figure 1 shows a boost converter used in the Toyota Prius hybrid electric vehicle for stepping up the battery voltage from 150-250 V to 500 V. The converter's switching frequency is $f_s = 20\text{kHz}$.

(5) Calculate the critical (maximum) inductance value of the output filter inductor to ensure that the converter operates in the discontinuous conduction mode for the given input voltage range and the load current up to $I_o = 10\text{A}$.

(5) Given the inductance value obtained in 1.1 calculate the duty cycle range to keep the output voltage V_o constant (500V) for the given range of the input voltage and the nominal current of $I_{nom} = 4\text{A}$.

(10) Calculate the required capacitance value of the output filter capacitor to ensure that at the nominal input voltage ($V_{in_{nom}} = 200\text{V}$) the peak-to-peak output voltage ripple is less than 2% of the nominal output voltage (V_o).



$V_d = 250$ V. Switching frequency is 30 kHz. The bridge is connected to a speed controlled dc machine. The armature inductance $L_a = 0,2$ mH. The armature resistance is negligible.

(a) The bridge is controlled to provide an average output voltage, $V_o = 200$ V. Find the duty ratios $D1$ and $D2$ and the ripple frequency of the two control principles. Sketch $v_o(t)$ for the two control principles.

At a given speed, the back-emf $E_a = 200$ V. Unipolar PWM is used.

(b) The armature current, i_a , is 1 A, find the maximum and the minimum instantaneous armature current. Sketch the armature current, $i_a(t)$. Indicate which of the power semiconductors are conducting. Also sketch $i_d(t)$.

(c) As in Problem (b), with $i_a = 20$ A.

(d) As in Problem (b), except bipolar PWM is used.

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