Traffic Flow Theory and Simulation

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Lecture 8
Influence of Trucks, Lagrangian Coordinates and MFD
Influence of trucks and Lagrangian coordinates

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Recap of last time: Godunov scheme

- **Demand** of cell A and **supply** of cell B

  \[
  D_L = \begin{cases} 
  q_A & k < k_c \\
  c & k \geq k_c 
  \end{cases} \quad S_R = \begin{cases} 
  c & k < k_c \\
  q_r & k \geq k_c 
  \end{cases}
  \]

- \( D_L \) = maximum number of vehicles that can flow out of L (bounded by the capacity of the road)
- \( S_R \) = maximum number of vehicles that can flow into R (bounded by road capacity and the space becoming available during one time-step)
- Actual flow at \( x=0 \): \( \min(D_L, S_R) \)
Godunov graphically

- Flow based on Demand & Supply
- $\Rightarrow$ fundamental diagram
Example application Godunov
Eulerian & Lagrangian Coordinates

At a fixed point...

Travel along with moving vehicles...

Influence of trucks and Lagrangian coordinates
Eulerian & Lagrangian Coordinates

- **Euler**: coordinates fixed in space \((x, t)\)
- **Lagrange**: coordinates move with traffic \((n, t)\)

Leonhard Euler

Joseph Louis Lagrange
Lagrangian formulation

Conservation equation (Eulerian)

\[ \frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0 \]

\( \rho \) density [veh/m]

\( q \) flow [veh/s]

Lagrangian formulation

\[ \frac{D_s}{D_t} + \frac{\partial v(s)}{\partial n} = 0 \]

\( s = 1 / \rho \) spacing [m/veh]

\( v = q / \rho \) speed [m/s]

\( n \) vehicle number [veh]

\( D/D_t: \) Lagrangian time derivative

(distinguish from Eulerian case)
Interpretation and Derivation
Langrangian Formulation

Variation of platoon length

\[ s_{\text{new}} - s_{\text{old}} = (v_{\text{first}} - v_{\text{last}}) dt \]

\[ \frac{ds}{dt} = - \frac{dv}{dn} \]

\[ \frac{\partial s}{\partial t} + \frac{\partial v}{\partial n} = 0 \]
Fundamental relations

Eulerian (Daganzo) Lagrangian

\[ s = \frac{1}{\rho} \]
\[ v = \frac{q}{\rho} = q \cdot s \]
\[ q = \rho \cdot v \]

What is \( s_{\text{min}} \)?
A) \( \sim 1.5 \) seconds
B) \( \sim 100 \) km/h
C) \( \sim 5 \) meters
Information propagation in Lagrangian coordinates

Traffic characteristics move in groups of vehicles in the downstream direction (increasing vehicle number instead of space)

Vehicles only react to vehicles in front of them

- Numerical solution: An upwind scheme [less time-consuming]
What happens if \( dn = 1 \)?

- In Lagrangian coordinates, one studies platoons of \( dn \) vehicles
- Will the model give output if \( dn = 1 \)
  - A. No, vehicles are not conserved
  - B. Yes, but the results are non-sensible (negative spacings)
  - C. Yes, we obtain a microscopic model
What happens if $dn=1$?

- Yes, we obtain a microscopic model
- The choice of the fundamental relation is important
- Microscopic interpretation of macroscopic model
  (See Newell’s simplified model, later in the course)

- Also possible:
  $dn=3.5$, $dn=0.1$, $dn=\pi$

- Not possible:
  $dn=0$, $dn<0$
Accurate simulation results

**Euler**

**Lagrange**
Example application Godunov
Advantages of Lagrangian formulation

- Easy numerical discretization
- Fast simulation and accurate results
- Real-time applications
- Run iteratively in real time to test best control setting
- Easy switching between macro and micro level (dn=1)
Multiple vehicle classes in traffic
Macroscopic modeling of heterogeneity (1) class-specific conservation laws / FD

- Conservation of vehicles equation (Class-specific)

\[
\frac{\partial k_u}{\partial t} + \frac{\partial q_u}{\partial x} = 0
\]

\[
q_u = k_u v_u
\]

\[
v_u = V_u (k),
\]

\[
k = \sum_{U} k_u
\]
Macroscopic modeling of heterogeneity (2) PCE values

- Usually heavy vehicles are considered the equivalent of $X$ person cars
  E.g. 1 truck = $X$ pce (person-car equivalents)
- Ideas on which factors determine $X$?
  - Geometry: slope, grade, curvature
  - Vehicle characteristics
  - Truck percentage itself
  - **Speed!** – we will return to this shortly …
Spacing and the effect of heavy vehicles

- Spacing under light traffic

Gross distance gap trucks

Gross distance gap passenger cars
Spacing and the effect of heavy vehicles

- Spacing under congested traffic

- Relative space occupation trucks under congestion is larger (in the limit: think of a parking lot)
Fastlane equations and basic ideas

- Class-specific conservation of vehicles equation
  \[
  \frac{\partial k_u}{\partial t} + \frac{\partial q_u}{\partial x} = 0
  \]
  \[q_u = k_u v_u\]

- Class specific equilibrium relation – in Fastlane function of effective density in pce
  \[v_u = V_u (K),\]
  \[K = \sum_{U} \eta_u k_u\]
State-dependent pce values

Pce value is dynamic:

\[ \eta_u(t,x) = \frac{s_u + T_u \cdot v_u(t,x)}{s_{u_0} + T_{u_0} \cdot v_{u_0}(t,x)} \]
State-dependent pcu values

Illustration of pcu under different speeds

\[ \eta_u(t, x) = \frac{s_u + T_u \cdot v_u(t, x)}{s_{u0} + T_{u0} \cdot v_{u0}(t, x)} \]

Maximum speed truck = 85 km/h

Parameters

- \( s_{p,\text{car}} = 7.5 \text{ [m]} \), \( T_{p,\text{car}} = 1.2 \text{ [s]} \)
- \( s_{\text{truck}} = 15.0 \text{ [m]} \), \( T_{\text{truck}} = 1.8 \text{ [s]} \)
State-dependent pcu values

Illustration of pcu under different speeds

Maximum speed truck = 85 km/h

Parameters

$s_{\text{p.car}} = 5.0 \text{ [m]}$, $T_{\text{p.car}} = 1.2 \text{ [s]}$
$s_{\text{truck}} = 18.0 \text{ [m]}$, $T_{\text{truck}} = 2.0 \text{ [s]}$

\[ \eta_u(t, x) = \frac{s_u + T_u \cdot v_u(t, x)}{s_{u_0} + T_{u_0} \cdot v_{u_0}(t, x)} \]
Example (1)

Consider straight 25 km two-lane freeway stretch

- Platoon of trucks (1500 veh/h) enters 9:00-9:01
- Platoon of person cars (1500 veh/h) enters 9:04-9:05
Example (2)

Same straight 25 km two-lane freeway stretch
- Platoon trucks + cars with fixed composition enters at origin 8:30-9:00
- Bottleneck (e.g. lane-blockage) halfway
Example (3)

Same straight 25 km two-lane freeway stretch
- Platoon trucks + cars with fixed composition enters at origin 8:30-9:00
- Bottleneck (e.g. lane-blockage) halfway
- **Effect of dynamic PCE values**

Static case:

\[ \eta_u(t, x) = \eta_u = \frac{s_u}{s_{u0}} \]
Example (3)

Same straight 25 km two-lane freeway stretch
- Platoon trucks + cars with fixed composition enters at origin 8:30-9:00
- Bottleneck (e.g. lane-blockage) halfway
- **Effect of dynamic PCE values**

\[
\eta_u(t, x) = \frac{s_u + T_u \cdot v_u(t, x)}{s_{u_0} + T_{u_0} \cdot v_{u_0}(t, x)}
\]
Learning goals

• Now, you can…
  1. … describe traffic in Lagrangian coordinates (but the exam will not question dynamic equations)
  2. … explain the advantages of Lagrangian formulation
  3. … comment on the effect of traffic heterogeneity on traffic operations and models
  4. … make calculations using (dynamic) pce-values
Same traffic model and problems

Solution to Acc. Fan & Shockwave by MoC in Eulerian Coordinates

Solution to Acc. Fan & Shockwave by MoC in Lagrangian Coordinates
(Vertical axis: vehicle number)
Advantages of Lagrangian formulation

- Relate to simulations???
- Discretized KW model (Euler)
  - space is discretized into cells of length $dx$
  - time is discretized into time period of size $dt$
- Numerical solution: Godunov scheme (min supply demand)

\[
\frac{\rho_{i+1}^i - \rho_i^i}{dt} + \frac{q_{i+\frac{1}{2}} - q_{i-\frac{1}{2}}}{dx} = 0
\]
Introduction to the Macroscopic Fundamental Diagram

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Victor L. Knoop
Simple road with increasing demand

What happens if the demand increases
What observations can be made?

A) Whole FD
B) Increasing part of FD (free flow)
C) Decreasing part of FD (congestion)
Simple road with varying demand

Flow $q$ (veh/h) vs. Density $k$ (veh/km)
Not so simple road

- Origins and destinations everywhere
- By increasing input \(\Rightarrow\) congestion
In an area

- Make an average fundamental diagram
  (but the mechanism behind it differs from FD)
Averaging traffic states

Flow $q$ (veh/h) vs. Density $k$ (veh/km)
Relation performance - production

Correlation - Geroliminis and Daganzo (2008)

Outbound flow (veh/s)

Average flow (veh/s)
Perimeter control

- Reduce input to an area to improve throughput
Exercise for at home (exam practice)

• Consider ring road slide 5
• Suppose
  - an outflow governed by a MFD (fig b)
  - instantaneous density changes
  - an inflow curve A (see picture)

• Construct the cumulative curves, and calculate the delay
• How can this be improved with inflow reduction (and by how much)
• (See fig for answer, Daganzo2005)
Network periodic boundaries
Build up of congestion
Influence of trucks and Lagrangian coordinates
Phenomena

- Congestion triggered at nucleation points
- Performance decreases with congestion (but average density remains constant)

- What does this mean for the MFD
  a) Not influenced
  b) Cannot hold
  c) Needs to be modified
Generalised Macroscopic Fundamental Diagram

Spread

Average density
Create subnetworks
Assume subnetwork speeds

1. Shortest path (distance)
2. Shortest path (time)
3. Area-based
   a) Average speed
   b) Macroscopic Fundamental Diagram approaches
      - Same as links
      - Lower in flow
      - With tail
Build up of congestion
Build up of congestion
Good network performance

- Less than with “full routing”
- Factor 100 less data / computations needed
Which MFD works best

Used macroscopic fundamental diagrams

- Subnetwork accumulation routing
- Subnetwork accumulation safe routing
- Drake accumulation routing

Too optimistic
Speed 0 from too low accumulations: no alternatives
Tail useful (but not necessity “Drake”)
Summary + learning goals

• A macroscopic fundamental diagram exists
• It lies lower than the regular fundamental diagram, due to state mixing
• Traffic states are locally correlated

• You can:
  1. Analyse and comment on the form of the MFD, based on the underlying queuing patterns
  2. Make the question at slide 9