Traffic Flow Theory & Simulation

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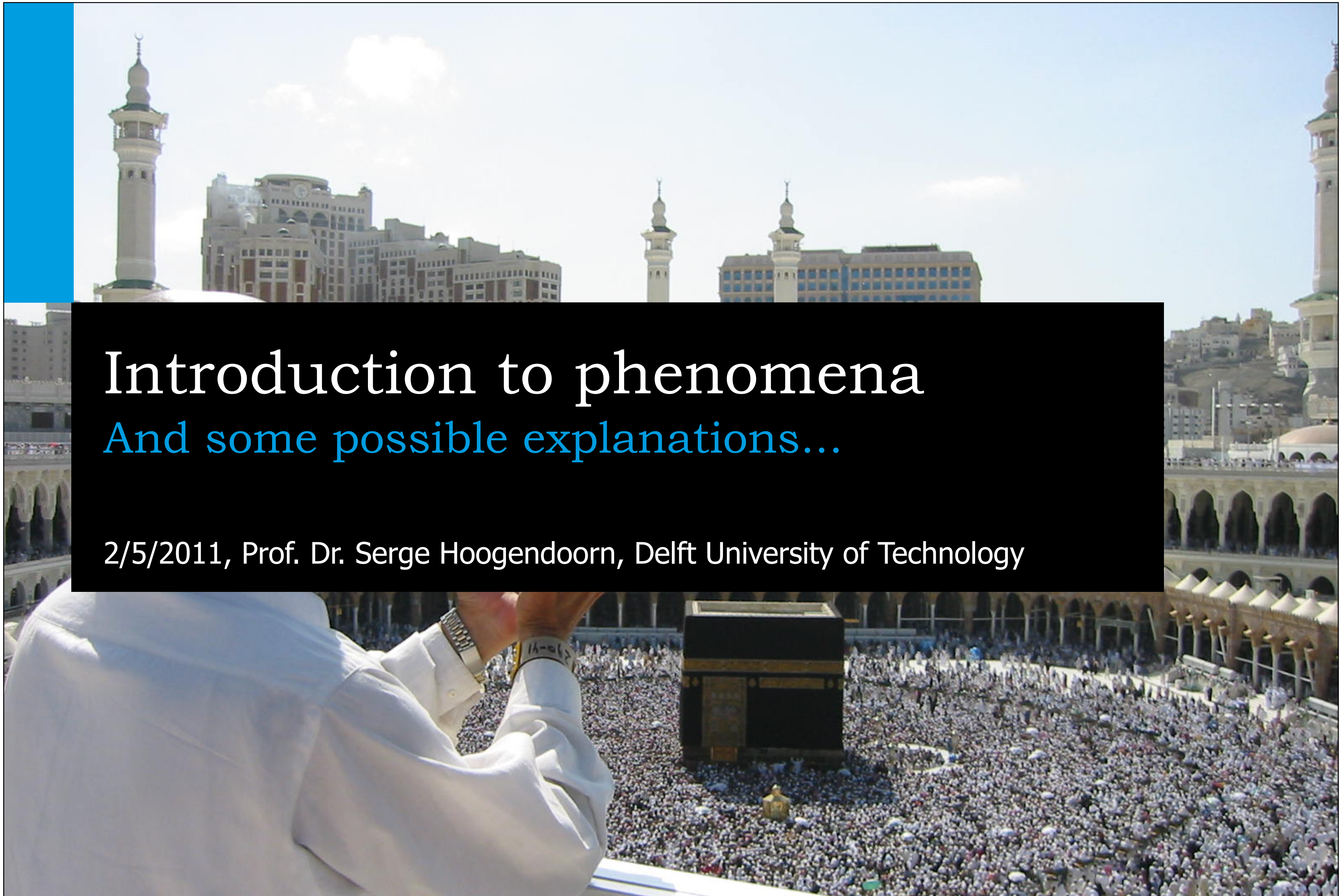
Lecture 7
Introduction to Phenomena



Introduction to phenomena

And some possible explanations…

2/5/2011, Prof. Dr. Serge Hoogendoorn, Delft University of Technology

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The story so far...

- Theory and models to explain traffic flow operations, specifically queuing phenomena, using:
  - Queuing Theory
  - Shockwave theory
  - **Kinematic Wave theory**
- Kinematic Wave theory uses:
  - Conservation of vehicle equation
  - Assumption that traffic behaves according to fundamental diagram

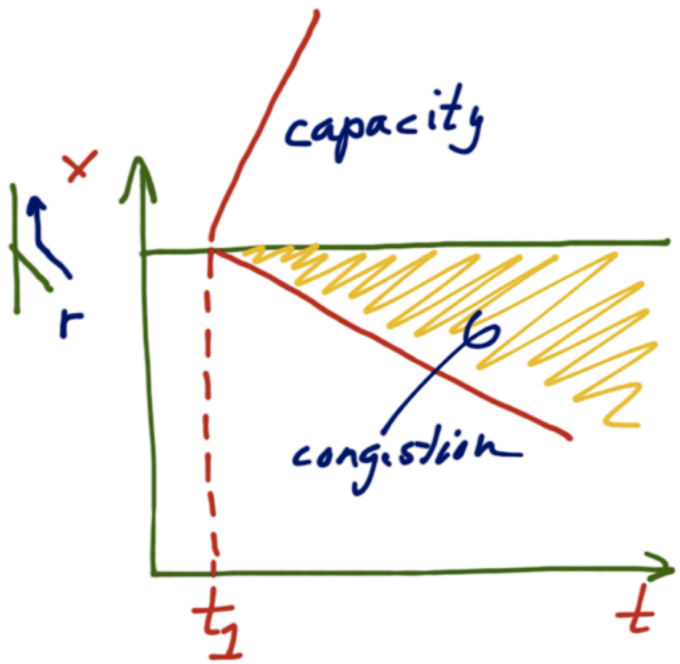
$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = r - s \quad \text{and} \quad q = Q(k)$$

Example

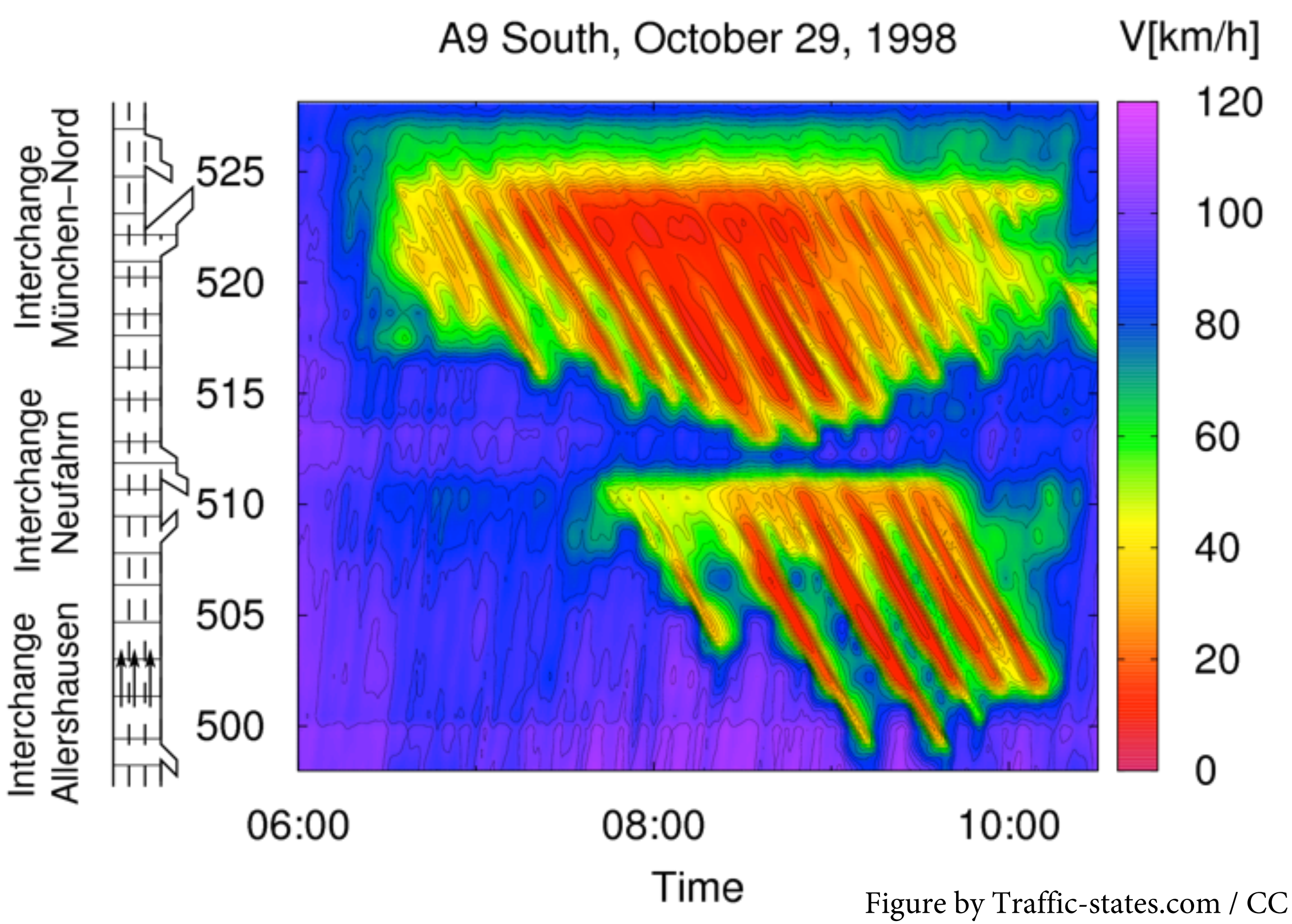
- Consider an over-saturated on-ramp with on-ramp flow r
- Over-saturation starts at t1

- Which additional assumption do we need to predict what will happen using KWT?
- What will happen according to KWT? I.e. which traffic state occurs upstream of on-ramp

- Is this what will happen in reality?



Example A5



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Are these findings in line with KWT?

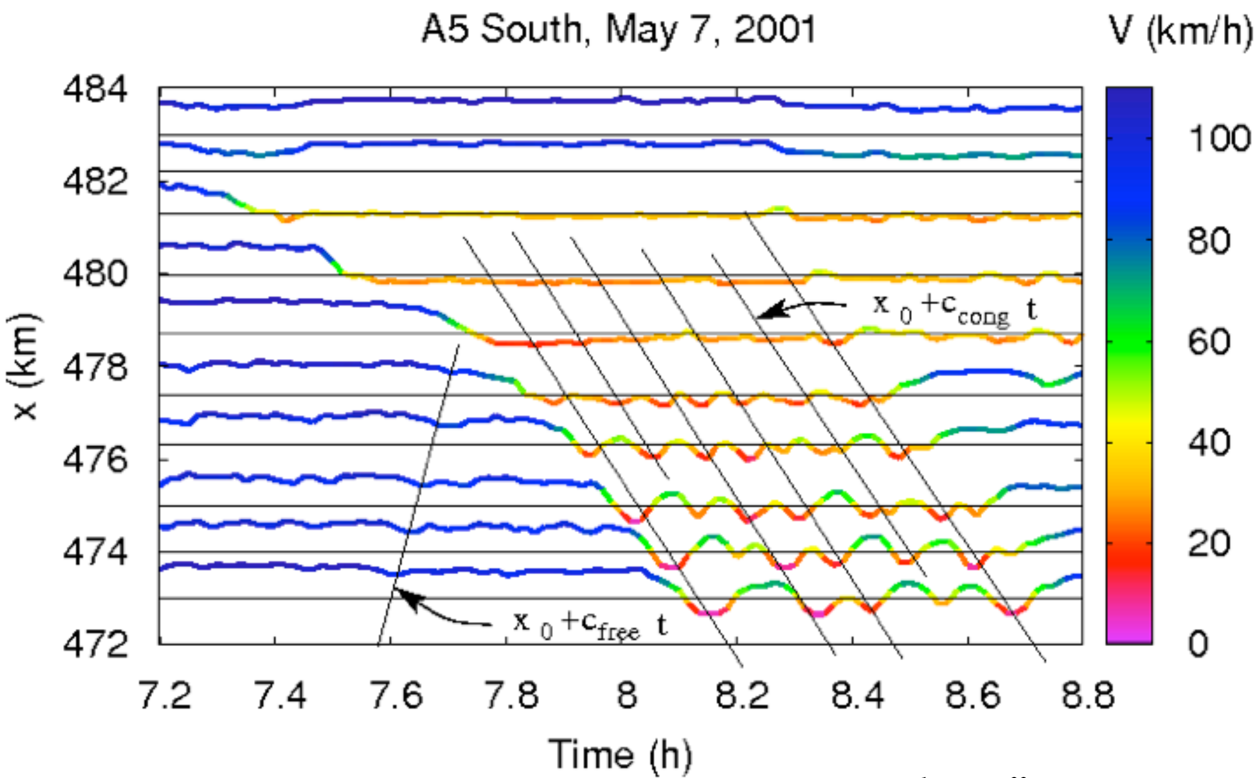
- What do we know about the movement of (small) disturbances in the flow?

- What does KWT predict:
  - In terms of their speed?
  - In terms of their amplitude?

- What about traffic data?

Occurrence of moving jams

Growing amplitude of perturbations… $A(t) = A_0 \cdot e^{\sigma(t-t_0)}$



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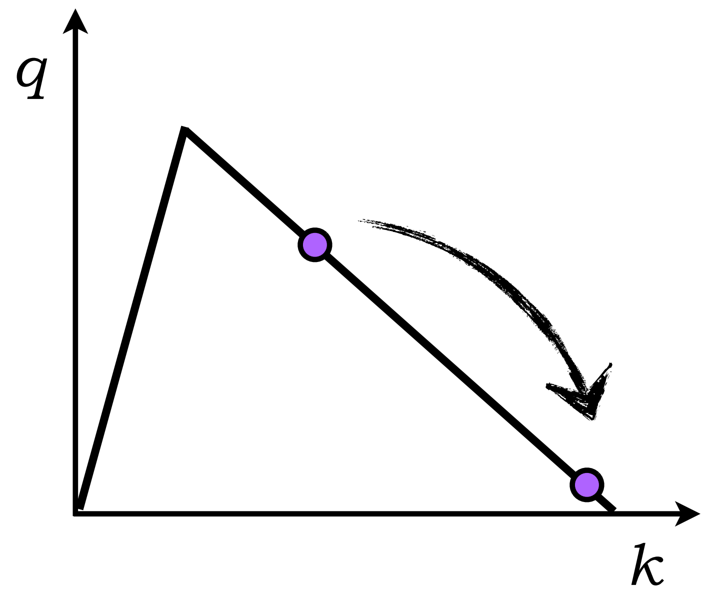
Observations

- In certain high density regions, traffic is unstable
- Small disturbances grow quickly and become wide moving jams

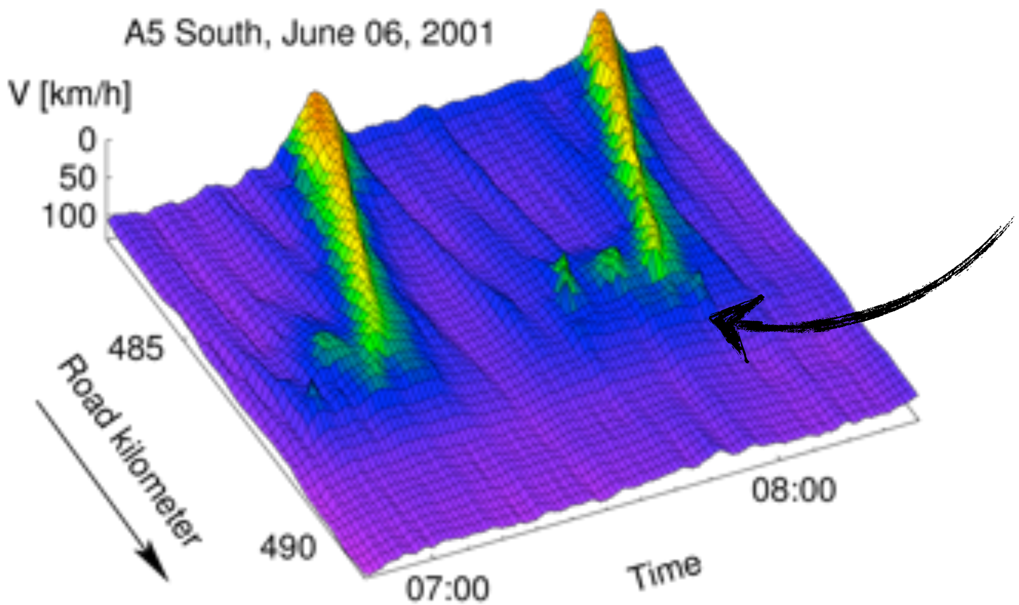
- **Phase transition** from a congested state to a wide moving jam

- Other phase transitions observed in reality?



Isolated moving jams

- Free flow to wide-moving jams (isolated moving jams)



Critical flow conditions: *high flows, high speeds, density near critical density*

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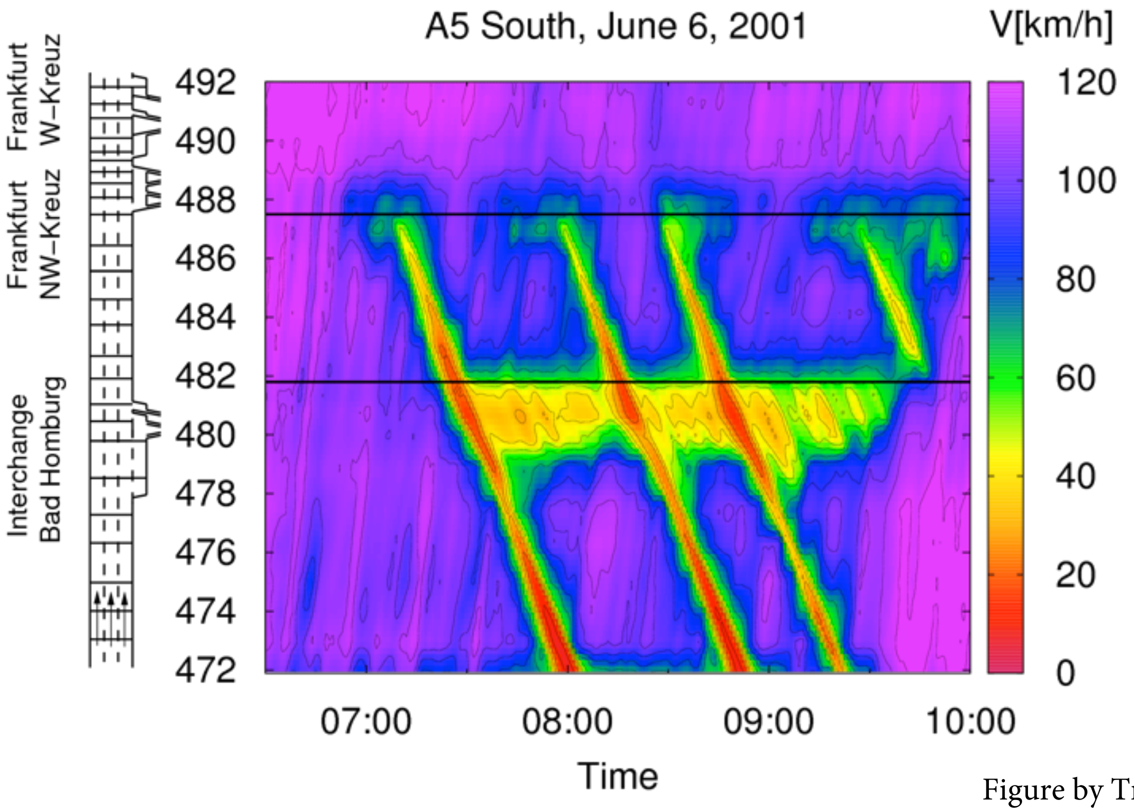
Triggered standing queues

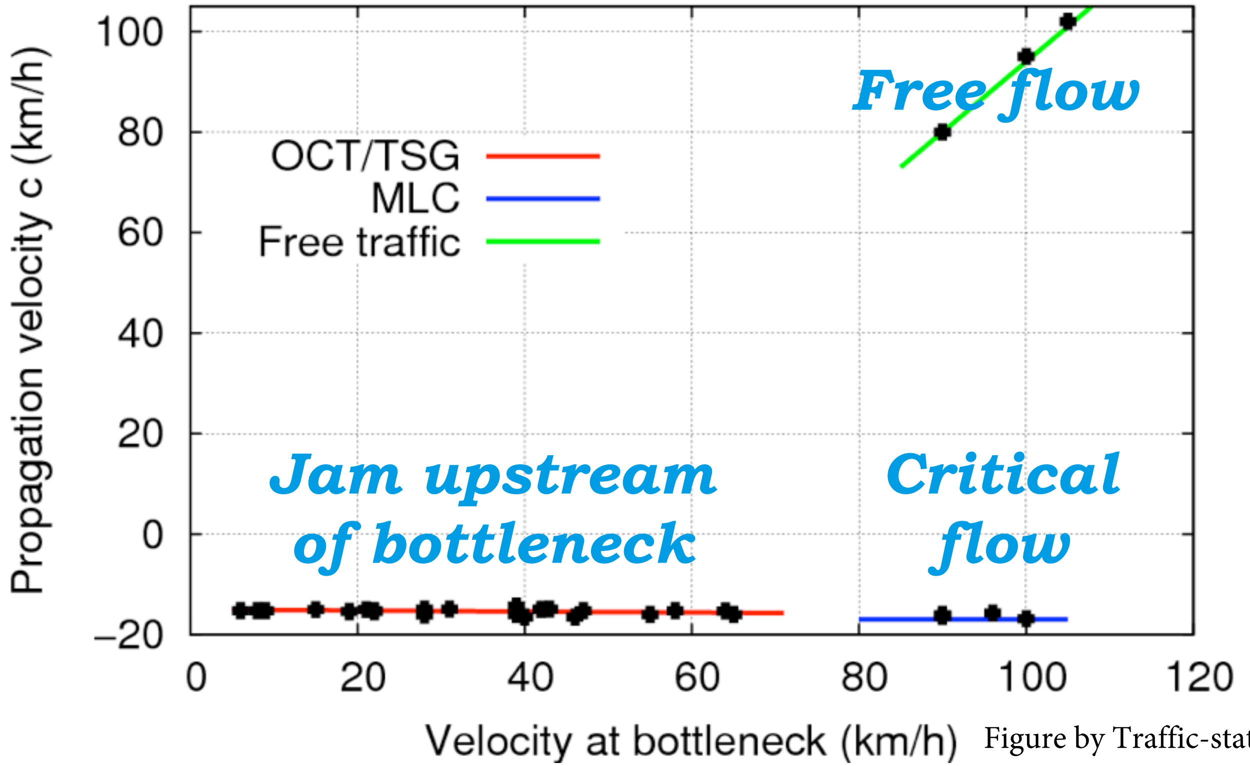


***Critical flow conditions**:*
high flows,
high speeds,
density near critical density

How can you explain this 'triggered' congestion?

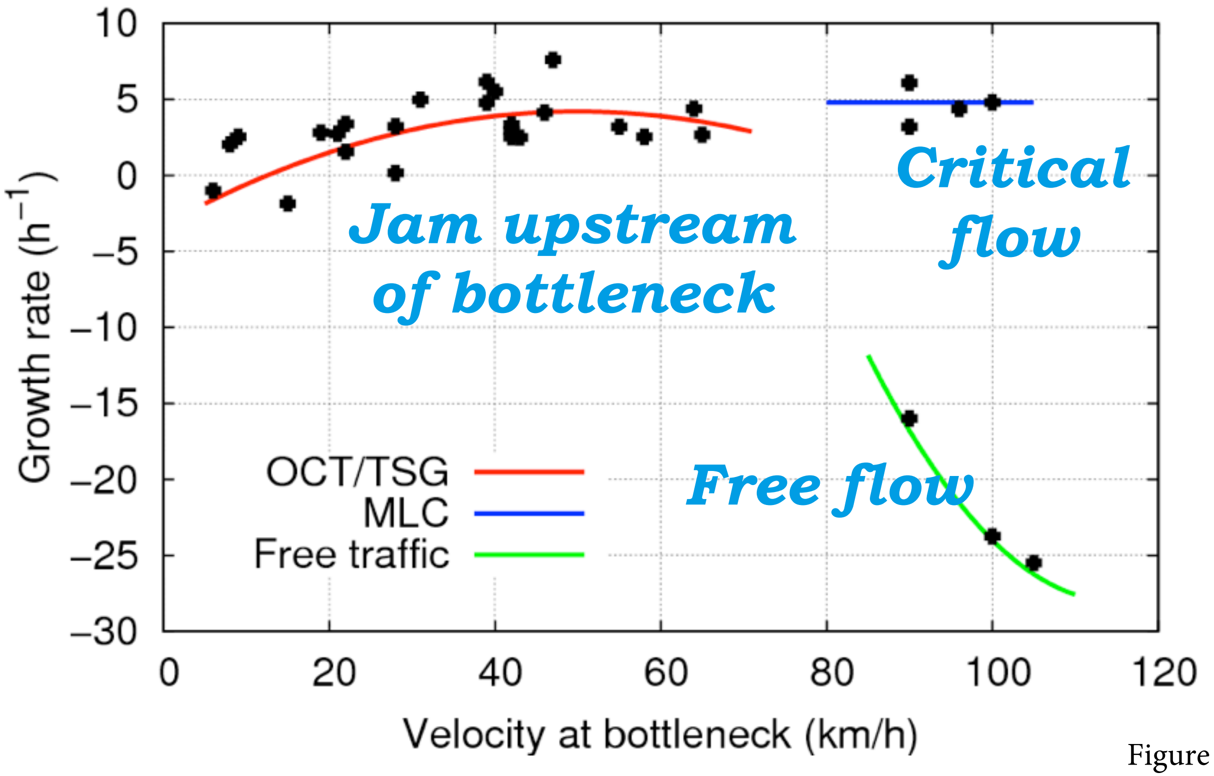
Some empirical features

Perturbation speeds for different regimes…



Quantifying stability

Figure shows growth rate σ for different regimes...



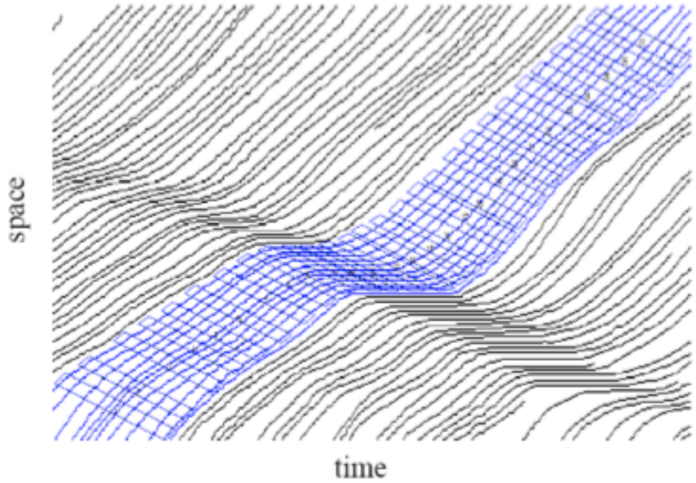
Recall:

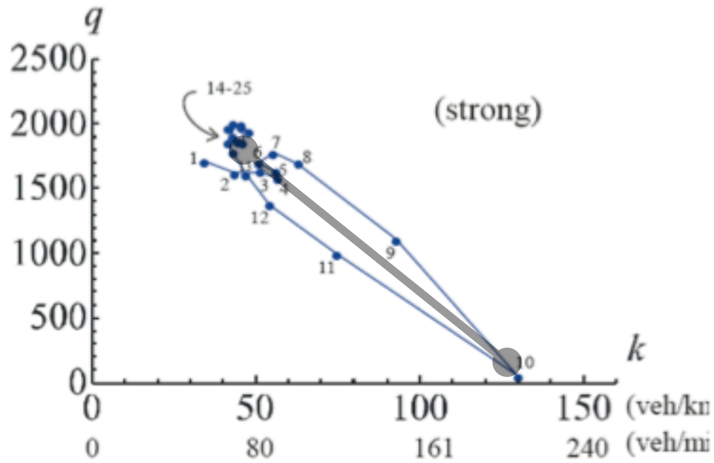
$$A(t) = A_0 \cdot e^{\sigma(t-t_0)}$$

What does the sign of the growth rate imply?

What about the states?

- Transient (intermediate) states are generally not on the FD
- Consider traffic state dynamics of area indicated by trapezoid*
- When driving into congestion, points are 'above FD', when driving out of congestion, points are 'below FD' (= hysteresis)

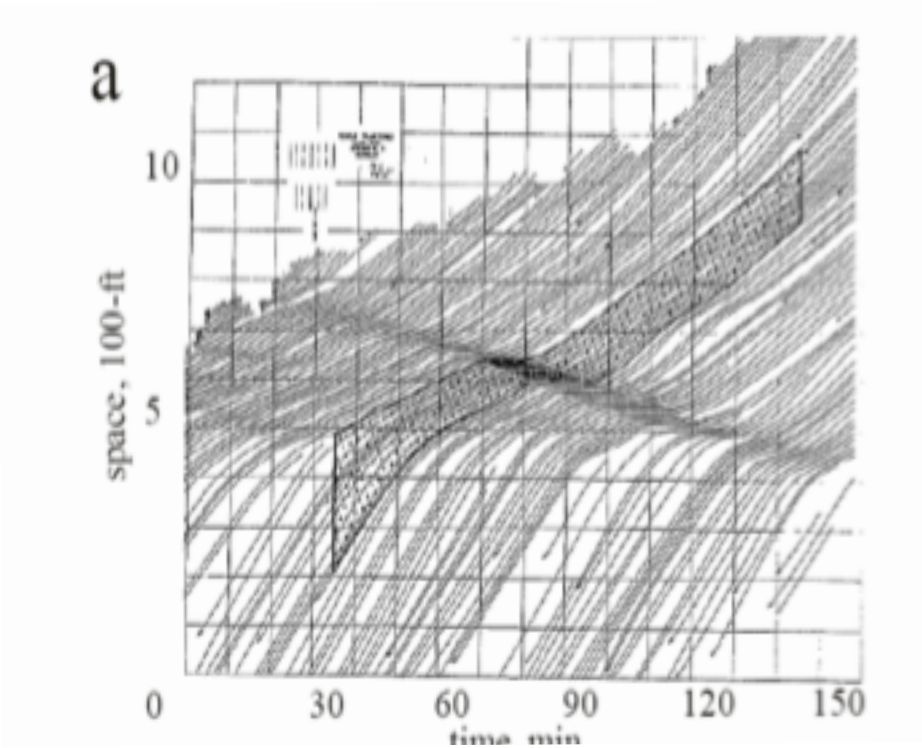




Understanding hysteresis

- Original study of Treiterer and Myers (1974) showed quite different results
- In this study, a rectangular grid to determine the traffic states from the trajectories

- Think of what the impact will be of this by drawing trajectories of vehicles going to a wide moving jam

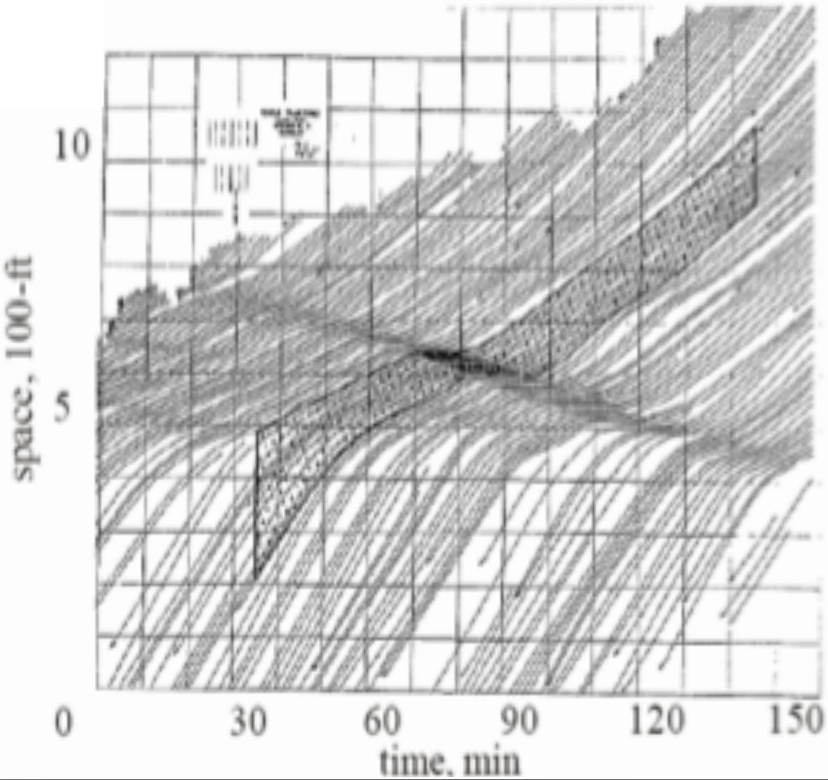


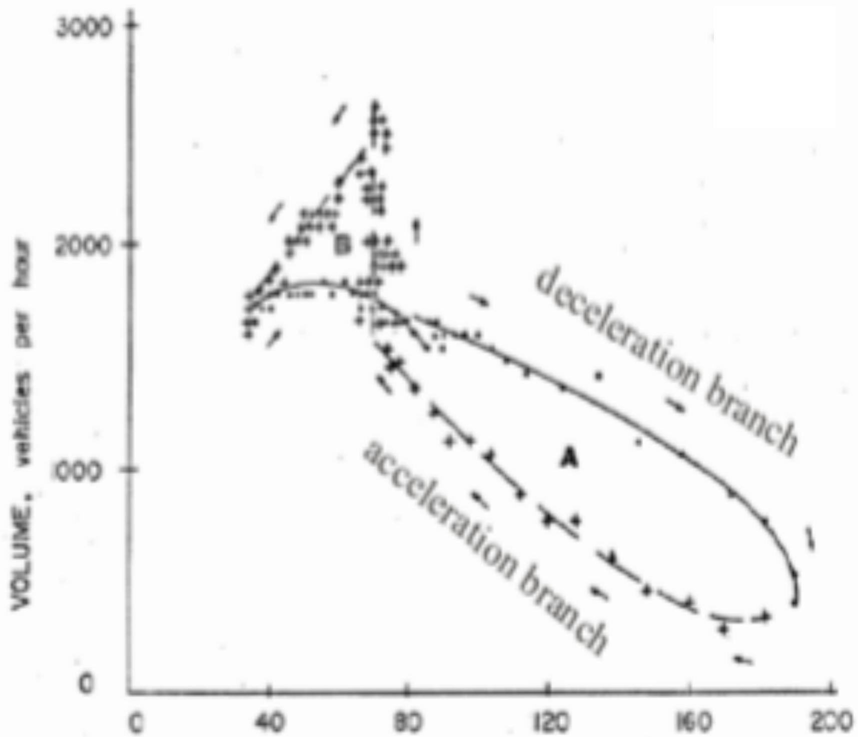
Methodology: Analyze Trajectories

- <u>Classic Method</u>: track platoon

(Treiterer and Myers, 1974)

Result:
strong hysteresis





Methodology: Analyze Trajectories

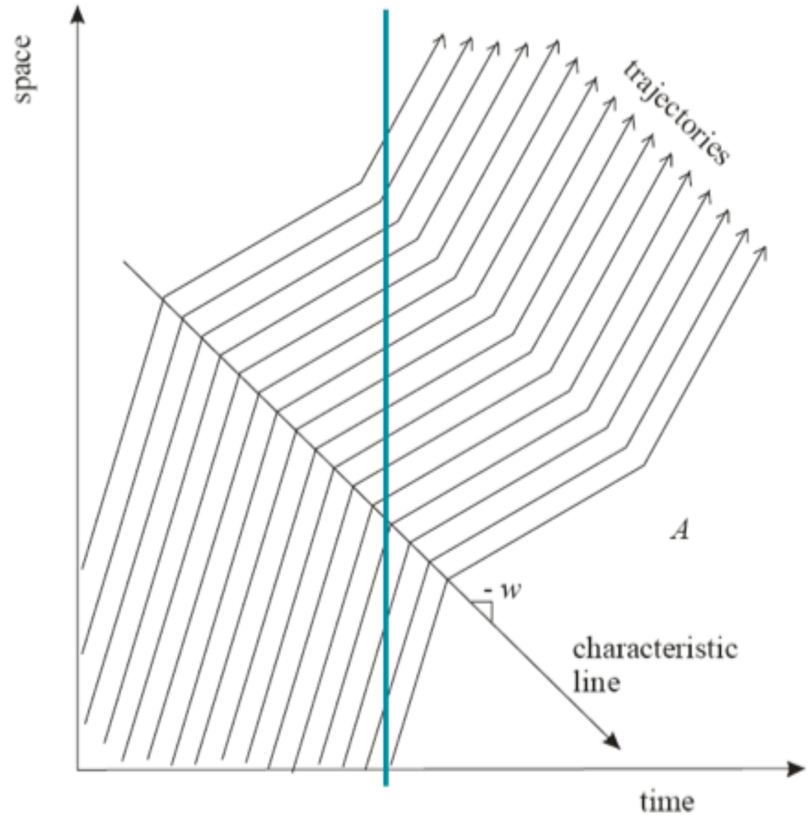
- Classic Method: track conditions in the platoon at different time instants

Result:
strong hysteresis

Problem:

- averaging over different traffic states ☹
  (like loop detectors)

  How can we improve this?



Methodology: Analyze Trajectories

- New Method: follow waves + Edie

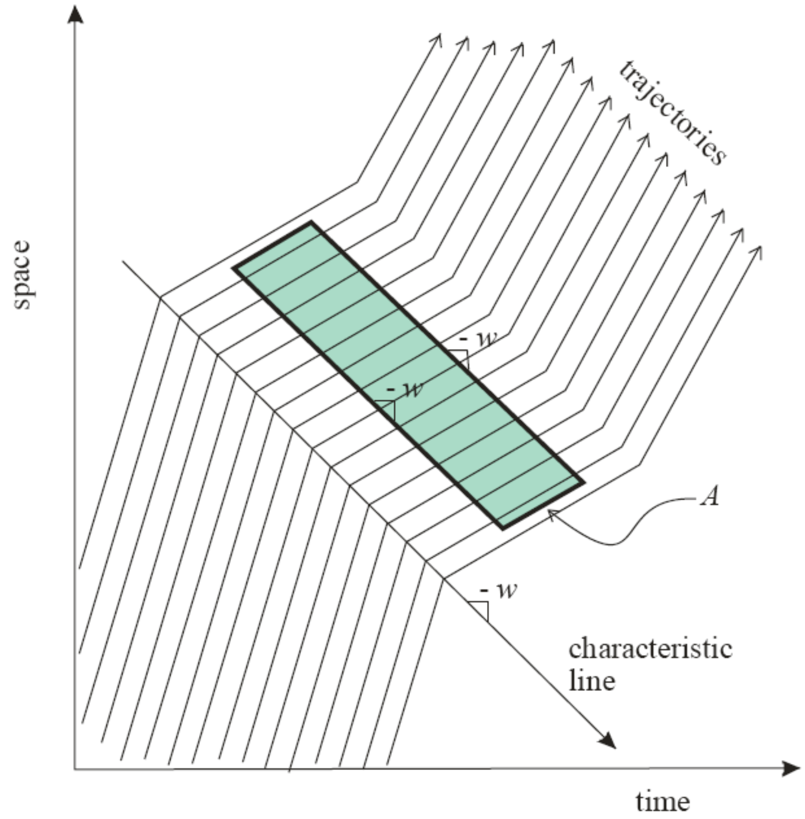
$$k = \sum_{i=1}^{n} t_i / |A|,$$

$$q = \sum_{i=1}^{n} x_i / |A|,$$

$$v = q/k = \sum_{i=1}^{n} t_i / \sum_{i=1}^{n} x_i,$$

area A dependent on trajectories

➔ Averaging over stationary traffic state ☺



In sum…

- We observe various **unstable** (or 'meta-stable') **traffic states** in
  - critical (free-flow) conditions and in
  - congested (synchronized) flow (standing queues, queues due to MB)
- Perturbations (small / considerable) trigger a so-called **phase-transition** (critical flow to wide-moving jams or congested flow to wide-moving jams)
- Moving jams can trigger (standing) queues due to **capacity drop**
- **Hysteresis phenomena** (transient states not on the FD)

- For today, two main questions:
  - Can we **explain** these phenomena?
  - Can we **model** these phenomena?

Hysteresis

Simple mathematical model…

Mathematical preliminaries

Taylor series expansion

- Model derivation approach based on simple 'microscopic' model
- Use of Taylor series expansion and chain rule

- Taylor series:

$$f(x+\delta)=f(x)+\delta\frac{d}{dx}f(x)+\frac{1}{2}\delta^2\frac{d^2}{dx^2}f(x)+\ldots$$

$$f(x+\delta)=f(x)+\delta\frac{d}{dx}f(x)+O(\delta^2)$$

- For multi-dimensional functions, we have:

$$f(x+\delta_x,y+\delta_y)\approx f(x)+\delta_x\frac{\partial}{\partial x}f(x,y)+\delta_y\frac{\partial}{\partial y}f(x,y)$$

Quasi-microscopic model I

- Basic driving rule: drivers adapt their speed u based on the local spacing s = 1/k, as reflected by the function U

$$u(t,x) = U(1/s(t,x)) = U(k(t,x))$$

- How can we improve this model?

- Assume that drivers ‘anticipate’ on downstream conditions with a certain anticipation distance:

$$u(t,x) = U(k(t,x+\Delta))$$

- Consider resulting model derivation...

Quasi-microscopic model I

- Resulting model:

$$u(t,x) \approx U(k(t,x)) + \Delta \cdot \frac{\partial k}{\partial x} \cdot \frac{d}{dk} U(k(t,x))$$

- Interpretation?

- What happens when a driver moves into a congestion region?
- Which (transient) states would you observe in the phase plane?

- Model describes so-called **anticipation dominated** driving behavior

Quasi microscopic model II

- Assumption: drivers have a delayed reaction to changing driving conditions:

$$u(t+\tau, x(t+\tau)) = U(k(t,x))$$

- Apply Taylors rule to derive:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{U(k)-u}{\tau}$$

- Interpretation?
- What happens when a driver moves into a congestion region?
- Which (transient) states would you observe in the phase plane?
- Model describes **reaction dominated** driving behavior

Combining the models...

- Combining models I (anticipation) and II (reaction) yields:

$$u(t+\tau, x(t+\tau)) = U(k(t, x+\Delta))$$

- What happens when a driver moves into a congestion region?
  What happens when a driver moves out of a congested region?
- Which (transient) states would you observe in the phase plane?

Payne model

- Note that for the combined model, we can derive the PDE

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + c(k)\frac{\partial k}{\partial x} = \frac{U(k) - u}{\tau} \quad \text{with} \quad c(k) = -\frac{\Delta}{\tau}\frac{dU}{dk} > 0$$

- Interpretation of the terms?
- Together with the conservation of vehicle equation

$$\frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} = 0$$

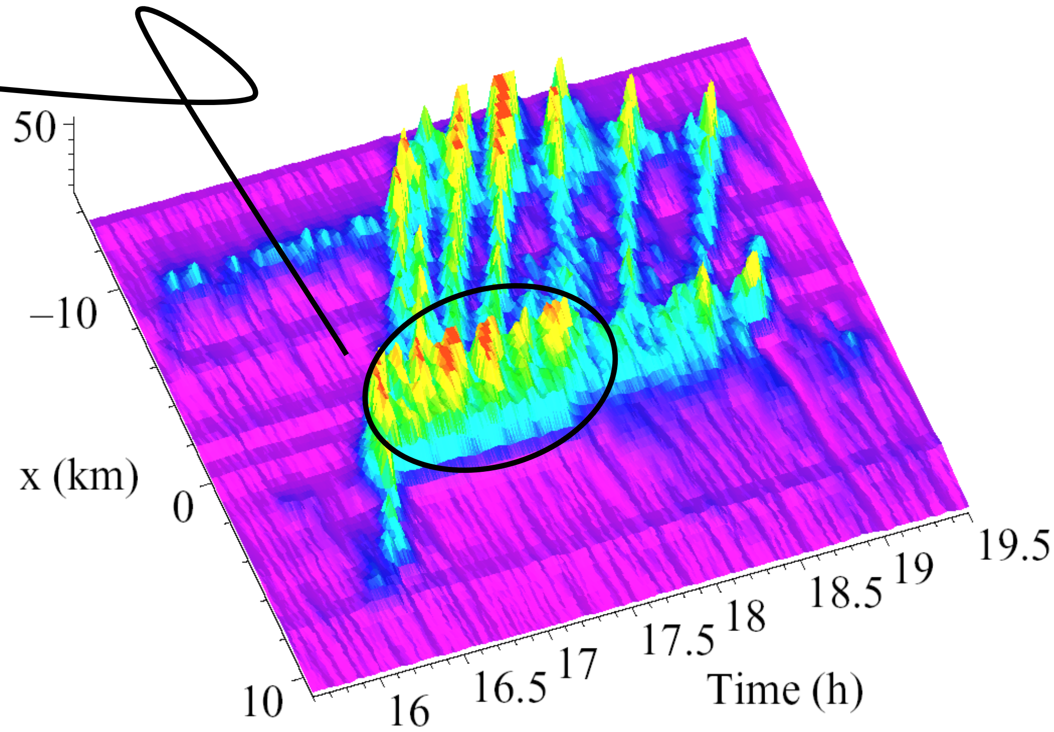
we get the so-called Payne model (Payne,1979)

Kerner's Theory

Three phase theory

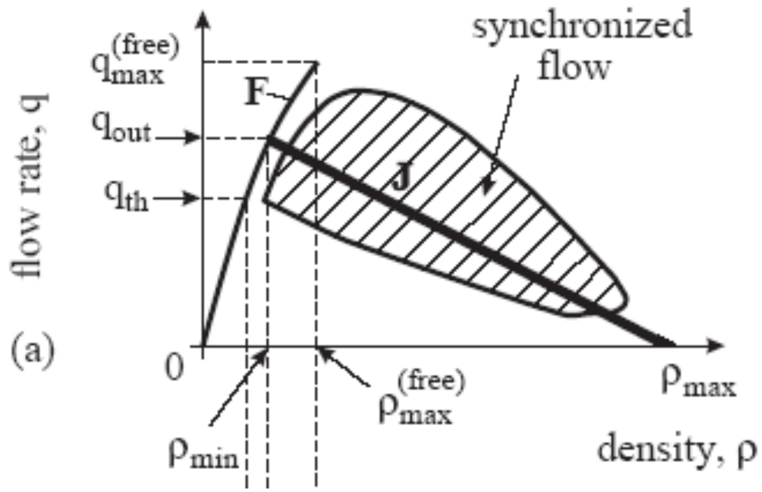
Spontaneous phase transitions

- Consider conditions upstream of active bottleneck
- KWT does not predict phase transitions
- Theory of Kerner qualitatively describes phase transitions in unstable and meta-stable traffic flow operations



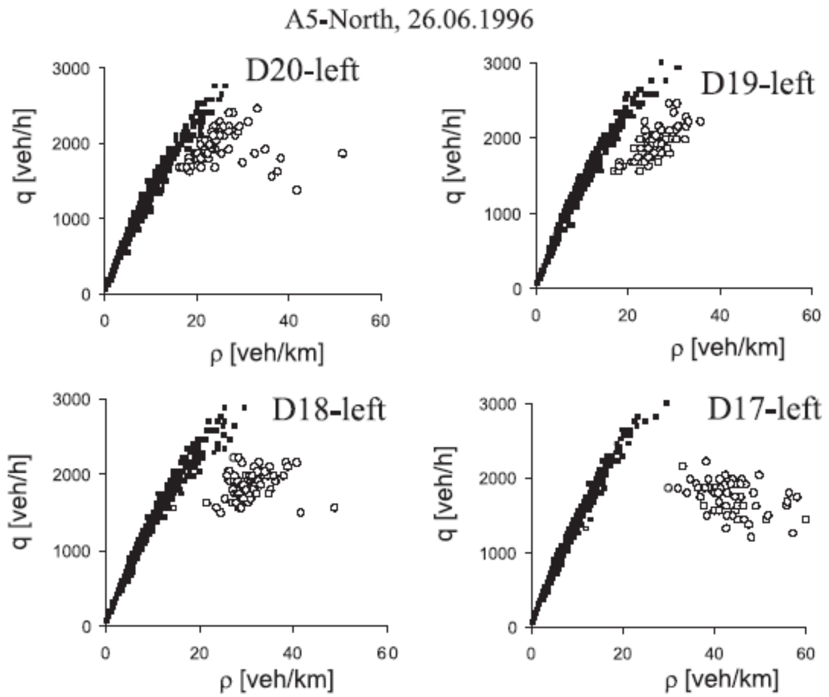
Traffic theory of Kerner

- Three phase (state) theory of traffic flow:
  - Free flow (the F line)
  - Synchronized flow (density > critical density, but less than jam density); (shaded area)
  - Wide moving jams (density = jam density) (the J-line)

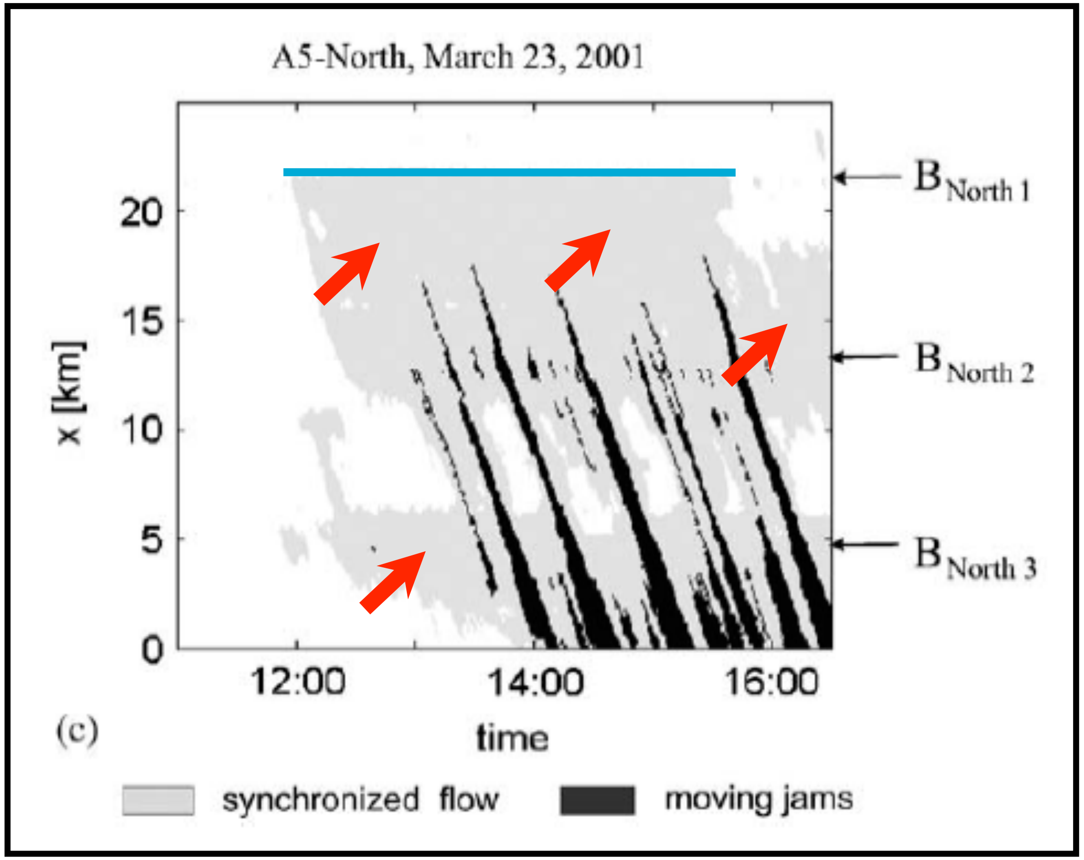


Traffic theory of Kerner

- Synchronized flow
  - Occurs at bottlenecks (comparable to regular queues)
  - Head of the queue is generally stationary
  - Congested traffic state
  - Multiple stationary states in congested branch, which is an area rather than a line
  - Little lane changing, speed of lanes are nearly equal

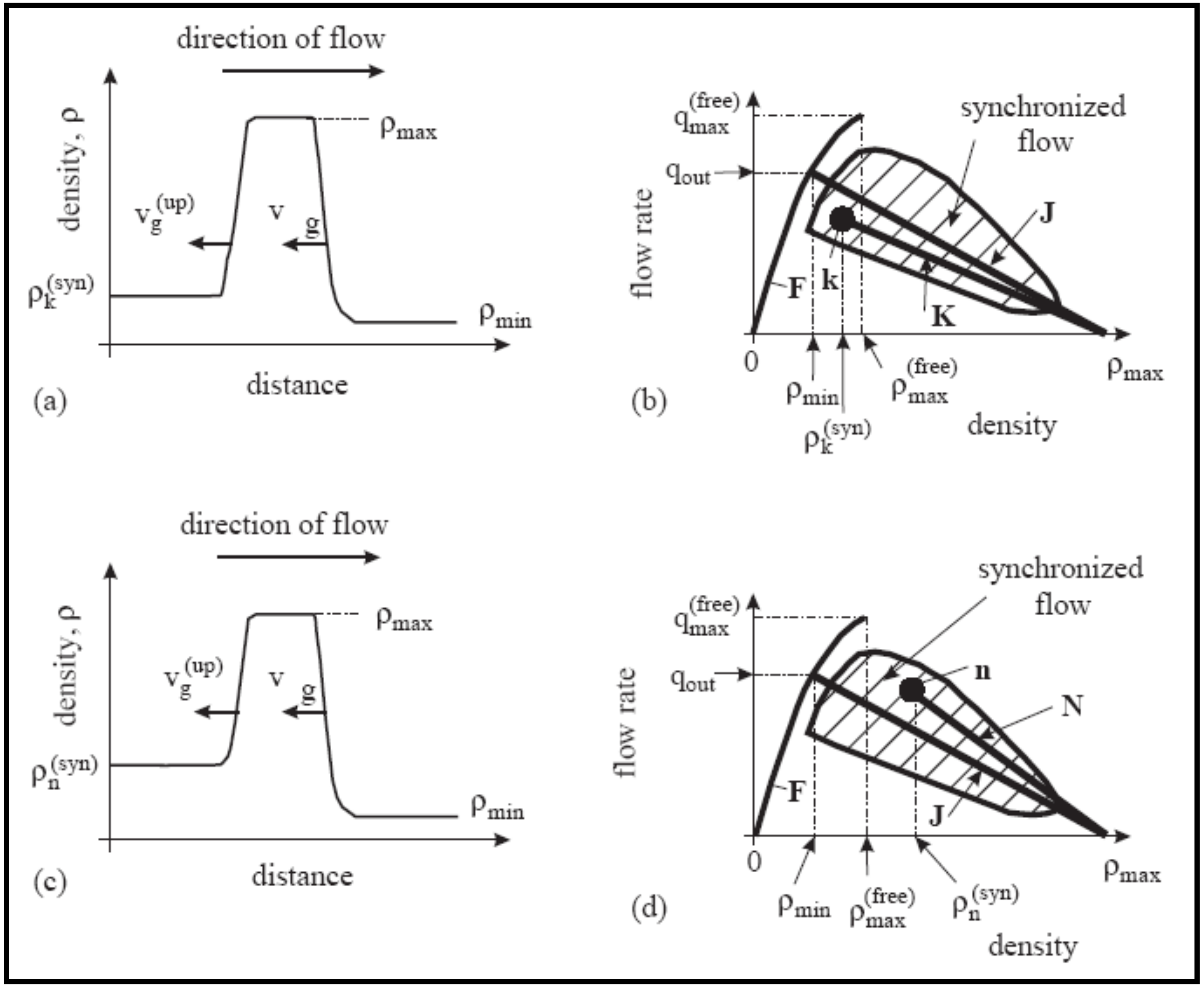


Traffic theory of Kerner



Traffic theory of Kerner

- Dynamic properties of ‘wide moving jam’
  - Density in wide moving jam equals the jam density, vehicles inside the queue are standing still
  - Density upstream equals critical density ρmin
  - Head of queue is moving at a constant speed
  - Wide moving jam can move through other disturbances


direction of flow
density, ρ
ρmax
vg(up)
vg
ρk(syn)
ρmin
distance
(a)
flow rate
qmax(free)
qout
synchronized flow
J
k
F
K
0
ρmin
ρmax(free)
ρk(syn)
ρmax
density
(b)
direction of flow
density, ρ
ρmax
vg(up)
vg
ρn(syn)
ρmin
distance
(c)
flow rate
qmax(free)
qout
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ρmax(free)
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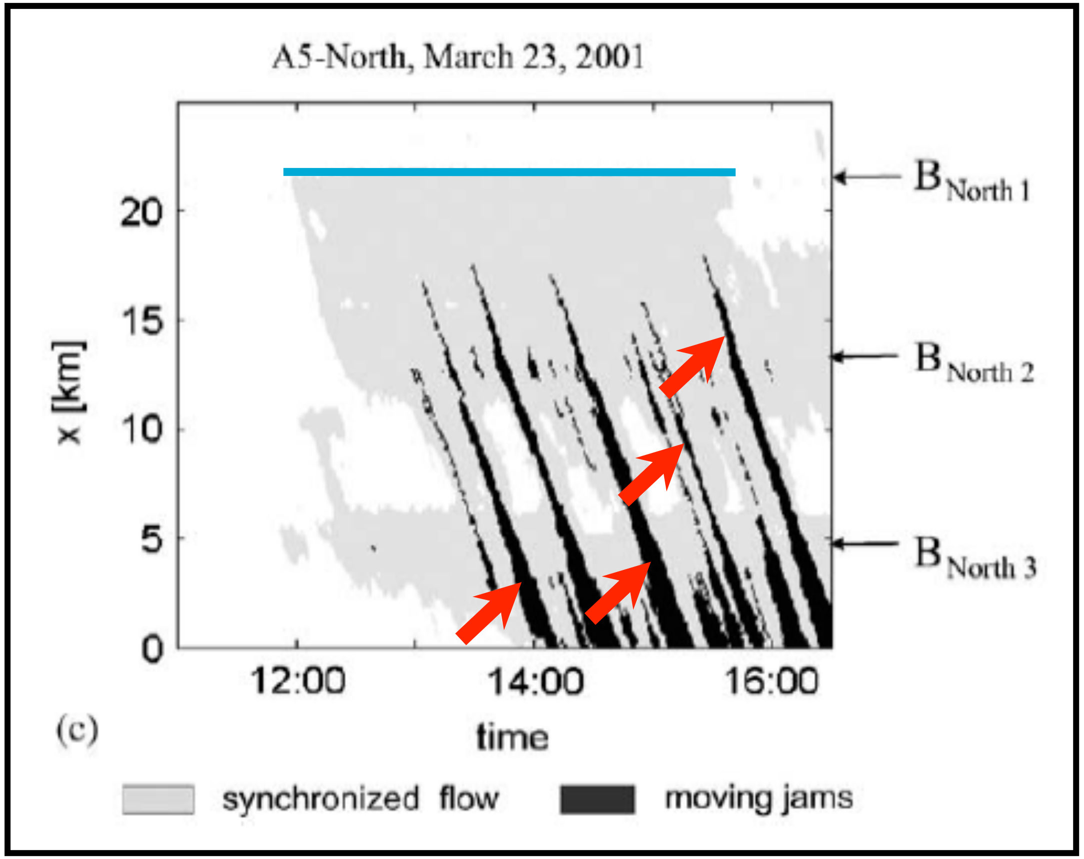
density
(d)

Traffic theory of Kerner



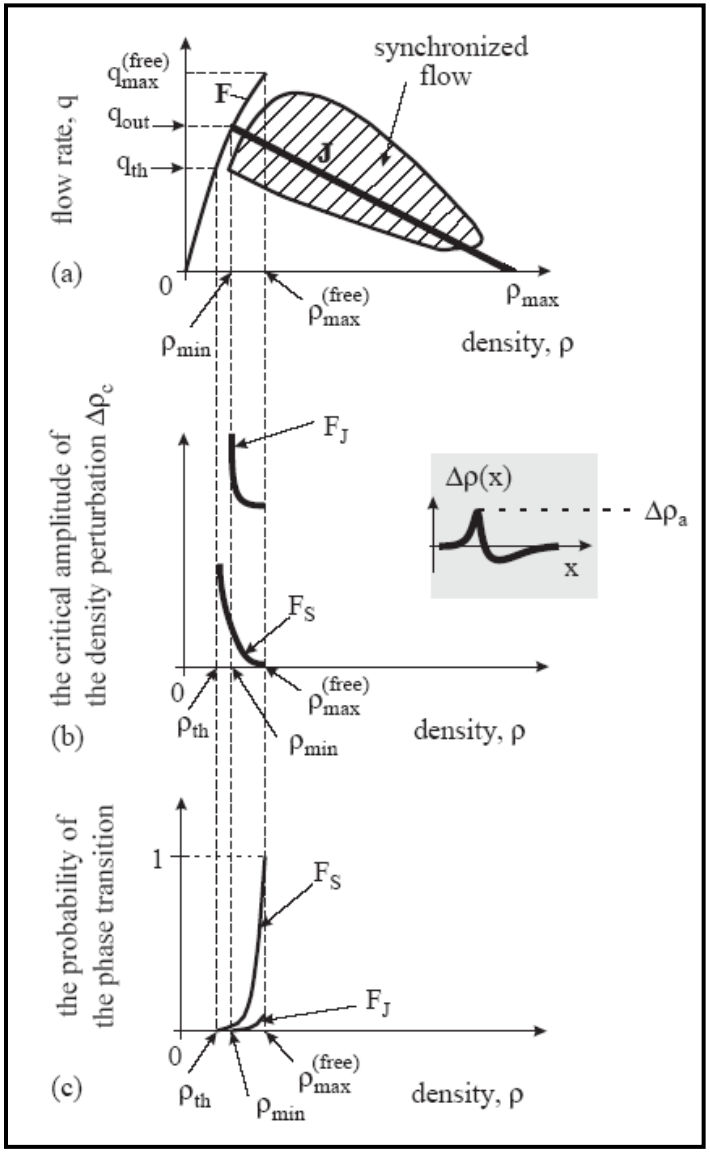
Traffic theory of Kerner

- 'Spontaneous' transitions from one state to another, also referred to as self-organisation

- Stable traffic conditions
  - Disturbances will not yield a phase transition
- Metastability:
  - Small disturbances are damped out, large disturbances cause a phase transition
- Instability:
  - Any disturbance causes a phase transition

Phase-transitions

- Figure (b) shows critical densities causing transition from free flow to synchronized flow or to jammed flow (example)
- Figure (c) shows corresponding transition probabilities (determined empirically from data on motorway traffic fluctuations

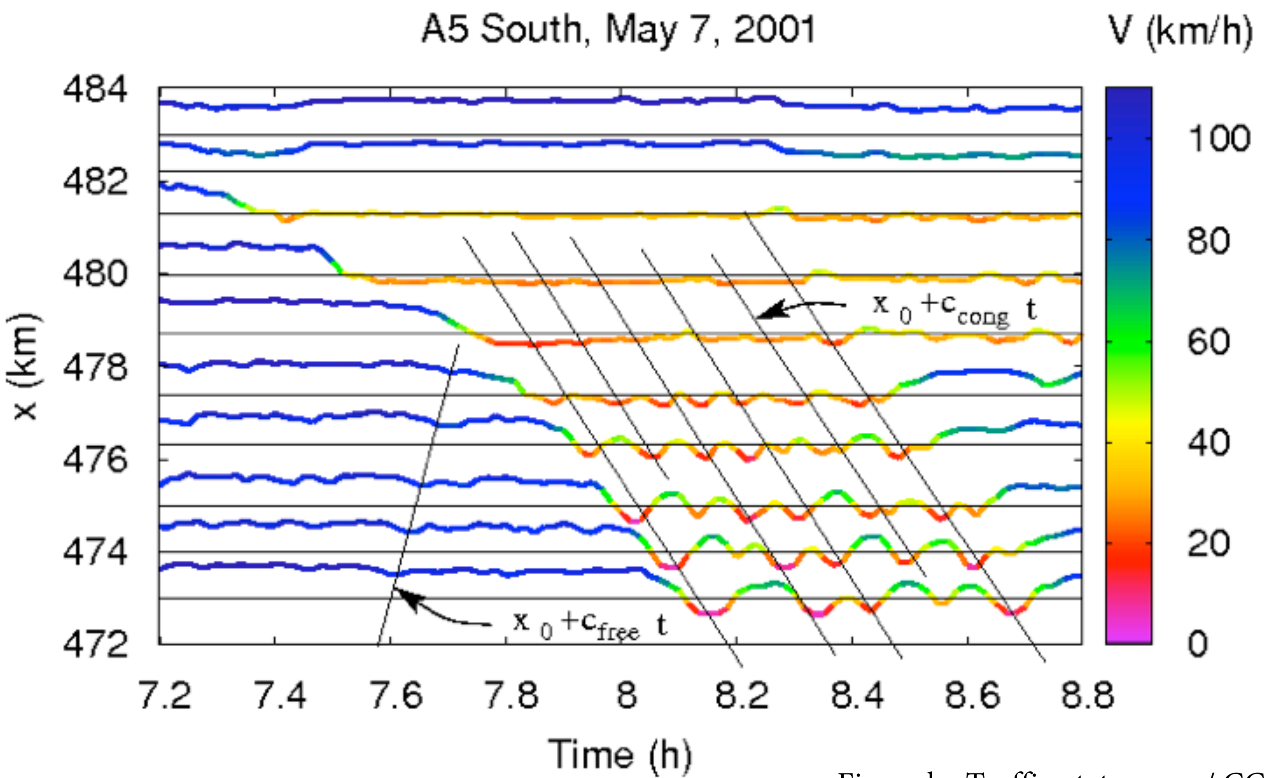


Instability

...macroscopic and microscopic explanations and (simple) models...

Occurrence of moving jams

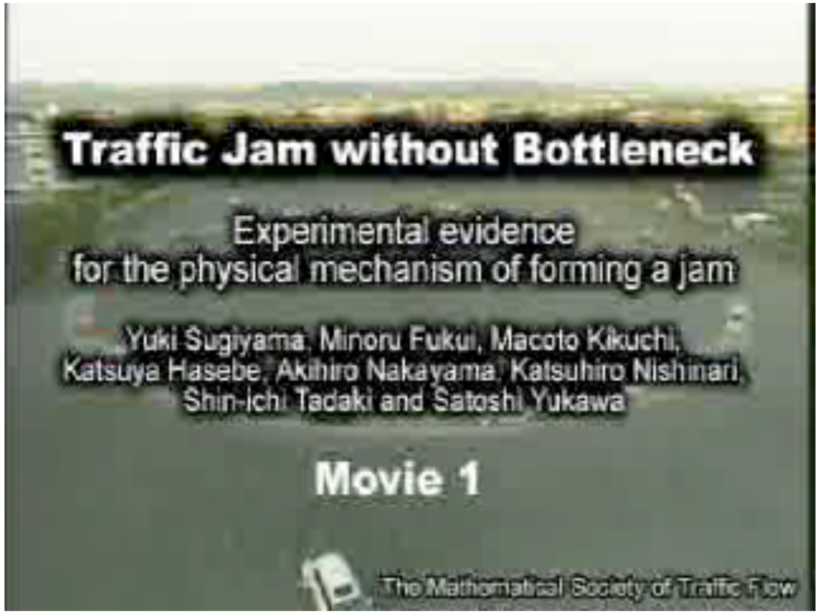
Growing amplitude of perturbations… $A(t) = A_0 \cdot e^{\sigma(t-t_0)}$



Instability and wide moving jams

Emergence and dynamics of start-stop waves

- In certain density regimes, traffic is highly unstable
- So called 'wide moving jams' (start-stop waves) self-organize frequently (1-3 minutes) in these high density regions



Payne model and instability

- Payne's model (1979) was the first second-order model

$$\frac{\partial k}{\partial t}+\frac{\partial (ku)}{\partial x}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\underbrace{\frac{U(k)-u}{\tau}+\frac{1}{2k\tau}\frac{dU}{dk}\frac{\partial k}{\partial x}}_{=A(k,u)}$$

where $\Delta = 1/2k$

- A = A(k,u) describes the acceleration along trajectories governed by relaxation to equilibrium speed and anticipation

Payne and Instability

Linear stability analysis

- Consider an equilibrium solution (k_e,u_e) of the Payne model
- Note that A = 0 in case of equilibrium
- In a linear stability analysis, we consider small perturbations on this solution, i.e.:

$$k = k_e + \delta k \quad \text{and} \quad u = u_e + \delta u$$

- Derive dynamic equations for the perturbations $(\delta k, \delta u)$ and determine if these will either damp out or become larger over time
- For the Payne model, we can derive conditions for stability:

$$k \cdot U'(k)^2 \leq -\frac{U'(k)}{2k\tau}$$

Example: Greenshields

- For Greenshields: $U(k) = u_0\left(1 - \dfrac{k}{k_{jam}}\right) \quad \Rightarrow \quad U'(k) = -u_0 / k_{jam}$
- Substitution yields:

$$k \cdot (u_0 / k_{jam})^2 \leq \frac{u_0 / k_{jam}}{2k\tau}$$

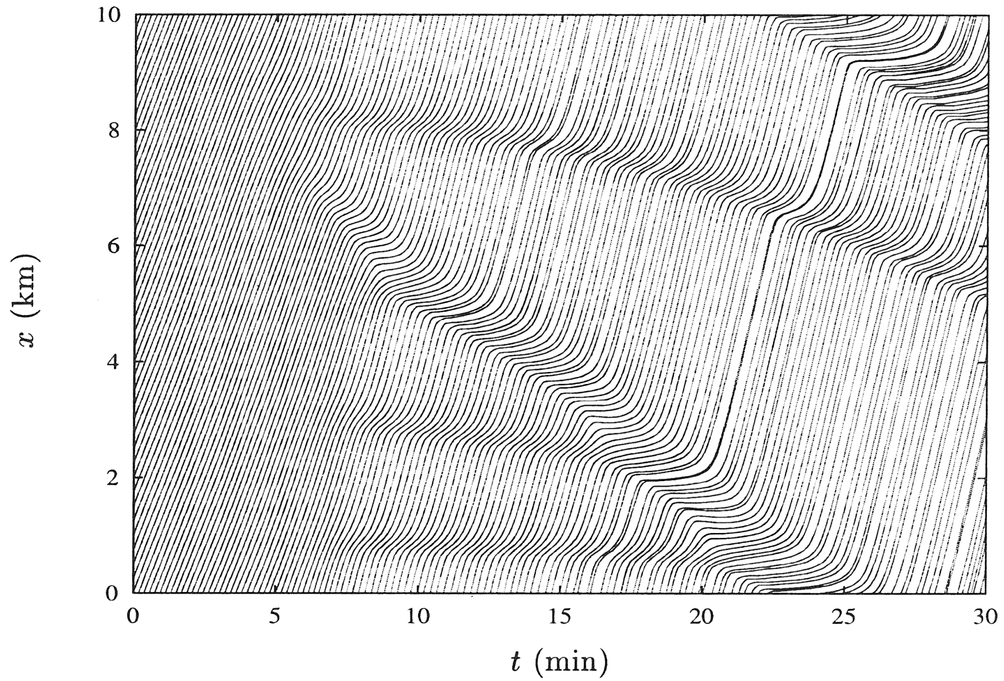
$$\Rightarrow$$

$$k \leq \sqrt{\frac{k_{jam}}{2\tau u_0}}$$

- In other words, traffic instability occurs for sufficiently high densities

Example Payne model

- Example predicted flow conditions in case of initially homogeneous state



Instabilities in microscopic models...

A more or less generic car-following model...

- Considered class of car following models describe acceleration as a function of distance, speed, rel. speed

$$a_i = f(s_i, \Delta v_i, v_i) \text{ where } \Delta v_i = \frac{d}{dt} s_i = v_{i-1} - v_i$$

- Example: Intelligent Driver Model (IDM):

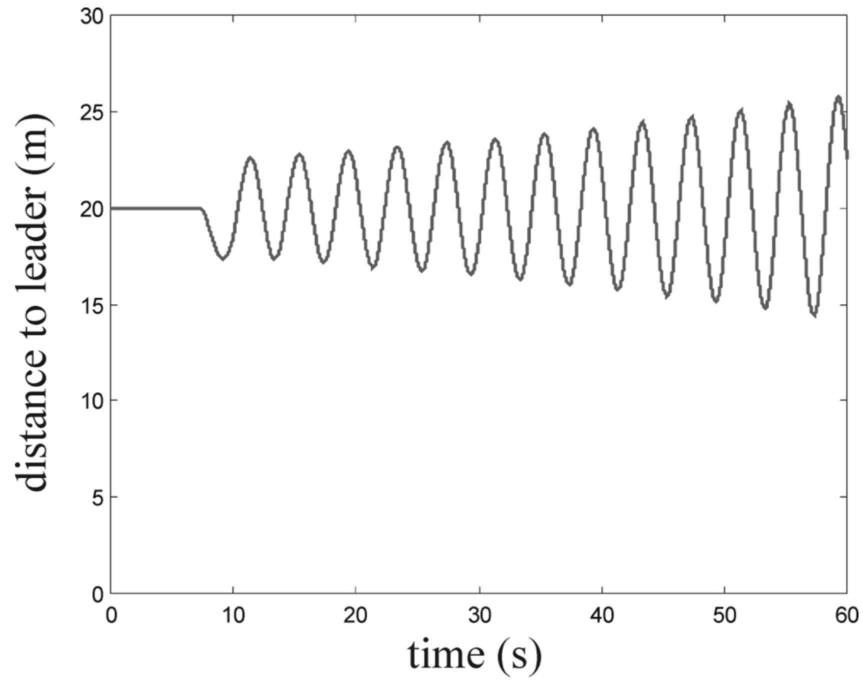
$$a = \alpha \left[1 - \left(\frac{v}{v_0} \right)^{\delta} - \left(\frac{s_*(v, \Delta v)}{s - l} \right)^2 \right]$$

$$\text{where } s_* = s_0 + s_1 \sqrt{\frac{v}{v_0}} + \tau v - \frac{v \Delta v}{2\sqrt{ab}}$$

Car-following stability analysis

- **Local stability** (studied from the fifties) describes how a follower reacts on perturbation of leader
- Example shows local instability of leader-follower pair

- Does this make any sense for a realistic model?
- How to analyze?



Analyzing local stability

- Assuming a leader-follower pair in equilibrium
- Consider disturbance by follower (leader does not react):

$$s(t) = s_* + y(t) \text{ and } v(t) = v_* + u(t)$$

- What are the dynamics of the disturbance?
- Linearized system:

$$\frac{d}{dt}\begin{pmatrix} y \\ u \end{pmatrix} = A \cdot \begin{pmatrix} y \\ u \end{pmatrix} \text{ where } A = \begin{pmatrix} 0 & -1 \\ f_s & f_v - f_{\Delta v} \end{pmatrix}$$

- Eig(A), solution of $\lambda^2 + (f_{\Delta v} - f_v)\lambda + f_s = 0$ determines stab.

Analyzing local stability

- We get the following solutions:

$$\lambda_{1,2} = \frac{-(f_{\Delta v} - f_v) \pm \sqrt{(f_{\Delta v} - f_v)^2 - 4f_s}}{2}$$

- Necessary conditions for stability: Re(λ) < 0
  - When would we have Re(λ) > 0, small disturbances grown over time (and actually become infinitely large)

- No oscillations in the solution Im(λ) = 0

Linear stability analysis

- Criteria for linear stability?
- For the simple car-following model: $a = f(s, v, \Delta v)$
  we find:

$$f_s > 0 \quad \Rightarrow \quad f_v < f_{\Delta v}$$

- Are these reasonable assumptions?

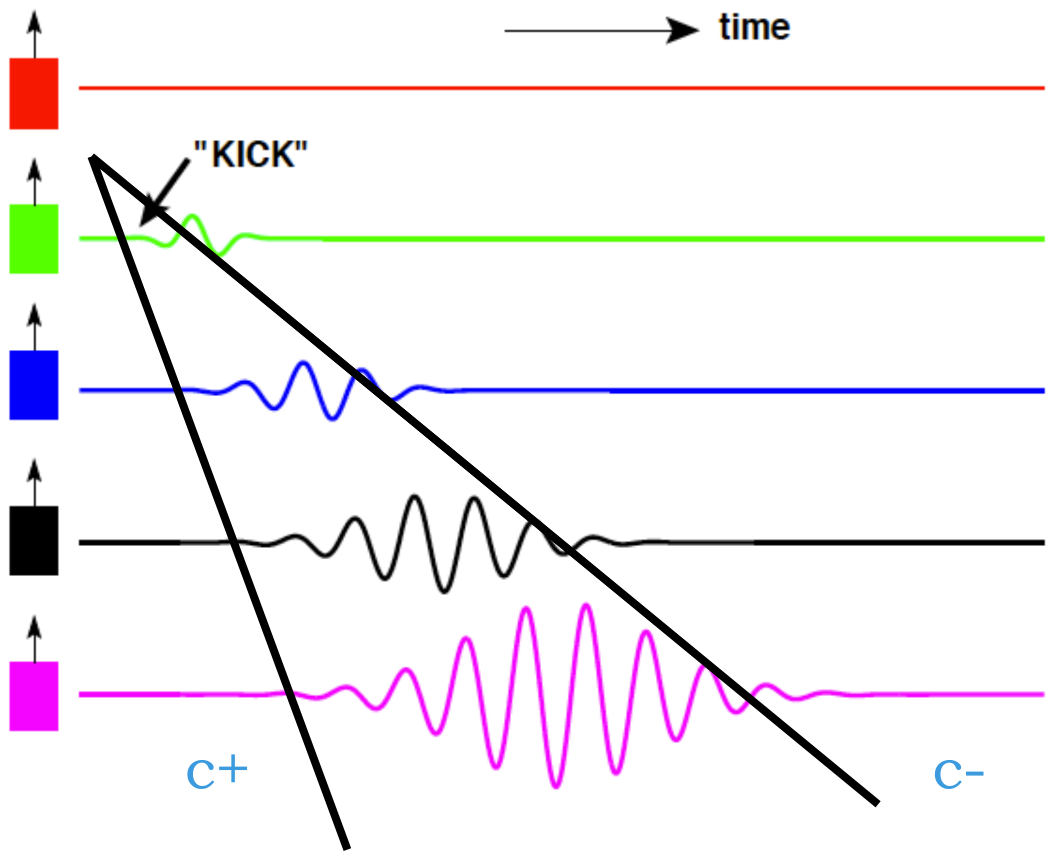
- No oscillations in response of follower?

$$f_s < \frac{1}{4}(f_v - f_{\Delta v})^2$$

- Does local instability entail realistic driving behavior?

More sensible definitions of stability

- String stability and instability



Stability tests for your model...

- For **local stability**, we need to test for solutions of eigenvalue problem:

$$\lambda^2 + (f_{\Delta v} - f_v)\lambda + f_s = 0$$

- Sign of real-part of the roots tells you if there is local stability or not (Re(λ) < 0)

- For **string stability**, we need to test the following:

$$\lambda_2 = \frac{f_s}{f_v^3}\left(\frac{f_v^2}{2} - f_{\Delta v} f_v - f_s\right)$$

- Is string stability realistic?

Exercise

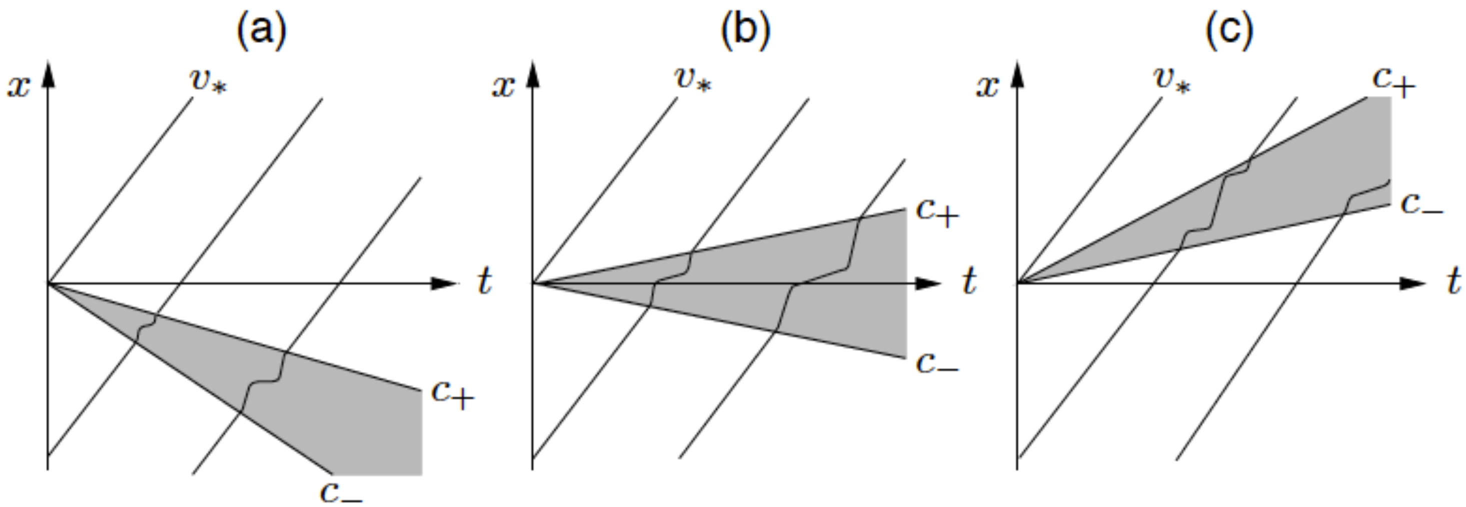
- Consider the IDM:

$$a = \alpha\left[1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s_*(v,\Delta v)}{s-l}\right)^2\right] \text{ where } s_* = s_0 + s_1\sqrt{\frac{v}{v_0}} + \tau v - \frac{v\Delta v}{2\sqrt{ab}}$$

- Can you determine the fundamental diagram? Which conditions will occur in case of equilibrium?
- Can you derive conditions for local stability?
- Can you derive conditions for string stability?

Types of string instability

- Three kinds...



Capacity drop

…micro and macro explanations…

Capacity drop on motorways

- How can we identify the capacity drop?
  - Cumulative curves and slanted cumulative curves
- Size and relevance of the phenomenon
  - How large is the capacity drop?
  - Why is it important and what are possible mitigating actions

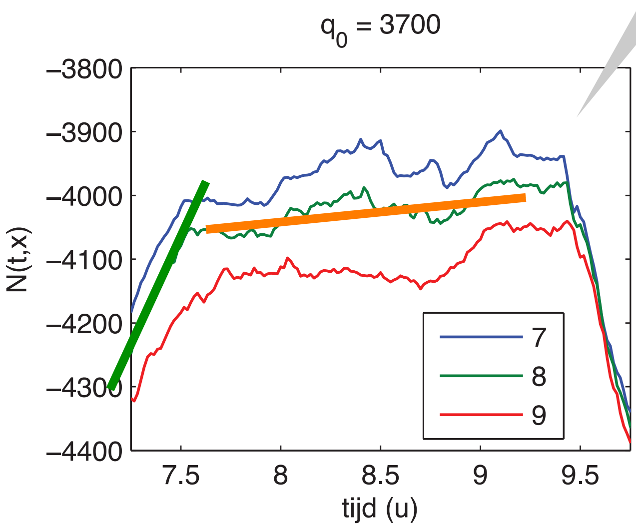
- What are possible explanations?
  - Slow vehicles with bounded accelerations changing lanes
  - Slow reaction on downstream conditions

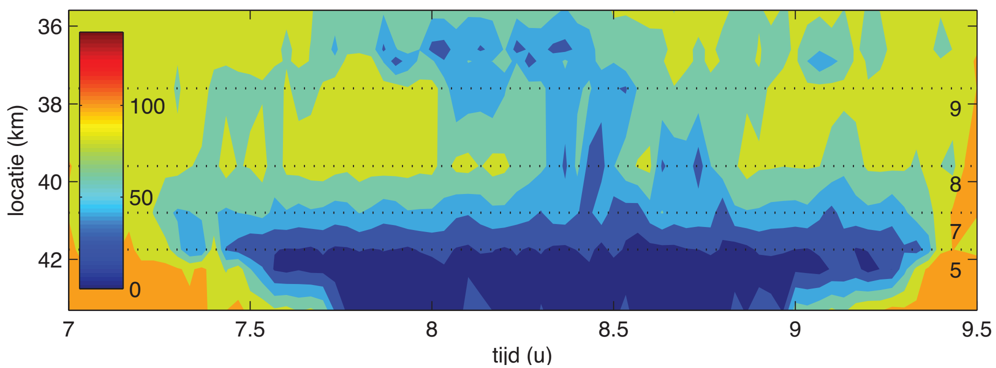
Capacity drop

Two capacities

- Free flow cap > queue-dis
- Use of (slanted cumulative
  clearly reveals this
- N(t,x) = #vehicles passinç x until t
- Slope = flow

- Capacity = slope of line (+ ref value of 3700 vtg/h)
- Free flow capacity = 4200 vtg/h
- Q-discharge rate = 3750 vtg/h
- Capacity drop = 11%



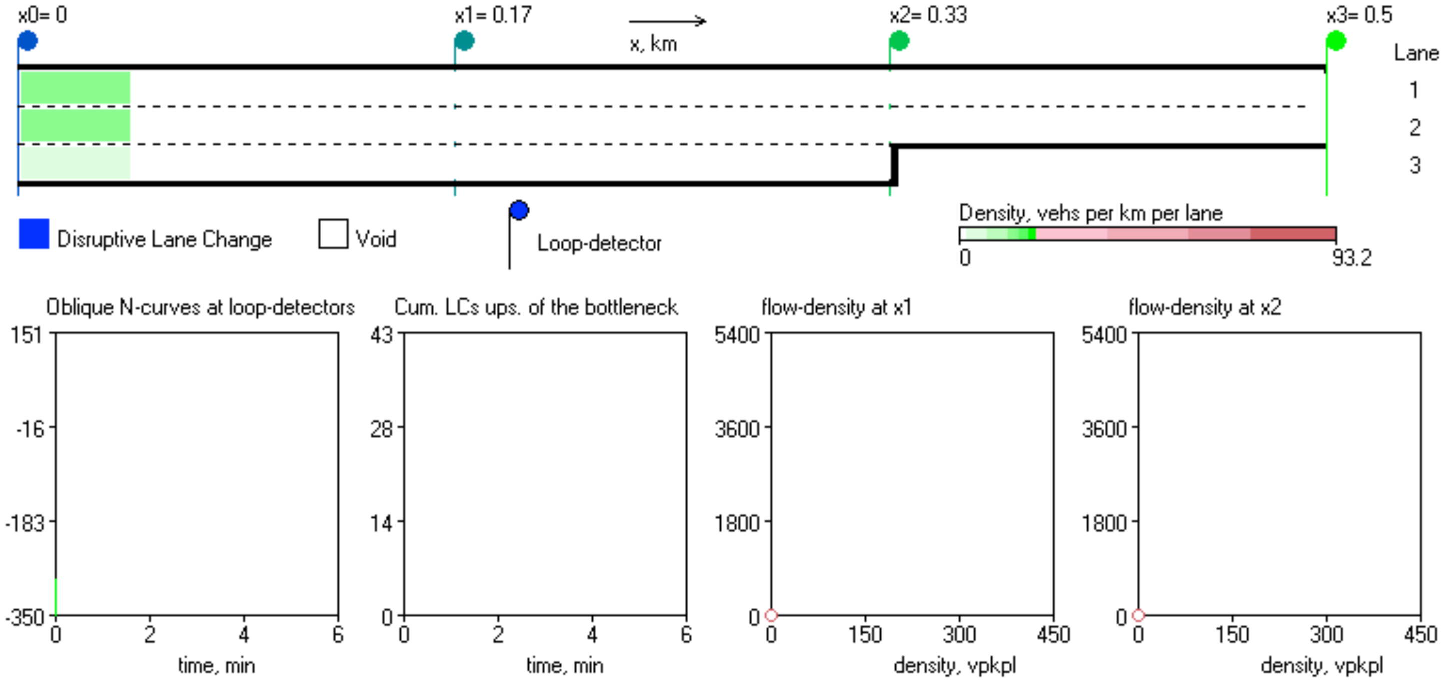


$$N'(t,x) = N(t,x) - q_0 \cdot t$$

Capacity drop

Possible causes: overtaking & moving bottleneck

- Overtaking by slow moving vehicles
- Moving bottlenecks with empty spaces in front



Capacity drop

Possible causes: differences in acceleration

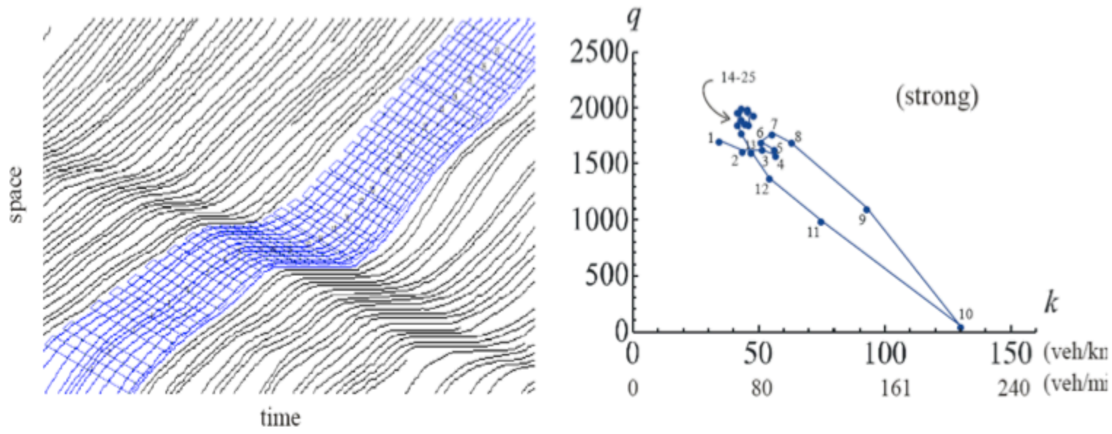
- Vehicles (person-cars, trucks) have different acceleration characteristics
- Consider vehicles flowing out of wide-moving jam
- What will these differences result in?

- Slower accelerating vehicles result in platooning effects, with slow moving vehicles are platoon leaders that leave a void in front that will not be filled

Capacity drop

Possible causes: retarded reaction (hysteresis)

- "hysteresis is manifested as a generally different behavior displayed by (a platoon of) drivers after emerging from a disturbance as compared to the behavior of the same vehicles approaching the disturbance"
- Explanation by (in-) balance between anticipation and delayed reaction shows different hysteresis curves



Think about modeling

Suitability of modeling paradigms…

- Which models do you know that can describe / deal with the capacity drops

- Which of the modeling paradigms are (in principle) suitable to describe the capacity drop?
  - First-order theory and shockwave theory
  - Queuing models
  - Other continuum traffic flow models (Payne)
  - Microscopic traffic simulation models

Summary

Phenomena and models...

Today's lecture

- Discuss higher-order phenomena in traffic flow:
  - Capacity drop
  - Hysteresis
  - Traffic instability
- Discuss underlying explanations
- Show possible modeling approaches:
  - Higher-order models
  - Microscopic models

- In coming lectures, microscopic (car-following) models are discussed in more detail

Models and phenomena

	Newell [8]	Newell + TA [5]	OVM [1]	IDM [11]	LWR [7, 10]	LWR + CD [2]	LWR + BA [6]	Payne [9]	Payne + Hyst. [13]
Capacity drop	-	0	0	+	-	+	0	0	+
Emergence of s&g	-	+	+	+	-	-	-	-	+
Propagation of s&g	+	+	+	+	0	0	0	0	0
Instability	-	+	+	+	-	-	-	-	+
Scatter in density-flow plane	-	+	+	+	-	+	+	+	+