Offshore Hydromechanics Part 2

Ir. Peter Naaijen

7. Summary and Internal Forces

OE 4630 2012 - 2013
Offshore Hydromechanics, lecture 1

Take your laptop, i- or whatever smart-phone and go to:
www.rwpoll.com
Login with session ID

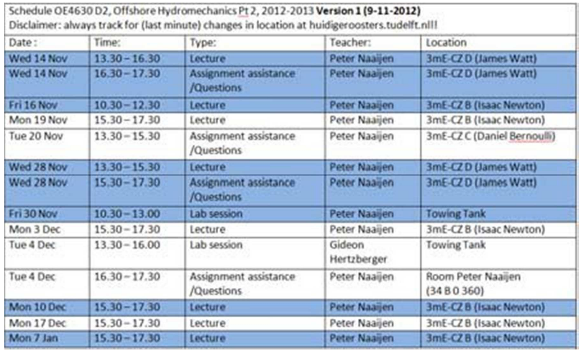
Teacher module II:
- Ir. Peter Naaijen
- p.naaijen@tudelft.nl
- Room 34 B-0-360 (next to towing tank)

Book:
- Offshore Hydromechanics, by J.M.J. Journee & W.W.Massie

Useful weblinks:
- http://www.shipmotions.nl
- Blackboard

OE4630 module II course content

- +/- 7 Lectures
- Bonus assignments (optional, contributes 20% of your exam grade)
- Laboratory Excercise (starting 30 nov)
  - 1 of the bonus assignments is dedicated to this exercise
  - Groups of 7 students
  - Subscription available soon on BB
- Written exam



Schedule OE4630 D2, Offshore Hydromechanics Pt 2, 2012-2013 **Version 1 (9-11-2012)**
Disclaimer: always track for (last minute) changes in location at huidigeroosters.tudelft.nl!!

Date :	Time:	Type:	Teacher:	Location
Wed 14 Nov	13.30 – 16.30	Lecture	Peter Naaijen	3mE-CZ D (James Watt)
Wed 14 Nov	16.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ D (James Watt)
Fri 16 Nov	10.30 – 12.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 19 Nov	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Tue 20 Nov	13.30 – 15.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ C (Daniel Bernoulli)
Wed 28 Nov	13.30 – 15.30	Lecture	Peter Naaijen	3mE-CZ D (James Watt)
Wed 28 Nov	15.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	3mE-CZ D (James Watt)
Fri 30 Nov	10.30 – 13.00	Lab session	Peter Naaijen	Towing Tank
Mon 3 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Tue 4 Dec	13.30 – 16.00	Lab session	Gideon Hertzberger	Towing Tank
Tue 4 Dec	16.30 – 17.30	Assignment assistance /Questions	Peter Naaijen	Room Peter Naaijen (34 B 0 360)
Mon 10 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 17 Dec	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)
Mon 7 Jan	15.30 – 17.30	Lecture	Peter Naaijen	3mE-CZ B (Isaac Newton)

Lecture notes:

- Disclaimer: Not everything you (should) learn is in the lecture notes (lees: niet alles wat op het tentamen gevraagd kan worden staat in diktaat…) →

Make personal notes during lectures!!

- Don’t save your questions ‘till the break →

Ask if anything is unclear

Learning goals Module II, behavior of floating bodies in waves

• Definition of ship motions **Motion Response in regular waves:** • How to use RAO’s • Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces • How to solve RAO’s from the equation of motion **Motion Response in irregular waves:** •How to determine response in irregular waves from RAO’s and wave spectrum without forward speed		
3D linear Potential Theory •How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential •How to determine Velocity Potential		
Motion Response in irregular waves: • How to determine response in irregular waves from RAO’s and wave spectrum with forward speed • Make down time analysis using wave spectra, scatter diagram and RAO’s	Ch. 8	
Structural aspects: • Calculate internal forces and bending moments due to waves		
Nonlinear behavior: • Calculate mean horizontal wave force on wall • Use of time domain motion equation	Ch.6	

Introduction

[3]

Introduction

Offshore → oil resources have to be explored in deeper water → floating structures instead of bottom founded

[4]

Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Inertia forces on sea fastening due to accelerations:

Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Direct wave induced structural loads


[5]

Minimum required air gap to avoid wave damage

Introduction

Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations

[5]

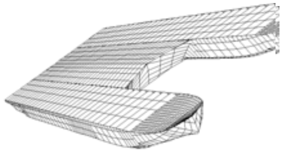
[5]

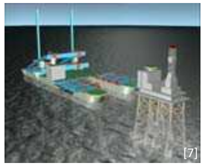

[6]

Introduction

Decommissioning / Installation / Pipe laying → Excalibur / Allseas 'Pieter Schelte'

- Motion Analysis

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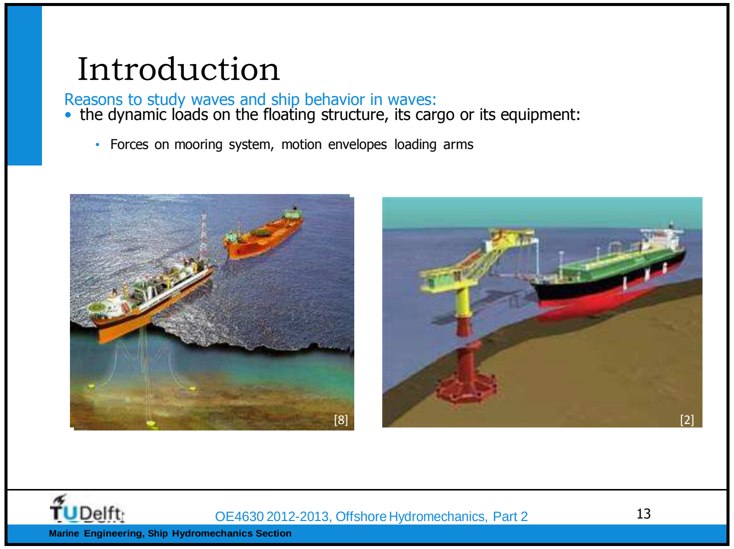


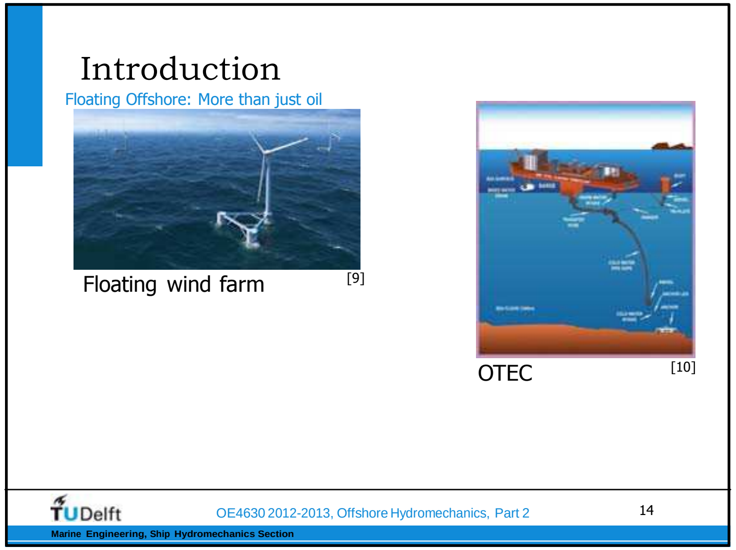
Introduction

Reasons to study waves and ship behavior in waves:

- the dynamic loads on the floating structure, its cargo or its equipment:
  - Forces on mooring system, motion envelopes loading arms

[8]

[2]

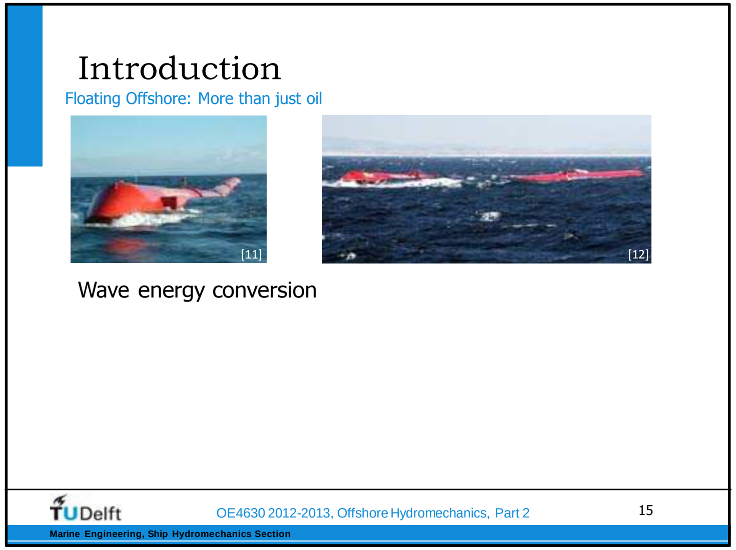


Introduction

Floating Offshore: More than just oil

Floating wind farm [9]

OTEC [10]



Introduction

Floating Offshore: More than just oil

[11]

[12]

Wave energy conversion

Introduction

Floating Offshore: More than just oil

Mega Floaters

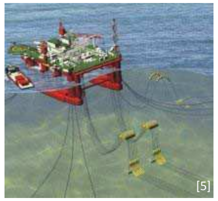
Introduction

Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations
- Downtime analysis

Winter Data of Areas 8, 9, 15 and 16 of the North Atlantic (Global Wave Statistics)

	T_2 (s)											
Hs (m)	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	Total
14.5	0	0	0	0	2	30	154	362	466	370	202	1586
13.5	0	0	0	0	3	33	145	293	322	219	101	1116
12.5	0	0	0	0	7	72	289	539	548	345	149	1949
11.5	0	0	0	0	17	160	585	996	931	543	217	3449
10.5	0	0	0	1	41	363	1200	1852	1579	843	310	6189
9.5	0	0	0	4	109	845	2485	3443	2648	1283	432	11249
8.5	0	0	0	12	295	1996	5157	6323	4333	1882	572	20570
7.5	0	0	0	41	818	4723	10537	11242	6755	2594	703	37413
6.5	0	0	1	138	2273	10967	20620	18718	9665	3222	767	66371
5.5	0	0	7	471	6187	24075	36940	27702	11969	3387	694	111432
4.5	0	0	31	1586	15757	47072	56347	33539	11710	2731	471	169244
3.5	0	0	148	5017	34720	74007	64809	28964	7804	1444	202	217115
2.5	0	4	681	13441	56847	77259	45013	13962	2725	381	41	210354
1.5	0	40	2699	23284	47839	34532	11554	2208	282	27	2	122467
0.5	5	350	3314	8131	5858	1598	216	18	1	0	0	19491
Tota	5	394	6881	52126	170773	277732	256051	150161	61738	19271	4863	999995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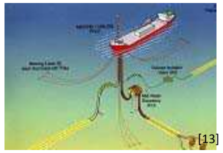
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Real-time motion prediction
Using X-band radar remote wave observation

Definitions & Conventions
Regular waves
Ship motions


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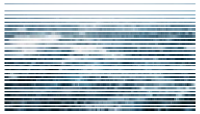
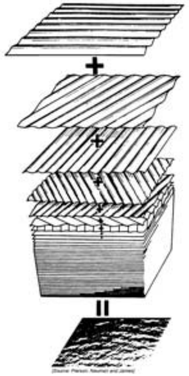
Irregular wind waves

apparently irregular but can be considered as a superposition of a finite number of regular waves, each having own frequency, amplitude and propagation direction

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Regular waves

(Ch.5 revisited)

Regular waves

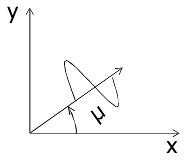
regular wave propagating in direction μ:

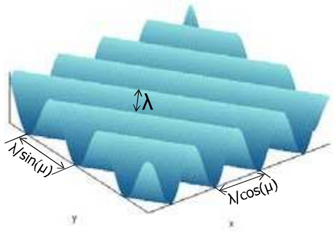
$$\zeta(t,x)=\zeta_a\cos(\omega t-kx\cos\mu-ky\sin\mu)$$

$$k=2\pi/\lambda$$

$$\omega=2\pi/T$$

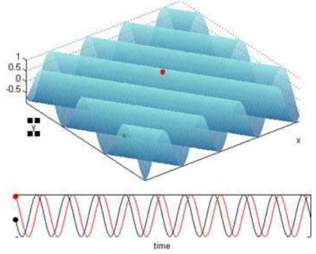
Linear solution Laplace equation





- Regular waves
- regular wave propagating in direction μ

$$\zeta(t,x)=\zeta_a\cos\underbrace{(\omega t-kx\cos\mu-ky\sin\mu)}_{\text{Phase angle }\varepsilon}$$



Phase angle wave at black dot with respect to wave at red dot:

$$\varepsilon_{\zeta,\zeta}=-k(x-x)\cos\mu-k(y-y)\sin\mu$$

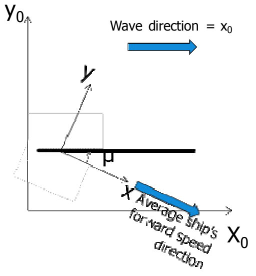
Co-ordinate systems

Definition of systems of axes

Earth fixed: (x_0, y_0, z_0)

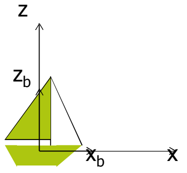
Following average ship position: (x,y,z)

wave direction with respect to ship’s exes system: μ



Behavior of structures in waves

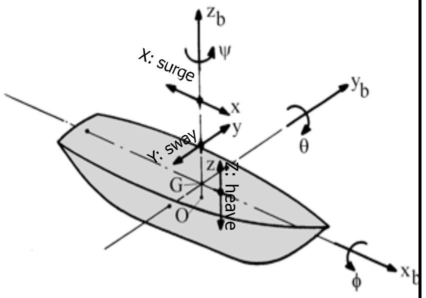
Ship's body bound axes system (x_b,y_b,z_b) follows all ship motions



Behavior of structures in waves

Definition of translations

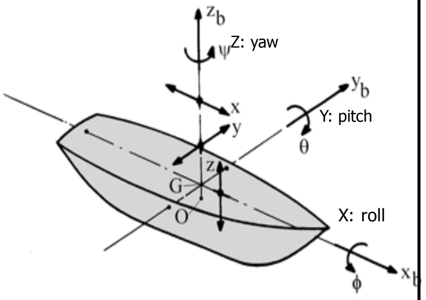
		NE	EN
1	x	Schrikken	Surge
2	y	Verzetten	Sway
3	z	Dompen	Heave



Behavior of structures in waves

Definition of rotations

		NE	EN
4	x	Slingeren	Roll
5	y	Stampen	Pitch
6	z	Gieren	Yaw



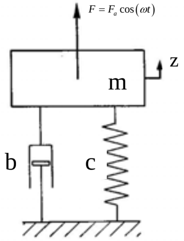
How do we describe ship motion response?

Rao's
Phase angles

Mass-Spring system:

$m\ddot{z}+b\dot{z}+cz=F_a\cos(\omega t)$ Motion equation

$z(t)=z_a\cos(\omega t+\varepsilon)$ Steady state solution



Motions of and about COG

Amplitude   Phase angle

$Surge(schrikken):\ x=x_a\cos\left(\omega t+\varepsilon_{x\zeta}\right)$

$Sway(verzetten):\ \mathrm{y}=y_a\cos\left(\omega t+\varepsilon_{y\zeta}\right)$

$Heave(dompen):\ z=z_a\cos\left(\omega t+\varepsilon_{z\zeta}\right)$

$Roll(rollen):\ \langle\text{phi}\rangle\phi=\phi_a\cos\left(\omega t+\varepsilon_{\phi\zeta}\right)$

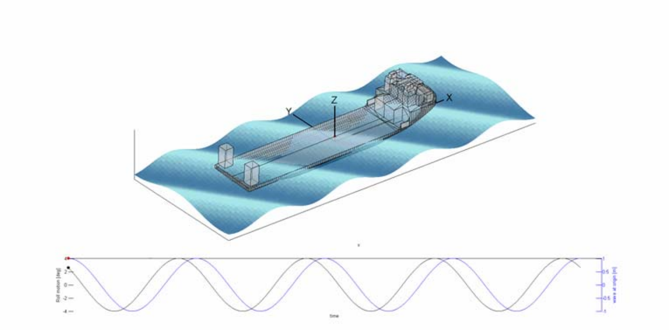
$Pitch(stampen):\ \langle\text{theta}\rangle\theta=\theta_a\cos\left(\omega t+\varepsilon_{\theta\zeta}\right)$

$Yaw(gieren):\ \langle\text{psi}\rangle\psi=\psi_a\cos\left(\omega t+\varepsilon_{\psi\zeta}\right)$

Phase angles ε are related to undisturbed wave at origin of steadily translating ship-bound system of axes (→COG)

Motions of and about COG

Phase angles ε are related to undisturbed wave at origin of steadily translating ship-bound system of axes (→COG)



Motions of and about COG

$Surge(schrikken):\ x=x_a\cos\left(\omega t+\varepsilon_{x\zeta}\right)$ $RAOSurge:\ \frac{x_a}{\zeta_a}(\omega,\mu)$

$Sway(verzetten):\ \mathrm{y}=y_a\cos\left(\omega t+\varepsilon_{y\zeta}\right)$ $RAOSway:\ \frac{y_a}{\zeta_a}(\omega,\mu)$

$Heave(dompen):\ z=z_a\cos\left(\omega t+\varepsilon_{z\zeta}\right)$ $RAOHeave:\frac{z_a}{\zeta_a}(\omega,\mu)$

$Roll(rollen):\ \langle\text{phi}\rangle\phi=\phi_a\cos\left(\omega t+\varepsilon_{\phi\zeta}\right)$ $RAORoll:\ \frac{\phi_a}{\zeta_a}(\omega,\mu)$

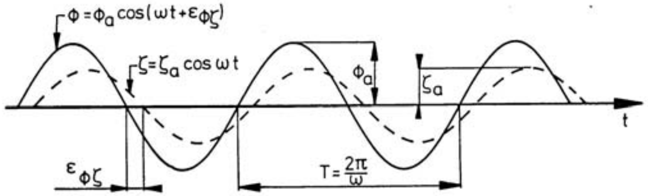
$Pitch(stampen):\ \langle\text{theta}\rangle\theta=\theta_a\cos\left(\omega t+\varepsilon_{\theta\zeta}\right)$ $RAOPitch:\frac{\theta_a}{\zeta_a}(\omega,\mu)$

$Yaw(gieren):\ \langle\text{psi}\rangle\psi=\psi_a\cos\left(\omega t+\varepsilon_{\psi\zeta}\right)$ $RAOYaw:\frac{\psi_a}{\zeta_a}(\omega,\mu)$

RAO and phase depend on:

- Wave frequency
- Wave direction

Example: roll signal



Displacement $\phi = \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right)$

Velocity… $\dot{\phi} = -\omega_e \phi_a \sin\left(\omega_e t + \varepsilon_{\phi\zeta}\right) = \omega_e \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta} + \pi/2\right)$

Acceleration… $\ddot{\phi} = -\omega_e^2 \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right) = \omega_e^2 \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta} + \pi\right)$

Motions of and about COG

1 *Surge*(*schrikken*): $x = x_a \cos\left(\omega_e t + \varepsilon_{x\zeta}\right)$
2 *Sway*(*verzetten*): $y = y_a \cos\left(\omega_e t + \varepsilon_{y\zeta}\right)$
3 *Heave*(*dompen*): $z = z_a \cos\left(\omega_e t + \varepsilon_{z\zeta}\right)$
4 *Roll*(*rollen*): $\langle \text{phi} \rangle \phi = \phi_a \cos\left(\omega_e t + \varepsilon_{\phi\zeta}\right)$
5 *Pitch*(*stampen*): $\langle \text{theta} \rangle \theta = \theta_a \cos\left(\omega_e t + \varepsilon_{\theta\zeta}\right)$
6 *Yaw*(*gieren*): $\langle \text{psi} \rangle \psi = \psi_a \cos\left(\omega_e t + \varepsilon_{\psi\zeta}\right)$

- Frequency of input (regular wave) and output (motion) is ALWAYS THE SAME !!
- Phase can be positive ! (shipmotion ahead of wave elevation at COG)
- Due to symmetry: some of the motions will be zero
- Ratio of motion amplitude / wave amplitude = RAO (Response Amplitude Operator)
- RAO's and phase angles depend on wave frequency and wave direction
- RAO's and phase angles must be calculated by dedicated software or measured by experiments
- Only some special cases in which 'common sense' is enough:

Consider Long waves relative to ship dimensions

What is the RAO of pitch in head waves ?

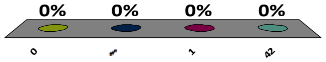
- Phase angle heave in head waves ?…
- RAO pitch in head waves ?…
- Phase angle pitch in head waves ?…
- Phase angle pitch in following waves ?…

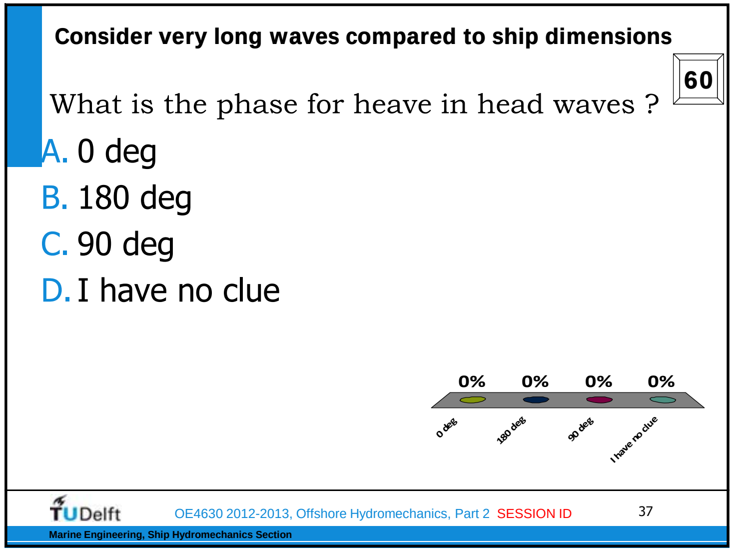
Consider very long waves compared to ship dimensions

60

What is the RAO for heave in head waves ?

A. 0
B. ∞
C. 1
D. 42



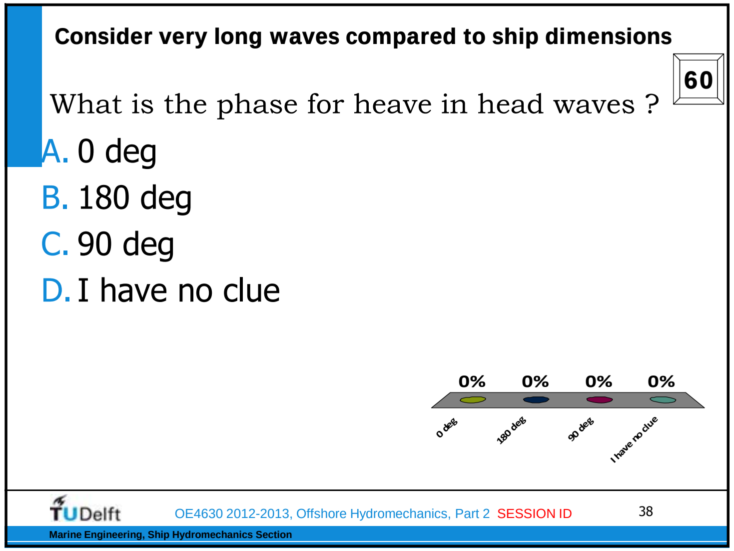


Consider very long waves compared to ship dimensions

60

What is the phase for heave in head waves ?

A. 0 deg
B. 180 deg
C. 90 deg
D. I have no clue



Consider very long waves compared to ship dimensions

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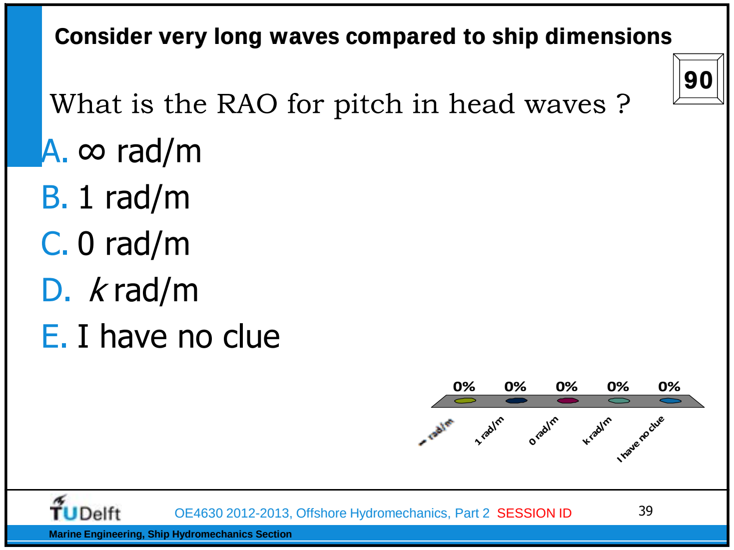
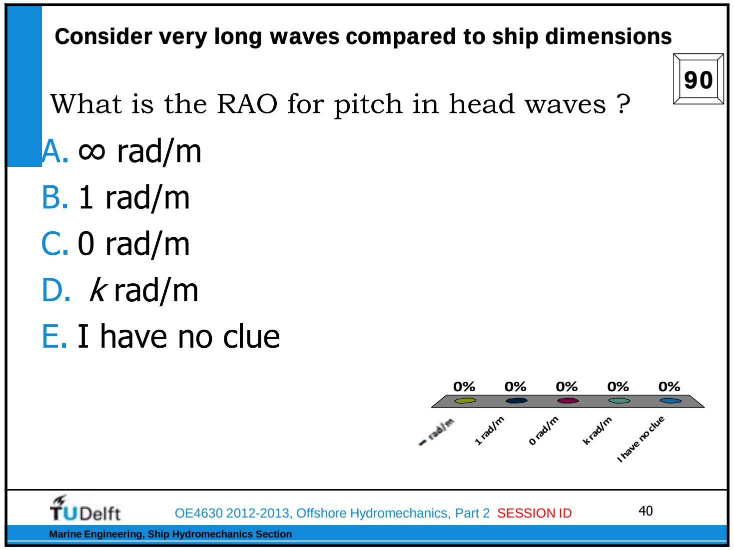


Consider very long waves compared to ship dimensions

90

What is the RAO for pitch in head waves ?

A. ∞ rad/m
B. 1 rad/m
C. 0 rad/m
D. *k* rad/m
E. I have no clue



Consider very long waves compared to ship dimensions

90

What is the RAO for pitch in head waves ?

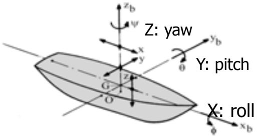
A. ∞ rad/m
B. 1 rad/m
C. 0 rad/m
D. *k* rad/m
E. I have no clue

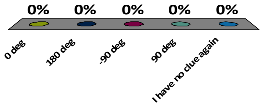
Consider very long waves compared to ship dimensions

What is the phase for pitch in head waves ?

120

A. 0 deg
B. 180 deg
C. -90 deg
D. 90 deg
E. I have no clue again





Consider very long waves compared to ship dimensions

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A. 0 deg
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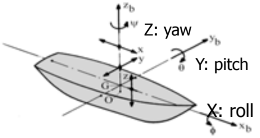
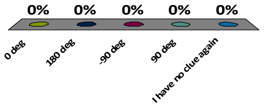
E. I have no clue again





Local motions (in steadily translating axes system)

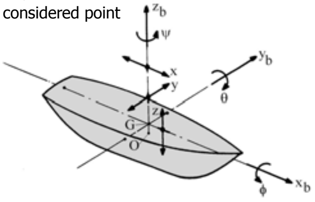
- **Only variations!!**
- **Linearized!!**

$$\begin{pmatrix} x_P(t) \\ y_P(t) \\ z_P(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} + \begin{pmatrix} 0 & -\psi(t) & \theta(t) \\ \psi(t) & 0 & -\phi(t) \\ -\theta(t) & \phi(t) & 0 \end{pmatrix} \cdot \begin{pmatrix} x_{bP} \\ y_{bP} \\ z_{bP} \end{pmatrix}$$

6 DOF Ship motions

Location considered point

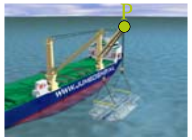
$$x_P(t) = x(t) - y_{bP}\psi(t) + z_{bP}\theta(t)$$
$$y_P(t) = y(t) + x_{bP}\psi(t) - z_{bP}\phi(t)$$
$$z_P(t) = z(t) - x_{bP}\theta(t) + y_{bP}\phi(t)$$



Local Motions

Example 3: horizontal crane tip motions

The tip of an onboard crane, location:
xb,yb,zb = -40, -9.8, 25.0



For a frequency ω=0.6 the RAO's and phase angles of the ship motions are:

SURGE		SWAY		HEAVE		ROLL		PITCH		YAW	
RAO -	phase degr	RAO -	phase degr	RAO -	phase degr	RAO deg/m	phase degr	RAO deg/m	phase degr	RAO deg/m	phase degr
1.014E-03	3.421E+02	5.992E-01	2.811E+02	9.991E-01	3.580E+02	2.590E+00	1.002E+02	2.424E-03	1.922E+02	2.102E-04	5.686E+01

Calculate the RAO and phase angle of the transverse horizontal motion (y-direction) of the crane tip.

Complex notation of harmonic functions

$$1\ Surge(schrikken):\quad x = x_a\cos\left(\omega_e t+\varepsilon_{x\zeta}\right)$$

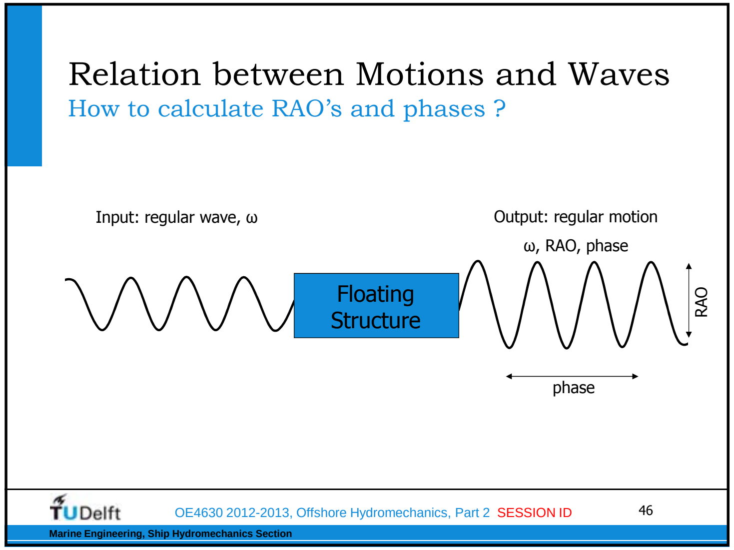
$$= \mathrm{Re}\left(x_a e^{i\left(\omega t+\varepsilon_{x\zeta}\right)}\right)$$

$$= \mathrm{Re}\left(\boxed{x_a e^{i\varepsilon_{x\zeta}}}\cdot e^{i\omega t}\right)$$

Complex motion amplitude

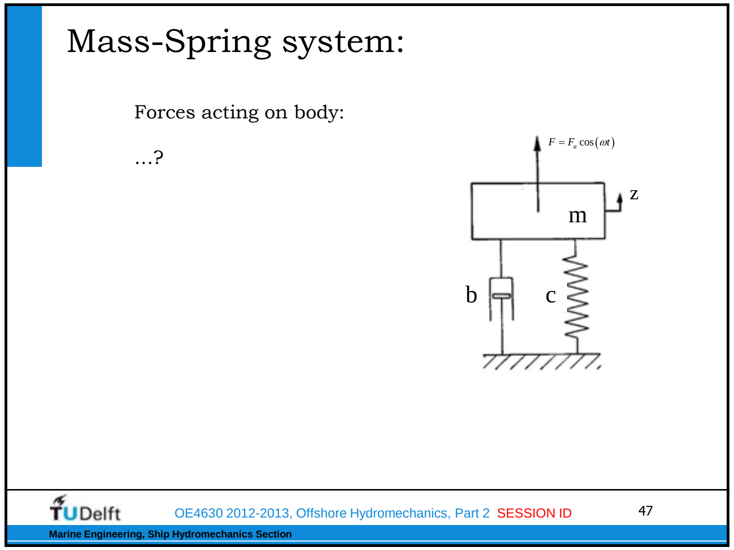
$$= \mathrm{Re}\left(\boxed{\hat{x}_a}\cdot e^{i\omega t}\right)$$

- :



Relation between Motions and Waves

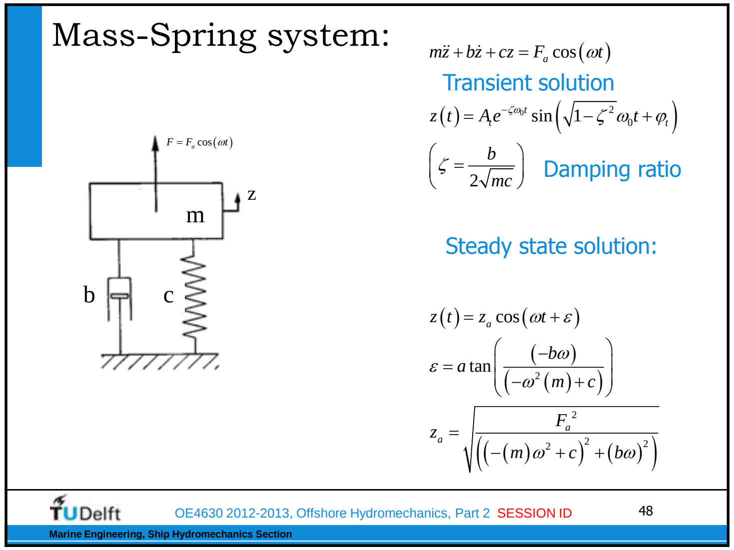
How to calculate RAO's and phases ?



Mass-Spring system:

Forces acting on body:

…?



Mass-Spring system:

$$m\ddot{z}+b\dot{z}+cz = F_a\cos\left(\omega t\right)$$

Transient solution

$$z(t) = Ae^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\,\omega_0 t+\varphi_t\right)$$

$$\left(\zeta = \frac{b}{2\sqrt{mc}}\right)$$ Damping ratio

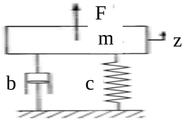
Steady state solution:

$$z(t) = z_a\cos\left(\omega t+\varepsilon\right)$$

$$\varepsilon = a\tan\left(\frac{(-b\omega)}{\left(-\omega^2(m)+c\right)}\right)$$

$$z_a = \sqrt{\frac{F_a^{\ 2}}{\left(\left(-(m)\omega^2+c\right)^2+(b\omega)^2\right)}}$$

Moving ship in waves:



$$m_3\ddot{z}+b_3\dot{z}+c_3z=F_{a3}\cos(\omega t)$$

Restoring coefficient for **heave** ?

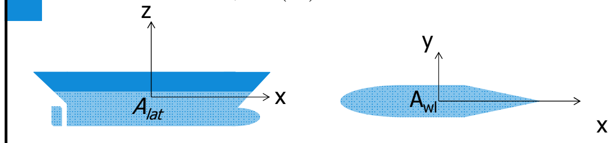
$$m_4\ddot{\varphi}+b_4\dot{\varphi}+c_4\varphi=F_{a4}\cos(\omega t)$$

Restoring coefficient for **roll** ?

m for **roll** ?

What is the hydrostatic spring coefficient for the sway motion ?

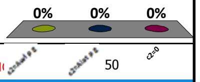
$$m_2\cdot\ddot{y}+b_2\cdot\dot{y}+c_2\cdot y=F_{a2}\cos(\omega t)$$



A. $c_2=A_{wl}\,\rho\,g$

B. $c_2=A_{lat}\,\rho\,g$

C. $c_2=0$



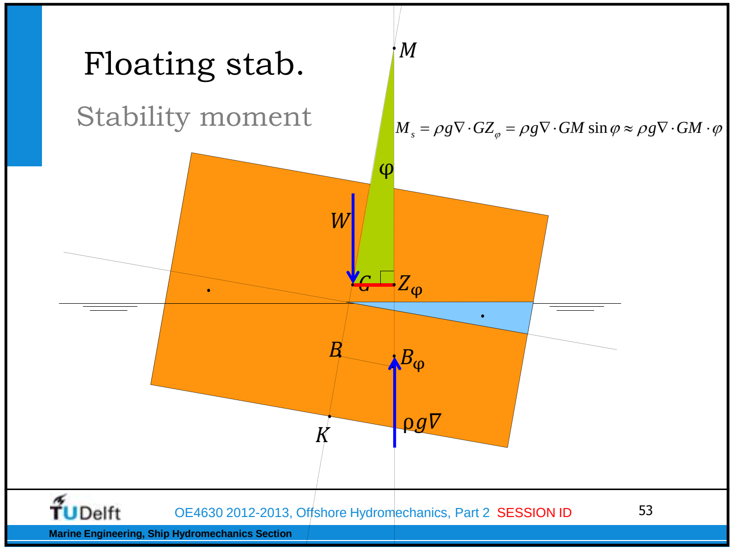
Non linear stability issue…

Roll restoring

Roll restoring coefficient:

$$c_4=\rho g\nabla\cdot GM$$

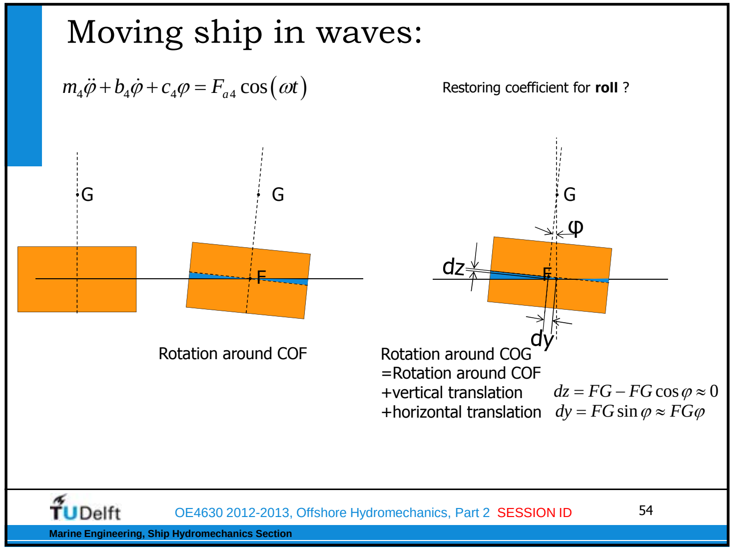
What is the point the ship rotates around statically speaking ? (Ch 2)



Floating stab.

Stability moment

$M_s = \rho g \nabla \cdot GZ_\varphi = \rho g \nabla \cdot GM \sin\varphi \approx \rho g \nabla \cdot GM \cdot \varphi$



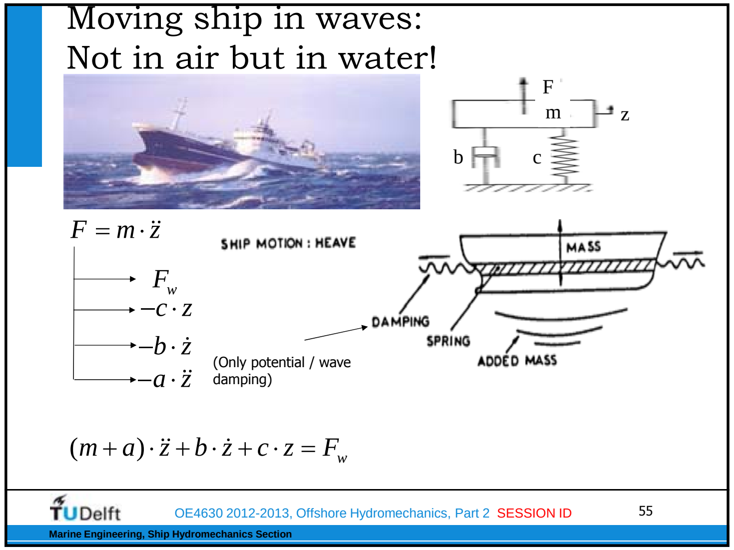
Moving ship in waves:

$m_4\ddot{\varphi} + b_4\dot{\varphi} + c_4\varphi = F_{a4}\cos(\omega t)$

Restoring coefficient for **roll** ?

Rotation around COF

Rotation around COG
=Rotation around COF
+vertical translation $dz = FG - FG\cos\varphi \approx 0$
+horizontal translation $dy = FG\sin\varphi \approx FG\varphi$



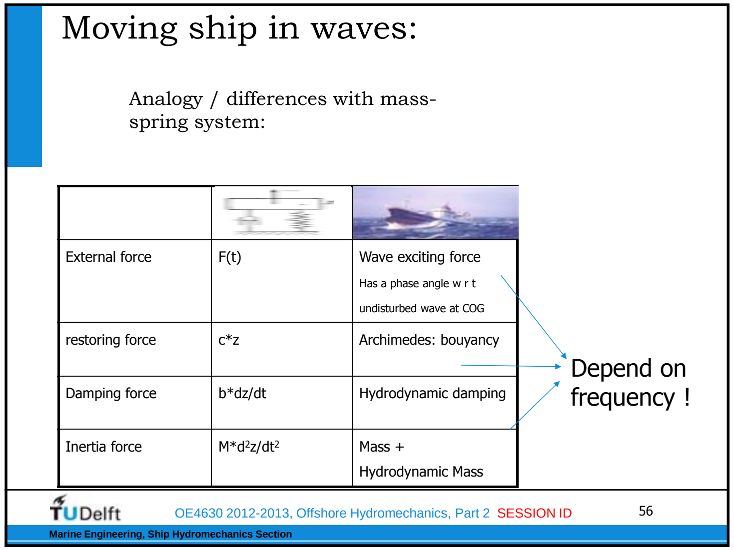
Moving ship in waves:
Not in air but in water!

$F = m \cdot \ddot{z}$

F_w
$-c \cdot z$
$-b \cdot \dot{z}$
$-a \cdot \ddot{z}$

(Only potential / wave damping)

$(m+a)\cdot\ddot{z} + b\cdot\dot{z} + c\cdot z = F_w$



Moving ship in waves:

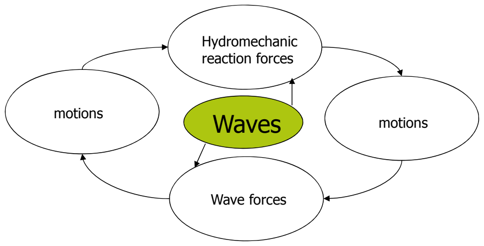
Analogy / differences with mass-spring system:

External force	F(t)	Wave exciting force Has a phase angle w r t undisturbed wave at COG
restoring force	c*z	Archimedes: bouyancy
Damping force	b*dz/dt	Hydrodynamic damping
Inertia force	M*d²z/dt²	Mass + Hydrodynamic Mass

Depend on frequency !

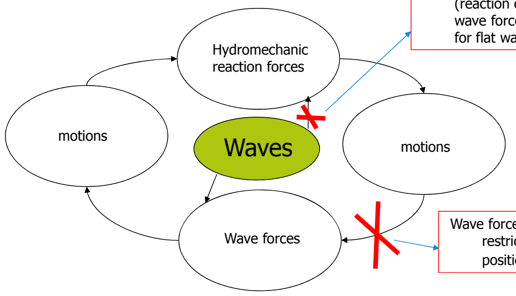
Moving ship in waves:

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$



Moving ship in waves:

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$



Hydromechanic reaction forces (reaction on calculated wave force)are calculated for flat water (**NO WAVES**)

Wave force is calculated for restricted ship in mean position: **No motions**

Right hand side of m.e.: Wave Exciting Forces

- Incoming: regular wave with given frequency and propagation direction
- Assuming the vessel is not moving

Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x)=\zeta_a\cos\left(\omega t-kx\cos\mu-ky\sin\mu\right)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

Back to Regular waves

regular wave propagating in direction μ

$$\zeta(t,x) = \zeta_a \cos\left(\omega t - kx\cos\mu - ky\sin\mu\right)$$

Linear solution Laplace equation

In order to calculate forces on immerged bodies:
What happens underneath free surface ?

Potential Theory

What is potential theory ?:
way to give a mathematical description of flowfield

Most complete mathematical description of flow is viscous Navier-Stokes equation:

Navier-Stokes vergelijkingen:

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\lambda\nabla\cdot V+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]$$

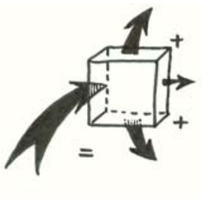
$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left(\lambda\nabla\cdot V+2\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]$$

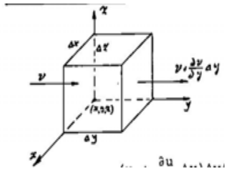
$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left(\lambda\nabla\cdot V+2\mu\frac{\partial w}{\partial z}\right)$$

(not relaxed)

Water is hard to compress, we will assume this is impossible →

Apply principle of continuity on control volume:





Continuity: what comes in,
must go out

This results in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

If in addition the flow is considered to be irrotational and non viscous →

Velocity potential function can be used to describe water motions
Main property of velocity potential function:

for potential flow, a function Φ(x,y,z,t) exists whose derivative in a certain arbitrary direction equals the flow velocity in that direction. This function is called the velocity potential.

From definition of velocity potential:

$$u=\frac{\partial \Phi}{\partial x}, v=\frac{\partial \Phi}{\partial y}, w=\frac{\partial \Phi}{\partial z}$$

Substituting in continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Results in Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2}+\frac{\partial^2 \Phi}{\partial y^2}+\frac{\partial^2 \Phi}{\partial z^2}=0$$

Summary

- Potential theory is mathematical way to describe flow

Important facts about velocity potential function Φ:

- definition: Φ is a function whose derivative in any direction equals the flow velocity in that direction
- Φ describes non-viscous flow
- Φ is a scalar function of space and time (NOT a vector!)

Summary

- Velocity potential for regular wave is obtained by
  - Solving Laplace equation satisfying:
    1. Seabed boundary condition
    2. Dynamic free surface condition

$$\Phi(x,y,z,t)=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx\cos\mu+ky\sin\mu-\omega t)$$

$$\Phi(x,y,z,t)=\frac{\zeta_a g}{\omega}\cdot\frac{\cosh(k(h+z))}{\cosh(kh)}\cdot\sin(kx\cos\mu+ky\sin\mu-\omega t)$$

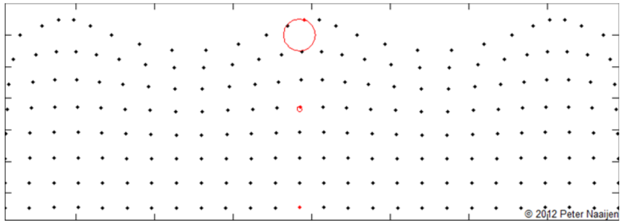
3. Kinematic free surface boundary condition results in:
   Dispersion relation = relation between wave frequency and wave length

$$\omega^2=kg\tanh(kh)$$

Water Particle Kinematics

trajectories of water particles in infinite water depth

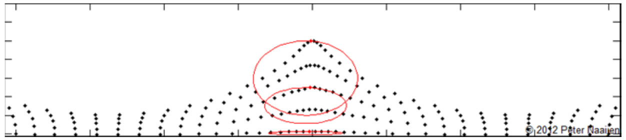
$$\Phi(x,y,z,t)=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx\cos\mu+ky\sin\mu-\omega t)$$



Water Particle Kinematics

trajectories of water particles in finite water depth

$$\Phi(x,y,z,t)=\frac{\zeta_a g}{\omega}\cdot\frac{\cosh(k(h+z))}{\cosh(kh)}\cdot\sin(kx\cos\mu+ky\sin\mu-\omega t)$$



Pressure

Pressure in the fluid can be found using Bernouilli equation for unsteady flow:

$$\frac{\partial\Phi}{\partial t}+\tfrac{1}{2}(u^2+w^2)+\frac{p}{\rho}+gz=0$$

$$p=-\rho\frac{\partial\Phi}{\partial t}\cancel{-\tfrac{1}{2}\rho(u^2+w^2)}-\rho gz$$

1^{st} order fluctuating pressure

2^{nd} order (small quantity squared=small enough to neglect)

Hydrostatic pressure (Archimedes)

Potential Theory

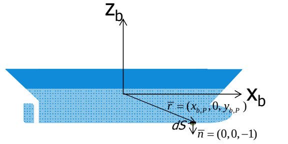
From all these velocity potentials we can derive:

- Pressure
- Forces and moments can be derived from pressures:

$$\bar{F} = -\iint_S \left(p \cdot \bar{n} \right) dS$$

$$\bar{M} = -\iint_S p \cdot \left(\bar{r} \times \bar{n} \right) dS$$

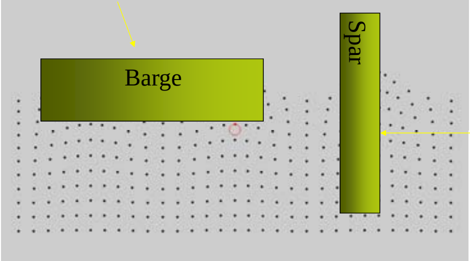
Verify these formulae (incl the signs!) yourself in order to understand them. Just check e.g. the force in heave direction (F_3) and the pitch moment (M_2) induced by a pressure on an infinite piece of hull surface *dS* at location $\bar{r}$:



Wave Force

Determination F_w

- Froude Krilov
- Diffraction

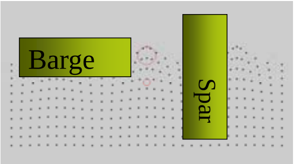


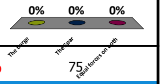
Which structure experiences the highest <u>vertical</u> wave load acc. to potential theory ?

A. The Barge

B. The Spar

C. Equal forces on both





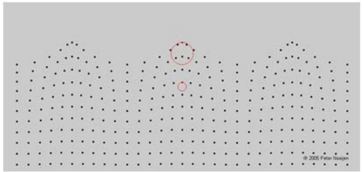
Flow superposition

$$(m+a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = \boxed{F_w}$$

Considering a fixed structure (ignoring the motions) we will try to find a description of the disturbance of the flow by the presence of the structure in the form of a velocity potential. We will call this one the diffraction potential and added to the undisturbed wave potential (for which we have an analytical expression) it will describe the total flow due to the waves.

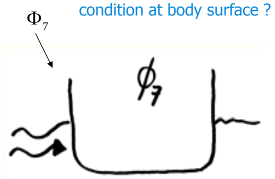
1. Flow due to Undisturbed wave

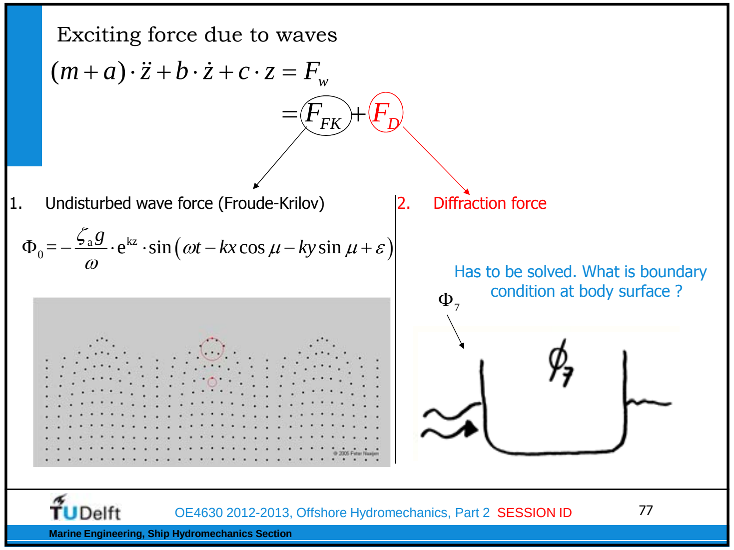
$$\Phi_0 = -\frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin\left(\omega t - kx\cos\mu - ky\sin\mu\right)$$



2. Flow due to Diffraction

Has to be solved. What is boundary condition at body surface ?





Exciting force due to waves

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$

$$=\boxed{F_{FK}}+\boxed{F_D}$$

1. Undisturbed wave force (Froude-Krilov)

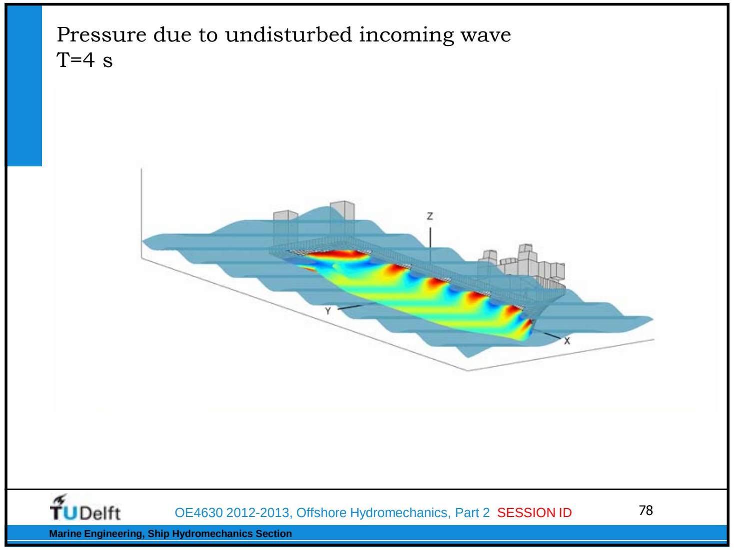
$$\Phi_0=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin\left(\omega t-kx\cos\mu-ky\sin\mu+\varepsilon\right)$$

2. Diffraction force

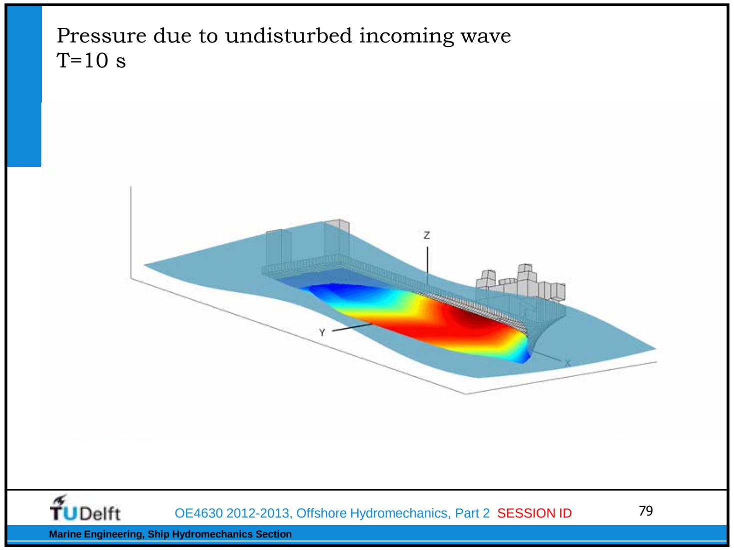
Has to be solved. What is boundary condition at body surface ?

Φ_7

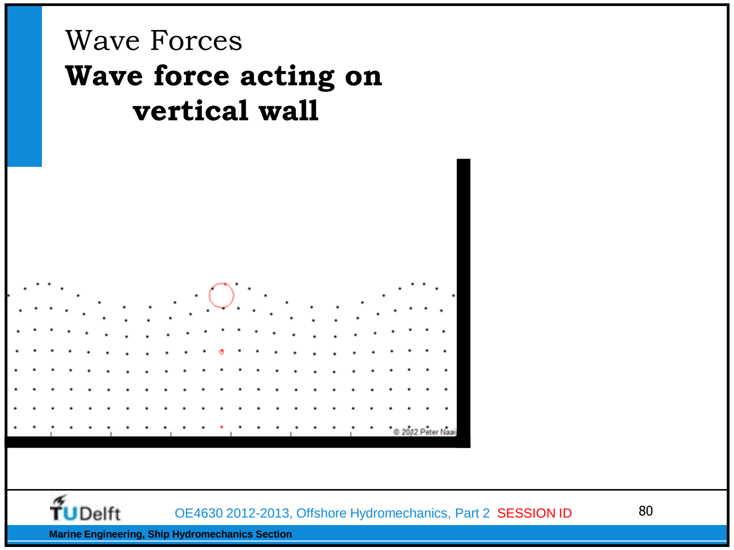
ϕ_7



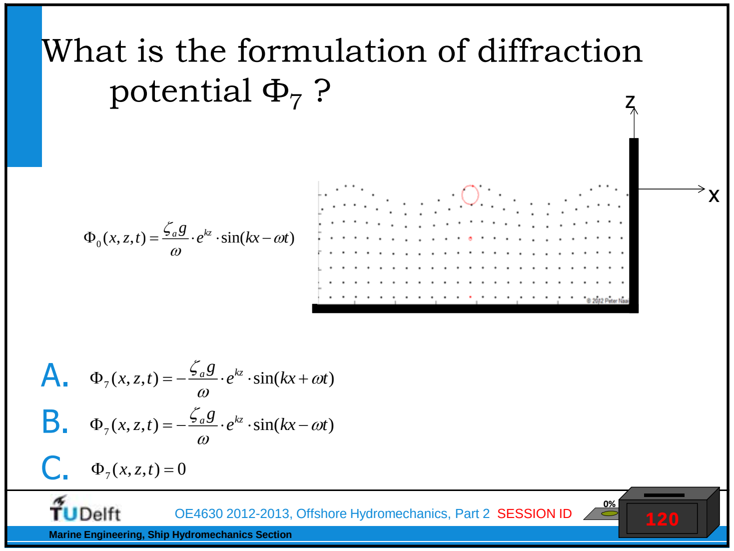
Pressure due to undisturbed incoming wave
T=4 s



Pressure due to undisturbed incoming wave
T=10 s



Wave Forces
Wave force acting on vertical wall



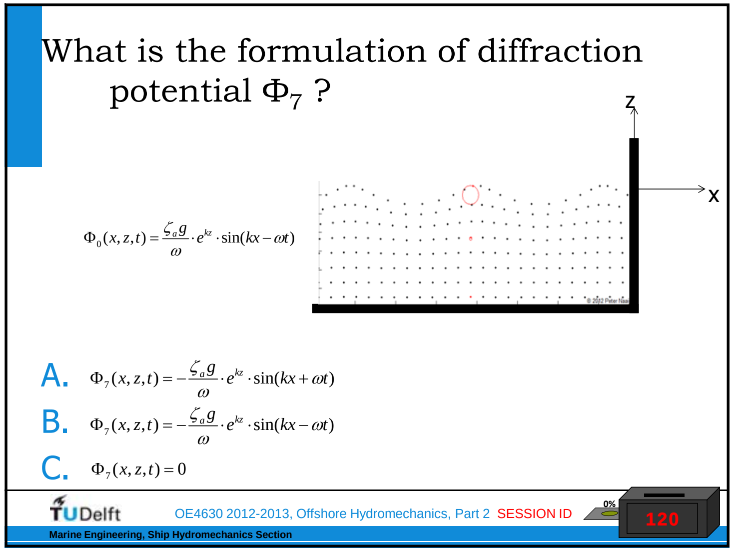
What is the formulation of diffraction potential Φ_7 ?

$$\Phi_0(x,z,t)=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx-\omega t)$$

A. $\Phi_7(x,z,t)=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx+\omega t)$

B. $\Phi_7(x,z,t)=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx-\omega t)$

C. $\Phi_7(x,z,t)=0$



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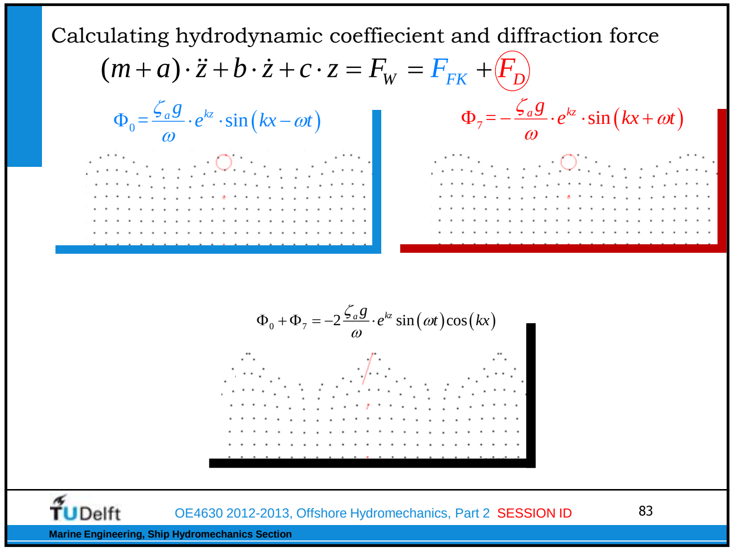


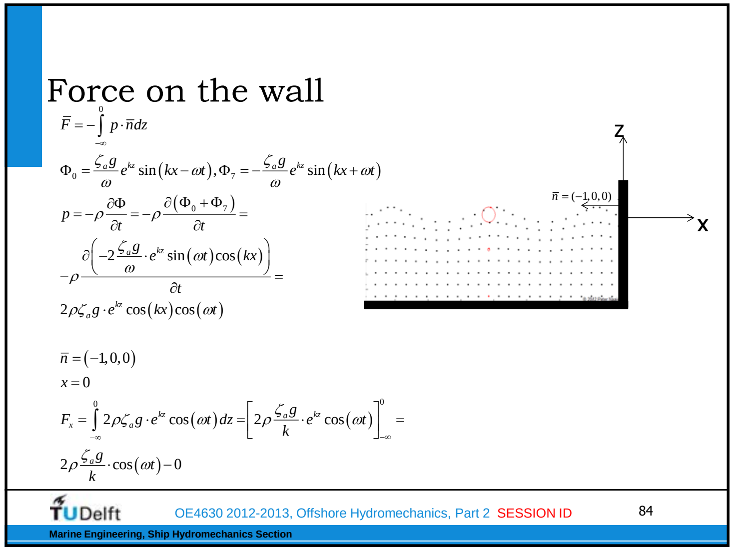
Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+\circled{F_D}$$

$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx-\omega t)$$

$$\Phi_7=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin(kx+\omega t)$$

$$\Phi_0+\Phi_7=-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)$$



Force on the wall

$$\bar{F}=-\int_{-\infty}^{0}p\cdot\bar{n}dz$$

$$\Phi_0=\frac{\zeta_a g}{\omega}e^{kz}\sin(kx-\omega t),\Phi_7=-\frac{\zeta_a g}{\omega}e^{kz}\sin(kx+\omega t)$$

$$p=-\rho\frac{\partial\Phi}{\partial t}=-\rho\frac{\partial(\Phi_0+\Phi_7)}{\partial t}=$$

$$-\rho\frac{\partial\left(-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin(\omega t)\cos(kx)\right)}{\partial t}=$$

$$2\rho\zeta_a g\cdot e^{kz}\cos(kx)\cos(\omega t)$$

$$\bar{n}=(-1,0,0)$$

$$x=0$$

$$F_x=\int_{-\infty}^{0}2\rho\zeta_a g\cdot e^{kz}\cos(\omega t)dz=\left[2\rho\frac{\zeta_a g}{k}\cdot e^{kz}\cos(\omega t)\right]_{-\infty}^{0}=$$

$$2\rho\frac{\zeta_a g}{k}\cdot\cos(\omega t)-0$$

Left hand side of m.e.: Hydromechanic reaction forces

- NO incoming waves:
- Vessel moves with given frequency

Recap: Motion equation

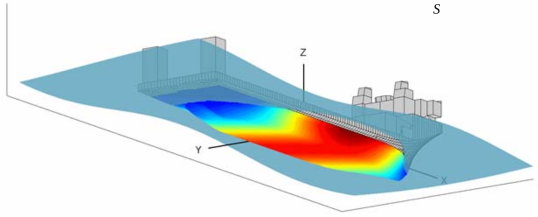
$$\underbrace{(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z}_{\text{Hydromechanic force depends on motion}}=\underbrace{+F_{FK}+F_D=F_W}_{\text{Wave Force independent of motion}}$$

Pressure / force due to undisturbed incoming wave
T=10 s

$$p=-\rho\frac{\partial\Phi}{\partial t}$$

$$\bar{F}=-\iint_S\left(p\cdot\bar{n}\right)dS$$

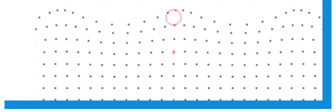
$$\bar{M}=-\iint_S p\cdot\left(\bar{r}\times\bar{n}\right)dS$$



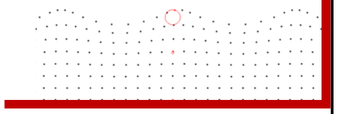
Calculating diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+\textcircled{F_D}$$

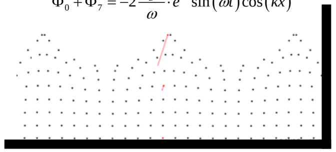
$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin\left(kx-\omega t\right)$$



$$\Phi_7=-\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin\left(kx+\omega t\right)$$



$$\Phi_0+\Phi_7=-2\frac{\zeta_a g}{\omega}\cdot e^{kz}\sin\left(\omega t\right)\cos\left(kx\right)$$



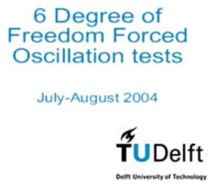
left hand side: reaction forces

$$\underbrace{(m+a)\cdot\ddot{z}+b\cdot\dot{z}}_{\text{Hydromechanic force depends on motion}}+c\cdot z=\underbrace{+F_{FK}+F_{D}=F_{W}}_{\text{Wave Force independent of motion}}$$

Hydrodynamic coefficients

Determination of a and b:

- Forced oscillation with known frequency and amplitude
- Measure Force needed to oscillate the model



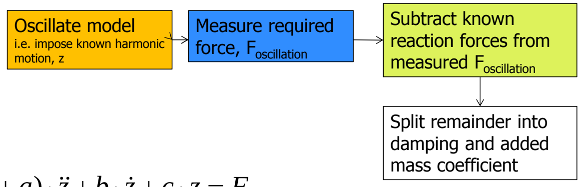
6 Degree of Freedom Forced Oscillation tests

July-August 2004

TUDelft
Delft University of Technology

Determine added mass and damping

Experimental procedure:



Oscillate model
i.e. impose known harmonic motion, z

Measure required force, $F_{oscillation}$

Subtract known reaction forces from measured $F_{oscillation}$

Split remainder into damping and added mass coefficient

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_{oscillation}$$

$$z=z_a\cos(\omega t),\dot{z}=-\omega z_a\sin(\omega t),\ddot{z}=-\omega^2 z_a\cos(\omega t)$$

$$\left(-\omega^2(m+a)+c\right)z_a\cos\omega t-\omega b z_a\sin\omega t=F_{a,osc}\cdot\cos\left(\omega t+\varepsilon_{F,z}\right)$$

$$-\omega^2 a z_a\cos\omega t-\omega b z_a\sin\omega t=F_{a,osc}\cdot\cos\left(\omega t+\varepsilon_{F,z}\right)+\left(\omega^2 m-c\right)z_a\cos\omega t$$

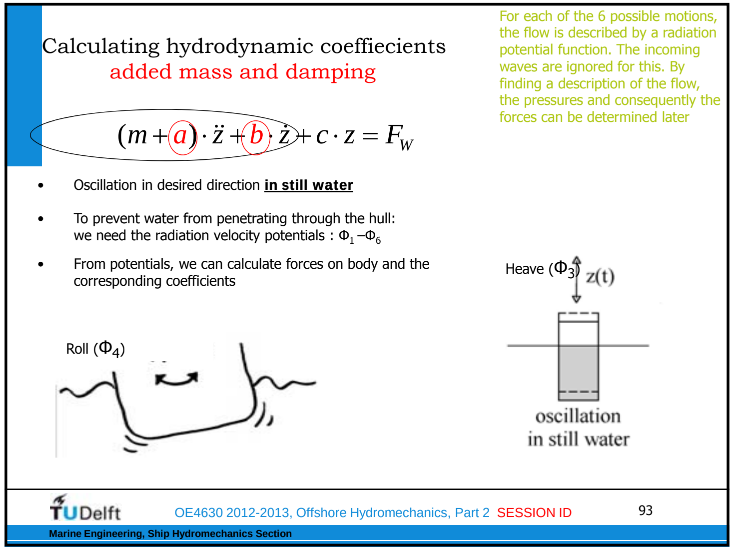
Equation of motion

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

Hydrodynamic coefficients:

a=added mass coefficient= force on ship per 1 m/s^2 acceleration →
a * acceleration = hydrodynamic inertia force

b=damping coefficient= force on ship per 1 m/s velocity →
b * velocity = hydrodynamic damping force



Calculating hydrodynamic coeffiecients added mass and damping

$$(m+\textcolor{red}{a})\cdot\ddot{z}+\textcolor{red}{b}\cdot\dot{z}+c\cdot z=F_W$$

For each of the 6 possible motions, the flow is described by a radiation potential function. The incoming waves are ignored for this. By finding a description of the flow, the pressures and consequently the forces can be determined later

- Oscillation in desired direction **in still water**
- To prevent water from penetrating through the hull: we need the radiation velocity potentials : $\Phi_1 - \Phi_6$
- From potentials, we can calculate forces on body and the corresponding coefficients

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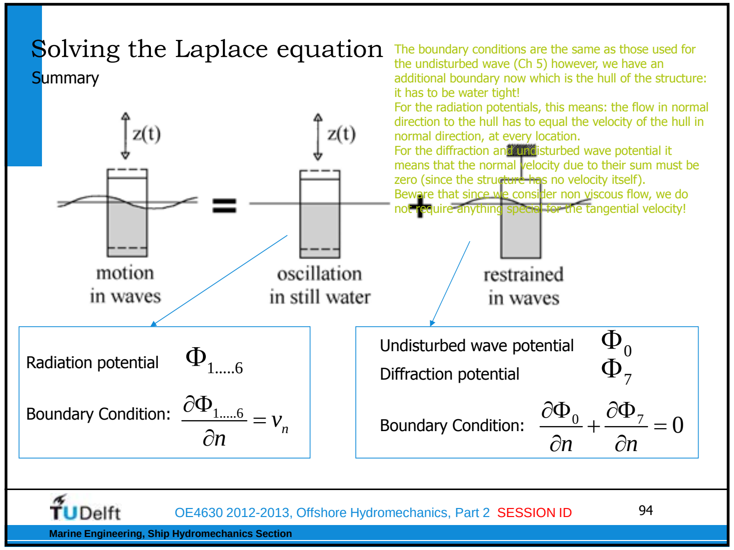


Solving the Laplace equation

Summary

The boundary conditions are the same as those used for the undisturbed wave (Ch 5) however, we have an additional boundary now which is the hull of the structure: it has to be water tight!
For the radiation potentials, this means: the flow in normal direction to the hull has to equal the velocity of the hull in normal direction, at every location.
For the diffraction and undisturbed wave potential it means that the normal velocity due to their sum must be zero (since the structure has no velocity itself).
Beware that since we consider non viscous flow, we do not require anything special for the tangential velocity!

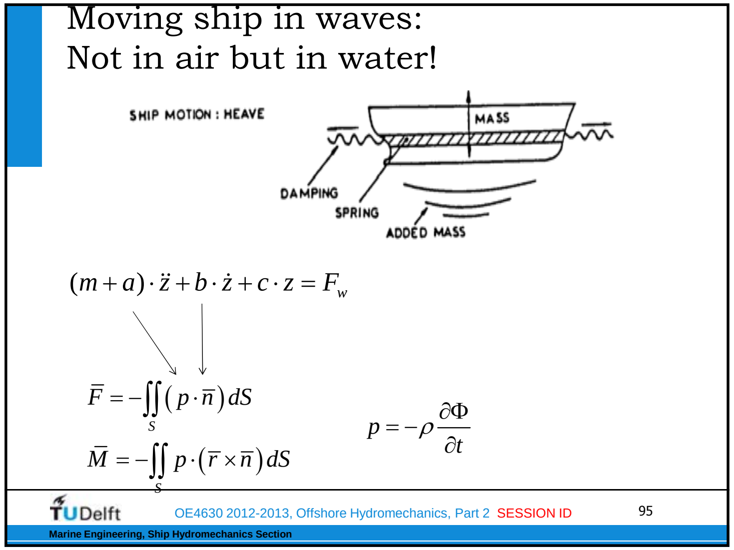
Radiation potential $\Phi_{1.....6}$

Boundary Condition: $\dfrac{\partial \Phi_{1.....6}}{\partial n} = v_n$

Undisturbed wave potential Φ_0

Diffraction potential Φ_7

Boundary Condition: $\dfrac{\partial \Phi_0}{\partial n} + \dfrac{\partial \Phi_7}{\partial n} = 0$



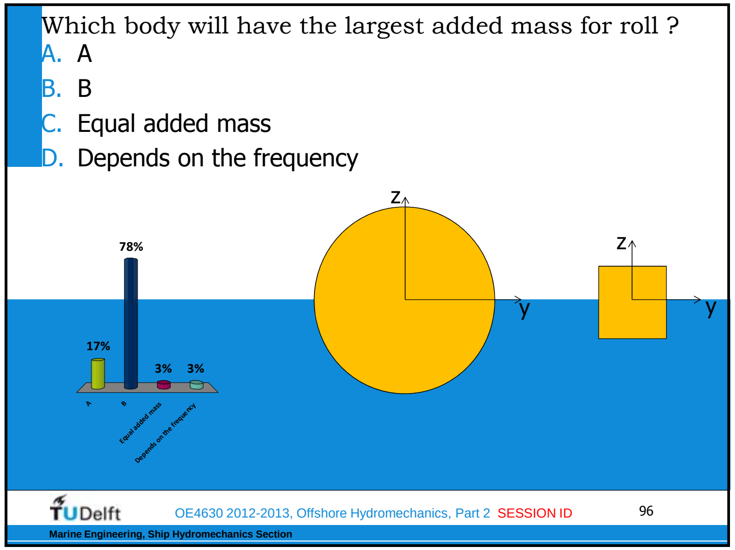
Moving ship in waves: Not in air but in water!

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_w$$

$$\bar{F}=-\iint_S (p\cdot\bar{n})\,dS$$

$$\bar{M}=-\iint_S p\cdot(\bar{r}\times\bar{n})\,dS$$

$$p=-\rho\frac{\partial\Phi}{\partial t}$$



Which body will have the largest added mass for roll ?

A. A
B. B
C. Equal added mass
D. Depends on the frequency

Equation of motion

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=+F_{FK}+F_D=F_W$$

To solve equation of motion for certain frequency:

- Determine spring coefficient:
  - c → follows from geometry of vessel
- Determine required hydro*dynamic* coefficients for desired frequency:
  - a, b → computer / experiment
- Determine amplitude and phase of F_w of regular wave with amplitude =1:
  - Computer / experiment: $F_w = F_{wa}\cos(\omega t+\varepsilon_{Fw,\xi})$
- As we consider the response to a regular wave with frequency ω:

  Assume steady state response: $z=z_a\cos(\omega t+\varepsilon_{z,\xi})$

  and substitute in equation of motion:

Equation of motion

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

$$z=z_a\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$$

$$\dot{z}=-z_a\omega\sin\left(\omega t+\varepsilon_{z,\zeta}\right)$$

$$\ddot{z}=-z_a\omega^2\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$$

$$\left(c-\omega^2(m+a)\right)\cdot z_a\cos\left(\omega t+\varepsilon_{z,\zeta}\right)+b\cdot-z_a\omega\sin\left(\omega t+\varepsilon_{z,\zeta}\right)=F_{Wa}\cos\left(\omega t+\varepsilon_{F_w,\zeta}\right)$$

Now solve the equation for the unknown motion amplitude z_a and phase angle $\varepsilon_{z,\xi}$

Equation of motion

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$

Now solve the equation for the unknown motion amplitude z_a and phase angle $\varepsilon_{z,\xi}$ for 1 frequency

System is linear

If wave amplitude doubles → wave force doubles → motion doubles

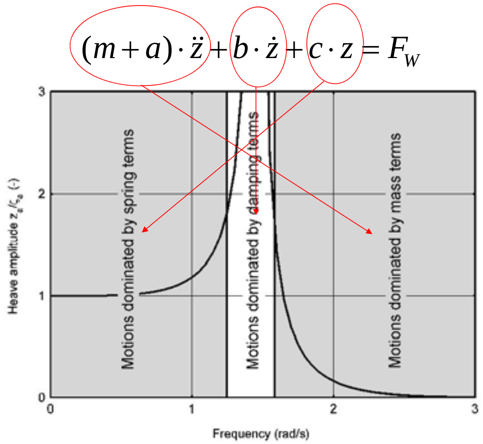
$$(m+a)\cdot\frac{\ddot{z}}{\zeta_a}+b\cdot\frac{\dot{z}}{\zeta_a}+c\cdot\frac{z}{\zeta_a}=\frac{F_W}{\zeta_a}$$

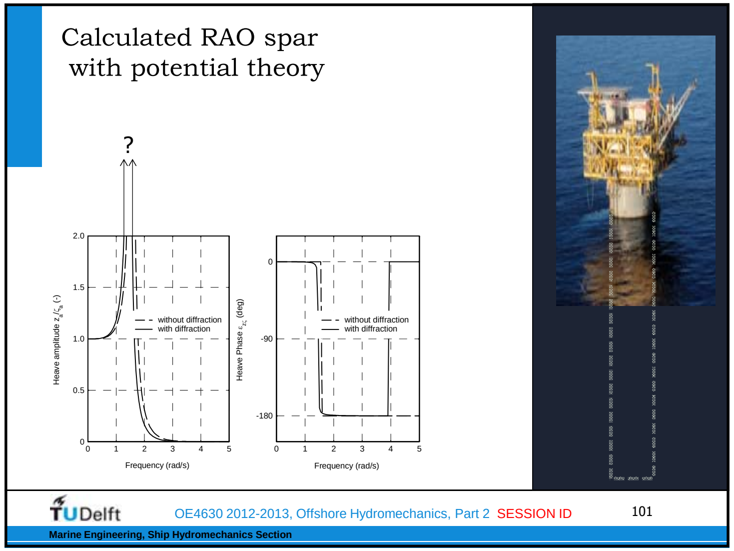
Substitue solution $\frac{z}{\zeta_a}=\frac{z_a}{\zeta_a}\cos\left(\omega t+\varepsilon_{z,\zeta}\right)$ and solve RAO and phase

---

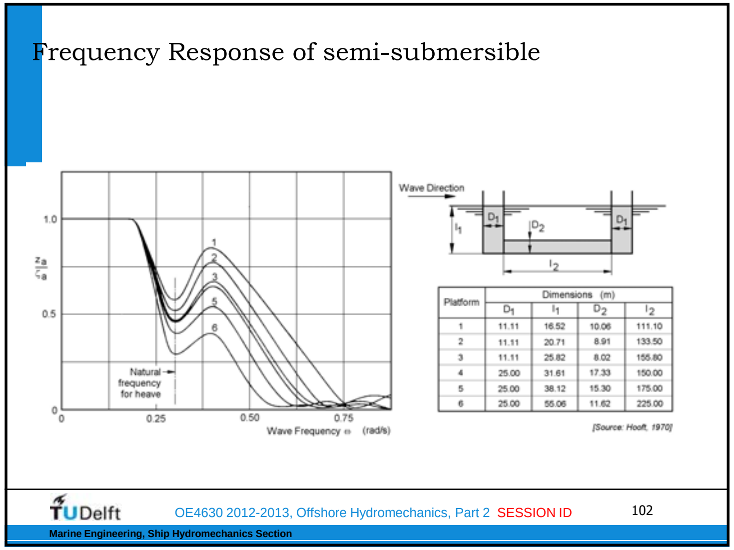
RAO

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W$$





Calculated RAO spar with potential theory



Frequency Response of semi-submersible

Platform	Dimensions (m)			
	D_1	l_1	D_2	l_2
1	11.11	16.52	10.06	111.10
2	11.11	20.71	8.91	133.50
3	11.11	25.82	8.02	155.80
4	25.00	31.61	17.33	150.00
5	25.00	38.12	15.30	175.00
6	25.00	55.06	11.62	225.00

[Source: Hooft, 1970]

What is 'linear' ???

R

1. Linear waves:
   - 'nice' regular harmonic (<u>cosine</u> shaped) waves
   - Wave steepness small: free surface boundary condition satisfied at <u>mean still water</u> level
     - Pressures and fluid velocities are proportional to wave elevation and have same frequency as elevation
2. linearised wave exciting force:
   - Wave force independent of motions
   - Wave force only on <u>mean</u> wetted surface

L

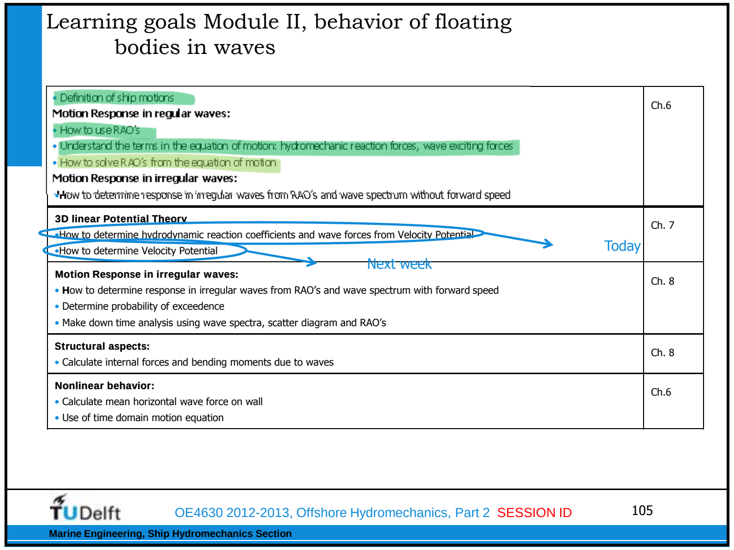
3. Motion amplitudes are small
   - Restoring force proportional to motion amplitude
   - Hydrodynamic reaction forces proportional to motion amplitude

Motions are proportional to wave height !

Motions have same frequency as waves

Learning goals Module II, behavior of floating bodies in waves

• Definition of ship motions **Motion Response in regular waves:** • How to use RAO's • Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces • How to solve RAO's from the equation of motion **Motion Response in irregular waves:** • How to determine response in irregular waves from RAO's and wave spectrum without forward speed	Ch.6
3D linear Potential Theory •How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential •How to determine Velocity Potential	Ch. 7
Motion Response in irregular waves: • How to determine response in irregular waves from RAO's and wave spectrum with forward speed • Determine probability of exceedence • Make down time analysis using wave spectra, scatter diagram and RAO's	Ch. 8
Structural aspects: • Calculate internal forces and bending moments due to waves	Ch. 8
Nonlinear behavior: • Calculate mean horizontal wave force on wall • Use of time domain motion equation	Ch.6



Learning goals Module II, behavior of floating bodies in waves

- Definition of ship motions **Motion Response in regular waves:** - How to use RAO's - Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces - How to solve RAO's from the equation of motion **Motion Response in irregular waves:** - How to determine response in irregular waves from RAO's and wave spectrum without forward speed	Ch.6
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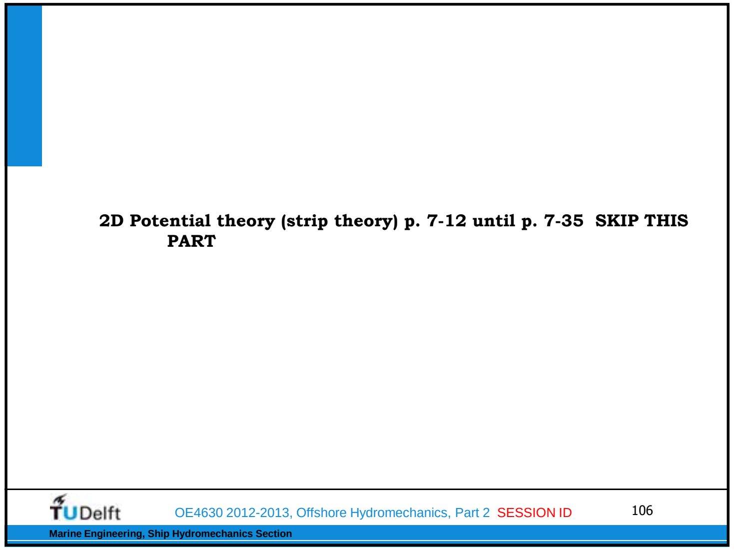
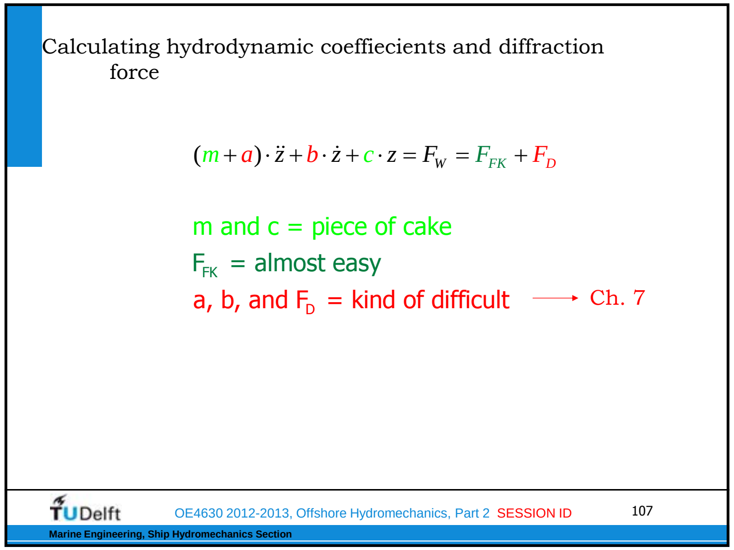


2D Potential theory (strip theory) p. 7-12 until p. 7-35 SKIP THIS PART



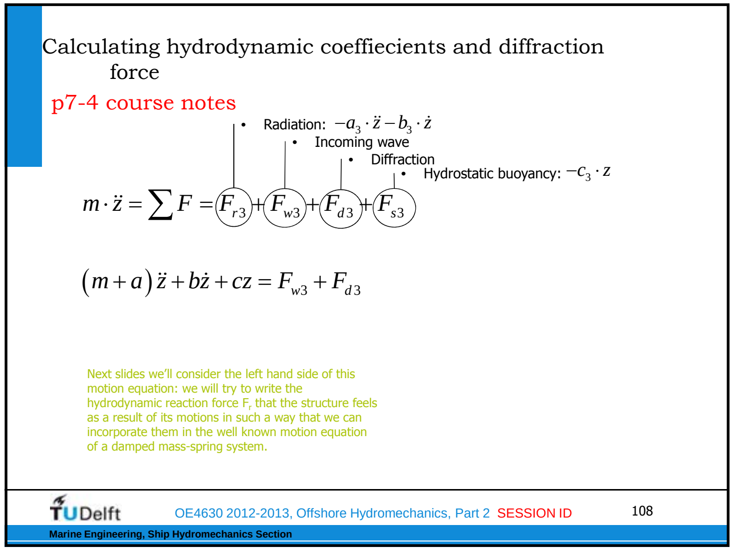
Calculating hydrodynamic coeffiecients and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

m and c = piece of cake

F_{FK} = almost easy

a, b, and F_D = kind of difficult → Ch. 7



Calculating hydrodynamic coeffiecients and diffraction force

p7-4 course notes

- Radiation: $-a_3\cdot\ddot{z}-b_3\cdot\dot{z}$
- Incoming wave
- Diffraction
- Hydrostatic buoyancy: $-c_3\cdot z$

$$m\cdot\ddot{z}=\sum F=F_{r3}+F_{w3}+F_{d3}+F_{s3}$$

$$(m+a)\ddot{z}+b\dot{z}+cz=F_{w3}+F_{d3}$$

Next slides we'll consider the left hand side of this motion equation: we will try to write the hydrodynamic reaction force F_r that the structure feels as a result of its motions in such a way that we can incorporate them in the well known motion equation of a damped mass-spring system.

Calculating hydrodynamic coeffiecients and diffraction force

$$m \cdot \ddot{z} = \sum F = F_{r3} + F_{w3} + F_{d3} + F_{s3}$$

Radiation Force: $F_{r3} = -a_3 \cdot \ddot{z} - b_3 \cdot \dot{z}$

To calculate force: first describe fluid motions due to given heave motion by means of radiation potential:

Potential theory
Radiation potential

$$(m+a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = F_w + F_d$$

Radiation potential heave $\Phi_3\left(x, y, z, t\right)$

= flow due to heave motion

Knowing the potential, calculating resulting force is straight forward:

$$\left.\begin{aligned} \bar{F} &= -\iint_S \left(p \cdot \bar{n}\right) dS \\ \bar{M} &= -\iint_S p \cdot \left(\bar{r} \times \bar{n}\right) dS \\ p &= -\rho \frac{\partial \Phi}{\partial t} \end{aligned}\right\} \quad \begin{aligned} \bar{F} &= \iint_S \left(\rho \frac{\partial \Phi}{\partial t} \cdot \bar{n}\right) dS \\ \bar{M} &= \iint_S \rho \frac{\partial \Phi}{\partial t} \cdot \left(\bar{r} \times \bar{n}\right) dS \end{aligned}$$

Potential theory
Radiation potential

$$(m+a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = F_w + F_d$$

Radiation potential heave $\Phi_3\left(x, y, z, t\right)$

= flow due to motions, larger motions → 'more' flow

Problem: But we don't know the motions !! (we need the flow to calculate the motions…and we need the motions to calculate the flow…)

Solution: radiation potential is written as function of motion:

Potential theory
Radiation potential

Solution: radiation potential is written as function of *velocity of the* motion

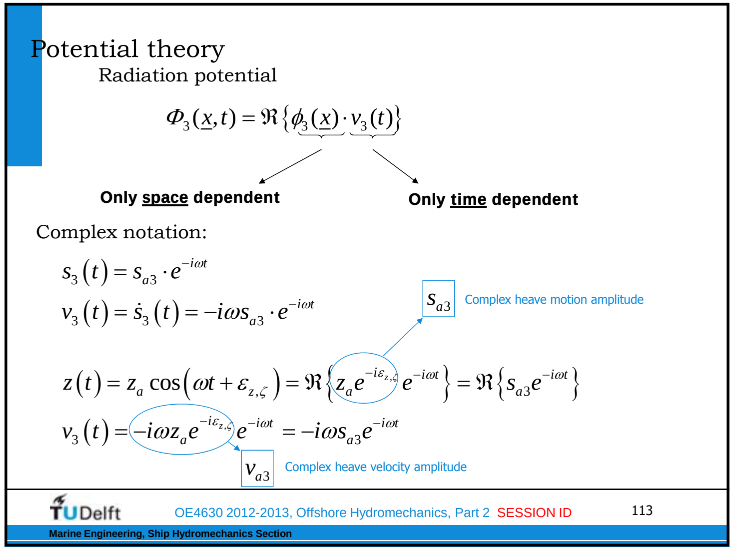
$$\mathit{\Phi}_3(\underline{x}, t) = \Re\left\{\underbrace{\phi_3(\underline{x})}\cdot\underbrace{v_3(t)}\right\}$$

P7-5 eq. 7.17

Only <u>space</u> dependent **Only <u>time</u> dependent**

Suppose we would know the velocity potential due to heave motion: Φ_3
Assuming linearity this will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Let's define the amplitude and the phase angle of this potential function to be related to the velocity of the heave motion ($\dot{z}$ or in complex notation: v_3).
So we write the potential function Φ_3 as a complex product of:
ϕ_3 (which can be considered as a complex transfer function between potential and heave velocity) and the heave velocity v_3



Potential theory

Radiation potential

$$\Phi_3(\underline{x},t)=\Re\{\underbrace{\phi_3(\underline{x})}\cdot\underbrace{v_3(t)}\}$$

Only space dependent **Only time dependent**

Complex notation:

$$s_3(t)=s_{a3}\cdot e^{-i\omega t}$$

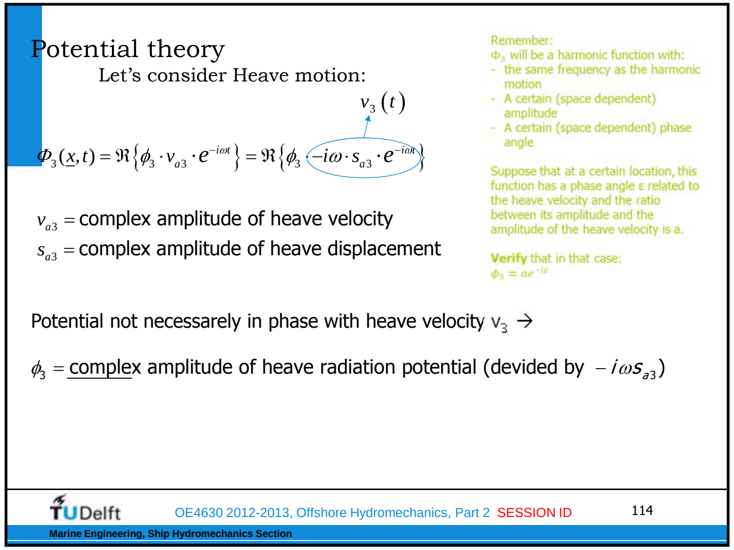
$$v_3(t)=\dot{s}_3(t)=-i\omega s_{a3}\cdot e^{-i\omega t}$$

$\boxed{s_{a3}}$ Complex heave motion amplitude

$$z(t)=z_a\cos\left(\omega t+\varepsilon_{z,\zeta}\right)=\Re\left\{z_a e^{-i\varepsilon_{z,\zeta}}e^{-i\omega t}\right\}=\Re\left\{s_{a3}e^{-i\omega t}\right\}$$

$$v_3(t)=-i\omega z_a e^{-i\varepsilon_{z,\zeta}}e^{-i\omega t}=-i\omega s_{a3}e^{-i\omega t}$$

$\boxed{v_{a3}}$ Complex heave velocity amplitude



Potential theory

Let's consider Heave motion:

$$\Phi_3(\underline{x},t)=\Re\left\{\phi_3\cdot v_{a3}\cdot e^{-i\omega t}\right\}=\Re\left\{\phi_3\cdot\overbrace{-i\omega\cdot s_{a3}\cdot e^{-i\omega t}}^{v_3(t)}\right\}$$

v_{a3} = complex amplitude of heave velocity

s_{a3} = complex amplitude of heave displacement

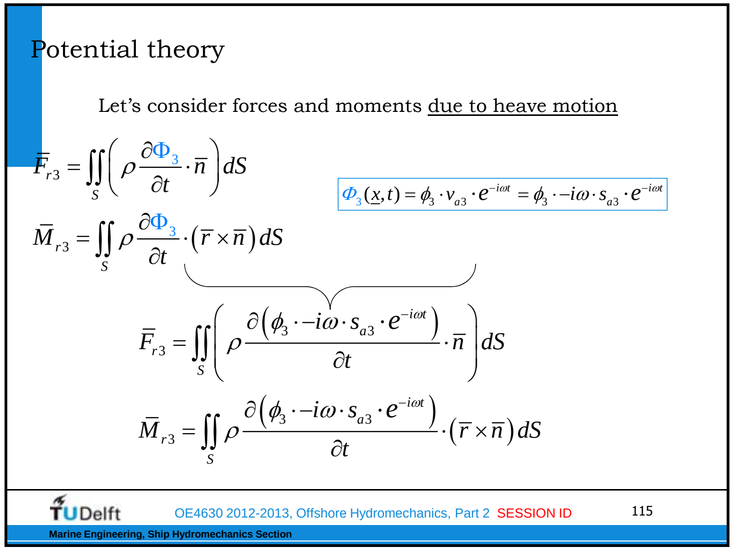
Remember:
Φ_3 will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Suppose that at a certain location, this function has a phase angle ε related to the heave velocity and the ratio between its amplitude and the amplitude of the heave velocity is a.

Verify that in that case:
$\phi_3 = ae^{-i\varepsilon}$

Potential not necessarely in phase with heave velocity v_3 →

ϕ_3 = <u>complex</u> amplitude of heave radiation potential (devided by $-i\omega s_{a3}$)



Potential theory

Let's consider forces and moments <u>due to heave motion</u>

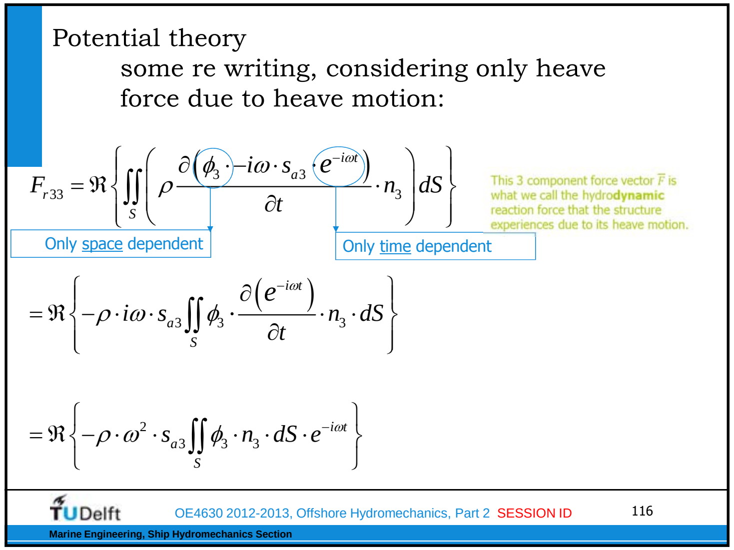
$$\bar{F}_{r3}=\iint_S\left(\rho\frac{\partial\Phi_3}{\partial t}\cdot\bar{n}\right)dS$$

$$\boxed{\Phi_3(\underline{x},t)=\phi_3\cdot v_{a3}\cdot e^{-i\omega t}=\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}}$$

$$\bar{M}_{r3}=\iint_S\rho\frac{\partial\Phi_3}{\partial t}\cdot(\bar{r}\times\bar{n})dS$$

$$\bar{F}_{r3}=\iint_S\left(\rho\frac{\partial\left(\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}\right)}{\partial t}\cdot\bar{n}\right)dS$$

$$\bar{M}_{r3}=\iint_S\rho\frac{\partial\left(\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}\right)}{\partial t}\cdot(\bar{r}\times\bar{n})dS$$



Potential theory

some re writing, considering only heave force due to heave motion:

$$F_{r33}=\Re\left\{\iint_S\left(\rho\frac{\partial\left(\phi_3\cdot -i\omega\cdot s_{a3}\cdot e^{-i\omega t}\right)}{\partial t}\cdot n_3\right)dS\right\}$$

Only <u>space</u> dependent

Only <u>time</u> dependent

This 3 component force vector $\bar{F}$ is what we call the hydro**dynamic** reaction force that the structure experiences due to its heave motion.

$$=\Re\left\{-\rho\cdot i\omega\cdot s_{a3}\iint_S\phi_3\cdot\frac{\partial\left(e^{-i\omega t}\right)}{\partial t}\cdot n_3\cdot dS\right\}$$

$$=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}$$

Potential theory

Radiation Force due to heave motion is 3 component vector:

$$F_{r13}=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_1\cdot dS\cdot e^{-i\omega t}\right\}$$ **Surge force due to heave motion**

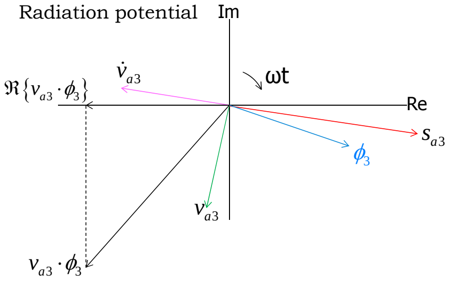
$$F_{r23}=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_2\cdot dS\cdot e^{-i\omega t}\right\}$$ **Sway force due to heave motion**

$$F_{r33}=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}$$ **Heave force due to heave motion**

In the following, only heave force due to heave motion is considered: F_{r33}

Potential theory

Radiation potential



$$F_{r33}=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}$$

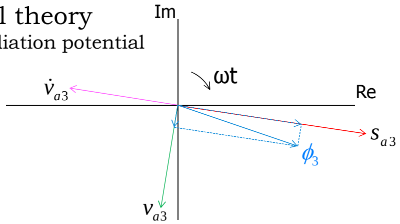
= Radiation force in heave direction, due to heave motion

$$m\cdot\ddot{z}\qquad +c\cdot z\qquad +F_{r33}=F_{w3}+F_{d3}=$$

$$m\cdot\dot{v}_{a3}\cdot e^{-i\omega t}+c\cdot s_{a3}\cdot e^{-i\omega t}+F_{r33}$$

Potential theory

Radiation potential



$$F_{r33}=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}$$

= Radiation force in heave direction, due to heave motion

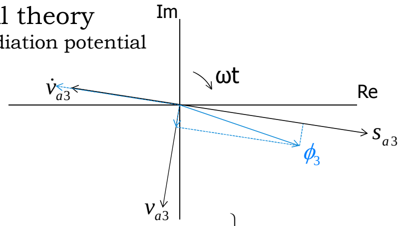
$$m\cdot\ddot{z}\qquad +c\cdot z\qquad +F_{r33}=F_{w3}+F_{d3}$$

$$\Re\left\{m\cdot\boxed{\dot{v}_{a3}}\cdot e^{-i\omega t}+c\cdot\boxed{s_{a3}}\cdot e^{-i\omega t}+\underset{\swarrow\ \searrow}{\boxed{F_{r33}}}\right\}=F_{w3}+F_{d3}$$

$$\boxed{\begin{array}{l}-a\cdot\ddot{z}=\\ \Re\left\{-a\cdot\boxed{\dot{v}_{a3}}\cdot e^{-i\omega t}\right\}\end{array}}\quad\boxed{\begin{array}{l}-b\cdot\dot{z}=\\ \Re\left\{-b\cdot\boxed{v_{a3}}\cdot e^{-i\omega t}\right\}\end{array}}$$

Potential theory

Radiation potential

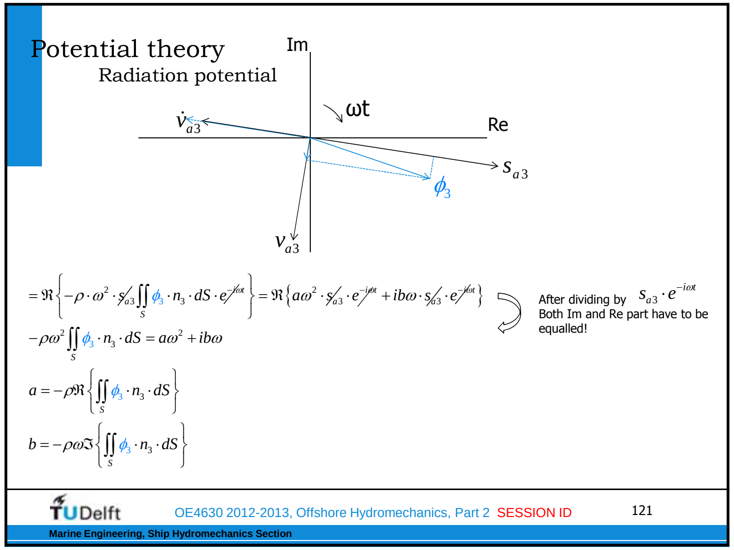


$$F_{r33}=\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}=\Re\left\{-a\cdot\dot{v}_{a3}\cdot e^{-i\omega t}-b\cdot v_{a3}\cdot e^{-i\omega t}\right\}$$

$$=\Re\left\{a\cdot\omega^2\cdot s_{a3}\cdot e^{-i\omega t}+b\cdot i\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$$

$$=\Re\left\{a\omega^2\cdot s_{a3}\cdot e^{-i\omega t}+ib\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$$

$$\Re\left\{-\rho\cdot\omega^2\cdot s_{a3}\iint_S\phi_3\cdot n_3\cdot dS\cdot e^{-i\omega t}\right\}=\Re\left\{a\omega^2\cdot s_{a3}\cdot e^{-i\omega t}+ib\omega\cdot s_{a3}\cdot e^{-i\omega t}\right\}$$



Potential theory

Radiation potential

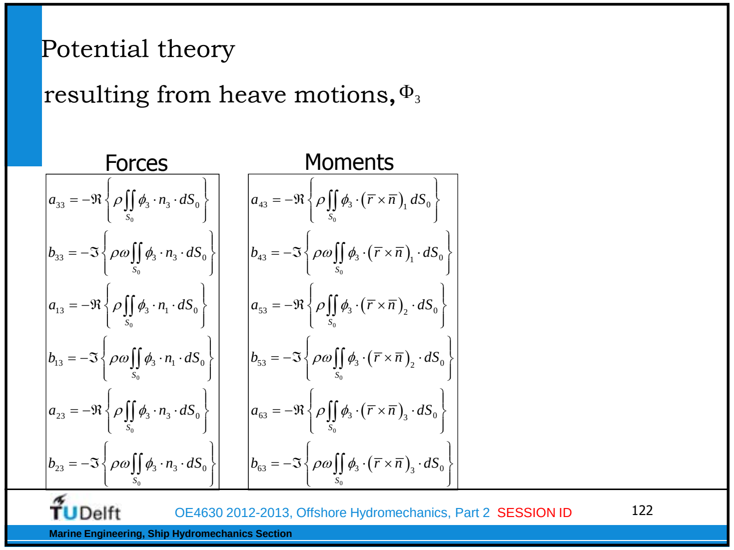
$$= \Re\left\{-\rho\cdot\omega^2\cdot\cancel{s}_{a3}\iint_S \phi_3\cdot n_3\cdot dS\cdot\cancel{e^{-i\omega t}}\right\} = \Re\left\{a\omega^2\cdot\cancel{s}_{a3}\cdot\cancel{e^{-i\omega t}} + ib\omega\cdot\cancel{s}_{a3}\cdot\cancel{e^{-i\omega t}}\right\}$$

After dividing by $s_{a3}\cdot e^{-i\omega t}$
Both Im and Re part have to be equalled!

$$-\rho\omega^2\iint_S \phi_3\cdot n_3\cdot dS = a\omega^2 + ib\omega$$

$$a = -\rho\Re\left\{\iint_S \phi_3\cdot n_3\cdot dS\right\}$$

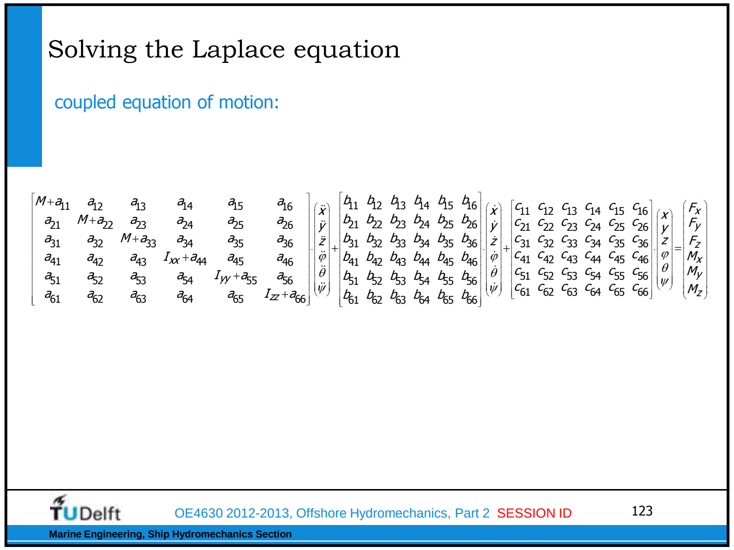
$$b = -\rho\omega\Im\left\{\iint_S \phi_3\cdot n_3\cdot dS\right\}$$



Potential theory

resulting from heave motions, Φ_3

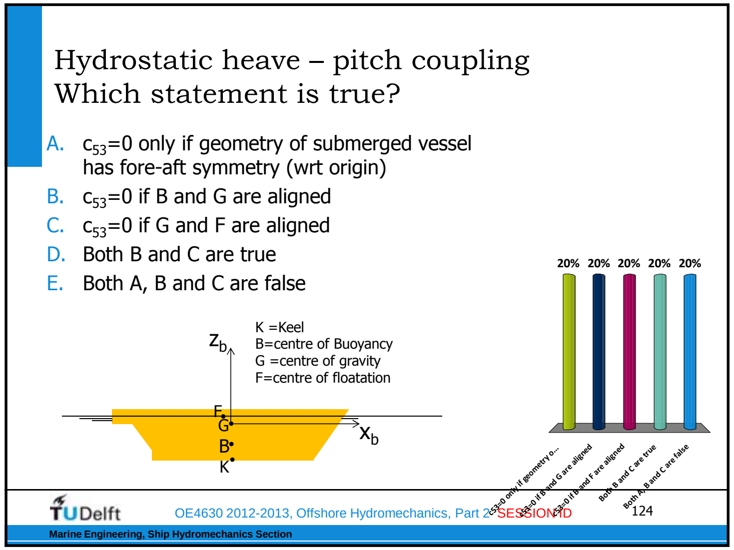
Forces	Moments
$a_{33} = -\Re\left\{\rho\iint_{S_0}\phi_3\cdot n_3\cdot dS_0\right\}$	$a_{43} = -\Re\left\{\rho\iint_{S_0}\phi_3\cdot(\overline{r}\times\overline{n})_1\,dS_0\right\}$
$b_{33} = -\Im\left\{\rho\omega\iint_{S_0}\phi_3\cdot n_3\cdot dS_0\right\}$	$b_{43} = -\Im\left\{\rho\omega\iint_{S_0}\phi_3\cdot(\overline{r}\times\overline{n})_1\cdot dS_0\right\}$
$a_{13} = -\Re\left\{\rho\iint_{S_0}\phi_3\cdot n_1\cdot dS_0\right\}$	$a_{53} = -\Re\left\{\rho\iint_{S_0}\phi_3\cdot(\overline{r}\times\overline{n})_2\cdot dS_0\right\}$
$b_{13} = -\Im\left\{\rho\omega\iint_{S_0}\phi_3\cdot n_1\cdot dS_0\right\}$	$b_{53} = -\Im\left\{\rho\omega\iint_{S_0}\phi_3\cdot(\overline{r}\times\overline{n})_2\cdot dS_0\right\}$
$a_{23} = -\Re\left\{\rho\iint_{S_0}\phi_3\cdot n_3\cdot dS_0\right\}$	$a_{63} = -\Re\left\{\rho\iint_{S_0}\phi_3\cdot(\overline{r}\times\overline{n})_3\cdot dS_0\right\}$
$b_{23} = -\Im\left\{\rho\omega\iint_{S_0}\phi_3\cdot n_3\cdot dS_0\right\}$	$b_{63} = -\Im\left\{\rho\omega\iint_{S_0}\phi_3\cdot(\overline{r}\times\overline{n})_3\cdot dS_0\right\}$



Solving the Laplace equation

coupled equation of motion:

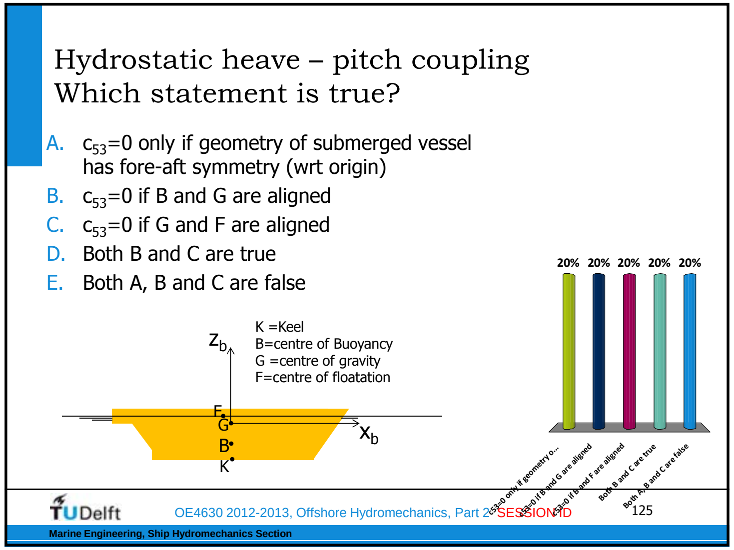
$$\begin{bmatrix} M+a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & M+a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & M+a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & I_{xx}+a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & I_{yy}+a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & I_{zz}+a_{66} \end{bmatrix}\cdot\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\\ \ddot{\varphi}\\ \ddot{\theta}\\ \ddot{\psi}\end{pmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix}\cdot\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{pmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix}\cdot\begin{pmatrix}x\\ y\\ z\\ \varphi\\ \theta\\ \psi\end{pmatrix} = \begin{pmatrix}F_x\\ F_y\\ F_z\\ M_x\\ M_y\\ M_z\end{pmatrix}$$



Hydrostatic heave – pitch coupling
Which statement is true?

A. c_{53}=0 only if geometry of submerged vessel has fore-aft symmetry (wrt origin)
B. c_{53}=0 if B and G are aligned
C. c_{53}=0 if G and F are aligned
D. Both B and C are true
E. Both A, B and C are false

K =Keel
B=centre of Buoyancy
G =centre of gravity
F=centre of floatation



Hydrostatic heave – pitch coupling Which statement is true?

A. c_{53}=0 only if geometry of submerged vessel has fore-aft symmetry (wrt origin)
B. c_{53}=0 if B and G are aligned
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K =Keel
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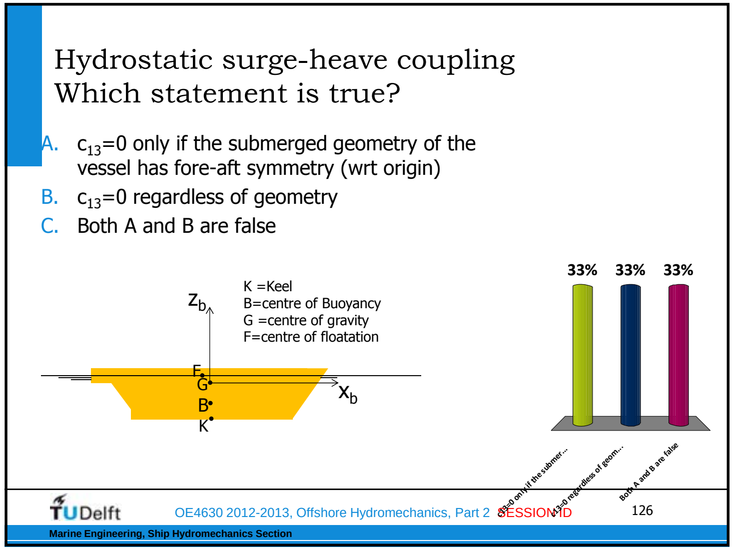
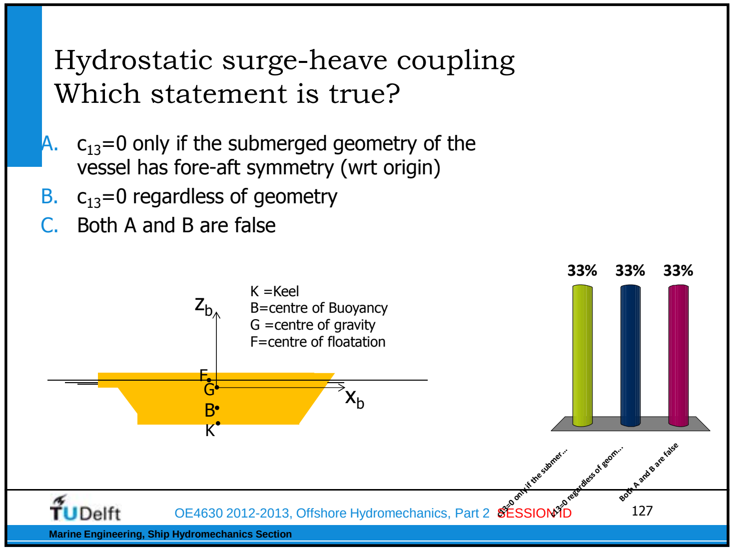


Hydrostatic surge-heave coupling Which statement is true?

A. c_{13}=0 only if the submerged geometry of the vessel has fore-aft symmetry (wrt origin)
B. c_{13}=0 regardless of geometry
C. Both A and B are false

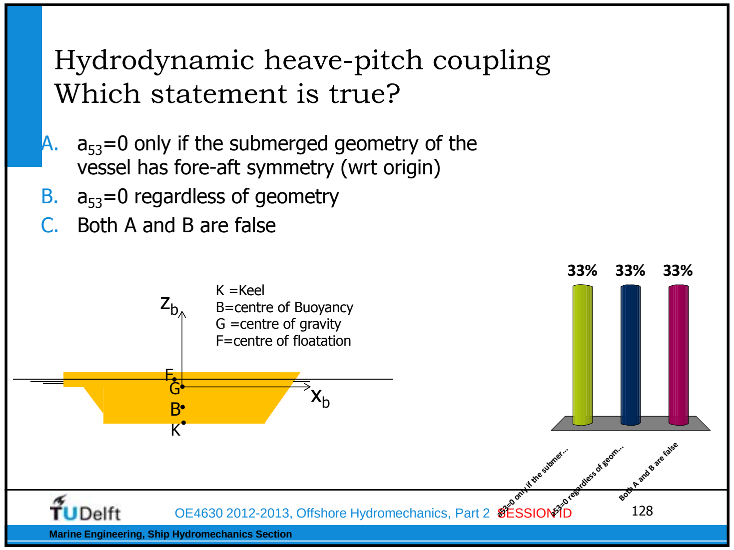
K =Keel
B=centre of Buoyancy
G =centre of gravity
F=centre of floatation



Hydrostatic surge-heave coupling Which statement is true?

A. c_{13}=0 only if the submerged geometry of the vessel has fore-aft symmetry (wrt origin)
B. c_{13}=0 regardless of geometry
C. Both A and B are false

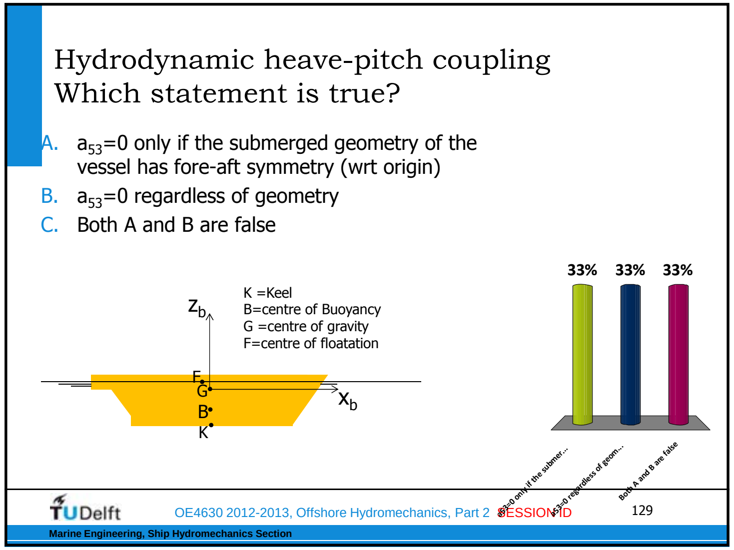
K =Keel
B=centre of Buoyancy
G =centre of gravity
F=centre of floatation



Hydrodynamic heave-pitch coupling Which statement is true?

A. a_{53}=0 only if the submerged geometry of the vessel has fore-aft symmetry (wrt origin)
B. a_{53}=0 regardless of geometry
C. Both A and B are false

K =Keel
B=centre of Buoyancy
G =centre of gravity
F=centre of floatation



Hydrodynamic heave-pitch coupling
Which statement is true?

A. a_{53}=0 only if the submerged geometry of the vessel has fore-aft symmetry (wrt origin)

B. a_{53}=0 regardless of geometry

C. Both A and B are false

K =Keel
B=centre of Buoyancy
G =centre of gravity
F=centre of floatation

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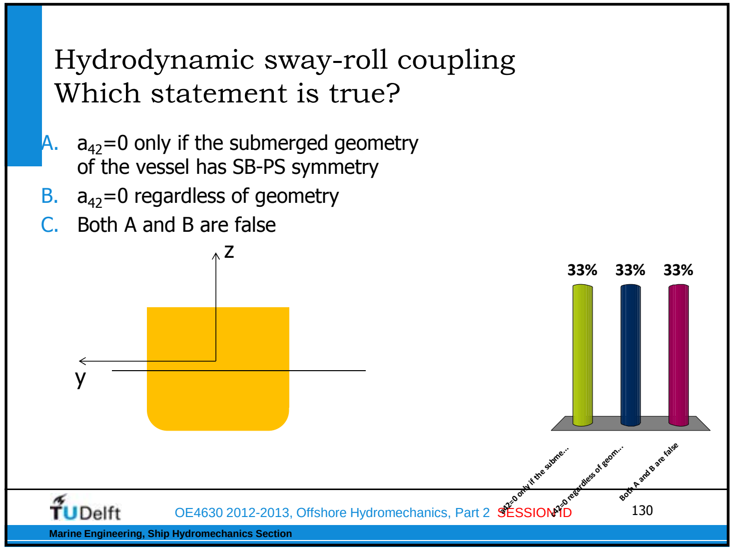
Marine Engineering, Ship Hydromechanics Section



Hydrodynamic sway-roll coupling
Which statement is true?

A. a_{42}=0 only if the submerged geometry of the vessel has SB-PS symmetry

B. a_{42}=0 regardless of geometry

C. Both A and B are false

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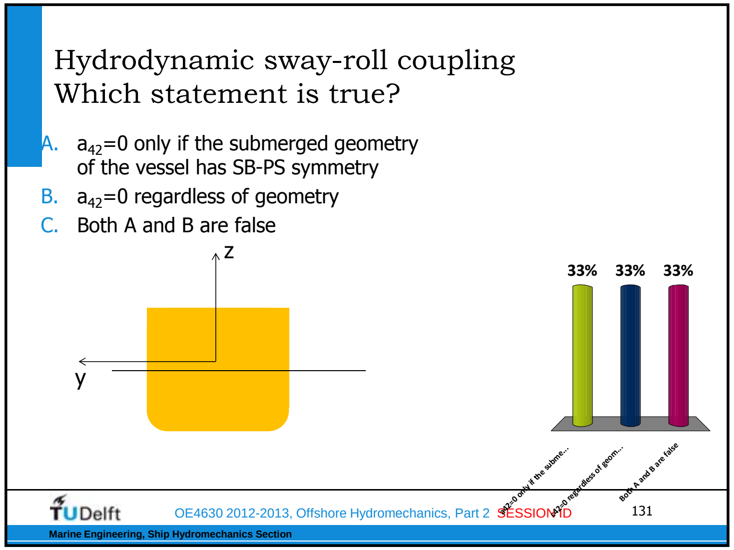
Marine Engineering, Ship Hydromechanics Section



Hydrodynamic sway-roll coupling
Which statement is true?

A. a_{42}=0 only if the submerged geometry of the vessel has SB-PS symmetry

B. a_{42}=0 regardless of geometry

C. Both A and B are false

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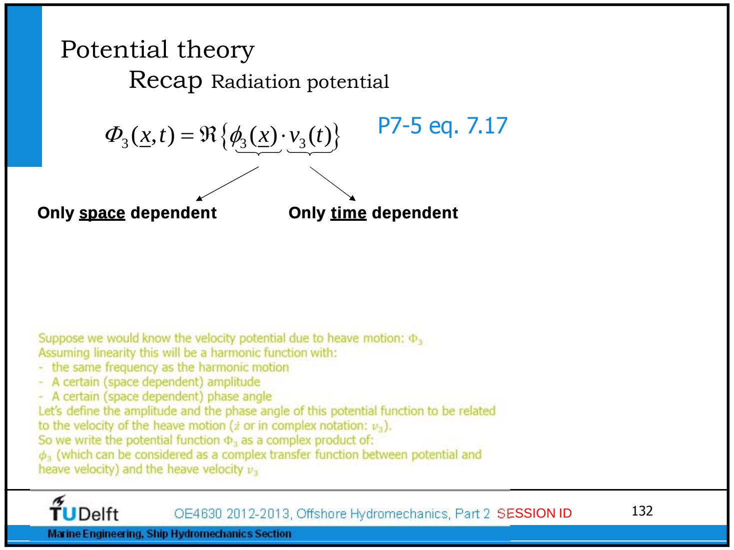
Marine Engineering, Ship Hydromechanics Section



Potential theory
Recap Radiation potential

$$\Phi_3(\underline{x},t) = \Re\{\underbrace{\phi_3(\underline{x})}\cdot\underbrace{v_3(t)}\}$$

P7-5 eq. 7.17

Only space dependent **Only time dependent**

Suppose we would know the velocity potential due to heave motion: Φ_3
Assuming linearity this will be a harmonic function with:
- the same frequency as the harmonic motion
- A certain (space dependent) amplitude
- A certain (space dependent) phase angle

Let's define the amplitude and the phase angle of this potential function to be related to the velocity of the heave motion ($\dot{z}$ or in complex notation: v_3).
So we write the potential function Φ_3 as a complex product of:
ϕ_3 (which can be considered as a complex transfer function between potential and heave velocity) and the heave velocity v_3

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Marine Engineering, Ship Hydromechanics Section

Potential theory

Radiation potential

$$a_{33} = -\rho\Re\left\{\iint_S \phi_3 \cdot n_3 \cdot dS\right\}$$

$$b_{33} = -\rho\omega\Im\left\{\iint_S \phi_3 \cdot n_3 \cdot dS\right\}$$

Potential theory

resulting from heave motions, Φ_3

Forces

$$a_{33} = -\Re\left\{\rho\iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$$

$$b_{33} = -\Im\left\{\rho\omega\iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$$

$$a_{13} = -\Re\left\{\rho\iint_{S_0} \phi_3 \cdot n_1 \cdot dS_0\right\}$$

$$b_{13} = -\Im\left\{\rho\omega\iint_{S_0} \phi_3 \cdot n_1 \cdot dS_0\right\}$$

$$a_{23} = -\Re\left\{\rho\iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$$

$$b_{23} = -\Im\left\{\rho\omega\iint_{S_0} \phi_3 \cdot n_3 \cdot dS_0\right\}$$

Moments

$$a_{43} = -\Re\left\{\rho\iint_{S_0} \phi_3 \cdot \left(\overline{r} \times \overline{n}\right)_1 dS_0\right\}$$

$$b_{43} = -\Im\left\{\rho\omega\iint_{S_0} \phi_3 \cdot \left(\overline{r} \times \overline{n}\right)_1 \cdot dS_0\right\}$$

$$a_{53} = -\Re\left\{\rho\iint_{S_0} \phi_3 \cdot \left(\overline{r} \times \overline{n}\right)_2 \cdot dS_0\right\}$$

$$b_{53} = -\Im\left\{\rho\omega\iint_{S_0} \phi_3 \cdot \left(\overline{r} \times \overline{n}\right)_2 \cdot dS_0\right\}$$

$$a_{63} = -\Re\left\{\rho\iint_{S_0} \phi_3 \cdot \left(\overline{r} \times \overline{n}\right)_3 \cdot dS_0\right\}$$

$$b_{63} = -\Im\left\{\rho\omega\iint_{S_0} \phi_3 \cdot \left(\overline{r} \times \overline{n}\right)_3 \cdot dS_0\right\}$$

Solving the Laplace equation

coupled equation of motion:

$$\begin{bmatrix} M+a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & M+a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & M+a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & I_{xx}+a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & I_{yy}+a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & I_{zz}+a_{66} \end{bmatrix} \cdot \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix} \cdot \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ \varphi \\ \theta \\ \psi \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{pmatrix}$$

Potential theory

Now let's consider the right hand side of the motion equation:
the excitation forces that the structure feels due to the waves.

Radiation potential $\Phi_{1.....6}$

Boundary Condition: $\frac{\partial\Phi_{1.....6}}{\partial n} = v_n$

Undisturbed wave potential Φ_0

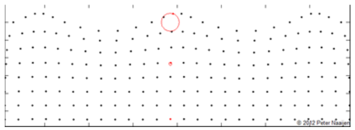
Diffraction potential Φ_7

Boundary Condition: $\frac{\partial\Phi_0}{\partial n} + \frac{\partial\Phi_7}{\partial n} = 0$

Calculating hydrodynamic coeffiecient and diffraction force

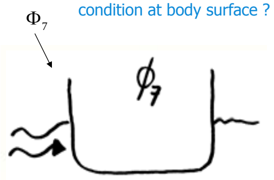
$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

1. Undisturbed wave force (Froude-Krilov)
   Potential is known from Ch. 5:



2. Diffraction force

Has to be solved. What is boundary condition at body surface ?



Calculating hydrodynamic coeffiecient and diffraction force

$$F_{FK}+F_D$$

Linear relation between undisturbed wave and diffraction potential →

$$\boxed{\Phi_7=\phi_7\cdot\dot{\zeta}=\phi_7\cdot-i\omega\cdot\zeta_a\cdot e^{-i\omega t}=\phi_7\cdot-i\omega\cdot\zeta_0\cdot e^{-i\omega t}}$$

Notation p 7-39, 7-40:

$\zeta_0=\zeta_7=$ amplitude undisturbed wave (at origin, so real)
$\zeta_{1\ldots6}=$ amplitude motions (complex)

Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z=F_W=F_{FK}+F_D$$

Undisturbed wave force (Froude-Krilov)

$$\zeta=\zeta_a\cos\left(kx\cos(\mu)+ky\sin(\mu)-\omega t\right)$$

$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin\left(kx\cos(\mu)+ky\sin(\mu)\text{-}\omega t\right)$$

$$=\Re\left\{-i\frac{\zeta_0 g}{\omega}\cdot e^{kz}\cdot e^{ik\left(x\cdot\cos(\mu)+y\cdot\sin(\mu)\right)}\cdot e^{-i\omega t}\right\}$$

Analogue to the radiation potential we write the known undisturbed wave potential function as a transfer function Φ_0 multiplied with the velocity of the undisturbed wave elevation at the origin of the axes system.

$$=\Re\left\{-i\omega\cdot\frac{g}{\omega^2}\cdot e^{kz}\cdot e^{ik\left(x\cdot\cos(\mu)+y\cdot\sin(\mu)\right)}\cdot\zeta_0\cdot e^{-i\omega t}\right\}$$

$$=\Re\left\{\phi_0\cdot-i\omega\cdot\zeta_0\cdot e^{-i\omega t}\right\}\longrightarrow$$

Velocity of wave elevation at origin of axes system (COG) in complex notation

Error p.7-39 eq. 7.151!!

$$\phi_0(x,y,z)=\frac{g}{\omega^2}e^{kz}\cdot e^{i\left(kx\cos(\mu)+ky\sin(\mu)\right)}$$

Calculating hydrodynamic coeffiecient and diffraction force

$$\Phi_0=\frac{\zeta_a g}{\omega}\cdot e^{kz}\cdot\sin\left(kx\cos(\mu)+ky\sin(\mu)\text{-}\omega t\right)$$

$$=\Re\left\{\phi_0(x)\cdot\dot{\zeta}(t)\right\}\longrightarrow$$

Velocity of wave elevation at origin of axes system (COG) in complex notation

$$\phi_0(x,y,z)=\frac{g}{\omega^2}e^{kz}\cdot e^{i\left(kx\cos(\mu)+ky\sin(\mu)\right)}$$

$$\dot{\zeta}(t)=-i\omega\cdot\zeta_0\cdot e^{-i\omega t}$$

Calculating hydrodynamic coeffiecient and diffraction force

Same for diffraction potential:

$$\Phi_7 = \Re\left\{\phi_7\left(\underline{x}\right)\cdot\dot{\zeta}\left(t\right)\right\}$$

$$\phi_7 = ?$$

$$\dot{\zeta}\left(t\right) = -i\omega\cdot\zeta_0\cdot e^{-i\omega t}$$

Calculating hydrodynamic coeffiecient and diffraction force

$$(m+a)\cdot\ddot{z}+b\cdot\dot{z}+c\cdot z = F_W = F_{FK} + F_D$$

Analogue to the radiation potential we write the known undisturbed wave potential function as a transfer function Φ_0 multiplied with the velocity of the undisturbed wave elevation at the origin of the axes system.
We also do this for the unknown diffraction potential whose transfer function we call Φ_7

$$\boxed{\Phi_0 + \Phi_7 = -i\omega\cdot\left(\phi_0 + \phi_7\right)\cdot\zeta_0\cdot e^{-i\omega t}}$$

Pressure:

$$\boxed{\begin{gathered} p_w = -\rho\frac{\partial\left(\Phi_0+\Phi_7\right)}{\partial t} = \rho\omega^2\cdot\left(\phi_0+\phi_7\right)\cdot\zeta_0\cdot e^{-i\omega t} \\ = \text{pressure due to incoming and diffracted wave}\end{gathered}}$$

Potential Theory

Forces and moments can be derived from pressures:

$$\overline{F} = -\iint_S \left(p\cdot\overline{n}\right)dS$$

$$\overline{M} = -\iint_S p\cdot\left(\overline{r}\times\overline{n}\right)dS$$

Knowing the potentials, pressures due to incoming and diffracted wave can be determined. Integrating these acc to the equations here finally gives the wave exciting forces.

Learning goals Module II, behavior of floating bodies in waves

• Definition of ship motions **Motion Response in regular waves:** • How to use RAO's • Understand the terms in the equation of motion: hydromechanic reaction forces, wave exciting forces • How to solve RAO's from the equation of motion **Motion Response in irregular waves:** • How to determine response in irregular waves from RAO's and wave spectrum without forward speed	Ch.6
3D linear Potential Theory • How to determine hydrodynamic reaction coefficients and wave forces from Velocity Potential • How to determine Velocity Potential now!	Ch. 7
Motion Response in irregular waves: • How to determine response in irregular waves from RAO's and wave spectrum with forward speed • Determine probability of exceedence • Make down time analysis using wave spectra, scatter diagram and RAO's	Ch. 8
Structural aspects: • Calculate internal forces and bending moments due to waves	Ch. 8
Nonlinear behavior: • Calculate mean horizontal wave force on wall • Use of time domain motion equation	Ch.6

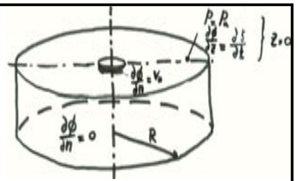
Potential Theory

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

So this is the differential equation we have to solve

What are the boundary conditions ?

Potential Theory



Boundary Conditions:

- At sea bottom: Sea bed is watertight
- At free surface:
  - p = $p_{atmospheric}$ (dynamic bc)
  - Water particles cannot leave free surface (kinematic bc)
- At ship hull: ship is watertight (that's what it's a ship for isn't it!)
- Far far away from the ship: no disturbances due to the ship's presence

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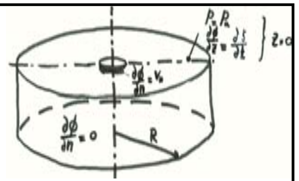
Potential Theory



Boundary Conditions:

- At sea bottom: Sea bed is watertight $\frac{\partial\Phi}{\partial n}=0$ at z = -h
- At free surface:
  - p = $p_{atmospheric}$ (dynamic bc)
  - Water particles cannot leave free surface (kinematic bc)

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0 \text{ at z = 0}$$

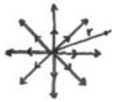
- At ship hull: ship is watertight (that's what it's a ship for!) $\frac{\partial\Phi}{\partial n}=v_n$ at S_0
- Far far away from the ship: no disturbances due to the ship's presence $\lim_{R\to\infty}\Phi=0$

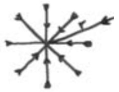
Solving the Laplace equation

Q: How to create the potential flows ?

A: Use of basic potential flow elements: *source-sheet* on the hull

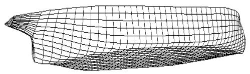
Recall Ch. 3: *point source*
3D

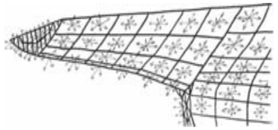


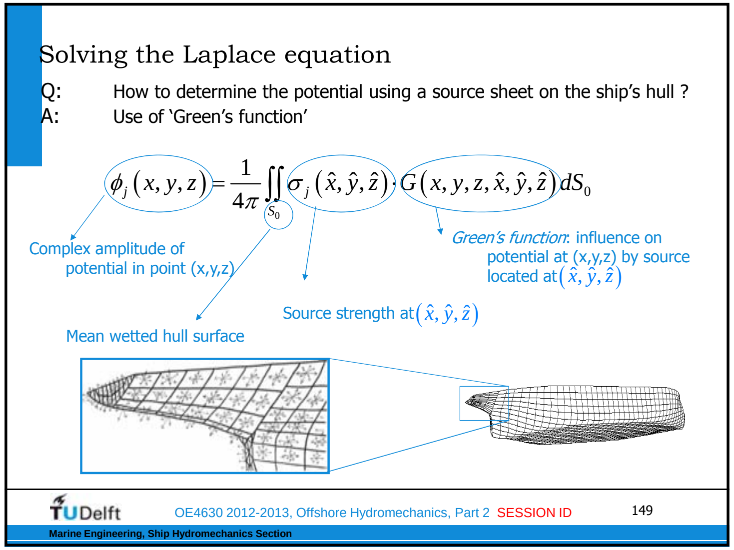


Source: $\phi=\frac{-\sigma}{4\pi r}, v_r=\frac{\sigma}{4\pi r^2}$

Sink σ is negative







Solving the Laplace equation

Q: How to determine the potential using a source sheet on the ship's hull ?
A: Use of 'Green's function'

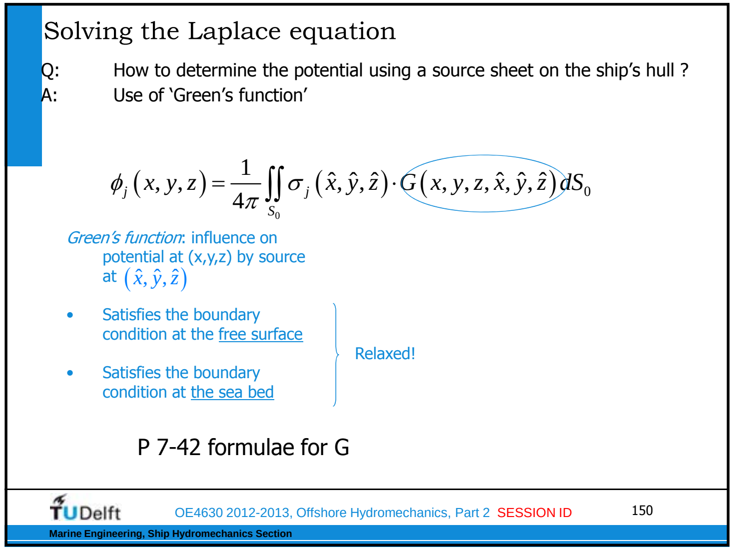
$$\phi_j(x,y,z)=\frac{1}{4\pi}\iint_{S_0}\sigma_j(\hat{x},\hat{y},\hat{z})\cdot G(x,y,z,\hat{x},\hat{y},\hat{z})\,dS_0$$

Complex amplitude of potential in point (x,y,z)

Mean wetted hull surface

Source strength at $(\hat{x},\hat{y},\hat{z})$

Green's function: influence on potential at (x,y,z) by source located at $(\hat{x},\hat{y},\hat{z})$



Solving the Laplace equation

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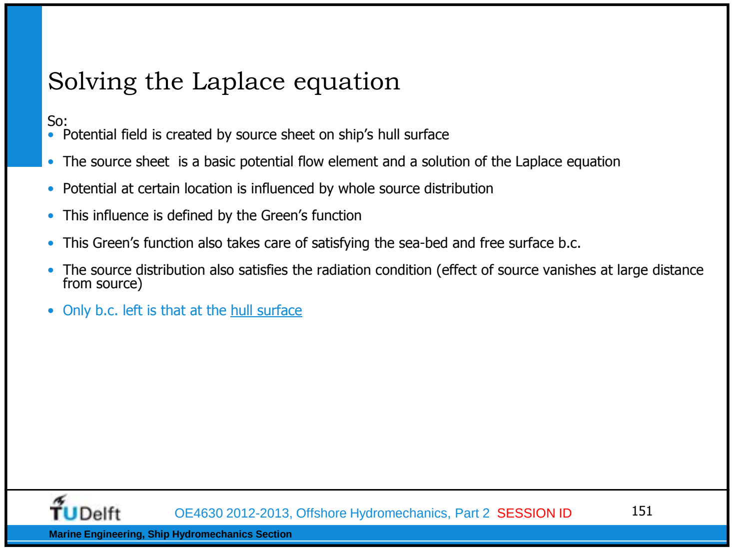
Green's function: influence on potential at (x,y,z) by source at $(\hat{x},\hat{y},\hat{z})$

- Satisfies the boundary condition at the free surface
- Satisfies the boundary condition at the sea bed

Relaxed!

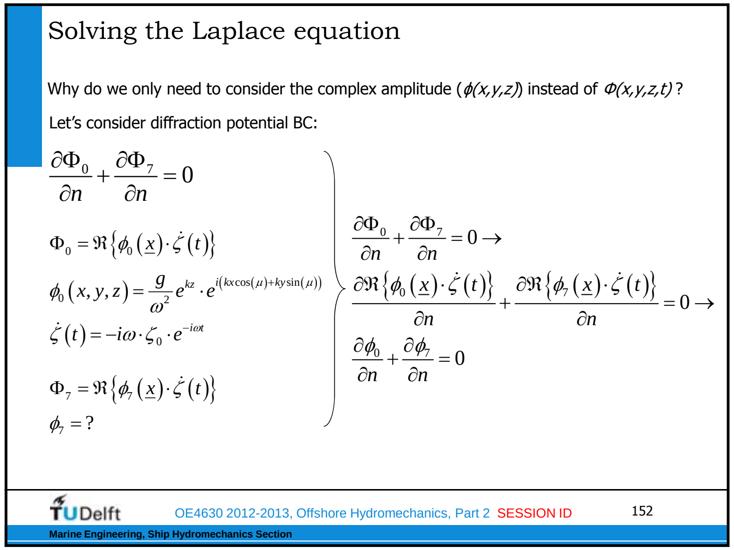
P 7-42 formulae for G



Solving the Laplace equation

So:

- Potential field is created by source sheet on ship's hull surface
- The source sheet is a basic potential flow element and a solution of the Laplace equation
- Potential at certain location is influenced by whole source distribution
- This influence is defined by the Green's function
- This Green's function also takes care of satisfying the sea-bed and free surface b.c.
- The source distribution also satisfies the radiation condition (effect of source vanishes at large distance from source)
- Only b.c. left is that at the hull surface



Solving the Laplace equation

Why do we only need to consider the complex amplitude (*ϕ(x,y,z)*) instead of *Φ(x,y,z,t)* ?

Let's consider diffraction potential BC:

$$\frac{\partial\Phi_0}{\partial n}+\frac{\partial\Phi_7}{\partial n}=0$$

$$\Phi_0=\Re\left\{\phi_0(\underline{x})\cdot\dot{\zeta}(t)\right\}$$

$$\phi_0(x,y,z)=\frac{g}{\omega^2}e^{kz}\cdot e^{i(kx\cos(\mu)+ky\sin(\mu))}$$

$$\dot{\zeta}(t)=-i\omega\cdot\zeta_0\cdot e^{-i\omega t}$$

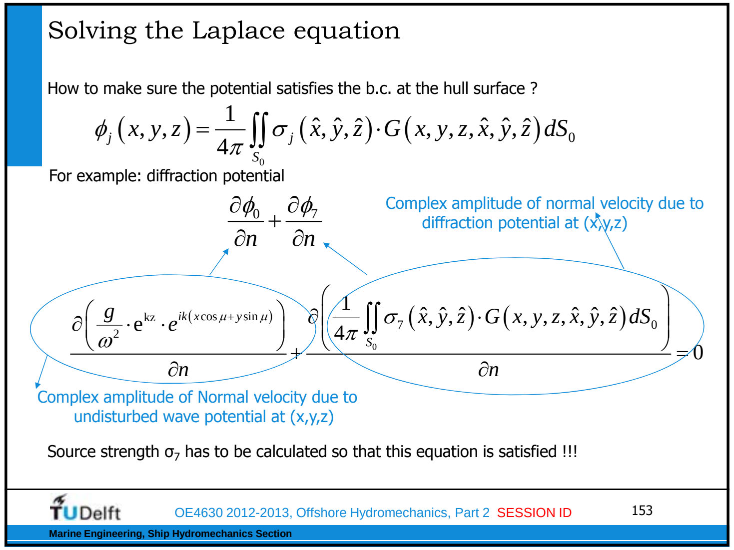
$$\Phi_7=\Re\left\{\phi_7(\underline{x})\cdot\dot{\zeta}(t)\right\}$$

$$\phi_7=?$$

$$\frac{\partial\Phi_0}{\partial n}+\frac{\partial\Phi_7}{\partial n}=0\rightarrow$$

$$\frac{\partial\Re\left\{\phi_0(\underline{x})\cdot\dot{\zeta}(t)\right\}}{\partial n}+\frac{\partial\Re\left\{\phi_7(\underline{x})\cdot\dot{\zeta}(t)\right\}}{\partial n}=0\rightarrow$$

$$\frac{\partial\phi_0}{\partial n}+\frac{\partial\phi_7}{\partial n}=0$$



Solving the Laplace equation

How to make sure the potential satisfies the b.c. at the hull surface ?

$$\phi_j(x,y,z)=\frac{1}{4\pi}\iint_{S_0}\sigma_j(\hat{x},\hat{y},\hat{z})\cdot G(x,y,z,\hat{x},\hat{y},\hat{z})\,dS_0$$

For example: diffraction potential

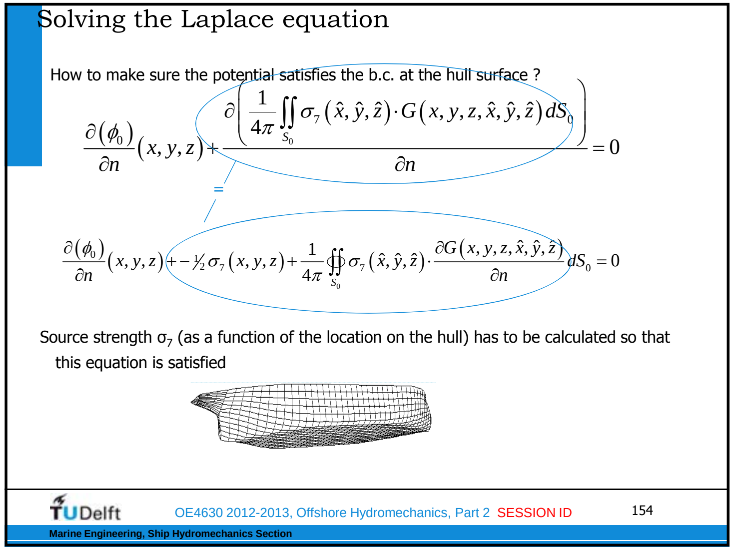
$$\frac{\partial\phi_0}{\partial n}+\frac{\partial\phi_7}{\partial n}$$

Complex amplitude of normal velocity due to diffraction potential at (x,y,z)

$$\frac{\partial\left(\frac{g}{\omega^2}\cdot \mathrm{e}^{\mathrm{kz}}\cdot e^{ik(x\cos\mu+y\sin\mu)}\right)}{\partial n}+\frac{\partial\left(\frac{1}{4\pi}\iint_{S_0}\sigma_7(\hat{x},\hat{y},\hat{z})\cdot G(x,y,z,\hat{x},\hat{y},\hat{z})\,dS_0\right)}{\partial n}=0$$

Complex amplitude of Normal velocity due to undisturbed wave potential at (x,y,z)

Source strength σ$_7$ has to be calculated so that this equation is satisfied !!!



Solving the Laplace equation

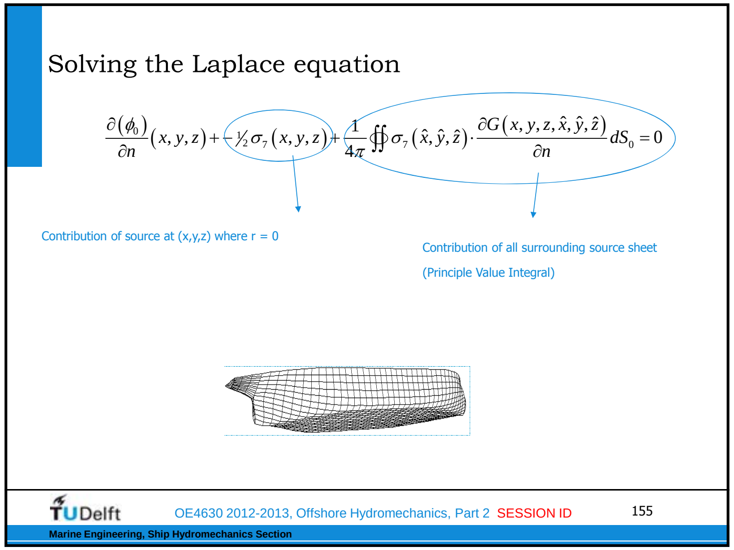
How to make sure the potential satisfies the b.c. at the hull surface ?

$$\frac{\partial(\phi_0)}{\partial n}(x,y,z)+\frac{\partial\left(\frac{1}{4\pi}\iint_{S_0}\sigma_7(\hat{x},\hat{y},\hat{z})\cdot G(x,y,z,\hat{x},\hat{y},\hat{z})\,dS_0\right)}{\partial n}=0$$

=

$$\frac{\partial(\phi_0)}{\partial n}(x,y,z)+-\tfrac{1}{2}\sigma_7(x,y,z)+\frac{1}{4\pi}\oiint_{S_0}\sigma_7(\hat{x},\hat{y},\hat{z})\cdot\frac{\partial G(x,y,z,\hat{x},\hat{y},\hat{z})}{\partial n}dS_0=0$$

Source strength σ$_7$ (as a function of the location on the hull) has to be calculated so that this equation is satisfied



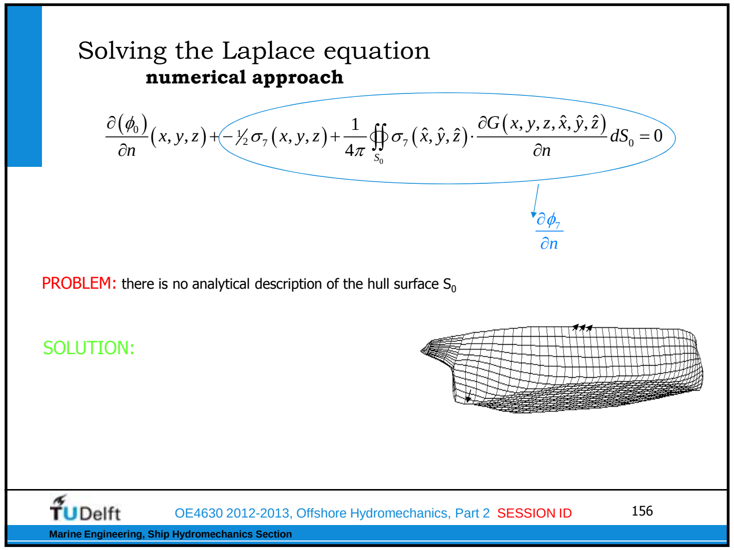
Solving the Laplace equation

$$\frac{\partial(\phi_0)}{\partial n}(x,y,z)+-\tfrac{1}{2}\sigma_7(x,y,z)+\frac{1}{4\pi}\oiint_{S_0}\sigma_7(\hat{x},\hat{y},\hat{z})\cdot\frac{\partial G(x,y,z,\hat{x},\hat{y},\hat{z})}{\partial n}dS_0=0$$

Contribution of source at (x,y,z) where r = 0

Contribution of all surrounding source sheet

(Principle Value Integral)



Solving the Laplace equation

numerical approach

$$\frac{\partial(\phi_0)}{\partial n}(x,y,z)+\underbrace{-\tfrac{1}{2}\sigma_7(x,y,z)+\frac{1}{4\pi}\oiint_{S_0}\sigma_7(\hat{x},\hat{y},\hat{z})\cdot\frac{\partial G(x,y,z,\hat{x},\hat{y},\hat{z})}{\partial n}dS_0}_{\frac{\partial\phi_7}{\partial n}}=0$$

PROBLEM: there is no analytical description of the hull surface S$_0$

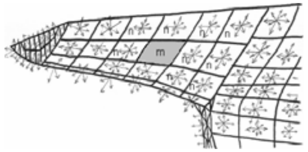
SOLUTION:

Solving the Laplace equation
numerical approach

$$\frac{\partial(\phi_0)}{\partial n}(x,y,z)+-\tfrac{1}{2}\sigma_7(x,y,z)+\frac{1}{4\pi}\oiint_{S_0}\sigma_7(\hat{x},\hat{y},\hat{z})\cdot\frac{\partial G(x,y,z,\hat{x},\hat{y},\hat{z})}{\partial n}dS_0=0$$

$$\frac{\partial\phi_7}{\partial n}$$

$$-\tfrac{1}{2}\sigma_{m7}(x,y,z)+\frac{1}{4\pi}\sum_{n=1}^{N}\sigma_{n7}\cdot\frac{\partial G_{mn}}{\partial n}\Delta S_n=-\left(\frac{\partial(\phi_0)}{\partial n}\right)_m$$

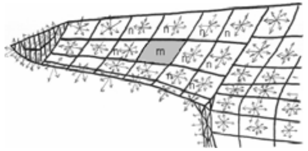


- Centroid of panel m is called collocation point (x,y,z)
- Only here boundary condition is satisfied.
- Every panel has 1 collocation point
- Source strength is constant on each panel
- In summation: n≠m (numerical form of P.V integral)

$$\boxed{-\tfrac{1}{2}\sigma_{m7}+\frac{1}{4\pi}\sum_{n=1}^{N}\sigma_{n7}\cdot\frac{\partial G_{mn}}{\partial n}\Delta S_n=-\left(\frac{\partial(\Phi_0)}{\partial n}\right)_m}$$

This equation must be solved for every panel m

Taking into account sources on all other panels



$$\begin{pmatrix}A_{11} & \cdots & A_{1N}\\ \vdots & \ddots & \vdots\\ A_{N1} & \cdots & A_{NN}\end{pmatrix}\cdot\begin{pmatrix}\sigma_{1,7}\\ \vdots\\ \sigma_{N,7}\end{pmatrix}=\begin{pmatrix}-\left(\frac{\partial(\phi_0)}{\partial n}\right)_1\\ \vdots\\ -\left(\frac{\partial(\phi_0)}{\partial n}\right)_N\end{pmatrix}$$

$A_{nn}=-\frac{1}{2}$ (influence of source at panel n on $\frac{\partial\phi_7}{\partial n}$ at its own collocation point)

$A_{mn}=\frac{1}{4\pi}\frac{\partial G_{mn}}{\partial n}\Delta S_n$(influence of source at panel n on $\frac{\partial\phi_7}{\partial n}$ at collocation point m)

$\sigma_{n,7}$ = unknown source strength of diffraction potential at panel n

---

Potential theory
Radiation potentials

$$\Phi_{rj}(\underline{x},t)=\phi_j(\underline{x})\cdot v_j(t)=\Re\left(\underbrace{\phi_j(\underline{x})}\cdot\underbrace{-i\omega s_{aj}e^{-i\omega t}}\right)$$

Only space dependent **Only time dependent**

Boundary condition at the hull surface:

Flow velocity in normal direction

$$\frac{\partial\Phi_j}{\partial n}=\frac{\partial\phi_j}{\partial n}v_j=v_n=v_j\cdot f_j$$

Local body velocity in normal direction

$$\frac{\partial\phi_j}{\partial n}=f_j$$

$$\left\{\begin{aligned}surge:&\ f_1=\cos(n,x)\\ sway:&\ f_2=\cos(n,y)\\ heave:&\ f_3=\cos(n,z)\\ roll:&\ f_4=y\cos(n,z)-z\cos(n,y)\\ pitch:&\ f_5=z\cos(n,x)-x\cos(n,z)\\ yaw:&\ f_6=x\cos(n,y)-y\cos(n,x)\end{aligned}\right.$$

P 7-3

Potential theory
Radiation potential

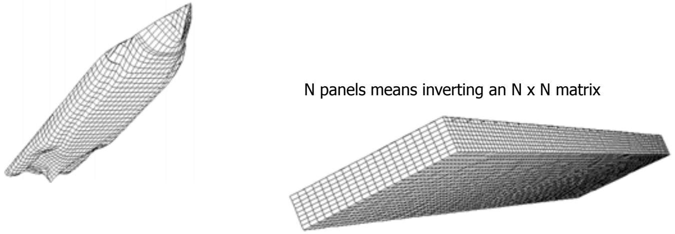
$$\frac{\partial\phi_j}{\partial n}=f_j$$

$$\boxed{-\tfrac{1}{2}\sigma_{mj}+\frac{1}{4\pi}\sum_{n=1}^{N}\sigma_{nj}\cdot\frac{\partial G_{mn}}{\partial n}\Delta S_n=f_{mj}}$$

$$\begin{pmatrix}A_{11} & \cdots & A_{1N}\\ \vdots & \ddots & \vdots\\ A_{N1} & \cdots & A_{NN}\end{pmatrix}\cdot\begin{pmatrix}\sigma_{1,j}\\ \vdots\\ \sigma_{N,j}\end{pmatrix}=\begin{pmatrix}(f_j)_1\\ \vdots\\ (f_j)_N\end{pmatrix}$$

- j indicates which radiation potential is considered: j = 1....6
- $A_{nn}=-\frac{1}{2}$ (influence of source at panel n on $\frac{\partial\phi_j}{\partial n}$ at its own collocation point)
- $A_{mn}=\frac{1}{4\pi}\frac{\partial G_{mn}}{\partial n}\Delta S_n$(influence of source at panel n on $\frac{\partial\phi_j}{\partial n}$ at collocation point m)
- $\sigma_{n,j}$ = unknown source strength of radiation potential (j=1...6) at panel n
- $(f_j)_m$ local normal direction due to motion in direction j at panel m

Saving calculation time by use of symmetry



N panels means inverting an N x N matrix

How many source strengths have to be calculated to determine 6DOF motion RAO's for 5 wave directions and 40 frequencies ?

A	1400 x N
B	440 x N
C	245 x N

Internal Loads

- Static
- Dynamic

Internal Loads (static)

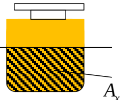
Global vertical force equilibrium:

$m = \nabla \cdot \rho$ mass = displacement

The '-sign indicates sectional values! E.g. m'(x_b)=mass per unit length [kg/m] at longitudinal location x_b

$$m = \int_{stern}^{bow} m'(x_b)\,dx_b \quad \text{mass}$$

$$= \rho \int_{stern}^{bow} A_x(x_b)\,dx_b = \rho\nabla \quad \text{displacement}$$



A_x = submerged area of cross section

$$x_G = \frac{1}{m}\int_{stern}^{bow} m'(x_b)\cdot x_b \cdot dx_b \quad \text{centre of gravity}$$

$$x_B = \frac{1}{\nabla}\int_{stern}^{bow} A_x(x_b)\cdot x_b \cdot dx_b \quad \text{centre of bouyancy}$$

$$k_{xx}{}^2 = \frac{1}{m}\int_{stern}^{bow} k_{xx}'(x_b)^2 \cdot m'(x_b)\,dx_b \quad \text{roll radius of gyration}$$

$$k_{yy}{}^2 = k_{zz}{}^2 = \frac{1}{m}\int_{stern}^{bow} m'(x_b)\cdot x_b{}^2 dx_b \quad \text{pitch / yaw radius of gyration}$$

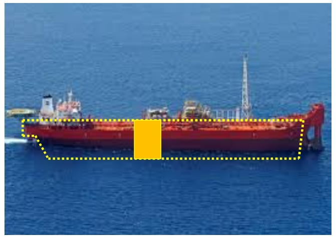
Internal Loads (static)

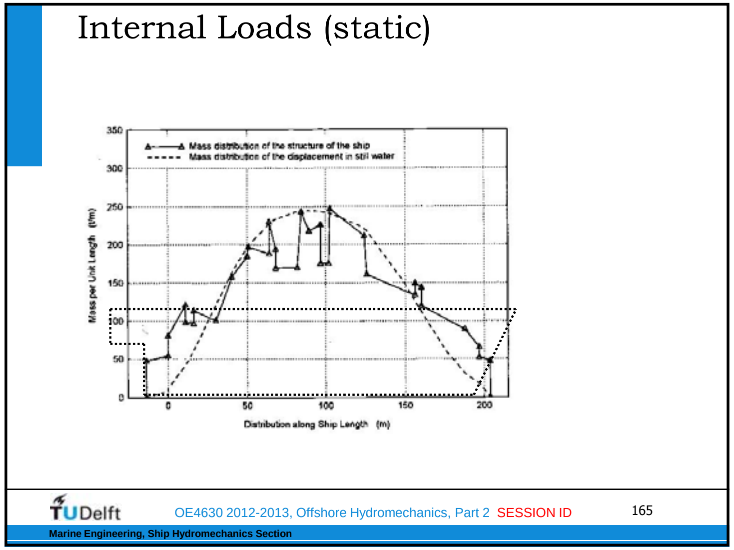
Global vertical force equilibrium:

$m = \nabla \cdot \rho$ mass = displacement

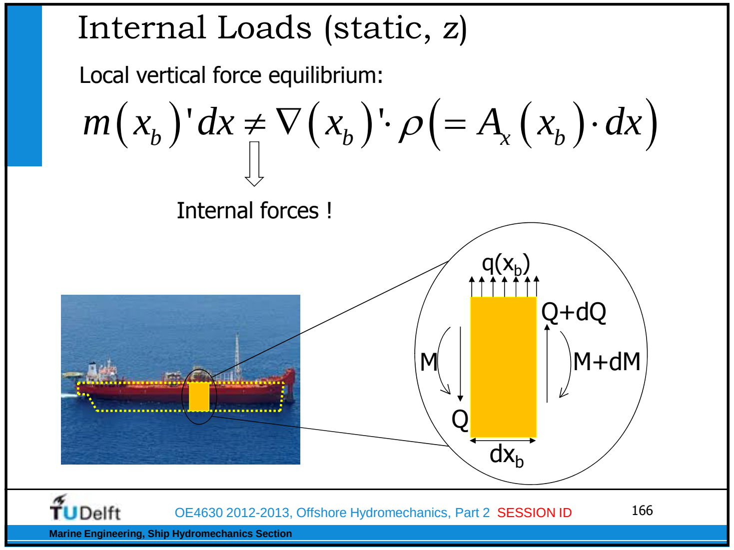
Local vertical force equilibrium (forces on a section of length dx):

$$m(x_b)'dx \stackrel{?}{=} \nabla(x_b)'\cdot\rho\left(= A_x(x_b)\cdot dx\right)$$





Internal Loads (static)



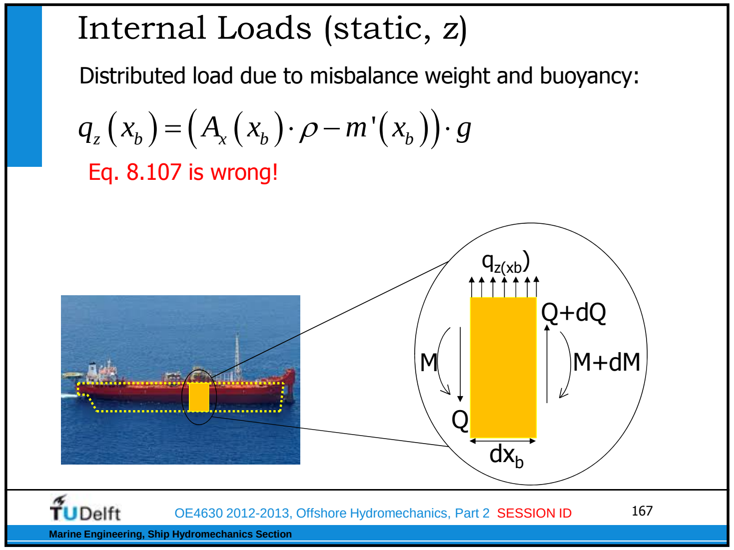
Internal Loads (static, z)

Local vertical force equilibrium:

$$m\left(x_b\right)'dx \neq \nabla\left(x_b\right)'\cdot\rho\left(= A_x\left(x_b\right)\cdot dx\right)$$

$$\Downarrow$$

Internal forces !

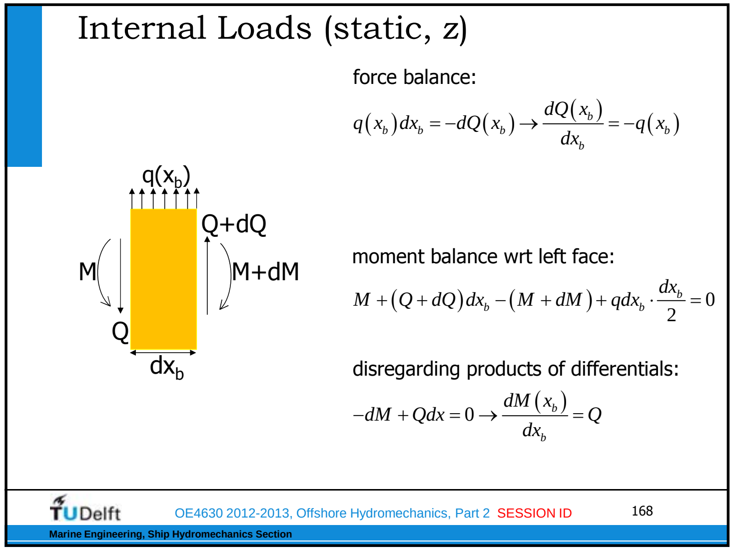


Internal Loads (static, z)

Distributed load due to misbalance weight and buoyancy:

$$q_z\left(x_b\right) = \left(A_x\left(x_b\right)\cdot\rho - m'\left(x_b\right)\right)\cdot g$$

Eq. 8.107 is wrong!



Internal Loads (static, z)

force balance:

$$q\left(x_b\right)dx_b = -dQ\left(x_b\right) \rightarrow \frac{dQ\left(x_b\right)}{dx_b} = -q\left(x_b\right)$$

moment balance wrt left face:

$$M + \left(Q + dQ\right)dx_b - \left(M + dM\right) + qdx_b \cdot \frac{dx_b}{2} = 0$$

disregarding products of differentials:

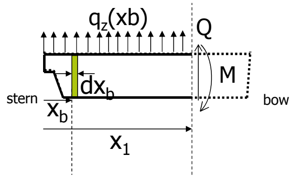
$$-dM + Qdx = 0 \rightarrow \frac{dM\left(x_b\right)}{dx_b} = Q$$

Internal Loads (static, z)

shear force due to $dq(x_b)$:

$$Q(x_1) = -\int_{stern}^{x_1} q_z(x_b)\,dx_b$$

Free body diagram at x=x_1

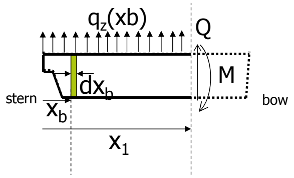


bending moment due to $dq_z(x_b)$:

$$M(x_1) = \int_{stern}^{x_1} Q(x_b)\,dx_b = \int_{stern}^{x_1}\int_{stern}^{x_b} -q_z(x^*)\,dx^*dx_b$$

$$M(x_1) = -\int_{stern}^{x_1} \underbrace{(x_1 - x_b)}_{\text{lever arm}} \cdot \overbrace{q_z(x_b)\,dx_b}^{\text{force}}$$

$$= \int_{stern}^{x_1} q_z(x_b)\cdot x_b dx_b \underbrace{- x_1\int_{stern}^{x_1} q_z(x_b)\cdot dx_b}_{Q(x_1)\cdot x_1}$$



Internal Loads (static, z)

shear force due to $dq(x_b)$:

$$Q(x_1) = -\int_{stern}^{x_1} q_z(x_b)\,dx_b$$

Free body diagram at x=x_1

bending moment due to $dq_z(x_b)$:

$$M(x_1) = \int_{stern}^{x_1} Q(x_b)\,dx_b = \int_{stern}^{x_1}\int_{stern}^{x_b} -q_z(x^*)\,dx^*dx_b$$

$$M(x_1) = -\int_{stern}^{x_1} \underbrace{(x_1 - x_b)}_{\text{lever arm}} \cdot \overbrace{q_z(x_b)\,dx_b}^{\text{force}}$$

$$= \int_{stern}^{x_1} q_z(x_b)\cdot x_b dx_b \underbrace{- x_1\int_{stern}^{x_1} q_z(x_b)\cdot dx_b}_{Q(x_1)\cdot x_1}$$

Internal Loads

- Static
- Dynamic ⇦

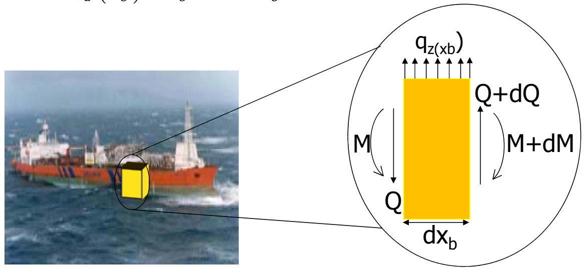
Internal Loads (dynamic, z-direction)

Newton’s second law applied to section of length dx:

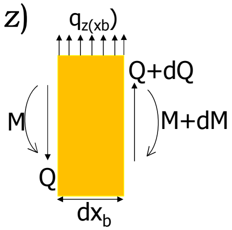
$$\sum F_3 = \qquad m'dx_b \qquad \cdot \qquad \ddot{z}'$$

vert. forces on section = Mass of section * vert. acceleration section

$$dQ + q_z(x_b)\cdot dx_b = m'dx_b \cdot \ddot{z}'$$



Internal Loads (dynamic, z)



$$dQ + q_z\left(x_b\right) \cdot dx_b = m' dx_b \cdot \ddot{z}'$$

Distributed load:

$$q_z\left(x_b\right) = \left(A_x\left(x_b\right) \cdot \rho - m'\left(x_b\right)\right) \cdot g + \boxed{?}$$

$$q_z\left(x_b\right) = \left(A_x\left(x_b\right) \cdot \rho - m'\left(x_b\right)\right) \cdot g + F_{waves}' \quad + F_{hydromechanic\ reaction}'$$

$$q_z\left(x_b\right) = \left(A_x\left(x_b\right) \cdot \rho - m'\left(x_b\right)\right) \cdot g + F_{w3}' \quad + F_{h3}'$$

This includes the static part. For a dynamic analysis this part is omitted:

$$q_z\left(x_b\right) = \cancel{\left(A_x\left(x_b\right) \cdot \rho - m'\left(x_b\right)\right) \cdot g} + F_{w3}' \quad + F_{h3}'$$

Internal Loads (dynamic, z)

distributed dynamic load = reaction +excitation:

$$q_z\left(x_b\right) = F'_{h3}\, dx_b + F'_{w3}\, dx_b$$

- reaction force (added mass, damping, restoring) ⇦
- excitation force (due to waves)

Internal Loads (dynamic, z)

reaction force (added mass, damping, restoring):

Hydrodynamic (a, and b)

Hydrostatic (c)

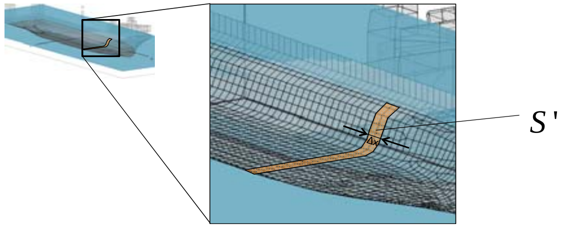
$$F'_{h3} dx_b = \boxed{F'_{r3}} + \boxed{F'_{s3}} = \boxed{\sum_{j=1}^{6} -a'_{3j} \cdot \ddot{x}_j + \sum_{j=1}^{6} -b'_{3j} \cdot \dot{x}_j} + \boxed{\sum_{j=1}^{6} -c'_{3j} \cdot x_j} =$$

$$\boxed{-\begin{pmatrix} a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \end{pmatrix}' dx_b \cdot \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} - \begin{pmatrix} b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{pmatrix}' dx_b \cdot \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}} \boxed{-\begin{pmatrix} c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \end{pmatrix}' dx_b \cdot \begin{pmatrix} x \\ y \\ z \\ \varphi \\ \theta \\ \psi \end{pmatrix}}$$

How to obtain sectional hydromechanic coefficients a', b', c' ?

Internal Loads (dynamic, z)

How to obtain sectional hydromechanic coefficients a', b', c' ?



To determine a' and b' coefficients:
Pressures due to motions:

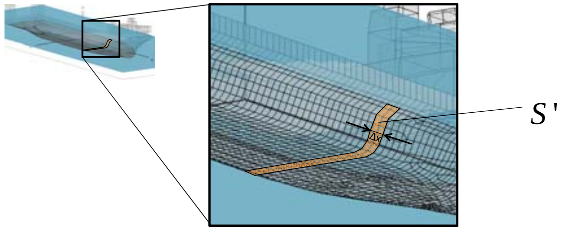
$$p_j = -\rho \frac{\partial \Phi_j}{\partial t}$$

Resulting radiation force in heave direction due to all possible motions found by integration over sectional submerged hull area, *S'*, only:

$$F'_{r3} = \frac{1}{\Delta x} \sum_{j=1}^{6} \iint_{S'} -p_j \cdot n_3 dS = \sum_{j=1}^{6} -a'_{3j} \cdot \ddot{x}_j + \sum_{j=1}^{6} -b'_{3j} \cdot \dot{x}_j$$

Internal Loads (dynamic, z)

Determining sectional hydromechanic coefficients:



Alternative to find restoring coefficients c in general:

$$c_{33} = \rho g \iint_S -n_3 dS \qquad c_{34} = \rho g \iint_S -y_b n_3 dS \qquad c_{35} = \rho g \iint_S x_b n_3 dS$$

For sectional restoring coefficient, simply replace *S* by *S′* and divide by *Δx!* :

$$c'_{33} = \frac{1}{\Delta x} \rho g \iint_{S'} -n_3 dS \qquad c'_{34} = \frac{1}{\Delta x} \rho g \iint_{S'} -y_b n_3 dS \qquad c'_{35} = \frac{1}{\Delta x} \rho g \iint_{S'} x_b n_3 dS$$

Internal Loads (dynamic, z)

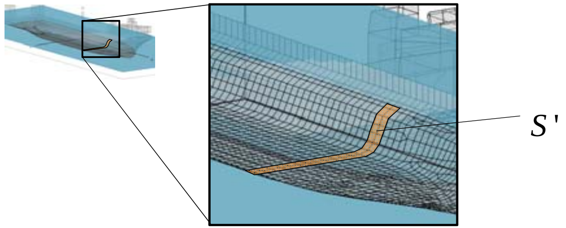
distributed dynamic load = reaction +excitation:

$$q_z(x_b) = F'_{h3}\, dx_b + F'_{w3}\, dx_b$$

- reaction force (added mass, damping, restoring):
- excitation force (due to waves) ⇦

Internal Loads (dynamic, z)

How to obtain sectional wave exciting force?



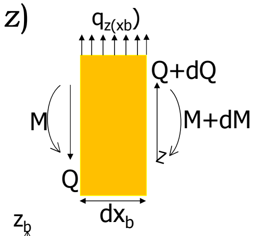
To determine F'_{w3}:
Pressures due to waves:

$$p = -\rho \frac{\partial(\Phi_0 + \Phi_7)}{\partial t}$$

Obtain sectional wave force by integrating over *S′* only:

$$F'_{w3} = \iint_{S'} -p \cdot n_3 dS$$

Internal Loads (dynamic, z)



Newton’s 2nd law:

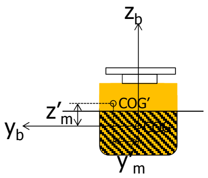
$$dQ + q_z(x_b) \cdot dx_b = m' dx_b \cdot \ddot{z}'$$

Dynamic distributed load:

$$q_z(x_b)\, dx_b = F'_{w3}\, dx_b + F'_{h3}\, dx_b$$

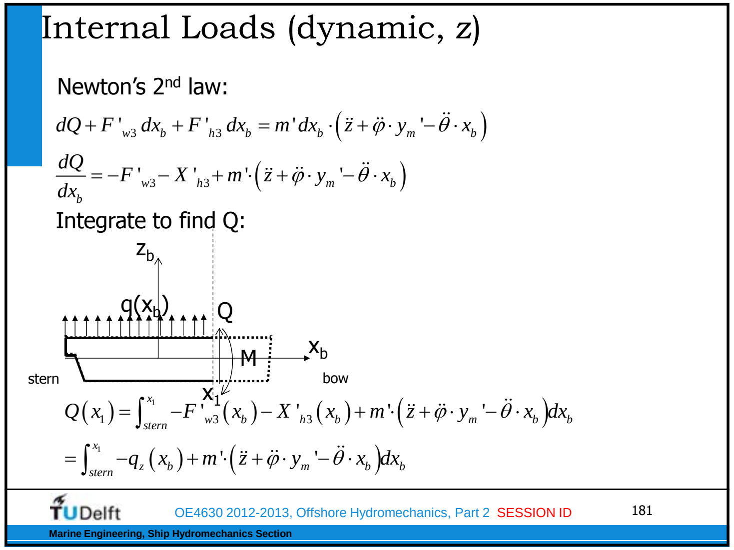
Local vert. acceleration:

$$\ddot{z}' = \ddot{z} + \ddot{\varphi} \cdot y_m' - \ddot{\theta} \cdot x_b'$$



Newton’s 2nd law:

$$dQ + F'_{w3}\, dx_b + F'_{h3}\, dx_b = m' dx_b \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m' - \ddot{\theta} \cdot x_b\right)$$



Internal Loads (dynamic, *z*)

Newton’s 2nd law:

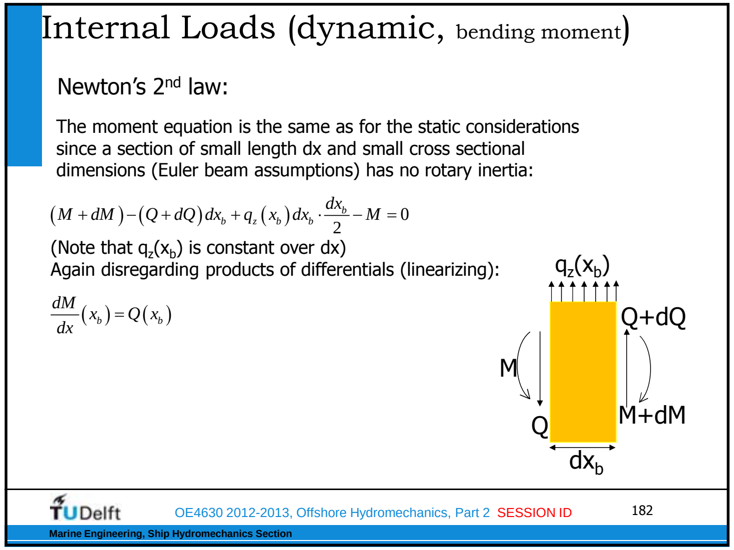
$$dQ + F'_{w3}\,dx_b + F'_{h3}\,dx_b = m'dx_b \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m{}' - \ddot{\theta} \cdot x_b\right)$$

$$\frac{dQ}{dx_b} = -F'_{w3} - X'_{h3} + m' \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m{}' - \ddot{\theta} \cdot x_b\right)$$

Integrate to find Q:

$$Q(x_1) = \int_{stern}^{x_1} -F'_{w3}(x_b) - X'_{h3}(x_b) + m' \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m{}' - \ddot{\theta} \cdot x_b\right) dx_b$$

$$= \int_{stern}^{x_1} -q_z(x_b) + m' \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m{}' - \ddot{\theta} \cdot x_b\right) dx_b$$



Internal Loads (dynamic, bending moment)

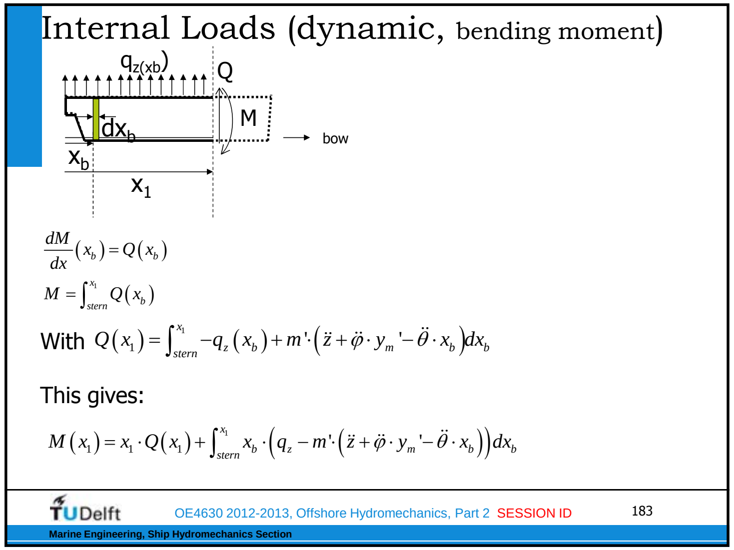
Newton’s 2nd law:

The moment equation is the same as for the static considerations since a section of small length dx and small cross sectional dimensions (Euler beam assumptions) has no rotary inertia:

$$(M + dM) - (Q + dQ)\,dx_b + q_z(x_b)\,dx_b \cdot \frac{dx_b}{2} - M = 0$$

(Note that $q_z(x_b)$ is constant over dx)
Again disregarding products of differentials (linearizing):

$$\frac{dM}{dx}(x_b) = Q(x_b)$$



Internal Loads (dynamic, bending moment)

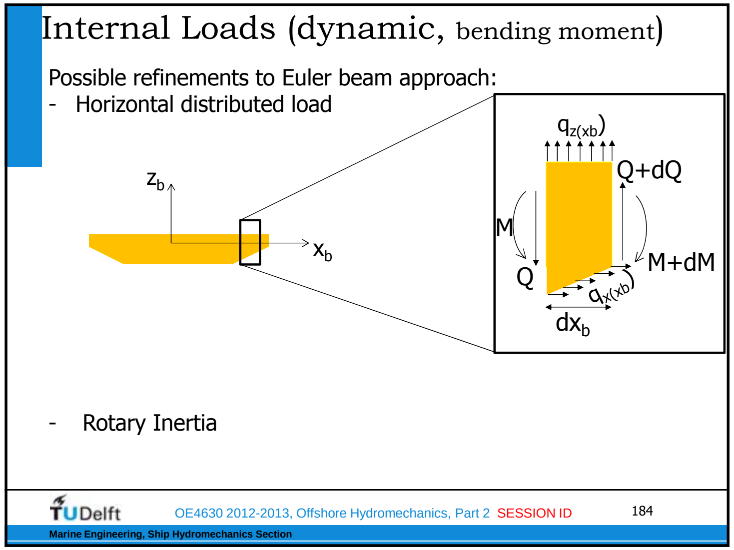
$$\frac{dM}{dx}(x_b) = Q(x_b)$$

$$M = \int_{stern}^{x_1} Q(x_b)$$

With $Q(x_1) = \int_{stern}^{x_1} -q_z(x_b) + m' \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m{}' - \ddot{\theta} \cdot x_b\right) dx_b$

This gives:

$$M(x_1) = x_1 \cdot Q(x_1) + \int_{stern}^{x_1} x_b \cdot \left(q_z - m' \cdot \left(\ddot{z} + \ddot{\varphi} \cdot y_m{}' - \ddot{\theta} \cdot x_b\right)\right) dx_b$$



Internal Loads (dynamic, bending moment)

Possible refinements to Euler beam approach:

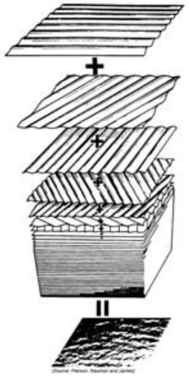
- Horizontal distributed load

- Rotary Inertia

Response in Irregular Waves

Irregular wind waves

apparently irregular but in fact a superposition of an infinite number of regular waves, each having own frequency, amplitude and propagation direction

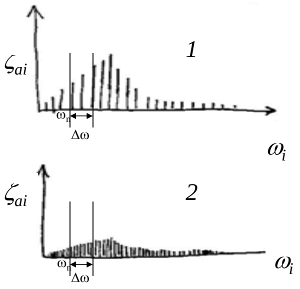


(Source: Pierson, Neumann and James)

Response in Irregular Waves:

Energy density spectrum,

1 direction only



Definition:

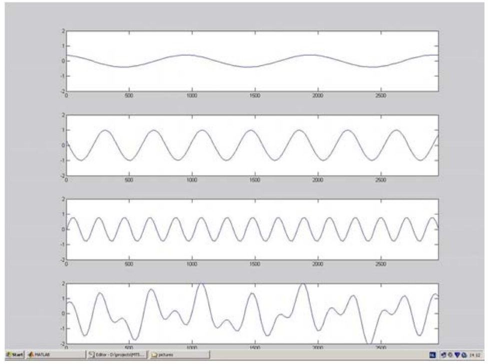
$$S_\zeta(\omega_n) = \frac{\sum\limits_{\omega_n}^{\omega_n+\Delta\omega} \frac{1}{2}\zeta_{a_n}^2(\omega)}{\Delta\omega}$$

$$\Delta\omega \to 0$$

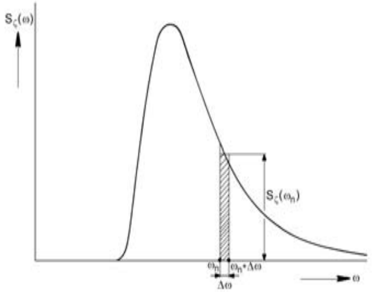
$$S_\zeta(\omega_n)\cdot d\omega = \tfrac{1}{2}\zeta_a^2(\omega_n)$$

Response in Irregular Waves:

Energy density spectrum, 1 direction only



Energy Density Spectrum



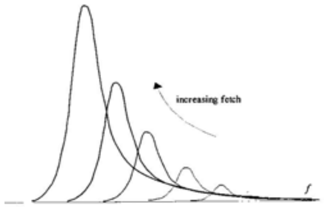
Shape is location / weather specific

Standard wave spectra

JONSWAP

Joint North Sea Wave Project

Late 60's effect of limited fetch was studied by extensive measurement program at German Bight



Measured spectra appeared to have a sharper peak than the PM spectrum. This is why the PM spectrum was adopted by means of a peak enhancement function.

$$S_\zeta(\omega) = \frac{320 \cdot H_{\frac{1}{3}}{}^2}{T_p{}^4} \cdot \omega^{-5} \cdot e^{\frac{-1950}{T_p{}^4} \cdot \omega^{-4}} \cdot \gamma^A$$

$\gamma^A =$ peak enhancement function

$\gamma = 3.3 =$ peak enhancement factor

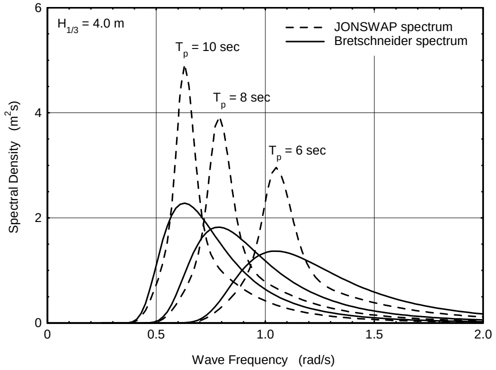
$$\mathrm{A} = \mathrm{e}^{-\left(\frac{\frac{\omega}{\omega_p}-1}{\sigma\sqrt{2}}\right)^2}$$

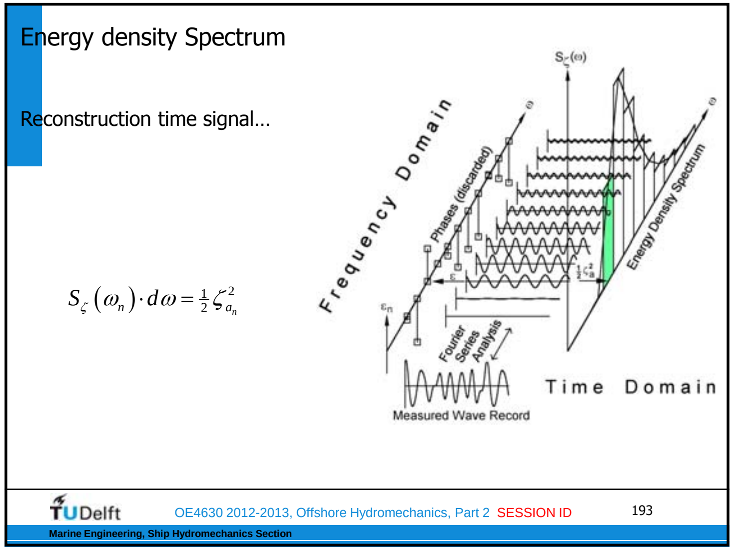
$\sigma =$ step function of ω:

for$\omega < \omega_p \quad \sigma = 0.07$

for$\omega > \omega_p \quad \sigma = 0.09$

Difference between spectra

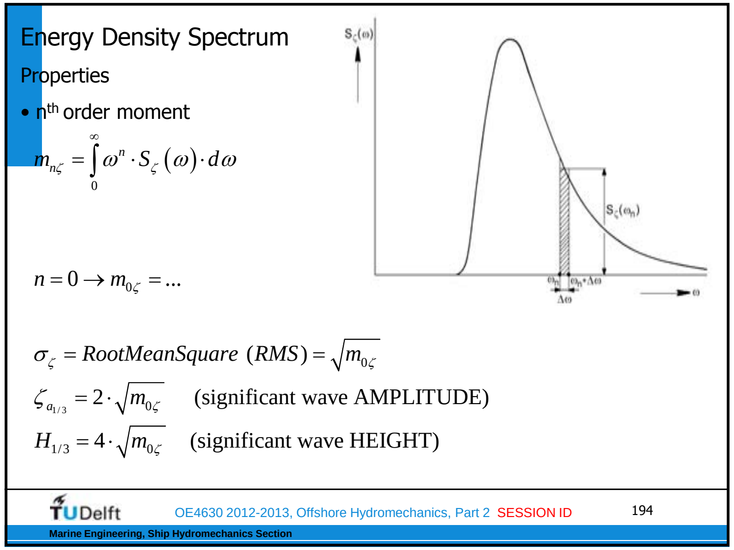




Energy density Spectrum

Reconstruction time signal…

$$S_\zeta(\omega_n)\cdot d\omega = \tfrac{1}{2}\zeta_{a_n}^2$$



Energy Density Spectrum

Properties

- n^{th} order moment

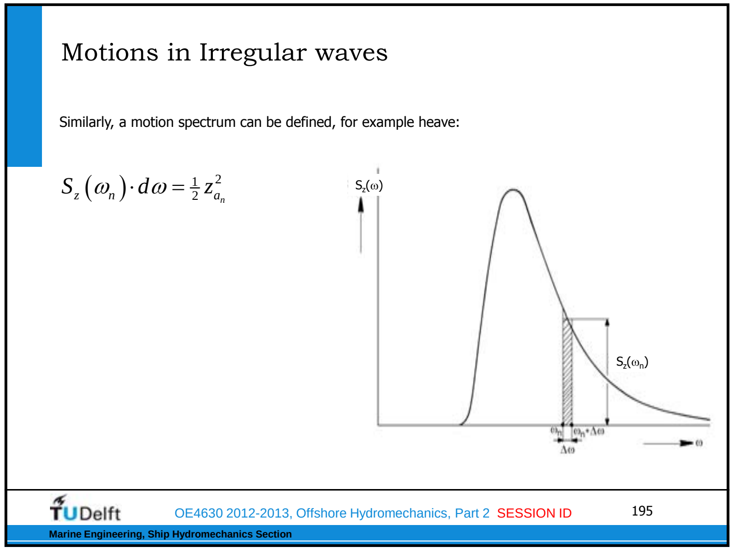
$$m_{n\zeta} = \int_0^\infty \omega^n \cdot S_\zeta(\omega)\cdot d\omega$$

$$n = 0 \rightarrow m_{0\zeta} = ...$$

$$\sigma_\zeta = RootMeanSquare\ (RMS) = \sqrt{m_{0\zeta}}$$

$$\zeta_{a_{1/3}} = 2\cdot\sqrt{m_{0\zeta}} \quad \text{(significant wave AMPLITUDE)}$$

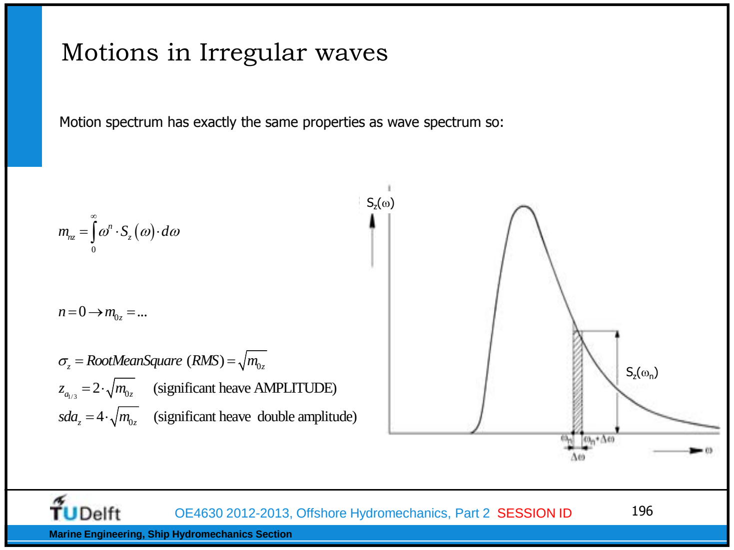
$$H_{1/3} = 4\cdot\sqrt{m_{0\zeta}} \quad \text{(significant wave HEIGHT)}$$



Motions in Irregular waves

Similarly, a motion spectrum can be defined, for example heave:

$$S_z(\omega_n)\cdot d\omega = \tfrac{1}{2} z_{a_n}^2$$



Motions in Irregular waves

Motion spectrum has exactly the same properties as wave spectrum so:

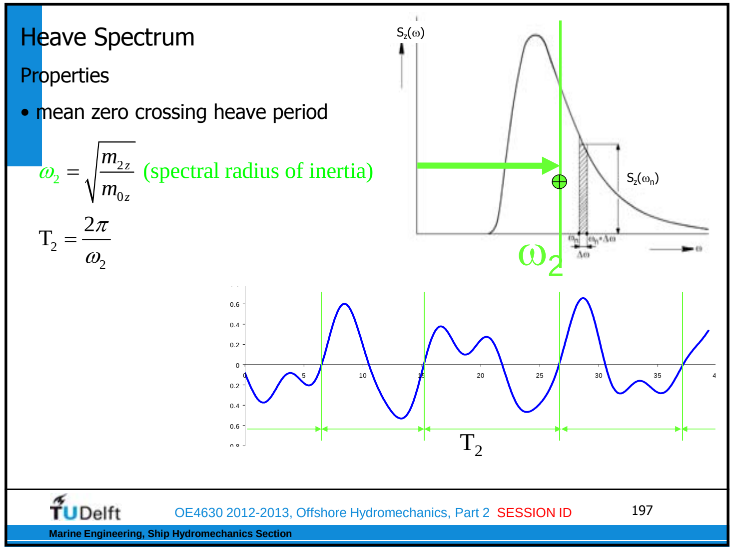
$$m_{nz} = \int_0^\infty \omega^n \cdot S_z(\omega)\cdot d\omega$$

$$n = 0 \rightarrow m_{0z} = ...$$

$$\sigma_z = RootMeanSquare\ (RMS) = \sqrt{m_{0z}}$$

$$z_{a_{1/3}} = 2\cdot\sqrt{m_{0z}} \quad \text{(significant heave AMPLITUDE)}$$

$$sda_z = 4\cdot\sqrt{m_{0z}} \quad \text{(significant heave double amplitude)}$$



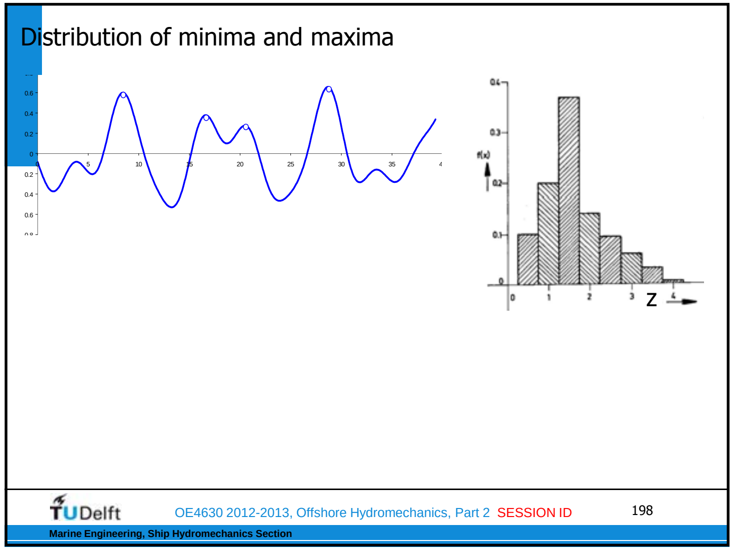
Heave Spectrum

Properties

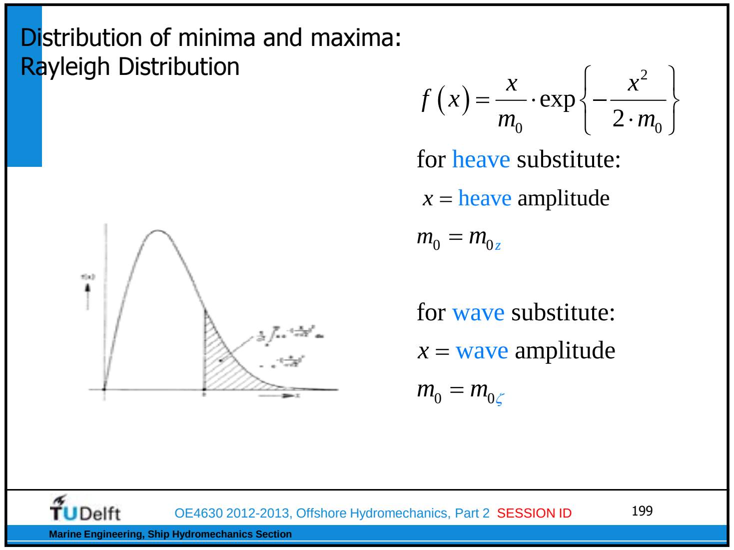
- mean zero crossing heave period

$$\omega_2 = \sqrt{\frac{m_{2z}}{m_{0z}}}$$ (spectral radius of inertia)

$$T_2 = \frac{2\pi}{\omega_2}$$



Distribution of minima and maxima



Distribution of minima and maxima: Rayleigh Distribution

$$f(x) = \frac{x}{m_0} \cdot \exp\left\{-\frac{x^2}{2 \cdot m_0}\right\}$$

for heave substitute:

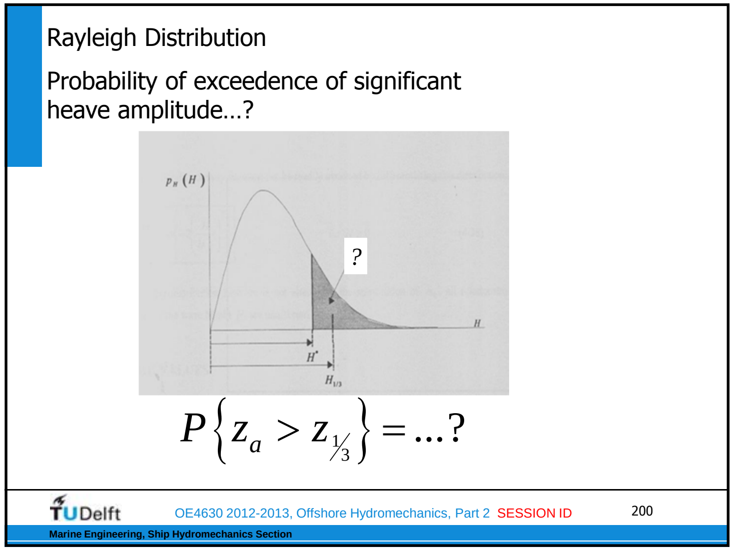
$x =$ heave amplitude

$m_0 = m_{0z}$

for wave substitute:

$x =$ wave amplitude

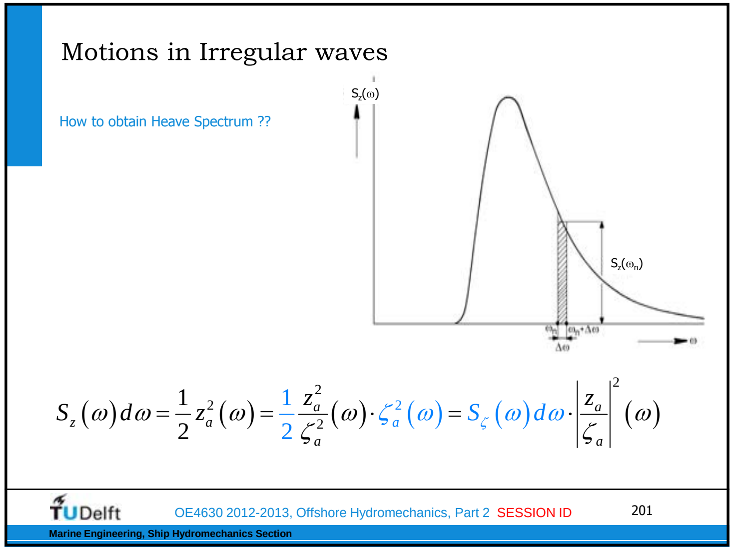
$m_0 = m_{0\zeta}$



Rayleigh Distribution

Probability of exceedence of significant heave amplitude…?

$$P\left\{z_a > z_{\frac{1}{3}}\right\} = \ldots ?$$



Motions in Irregular waves

How to obtain Heave Spectrum ??

$$S_z(\omega)d\omega = \frac{1}{2}z_a^2(\omega) = \frac{1}{2}\frac{z_a^2}{\zeta_a^2}(\omega)\cdot\zeta_a^2(\omega) = S_\zeta(\omega)d\omega\cdot\left|\frac{z_a}{\zeta_a}\right|^2(\omega)$$

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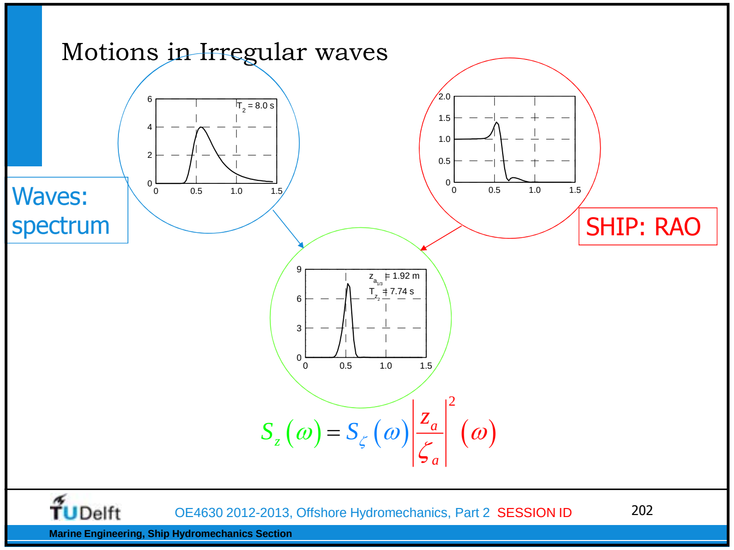
Marine Engineering, Ship Hydromechanics Section



Motions in Irregular waves

Waves: spectrum

SHIP: RAO

$$S_z(\omega) = S_\zeta(\omega)\left|\frac{z_a}{\zeta_a}\right|^2(\omega)$$

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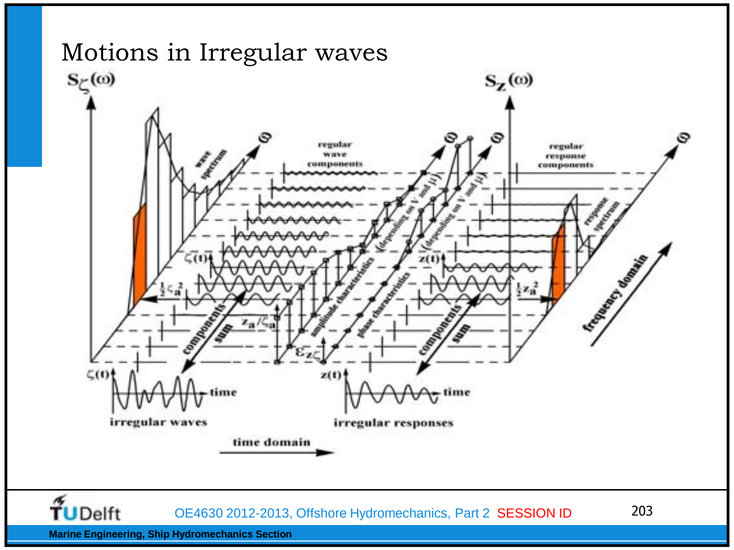

Motions in Irregular waves

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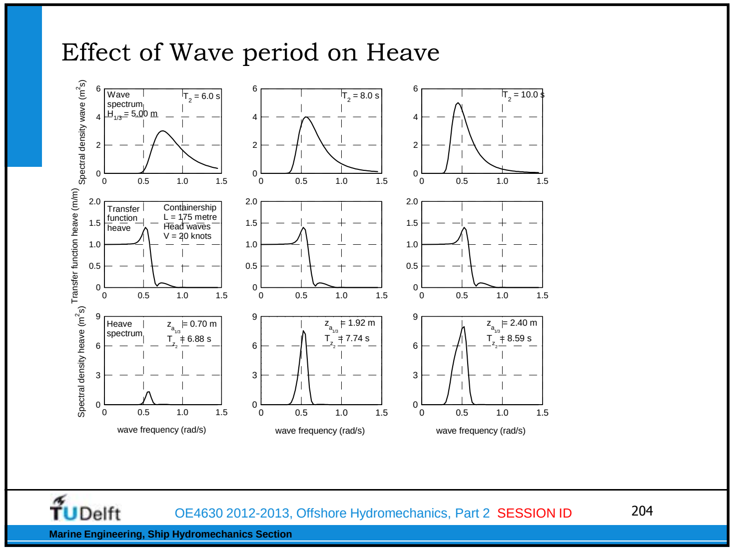

Effect of Wave period on Heave

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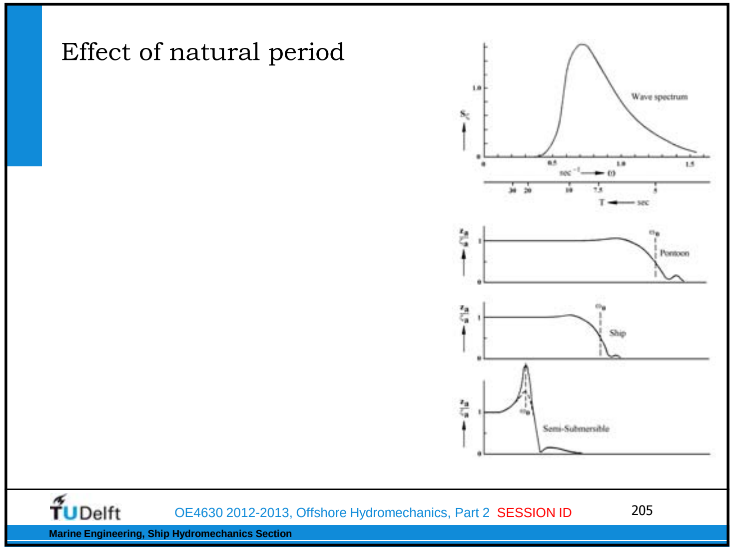
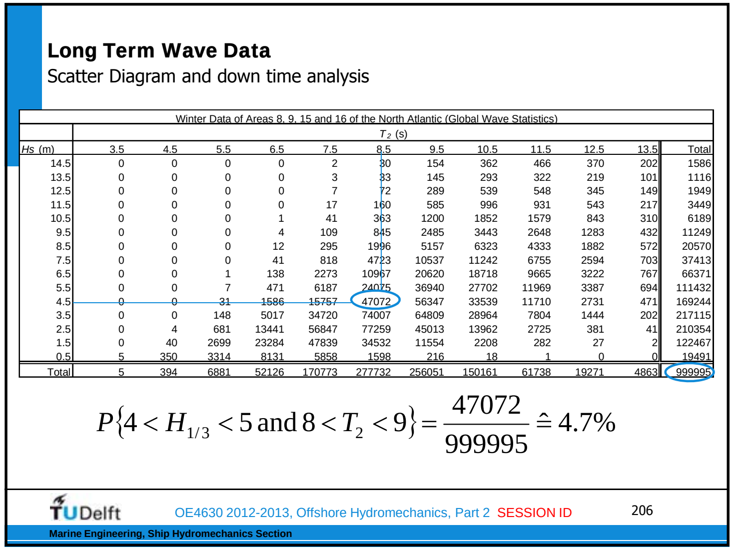


Effect of natural period



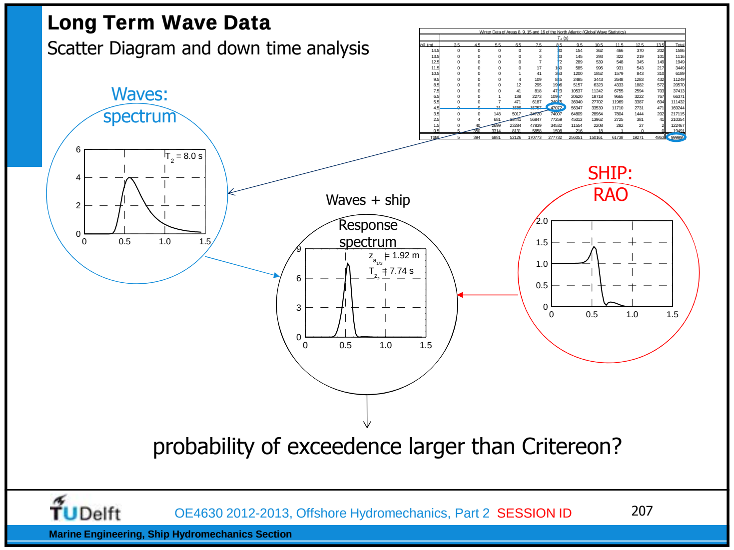
Long Term Wave Data

Scatter Diagram and down time analysis

Winter Data of Areas 8, 9, 15 and 16 of the North Atlantic (Global Wave Statistics)

	T_2 (s)											
Hs (m)	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	Total
14.5	0	0	0	0	2	30	154	362	466	370	202	1586
13.5	0	0	0	0	3	33	145	293	322	219	101	1116
12.5	0	0	0	0	7	72	289	539	548	345	149	1949
11.5	0	0	0	0	17	160	585	996	931	543	217	3449
10.5	0	0	0	1	41	363	1200	1852	1579	843	310	6189
9.5	0	0	0	4	109	845	2485	3443	2648	1283	432	11249
8.5	0	0	0	12	295	1996	5157	6323	4333	1882	572	20570
7.5	0	0	0	41	818	4723	10537	11242	6755	2594	703	37413
6.5	0	0	1	138	2273	10967	20620	18718	9665	3222	767	66371
5.5	0	0	7	471	6187	24075	36940	27702	11969	3387	694	111432
4.5	0	0	31	1586	15757	47072	56347	33539	11710	2731	471	169244
3.5	0	0	148	5017	34720	74007	64809	28964	7804	1444	202	217115
2.5	0	4	681	13441	56847	77259	45013	13962	2725	381	41	210354
1.5	0	40	2699	23284	47839	34532	11554	2208	282	27	2	122467
0.5	5	350	3314	8131	5858	1598	216	18	1	0	0	19491
Total	5	394	6881	52126	170773	277732	256051	150161	61738	19271	4863	999995

$$P\{4 < H_{1/3} < 5 \text{ and } 8 < T_2 < 9\} = \frac{47072}{999995} \mathrel{\hat{=}} 4.7\%$$



Long Term Wave Data

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Waves: spectrum

Waves + ship

Response spectrum

SHIP: RAO

probability of exceedence larger than Critereon?

Sources images

[1] Towage of SSDR Transocean Amirante, source: Transocean
[2] Tower Mooring, source: unknown
[3] Rogue waves, source: unknown
[4] Bluewater Rig No. 1, source: Friede & Goldman, LTD/GNU General Public License
[5] Source: unknown
[6] Rig Neptune, source: Seafarer Media
[7] Pieter Schelte vessel, source: Excalibur
[8] FPSO design basis, source: Statoil
[9] Floating wind turbines, source: Principle Power Inc.
[10] Ocean Thermal Energy Conversion (OTEC), source: Institute of Ocean Energy/Saga University
[11] ABB generator, source: ABB
[12] A Pelamis installed at the Agucadoura Wave Park off Portugal, source: S.Portland/Wikipedia
[13] Schematic of Curlew Field, United Kingdom, source: offshore-technology.com
[14] Ocean Quest Brave Sea, source: Zamakona Yards
[15] Medusa, A Floating SPAR Production Platform, source: Murphy USA