

Hydrological Measurements

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5. Interpolation Techniques



Interpolation techniques

Spatial interpolation techniques

- Inverse distance
- Thiessen polygons
- Contouring
- Kriging

Inverse distance

$$z'(x_0) = \sum \lambda_i * z(x_i)$$

$$\lambda_i = \frac{1/D_i^b}{\sum_{j=1}^n 1/D_j^b}$$

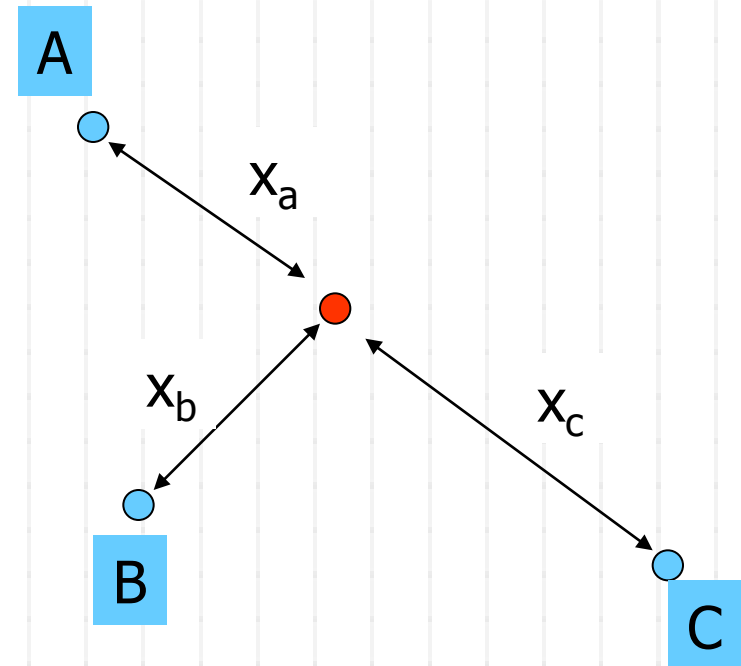
$z(x_i)$ = Observation in point x_i

λ_i = Weight for observation $z(x_i)$

$z'(x_0)$ = Estimate at x_0

D_i = Distance between x_0 and x_i

b = exponent (e.g. 2)



Inverse distance

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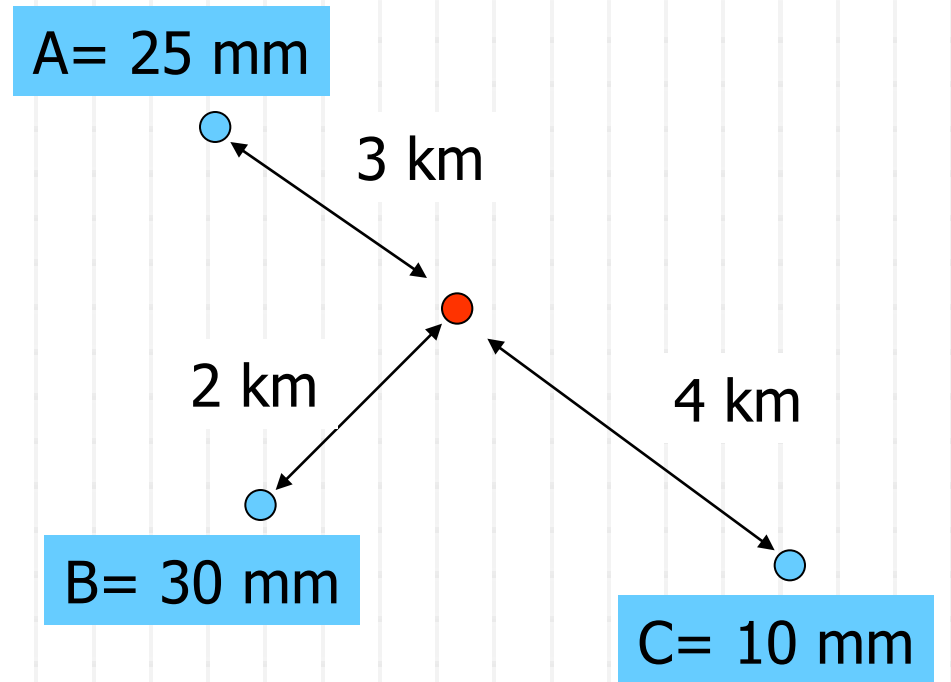
$z(x_i)$ = Observation in point x_i

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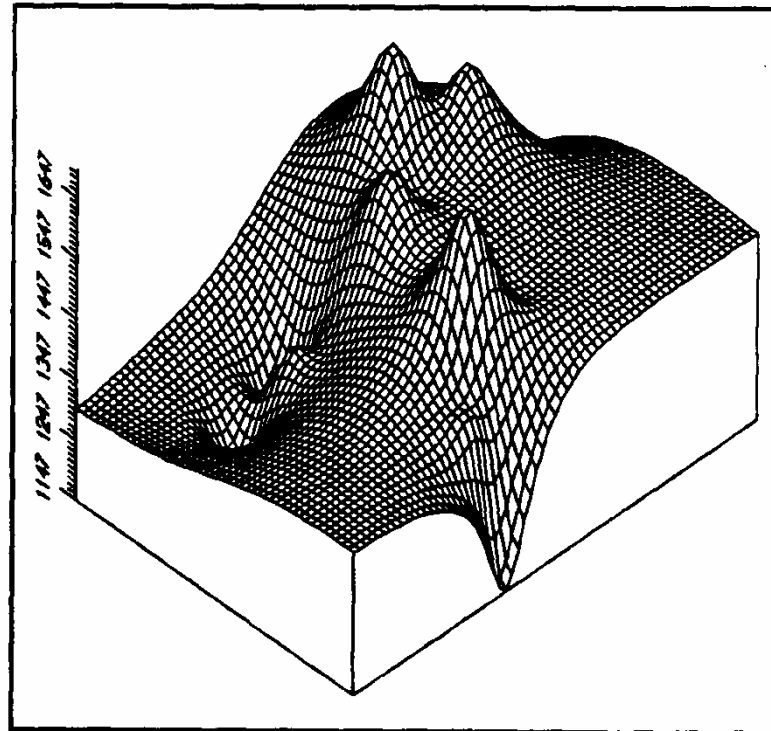
D_i = Distance between x_0 and x_i

b = exponent (e.g. 2)



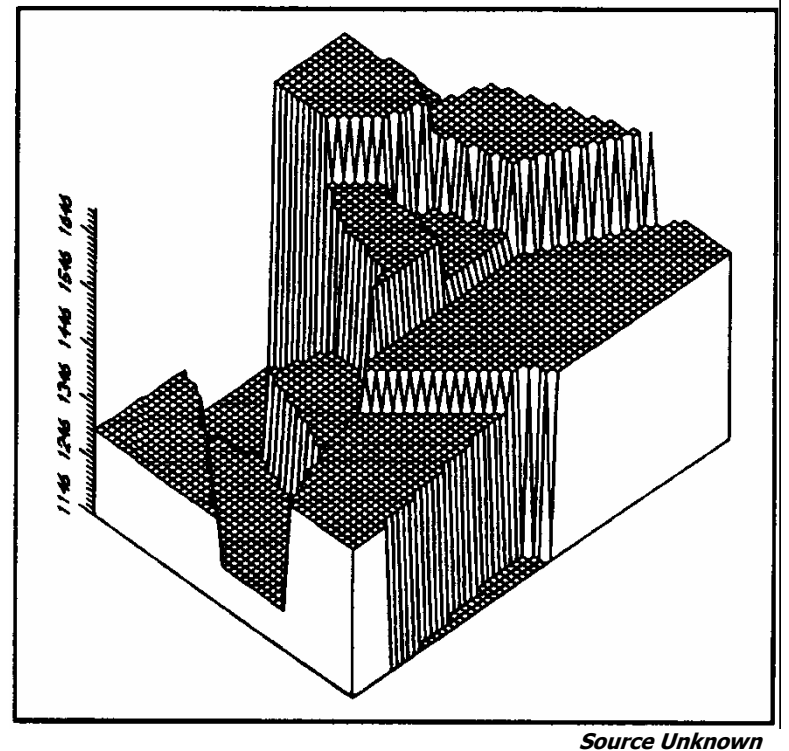
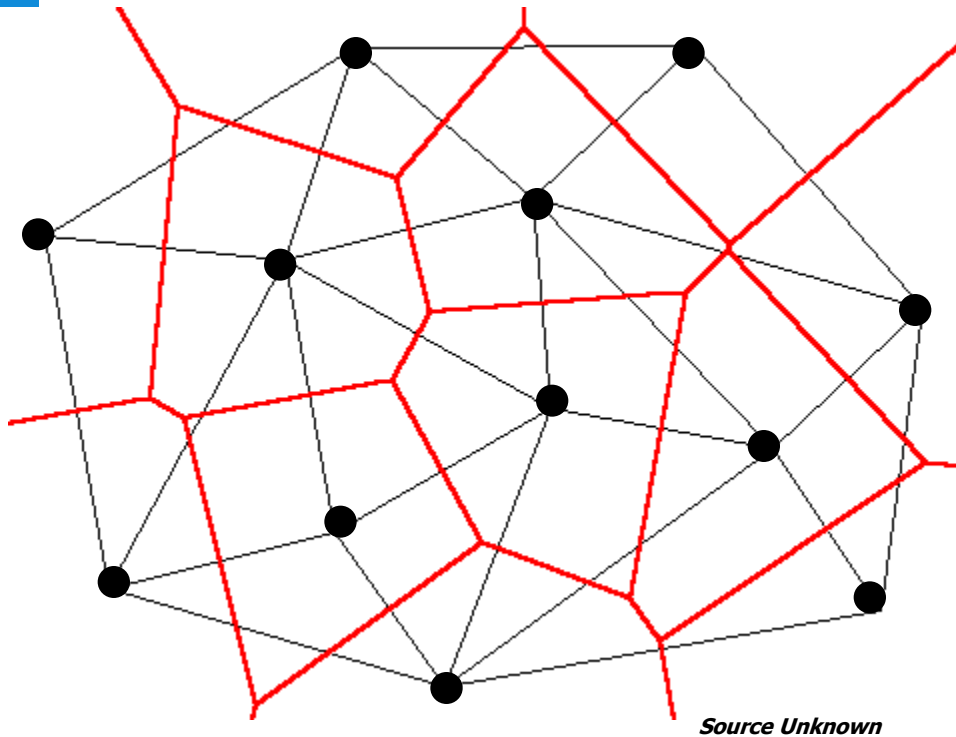
$P = 25.7$ mm

Inverse distance

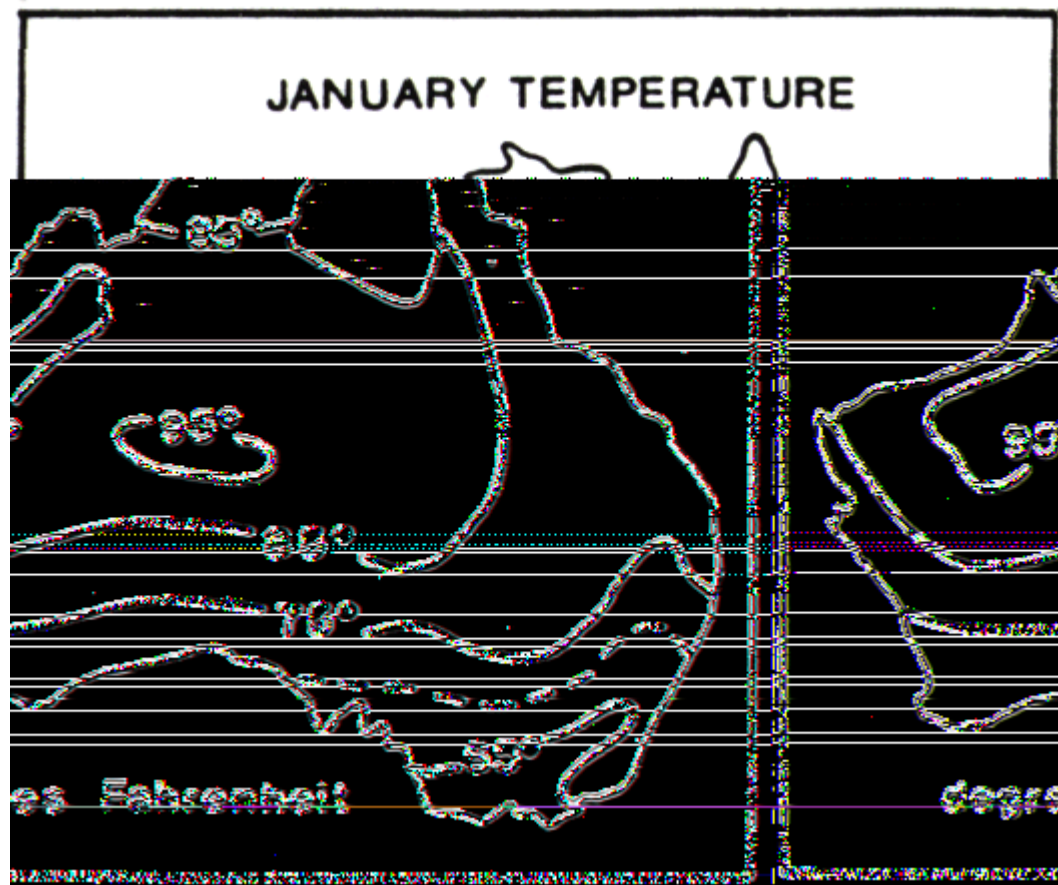


Source Unknown

Thiesen polygons



Contouring



Source Unknown

Kriging

$$z'(x_0) = \sum \lambda_i * z(x_i)$$



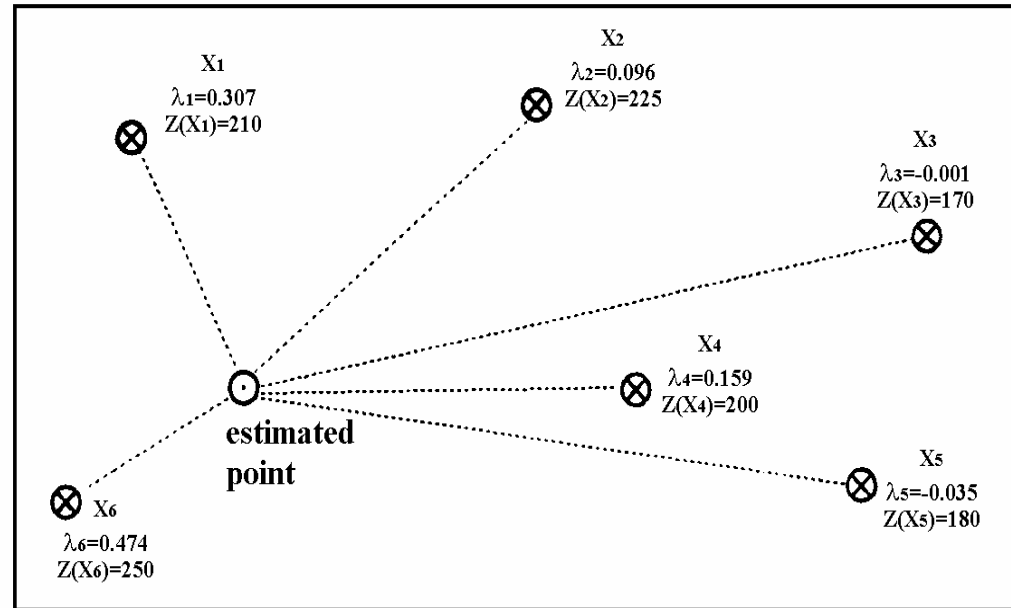
Minimizing the variance of
the error:

$$\varepsilon = z'(x_0) - z(x_0)$$

$z(x_i)$ = Observation in point x_i

λ_i = Weight for observation $z(x_i)$

$z'(x_0)$ = Estimate at x_0



Kriging

Ordinary Kriging: $\sum \lambda_i = 1$

$$\sigma_\varepsilon^2 = \sum \lambda_i * \gamma_i + \mu$$

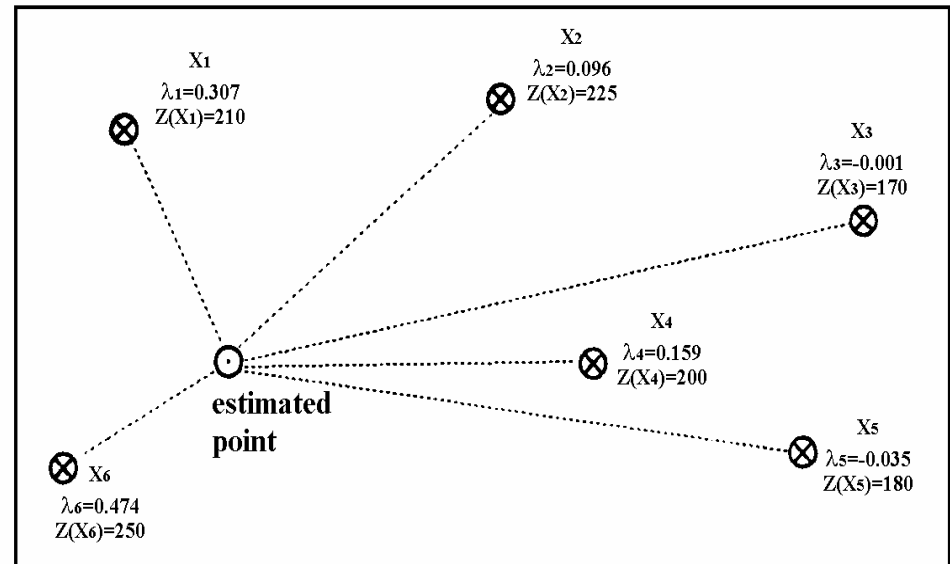
- σ_ε^2 = the variance for the error of the estimate at location x_0
- λ_i = weighing factor
- γ_i = the variance for the error from the observation at x_i
- μ = the Lagrange operator

Kriging

- Variance for the error:

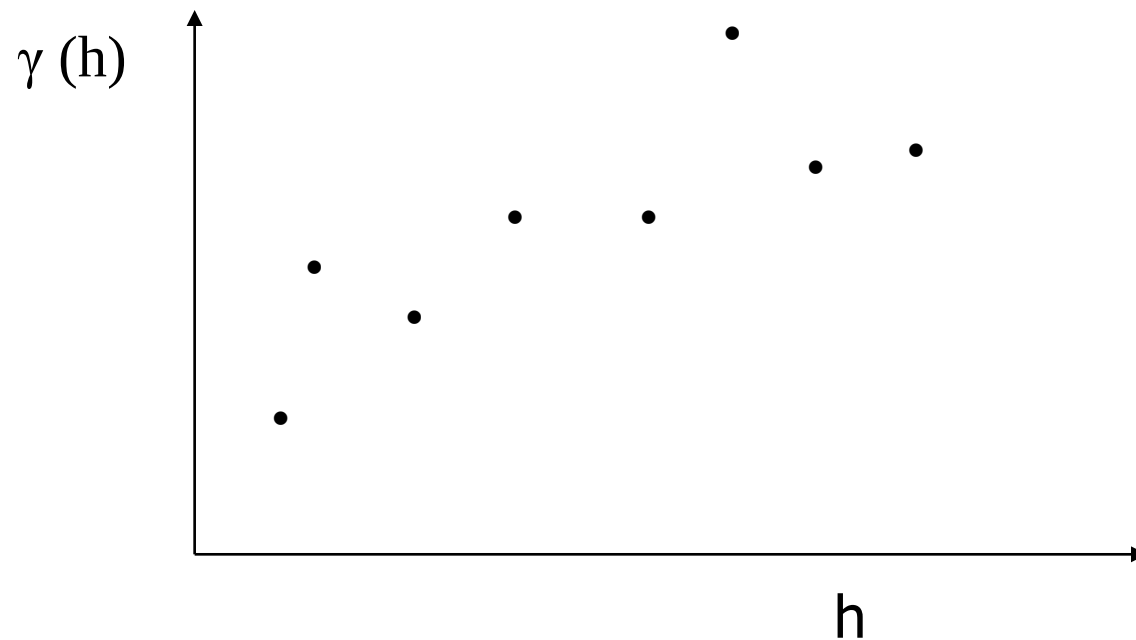
$$\gamma(h) = \frac{1}{2} * \frac{\sum (z(x+h) - z(x))^2}{N}$$

Data points all distance h apart (usually subdivided into classes)



Kriging

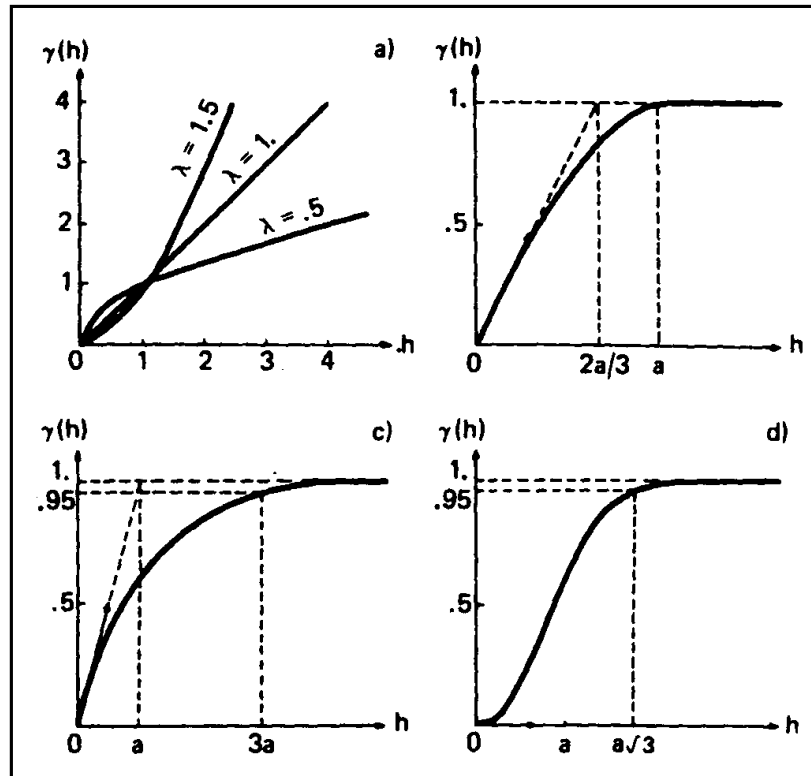
- Variogram:
$$\gamma(h) = \frac{1}{2} * \frac{\sum (z(x+h) - z(x))^2}{N}$$



Kriging

- Variogram models:

monomial



spherical

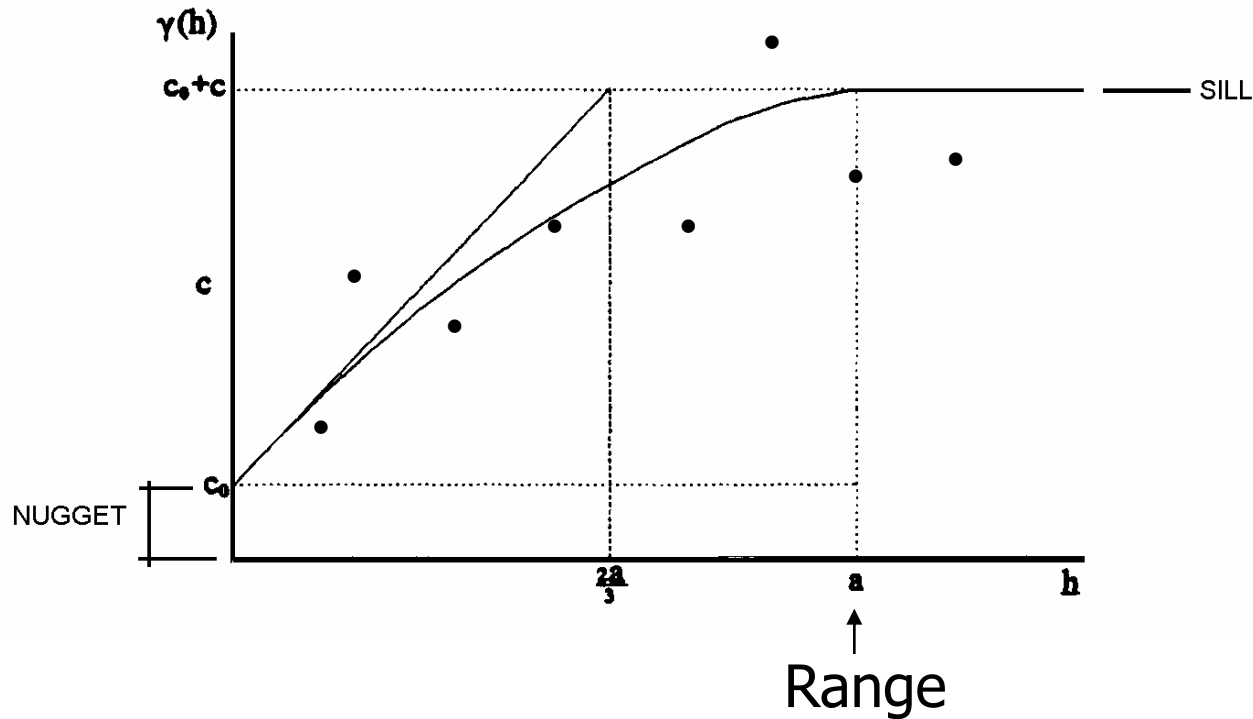
exponential

gaussian

Kriging

- Variogram

$$\gamma(h) = \frac{1}{2} * \frac{\sum (z(x+h) - z(x))^2}{N}$$



Kriging

- Solving ordinary Kriging:



$$\begin{bmatrix}
 \gamma(x_1, x_1) & \gamma(x_1, x_2) & \dots & \gamma(x_1, x_n) & 1 \\
 \gamma(x_2, x_1) & \gamma(x_2, x_2) & \dots & \gamma(x_2, x_n) & 1 \\
 \dots & & & & \\
 \dots & & & & \\
 \gamma(x_n, x_1) & \gamma(x_n, x_2) & \dots & \gamma(x_n, x_n) & 1 \\
 1 & 1 & \dots & 1 & 0
 \end{bmatrix}
 *
 \begin{bmatrix}
 \lambda_1 \\
 \lambda_2 \\
 \dots \\
 \dots \\
 \lambda_n \\
 \mu
 \end{bmatrix}
 =
 \begin{bmatrix}
 \gamma(x_0, x_1) \\
 \gamma(x_0, x_2) \\
 \dots \\
 \dots \\
 \gamma(x_0, x_n) \\
 1
 \end{bmatrix}$$

Existing network (EN)
?
? - EN

Validation Kriging results

- Cross validation: remove an existing observation
- Kriging Average Error (KAE):

$$KAE = \frac{1}{n} \sum_{i=1}^n \varepsilon_i(x_i) \qquad \varepsilon_i = z'(x_i) - z(x_i)$$

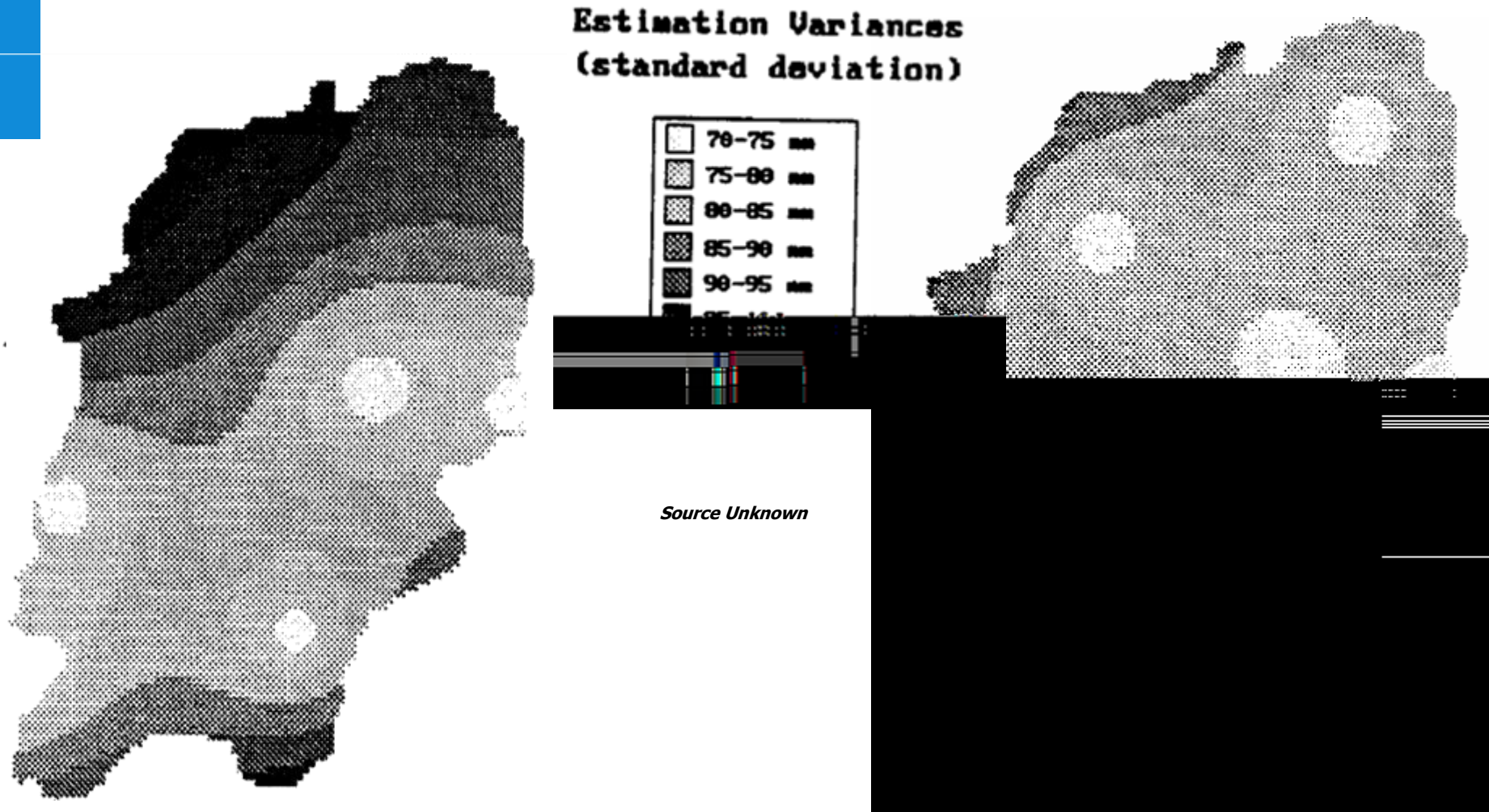
- Kriging Reduced Mean Square Error (KRMSE):

$$KRMSE = \frac{1}{n} \sum_{i=1}^n \left(\frac{\varepsilon_i(x_i)}{\sigma_i(x_i)} \right)^2 \qquad \varepsilon_i^2 = [z'(x_i) - z(x_i)]^2$$

Kriging: example cross validation

Station Name	UTM Easting [km]	UTM Northing [km]	Measured Mean Annual Rain [mm]	Estimated Mean Annual Rain [mm]	ε_i [mm]	σ_i [mm]	$(\varepsilon_i/\sigma_i)^2$
Murewa	422	8048	610	700	-90	130	0.5
Liemba	347	8003	640	560	80	120	0.4
Chiremwaremwa	382	7744	700	760	-60	150	0.2
Tandavel	475	7796	1150	950	200	140	2.0
Weleglegen	495	7808	1020	980	40	130	0.1
Makosa	447	8087	520	690	-170	140	1.5
Garuso	517	7902	1040	910	130	120	1.2
Mapai	397	7477	660	820	-160	200	0.6
Revue	503	7904	1000	900	100	130	0.6
Mazoi	500	8159	640	680	-40	160	0.1
				Sum	30		7.2

Kriging to improve your network



Concluding

- Inverse distance
- Thiessen polygons
- Contouring
- Kriging
 - With error estimation
 - Helps to improve your network