Pumping stations and water transport

Hydraulics: theoretical background

ct5550

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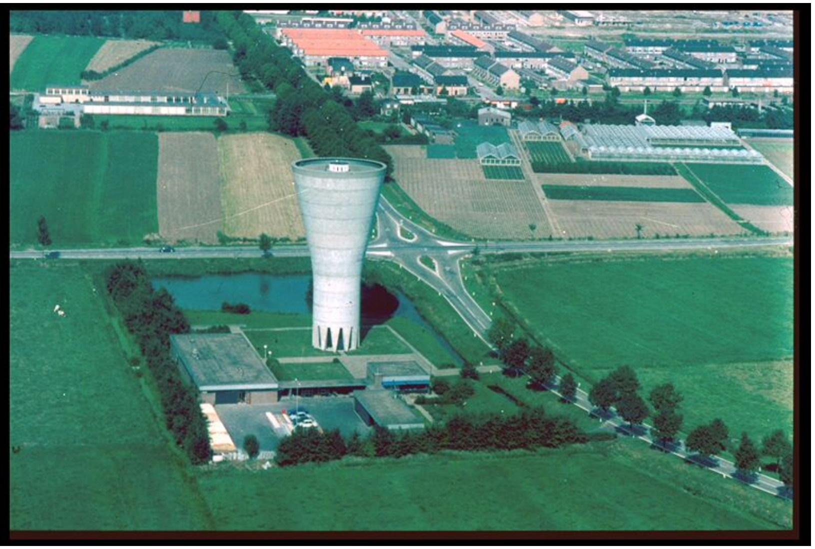
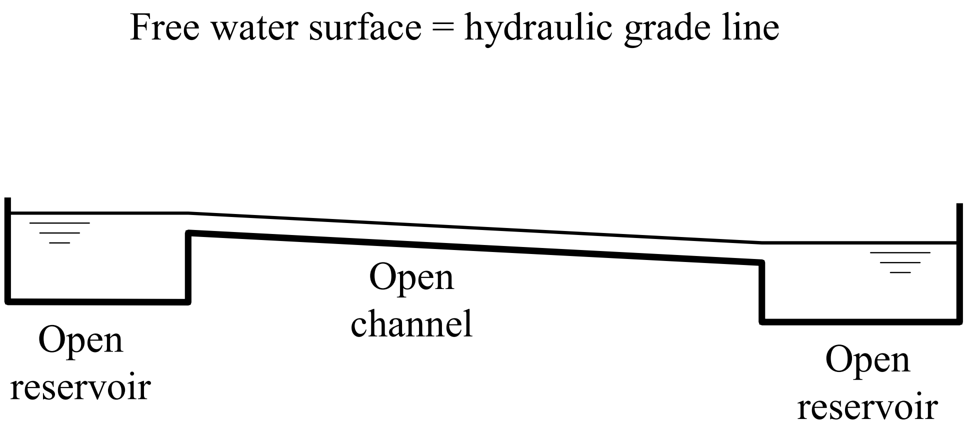
Delft University of Technology

Hydraulics of water transport through pipes: content

- Water transport through pipes
  - mathematical description
  - drinking water transport => rigid pipe
  - Sewerage transport => open channel flow
  - Water hammer
- Pumps and motors
- Network calculation

Water flows from a high level to a lower level





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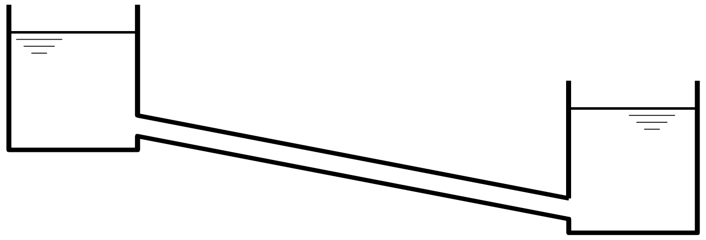
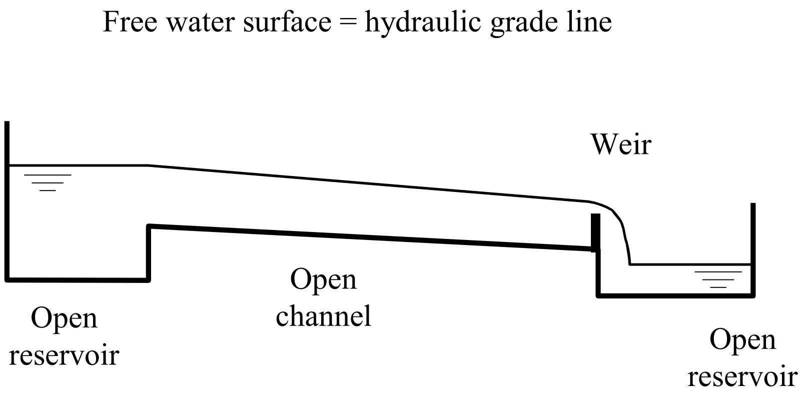
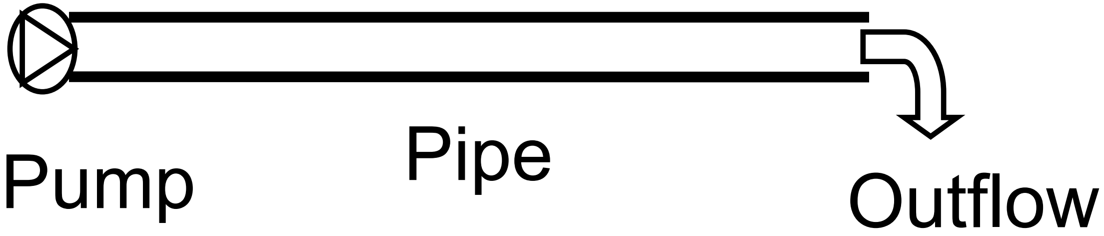
Water flows from a high level to a lower level







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Water transport through pipes flow types

- Closed pressurised flow
  - Fully filled closed pipes
  - Water pressure higher than atmospheric
- Open channel flow
  - Partly filled pipe
  - Free water surface
- Main difference: Open channel accommodates storage in profile

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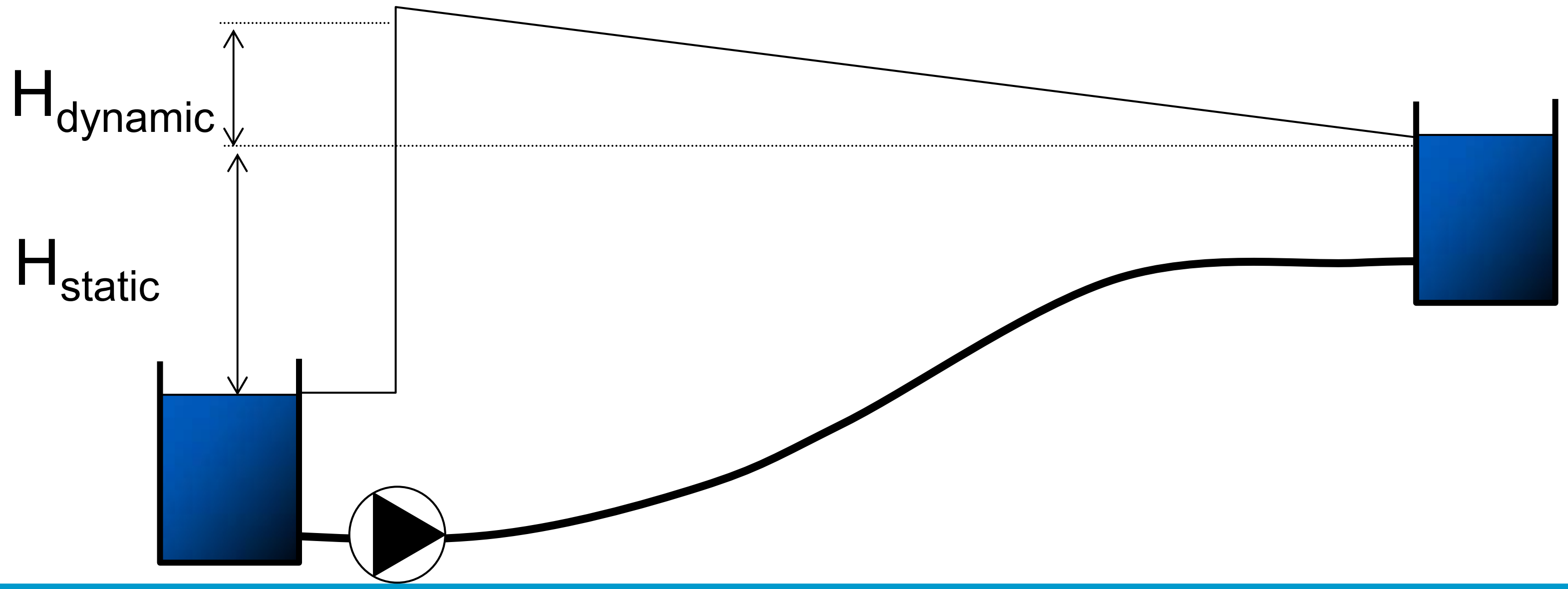
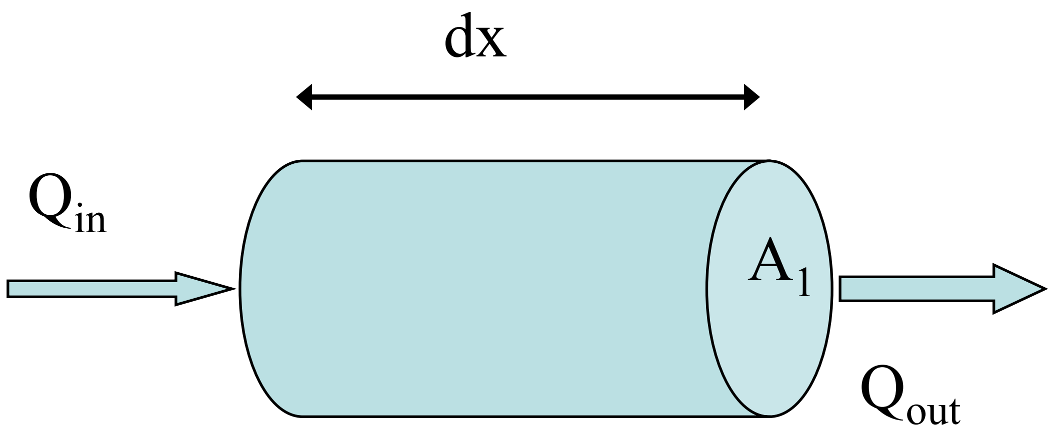
Heads in a pipe-pump system

- Static head to compensate level differences
- Dynamic head: to compensate
  - Friction loss ΔH_W
  - Deceleration loss ΔH_S
  - Local losses



Mathematical description

- Continuity equation:
  mass balance: Ingoing mass = outgoing mass over a certain period of time



Mathematical description

- Mass balance:
  Q_{in}*dt = Q_{out}* dt + dA* dx
  Incoming = outgoing + storage
- Dividing by ∂x and ∂t with limit transition:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

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Mathematical description

- Momentum balance

$$\underbrace{\frac{\partial Q}{\partial t}}_{1}+\underbrace{\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)}_{2}+\underbrace{gA\frac{\partial p}{\partial x}}_{3}+\underbrace{c\frac{|Q|Q}{AR}}_{4}=0$$

1: Acceleration term

2: Convective term

3: Gravitational/pressure term

4: Friction term

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Mathematical description

- Momentum balance

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial p}{\partial x}+c\frac{|Q|Q}{AR}=0$$

$$\mathrm{met}\ \ \mathrm{Q}=\mathrm{u}*\mathrm{A}$$

$$A\frac{\partial u}{\partial t}+u\frac{\partial A}{\partial t}+2Au\frac{\partial u}{\partial x}+u^2\frac{\partial A}{\partial x}+gA\frac{\partial p}{\partial x}+c\pi D|u|u=0$$

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Drinking water transport: fully filled closed pipes

- Two possible flow types:
  - Rapid changing boundaries for pressure and/or volume flow: water hammer
  - Slow changing boundaries for pressure and/or flow: friction flow
- Rigid column:
  - uniform and stationary flow $\partial Q/\partial t = 0$
  - prismatic pipe $\partial A/\partial x = 0$
  - water incompressible
  - elasticity pipe negligible $\partial A/\partial t = 0$
  - Newton's fluid

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Rigid column: Continuity becomes

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \xrightarrow{\frac{\partial A}{\partial t}=0} \frac{\partial Q}{\partial x}=0$$

And consequently

$$\frac{\partial Q}{\partial x}=0 \rightarrow \frac{\partial uA}{\partial x}=0 \rightarrow$$

$$A\frac{\partial u}{\partial x}+u\frac{\partial A}{\partial x}=0 \rightarrow \frac{\partial u}{\partial x}=0$$

Rigid column: momentum balance

$$A\frac{\partial u}{\partial t}+u\frac{\partial A}{\partial t}+2Au\frac{\partial u}{\partial x}+u^2\frac{\partial A}{\partial x}+gA\frac{\partial p}{\partial x}+c\pi D\left|u\right|u=0$$

$$\frac{\partial A}{\partial t}=0;\frac{\partial A}{\partial x}=0;\frac{\partial u}{\partial x}=0\Rightarrow$$

$$A\frac{\partial u}{\partial t}+gA\frac{\partial p}{\partial x}+c\pi D\left|u\right|u=0;\ \text{with } u=\frac{Q}{A}$$

$$\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{\partial p}{\partial x}+\frac{Q\left|Q\right|}{C^2A^2R}=0$$

Rigid column: Darcy Weissbach

$$\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{\partial p}{\partial x}+\frac{Q|Q|}{C^2A^2R}=0 \xrightarrow{\frac{\partial Q}{\partial t}=0 \text{ and integrating over L}}$$

$$\int_{x=o}^{x=L}\frac{\partial p}{\partial x}dx=-\int_{x=o}^{x=L}\frac{Q|Q|}{C^2A^2R}\Rightarrow$$

$$p_2-p_1=-\frac{Q|Q|}{C^2A^2R}\xrightarrow{\lambda=\frac{8g}{C^2}}$$

$$p_1-p_2=\lambda\frac{8L}{\pi^2 g}\frac{Q|Q|}{D^5}=0{,}0826\frac{\lambda L}{D^5}Q|Q|$$

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After some mathematical exercises

- Darcy Weissbach: No time dependency

$$\Delta H = H_2 - H_1 = \lambda \frac{L}{D} \frac{u^2}{2g}$$

$$= 0{,}0826 \frac{\lambda L}{D^5} Q^2$$

- White-Colebrook

$$\frac{1}{\sqrt{\lambda}} = -2 \log \left[\frac{k_N}{3D} + \frac{1}{0{,}32 \operatorname{Re} \sqrt{\lambda}} \right]$$

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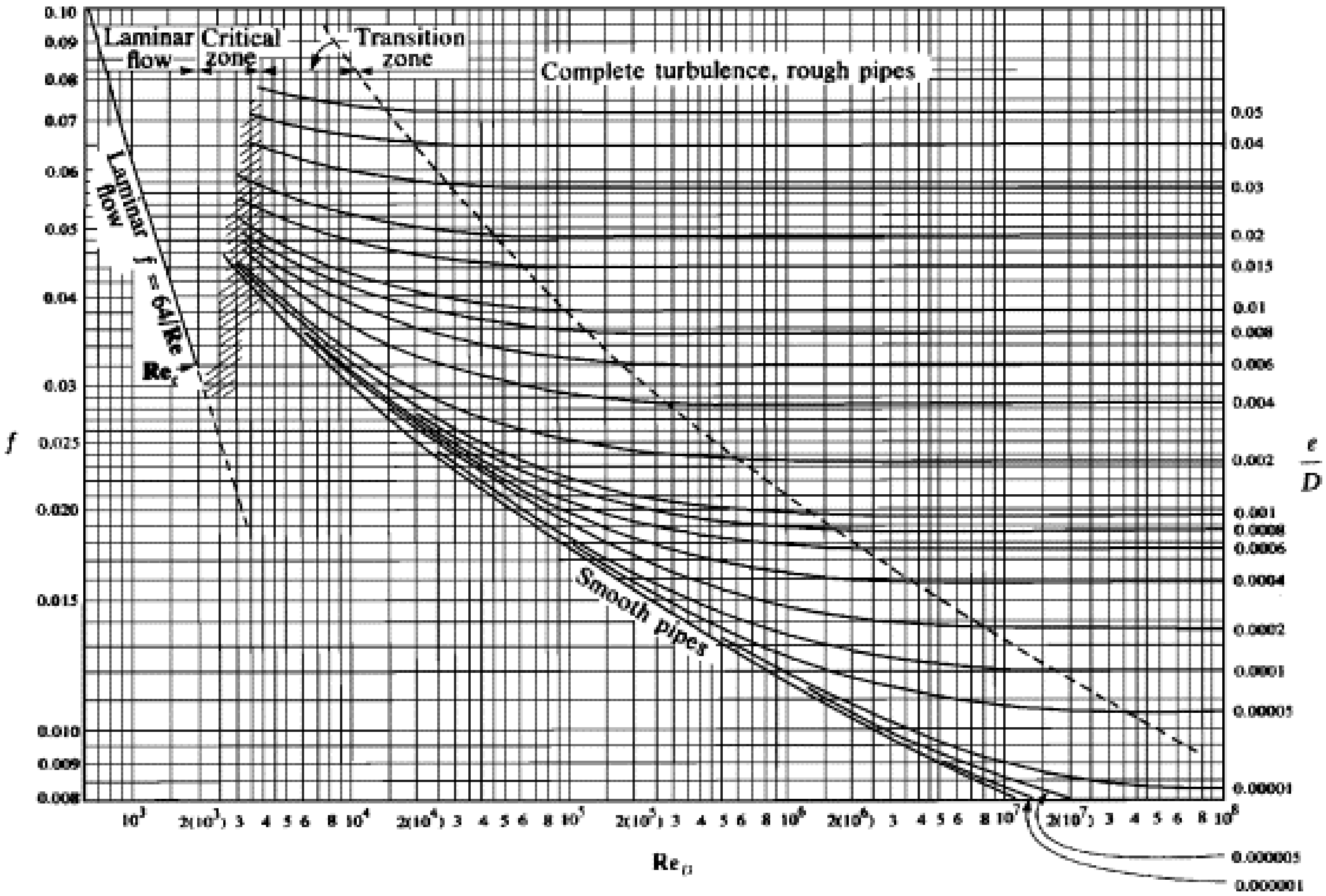
Moody diagram



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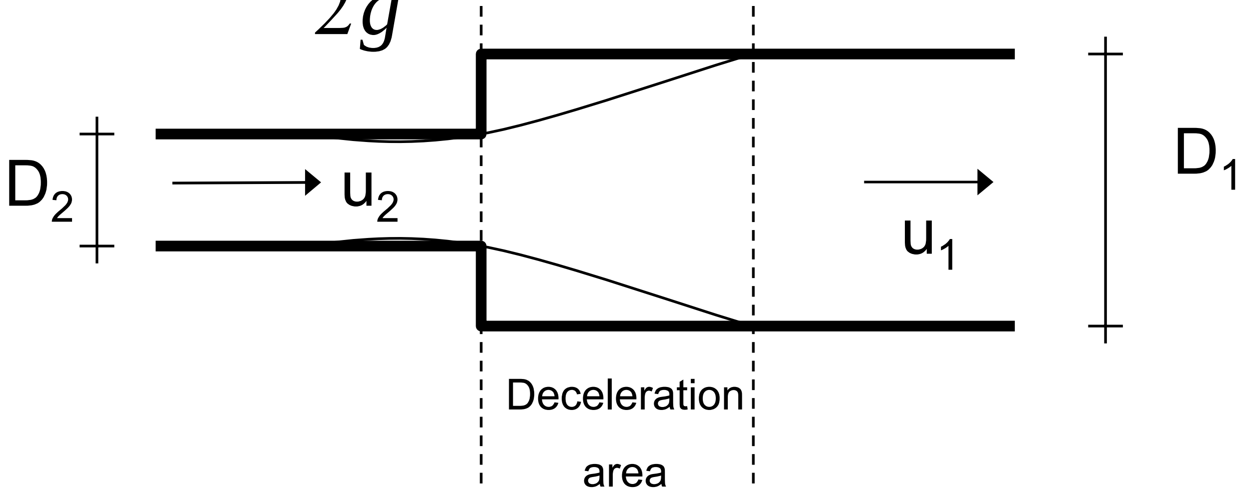
Local losses

- Energy loss due to deceleration and release of streamlines

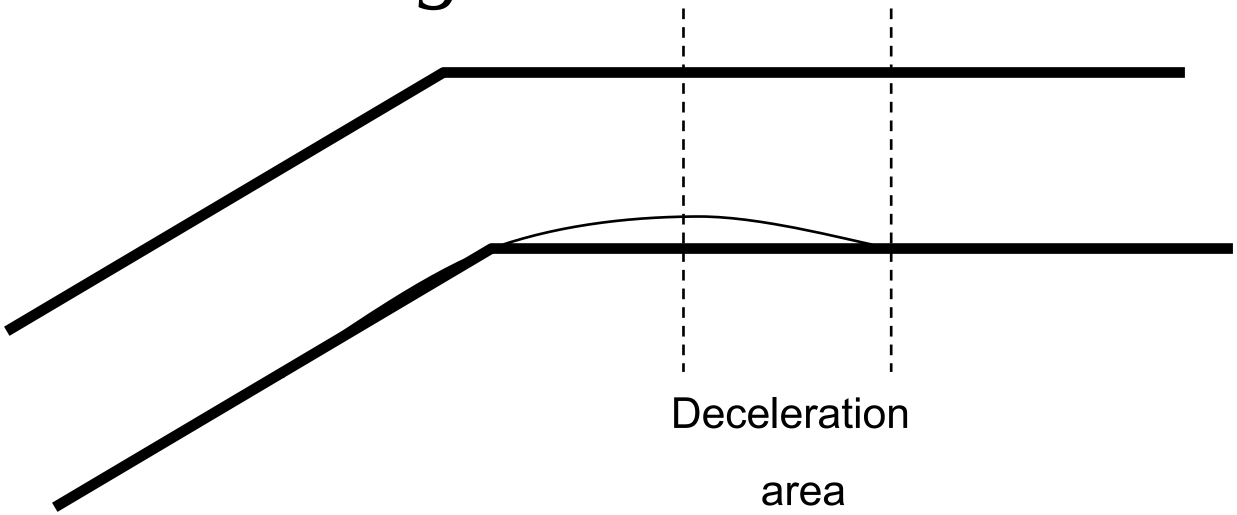
$$\Delta H = \xi \frac{u^2}{2g}$$

$$\frac{\partial A}{\partial x} \neq 0$$



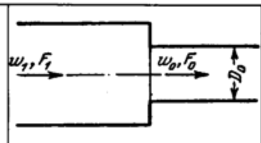
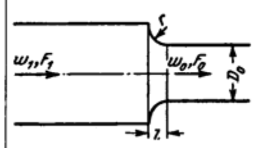
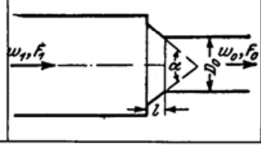
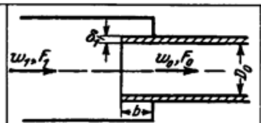
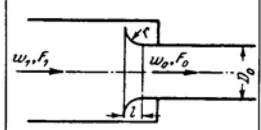
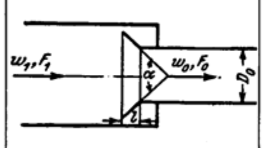
Local losses

$$\Delta H = \xi \frac{u^2}{2g}$$



Entrées avec retrécissement brusque
$Re = w_0 D_H/\nu > 10^4$

Chapitre III
Diagramme 3.9

Conditions à l'entrée	Schéma	Coefficient de perte de charge $\zeta = \frac{\Delta H}{\frac{\gamma w_0^2}{2g}}$
A. Section d'entrée dans une paroi frontale ($b/D_H = 0$) $D_H = 4F_0/\Pi_0$; Π_0 : périmètre		
Bord d'entrée à angle droit		$\zeta = 0{,}5\,(1 - F_0/F_1)$
Bord d'entrée arrondi		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f(b/D_H)$ sur le diagramme 3.3 (graphique *c*).
Bord d'entrée de forme conique		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f(\alpha^0, l/D_H)$ sur le diagramme 3.6.
B. Section d'entrée en avant de la paroi frontale ($b/D_H > 0$)		
Bord d'entrée effilé ou non		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f(\delta_1/D_H, b/D_H)$ sur le diagramme 3.1.
Bord d'entrée arrondi		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f(r/D_H)$ sur le diagramme 3.3 (graphiques *a* et *b*).
Bord d'entrée de forme conique		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f(\alpha^0, l/D_H)$ sur le diagramme 3.5 ; les valeurs de ν sont données dans le § 1.3, *b*).

Source:
Idel'cik

Elargissement brusque en aval d'un tronçon long et rectiligne, un diffuseur, etc, avec une répartition des vitesses suivant la loi exponentielle
Section circulaire ou rectangulaire Re $= w_0 D_H/\nu > 3{,}5\cdot 10^3$

Chapitre IV

Diagramme 4.2

$$\frac{w}{w_{max}} = \left(1 - \frac{y}{R_0}\right)^{\frac{1}{m}} ; \; m \geqslant 1$$

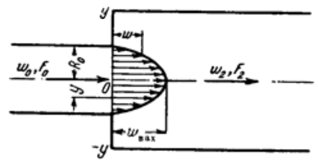
$$\zeta = \frac{\Delta H}{\frac{\gamma w_0^2}{2g}} = \frac{1}{n^2} + N - \frac{2M}{n}$$: est déterminé sur le graphique *a*).

$$M = \frac{(2m+1)^2 (m+1)}{4m^2 (m+2)}$$

$$N = \frac{(2m+1)^3 (m+1)^3}{4m^4 (2m+3)(m+3)}$$

sont déterminés sur le graphique *b*) ;

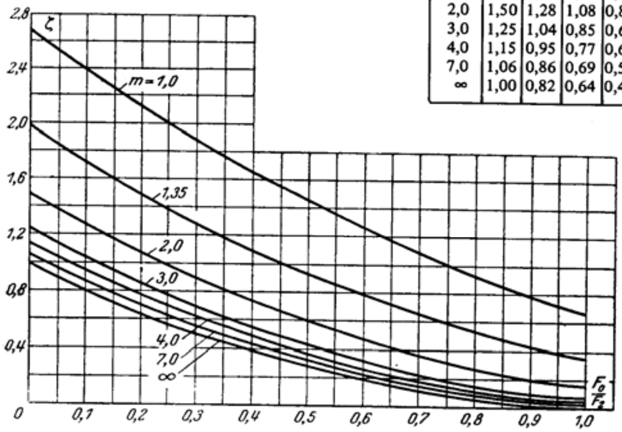
Les valeurs de ν sont données dans le paragraphe 1.3, *b*).



$D_H = 4F_0/\Pi_0$; Π_0 : périmètre ; $n = F_2/F_0$

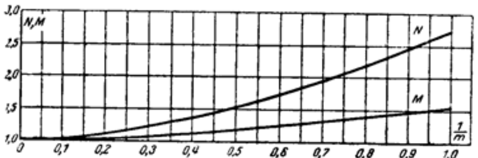
Valeurs de ζ

m	F_0/F_2									
	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	1,0
1,0	2,70	2,42	2,14	1,90	1,66	1,45	1,26	1,09	0,94	0,70
1,35	2,00	1,74	1,51	1,29	1,00	0,93	0,77	0,65	0,53	0,36
2,0	1,50	1,28	1,08	0,89	0,72	0,59	0,46	0,35	0,27	0,16
3,0	1,25	1,04	0,85	0,68	0,53	0,41	0,30	0,20	0,14	0,07
4,0	1,15	0,95	0,77	0,62	0,47	0,35	0,25	0,17	0,11	0,05
7,0	1,06	0,86	0,69	0,53	0,41	0,29	0,19	0,12	0,06	0,02
∞	1,00	0,82	0,64	0,48	0,36	0,25	0,16	0,09	0,04	0



a)

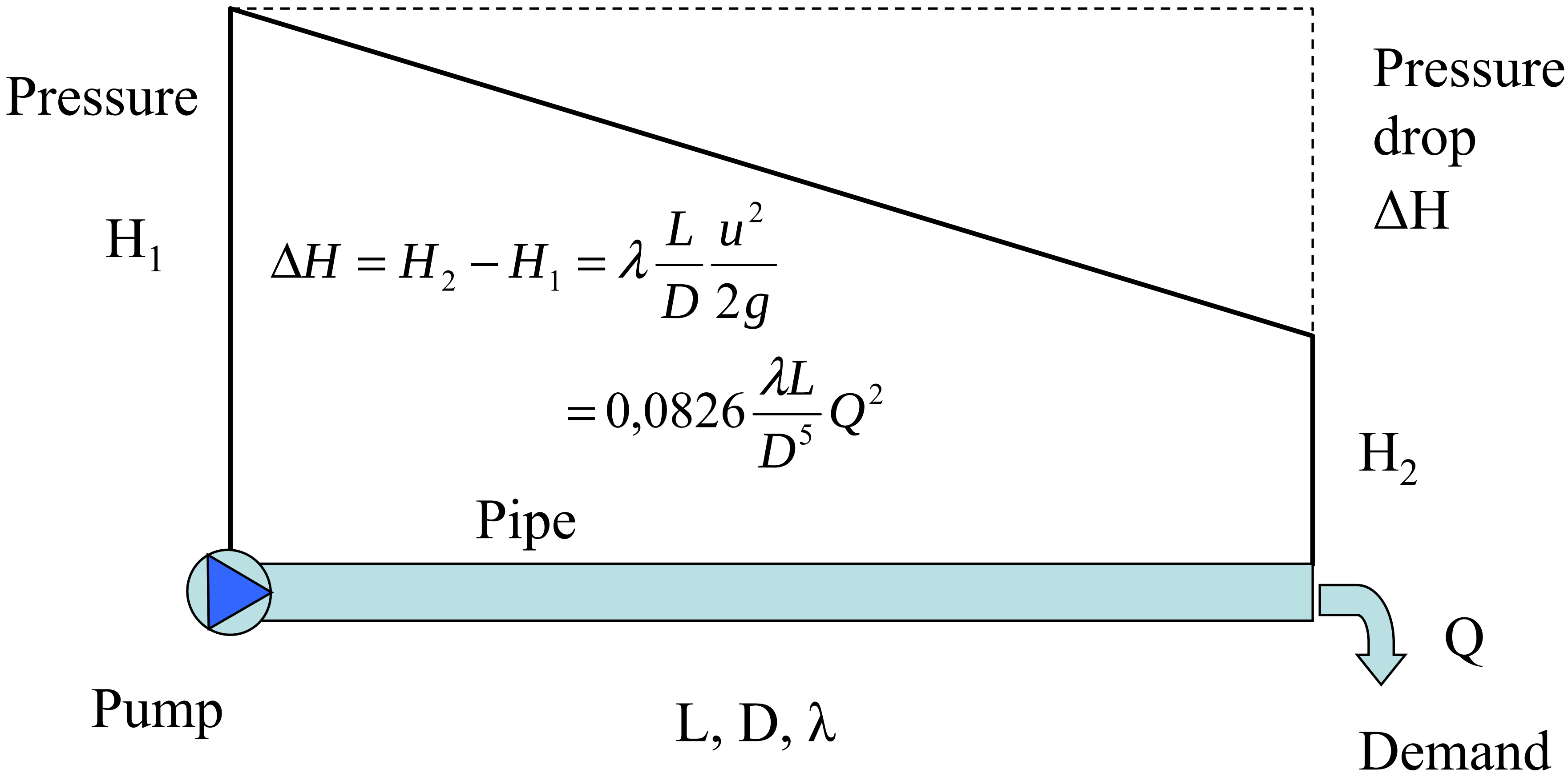
Source:
Idel'cik



b)

m	1,0	1,35	2,0	3,0	4,0	7,0	∞
N	2,70	2,00	1,50	1,25	1,15	1,06	1,0
M	1,50	1,32	1,17	1,09	1,05	1,02	1,0

Pressurised transport



$$\Delta H = H_2 - H_1 = \lambda \frac{L}{D}\frac{u^2}{2g}$$

$$= 0{,}0826 \frac{\lambda L}{D^5} Q^2$$

Sewer transport

- Three flow conditions occur:
  - Open channel flow
  - Fully filled closed pipe
  - Transition situation
- Modelling is very challenging

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Sewerage transport: open channel flow

$$\text{Mass balance}: \frac{\partial A(h)}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\text{Momentum equation}:$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial p}{\partial x} + c\frac{Q|Q|}{AR} = 0$$

$$\text{Flow surface dependant on width in time and place}:$$

$$A = B(x,t) * h(t)$$

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Sewer transport: open channel flow

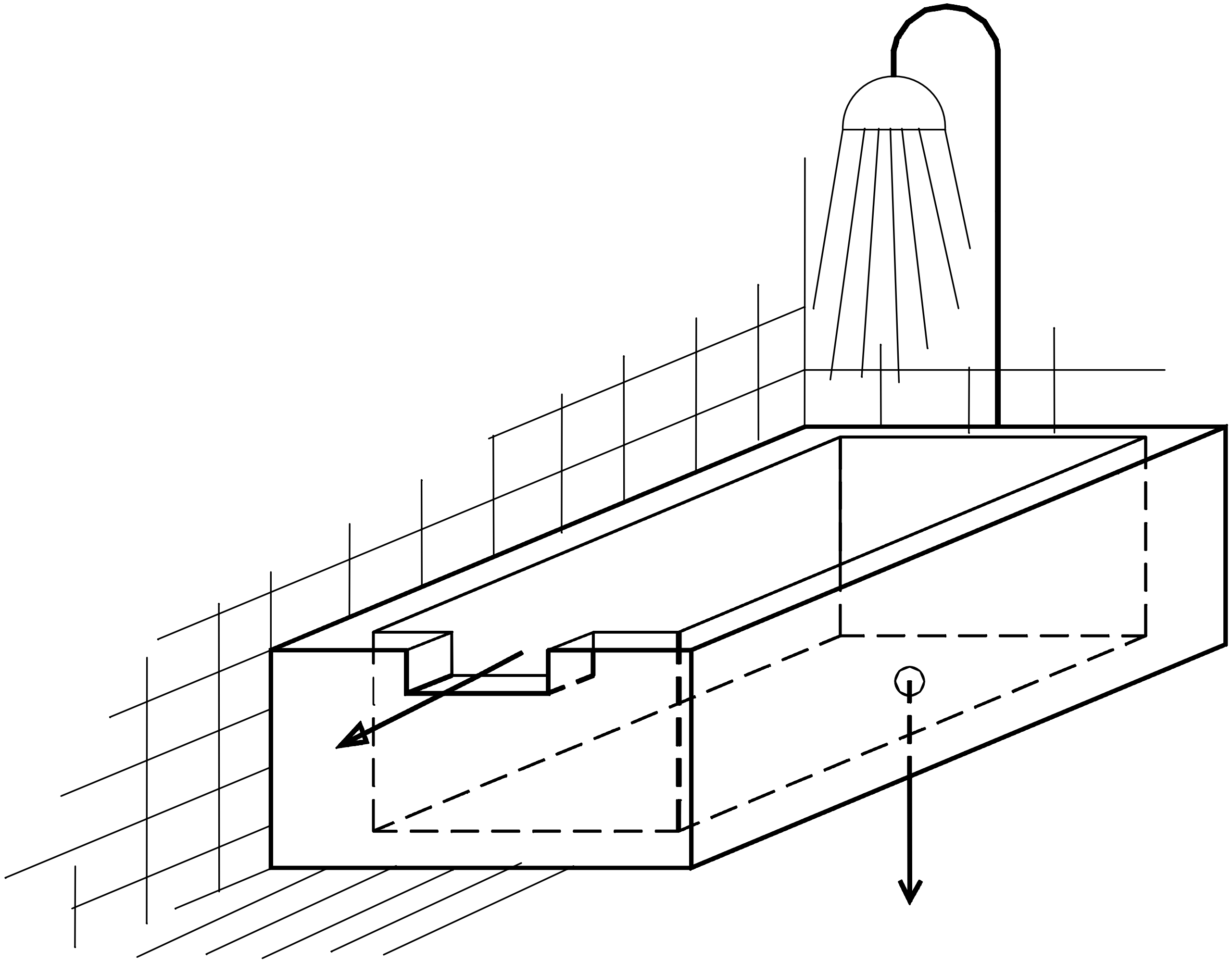
- Mass balance

$$B(h)=\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=0$$

- Continuity equation

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial h}{\partial x}+c\frac{|Q|Q}{AR}=0$$

Schematic model sewer system



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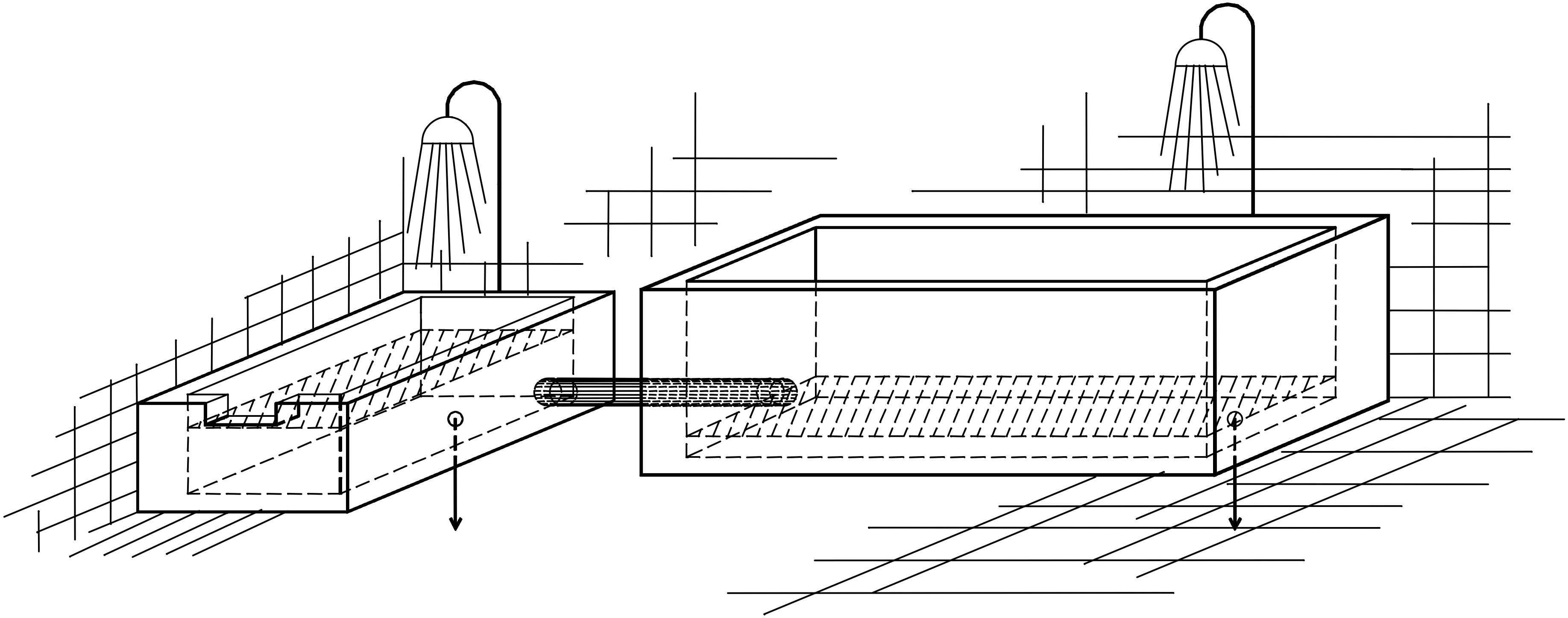
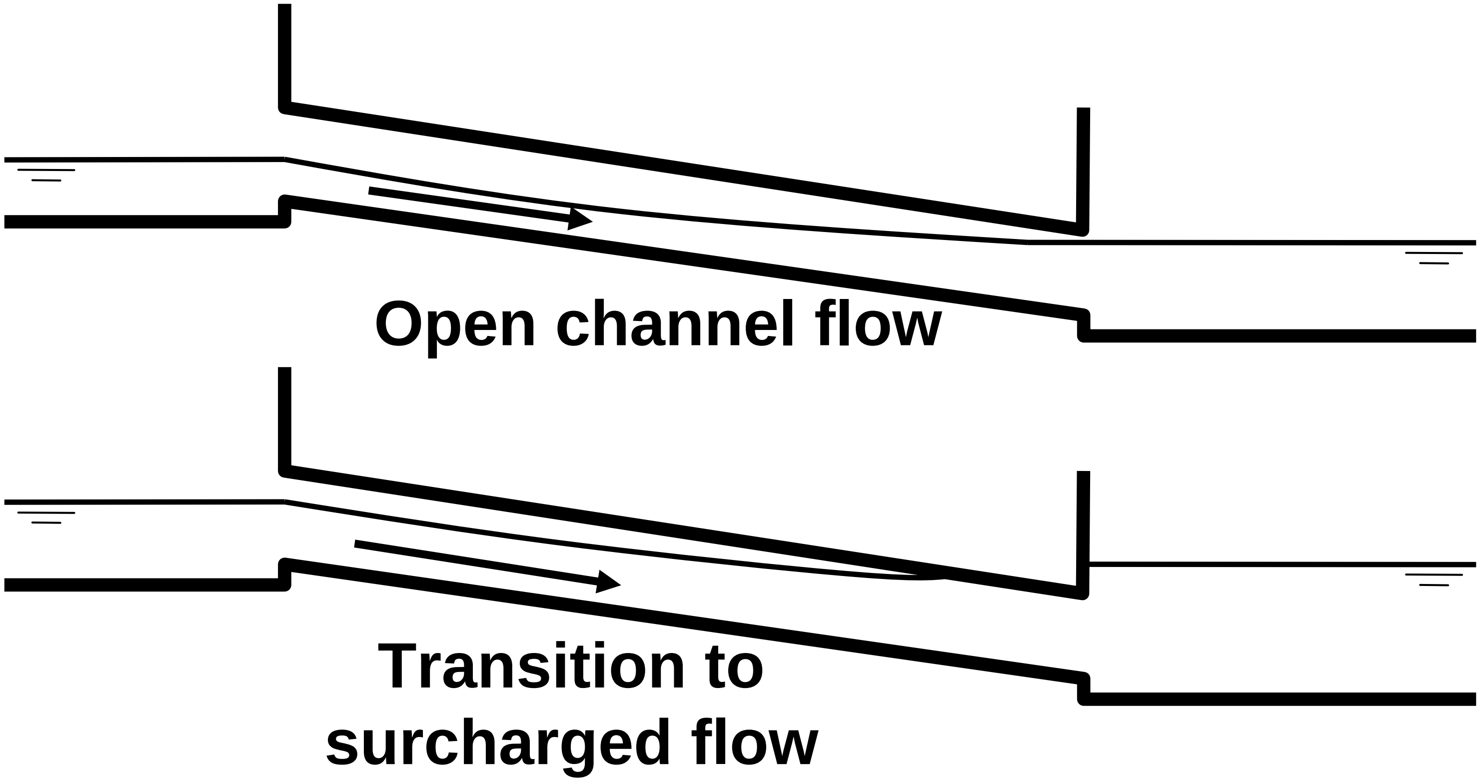
Schematic model urban drainage system



Transition between open channel and fully filled

- Physical (and mathematical) instability



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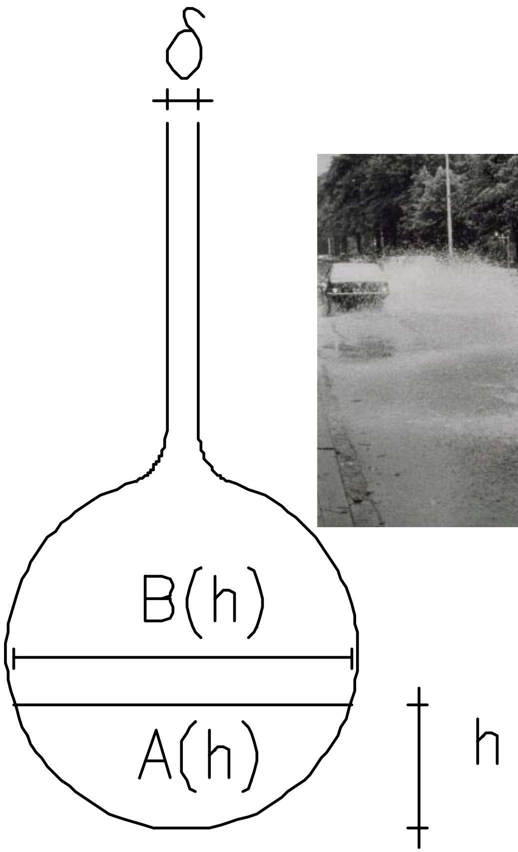
The Preissmann slot: preserve open channel flow

- δ is 0,1 to 5% of diameter
- Keeps open surface
- Valid through street level
- Introduces systematical error



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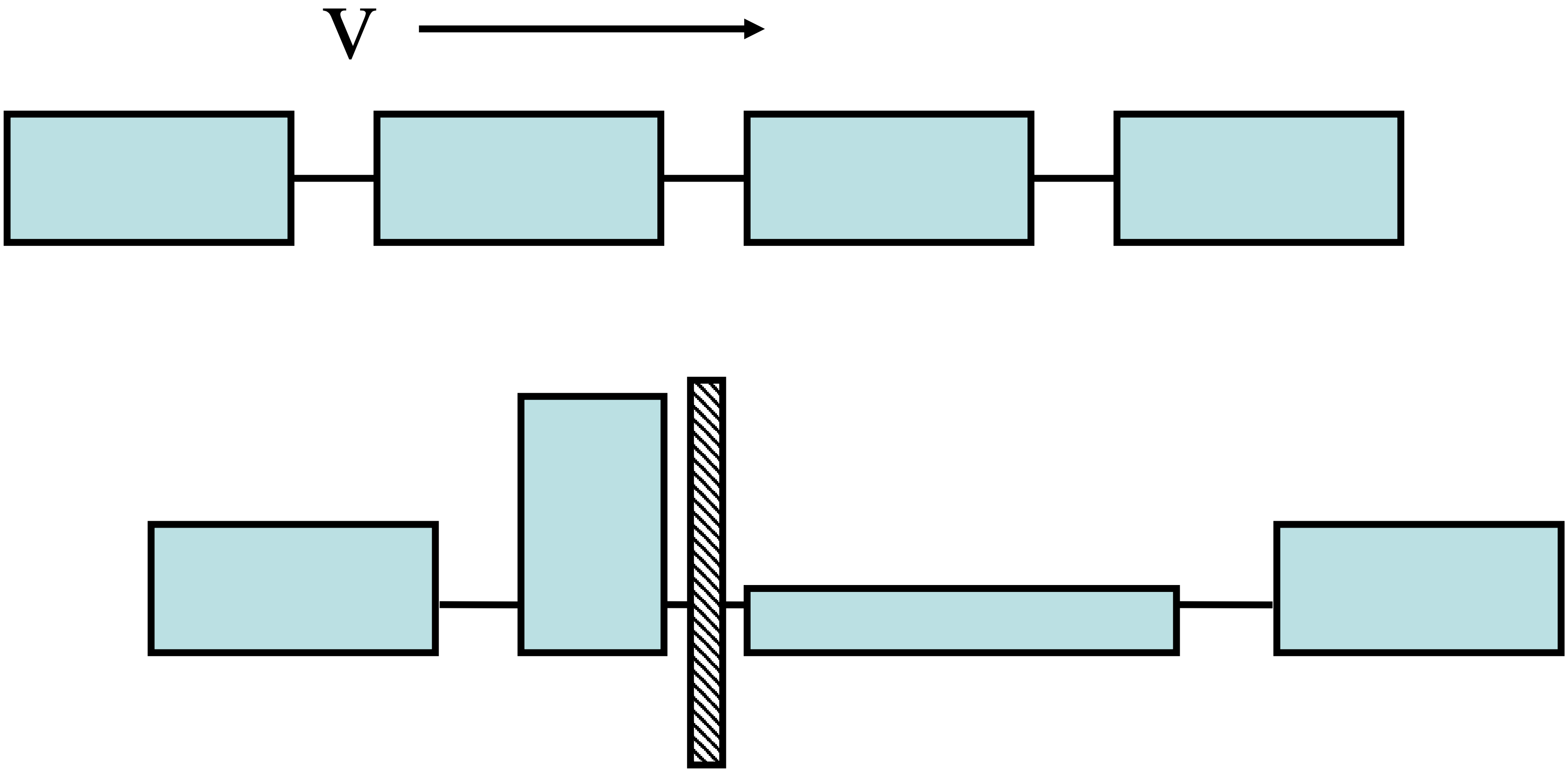
Water hammer: fully filled flow

- Sudden changes in boundary conditions:
  - Increase/decrease in velocity
  - Pump trip
- Take into account compressibility of water and elasticity of the pipe

Water hammer



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Water hammer

- Further explanation Ivo Pothof
  WL|Delft Hydraulics

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Summary

- Drinking water, normal conditions
  - Rigid column, closed pipes:
  - Darcy-Weissbach
  - Time independent
- Sewerage water, normal conditions
  - Open channel flow
  - Time dependant
  - Transition phase: Preissmann slot
- Water hammer
  - Time dependant
  - Special analysis
  - Practical and construction measures

To get some feeling of dimensions

- Assume a pipe
  - Length 5 km
  - Diameter 500 mm
  - Flow 400 m^3/h
- Pressure drop?

To get some more feeling

- Assume two pipes
- Connected parralel
- Same lambda’s, pressure drop, length
- Volume flow Q
- Diameters pipe 1 :D, pipe 2: 2*D

- What is the ratio between the flows through the pipes?

Few questions

- How much will pressure drop with doubling of flow?
- What effect has increasing roughness on pressure drop?
- What has more effect: increasing roughness or decreasing diameter?