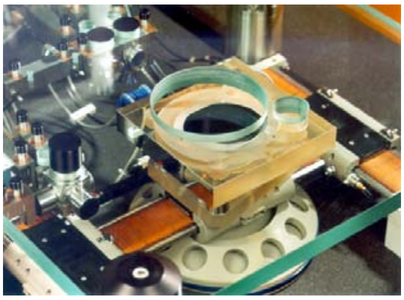
Mechatronic system design

Mechatronic system design wb2414-2013/2014
Course part 5



Motion control

Prof.ir. R.H.Munnig Schmidt
Mechatronic System Design

Lecture outline:

- What did you learn about PID - motion control so far?
- Introduction to motion control
- Feedforward control of piezoelectric scanner
- PD – feedback control of CD-player
- Sensitivity
- Stability and Robustness
- PID - feedback control of mass-spring system
- PID - feedback control of magnetic bearing
- How to deal with difficult plants having multiple eigenmodes

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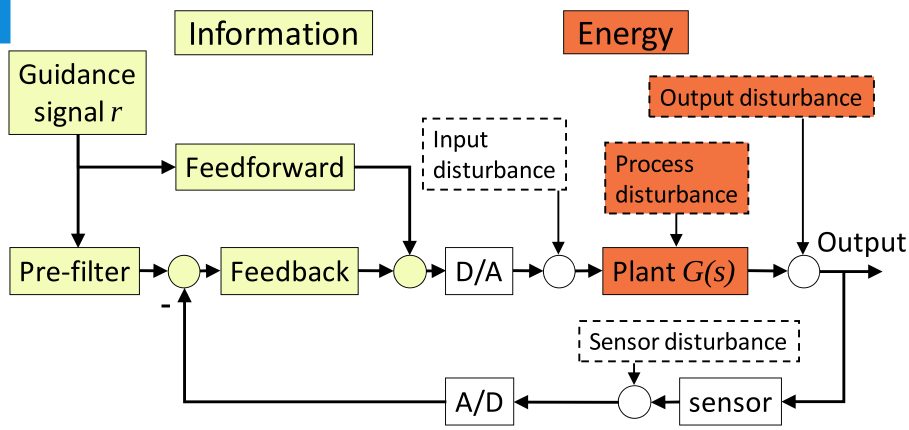
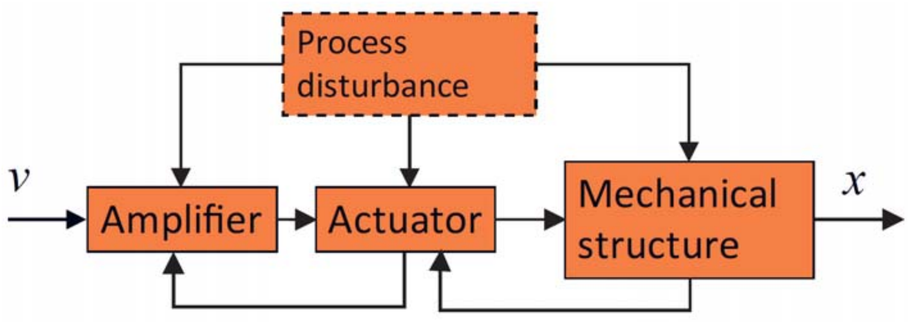
Motion Control:

- Control the trajectory of a machine
- Position control
- Velocity control (e.g. scanning)
- Path planning
- Disturbance rejection (vibrations from environment, imperfections of the guidance system, …)

A walk around the control loop



The plant is the process that needs to be controlled.



The successive order is fixed!

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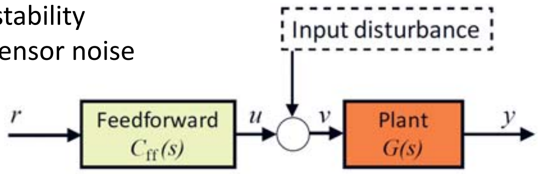
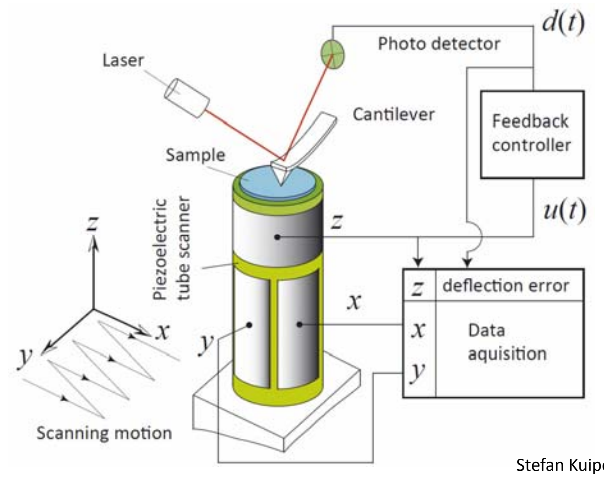
A walk around the control loop: Feedforward control

- requires no sensor
- predictable (if reference is known) (faster than pure feedback)
- cannot introduce instability
- no feeding back of sensor noise



- plant has to be stable
- no compensation of model uncertainties
- can only compensate for disturbances that are known (measured)

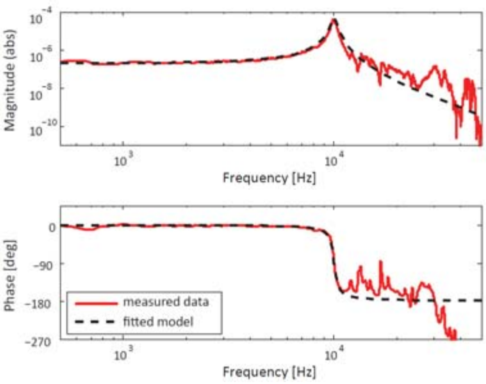
Feedforward control of a piezoelectric scanning unit



Stefan Kuiper TUD/TNO

Model-based open-loop feedforward control of the piezo scanner

Plant transfer function: $$G(s) = \frac{C_f}{\frac{s^2}{\omega_1^2} + 2\zeta_f \frac{s}{\omega_1} + 1} = \frac{C_f \omega_1^2}{s^2 + 2\zeta_f \omega_1 s + \omega_1^2}.$$



Problem is low damped first eigenmode

Model based open-loop feedforward control of the piezo scanner

Plant transfer function:

$$G(s)=\frac{C_\mathrm{f}}{\frac{s^2}{\omega_1^2}+2\zeta_\mathrm{f}\frac{s}{\omega_1}+1}=\frac{C_\mathrm{f}\omega_1^2}{s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2}.$$

Compensate dynamics by feedforward controller (pole-zero cancellation):

$$C_\mathrm{ff}(s)=\frac{s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2}{\omega_1^2}\quad\Rightarrow\quad G(s)C_\mathrm{ff}(s)=C_\mathrm{f}$$

Such a controller is unfortunately not realisable
(infinite gain at infinite frequencies)

In practice it can only be used to increase the damping

Add a well damped ζ=1 second order transfer function:

$$C_\mathrm{ff}(s)=\frac{s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2}{\omega_1^2}\quad\Rightarrow\quad C_\mathrm{ff}(s)=\frac{s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2}{s^2+2\cdot1\cdot\omega_1 s+\omega_1^2}.$$

And with an additional first order low pass filter against noise:

$$C_\mathrm{ff}(s)=\frac{s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2}{(s+\omega_1)\left(s^2+2\omega_1 s+\omega_1^2\right)}.$$

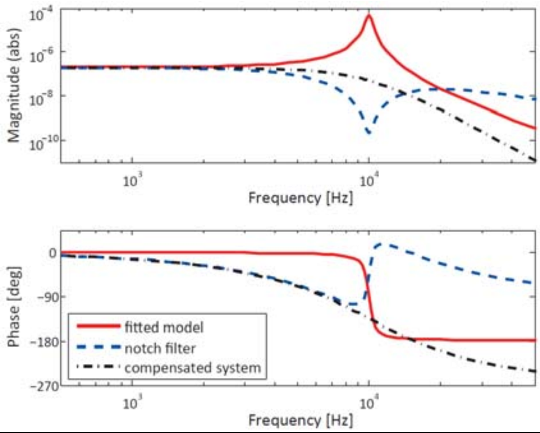
Which combines into:

$$G_\mathrm{t,ff}(s)=G(s)C_\mathrm{ff}(s)=\frac{C_\mathrm{f}}{\cancel{(s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2)}}\,\frac{\cancel{(s^2+2\zeta_\mathrm{f}\omega_1 s+\omega_1^2)}}{(s+\omega_1)\left(s^2+2\omega_1 s+\omega_1^2\right)}=$$

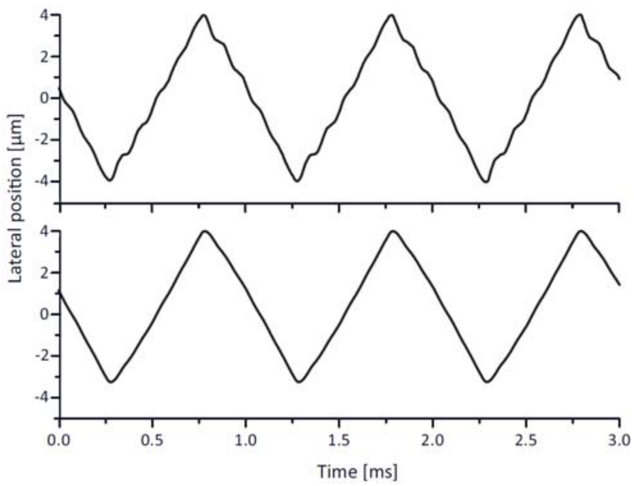
$$=\frac{C_\mathrm{f}}{(s+\omega_1)\left(s^2+2\omega_1 s+\omega_1^2\right)}$$

Resulting response

$$G_{t,ff}(s) = G(s)C_{ff}(s) = \frac{C_f}{(s+\omega_1)\left(s^2+2\omega_1 s+\omega_1^2\right)}$$



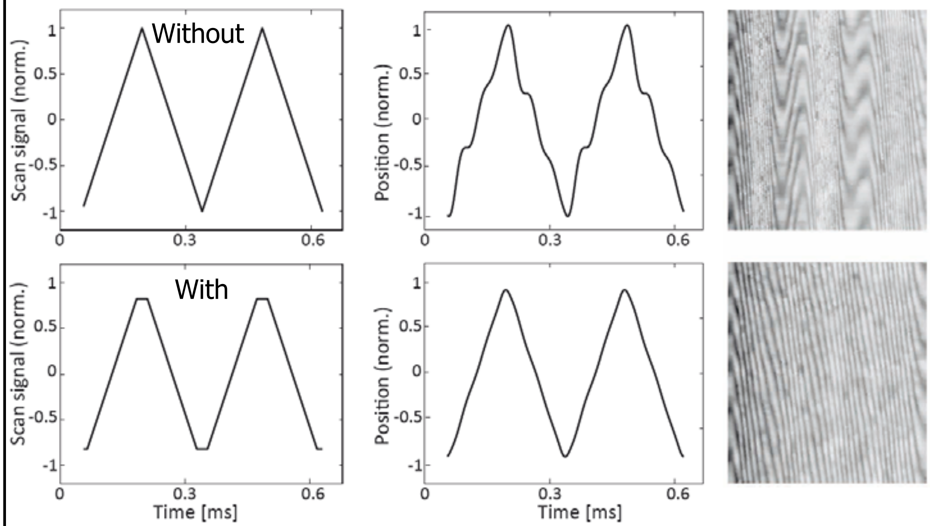
Triangular scanning movement



Without pole-zero cancellation

With pole-zero cancellation

Second method: input shaping



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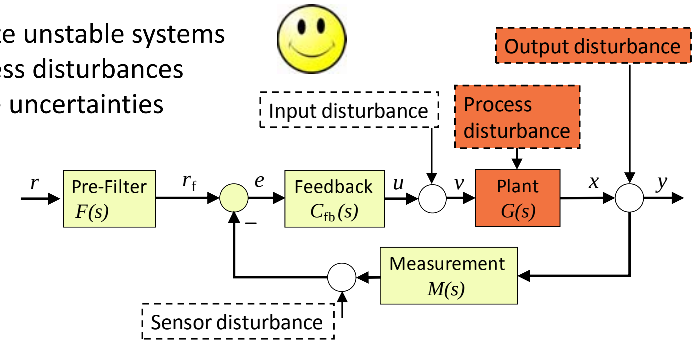
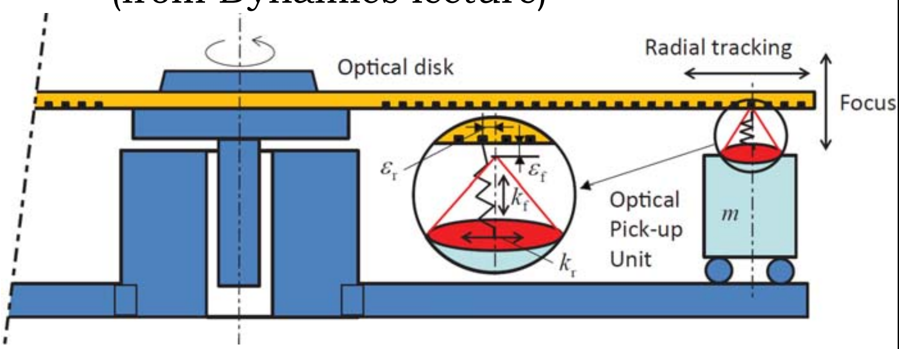
A walk around the control loop: Feedback control

- can stabilize unstable systems
- can suppress disturbances
- can handle uncertainties



- sensor required
- slower than FF (error has to occur first)
- (sensor) noise is also fed back
- can introduce instability
  (also of open-loop stable systems)

Active stiffness by feedback in a CD player, (from Dynamics lecture)

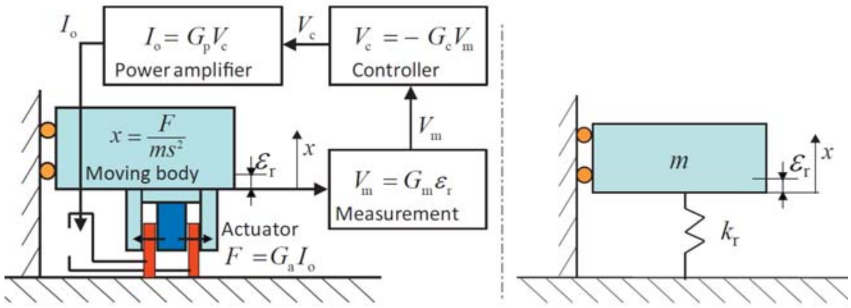


$$f_0 \geq f\sqrt{\frac{\hat{x}_r}{\varepsilon_r}} = 25\sqrt{\frac{200\cdot 10^{-6}}{0,2\cdot 10^{-6}}} = 800Hz$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_r}{m}} \Rightarrow k_r = 4\pi^2 m f_0^2 = 2.5\cdot 10^5 \text{ N/m}$$

- 200 µm radial vibrations at 25 Hz
- Mass lens: $10 \cdot 10^{-3}$ kg
- Max radial error: 0,2 µm

Virtual stiffness

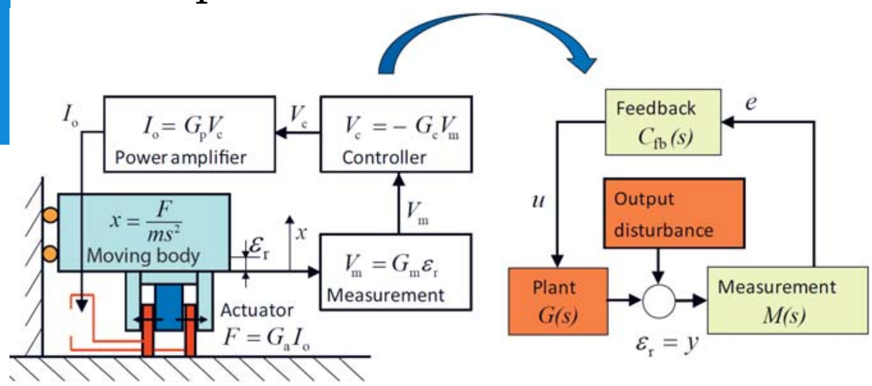


- Measure position
- Actuate with force proportional and opposite to the deviation (feedback!)

$$F = k_p \varepsilon_r = G_t \varepsilon_r = -G_m G_a G_p G_c \varepsilon_r$$

- Gives virtual spring stiffness $k_r = k_p = G_t = \frac{F}{\varepsilon_r}$

Servo position control



For the example:

$$m = 1 \cdot 10^{-2} \quad \text{kg}$$

$$G(s) = \frac{1}{ms^2} = \frac{100}{s^2}$$

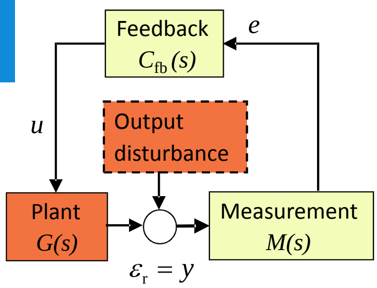
$$M(s) = 1$$

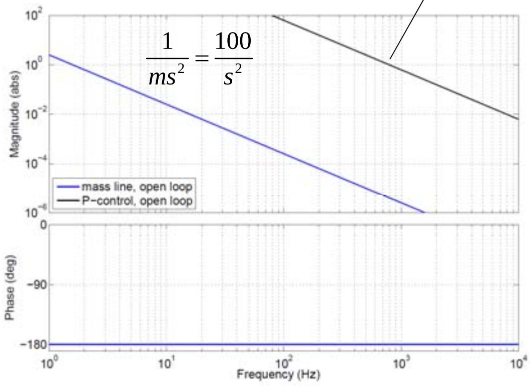
adding a P-gain (controller) …

$$C_{\text{fb}}(s) = C_p(s) = k_\text{p} = 2.5 \cdot 10^5$$

Open loop gain:

$$L_\text{p}(s) = G(s)C_{\text{fb}}(s)M(s) = k_\text{p}\frac{1}{ms^2} = \frac{2.5 \cdot 10^7}{s^2}$$



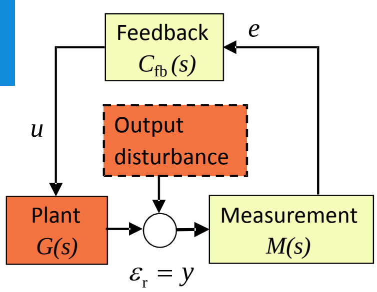


CD-player example:
control of lens position
m = 10 g

Is this system stable ?

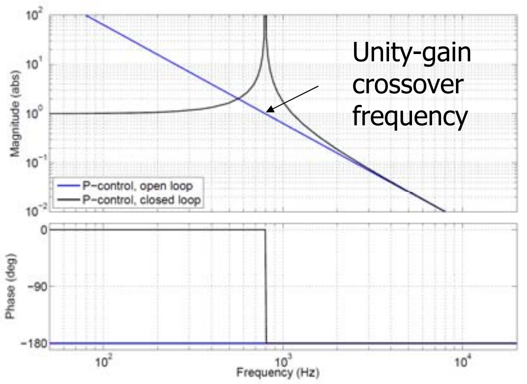
… closing the loop with P-controller

$$T_\text{p}(s) = \frac{L_\text{p}(s)}{L_\text{p}(s)+1} = \frac{1}{\frac{m}{k_\text{p}}s^2 + 1} = \frac{1}{-\frac{\omega^2}{\omega_0^2} + 1}$$

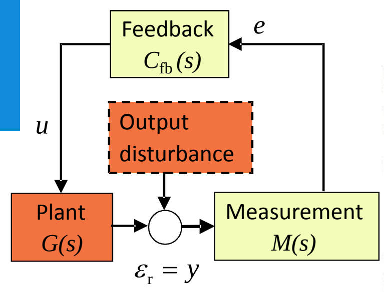


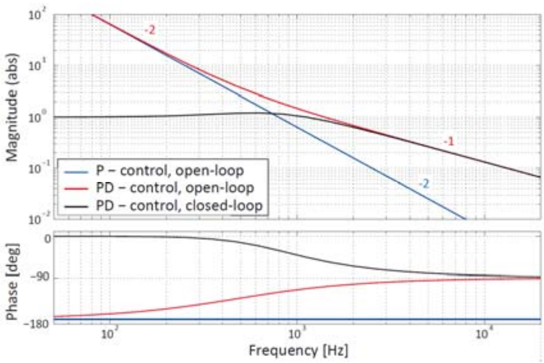
$$\omega_0 = \sqrt{\frac{k_\text{p}}{m}} = 5 \cdot 10^3$$

$$f_0 = \frac{1}{2\pi}\omega_0 = 800 \ \text{Hz}$$



We need some damping (PD-control)



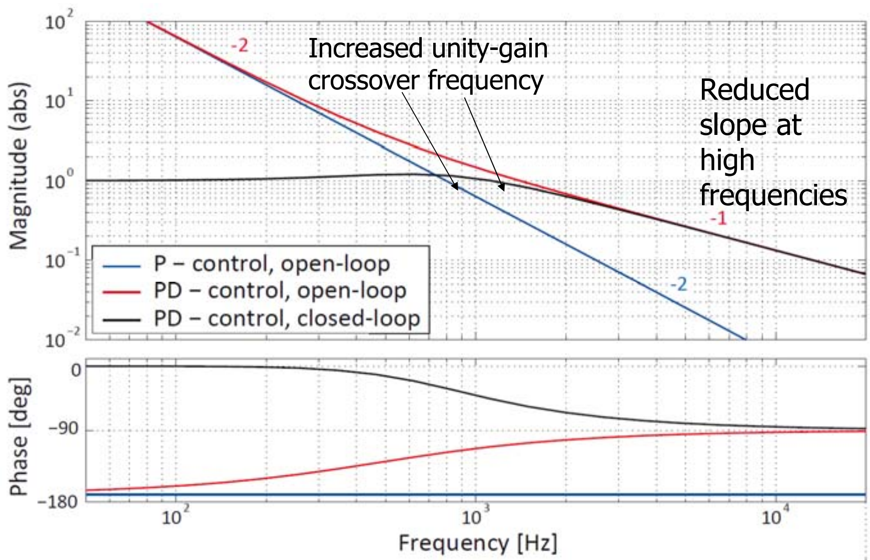


$$C_{fb}(s) = C_{pd}(s) = k_p + k_d s$$

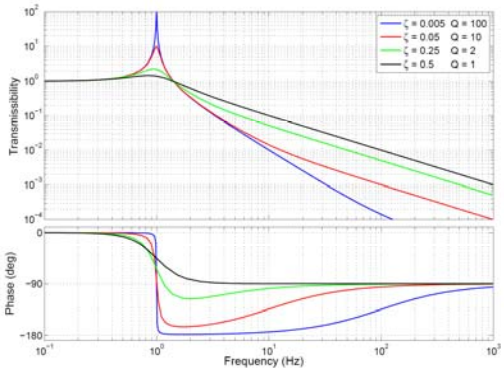
$$L_{pd}(s) = G(s)C_{pd}(s) = \frac{k_p + k_d s}{ms^2}$$

$$T_{pd}(s) = \frac{L_{pd}(s)}{L_{pd}(s)+1} = \frac{k_p + k_d s}{ms^2 + k_p + k_d s} = \frac{2j\varsigma\frac{\omega}{\omega_0}+1}{-\frac{\omega^2}{\omega_0^2}+2j\varsigma\frac{\omega}{\omega_0}+1} \qquad \frac{2\varsigma}{\omega_0} = \frac{k_d}{k_p}$$

Look better at the graph



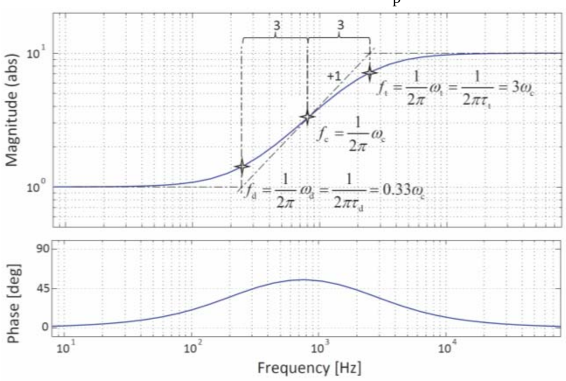
PD-control behaves like “transmissibility” and cannot be made in reality (not proper)



Reduced reduction of high frequencies by continuous D-action.
Infinite gain at infinite frequency!

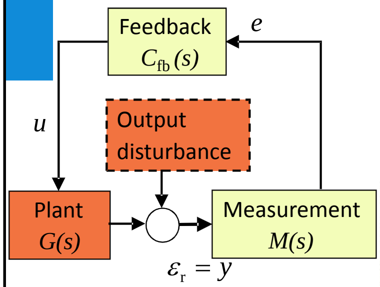
The D-action must be terminated at high frequencies and k_p must be reduced

$$C_{pd}(s) = k_p + k_d s = k_p(1+\frac{k_d}{k_p}s) = k_p(1+\tau_d s) = k_p(1+\frac{s}{\omega_d})$$



$$L_{pdt}(s) = \frac{\omega_d}{\omega_0}\frac{k_p}{ms^2}\frac{1+\tau_d s}{1+\tau_t s}$$

… closing the loop with tamed PD-controller



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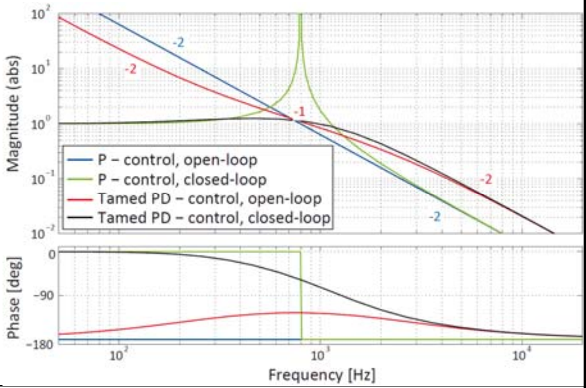
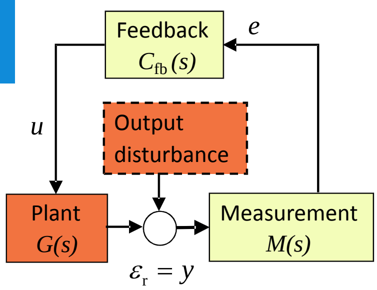
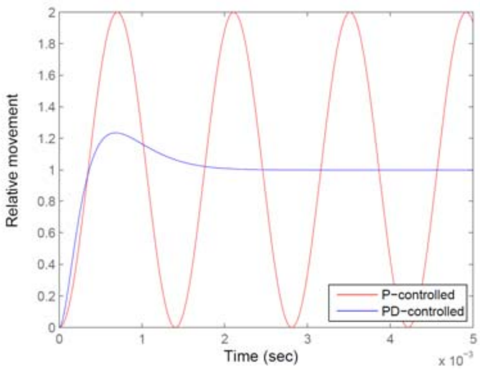


CD-player example:
control of lens position
m = 10 g

Closed-loop step response of PD-controlled mass





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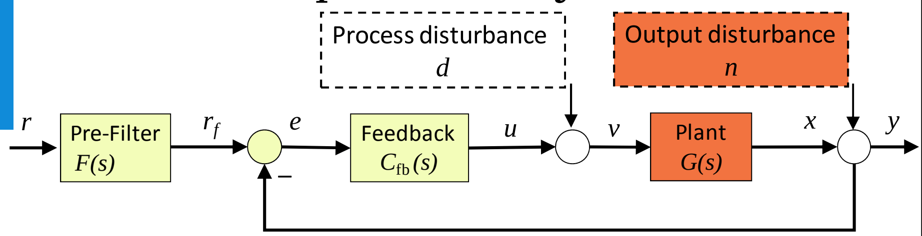
Feedback Loop Sensitivity Functions

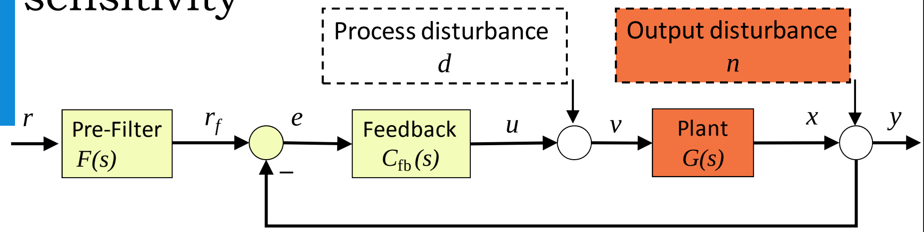


$$x = \frac{G}{1+GC}d - \frac{GC}{1+GC}n + \frac{GCF}{1+GC}r$$

$$y = \frac{G}{1+GC}d + \frac{1}{1+GC}n + \frac{GCF}{1+GC}r$$

$$u = -\frac{GC}{1+GC}d - \frac{C}{1+GC}n + \frac{CF}{1+GC}r$$

Response to reference, Complementary sensitivity



$$x = \frac{G}{1+GC}d - \frac{GC}{1+GC}n + \frac{GCF}{1+GC}r$$

$$y = \frac{G}{1+GC}d + \frac{1}{1+GC}n + \frac{GCF}{1+GC}r$$

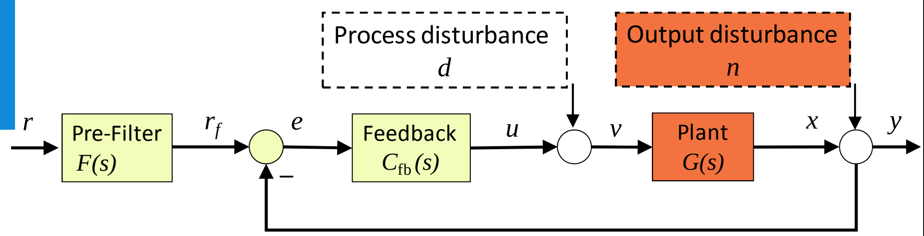
$$u = -\frac{GC}{1+GC}d - \frac{C}{1+GC}n + \frac{CF}{1+GC}r$$

If $F(s) = 1$

$$\frac{u}{d} = \frac{x}{n} = \frac{x}{r} = \frac{y}{r} = \frac{GC}{1+GC}$$

Complementary sensitivity function $T(s)$

Output disturbance rejection, sensitivity



$$x = \frac{G}{1+GC}d - \frac{GC}{1+GC}n + \frac{FCG}{1+GC}r$$

$$y = \frac{G}{1+GC}d + \frac{1}{1+GC}n + \frac{FCG}{1+GC}r$$

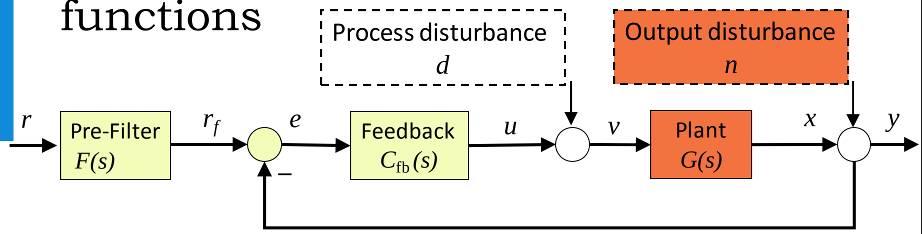
$$u = -\frac{GC}{1+GC}d - \frac{C}{1+GC}n + \frac{FC}{1+GC}r$$

If $F(s) = 1$

$$\frac{y}{n} = \frac{1}{1+GC}$$

Sensitivity function $T(s)$

Relation of the two sensitivity functions



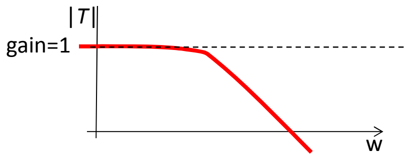
Complementary sensitivity

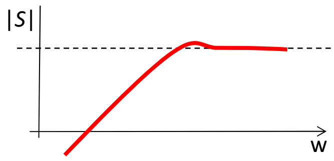
$$T(s) = \frac{G(s)C_{fb}(s)}{1+G(s)C_{fb}(s)}$$

$$T(s) + S(s) = 1$$

Sensitivity

$$S(s) = \frac{1}{1+G(s)C_{fb}(s)}$$





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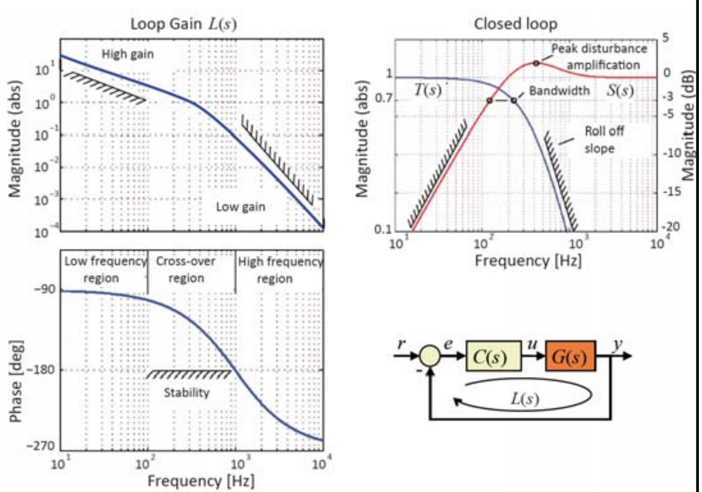
Stability and robustness

- When is a (closed-loop) system stable?
  - A system is asymptotically stable if and only if **all poles** are in the left half of the complex plane.
- When is a system robust?
  - Robustness of a feedback controlled system means that the closed-loop system can accept (some) model uncertainties (parameter variations).
- Both properties are investigated with the Bode and Nyquist plot.

Loop-Shaping design for stability and robustness

Shape the frequency response of the open-loop system such that:
• high gain (>>1) at low frequency
• low gain (<<1) at high frequency
• less than 180 degree phase lag in crossover region (unity-gain).

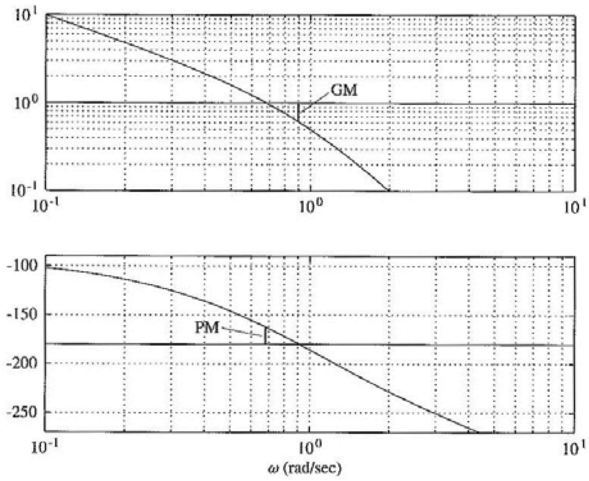
Robustness:
• Gain margin GM
• Phase margin PM



It happens around the unity-gain cross-over frequency

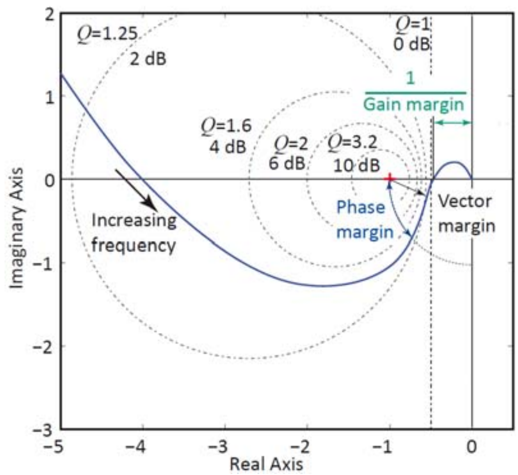
Stability: Is the unity-gain crossing of the *magnitude plot* at a frequency with less than *180º phase lag* ?

Robustness:
- Gain margin GM
- Phase margin PM



Nyquist Plot of open-loop transfer $L(s)$.

- Stability: The -1 point must be passed at the left side when increasing the frequency.
- Robustness:
  - Gain margin (GM), how much the gain can be increased until instability
  - Phase margin (PM), how much the phase lag can be increased until instability



The circle that is just passed at the left gives the peak value of the resonance after closing the loop

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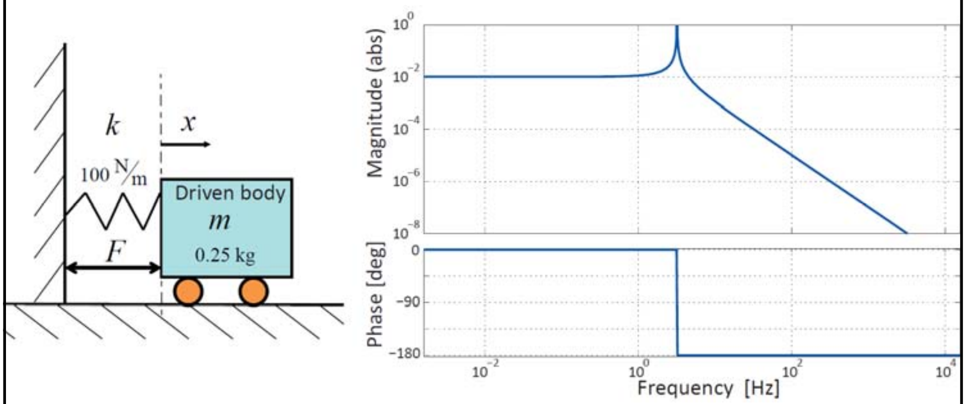
The plant, an undamped mass-spring system

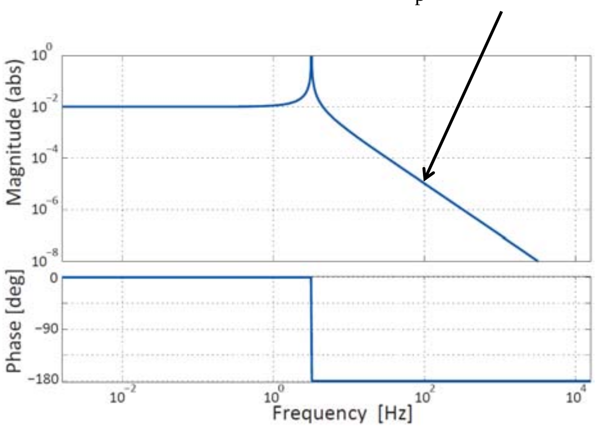


Specifications of closed loop system

- Bandwidth of 100 Hz (unity-gain cross-over frequency)
  - So far beyond the natural frequency on the mass line
- Asymptotically stable, all poles in the left half plane
- Sufficiently damped (is requirement!)
  - Specification, max Q = 1.3 (+3dB)
- Zero steady-state error (0 Hz)

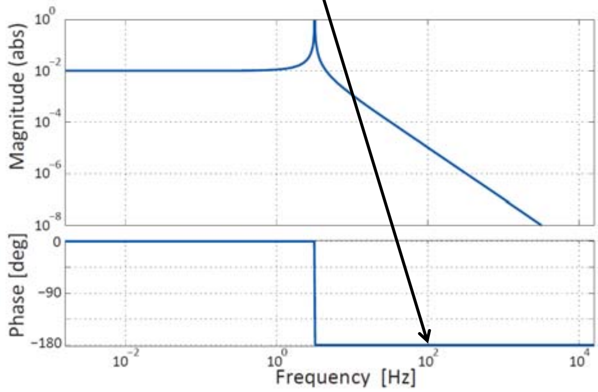
P-control

- Lift the response to unity gain @ 100 Hz => k_p~10^5



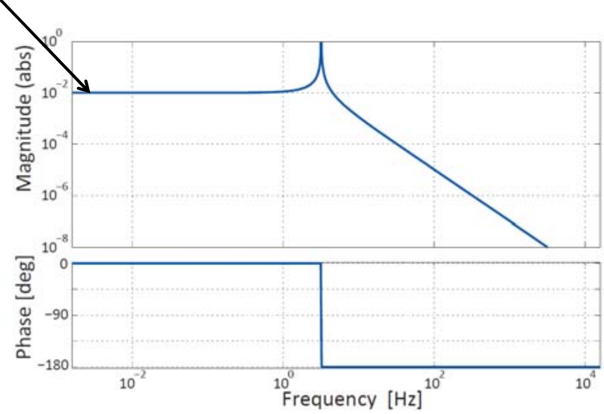
D-control for damping

- reduce the phase lag @100Hz
- Don't forget to reduce k_p



I-control for infinite gain at 0 Hz

- Sensitivity = $S(s) = \dfrac{1}{1+G(s)C_{fb}(s)}$



PID-control in math.

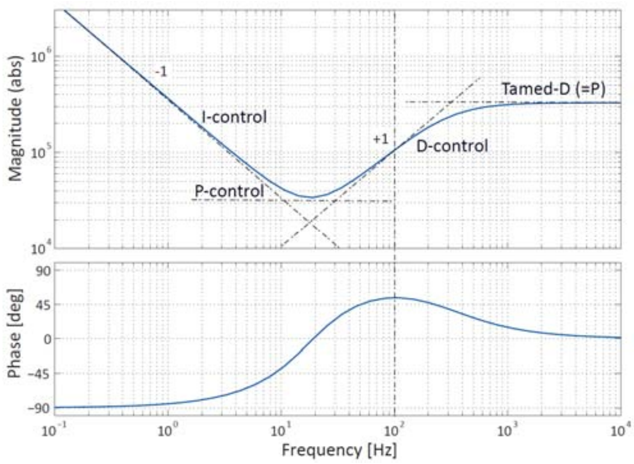
$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)\mathrm{d}\tau + k_d \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

$$u(s) = e(s)\left(k_p + \frac{k_i}{s} + k_d s\right)$$

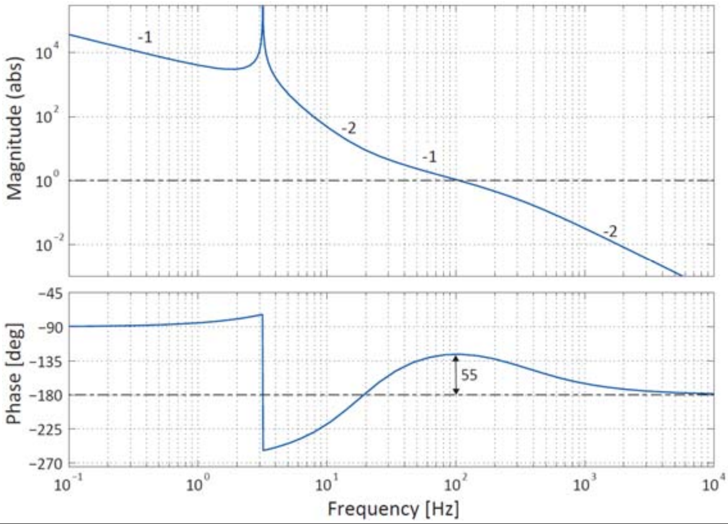
$$C_{pid}(s) = \frac{u(s)}{e(s)} = \left(k_p + \frac{k_i}{s} + k_d s\right)$$

$$C_{pid}(\omega) = \frac{u(\omega)}{e(\omega)} = \left(k_p - j\frac{k_i}{\omega} + jk_d\omega\right)$$

The bode plot of the designed PID controller



Open-loop controller and plant



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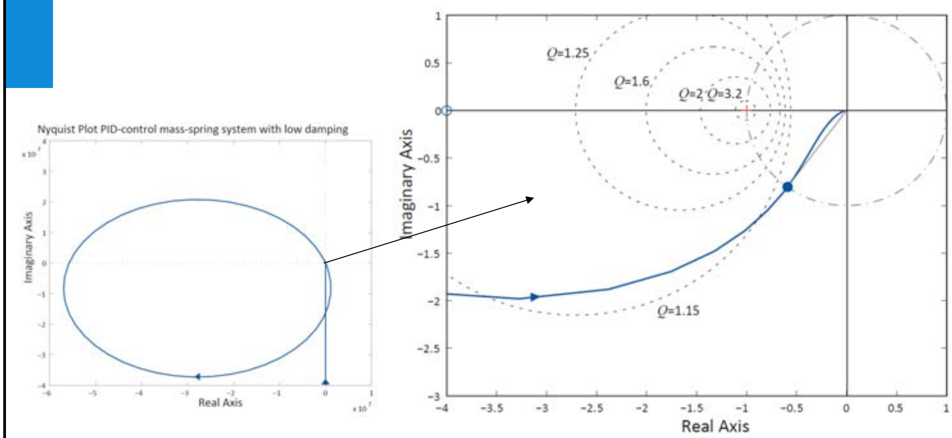
Nyquist plot



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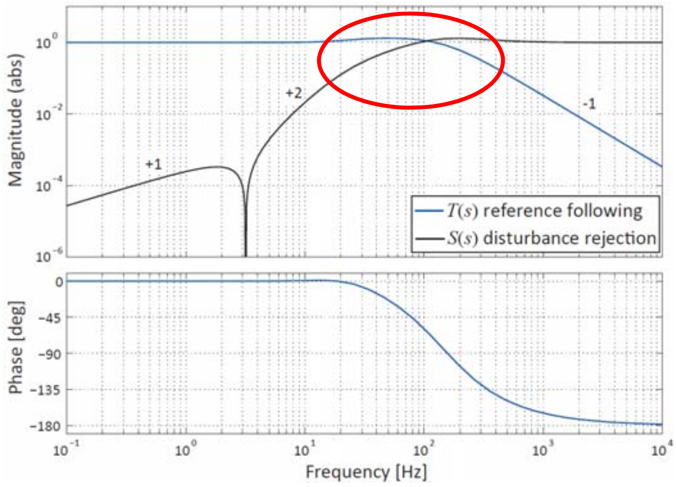
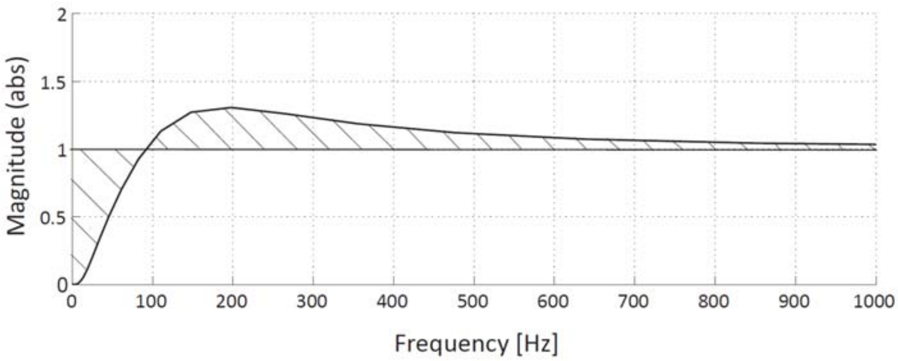
Closed loop total system



Increased sensitivity for disturbances above f_0 on a linear scale.



Waterbed effect due to Bode sensitivity integral!
Equals zero for system with 2 more poles than zeros

$$\int_0^\infty \ln|S(\omega)|\,\mathrm{d}\omega$$

Lecture outline:

- What did you learn about PID - motion control so far?
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- PD – feedback control of CD-player
- Sensitivity
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- PID - feedback control of mass-spring system
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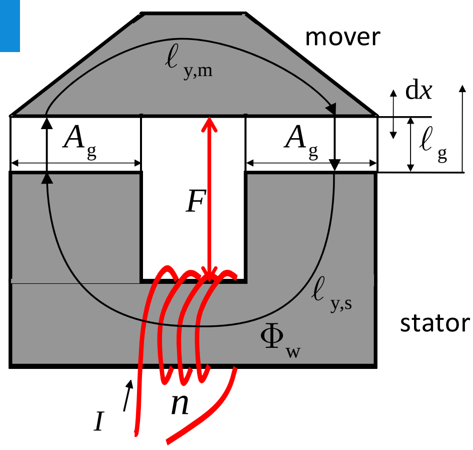
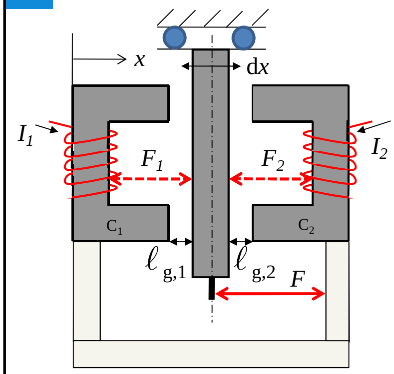
Magnetic bearings are often based on the variable reluctance actuator



Force based on energy balance
Energy stored in magnetic field

$$F = -\left(\frac{nI}{\ell_g}\right)^2 \frac{A_g \mu_0}{4}$$

Balancing and linearisation with two variable reluctance actuators



$$F = F_2 - F_1$$

$$= \left(\frac{nI_2}{\ell_{g,2}}\right)^2 \frac{A_g\mu_0}{4} - \left(\frac{nI_1}{\ell_{g,1}}\right)^2 \frac{A_g\mu_0}{4} = \frac{n^2 A_g \mu_0}{4}\left(\frac{I_2^2}{\ell_{g,2}^2} - \frac{I_1^2}{\ell_{g,1}^2}\right)$$

In the mid-position: $l_{g,1} = l_{g,2} = l_g$:

$$F = \frac{n^2 A_g \mu_0}{4}\left(\frac{I_2^2 - I_1^2}{\ell_g^2}\right)$$

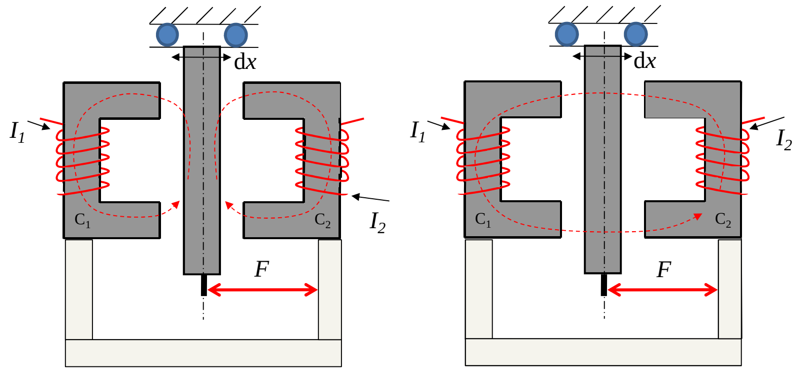
With: $I_2 = I_a + \Delta I$ and $I_1 = I_a - \Delta I$

$$F = \frac{n^2 A_g \mu_0}{4}\left(\frac{(I_a + \Delta I)^2 - (I_a - \Delta I)^2}{\ell_g^2}\right) = \frac{n^2 A_g \mu_0}{4}\left(\frac{4 I_a \Delta I}{\ell_g^2}\right)$$

$$\frac{F}{\Delta I} = \frac{I_a n^2 A_g \mu_0}{\ell_g^2}$$

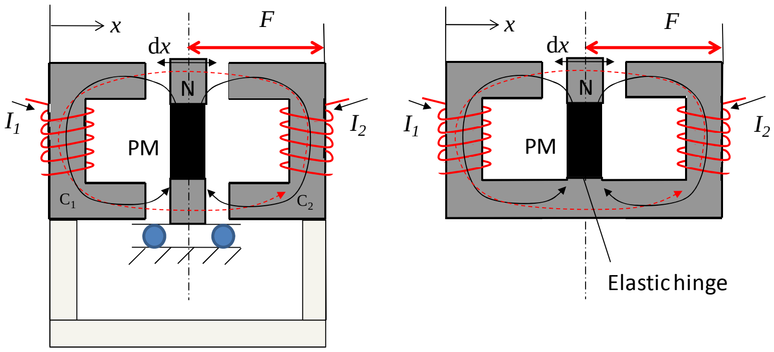
What is the force to position relation at a certain current level?

Flux in a balanced reluctance actuator depends on the current direction



Sharing the flux paths by reversing the current in one of the coils

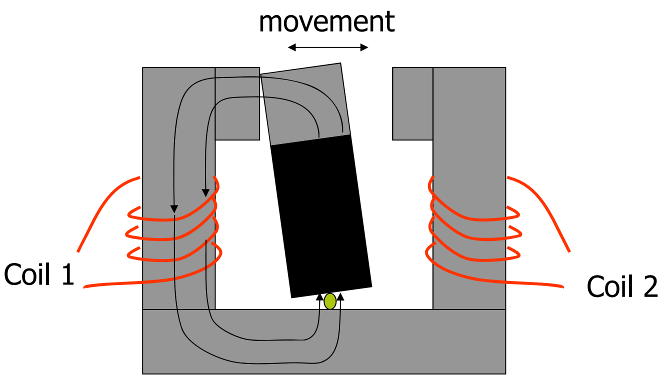
Inserting a permanent magnet in a double reluctance actuator



Creating the common mode flux with a permanent magnet

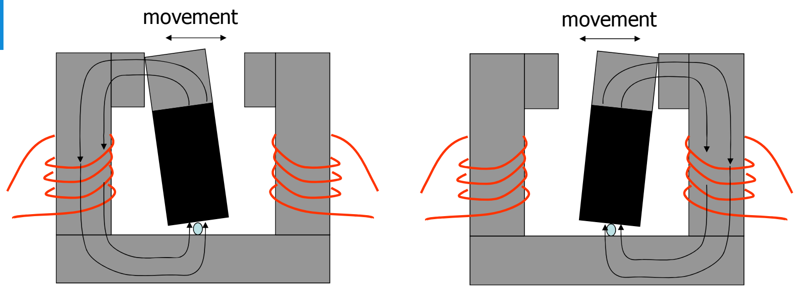
Hybrid actuator (right position)

Permanent magnet biased reluctance actuator



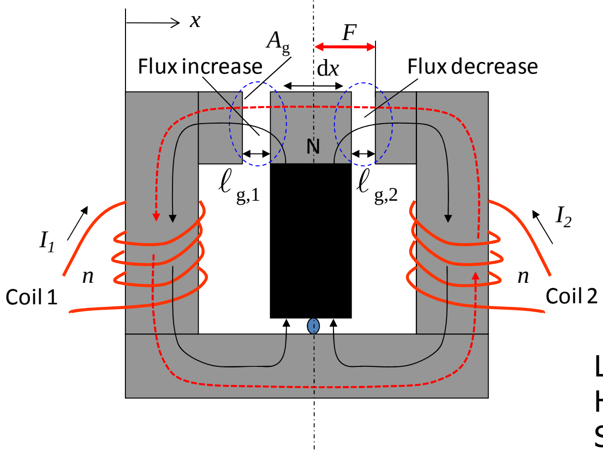
All PM flux passes through coil 1

Two ultimate positions



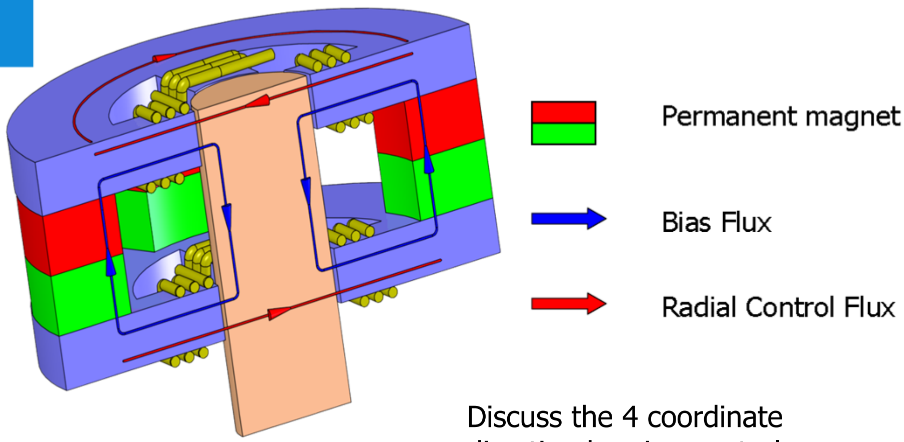
Ratio of PM flux through the coils depends on the position of the mover

Superposition of the PM and current induced magnetic flux



Large dΦ/dx
High force
Short stroke

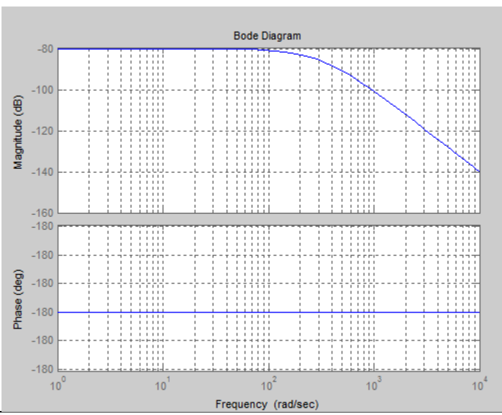
Hybrid actuator in Magnetic bearings

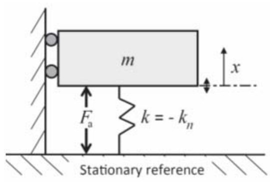


Discuss the 4 coordinate direction bearing control

Control of a magnetic bearing

- Bodeplot of negative stiffness





$$C_t(s) = \frac{x}{F} = \frac{1}{ms^2 + k} = \frac{1}{ms^2 - k_n}$$

$$k_n = 10000 \text{ N/m}$$

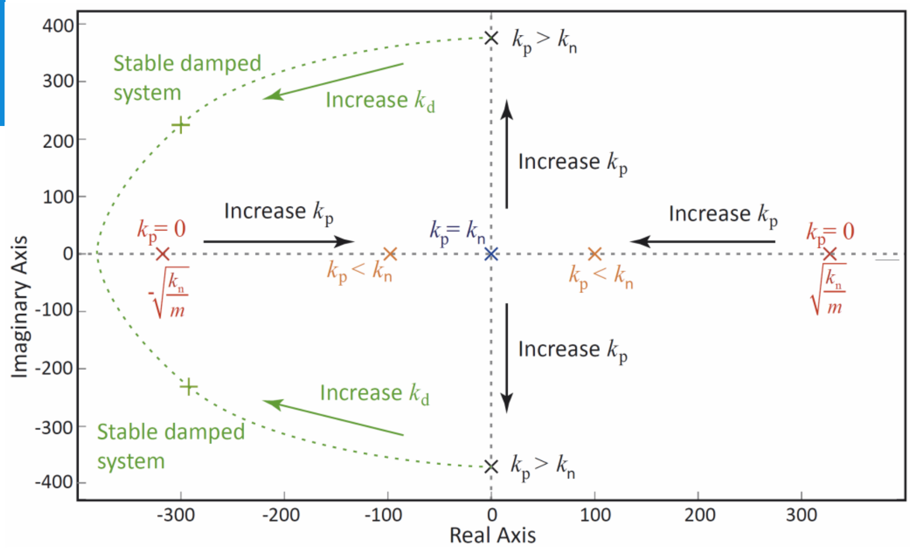
$$m = 0.1 \text{ kg}$$

$$C_t(\omega) = \frac{x}{F} = \frac{1}{-0.1\omega^2 - 1 \cdot 10^4}$$

Poles: $ms^2 - k_n = 0 \Rightarrow$

$$s = \pm\sqrt{\frac{k_n}{m}} = \pm 316$$

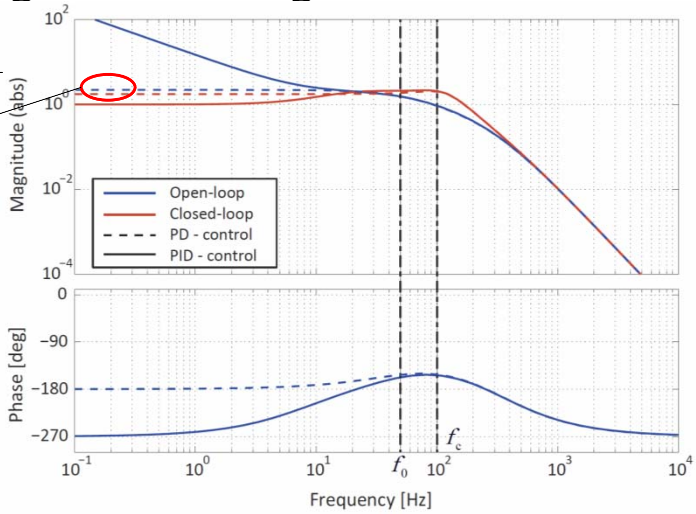
Shifting the pole to the left (root-locus)



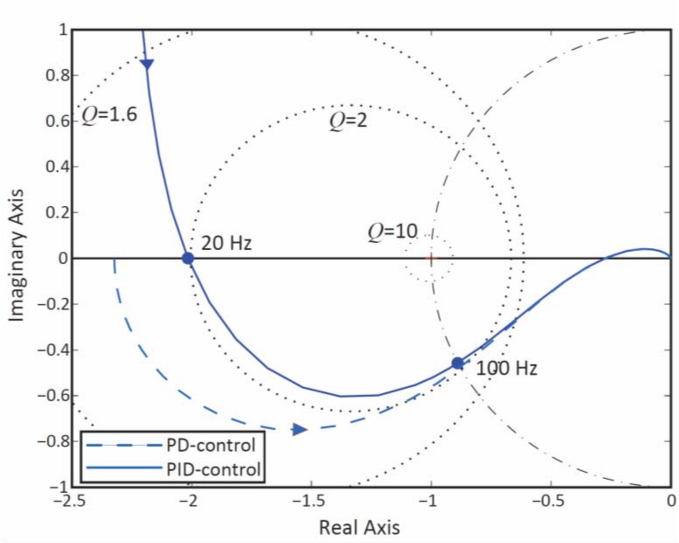
Bode plot of P(I)D-controlled magnetic bearing

$$C_{\text{plant+P}}(s) = k_p \frac{1}{ms^2 + k_n}$$

$$= \frac{k_p \frac{1}{k_n}}{\frac{m}{k_n} s^2 + 1} \approx \frac{2}{\frac{m}{k_n} s^2 + 1}$$



Nyquist explains what happens.



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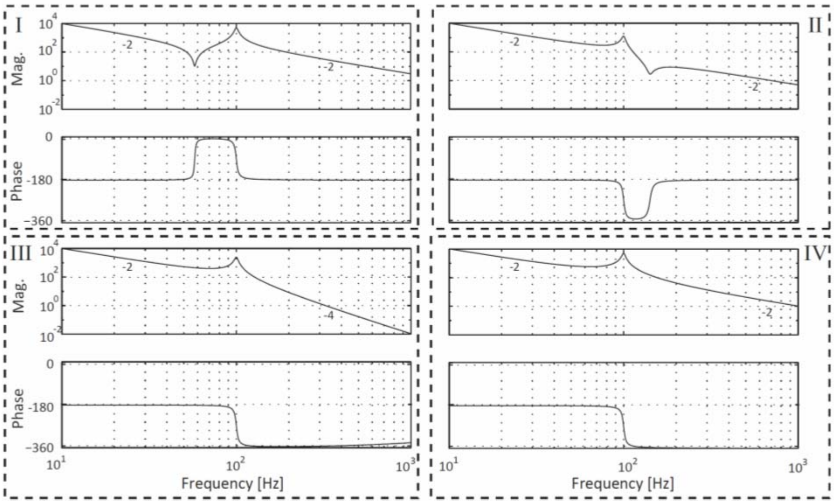
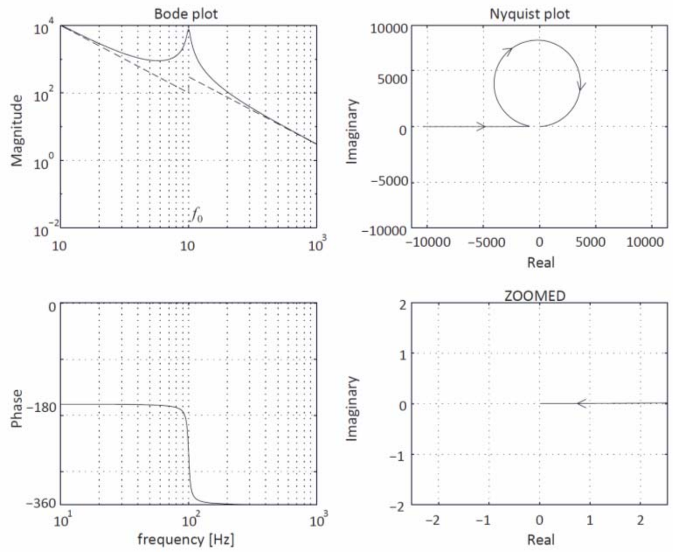
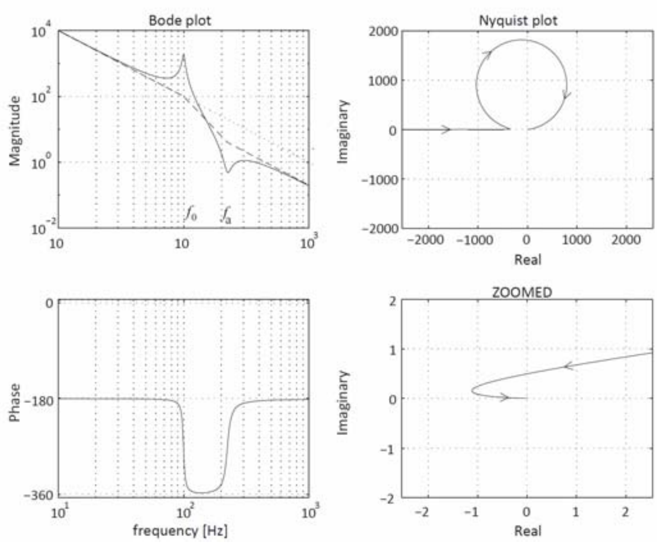
4 types of responses



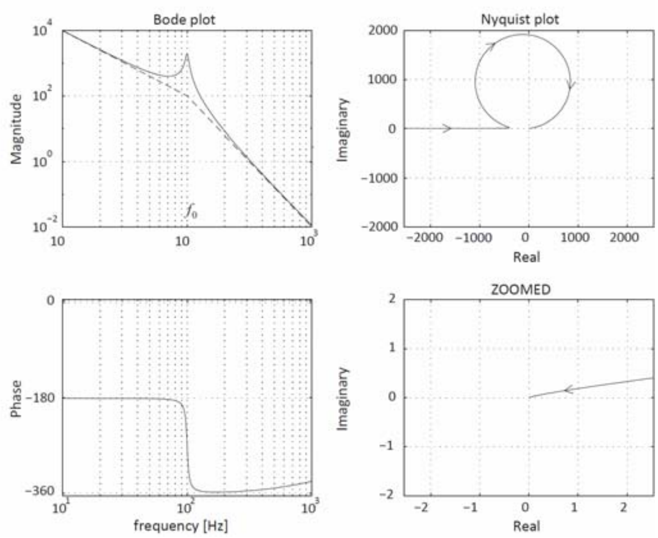
Type I: -2 slope, zero pair, pole pair, -2 slope



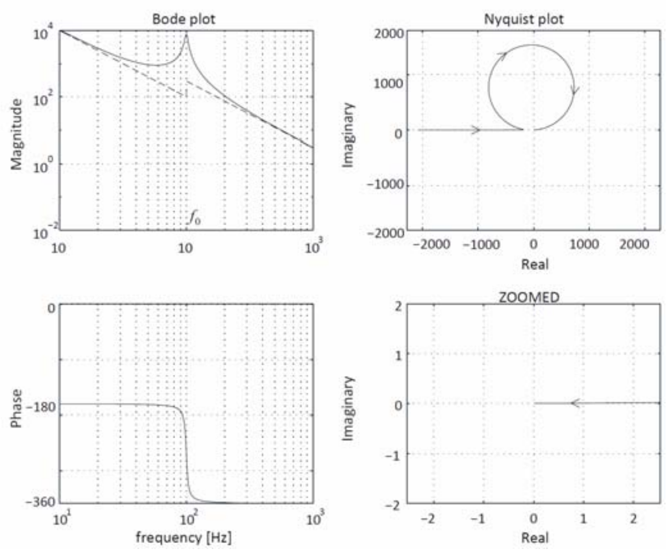
Type II: -2 slope, pole pair, zero pair, -2 slope



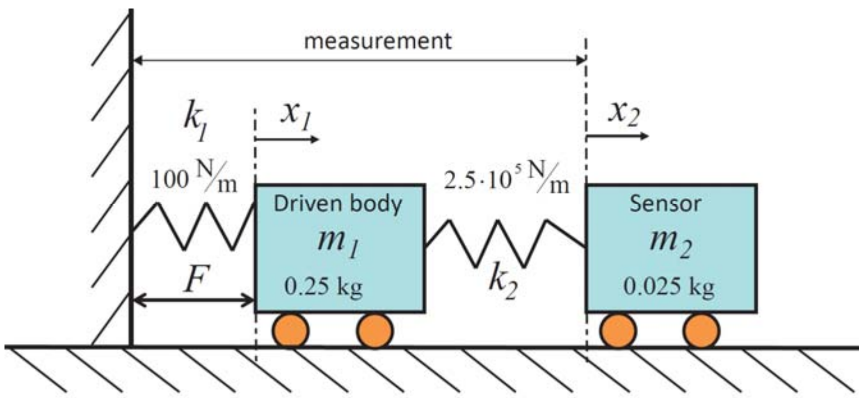
Type III: -2 slope, pole pair, -4 slope



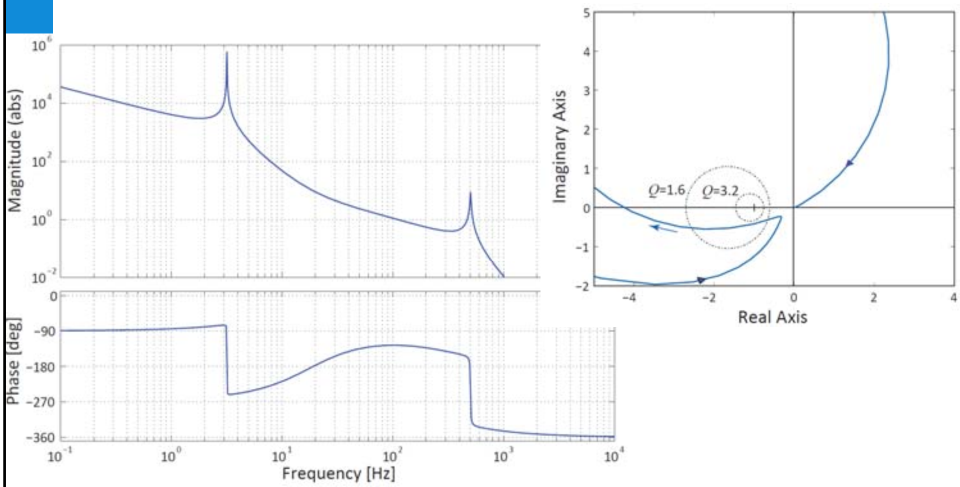
Type IV: -2 slope, pole pair, -2 slope



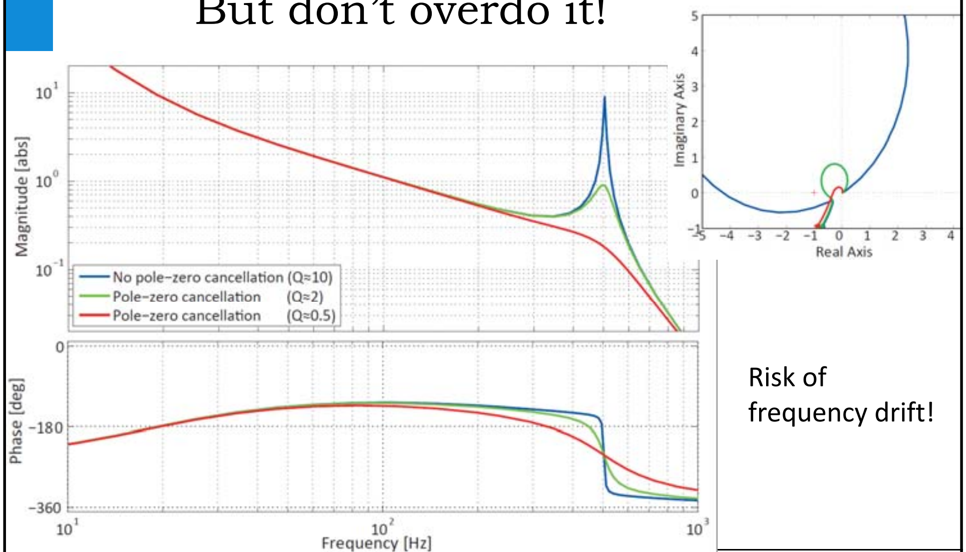
non-collocated measuring at non-rigid body



Bode and Nyquist with PID control open loop of sheet 48

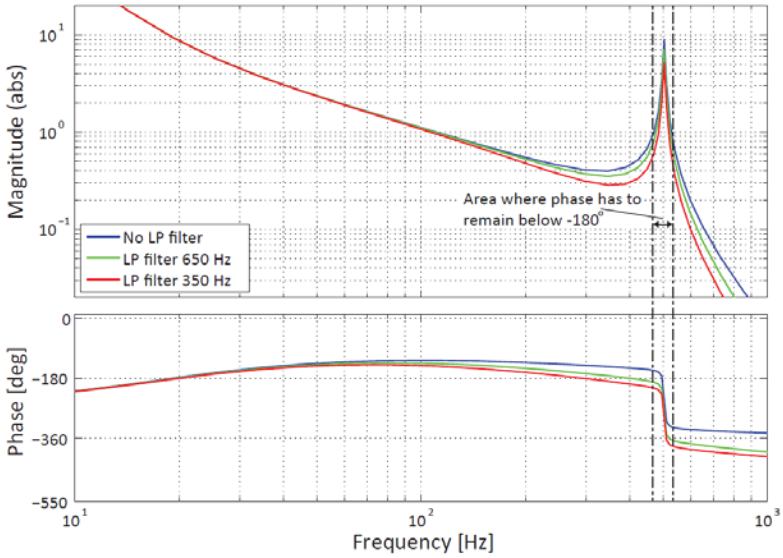


Pole-zero cancellation (notching) But don't overdo it!

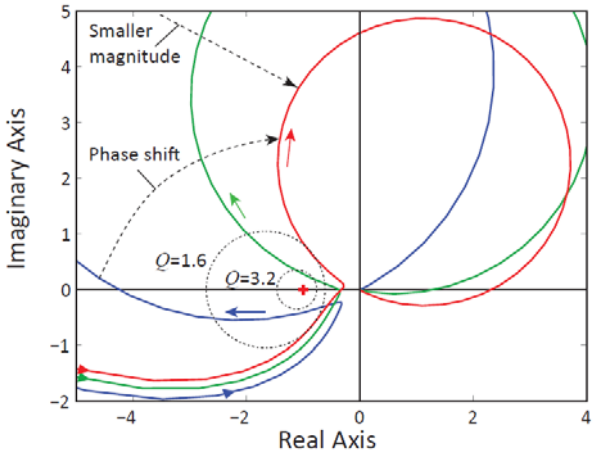


Risk of frequency drift!

Adding a low-pass filter (a pole)



Only Nyquist gives the real answer on stability



Optimisation with LPF corner frequency