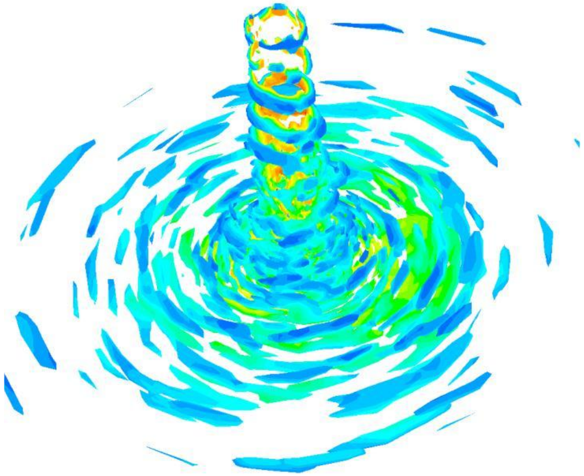
Hydrological Measurements

Prof. Wim Bastiaanssen

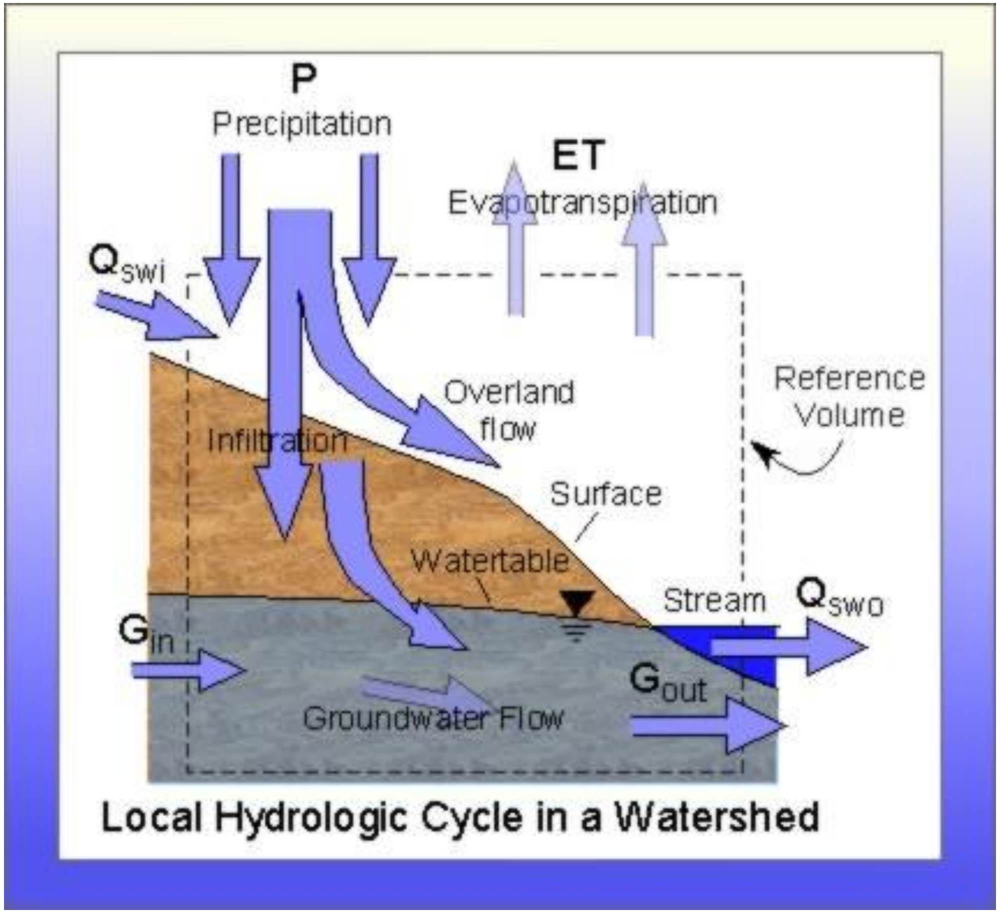
6. Modelling Evaporation

Modelling Evaporation



Prof. Wim Bastiaanssen

ET for hydrological studies



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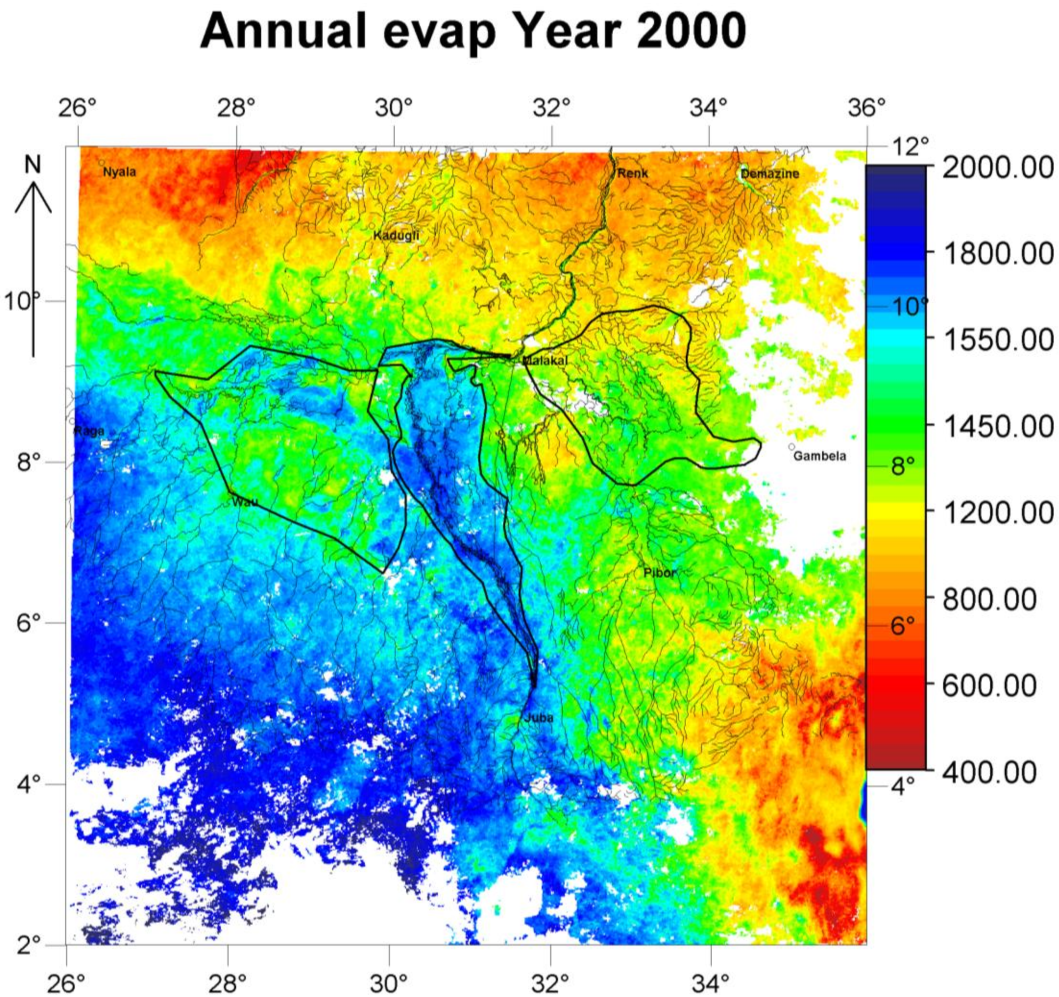
“actual” ET is unpredictable

EB can ‘see’ impacts on ET caused by:

- water shortage
- disease
- crop variety
- planting density
- cropping dates
- salinity
- management

ET for environmental studies



Annual evap Year 2000

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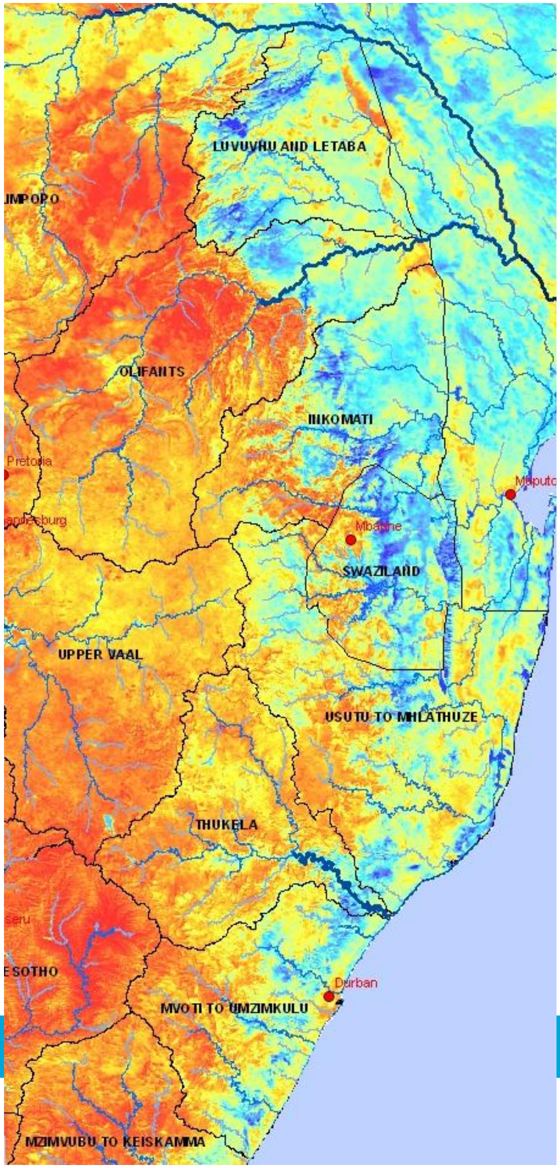
For solving international conflicts



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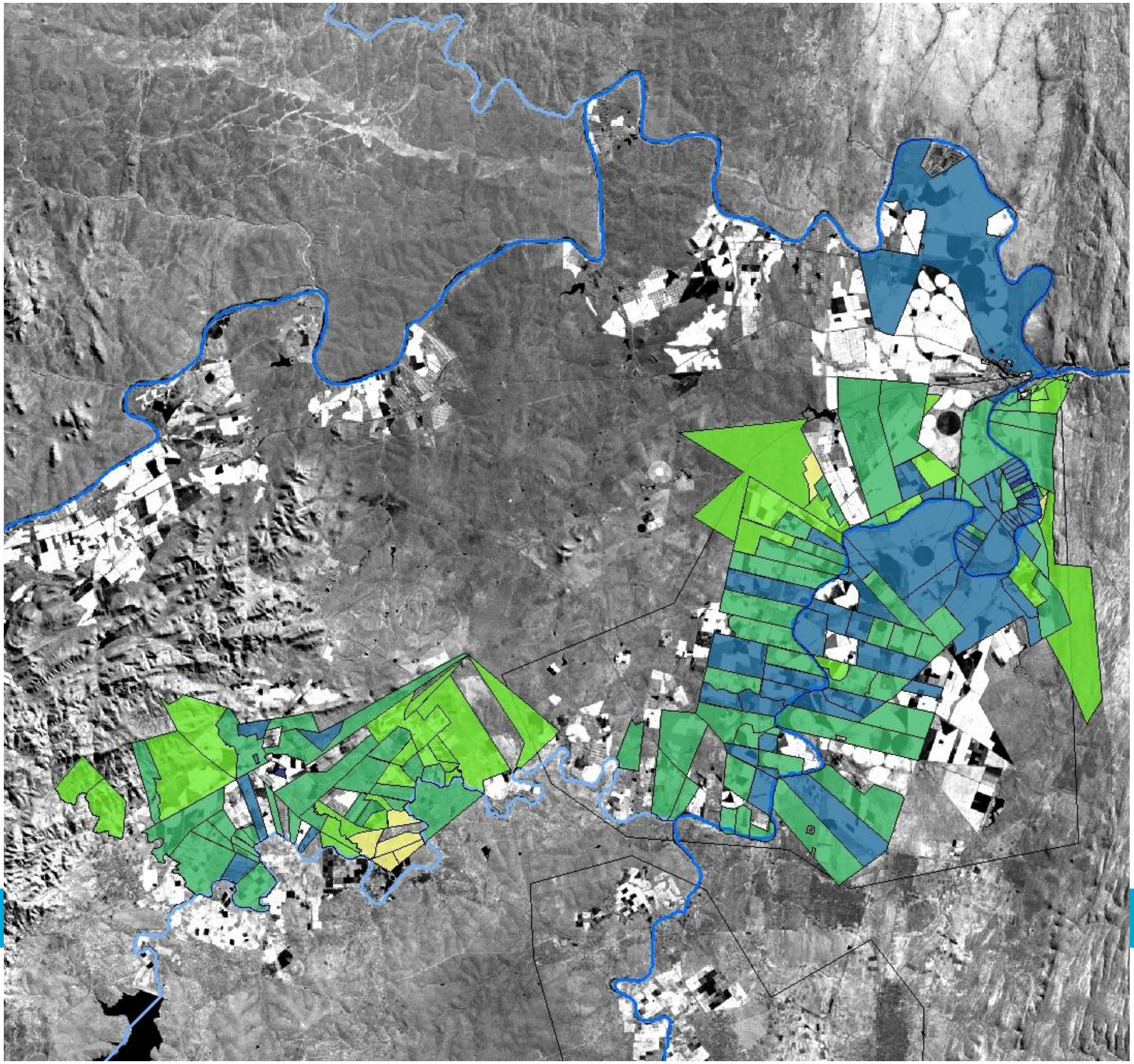
For verification of water use



Avg ETa per plot mm

🟨 501 - 750

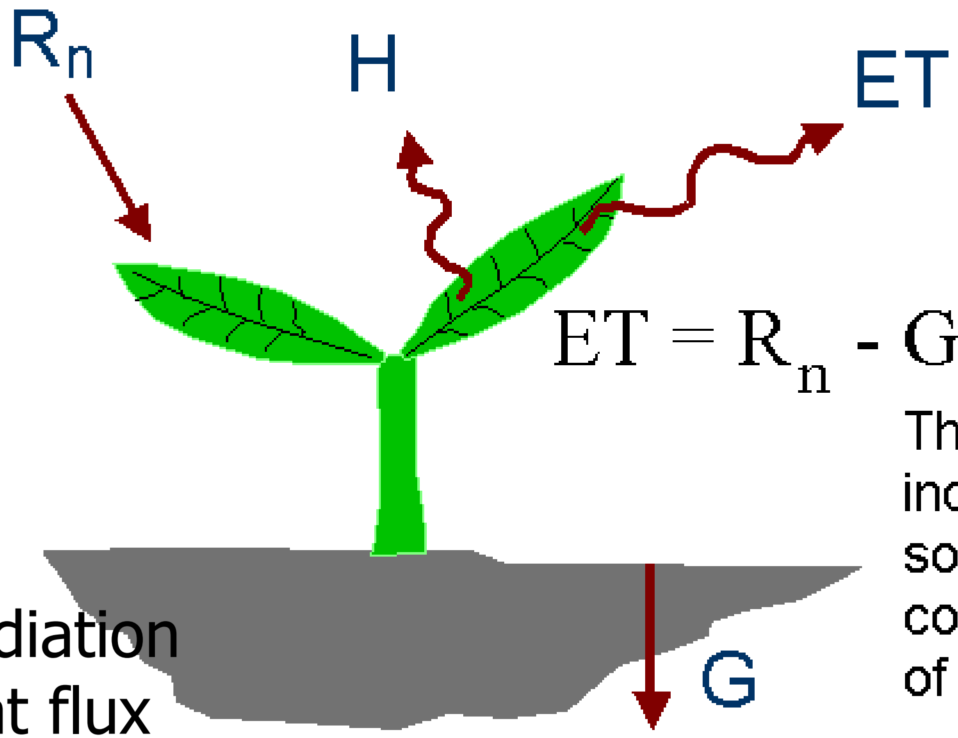
🟩 751 - 900

🟩 901 - 1,050

🟦 1,051 - 1,200

🟦 1,201 - 1,350

ET is calculated as a "residual" of the energy balance

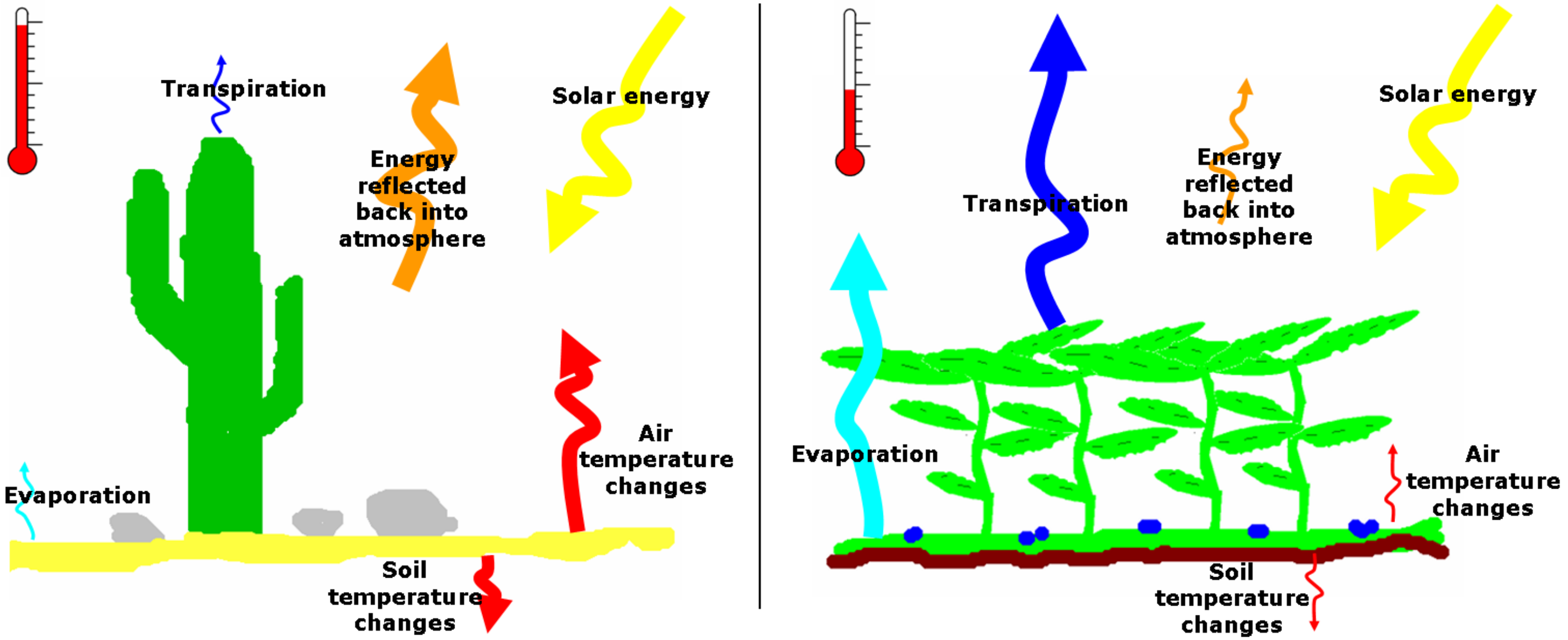


$$ET = R_n - G - H$$

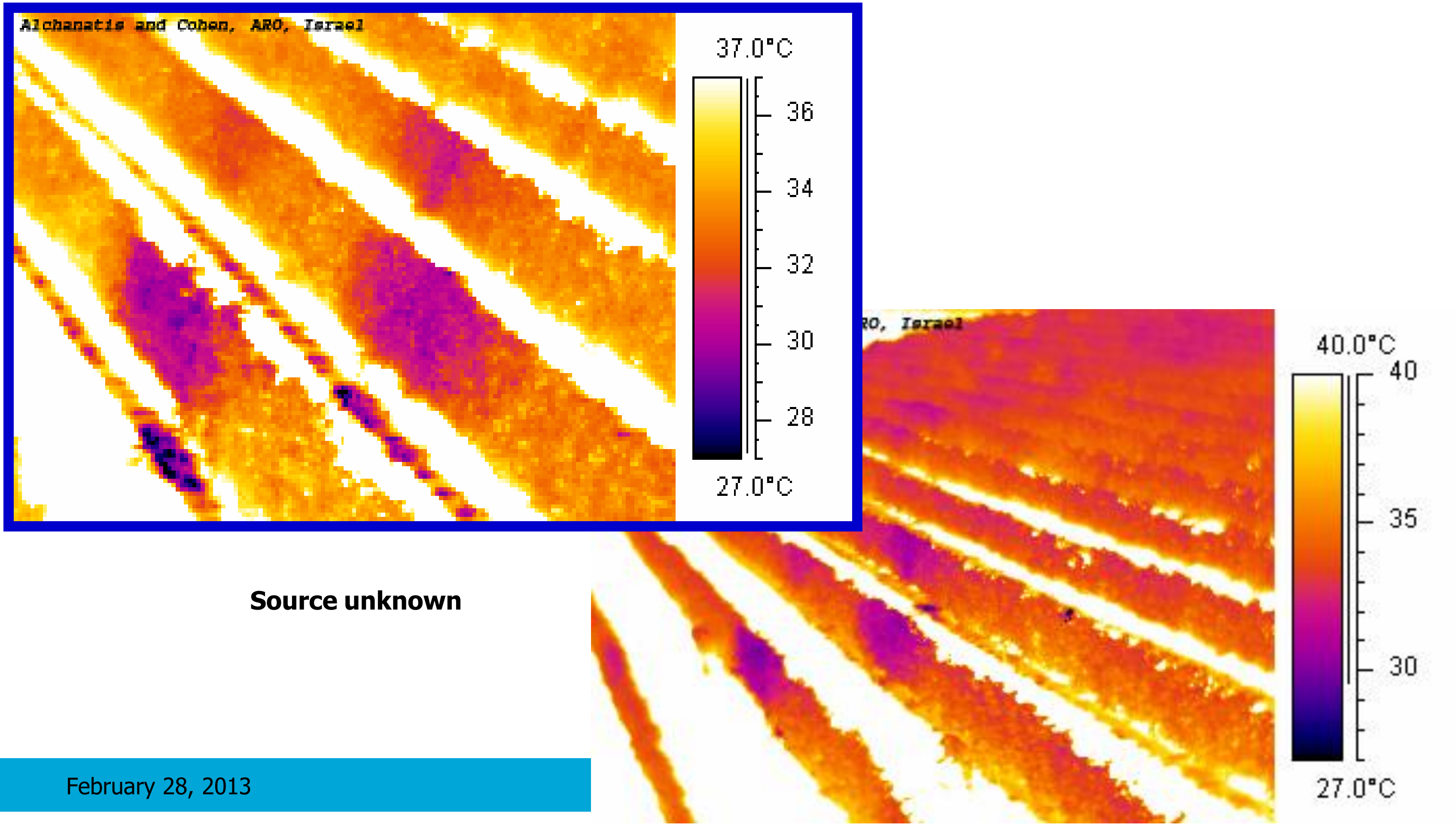
The energy balance includes all major sources (R_n) and consumers (ET, G, H) of energy

Rn=net radiation
G=soil heat flux
H=sensible heat flux

Temperature is a function of ET



Surface temperature is a reflection of soil moisture



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Latent heat of vaporization

TABLE 1
Conversion factors for evapotranspiration

	depth	volume per unit area		energy per unit area *
	mm day^{-1}	m^3 ha^{-1} day^{-1}	l s^{-1} ha^{-1}	MJ m^{-2} day^{-1}
1 mm day^{-1}	1	10	0.116	2.45
1 m^3 ha^{-1} day^{-1}	0.1	1	0.012	0.245
1 l s^{-1} ha^{-1}	8.640	86.40	1	21.17
1 MJ m^{-2} day^{-1}	0.408	4.082	0.047	1

* For water with a density of 1 000 kg m^{-3} and at 20°C.

EXAMPLE 1
Converting evaporation from one unit to another

On a summer day, net solar energy received at a lake reaches 15 MJ per square metre per day. If 80% of the energy is used to vaporize water, how large could the depth of evaporation be?

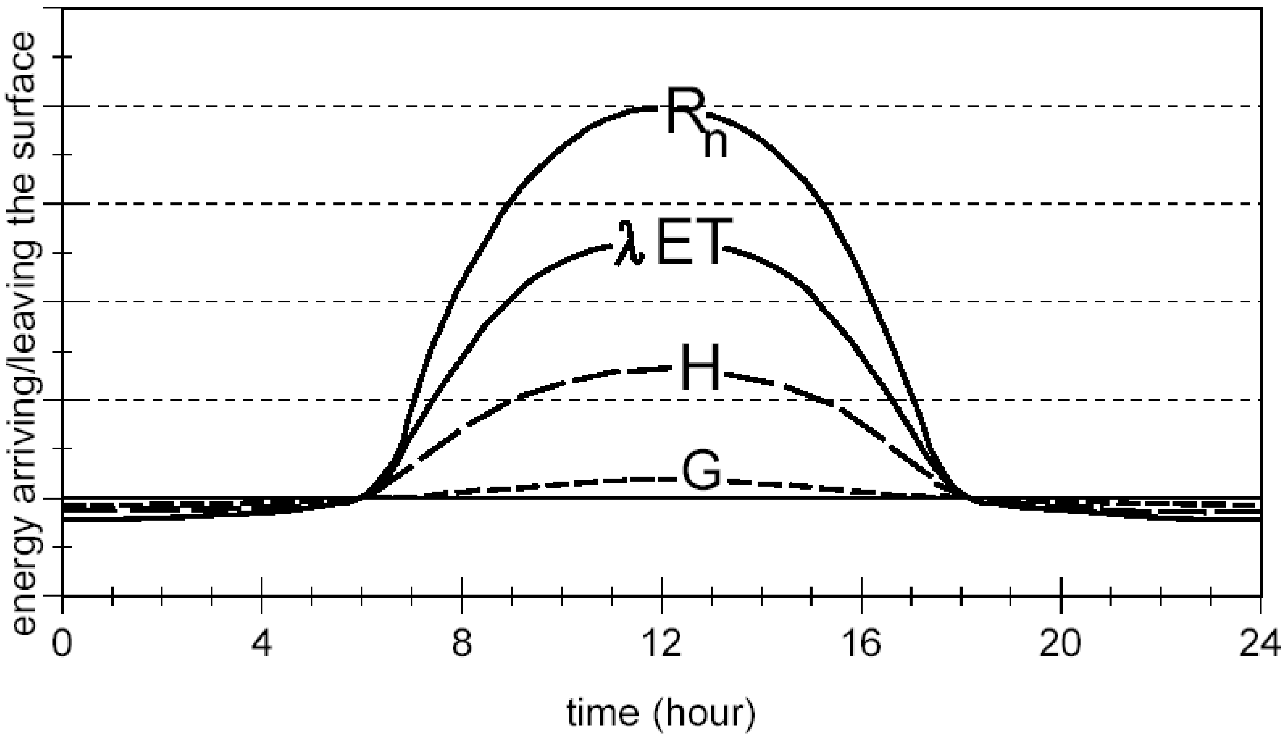
From Table 1:	1 MJ m^{-2} day^{-1} =	0.408	mm day^{-1}
Therefore:	0.8 x 15 MJ m^{-2} day^{-1} = 0.8 x 15 x 0.408 mm d^{-1} =	4.9	mm day^{-1}

The evaporation rate could be 4.9 mm/day

Daily energy balance

FIGURE 5

Schematic presentation of the diurnal variation of the components of the energy balance above a well-watered transpiring surface on a cloudless day



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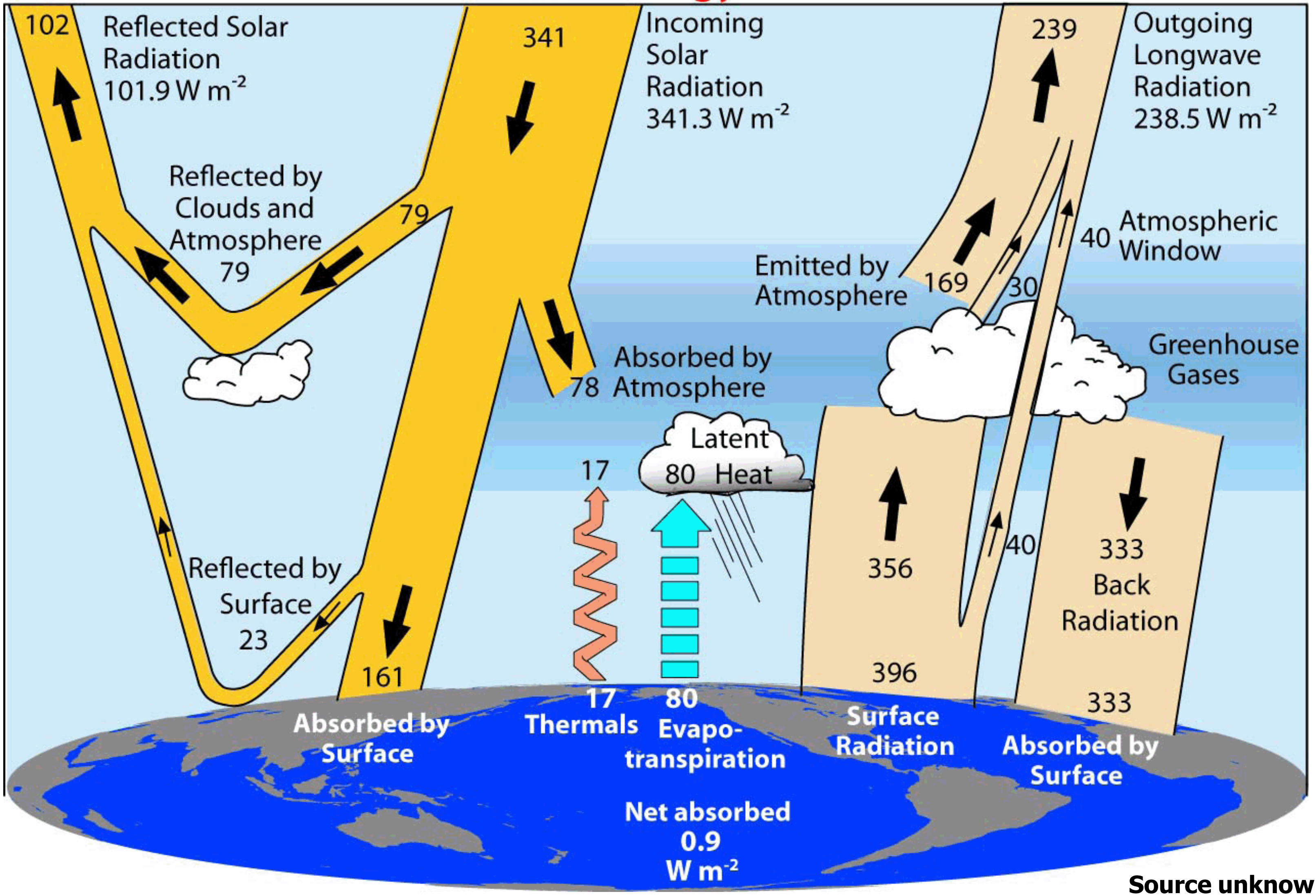
Global Energy Flows W m^{-2}



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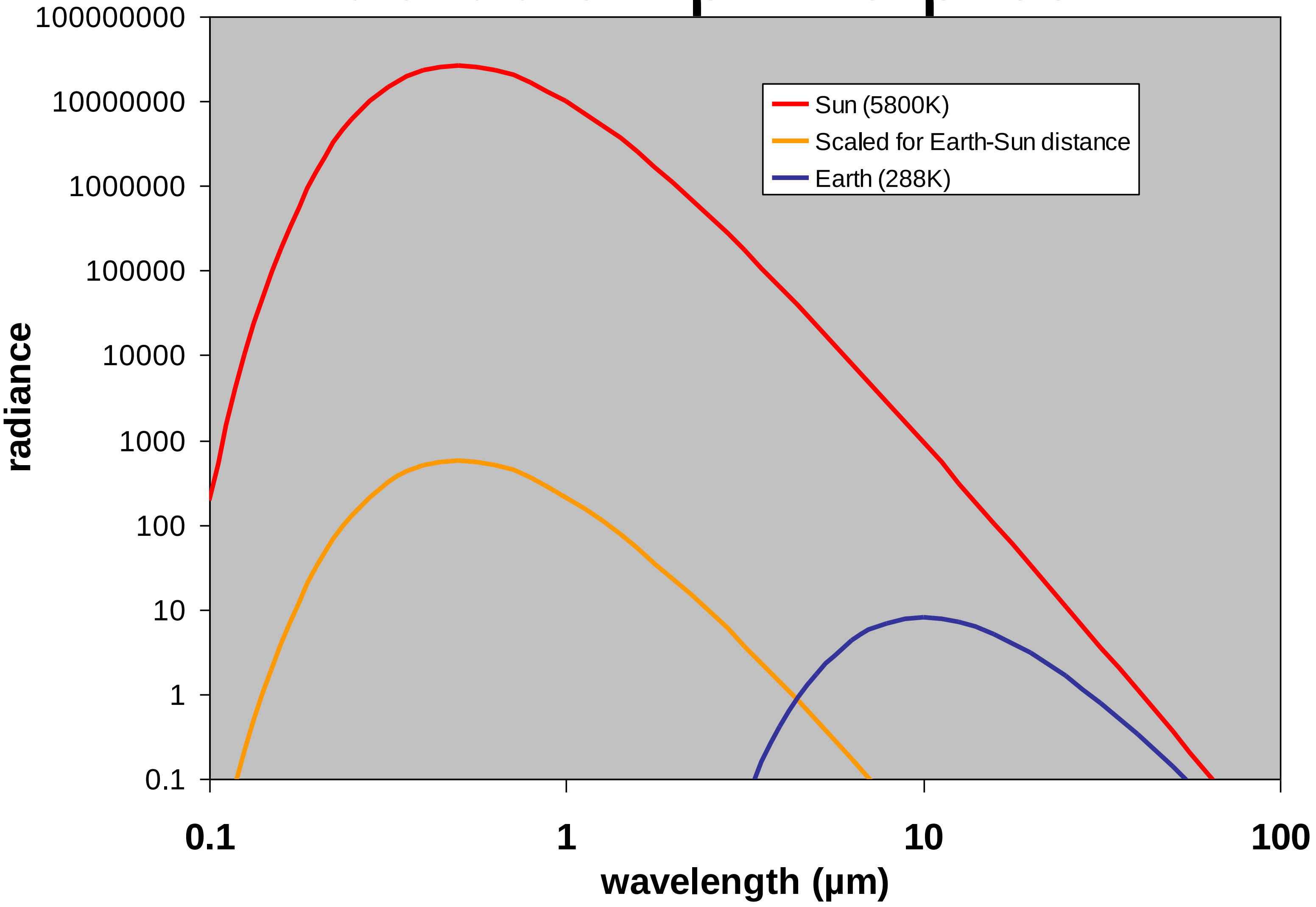
Radiation principles



Planck equation, details

Planck's equation (the spectral curves shown)

$$L_\lambda = \frac{2hc^2}{\lambda^5\left(e^x - 1\right)}, \text{ where } x = \frac{hc}{k\lambda T}$$

Stefan-Boltzmann equation

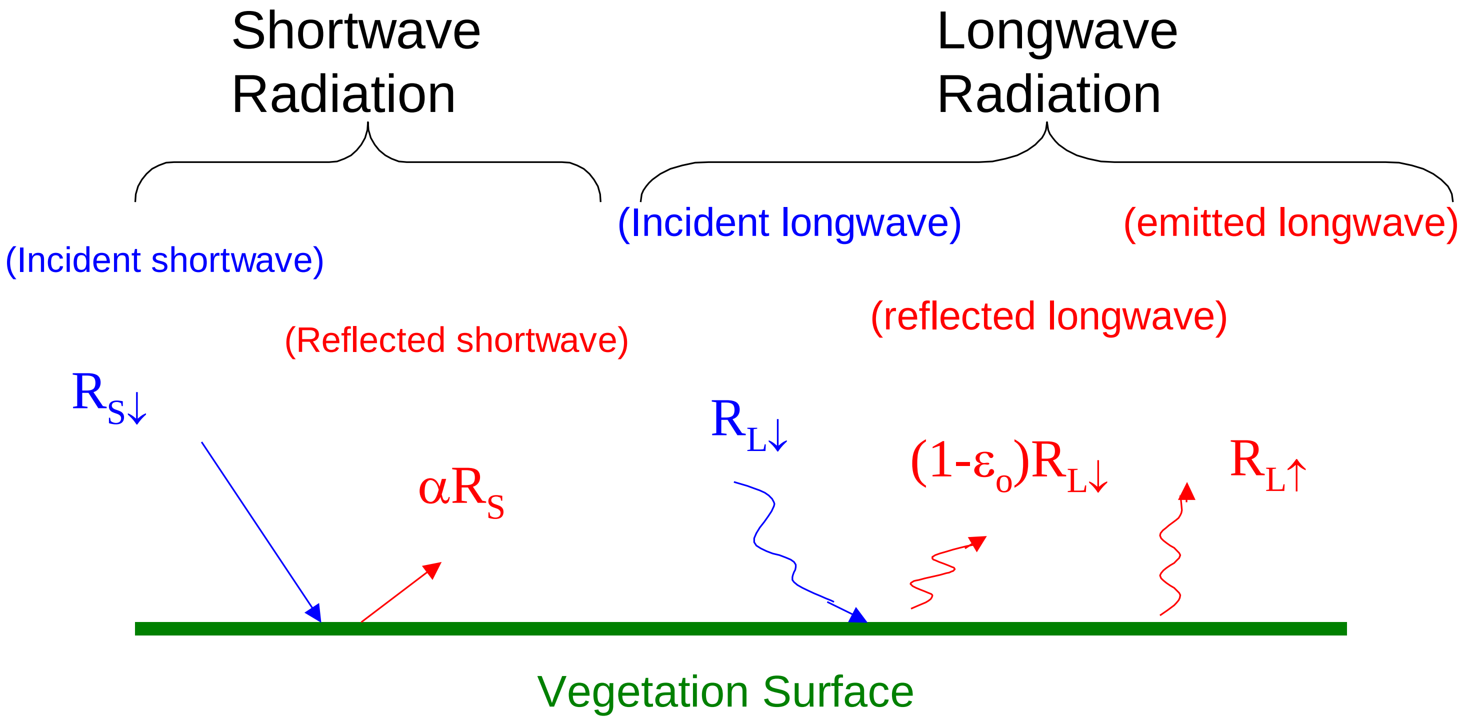
$$E = \pi \int_0^\infty L_\lambda d\lambda = \sigma T^4$$

Wien's displacement equation

$$\lambda_{\text{max}}(\mu\text{m}) = \frac{2897}{T}$$

c	speed of light	$3.00\times10^8\ \text{ms}^{-1}$
h	Planck's constant	$6.63\times10^{-34}\text{Js}$
k	Boltzmann's constant	$1.38\times10^{-23}\text{JK}^{-1}$
σ	Stefan-Boltzmann constant	$5.67\times10^{-8}\text{Wm}^{-2}\text{K}^{-4}$
L_λ	Spectral radiance	$\text{Wm}^{-2}\text{m}^{-1}\text{sr}^{-1}$

Surface Radiation Balance



Shortwave Radiation

Longwave Radiation

(Incident shortwave)

(Reflected shortwave)

(Incident longwave)

(emitted longwave)

(reflected longwave)

$R_{S\downarrow}$

αR_S

$R_{L\downarrow}$

$(1-\varepsilon_o)R_{L\downarrow}$

$R_{L\uparrow}$

Vegetation Surface

Net Surface Radiation = Gains – Losses

$$R_n = (1-\alpha)R_{S\downarrow} + R_{L\downarrow} - R_{L\uparrow} - (1-\varepsilon_o)R_{L\downarrow}$$

Net longwave radiation (1)

$$R_{nl} = \sigma\left[\frac{T_{max,K}{}^{4} + T_{min,K}{}^{4}}{2}\right]\left(0.34 - 0.14\sqrt{e_a}\right)\left(1.35\frac{R_s}{R_{so}} - 0.35\right) \qquad (39)$$

where	R_{nl}	net outgoing longwave radiation [MJ m^{-2} day^{-1}],
	σ	Stefan-Boltzmann constant [4.903 10^{-9} MJ K^{-4} m^{-2} day^{-1}],
	$T_{max,K}$	maximum absolute temperature during the 24-hour period [K = °C + 273.16],
	$T_{min,K}$	minimum absolute temperature during the 24-hour period [K = °C + 273.16],
	e_a	actual vapour pressure [kPa],
	R_s/R_{so}	relative shortwave radiation (limited to $\leq$ 1.0),
	R_s	measured or calculated (Equation 35) solar radiation [MJ m^{-2} day^{-1}],
	R_{so}	calculated (Equation 36 or 37) clear-sky radiation [MJ m^{-2} day^{-1}].

Net longwave radiation (2)

EXAMPLE 11

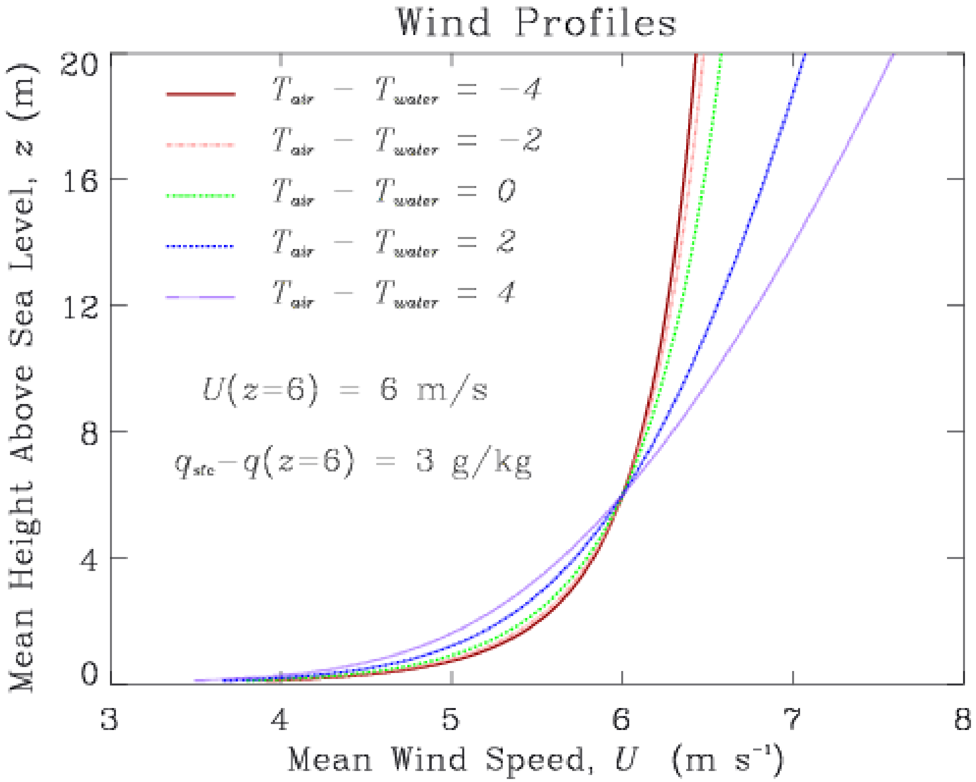
Determination of net longwave radiation

In Rio de Janeiro (Brazil) at a latitude of 22°54'S (= -22.70°), 220 hours of bright sunshine, a mean monthly daily maximum and minimum air temperature of 25.1 and 19.1°C and a vapour pressure of 2.1 kPa were recorded in May. Determine the net longwave radiation.

From Example 10:	R_s =	14.5	MJ m^{-2} day^{-1}
From Eq. 36:	$R_{so} = 0.75\ R_a = 0.75\ .\ 25.1 =$	18.8	MJ m^{-2} day^{-1}
From Table 2.8 or for:	σ =	4.903 10^{-9}	MJ K^{-4} m^{-2} day^{-1}
Then:	T_{max} = 25.1°C =	298.3	K
and:	$\sigma T_{max,K}{}^4$ =	38.8	MJ m^{-2} day^{-1}
and:	T_{min} = 19.1°C =	292.3	K
and:	$\sigma\ T_{min,K}{}^4$ = 35.8 MJ m^{-2} day^{-1}	35.8	MJ m^{-2} day^{-1}
and:	e_a =	2.1	kPa
and:	$0.34 - 0.14\ \sqrt{e_a}$ =	0.14	-
and:	R_s/R_{so} = (14.5)/(18.8)	0.77	-
-	1.35(0.77)-0.35 =	0.69	-
From Eq. 39:	R_{nl} = [(38.7 + 35.7)/2] (0.14) (0.69) =	3.5	MJ m^{-2} day^{-1}
From Eq. 20:	expressed as equivalent evaporation = 0.408 (3.5) =	1.4	mm/day

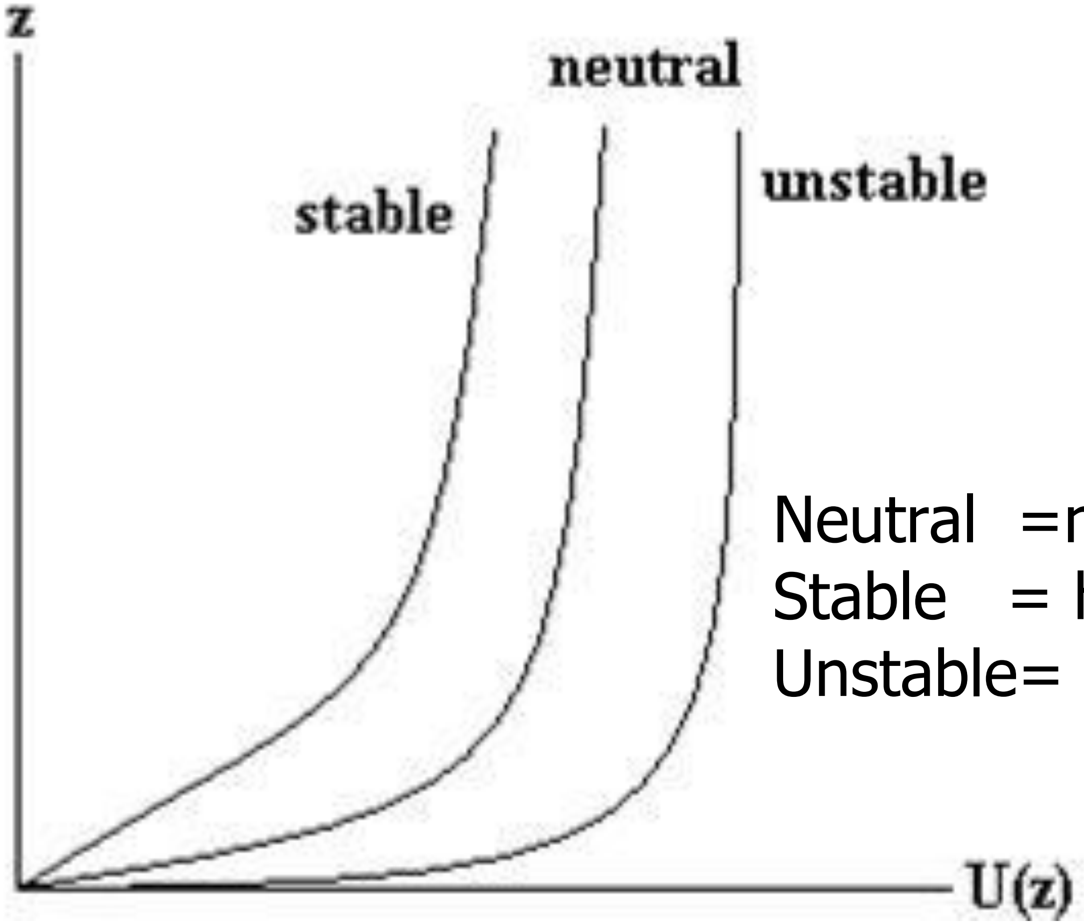
The net longwave radiation is 3.5 MJ m^{-2} day^{-1}.

Logarithmic wind profile

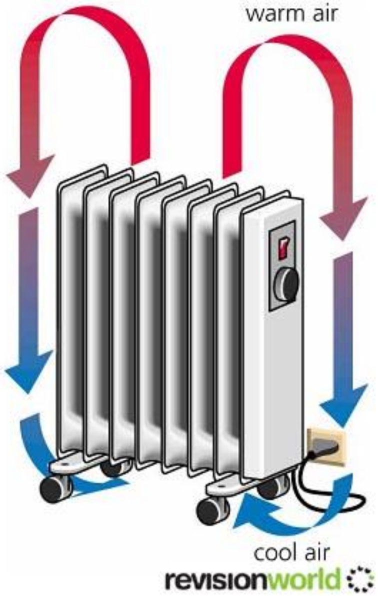


Effect of buoyancy on turbulent transport

height



wind speed



Neutral =no convection
Stable = heat towards land
Unstable= heat away from land

Vertical wind profile – neutral conditions

$$u_* = \frac{k \cdot u_{z1}}{\ln\left(\frac{z_1}{z_{om}}\right)}$$

Vertical wind profile – non neutral conditions

$$u_* = \frac{k \cdot u_{z1}}{\ln\left(\frac{z_1}{z_{om}}\right) - \psi_m\left(\frac{z_1}{L}\right) + \psi_m\left(\frac{z_{om}}{L}\right)}$$

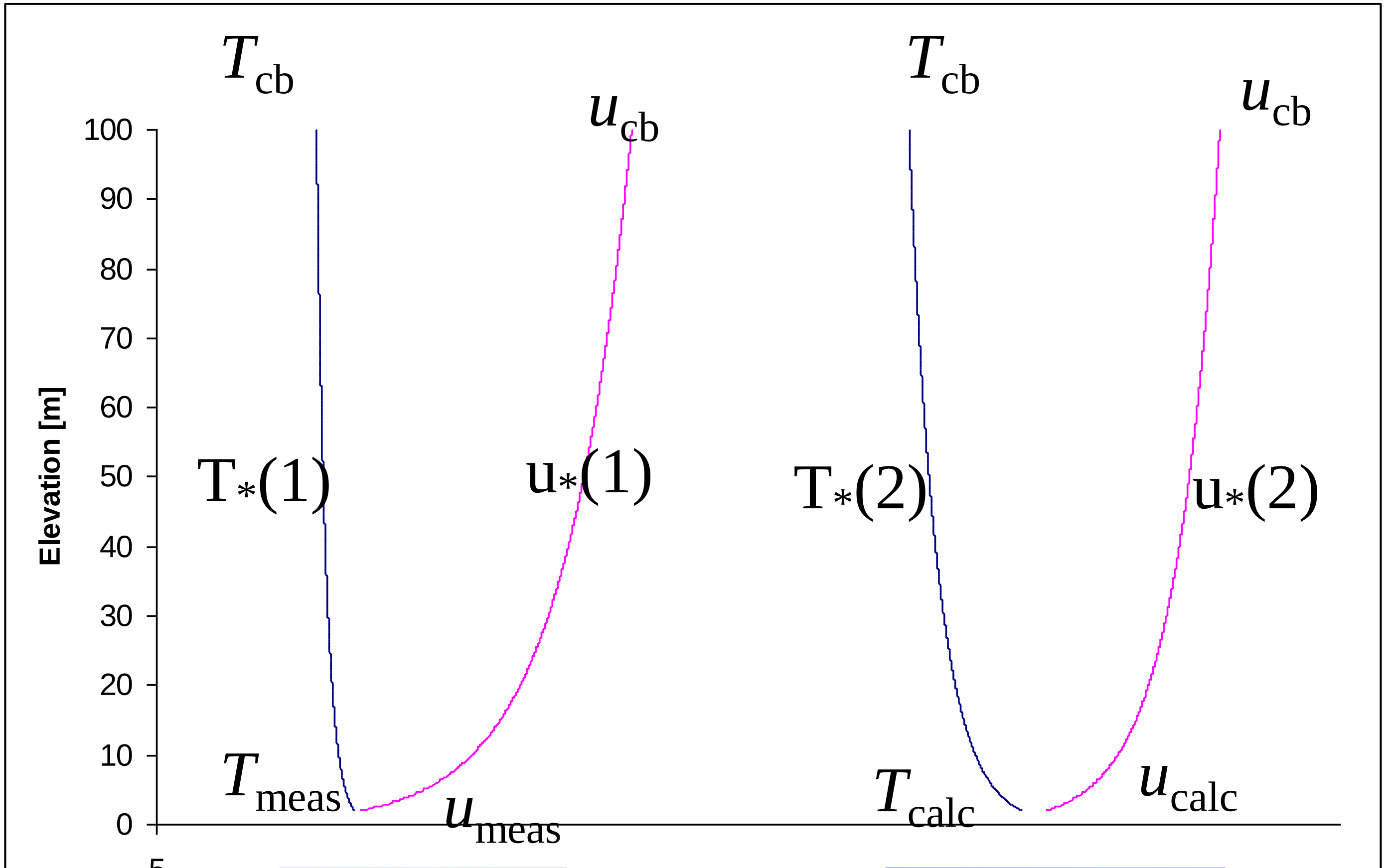
$$L = \frac{-\rho_a \cdot c_p \cdot T \cdot u_*^3}{k \cdot g \cdot H}$$

Monin Obukhov Length

$$\psi_m\left(\frac{z}{L}\right) = \begin{cases} L < 0 : 2 \cdot \ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2 \cdot \arctan(x) + \frac{\pi}{2} \\ L = 0 : 0 \\ L > 0 : -5 \cdot \frac{z}{L} \end{cases}$$

$$x = \sqrt[4]{1 - 16 \cdot \frac{z}{L}}$$

Flux – profile relationships for momentum, heat and vapor



Wind and temperature vertical profiles

$$u_{z2} = u_{z1} + \frac{u_*}{k}\left\{\ln\left(\frac{z_1}{z_2}\right) - \psi_m\left(\frac{z_1}{L}\right) + \psi_m\left(\frac{z_2}{L}\right)\right\}$$

$$T_{z2} = T_{z1} + \frac{T_*}{k}\left\{\ln\left(\frac{z_1}{z_2}\right) - \psi_h\left(\frac{z_1}{L}\right) + \psi_h\left(\frac{z_2}{L}\right)\right\}$$

$$\psi_h\left(\frac{z}{L}\right) = \begin{cases} L < 0 : 2\cdot\ln\left(\frac{1}{2} + \sqrt{\frac{1}{4} - 4\cdot\frac{z}{L}}\right) \\ L = 0 : 0 \\ L > 0 : -5\cdot\frac{z}{L} \end{cases}$$

Heat flux and scalars

$$T_* = \frac{-H}{\rho_a \cdot c_p \cdot u_*}$$

Sensible Heat Flux (H) written as Ohm's law

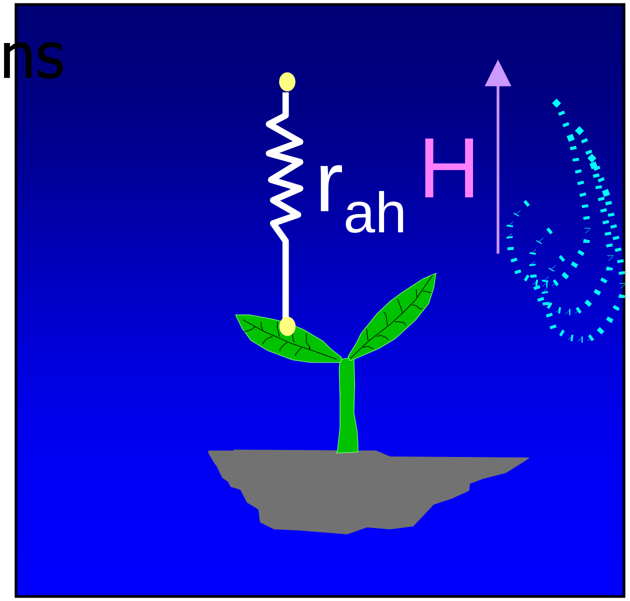
$$H = (\rho \times c_p \times dT) / r_{ah}$$

dT = the near surface temperature difference (K).

r_{ah} = the aerodynamic resistance to heat transport (s/m).

Neutral conditions

$$r_{ah} = \frac{\ln\left(\frac{z_2}{z_1}\right)}{u_* \times k}$$



u* =friction velocity [m/s]

Stability correction for buoyancy

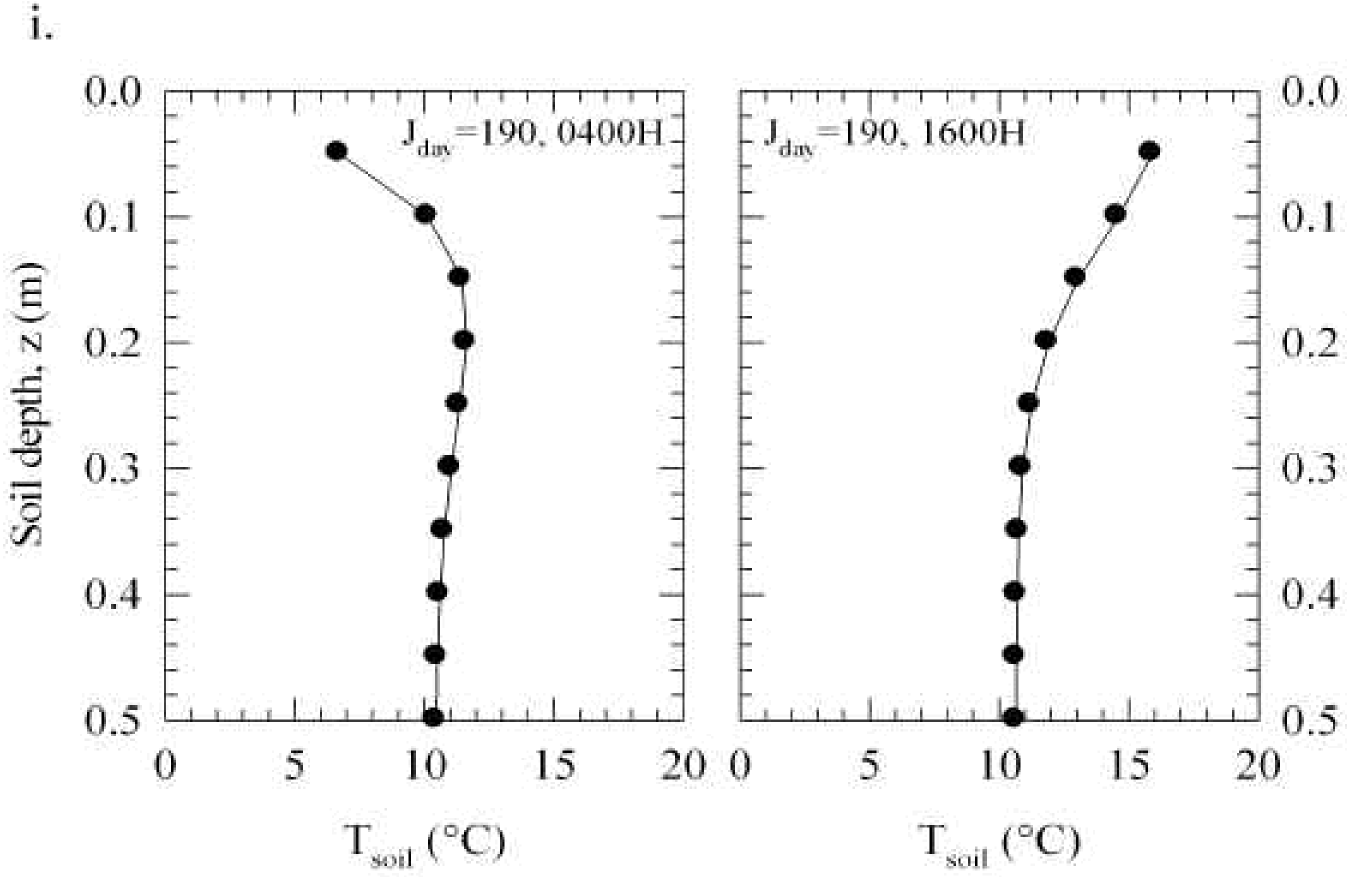
$$u^* = \frac{u_{200} k}{\ln\left(\frac{200}{z_{0m}}\right) - \Psi_{m(200m)} + \Psi_{m(z0m)}}$$

$$r_{ah} = \frac{\ln\left(\frac{z_2}{z_1}\right) - \Psi_{h(z_2)} + \Psi_{h(z_1)}}{u_* \times k}$$

Transfer equation sensible heat

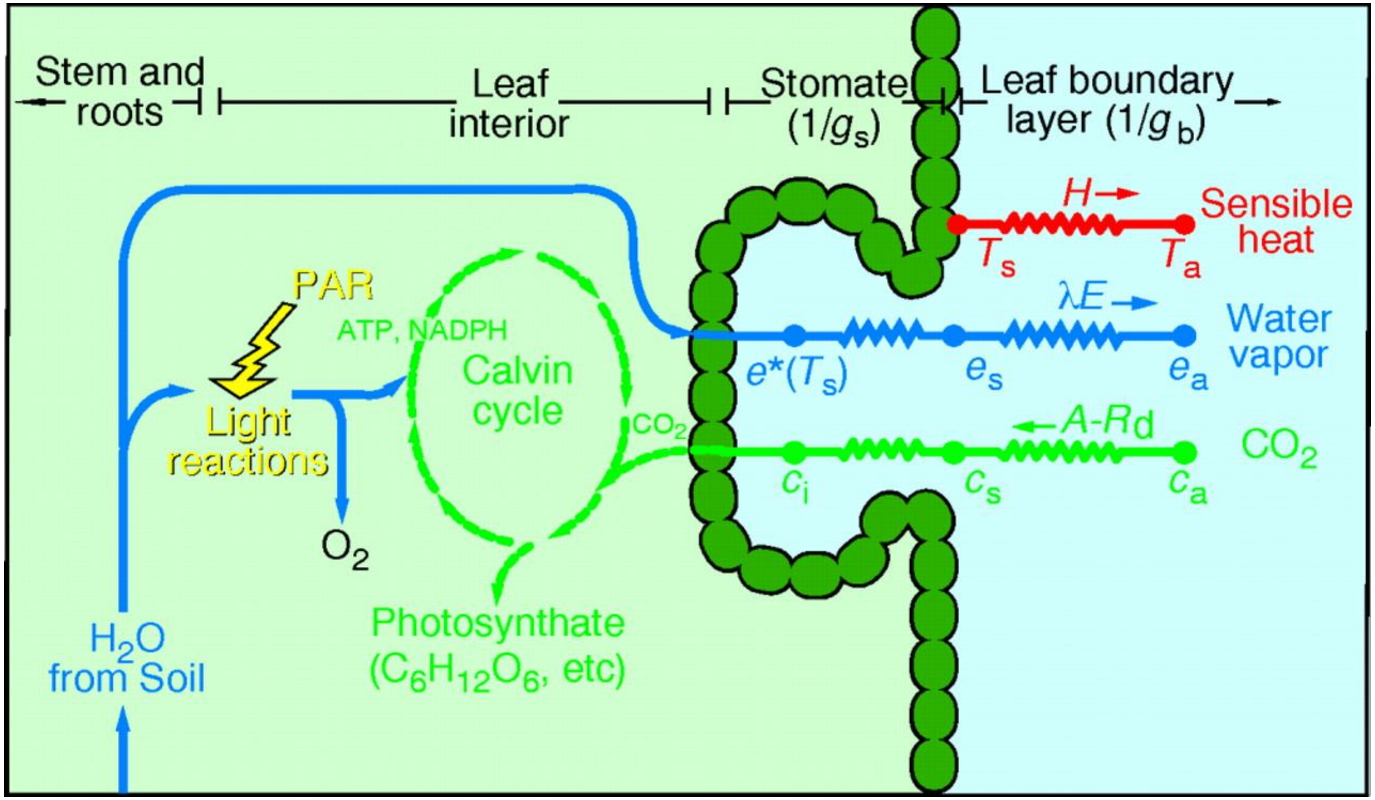
$$H = -\rho c_p u_* T_* = \rho c_p C_h U(T_0 - T_a) = \rho c_p[(T_0 - T_a)/r_{\mathrm{ah}}], \tag{3}$$

Soil heat flux



$$J_q = -K_q \frac{\partial T}{\partial x}$$

Transpiration process



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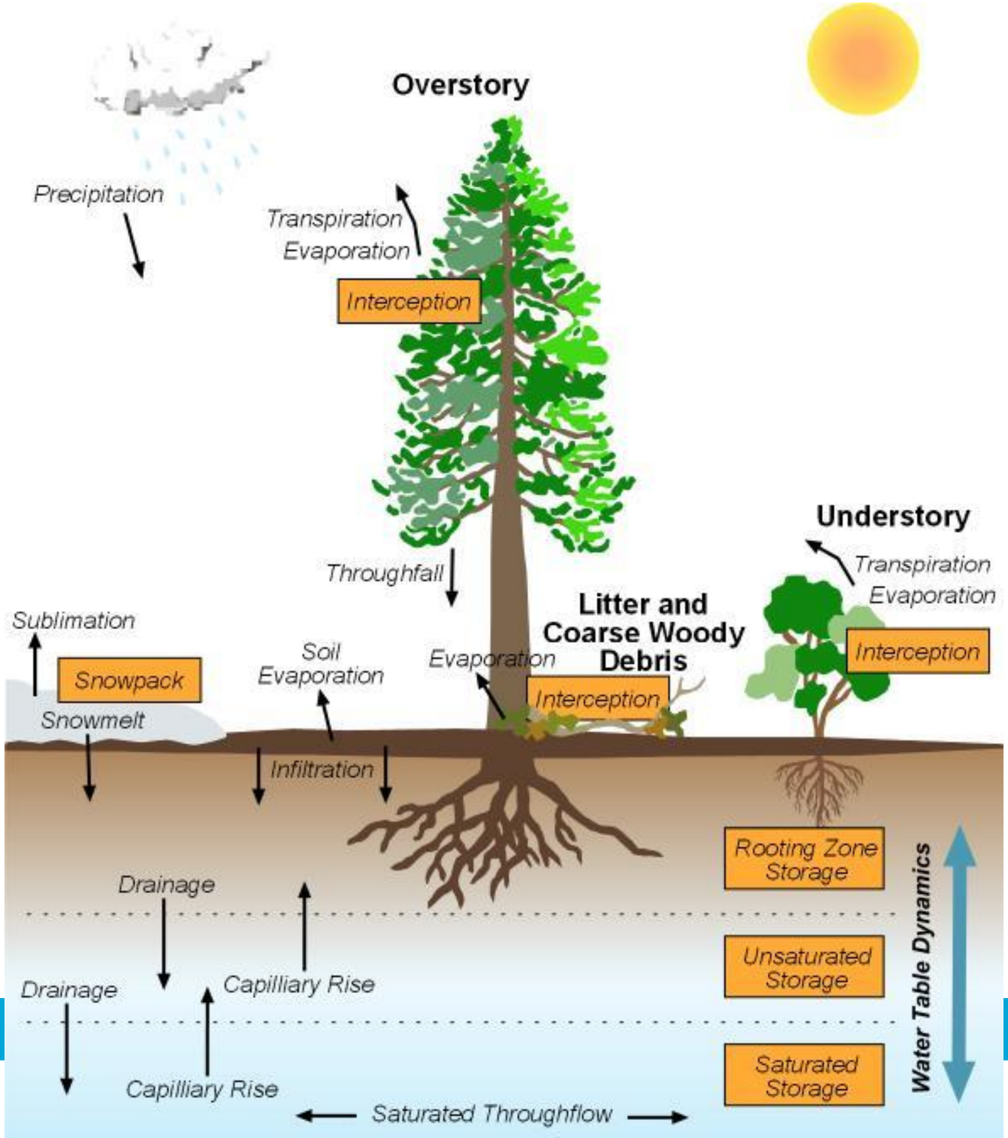
Soil evaporation process



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Transfer equation for latent heat

$$LE = \lambda \rho_{air} C_E u (q_{satTs} - q_a),$$

where

ρ_{air} is the density of moist air, kg/m^3,

C_E is a bulk transfer coefficient for water vapor, dimensionless,

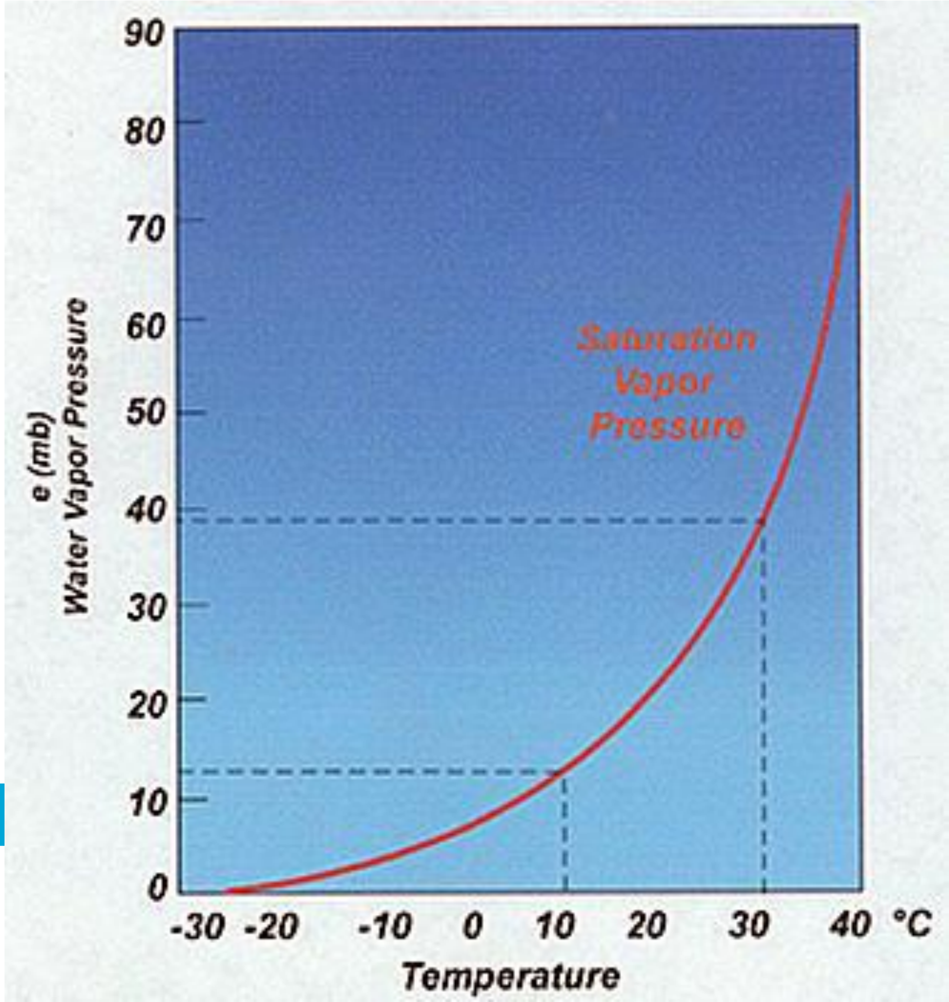
u is wind speed, in m/s,

q_{satTs} is saturated specific humidity at surface temperature, in kg/kg,

q_a is specific humidity at observation height, kg/kg.

Slope of the saturated vapor pressure curve

$$E_{sat}\ (T_0) = e_{sat}(T_a) + \mathrm{SLOPE}\ (T_0 - T_a)$$



Slope = Δ

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TUDelft

Penman – Monteith equation

$$LE = \frac{\Delta\,(R_n - G) + \rho\, c_p\, vpd / r_a}{\Delta + \gamma\,(1 + r_s / r_a)}$$

Bio-physical parameters (besides weather parameters)

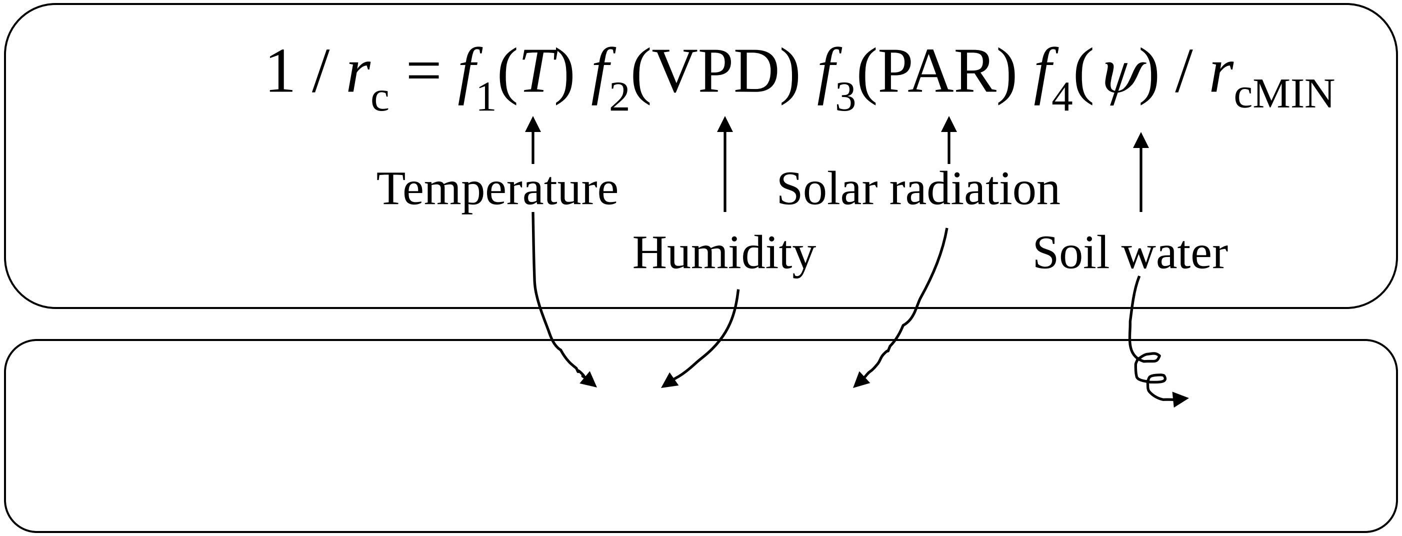
- Albedo
- Emissivity
- G/R_n
- Surface roughness, r_a
- Stomatal resistance, r_s
- LAI, r_s

Jarvis – Stewart model

Canopy resistance model:

$r_c = r_{smin} / LAI \{\phi_{par}\ \phi_{temp}\ \phi_{vpd}\ \phi_{mois})$

Soil moisture and surface resistance



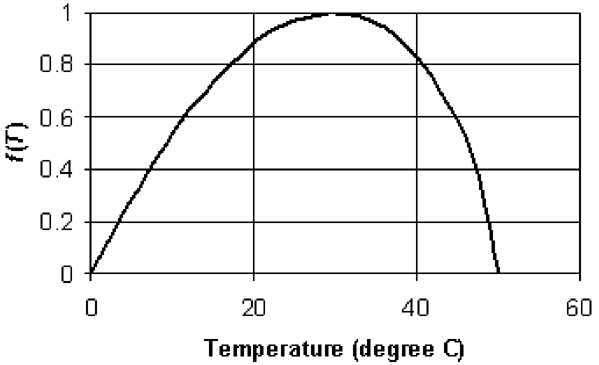
$$1 / r_c = f_1(T)\, f_2(\mathrm{VPD})\, f_3(\mathrm{PAR})\, f_4(\psi) / r_{\mathrm{cMIN}}$$

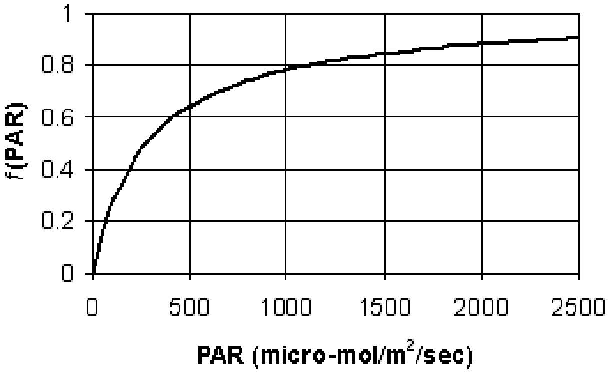
Temperature

Humidity

Solar radiation

Soil water



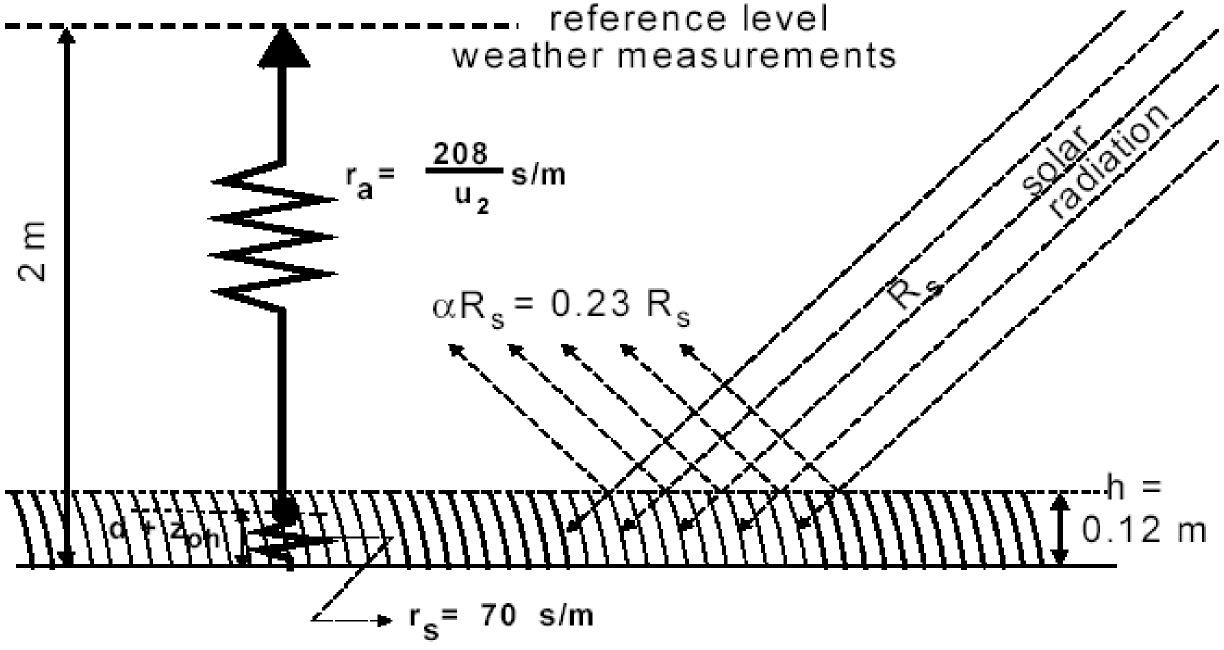


This solves the ET from MICW measurements

Reference ET

FIGURE 9

Characteristics of the hypothetical reference crop



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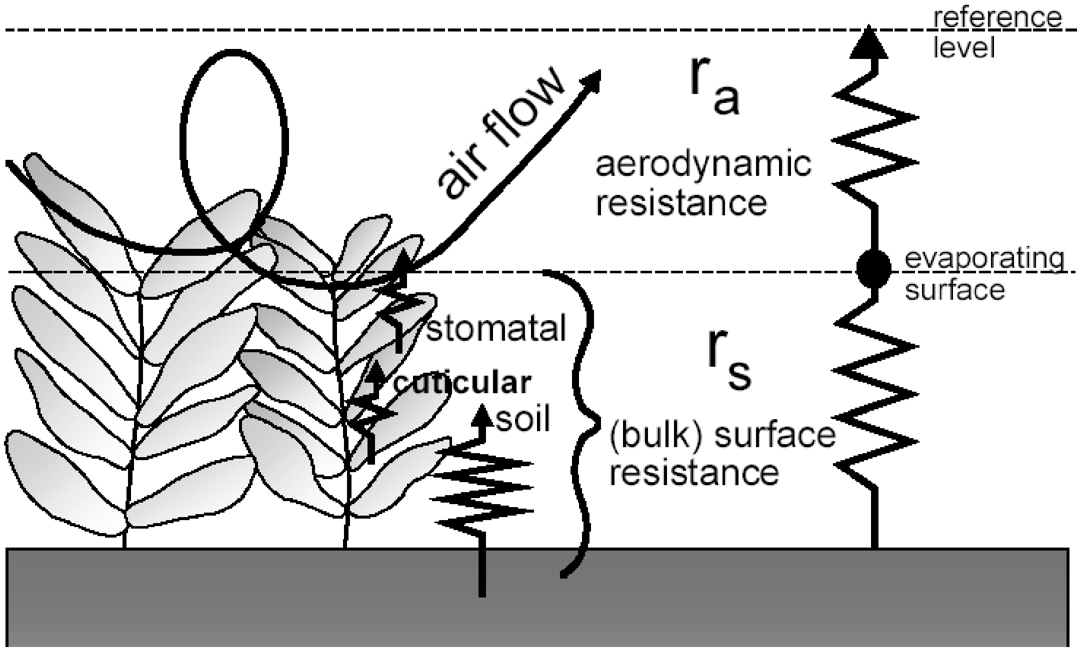
Penman-Monteith for ET_{ref} (ET_0)

FIGURE 7

Simplified representation of the (bulk) surface and aerodynamic resistances for water vapour flow



The Penman-Monteith form of the combination equation is:

$$\lambda ET = \frac{\Delta (R_n - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a}}{\Delta + \gamma \left(1 + \frac{r_s}{r_a}\right)} \qquad (3)$$

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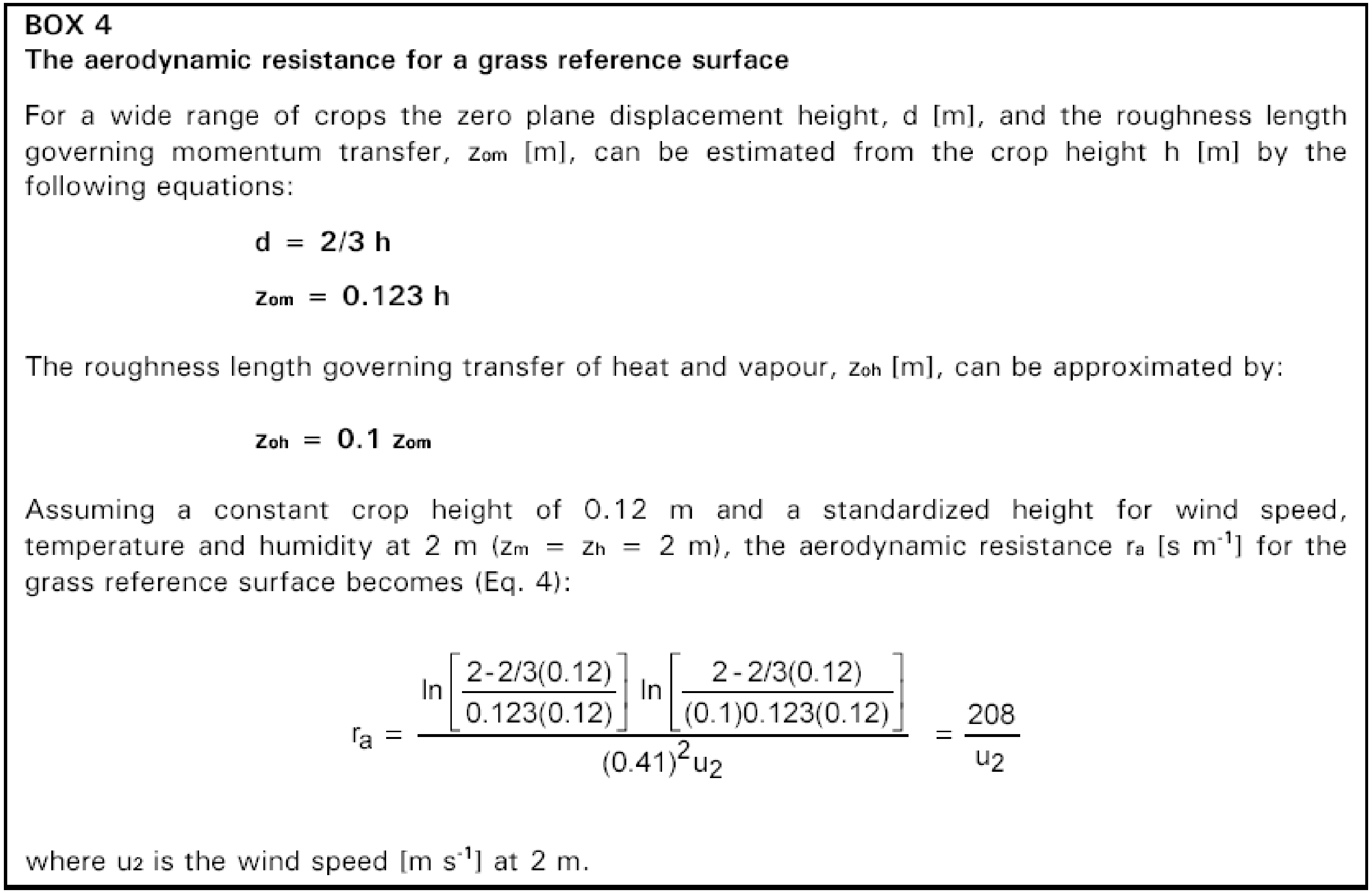
Aerodynamic resistance



BOX 4

The aerodynamic resistance for a grass reference surface

For a wide range of crops the zero plane displacement height, d [m], and the roughness length governing momentum transfer, z_{om} [m], can be estimated from the crop height h [m] by the following equations:

$$d = 2/3\ h$$

$$z_{om} = 0.123\ h$$

The roughness length governing transfer of heat and vapour, z_{oh} [m], can be approximated by:

$$z_{oh} = 0.1\ z_{om}$$

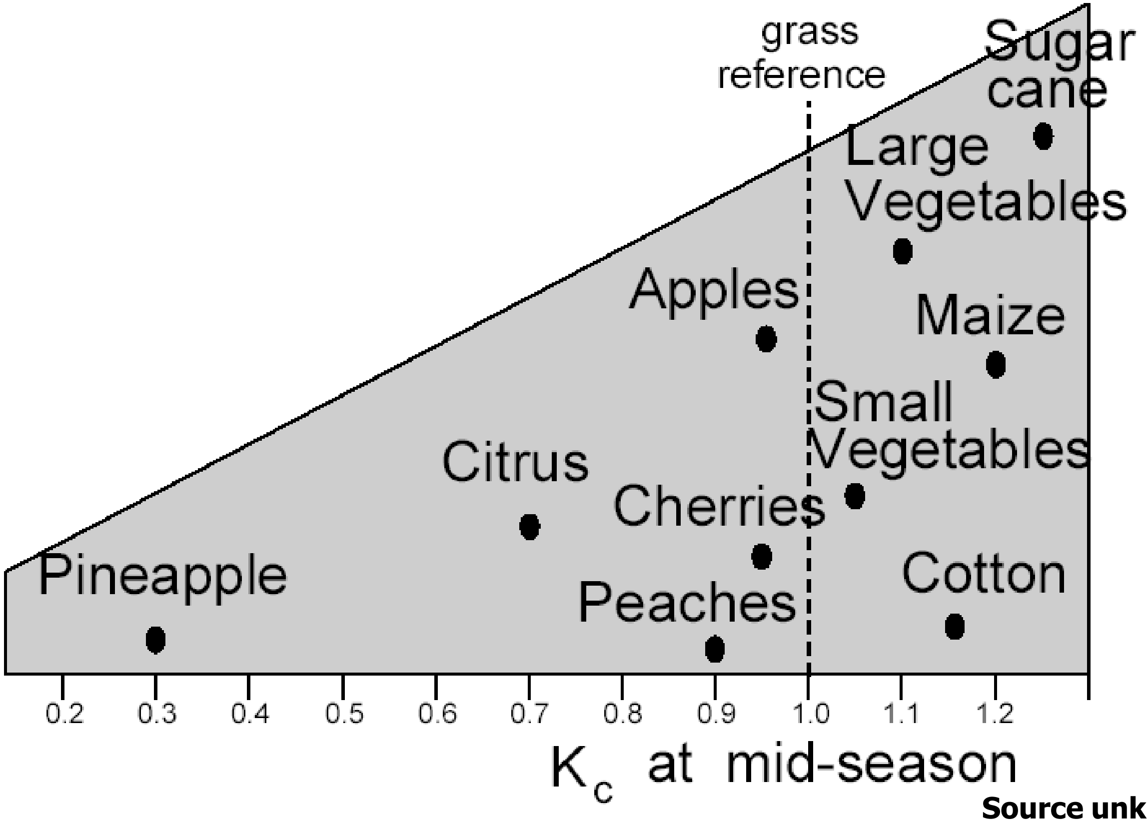
Assuming a constant crop height of 0.12 m and a standardized height for wind speed, temperature and humidity at 2 m ($z_m = z_h = 2$ m), the aerodynamic resistance r_a [s m^{-1}] for the grass reference surface becomes (Eq. 4):

$$r_a = \frac{\ln\left[\frac{2-2/3(0.12)}{0.123(0.12)}\right]\ln\left[\frac{2-2/3(0.12)}{(0.1)0.123(0.12)}\right]}{(0.41)^2 u_2} = \frac{208}{u_2}$$

where u_2 is the wind speed [m s^{-1}] at 2 m.

Potential ET for correction of grass

FIGURE 20
Typical K_c for different types of full grown crops



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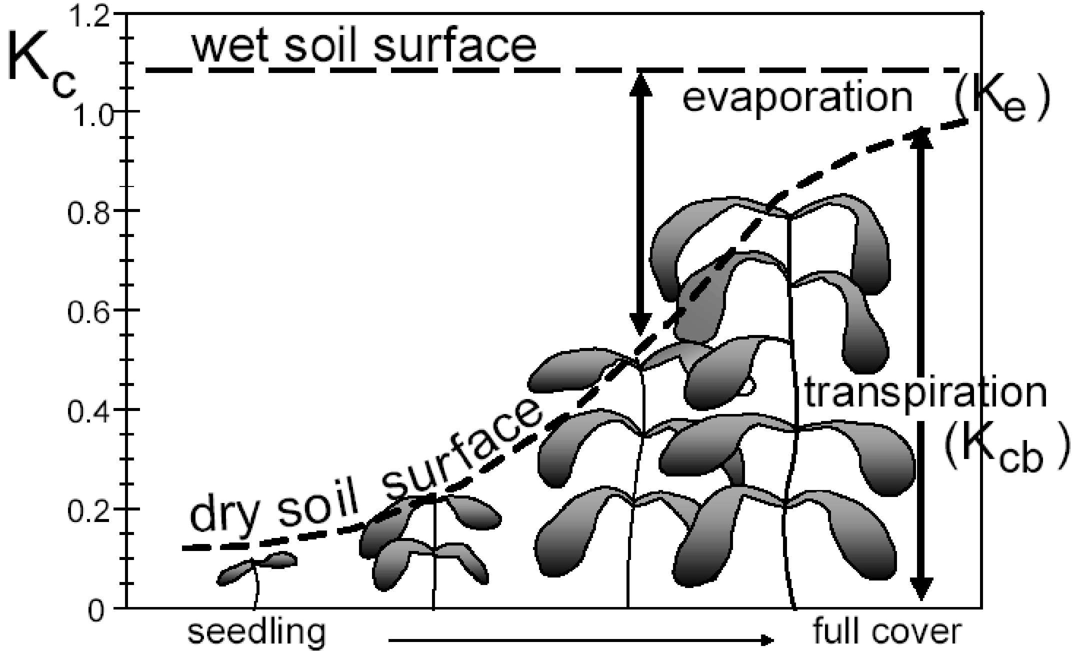
Crop coefficient

FIGURE 22

The effect of evaporation on K_c. The horizontal line represents K_c when the soil surface is kept continuously wet. The curved line corresponds to K_c when the soil surface is kept dry but the crop receives sufficient water to sustain full transpiration



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