Traffic Flow Theory and Simulation

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Lecture 6
Traffic Analysis: Characteristics

TU Delft

Traffic analyses - characteristics

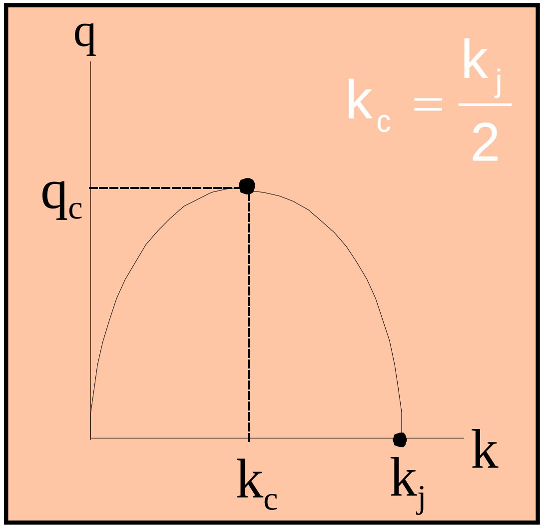
24-3-2014

Recap of shockwave theory

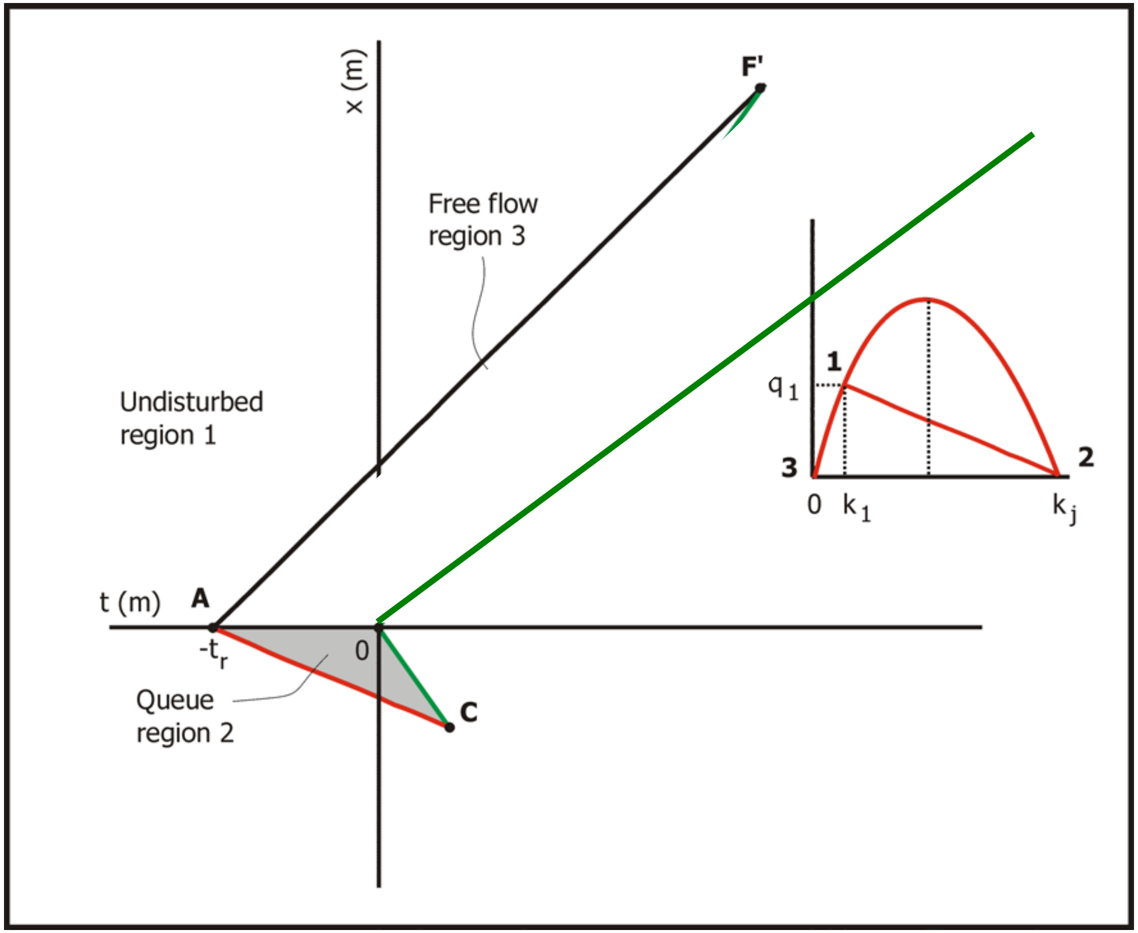
- Queuing at signalized intersection
- Upstream traffic demand equals q_1 (region 1)
- Initial conditions are free-flow (k_1,q_1) for all x until $t=-t_r$
- Assume Greenshields simplified fundamental diagram

Free speed

Jam density

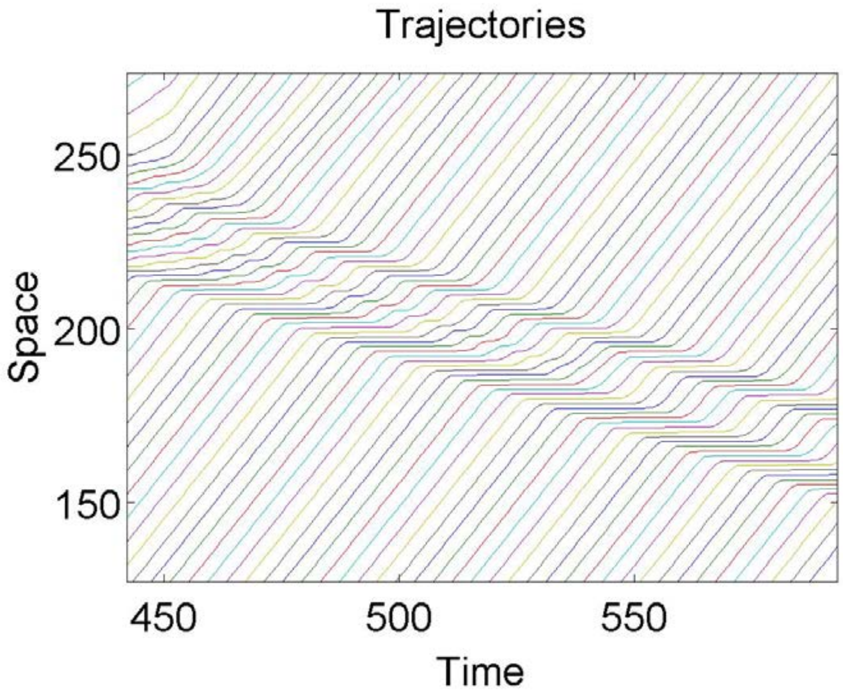


Queuing at signalized intersection (2)



Characteristics?

- Characteristic: line in x-t plane with constant properties



Method of Characteristics (MoC)

- Approach to solve traffic evolution analytically
  - Construct *characteristics* / *characteristic curves*: lines in the xt-plane with constant density (and speed, and flow)
  - Conditions known at boundaries, so we know them along the characterstics as well
  - Enough characteristics =>
    determine the traffic state everywhere

Method of characteristics - recepy

Characteristics are lines in the (x,t)-plane:

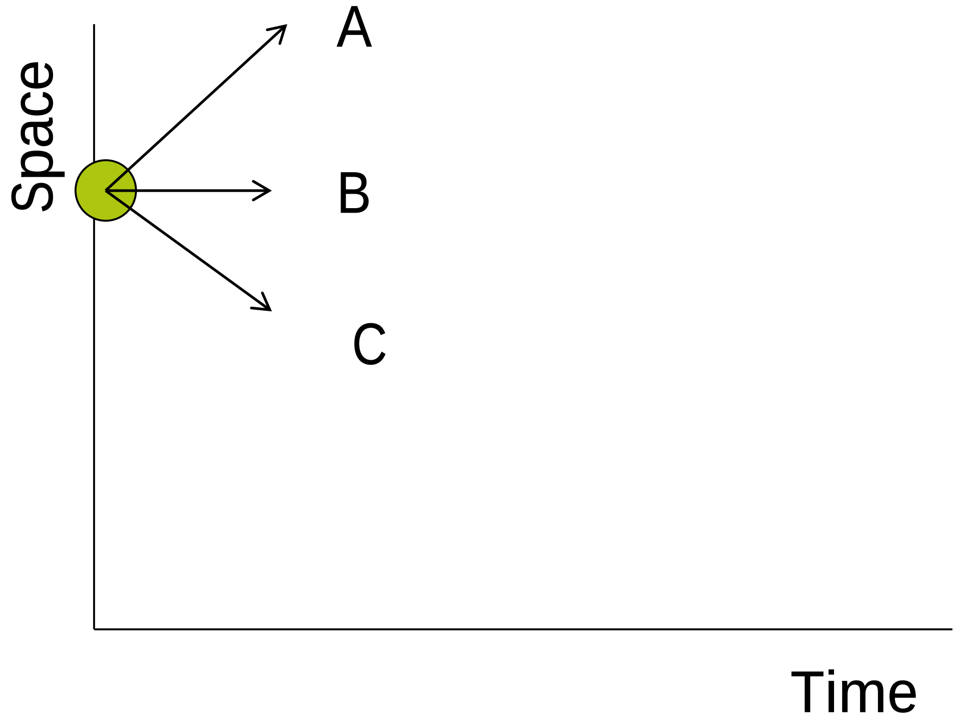
1. Slope of characteristic depends on the condition
2. Density is **constant** along characteristics
3. Combine 2+3: characteristics are straight lines from boundary

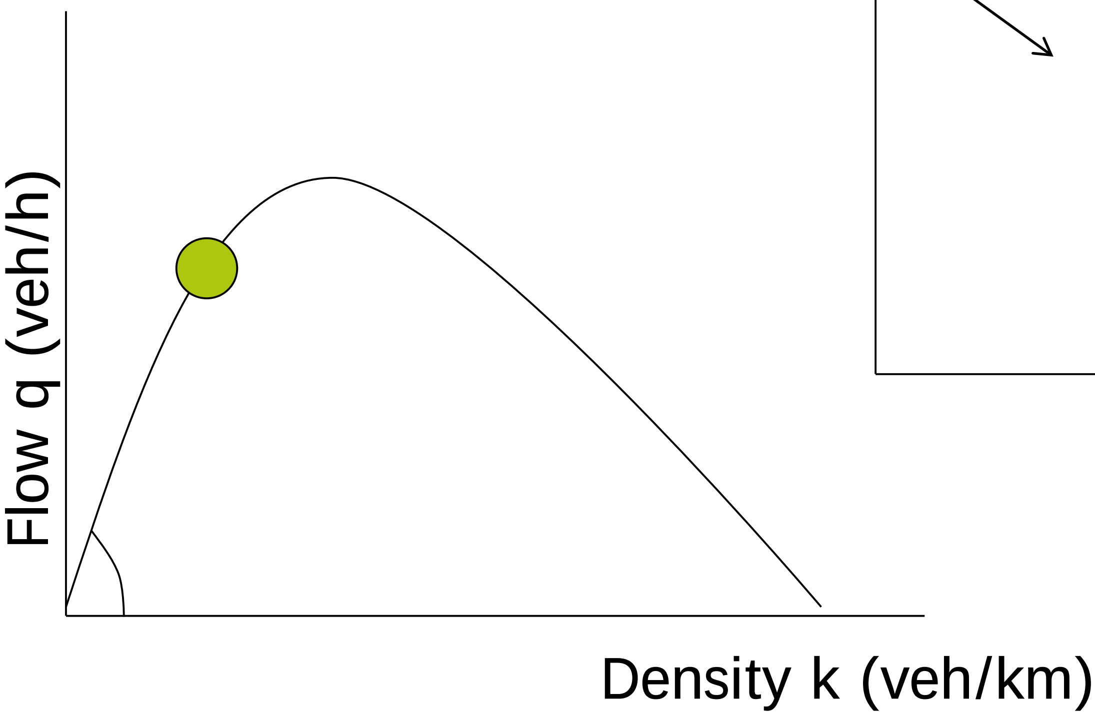
- Intersection of characteristics: shockwave!
  (use shockwave analysis to solve problem)

Speed of characteristic

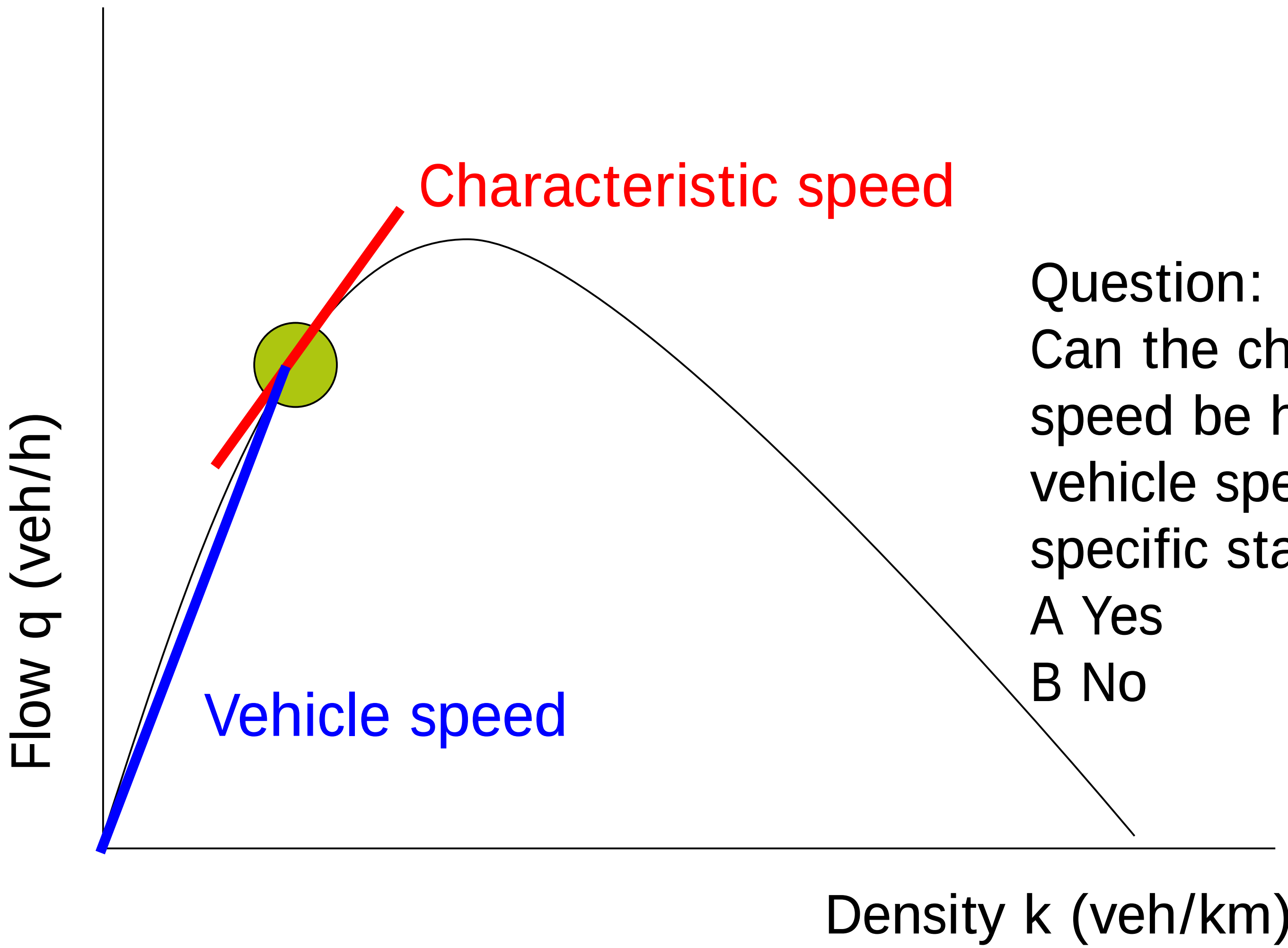
- In principle a shockwave with two sides (almost) the same condition

Question: where does the condition go?



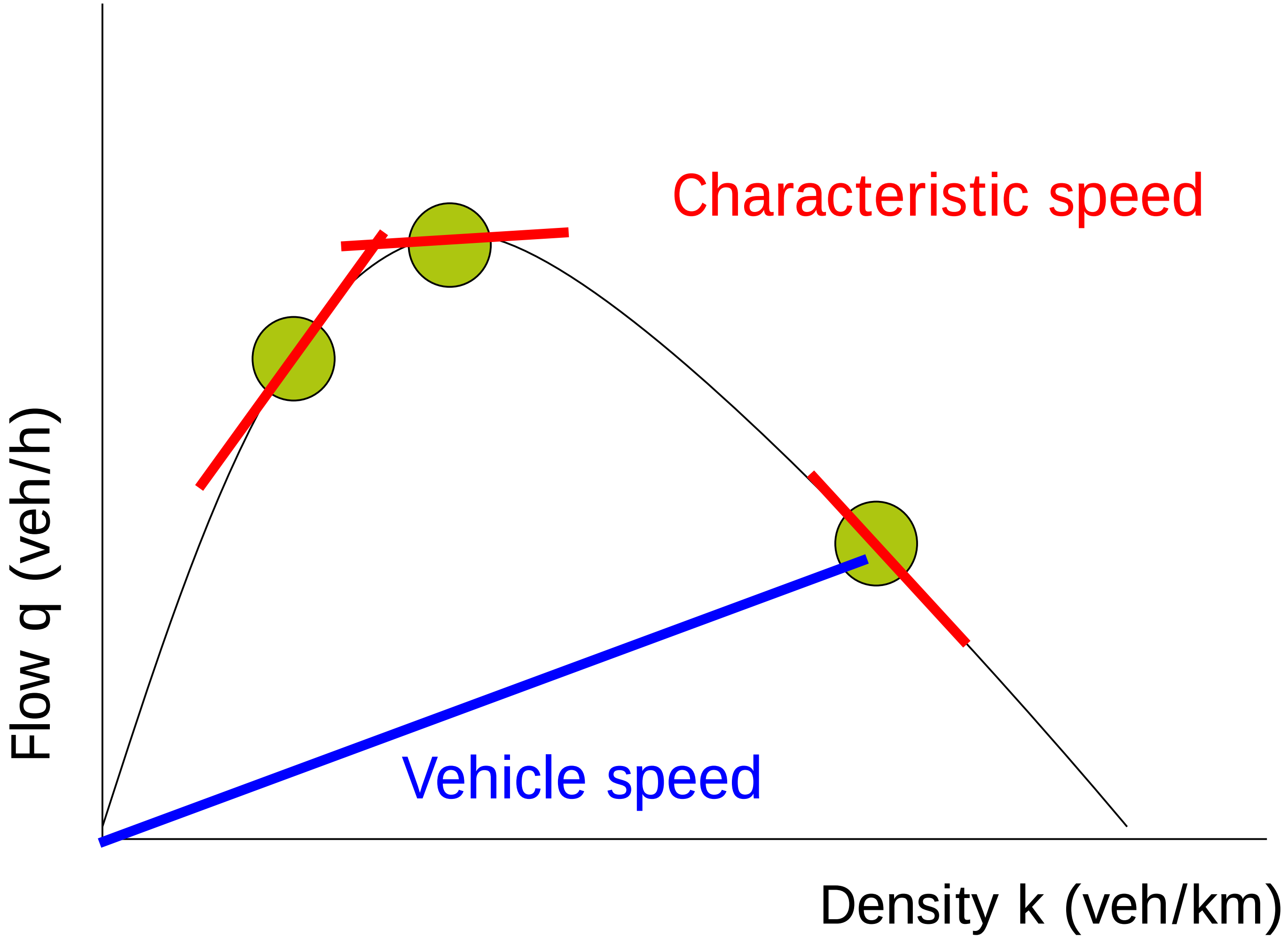


Two speeds

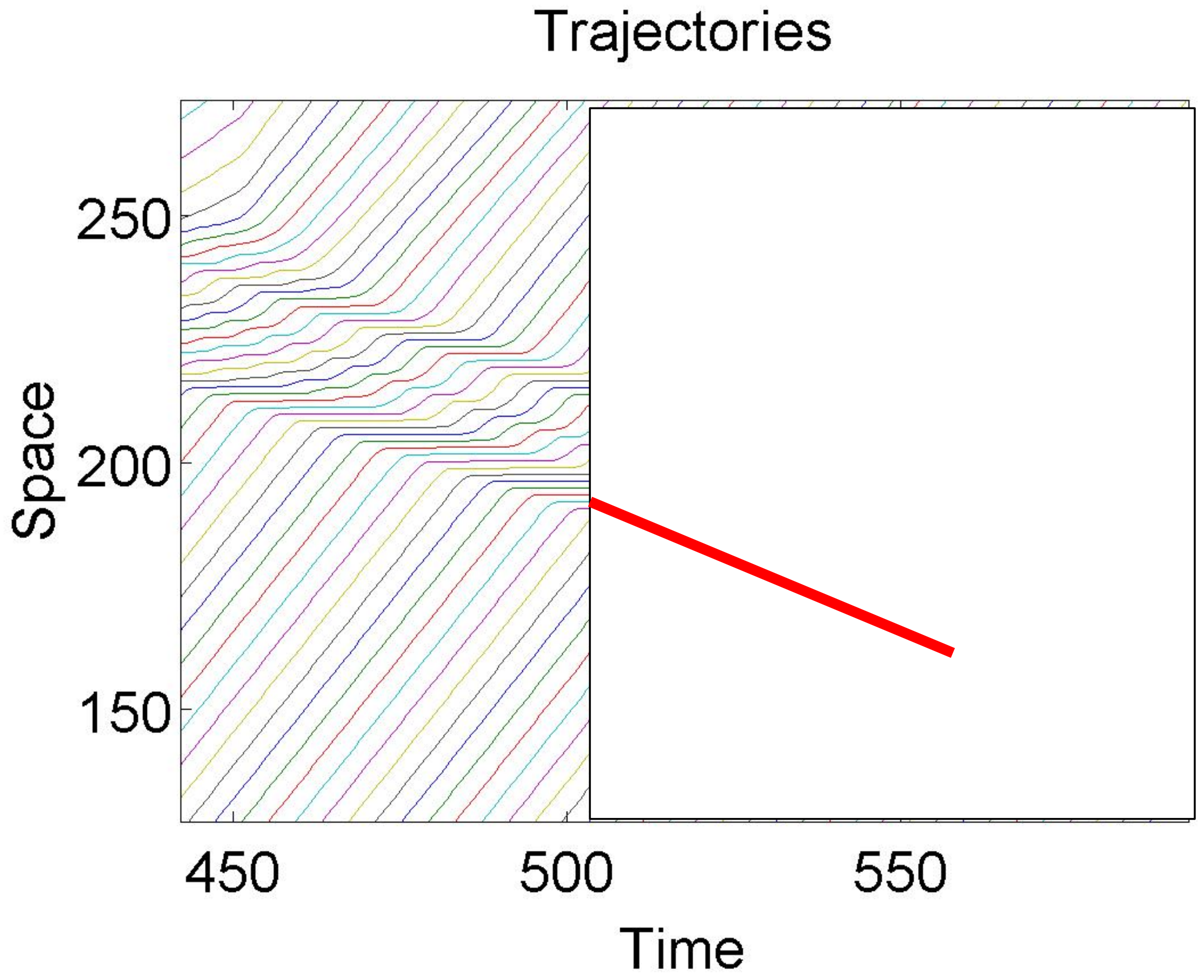


Question:
Can the characteristic speed be higher than the vehicle speed for a specific state?
A Yes
B No

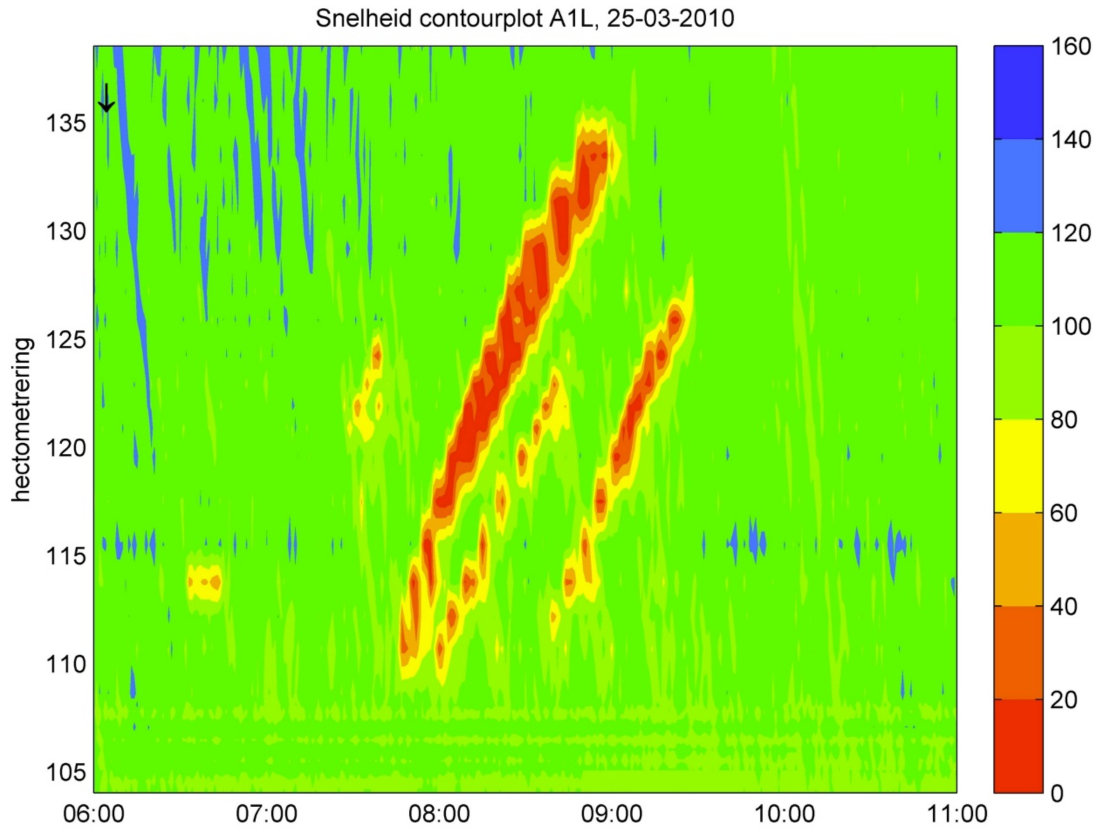
Two speeds


Characteristic speed
Flow q (veh/h)
Vehicle speed
Density k (veh/km)

Use: forward or backward analysis



Real-life example

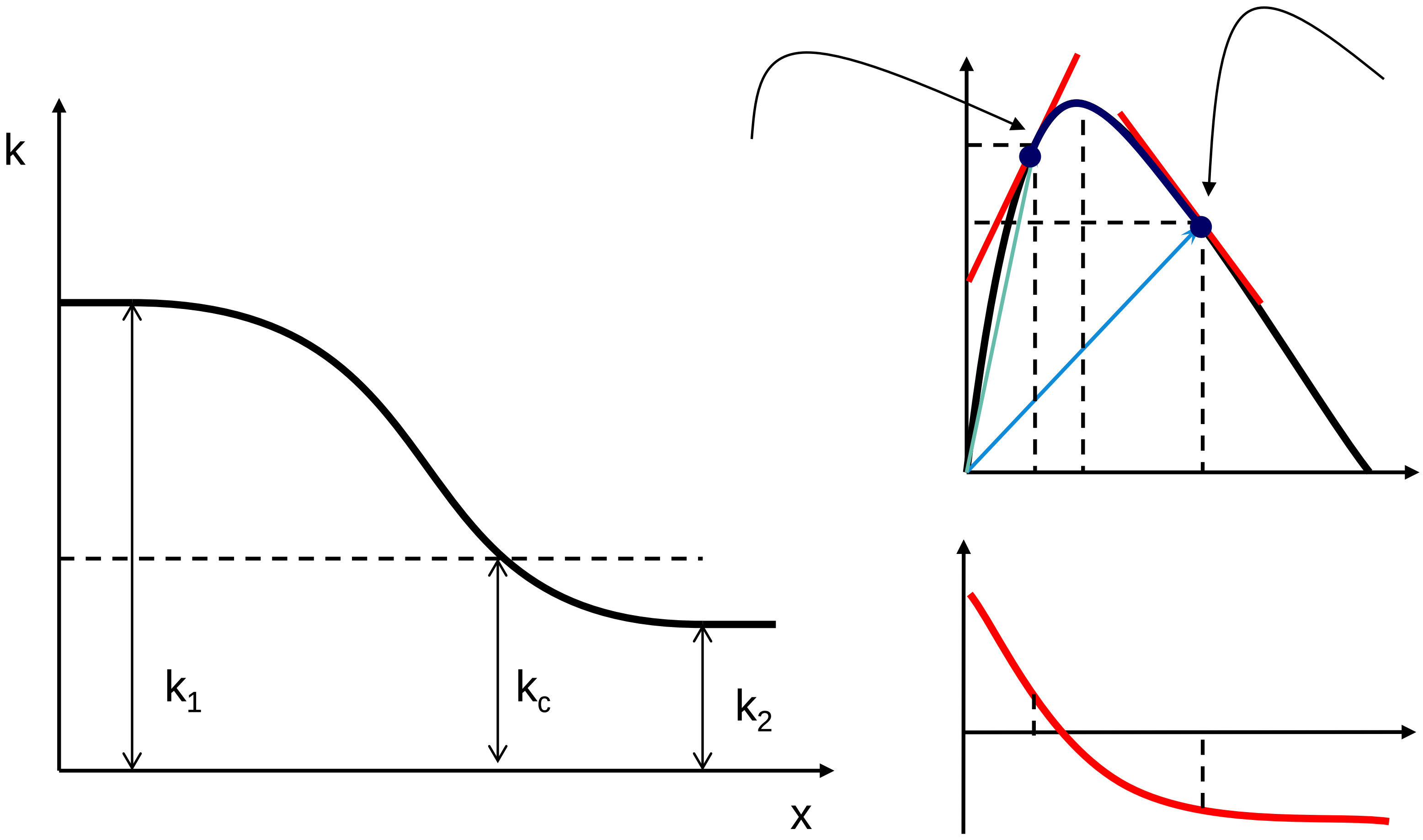


Acceleration fans and the formation of shockwaves

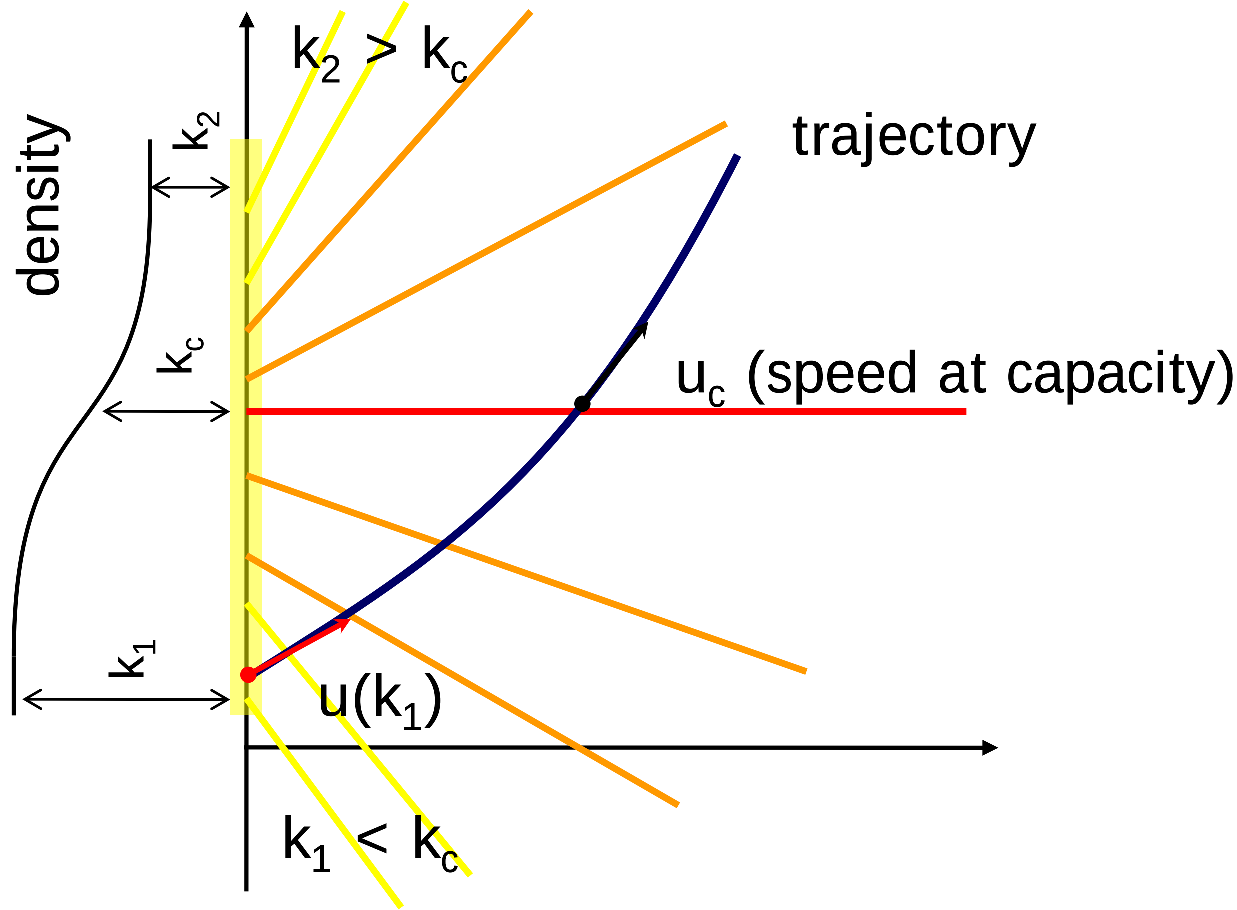
APPLICATIONS OF MOC

14

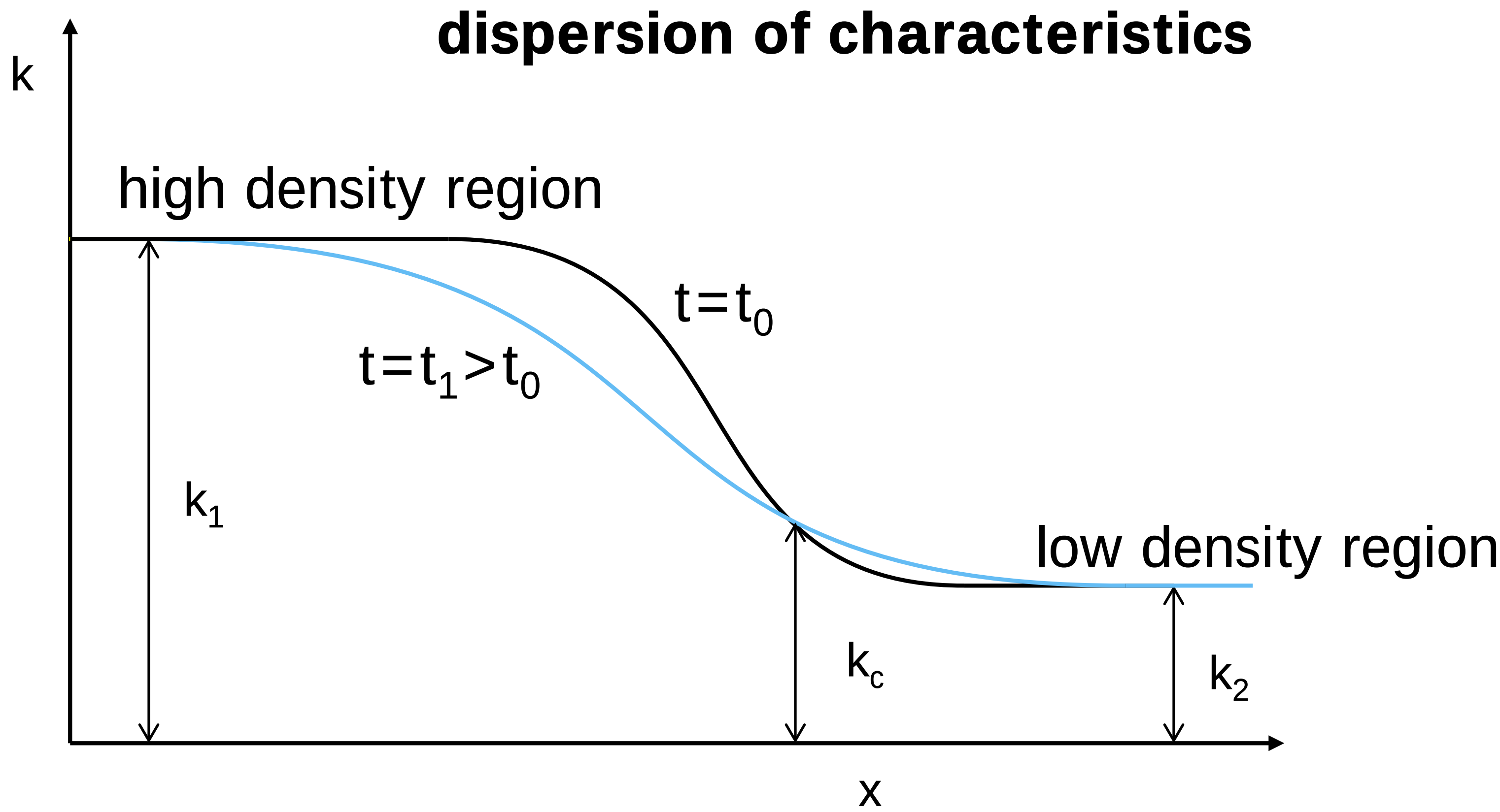
Application – acceleration fans



Application – acceleration fans (2)

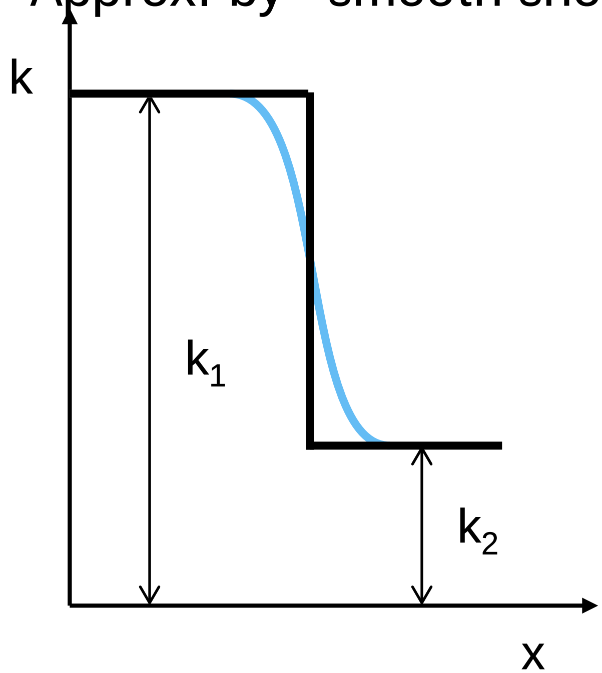


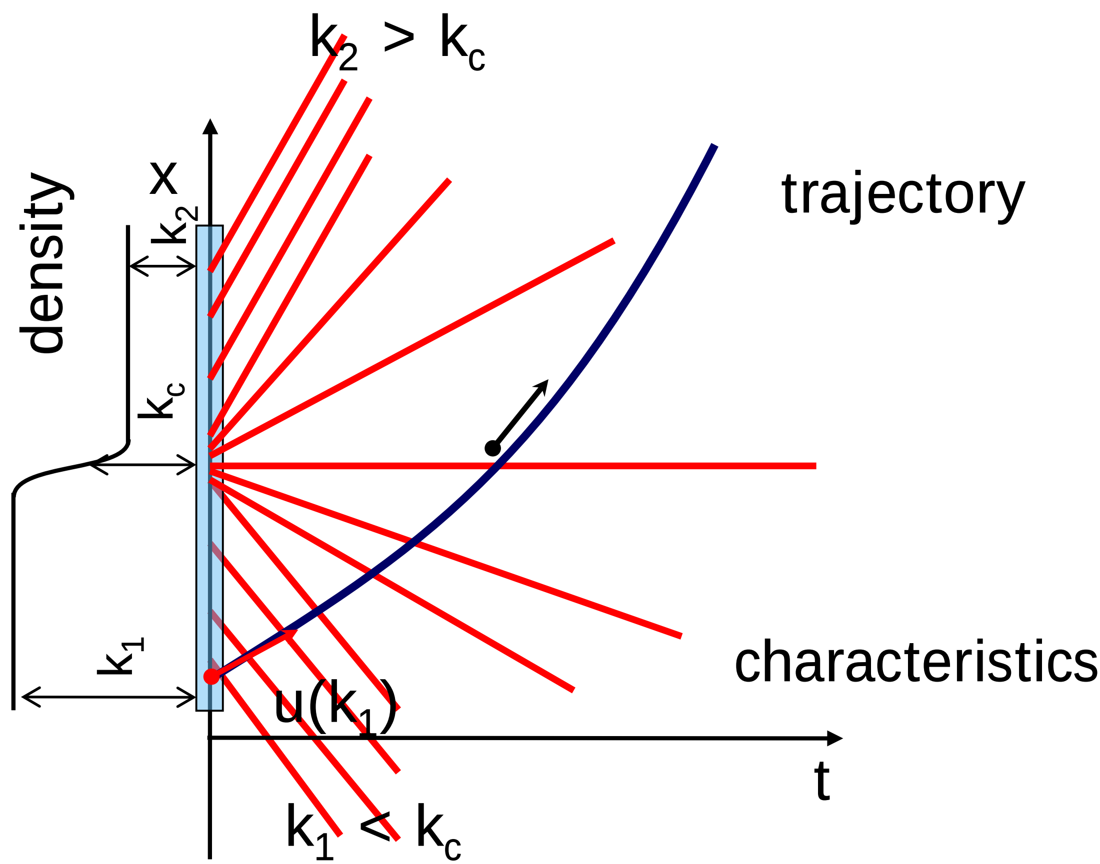
Application – acceleration fans (3)



Application – acceleration fan (4)

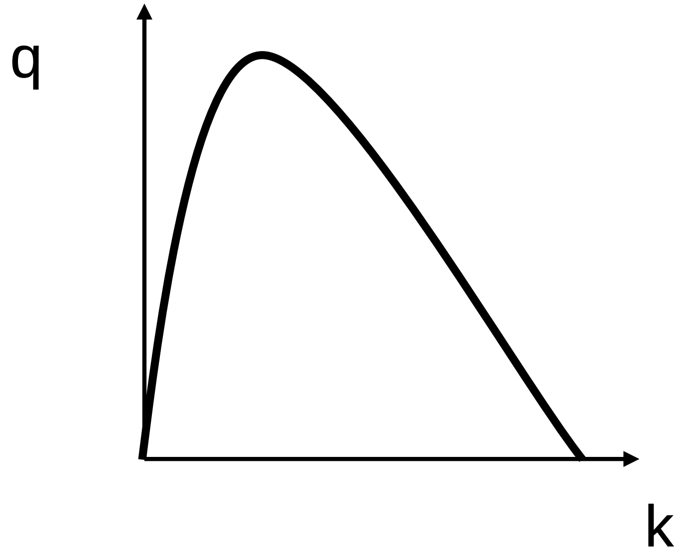
- Consider shock
- Approx. by ‘smooth shock’





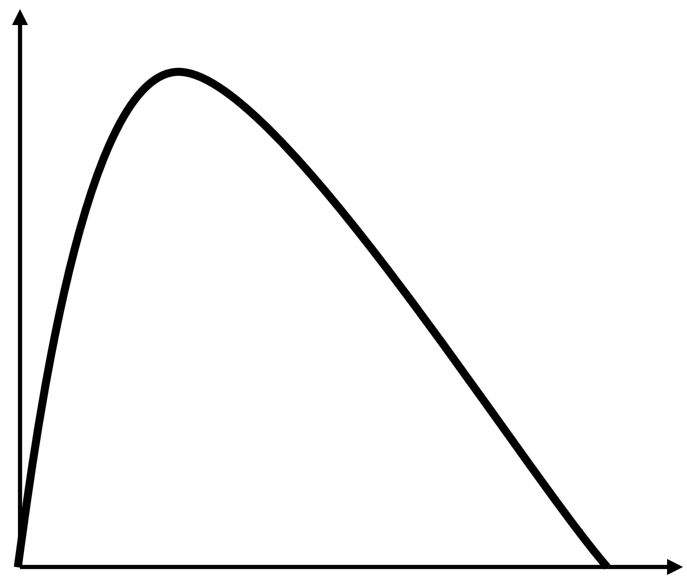
Question

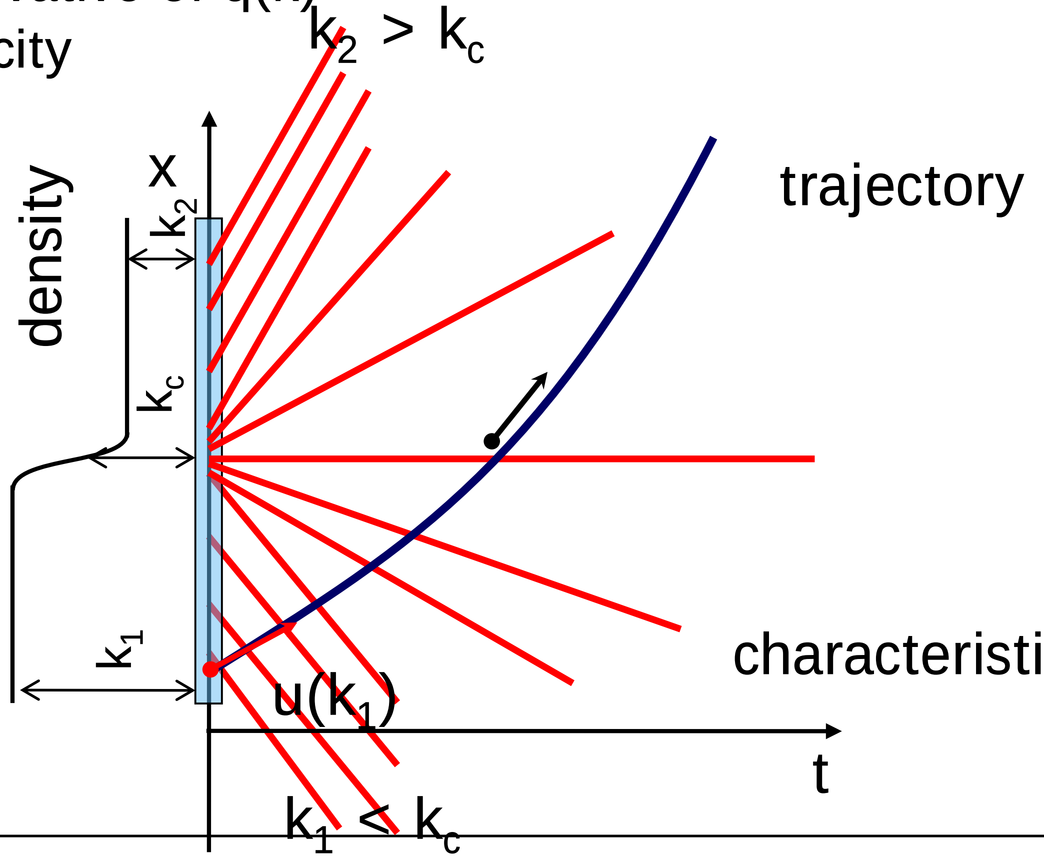
- There is a queue upstram of a traffic light
- The traffic light turns green
- What is the traffic state at the stop line
- Use this fundamental diagram:



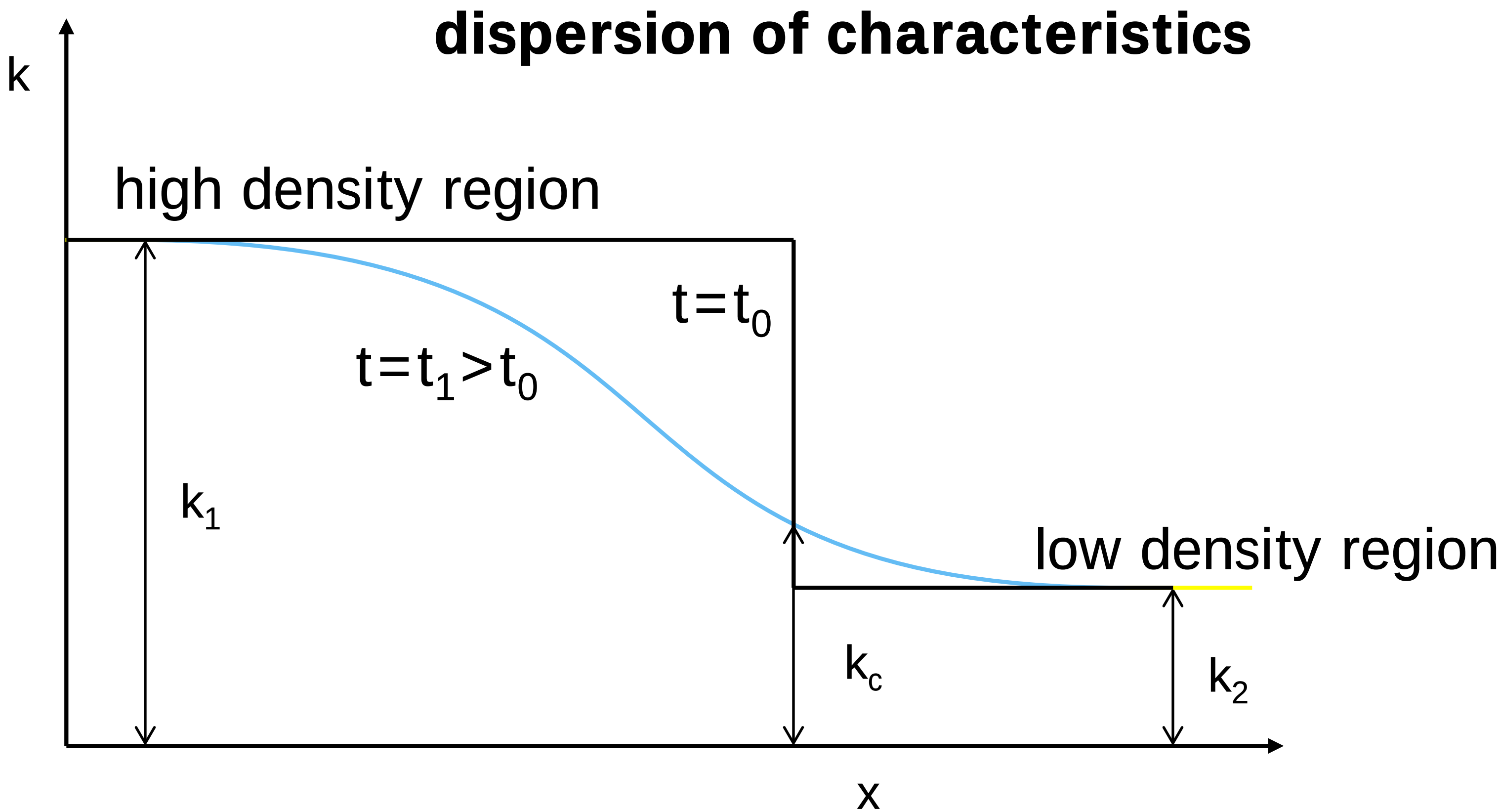
A. Undercritical
B. Overcritical
C. Critical = Capacity

- At t=0 “all” densities at stop line
- So => which density has a characteristic speed of 0
- Characteristic speed = derivative of q(k)
- => at d/dk(q)=0 => capacity





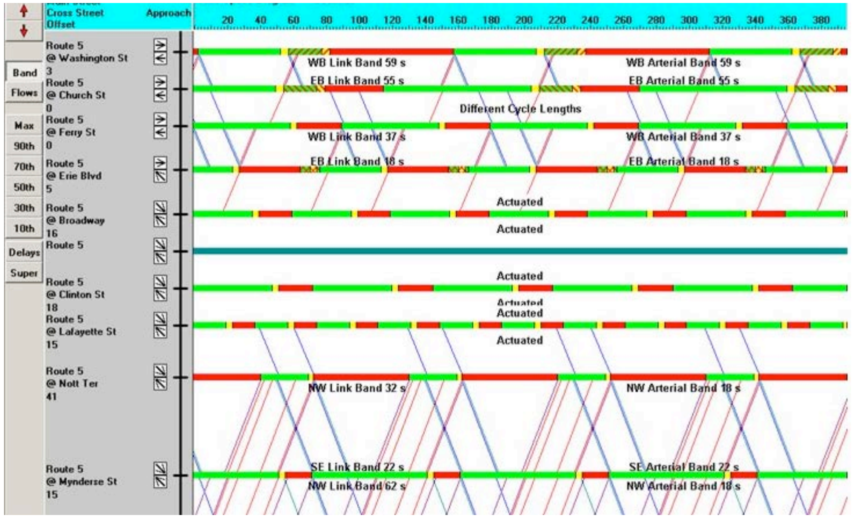
Application – acceleration fan (5)



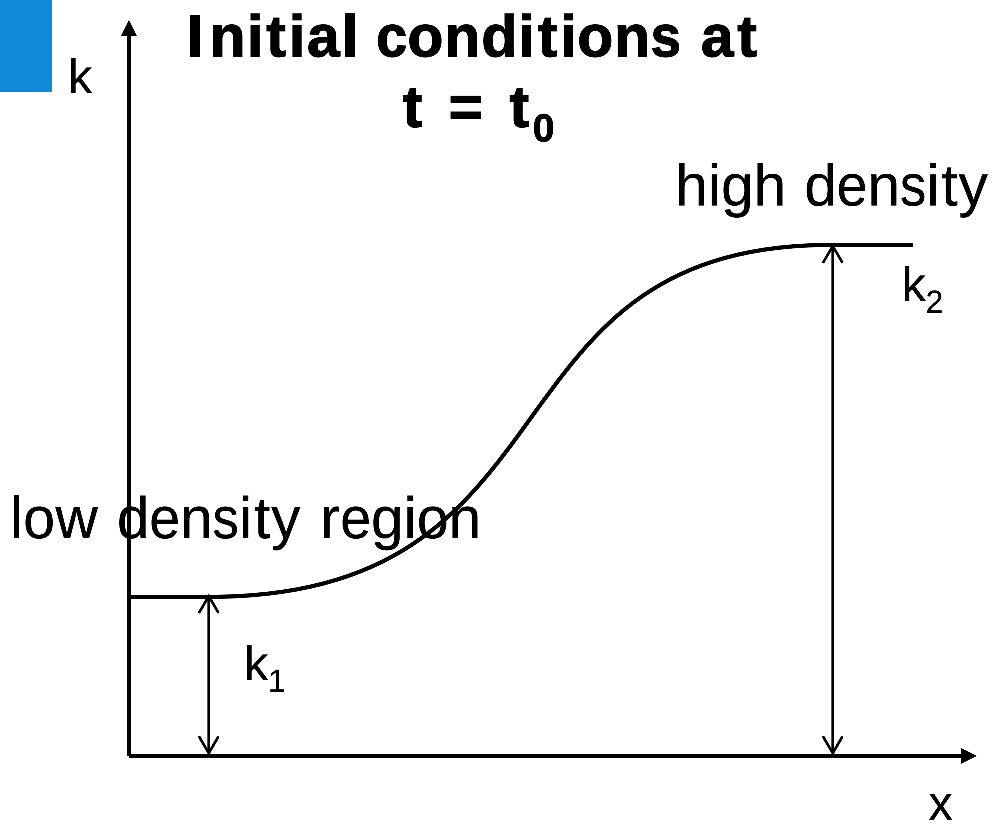
Link to practice – green wave

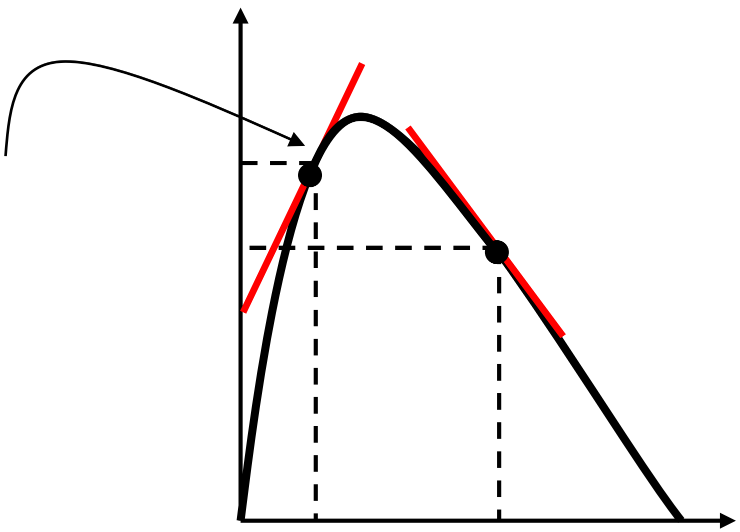
- After a red traffic light, keep the platoon of vehicles together and give them green throughout a section
- Avoid platoon dispersion
- Advise speed

Groene golf by rijkswaterstaat

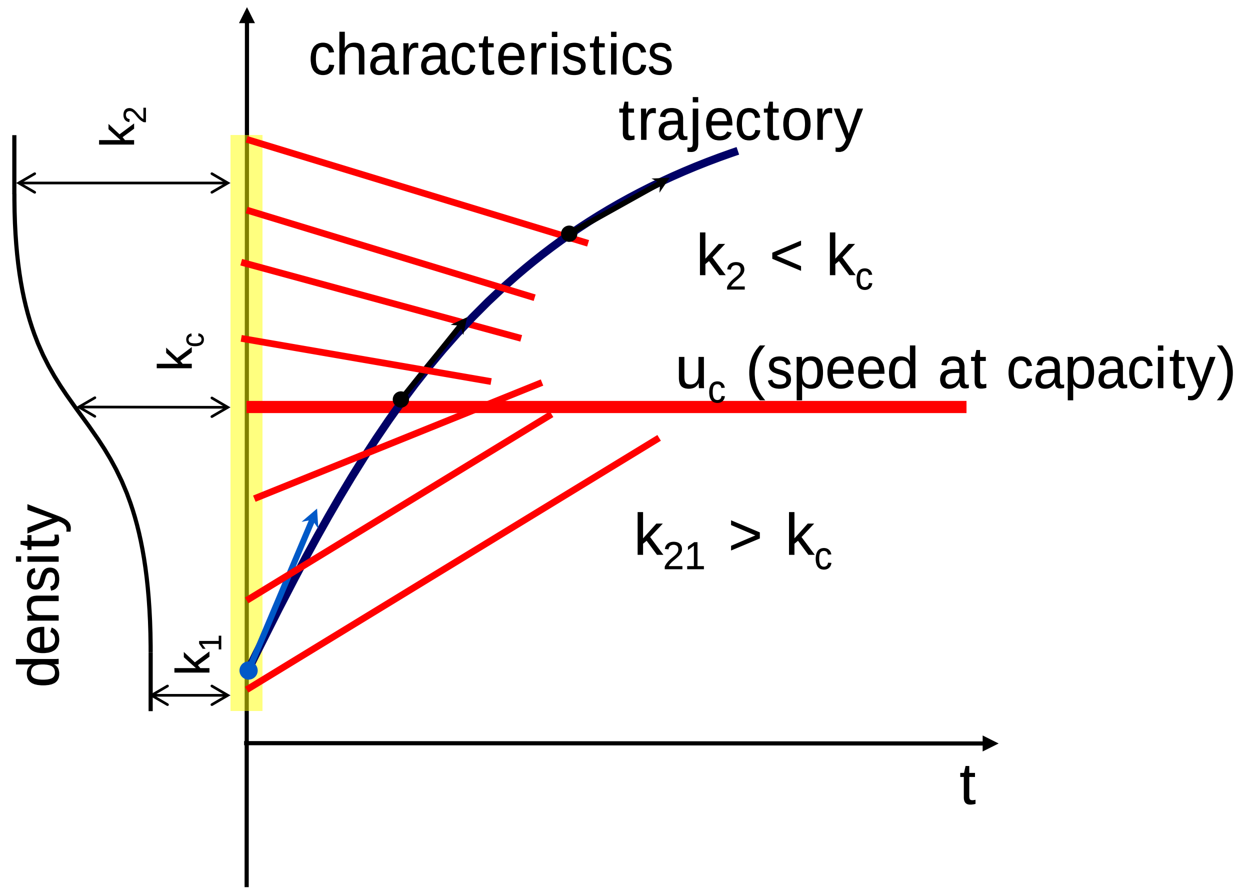


Application – deceleration fans

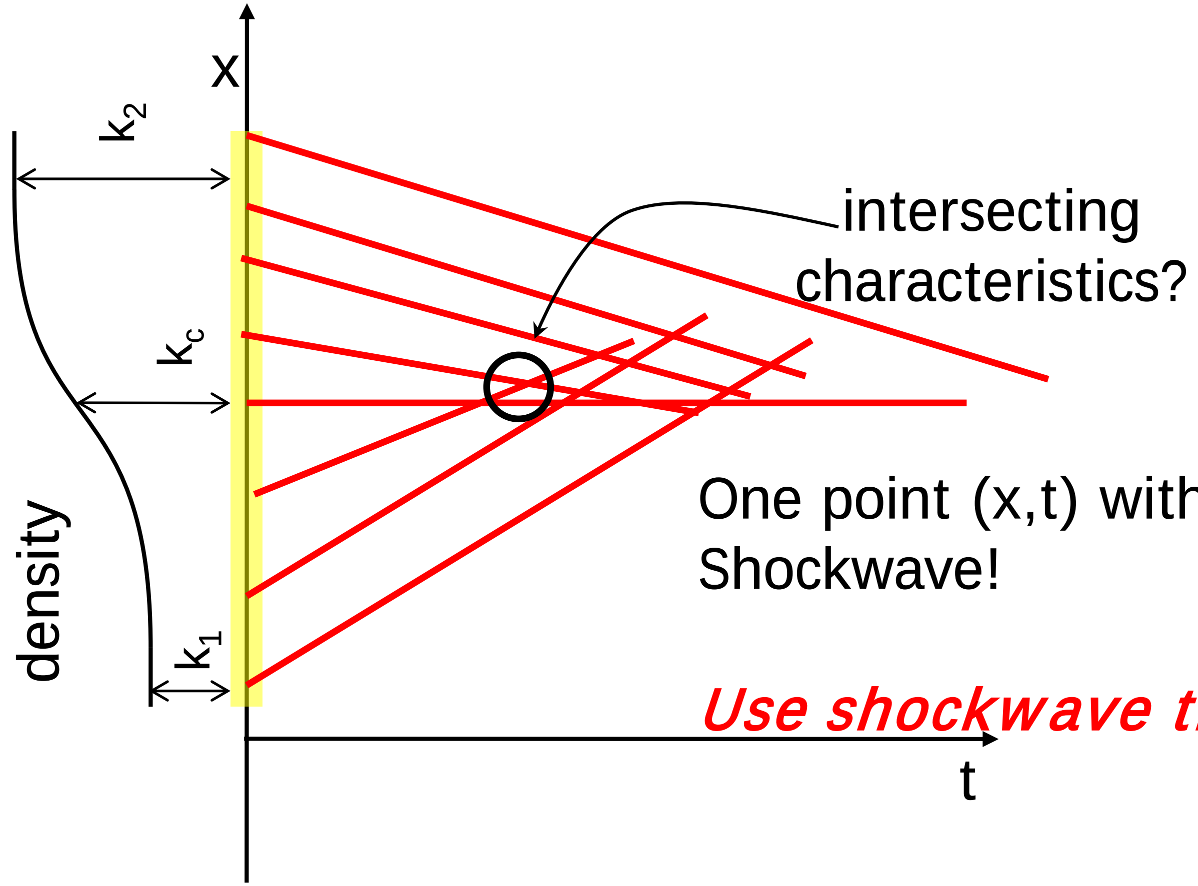




Application – deceleration fans (2)



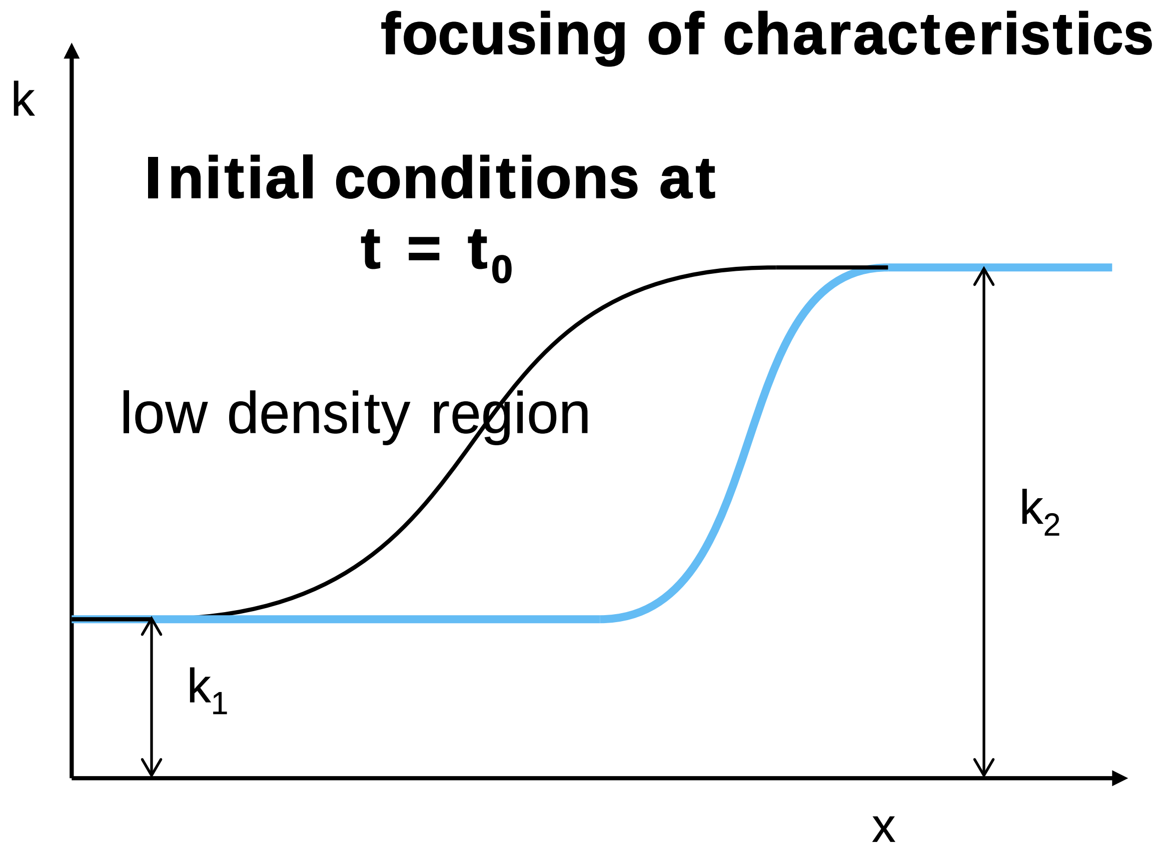
Formation of shocks



One point (x,t) with two values
Shockwave!

Use shockwave theory there

Formation of shocks (2)



In practice:

- Problem?

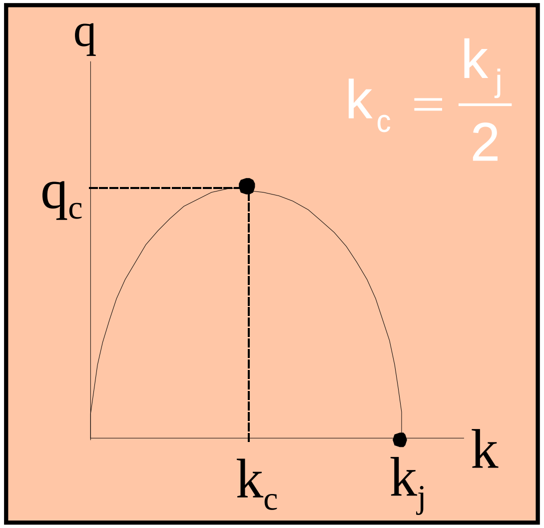
Photo by Wikipedia

Application of kinematic wave model

- Queuing at signalized intersection
- Upstream traffic demand equals q_1 (region 1)
- Initial conditions are free-flow (k_1,q_1) for all x until $t=-t_r$
- Assume Greenshields simplified fundamental diagram

Free speed

Jam density

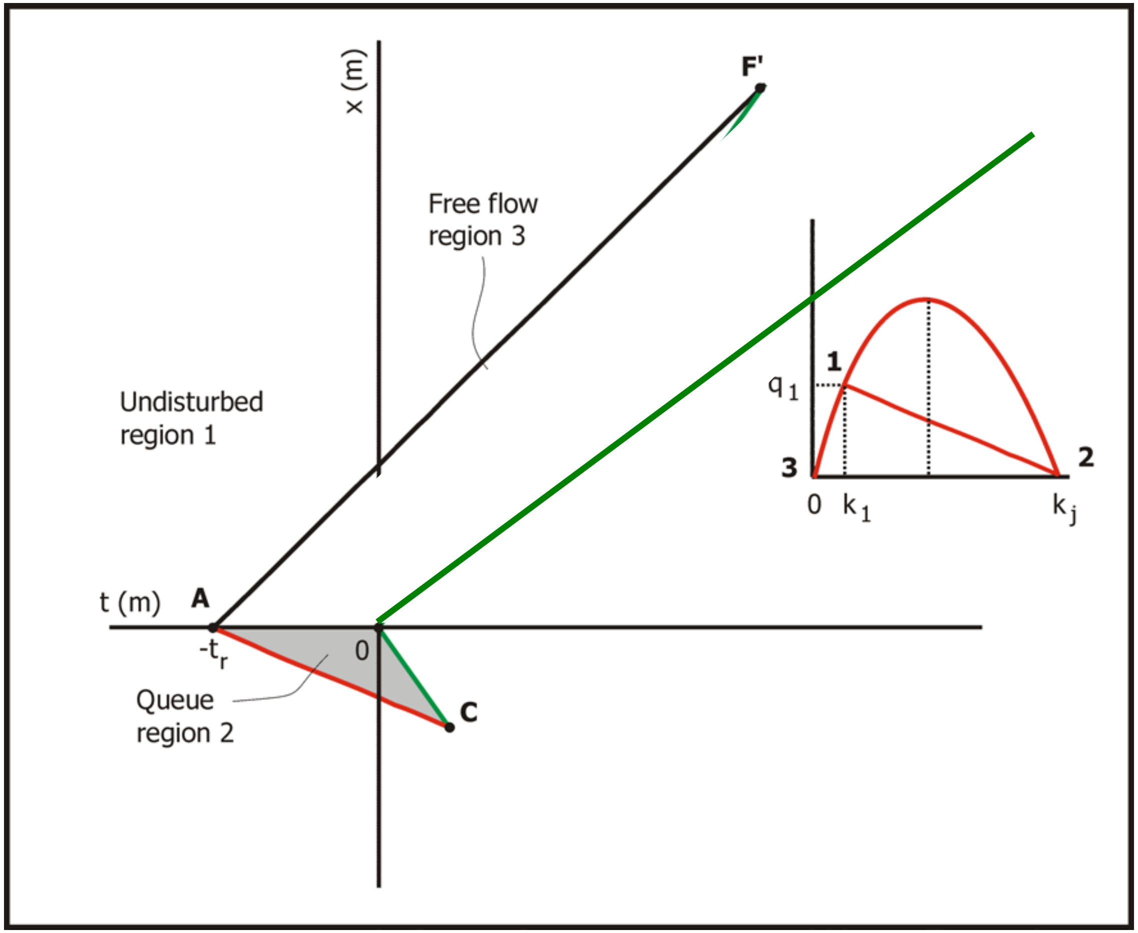


Queuing at signalized intersection

- Assume that at $t = -t_r$ a traffic light changes from green to red
- As a result, a queue will start to build up starting at the stop-line at $x = 0$ (region 2)
- Speed at which queue moves upstream can be determined using shockwave analysis

- Downstream of stopping line, free flow conditions (region 3)
- Two other shockwaves between:
  - Region 1 and 2
  - Region 1 and 3

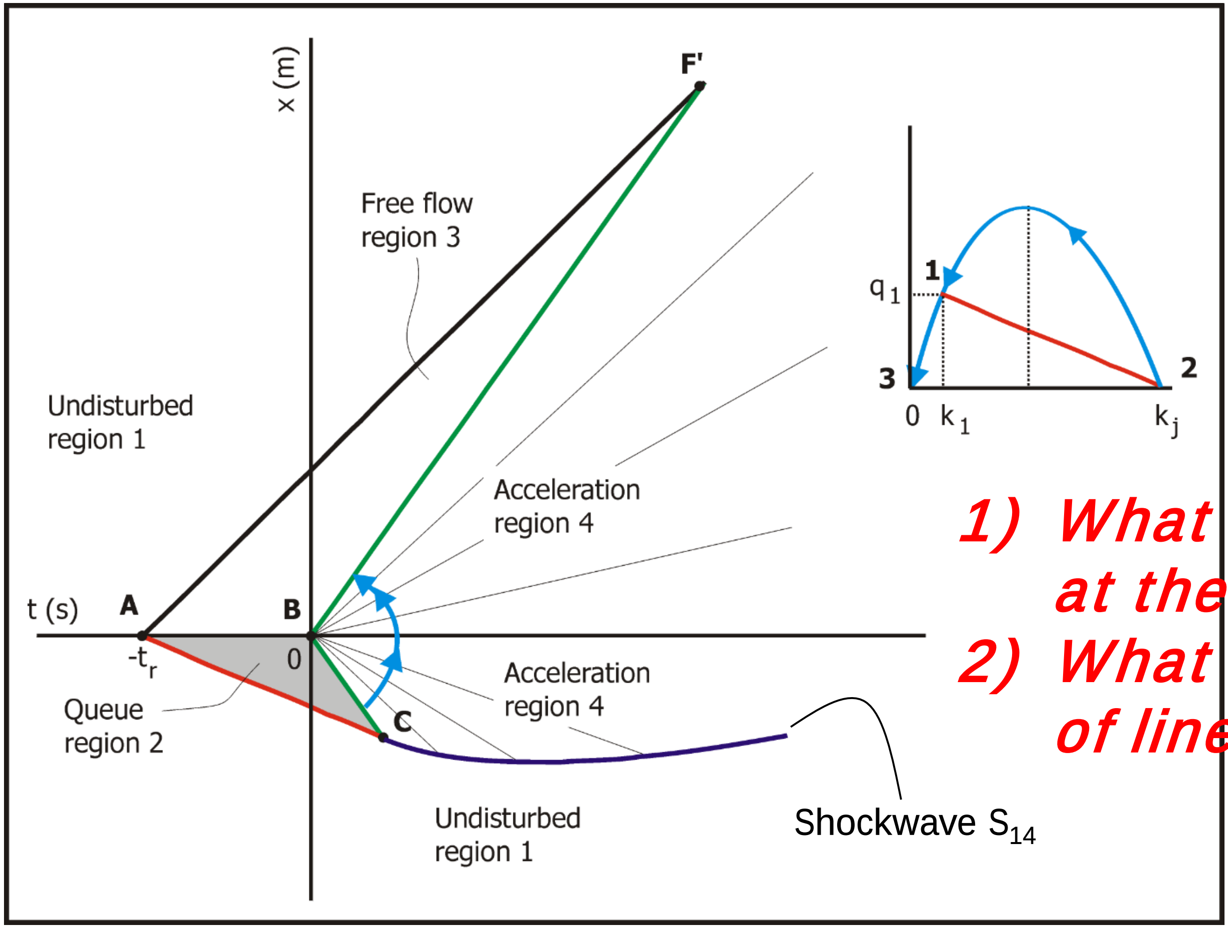
Queuing at signalized intersection (2)



Queuing at signalized intersection (3)

- At t = 0 red-phase ends. What will happen?
- Shockwave theory predicts that vehicles will drive away from the queue at capacity flow
- What will the kinematic wave model predict?
- Approximate shock at stop-line x=0 by 'smooth shock' yields description of acceleration fan

Queuing at signalized intersection (4)



1) What is the flow at the stop-line?

2) What is the slope of line B-C

Queuing at signalized intersection (5)

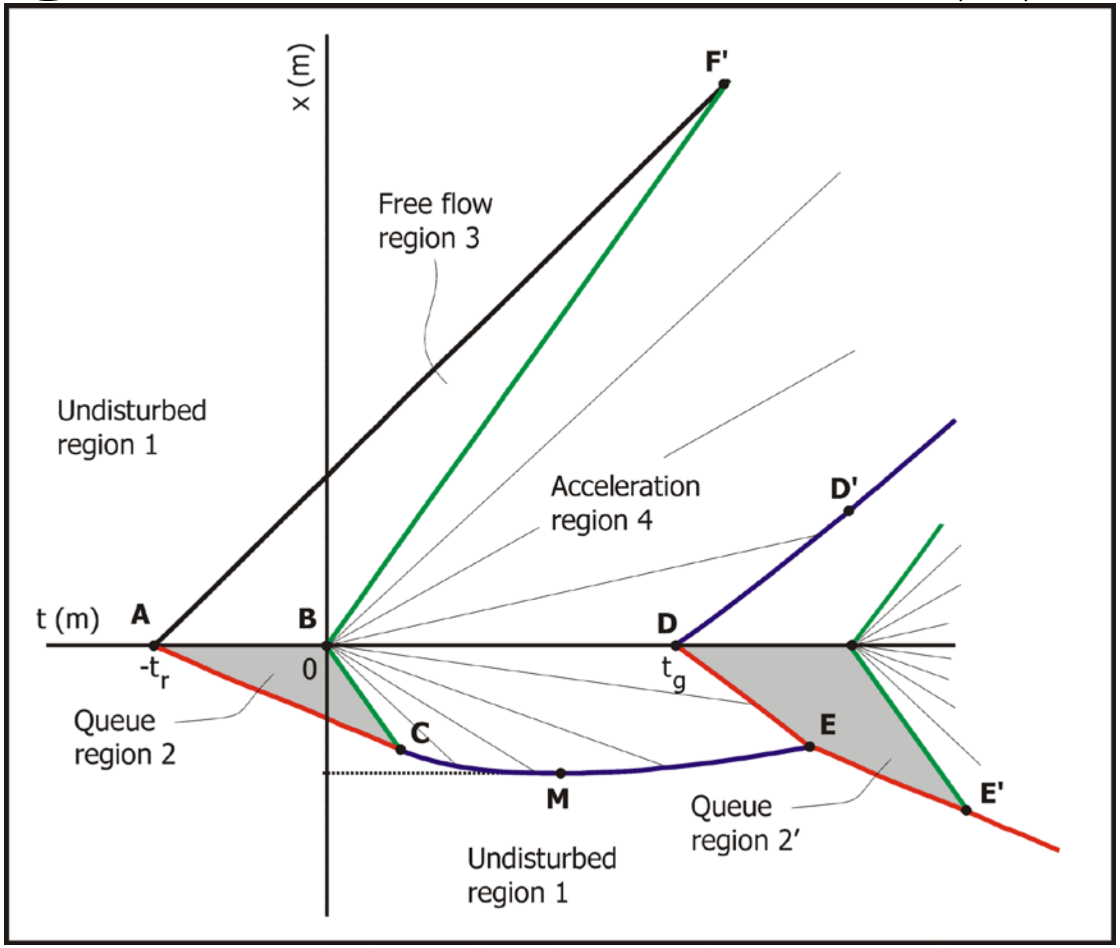
- How to determine dynamics of shock S_{14}?
- Consider any point (x,t) in the acceleration region 4
- This point lies of the characteristic emanating from the origin (0,0) thus having slope

- For Greenshields function we have

- Which enables us to determine the density at (x,t), and thus shockwave speed:
- Can be solved analytically (use Maple?) (see reader)

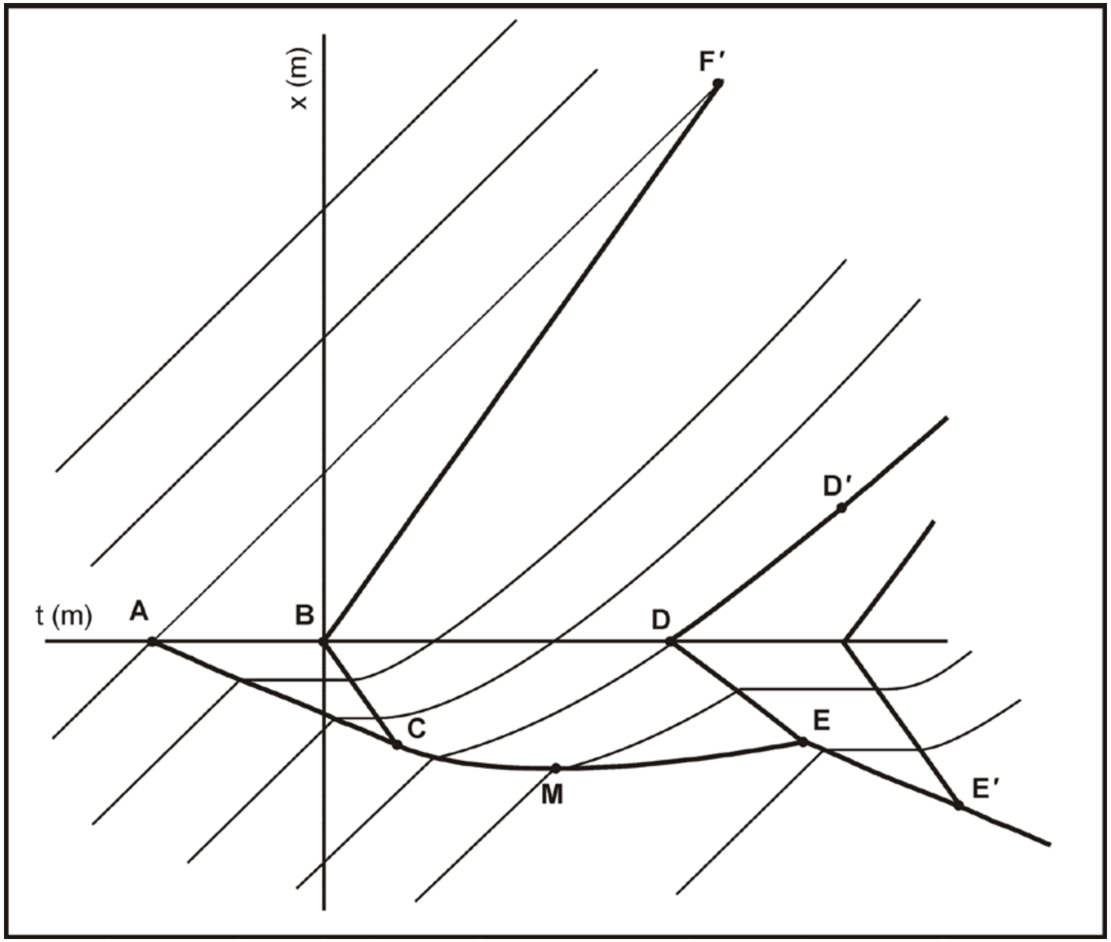
Queuing at signalized intersection (8)

- Assuming that red-phase occurs again during the acceleration phase
- Congestion region increases (queue grows as time goes by)



Queuing at signalized intersection (9)

- Example trajectories for signalized intersection



Queuing at signalized intersection (6)

- Thus for region 1:
- And for region 2:
- The speed of the shock S_{14} is thus

Queuing at signalized intersection (7)

- Shockwave trajectory $x_{CE}(t)$ starts at (x_C,t_C) and its slope is given by $dx_{CE}/dt = \omega_{14}$
- The shock S_{14} can thus be determined by solving the ODE

  subject to

- Can be solved analytically (use Maple?) (see reader)

Queuing at signalized intersection (8)

- How to determine point C?
- Point C represents the intersection of the shockwave S_{12} and the 'slowest' characteristic emanating from x = 0 (acceleration fan)
- This is the shockwave traveling at speed
- Point (x_C,t_C) is found easily (see reader)

Numerical solution approaches

Godunov scheme

24-3-2014

Why numerical solutions are needed

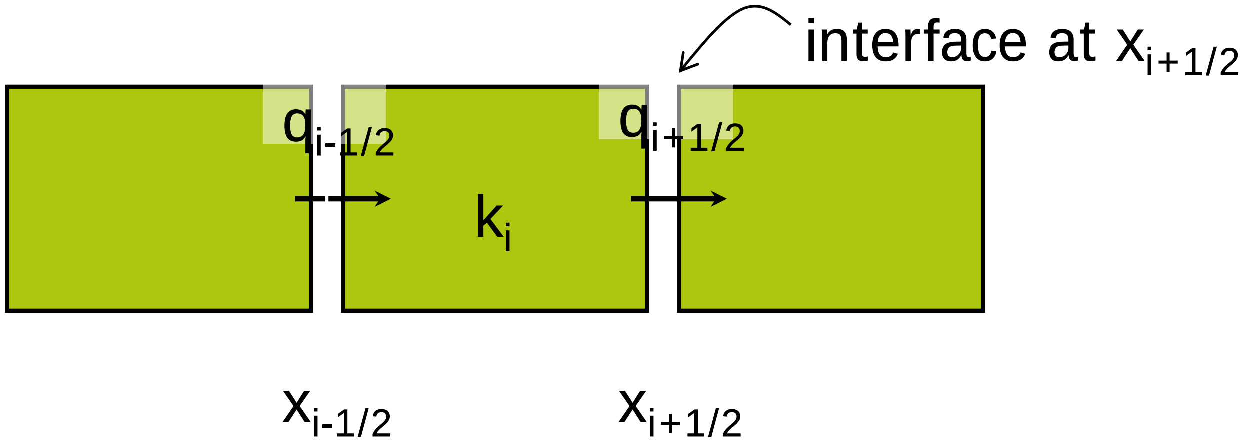
- Analytical solution to kinematic wave model is exact
- Only applicable in relatively simple situations, e.g. with respect to upstream traffic demand, off-ramps and on-ramps, etc.
- What to do when demand on main-road and on-ramps is changing dynamically? Use numerical approximations!

- Practical applications, e.g. use for network simulation (e.g. DSMART by Frank Zuurbier)

Basic principles

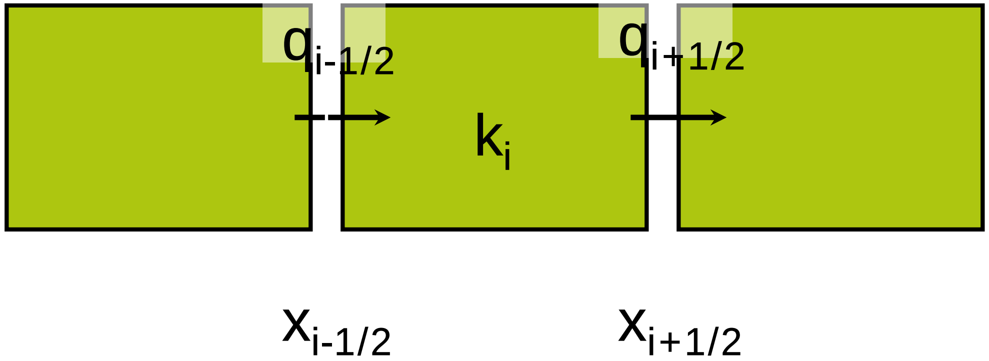
- Various approaches exist to solve kinematic wave model

- Simplest:
  Divide roadway into cells i, length dx
  - Divide time into steps with length dt



Assumptions

- Cells are homogeneous
- Within a time step (dt) , traffic flow is stationary
- Question: express $k_{i+1,j} = f(k_i,j,q_{i-1/2},q_{i+1/2},dt,dx)$



Basic principles (2)

- For the slides: this is the answer...

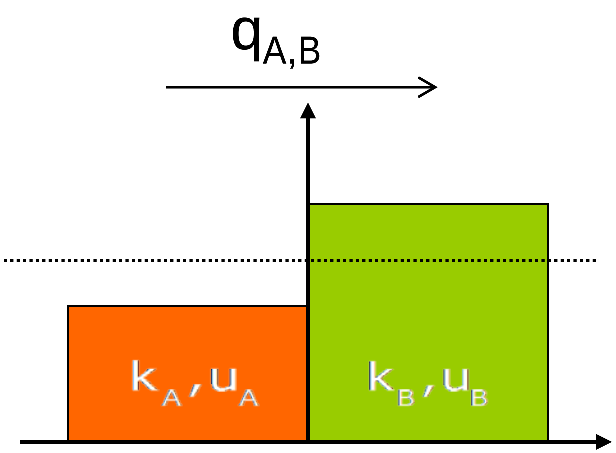
$$k_{i,j+1} = k_{i,j+1} + \frac{dt}{dx}\left(q_{i-1/2,j} - q_{i+1/2,j}\right)$$

- But how to get to:

- free-flow conditions:

What if congested?

- What determines the flow from A to B?
  - A – state in cell A
  - B – state in cell B
  - C - capactity



Traffic Light by OCAL

Combining: Godunov scheme (3)

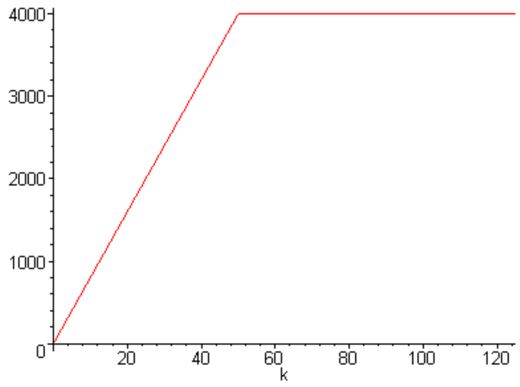
- *Demand* of region A and *supply* of region B

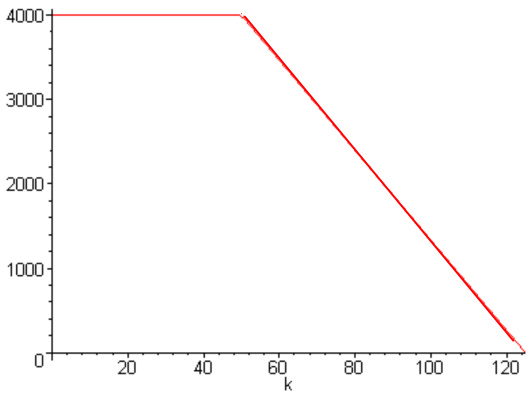
$$D_L := \begin{cases} q_A & k < k_c \\ c & k \geq k_c \end{cases} \qquad S_B := \begin{cases} c & k < k_c \\ q_B & k \geq k_c \end{cases}$$

- D_L = maximum number of vehicles that can flow out region L (bounded by the capacity of the road)
- S_R = maximum number of vehicles that can flow into region R (bounded by road capacity and the space becoming available during one time-step)
- Actual flow at x=0 : min(D_L,S_R)

Godunov graphically

- Flow based on
  Demand & Supply
- => fundamental diagram





Network: do this for all cells & times

- For all cells i, determine the cell demands for period j

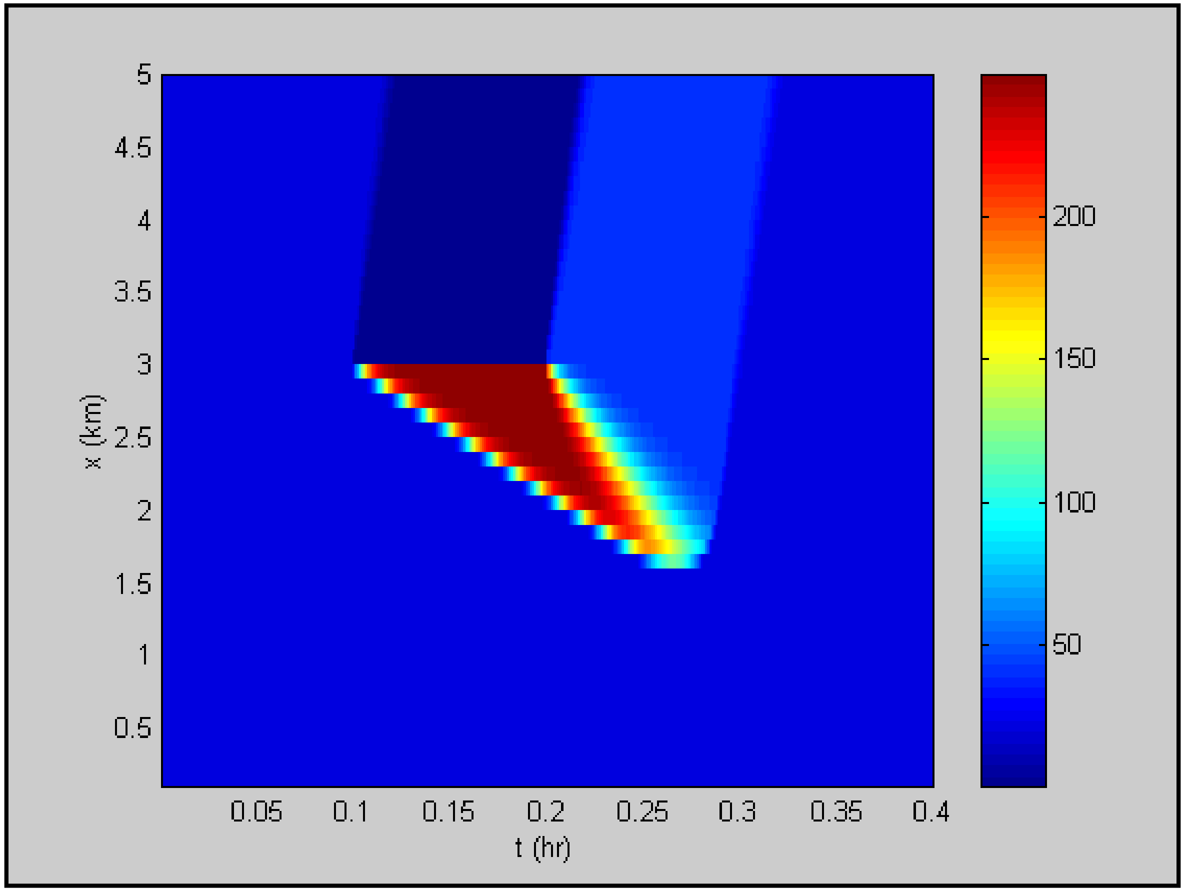
  and the cell supplies for period j

- The numerical flow between cell i and i+1 for period j becomes

Resulting traffic operations

- Movie

Example application Godunov



Summary – and learning goals

- Studied :
  - characteristics
  - Cell transmission model
- You can:
  - Follow traffic characteristics
  - Make traffic predicitons
    - Method of characteristics
      - +Shock wave theory when needed
    - Cell transmission model
      (euqations, interpretation of results)