Traffic Flow Theory and Simulation

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Lecture 8
Influence of Trucks, Lagranginan Coordinates and MFD

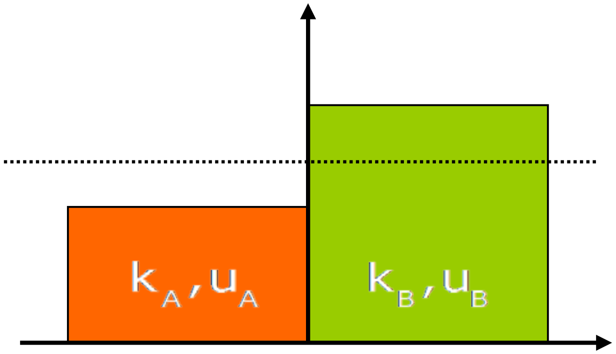
Influence of trucks and Lagranginan coordinates

24-3-2014

Recap of last time: Godunov scheme

- *Demand* of cell A and *supply* of cell B

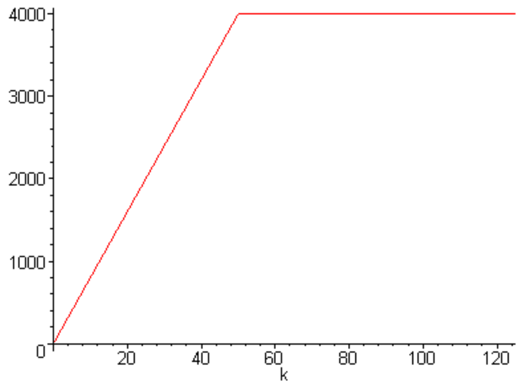
$$D_L := \begin{cases} q_A & k < k_c \\ C & k \geq k_c \end{cases} \qquad S_R := \begin{cases} C & k < k_c \\ q_R & k \geq k_c \end{cases}$$

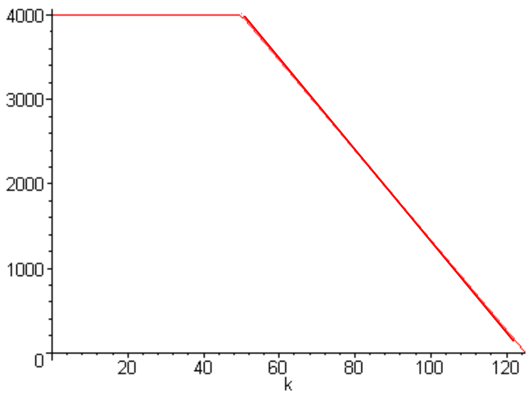


- D_L = maximum number of vehicles that can flow out of L (bounded by the capacity of the road)
- S_R = maximum number of vehicles that can flow into R (bounded by road capacity and the space becoming available during one time-step)
- Actual flow at x=0 : min(D_L,S_R)

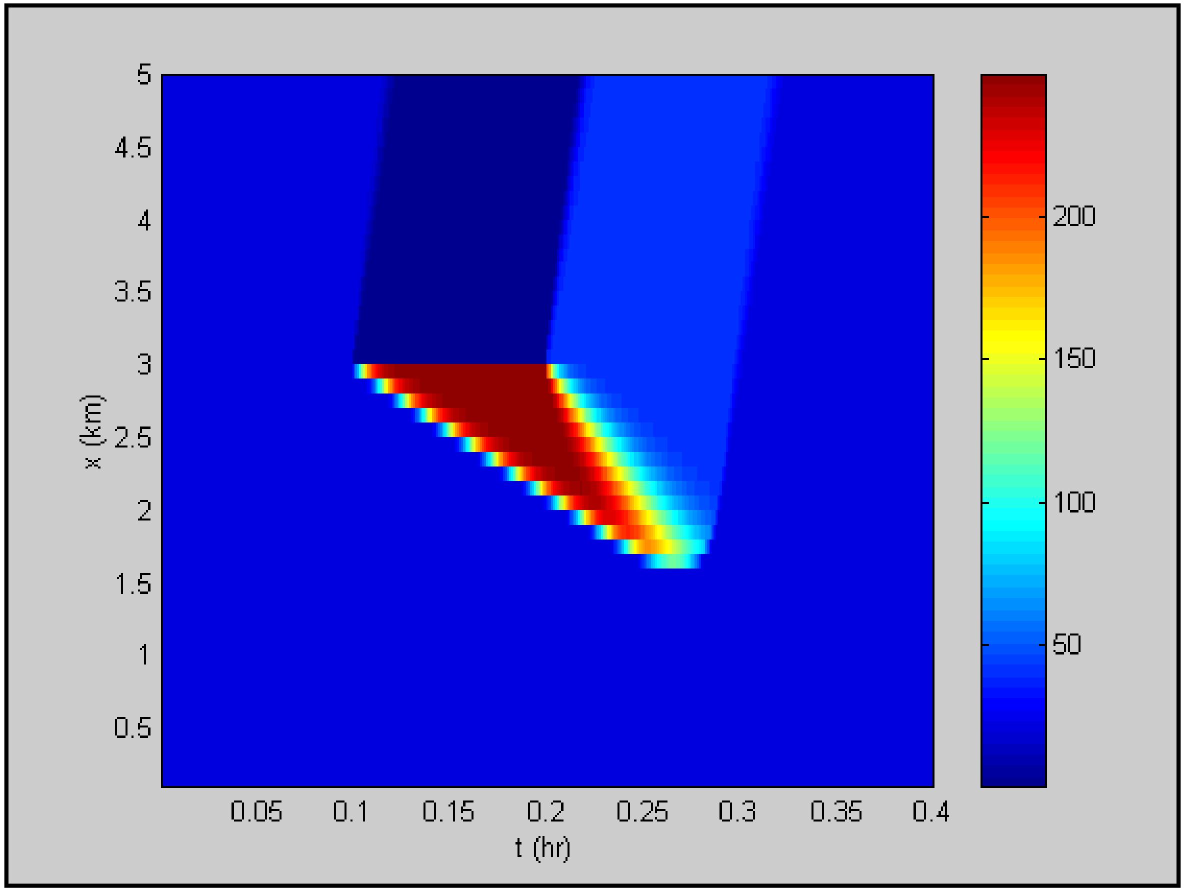
Godunov graphically

- Flow based on
  Demand & Supply
- => fundamental diagram

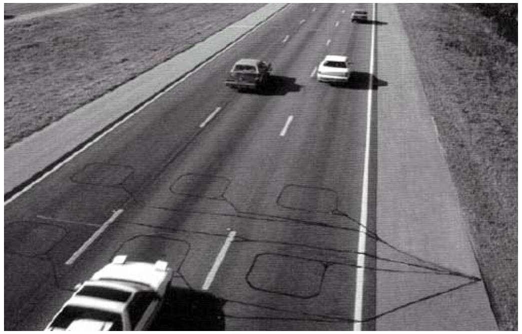




Example application Godunov



Eulerian & Lagrangian Coordinates



Inductive Loop Detector System by FHWA

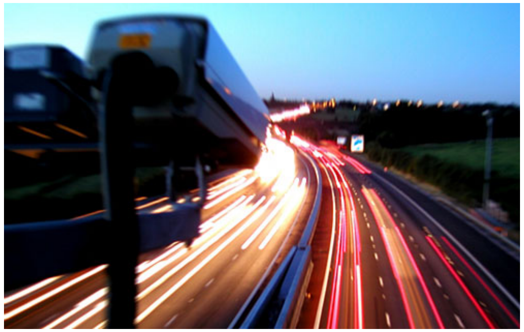
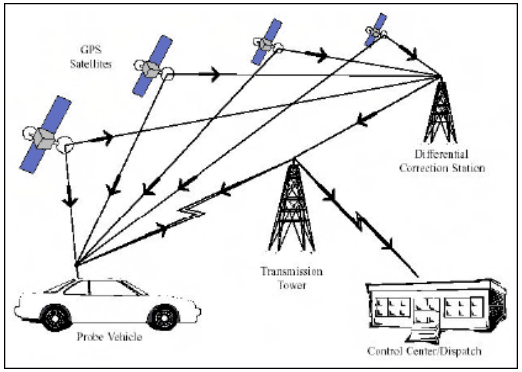


Photo by Seth Bloom

Photo by Wikipedia

Leonhard Euler

At a fixed point...



Typical Configuration for Satellite-based Probe Vehicle System by FHWA

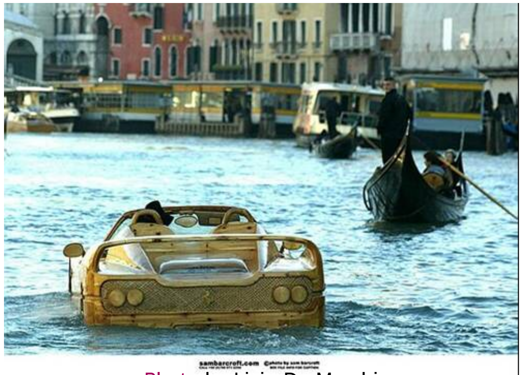


Photo by Livio De Marchi

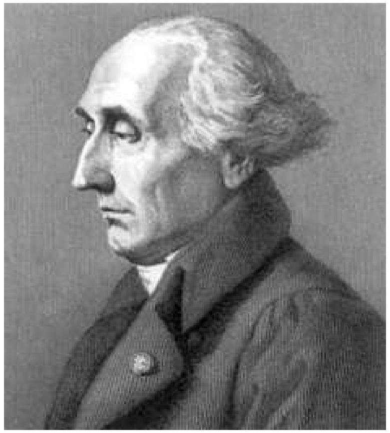
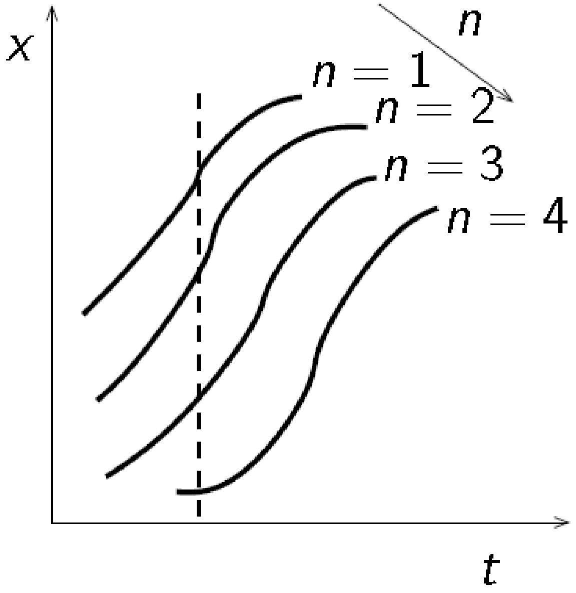


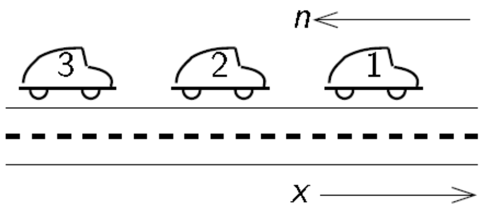
Photo by Wikipedia

Joseph Louis Lagrange

Travel along with moving vehicles...

Eulerian & Lagrangian Coordinates





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Leonhard Euler

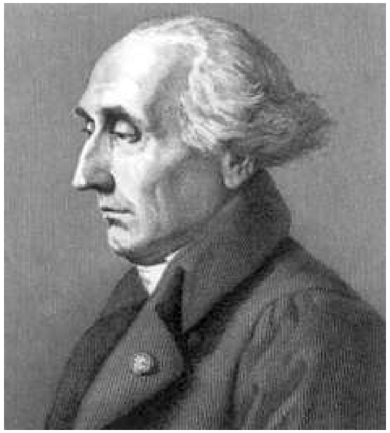


Photo by Wikipedia

Joseph Louis Lagrange

- Euler: coordinates fixed in space (x, t)
- Lagrange: coordinates move with traffic (n, t)

Lagrangian formulation

Conservation equation (Eulerian)

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

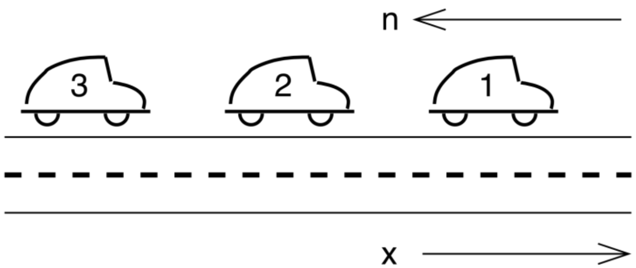
ρ	density	[veh/m]
q	flow	[veh/s]

Lagrangian formulation

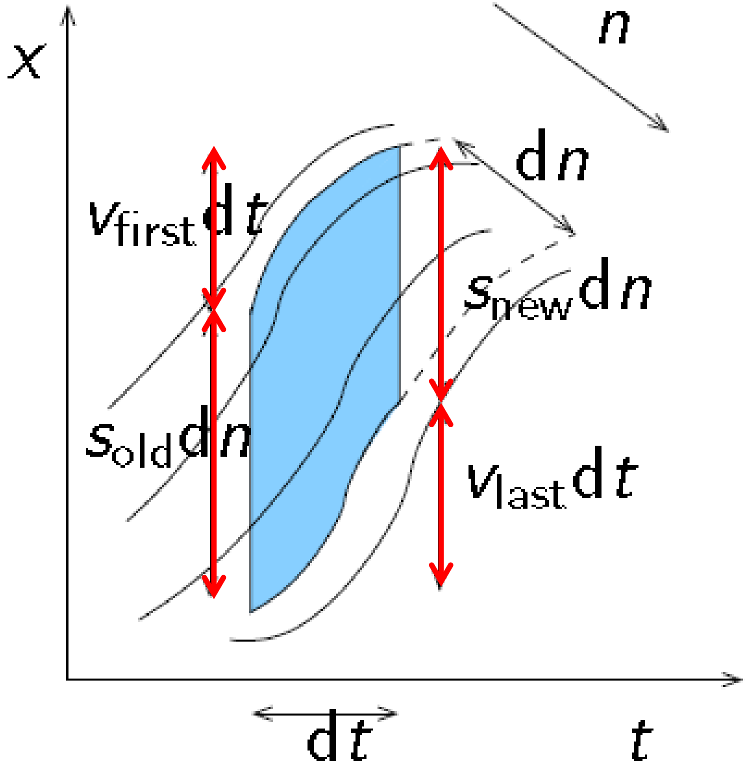
$$\frac{Ds}{Dt} + \frac{\partial v(s)}{\partial n} = 0$$

$s = 1 / \rho$	spacing	[m/veh]
$v = q / \rho$	speed	[m/s]
n	vehicle number	[veh]

D/Dt: Lagrangian time derivative
(distinguish from Eulerian case)



Interpretation and Derivation Langrangian Formulation



dn: platoon size
dt : time step

Variation of platoon length

final length = initial length
+distance travelled by first veh
- distance travelled by last veh

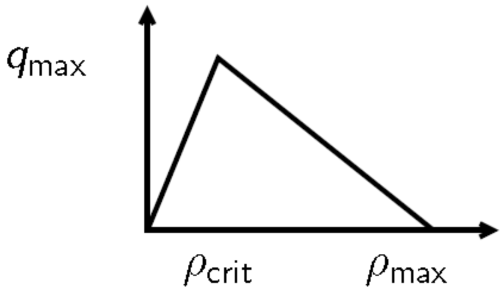
$s_{new}-s_{old}=(v_{first}-v_{last})dt$

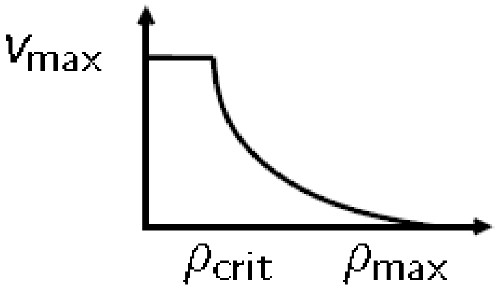
ds/dt = - (dv/dn)

$$\Rightarrow \quad \frac{\partial s}{\partial t}+\frac{\partial v}{\partial n}=0$$

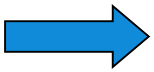
Fundamental relations

Euler (Daganzo)

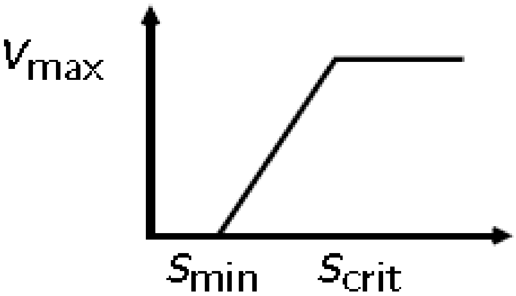




$s = 1 / \rho$
$v = q / \rho = q\, s$
$q = \rho\, v$



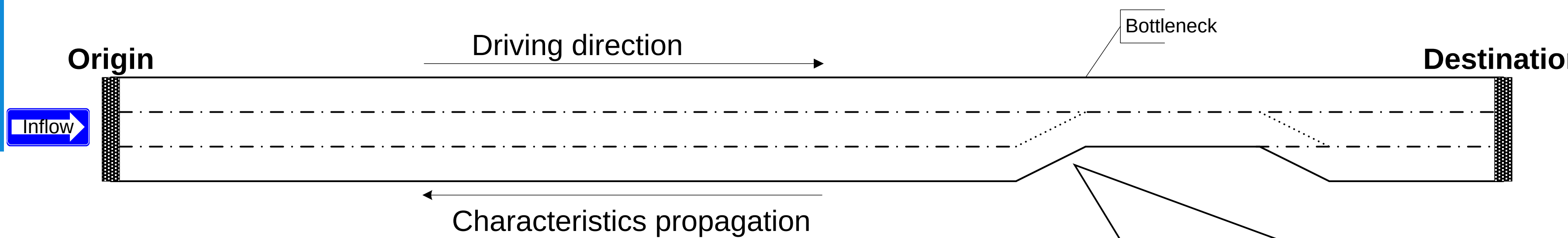
Lagrangian



What is s_{min}

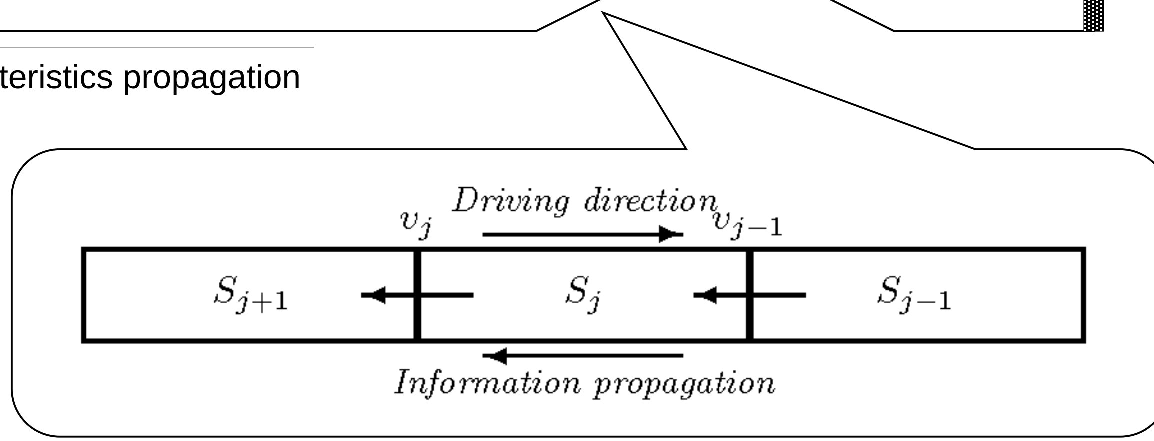
A) ~ 1.5 seconds
B) ~ 100 km/h
C) ~ 5 meters

Information propagation in Lagrangian coordinates



Upwind

$$\frac{s_{t+1}^{j} - s_{t}^{j}}{dt} + \frac{v_{t}^{j} - v_{t}^{j-1}}{dn} = 0$$



Traffic characteristics only move in the downstream direction (increasing vehicle number instead of space)

- Vehicles are clustered into groups of the vehicles
- Time is discretized into steps of dt seconds

Vehicles only react to vehicles in front of them

- ❑ Numerical solution: An upwind scheme [less time-consuming]

What happens if dn=1?

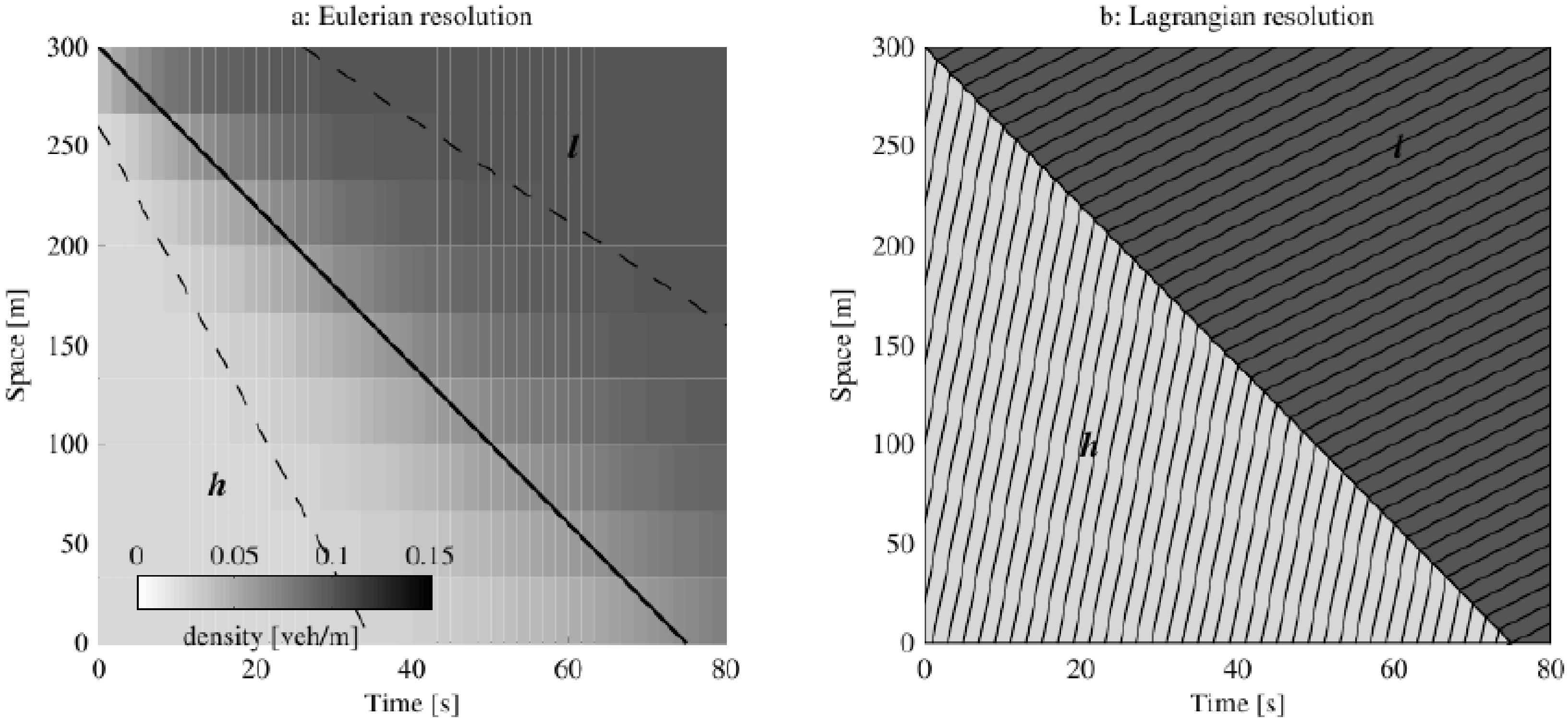
- In Lagrangian coordinates, one studies platoons of dn vehicles
- Will the model give output if dn=1
    A. No, vehicles are not conserved
    B. Yes, but the results are non-sensible (negative spacings)
    C. Yes, we obtain a microscopic model

What happens if dn=1?

- Yes, we obtain a microscopic model
- The choice of the fundamental relation is important
- Microscopic interpretation of macroscopic model
  (See Newell's simplified model, later in the course)

- Also possible:
  dn=3.5, dn=0.1, dn=π
- Not possible:
  dn=0, dn<0

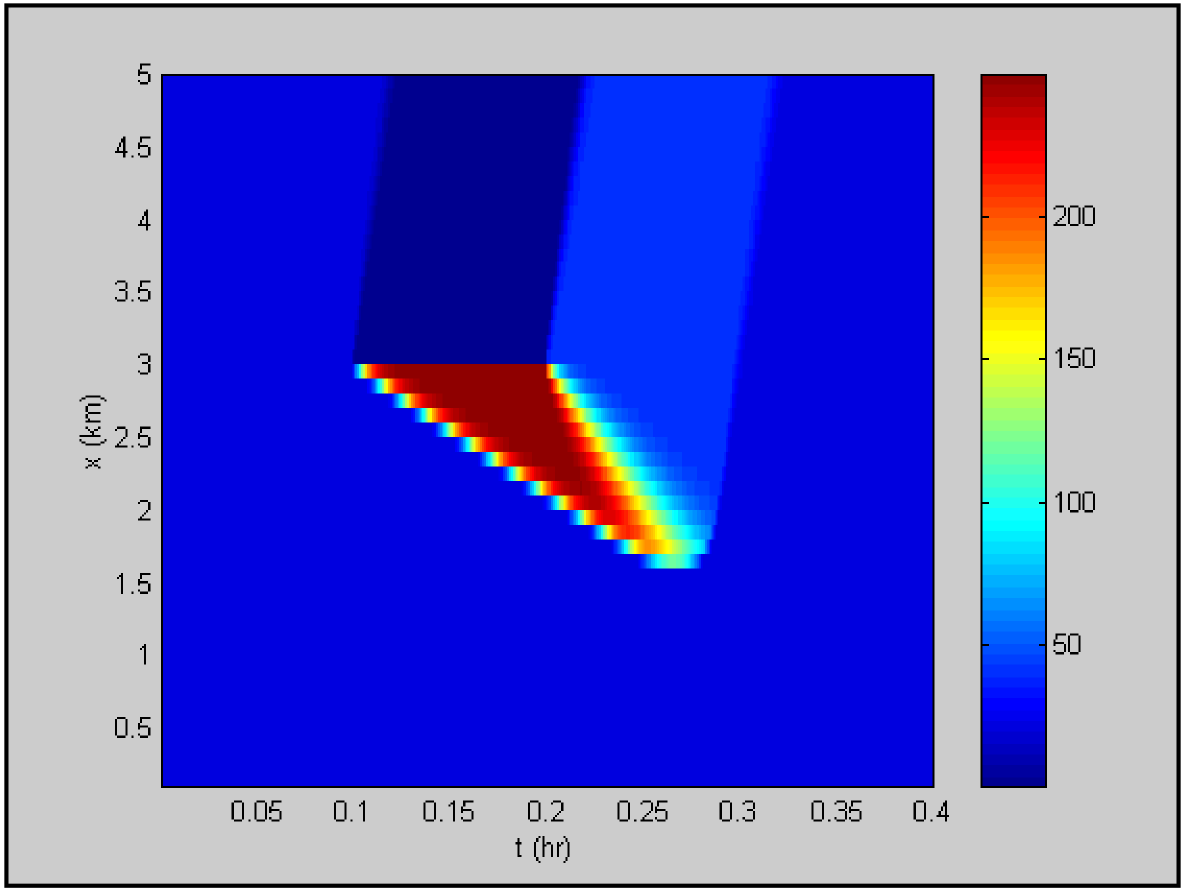
Accurate simulation results



Euler

Lagrange

Example application Godunov



Advantages of Lagrangian formulation

- Easy numerical discretization
- Fast simulation and accurate results
- Real-time applications
- Run iteratively in real time to test best control setting
- Easy switching between macro and micro level (dn=1)

Multiple vehicle classes in traffic

Source: unknown

Source: unknown

Macroscopic modeling of heterogeneity (1) class-specific conservation laws / FD

- Conservation of vehicles equation (Class-specific)

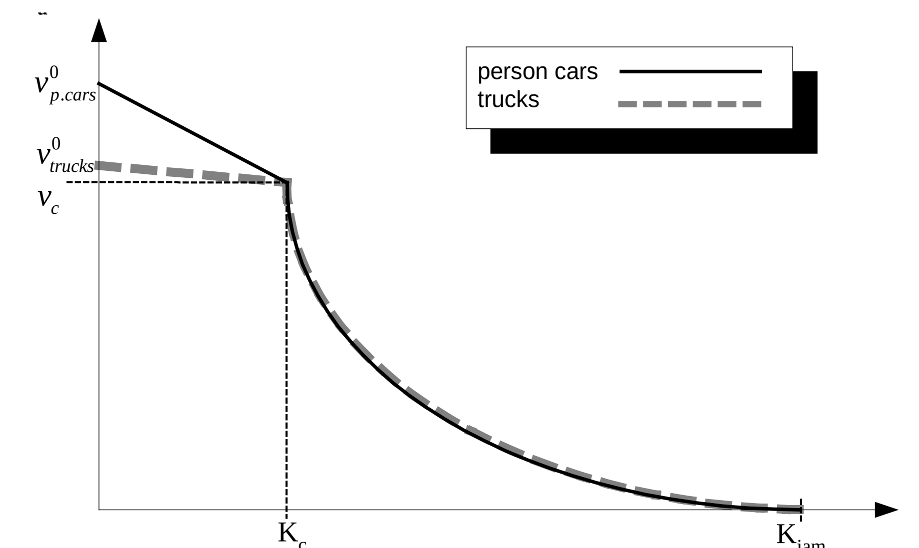
$$\frac{\partial k_u}{\partial t} + \frac{\partial q_u}{\partial x} = 0$$

$$q_u = k_u v_u$$

- Class specific equilibrium relation – usually function of the total density

$$v_u = V_u(k),$$

$$k = \sum_U k_u$$

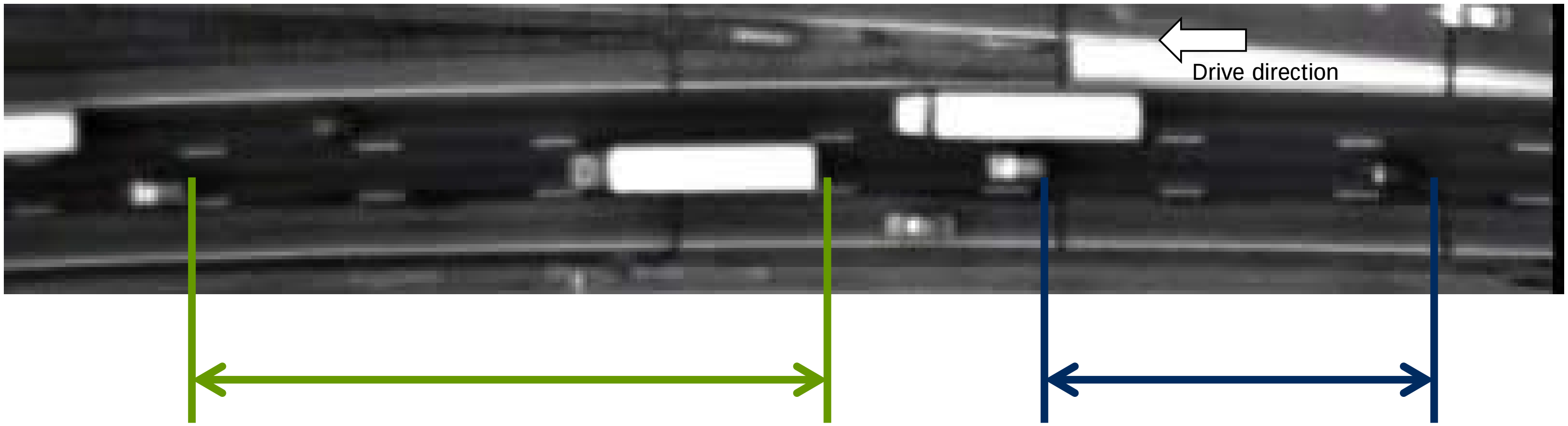


Macroscopic modeling of heterogeneity (2) PCE values

- Usually heavy vehicles are considered the equivalent of X person cars
  E.g. 1 truck = X pce (person-car equivalents)
- Ideas on which factors determine X ?
  - Geometry: slope, grade, curvature
  - Vehicle characteristics
  - Truck percentage itself
  - **Speed!** – we will return to this shortly ...

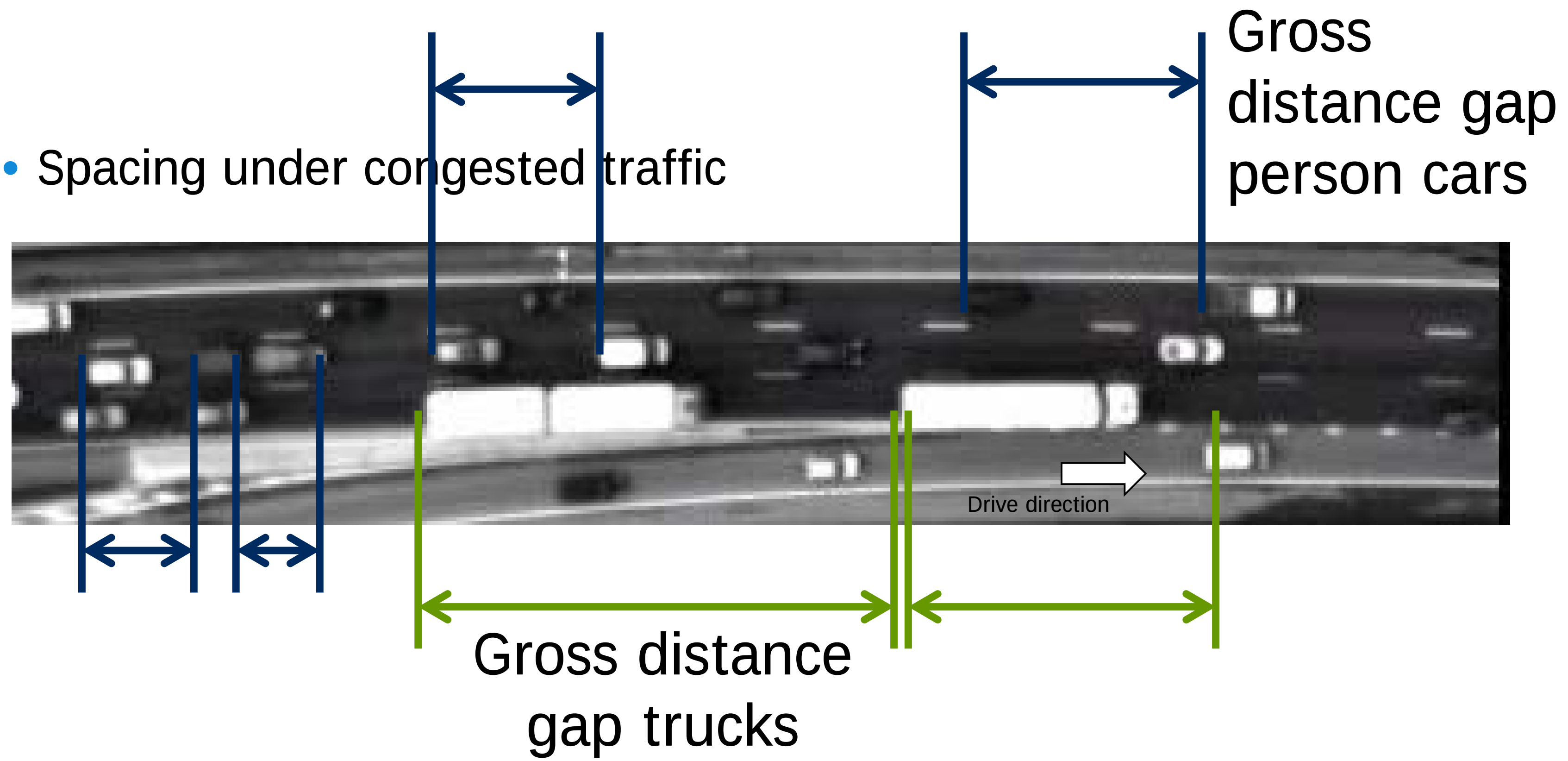
Spacing and the effect of heavy vehicles

- Spacing under light traffic



Spacing and the effect of heavy vehicles

- Spacing under congested traffic



- Relative space occupation trucks under congestion is larger (in the limit: think of a parking lot)

Fastlane equations and basic ideas

- Class-specific conservation of vehicles equation

$$\frac{\partial k_u}{\partial t} + \frac{\partial q_u}{\partial x} = 0$$

$$q_u = k_u v_u$$

- Class specific equilibrium relation – in Fastlane function of *effective* density in pce

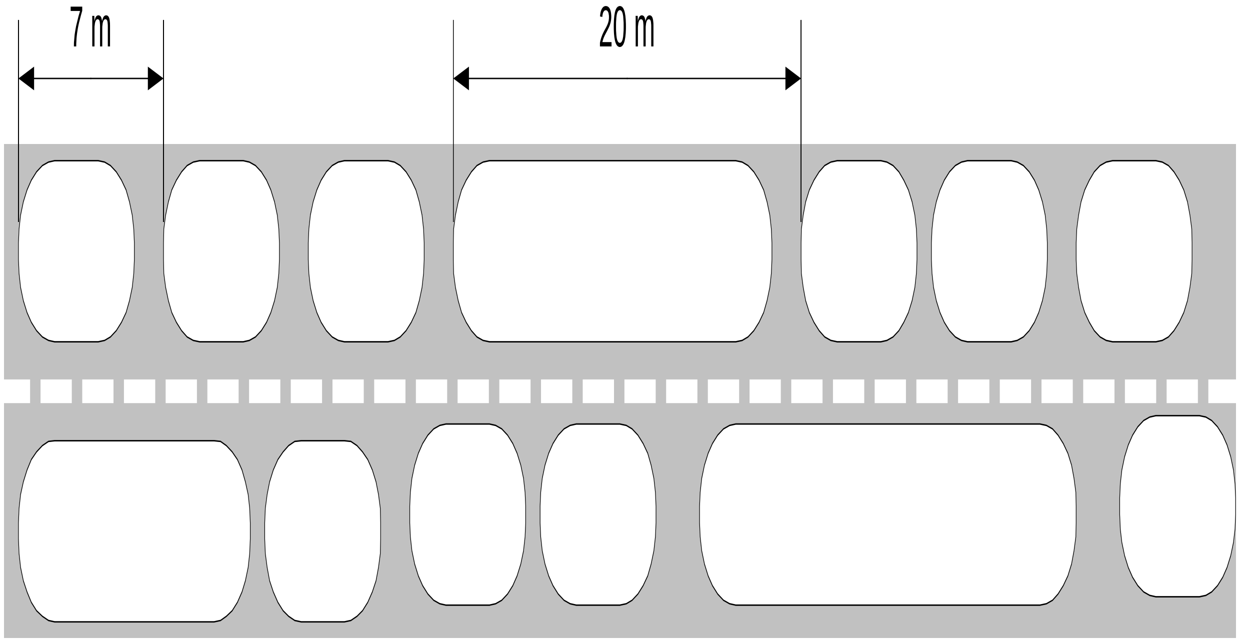
$$v_u = V_u\left(K\right),$$

$$K = \sum_U \eta_u k_u$$

State-dependent pce values

Pce value is dynamic:

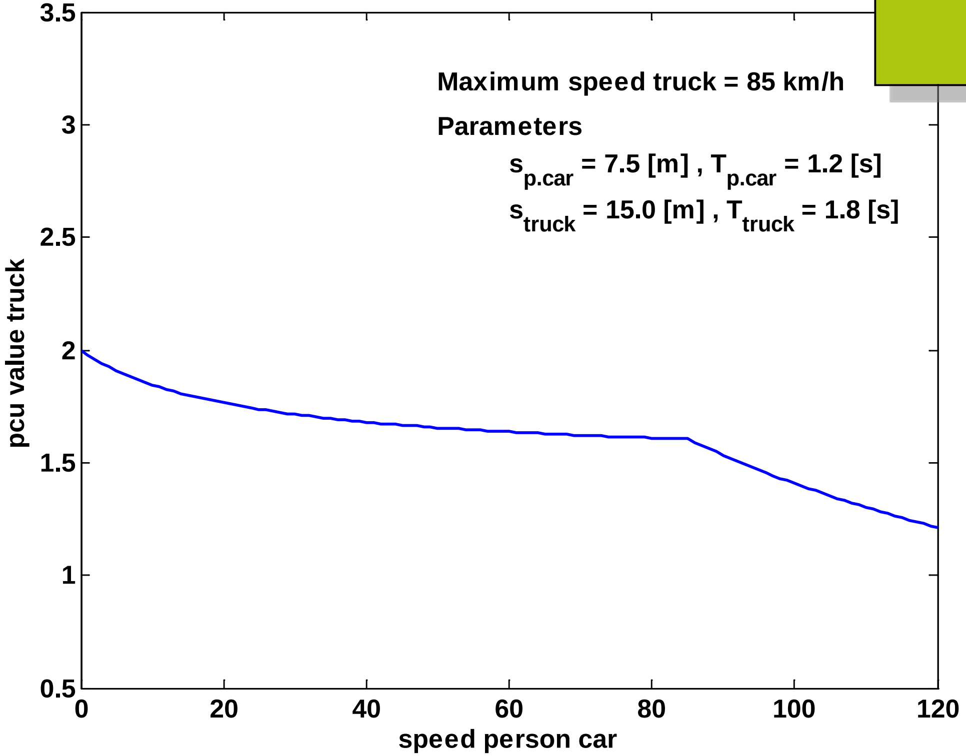
$$\eta_u(t,x) = \frac{s_u + T_u \cdot v_u(t,x)}{s_{u_0} + T_{u_0} \cdot v_{u_0}(t,x)}$$



State-dependent pcu values

Illustration of pcu under different speeds

$$\eta_u(t,x) = \frac{s_u + T_u \cdot v_u(t,x)}{s_{u_0} + T_{u_0} \cdot v_{u_0}(t,x)}$$



State-dependent pcu values

Illustration of pcu under different speeds

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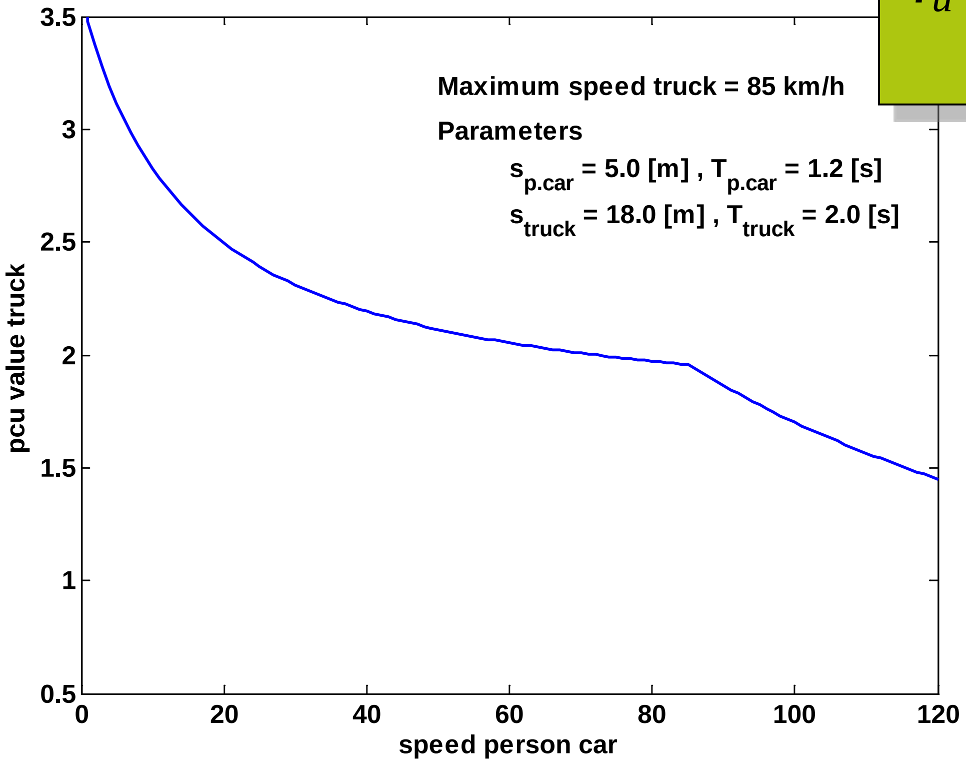
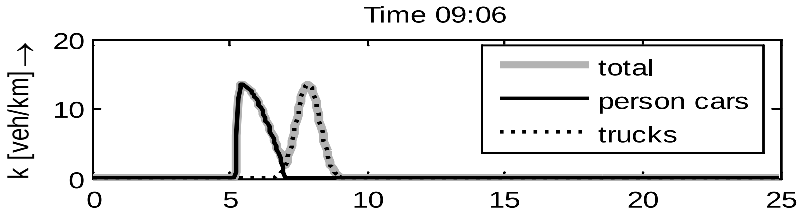




Example (1)

Consider straight 25 km two-lane freeway stretch

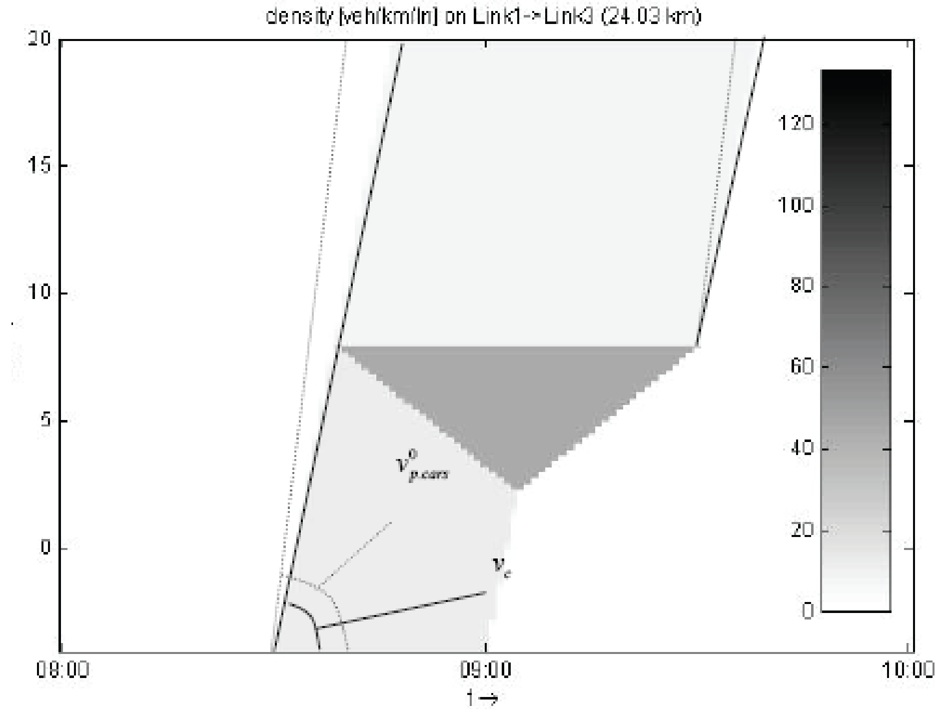
- Platoon of trucks (1500 veh/h) enters 9:00-9:01
- Platoon of person cars (1500 veh/h) enters 9:04-9:05

Example (2)

Same straight 25 km two-lane freeway stretch

- Platoon trucks + cars with fixed composition enters at origin 8:30-9:00
- Bottleneck (e.g. lane-blockage) halfway

90% trucks



(b) 10% passenger cars, 90% trucks (dynamic *pce-values*)

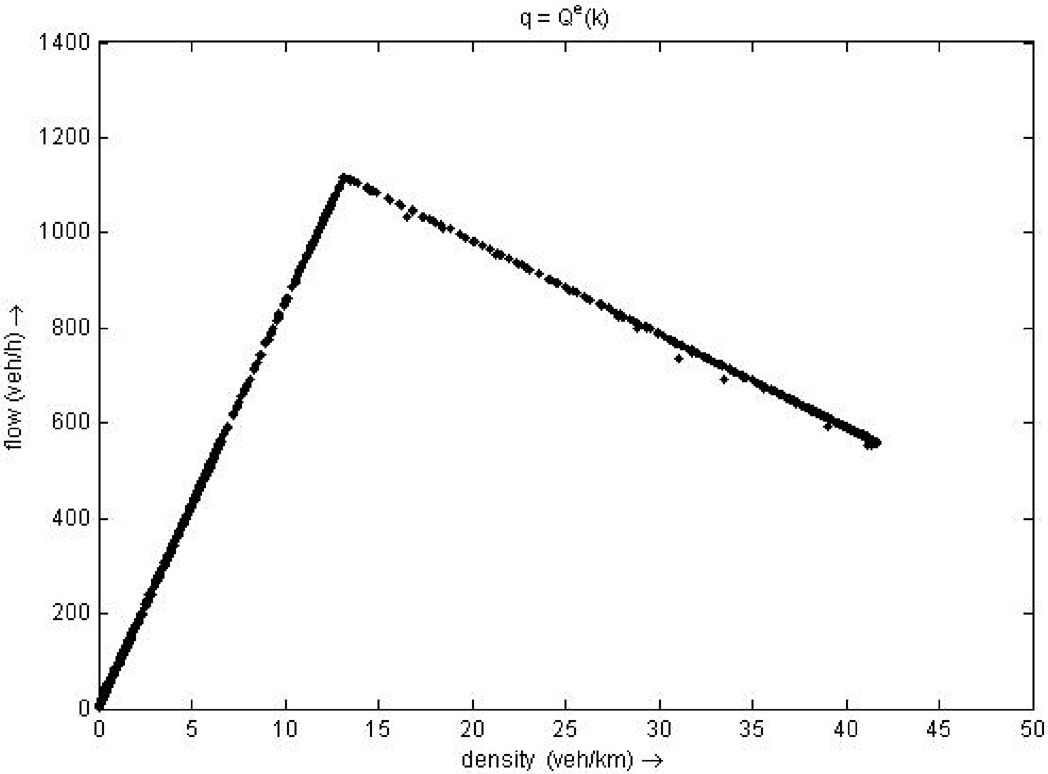
Example (3)

Static case:

$$\eta_u(t,x) = \eta_u = \frac{s_u}{s_{u_0}}$$

Same straight 25 km two-lane freeway stretch

- Platoon trucks + cars with fixed composition enters at origin 8:30-9:00
- Bottleneck (e.g. lane-blockage) halfway
- **Effect of dynamic PCE values**



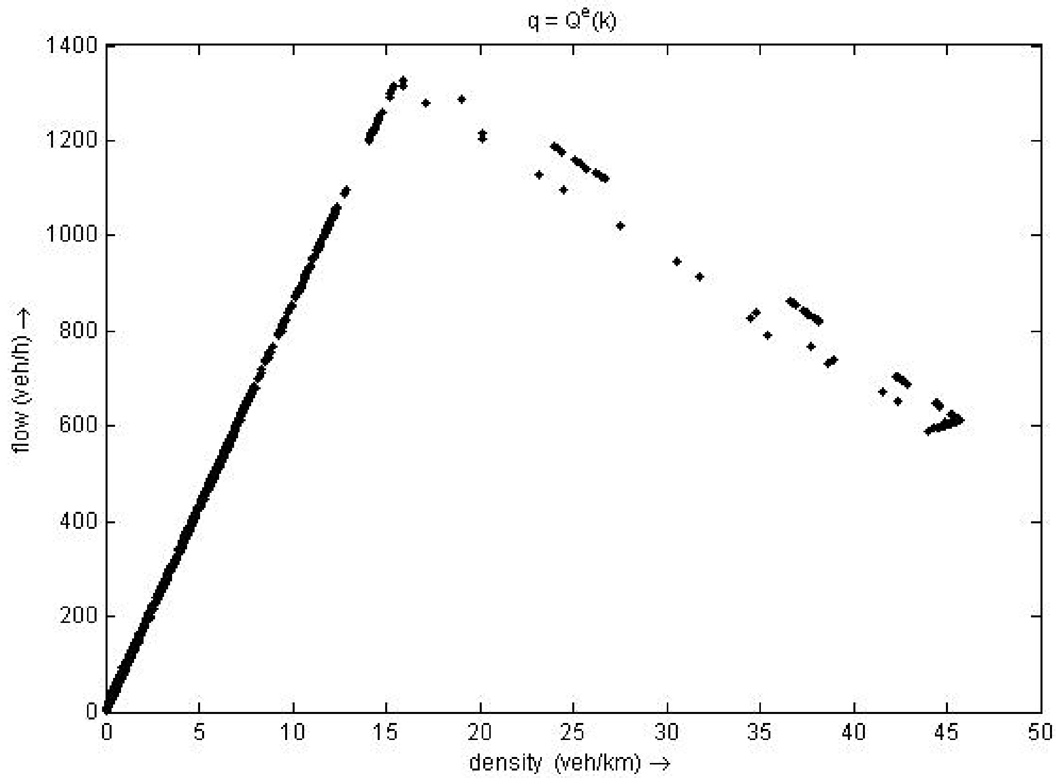
Example (3)

Dynamic case:

$$\eta_u(t,x) = \frac{s_u + T_u \cdot v_u(t,x)}{s_{u_0} + T_{u_0} \cdot v_{u_0}(t,x)}$$

Same straight 25 km two-lane freeway stretch

- Platoon trucks + cars with fixed composition enters at origin 8:30-9:00
- Bottleneck (e.g. lane-blockage) halfway
- **Effect of dynamic PCE values**

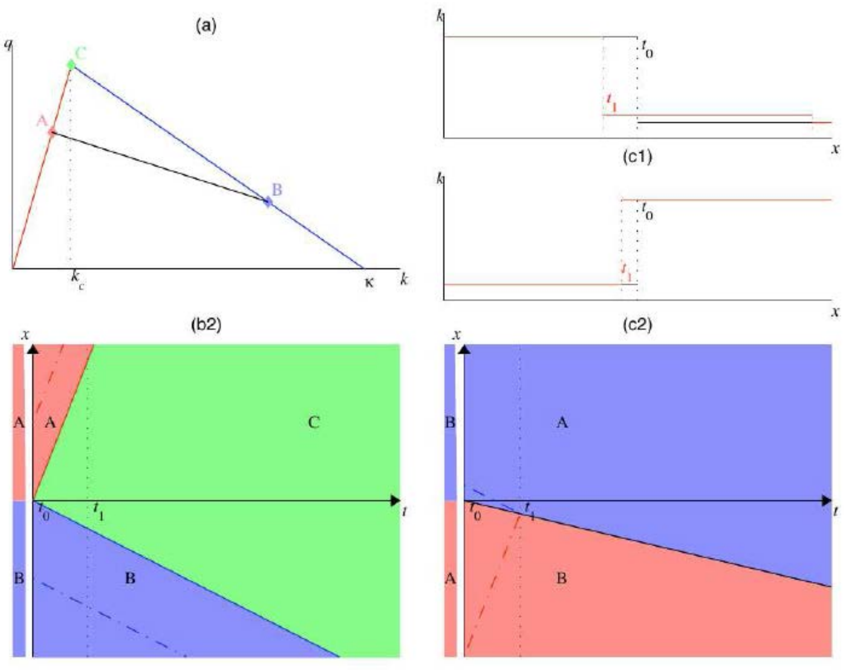


Learning goals

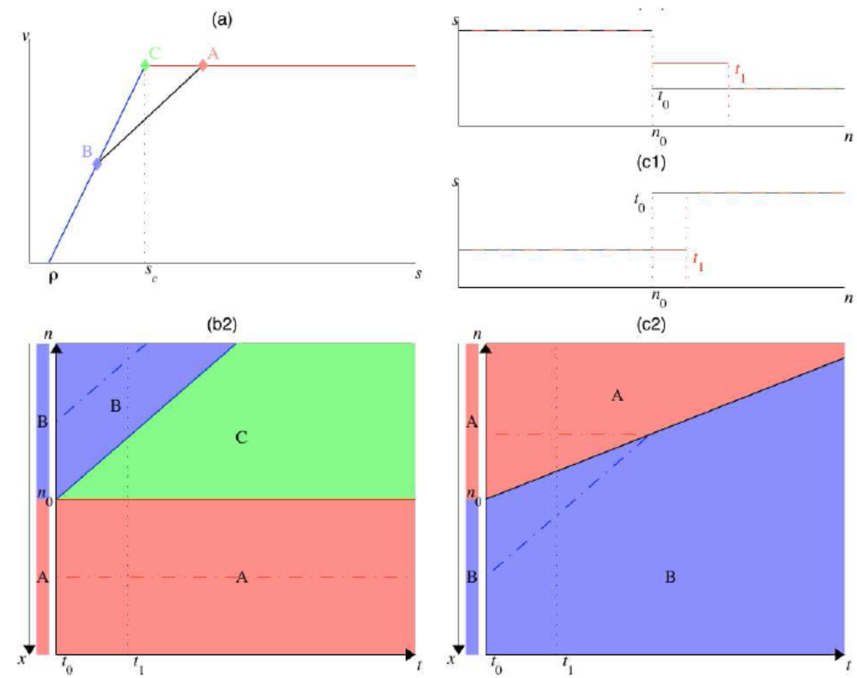
- Now, you can...

1. ... describe traffic in Lagrangian coordinates
   (but the exam will not question dynamic equations)
2. ... explain the advantages of Lagrangian formulation
3. ...comment on the effect of traffic heterogeneity on
   traffic operations and models
4. ...make calculations using (dynamic) pce-values

Same traffic model and problems



Solution to Acc. Fan & Shockwave by MoC in Eulerian Coordinates



Solution to Acc. Fan & Shockwave by MoC in Lagrangian Coordinates

(vertical axis: vehicle number)

Advantages of Lagrangian formulation

- ❑ Relate to simulations???

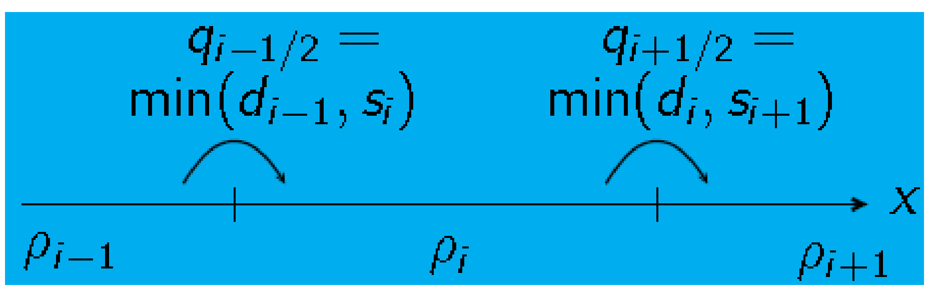
Downwind or Upwind

- ❑ Discretized KW model (Euler)

$$\frac{\rho_{t+1}^{i} - \rho_{t}^{i}}{dt} + \frac{q_{t}^{i+\frac{1}{2}} - q_{t}^{i-\frac{1}{2}}}{dx} = 0$$

  - space is discretized into cells of length dx
  - time is discretized into time period of size dt

- ❑ Numerical solution: Godunov scheme (min supply demand)



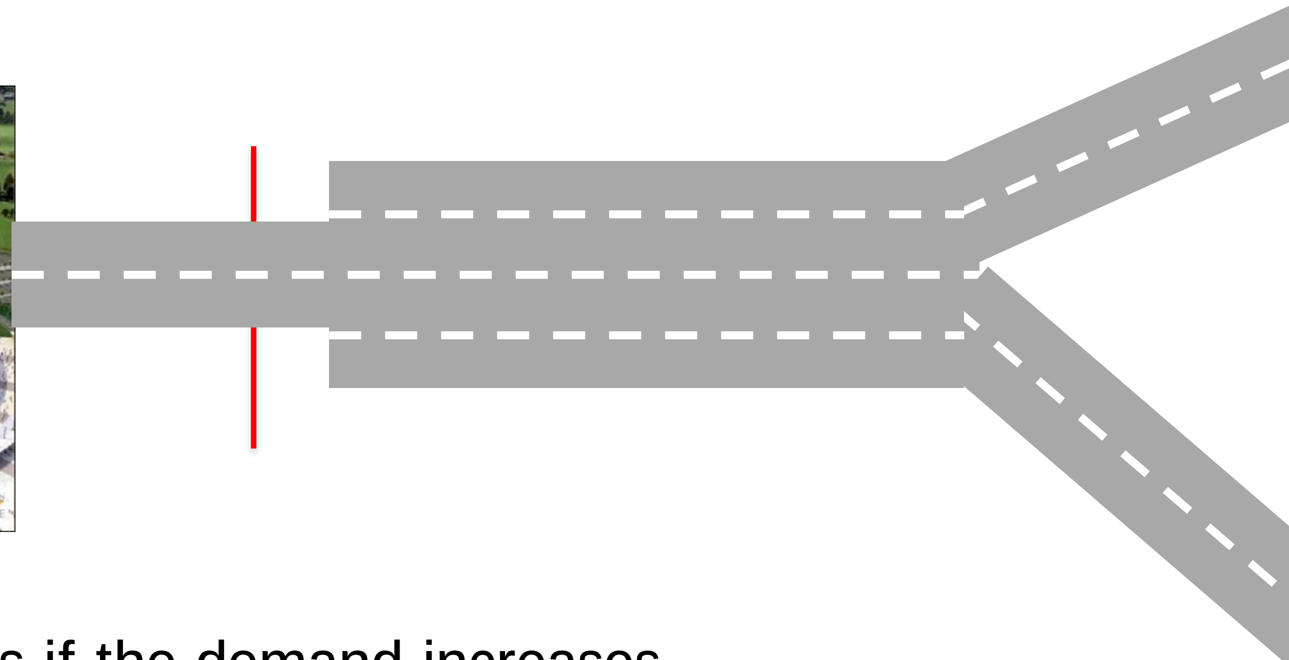
Introduction to the Macroscopic Fundamental Diag

24-3-2014

Victor L. Knoop

Simple road with increasing demand

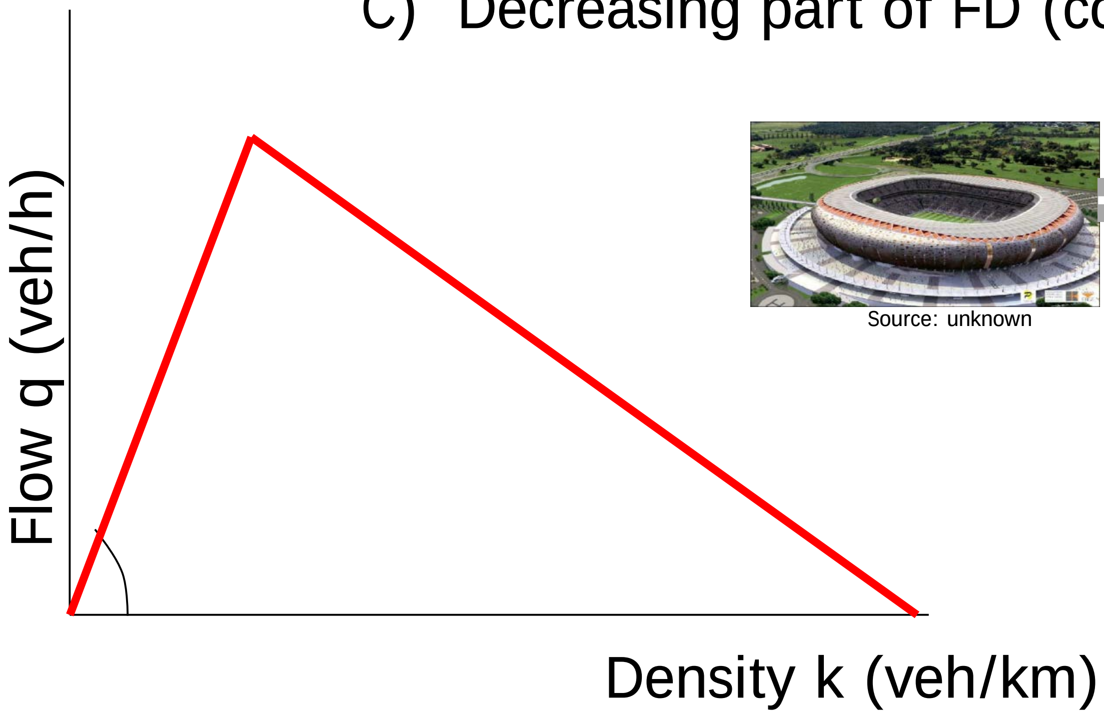
Source: unknown

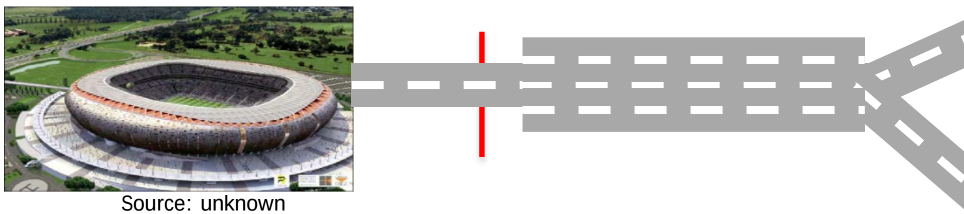


What happens if the demand increases

What observations can be made?

A) Whole FD
B) Increasing part of FD (free flow)
C) Decreasing part of FD (congestion)

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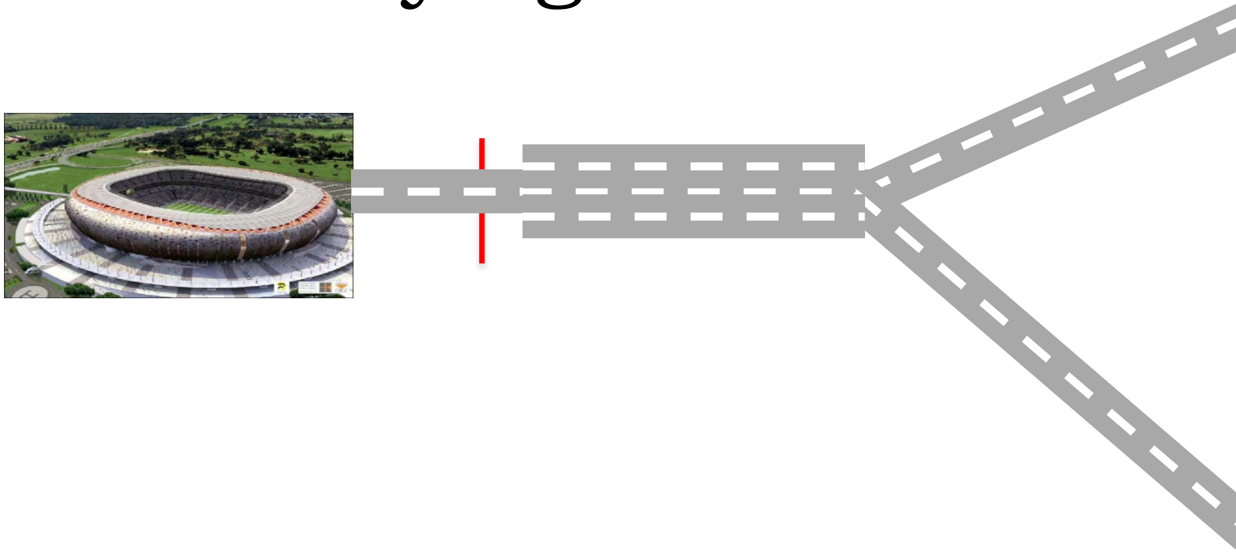
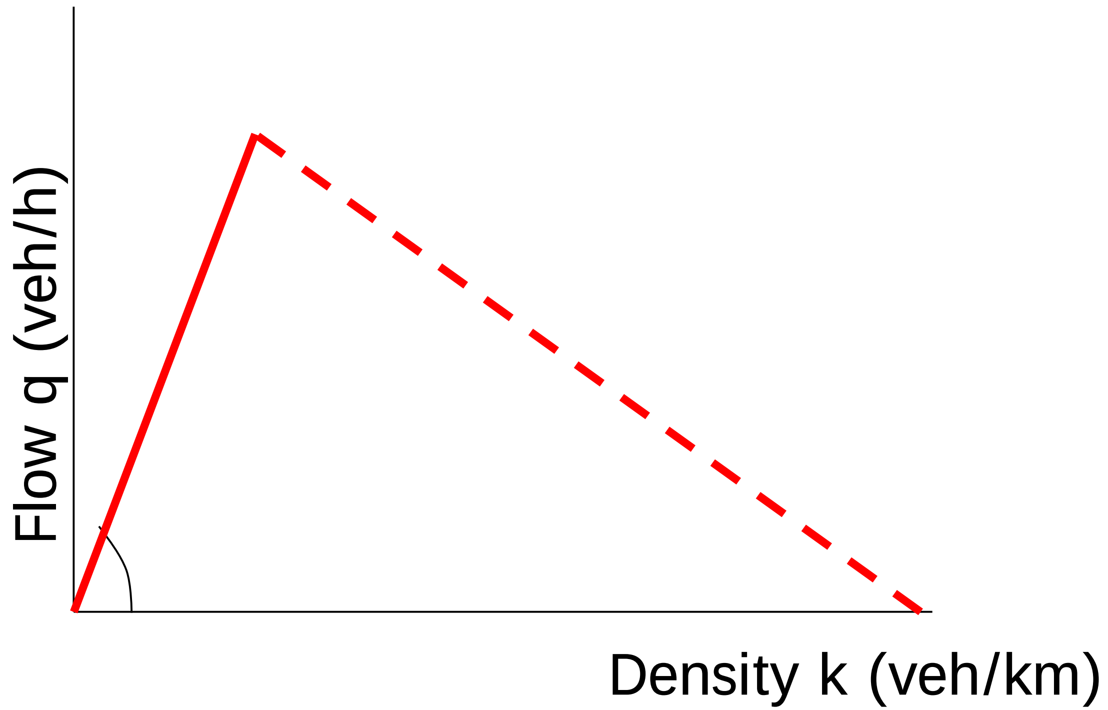
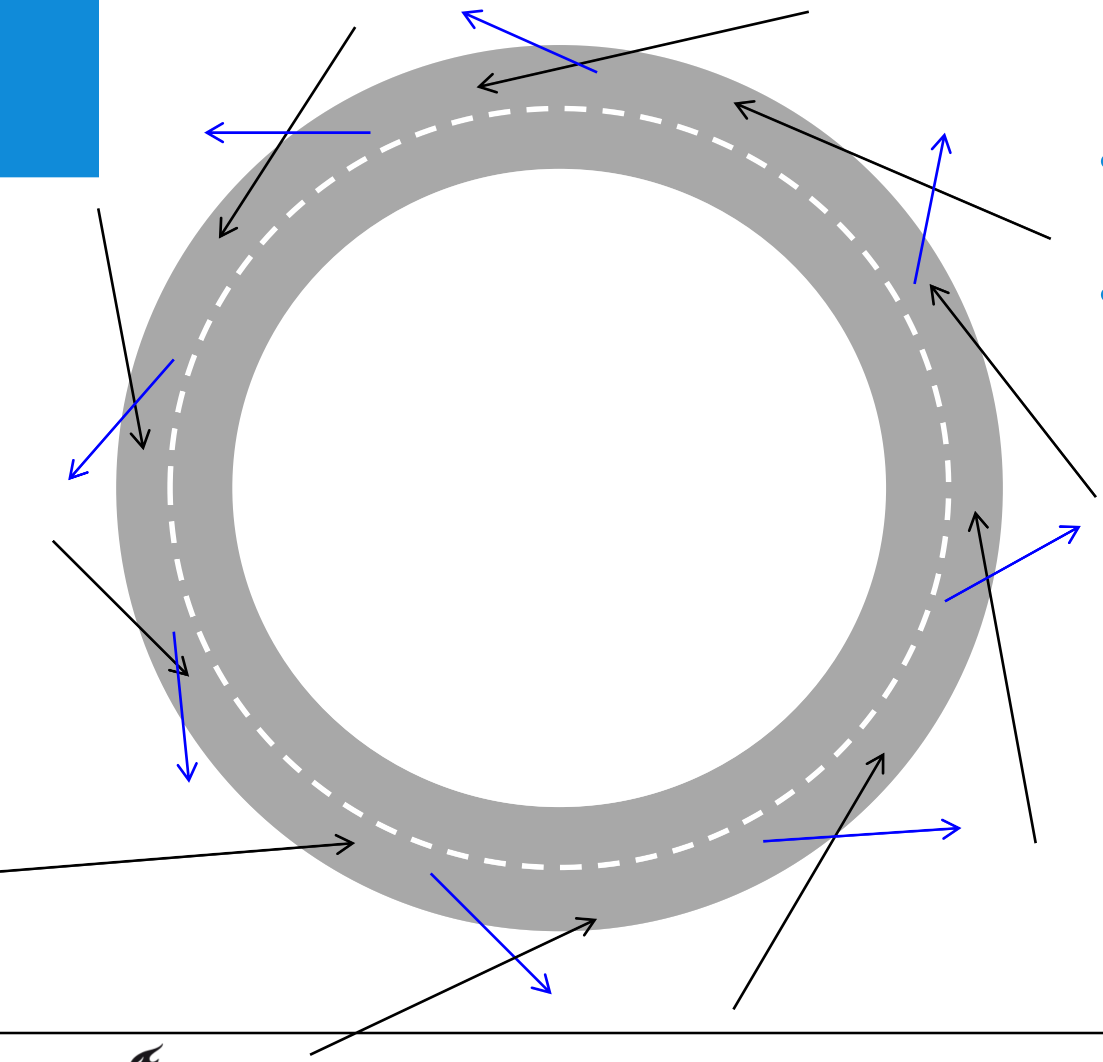
Simple road with varying demand





Not so simple road

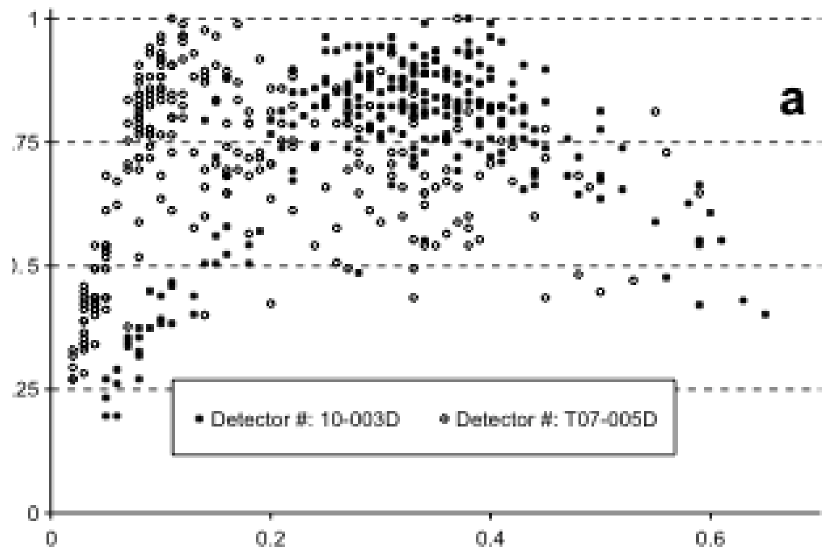


- Origins and destinations everywhere
- By increasing input => **congestion**

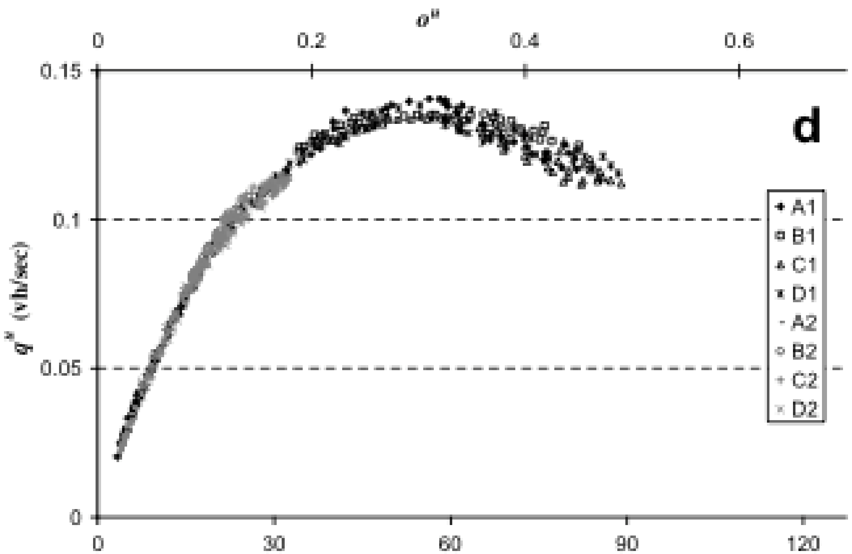
In an area

- Make an average fundamental diagram
  (but the mechanism behind it differs from FD)

(Average) flow

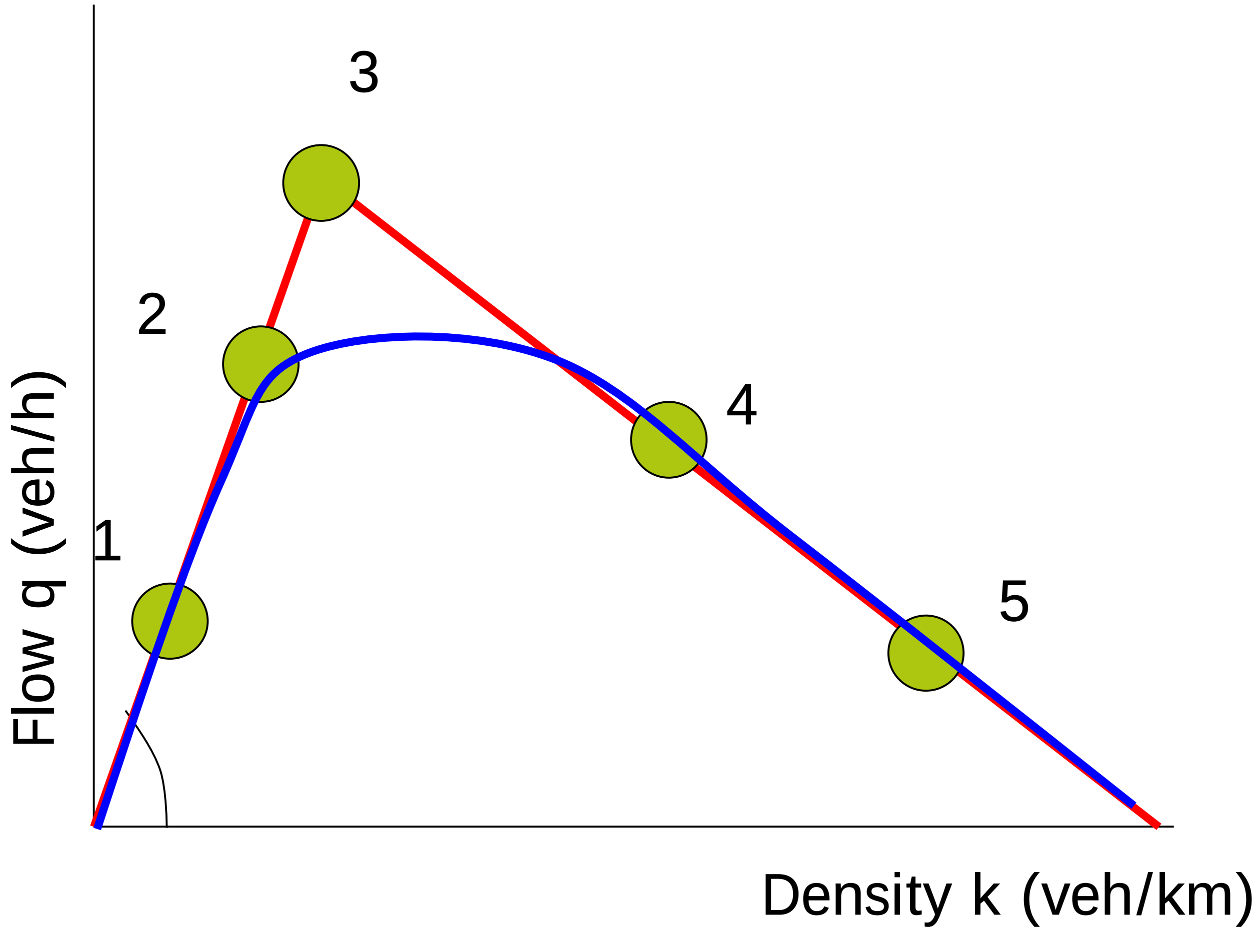


Density

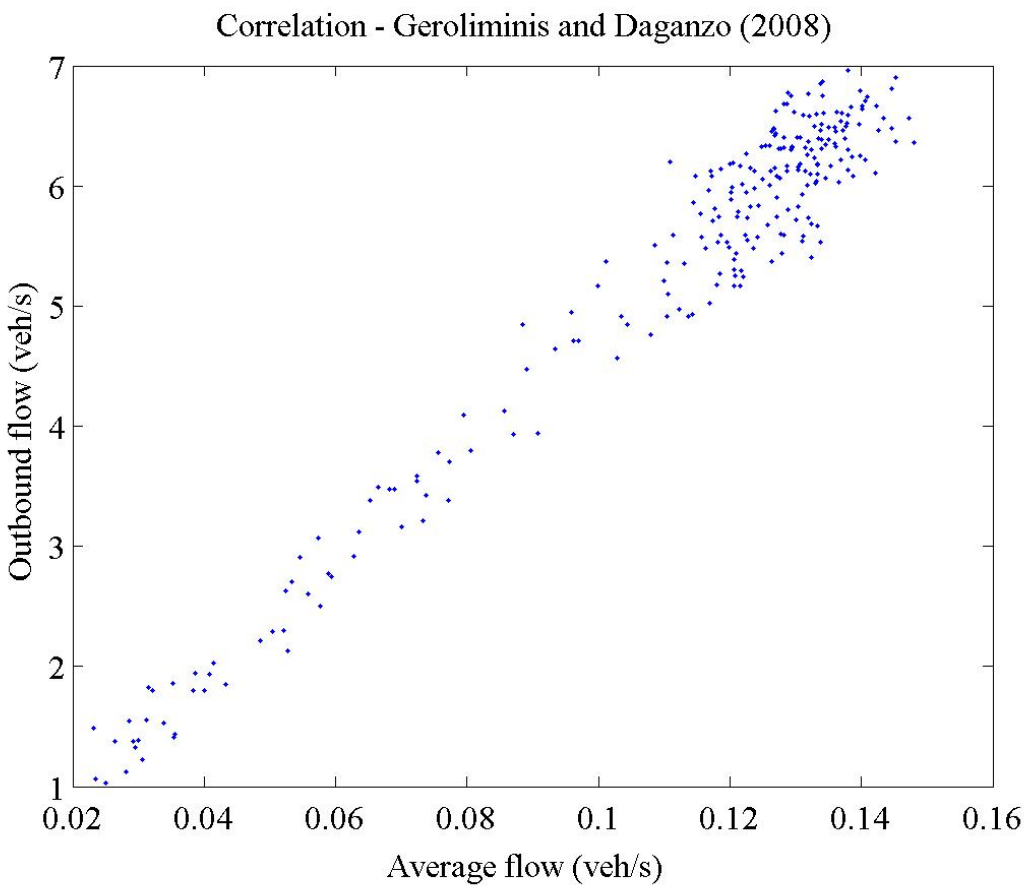


Average density

Averaging traffic states

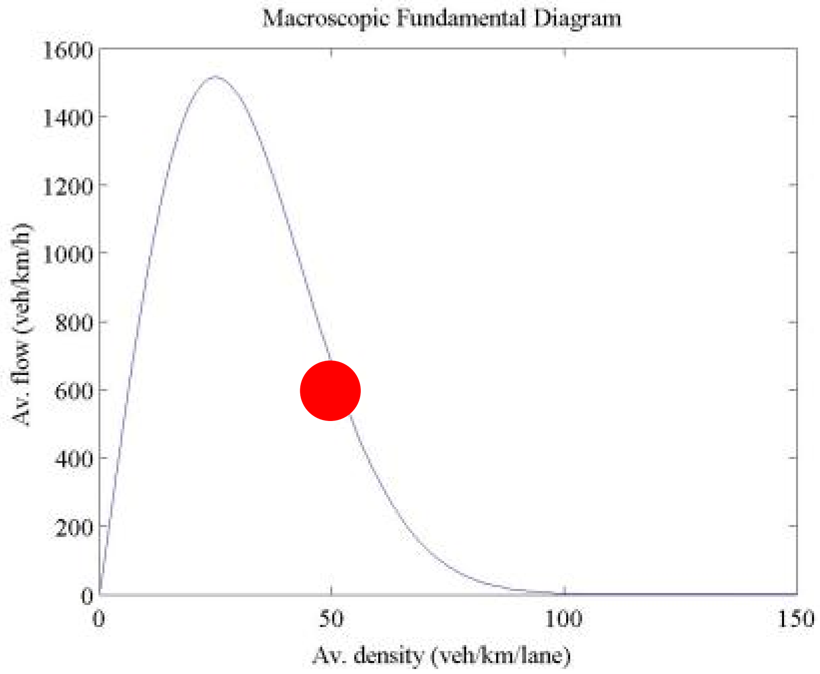


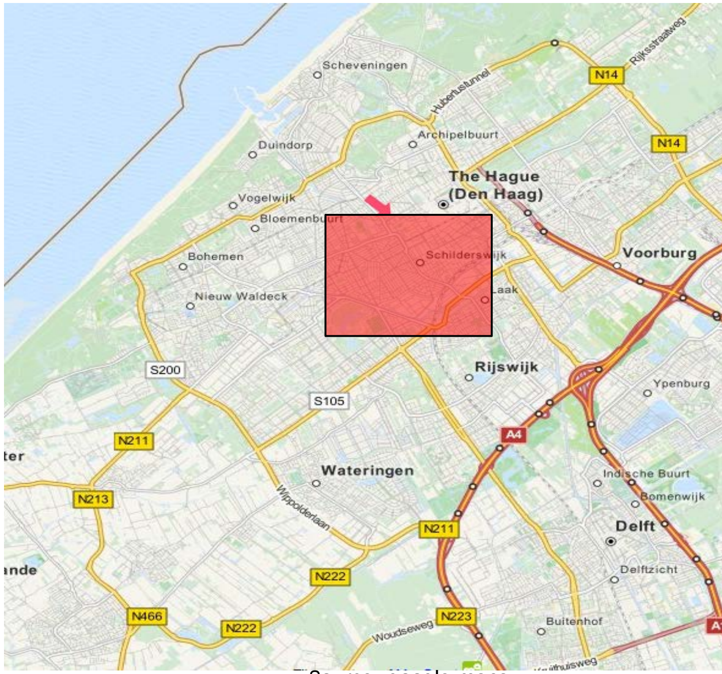
Relation performance - production



Perimeter control

- Reduce input to an area to improve throughput



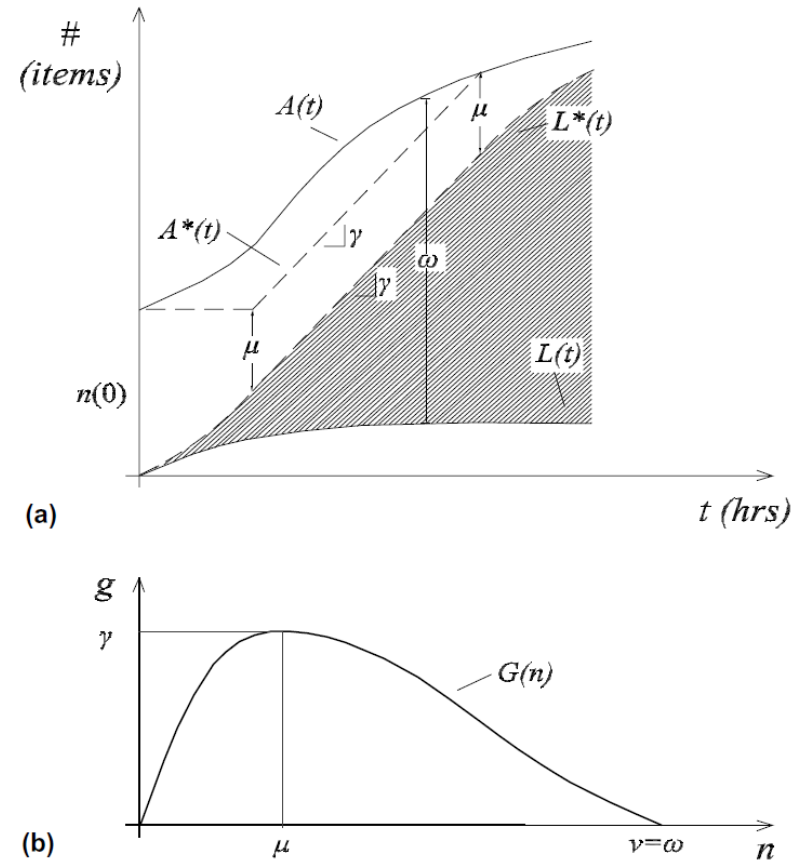


Source: google maps

Exercise for at home (exam practice)

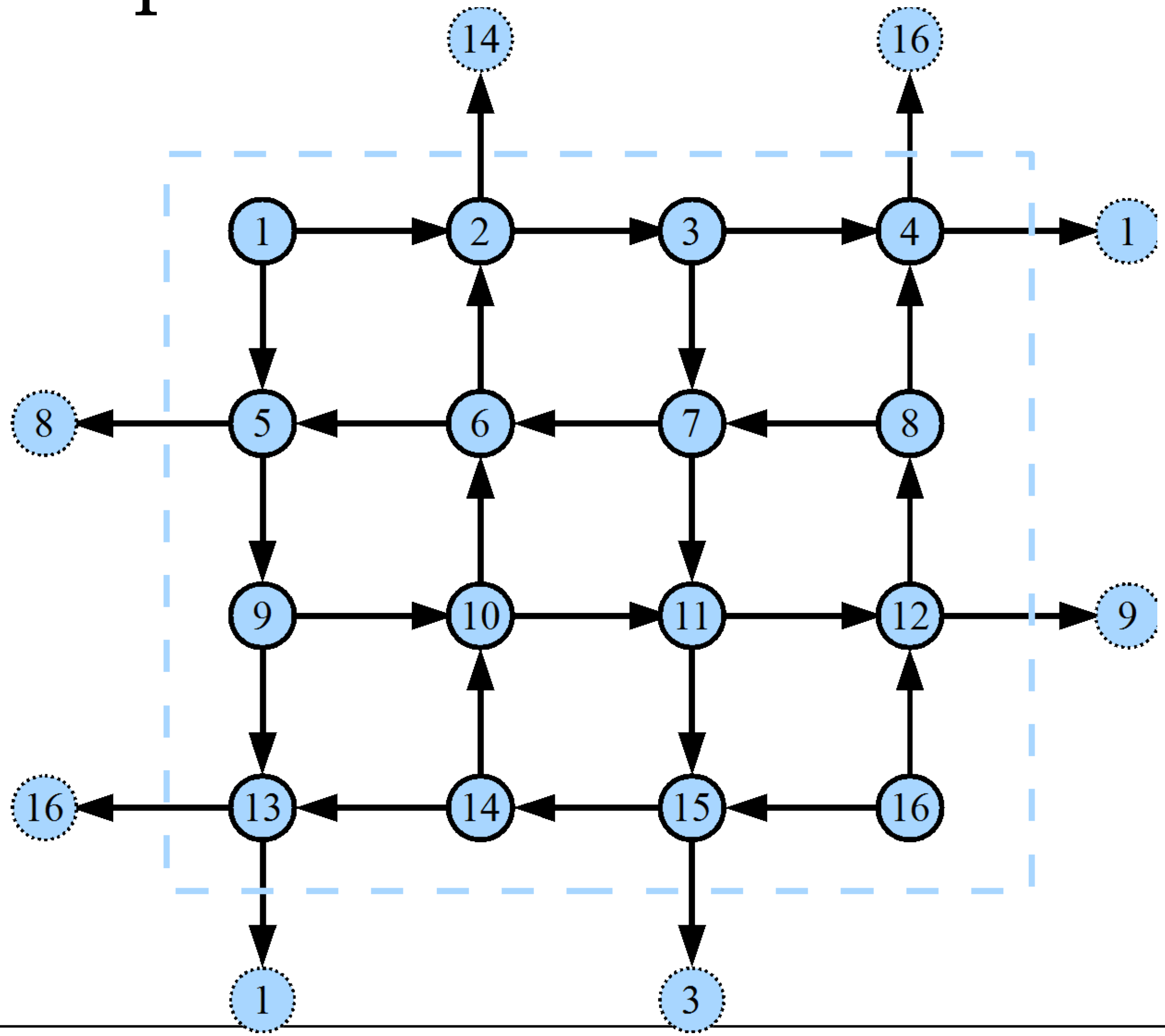
- Consider ring road slide 5
- Suppose
  -an outflow governed by a MFD (fig b)
  -instantaneous density changes
  -an inflow curve A (see picture)

- Construct the cumulative curves, and calculate the delay
- How can this be improved with inflow reduction (and by how much)
- (See fig for answer, Daganzo2005)

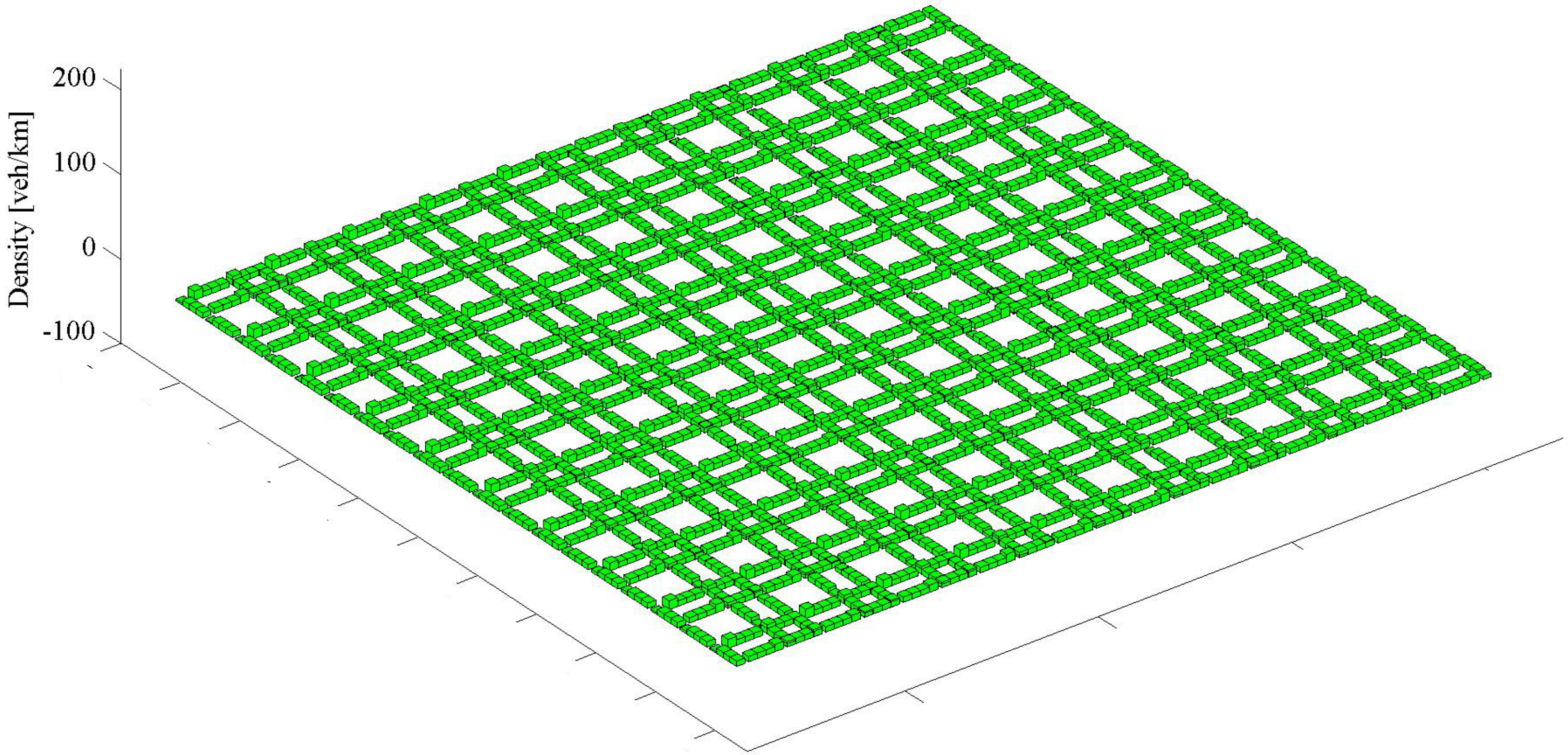


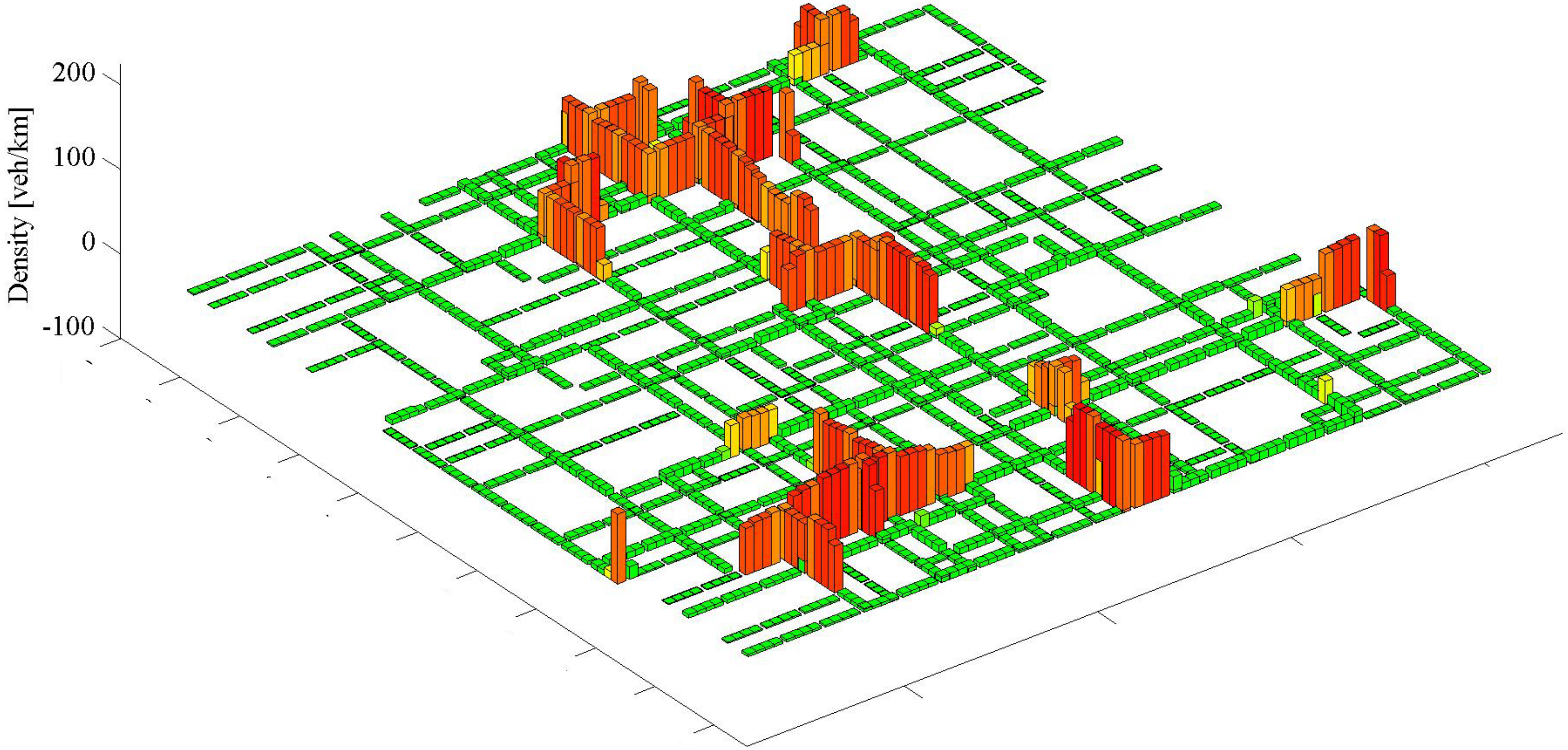
. 5. Gridlock development and control: (a) queuing diagram; (b) exit fu

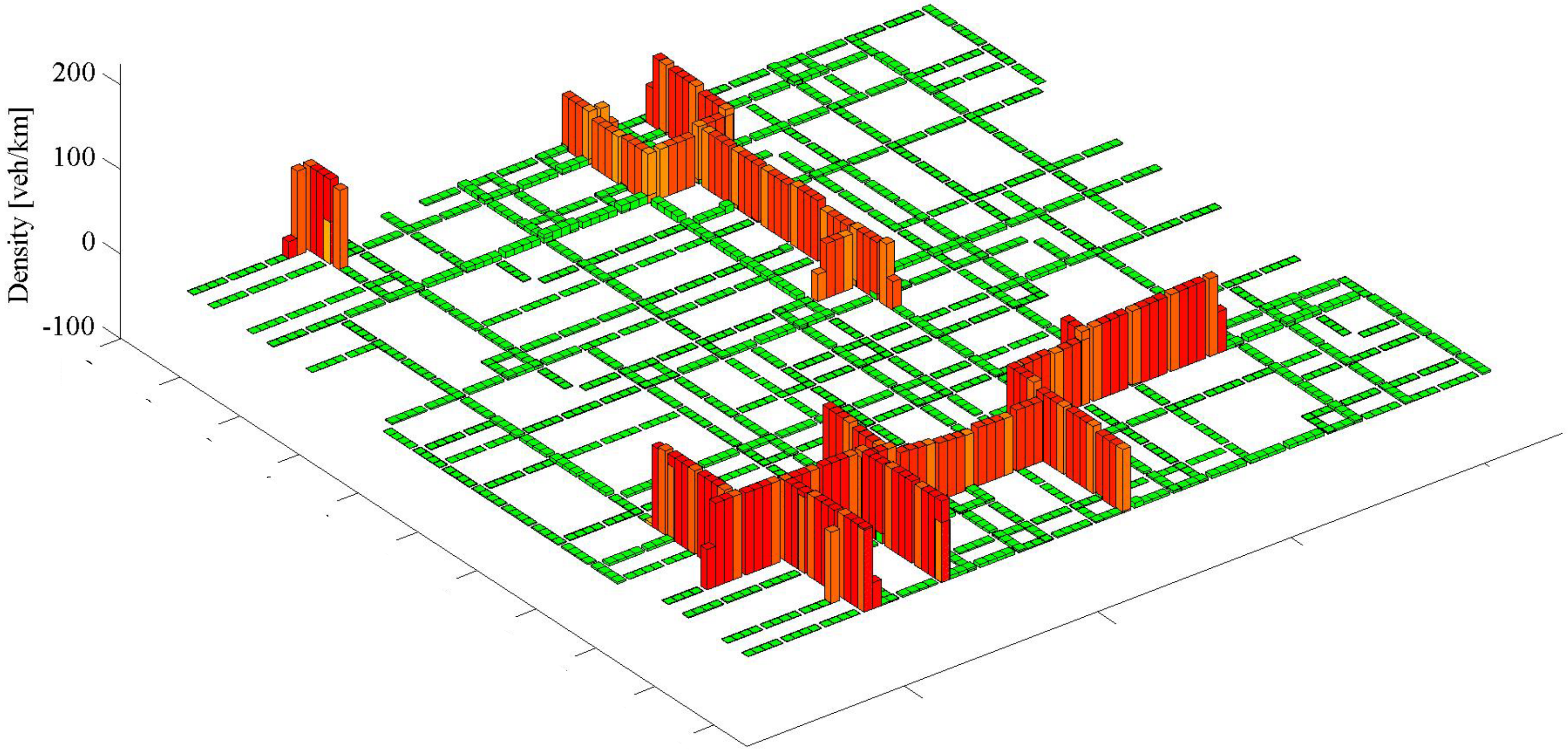
Network periodic boundaries



Build up of congestion




Density [veh/km]
200
100
0
-100

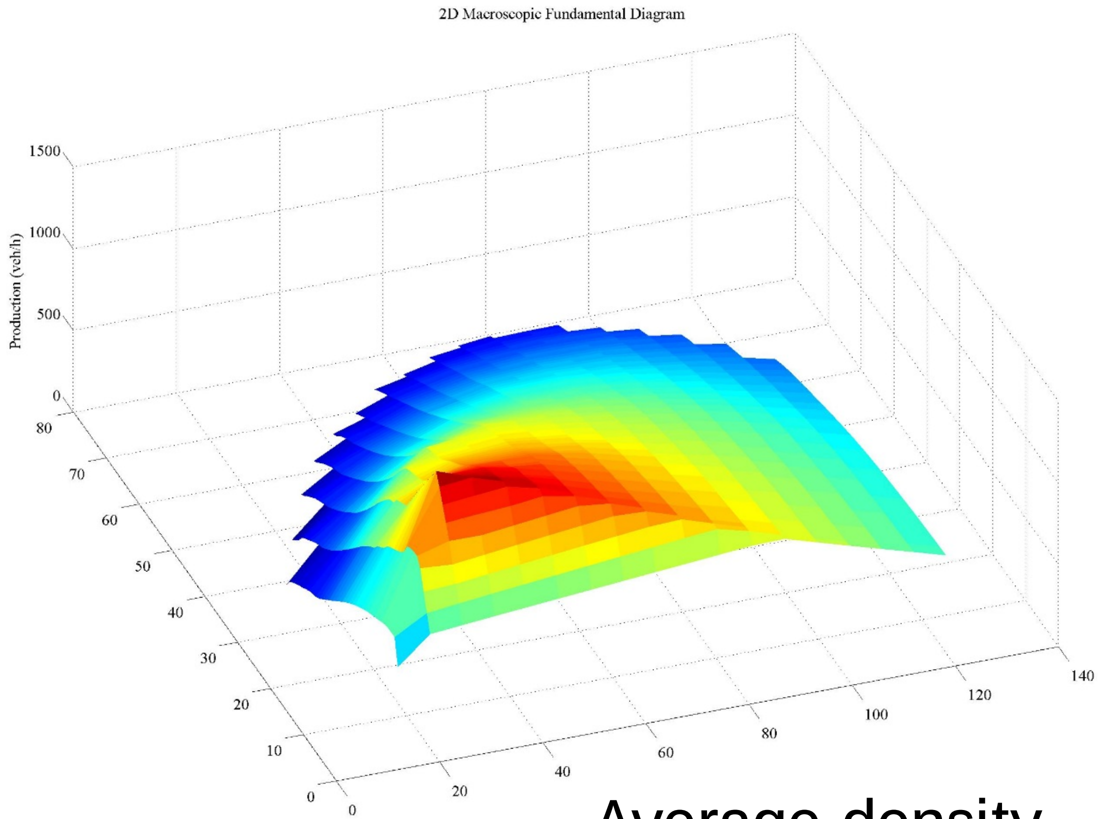

200
100
0
-100
Density [veh/km]

Phenomena

- Congestion triggered at nucleation points
- Performance descreases with congestion
  (but average density remains constant)

- What does this mean for the MFD
  a) Not influenced
  b) Cannot hold
  c) Needs to be modified

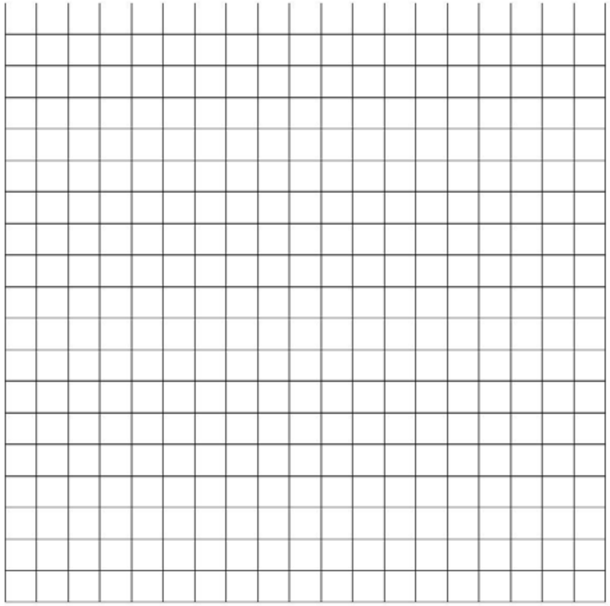
Generalised Macroscopic Fundamental Diagram

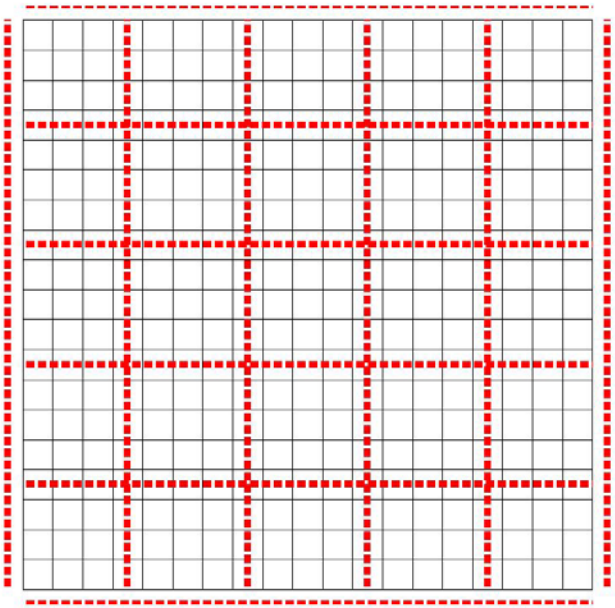


Spread

Average density

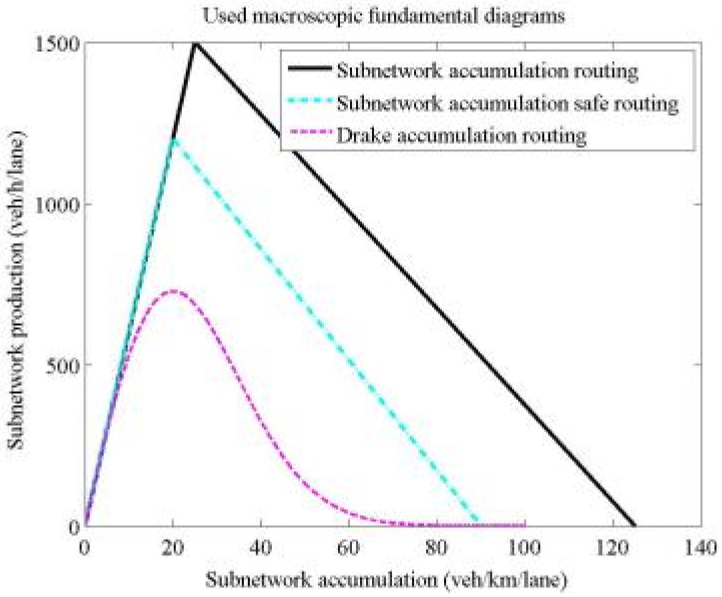
Create subnetworks



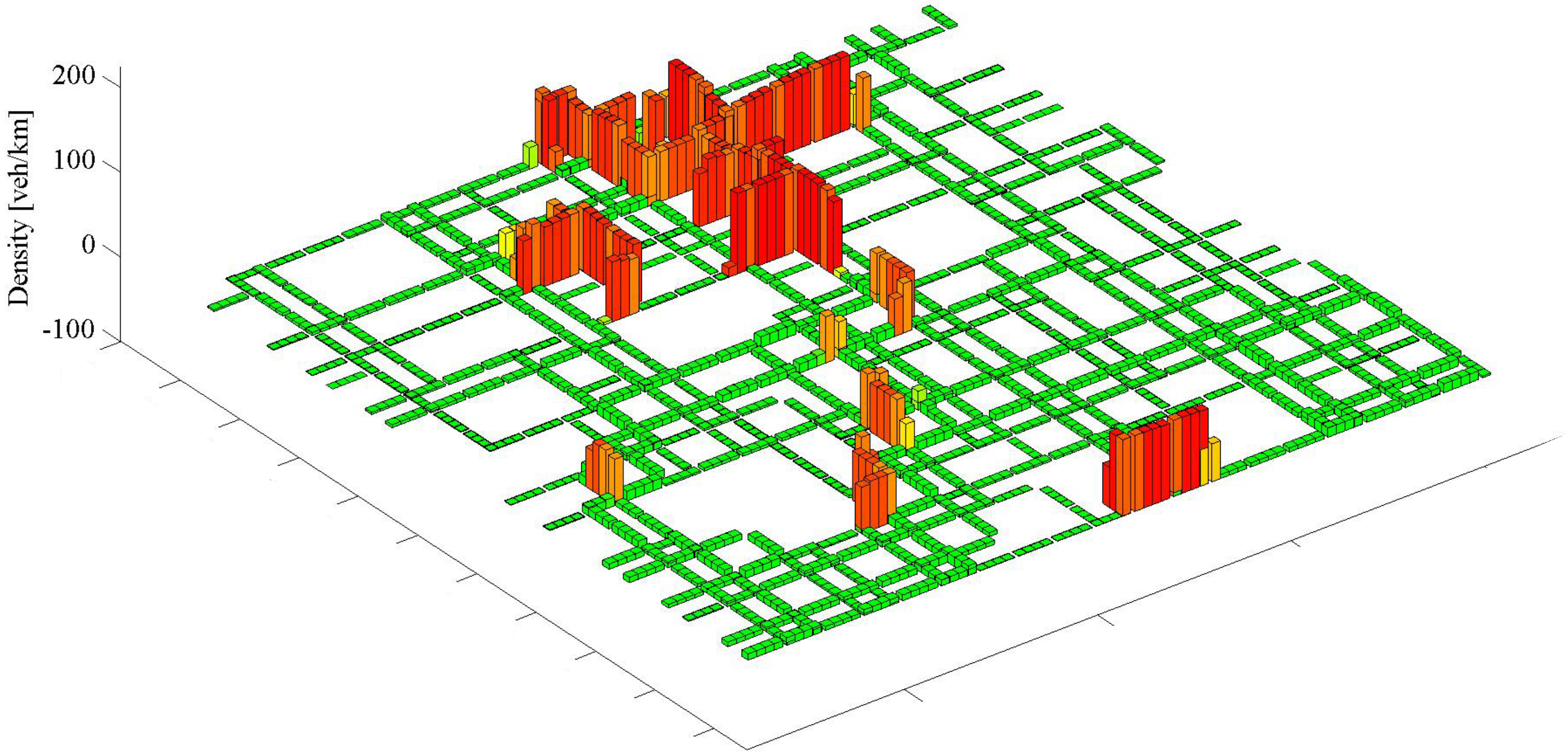


Assume subnetwork speeds

1. Shortest path (distance)
2. Shortest path (time)
3. Area-based
   a) Average speed
   b) Macroscopic Fundamental Diagram approaches
      - Same as links
      - Lower in flow
      - With tail



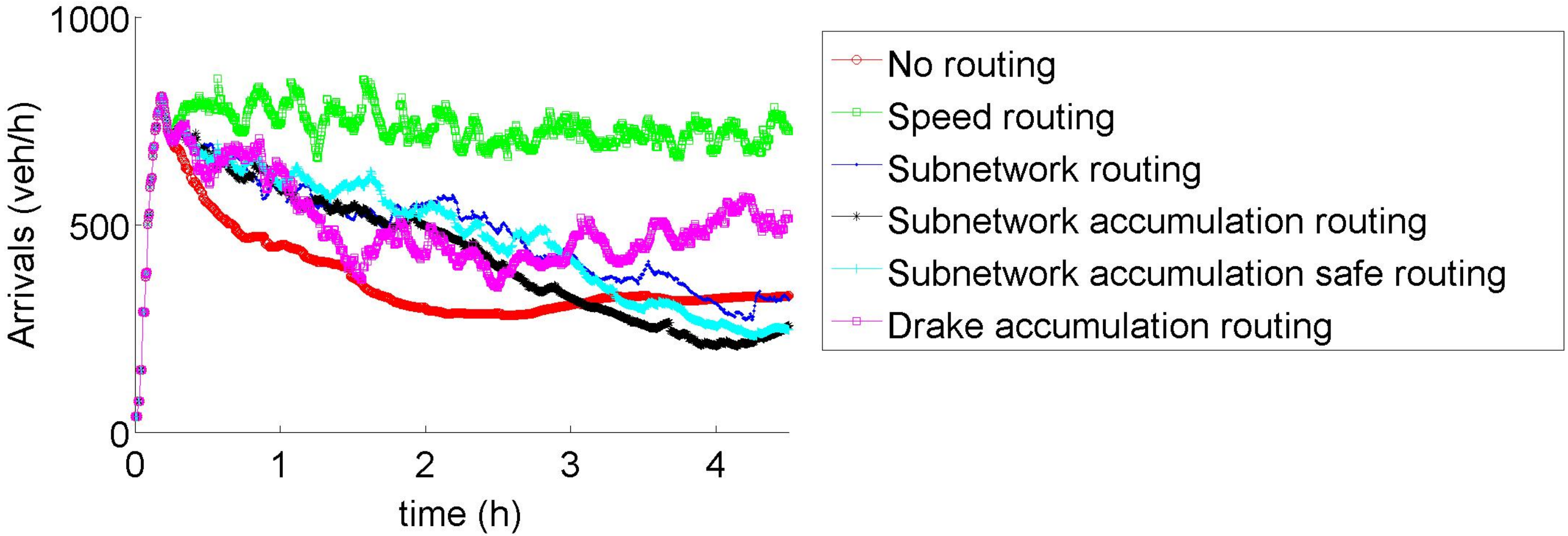
Build up of congestion



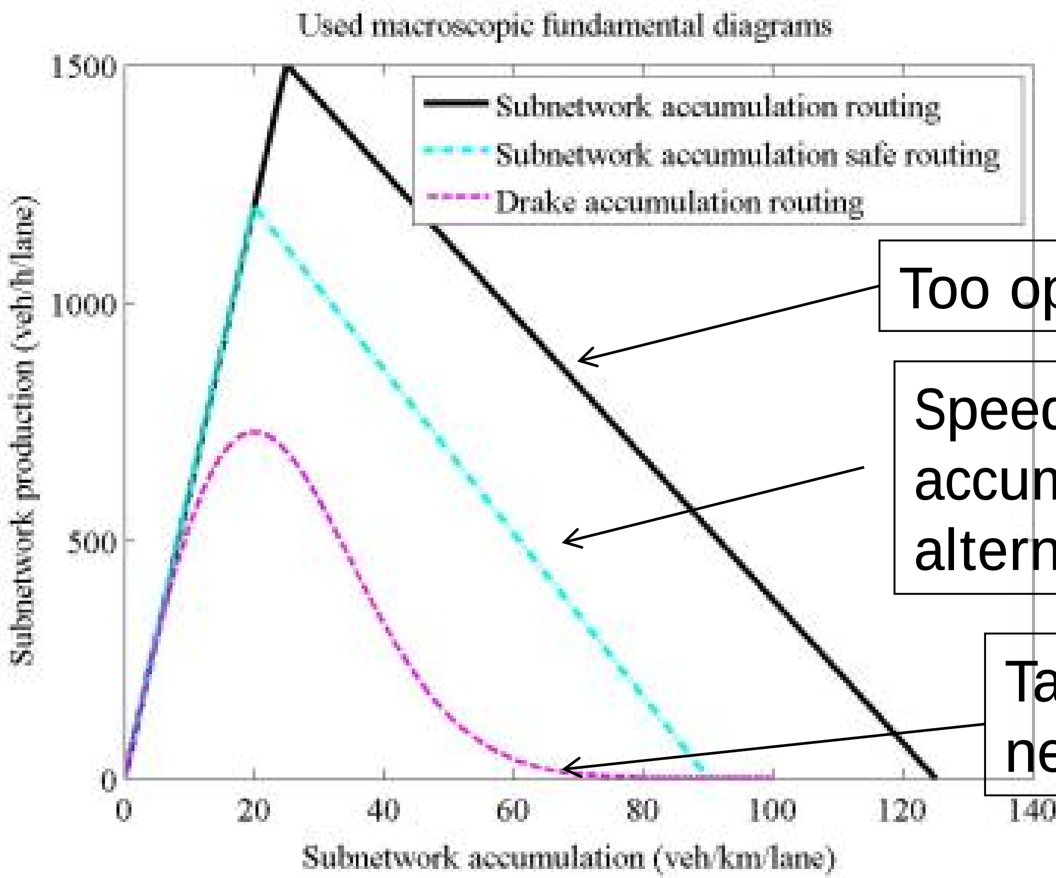
Build up of congestion

Good network performance

- Less than with “full routing”
- Factor 100 less data / computations needed



Which MFD works best



Too optimistic

Speed 0 from too low accumulations: no alternatives

Tail useful (but not necesserity “Drake”)

Summary + learning goals

- A macroscopic fundamental diagram exists
- It lies lower than the regular fundamental diagram, due to state mixing
- Traffic states are locally correlated

- You can:

1. Analyse and comment on the form of the MFD, based on the underlying queuing patterns
2. Make the question at slide 9