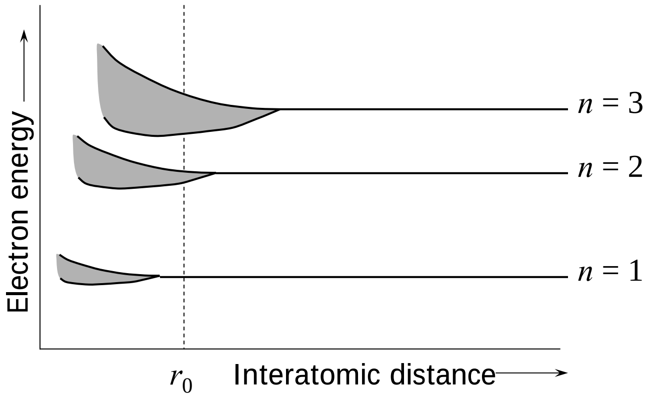
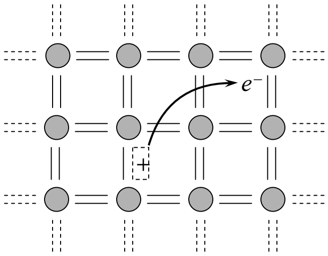
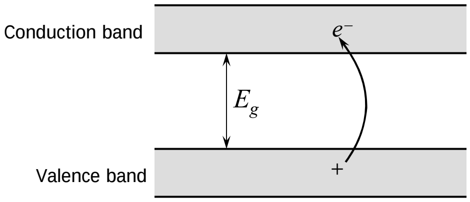
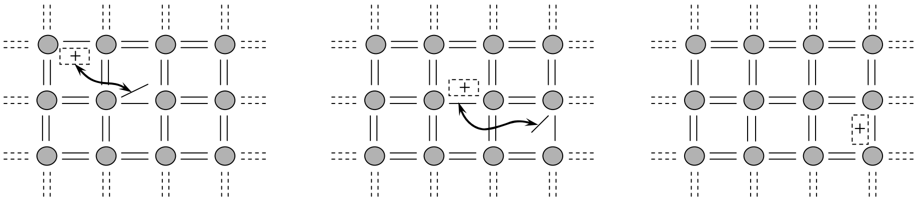
Semiconductor Components and Technology

ET4 249



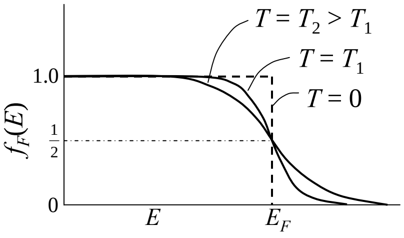


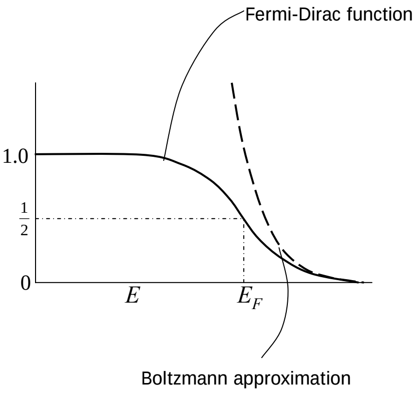


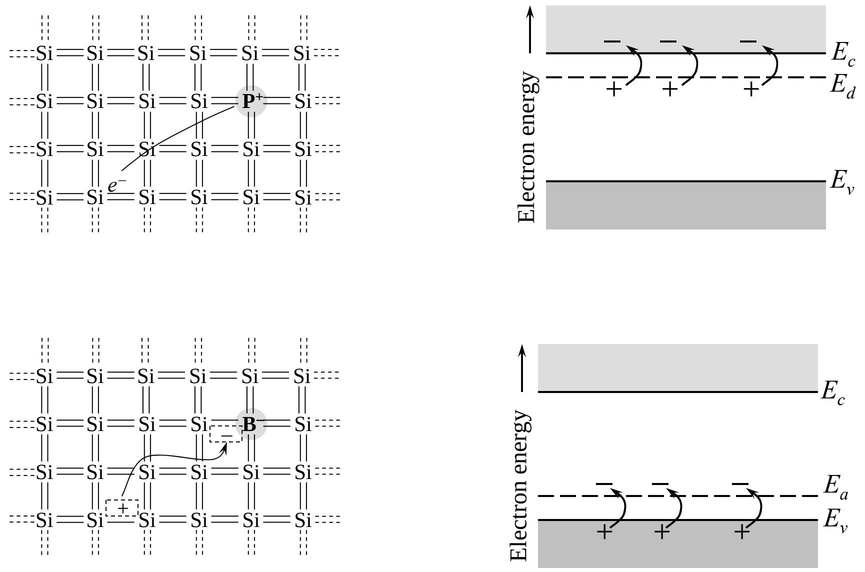


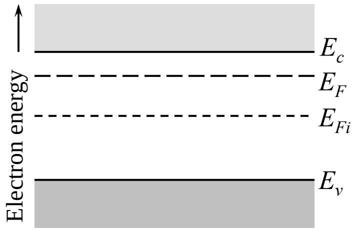
$$f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

$$f_F(E) \approx \exp\left[\frac{-(E-E_F)}{kT}\right]$$





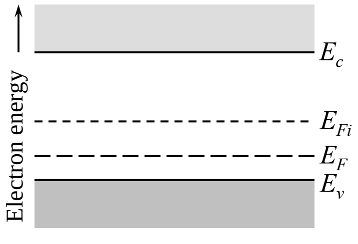


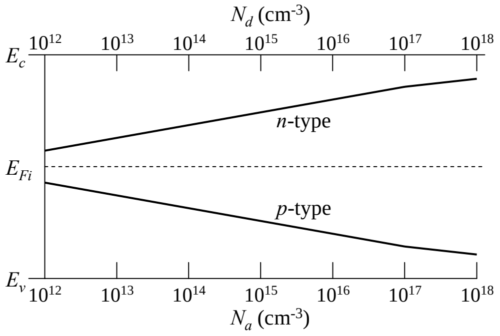


$$n_0 = \int g_c(E) f_F(E)\,dE \qquad = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$p_0 = \int g_v(E)\left[1 - f_F(E)\right]dE = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

Boltzmann approximation





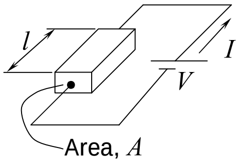
$$n_i^2 = N_C N_V \exp\left(-\frac{E_g}{kT}\right)$$ ← material characteristic

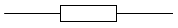
$$n_i^2 = n_0 p_0$$ ← Valid if material is in thermal equilibrium and if Boltzmann approximation is valid!

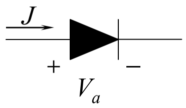
Application: Consider silicon with N_d = 10^{18} cm^{-3}. Determine the free charge carrier concentration in thermal equilibrium

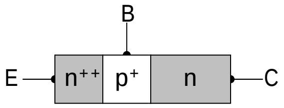
$$\left.\begin{array}{l} n_i = 1.5\times10^{10}\ \text{cm}^{-3};\quad n_i^2 = 2.25\times10^{20}\ \text{cm}^{-6} \\ n_0 \approx N_d = 1.0\times10^{18}\ \text{cm}^{-3} \end{array}\right\} \Rightarrow p_0 = \frac{n_i^2}{n_0} = 225\ \text{cm}^{-3}$$

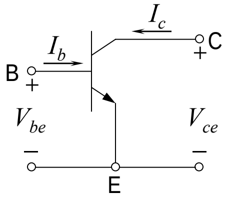
Conclusion: In this n-type material the free-holes concentration, minority charge carriers, is orders of magnitude smaller than that of electrons, the majority charge carriers.

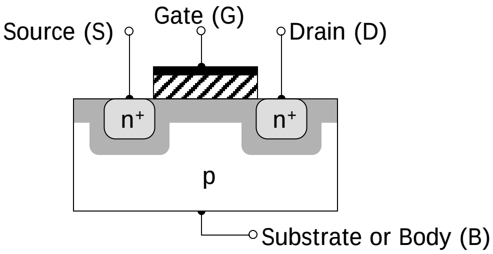


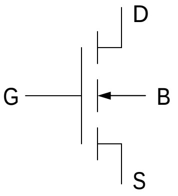






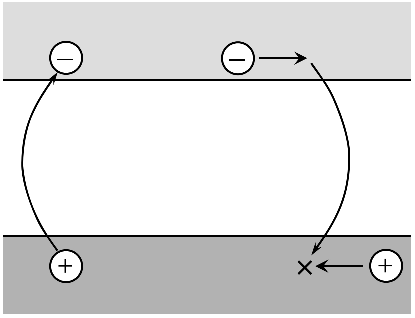






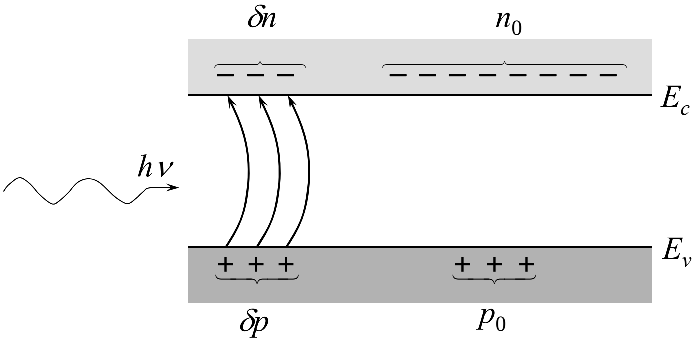
Generation/Recombination

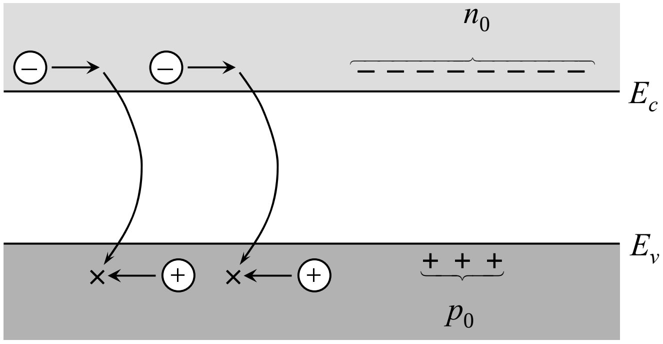
- In thermal equilibrium: thermal generation is equal to thermal recombination.
- In *static* equilibrium: *excess* generation equal to *excess* recombination.



Two generation-recombination processes

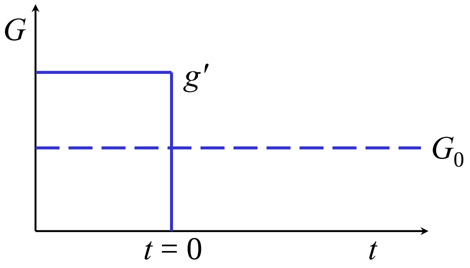
- Direct generation-recombination: generation and recombination of electron-hole pairs (thermal and as a result of other energy supply) across the forbidden energy band gap ⇒ “band-to-band recombination”.
- Generation and recombination via energy states in the forbidden energy band gap.





Study of recombination

Study recombination by investigating the effect of switching off a disturbance ⇒ find the relaxation time (life time) of the system to return to thermal equilibrium.



Determination of direct recombination

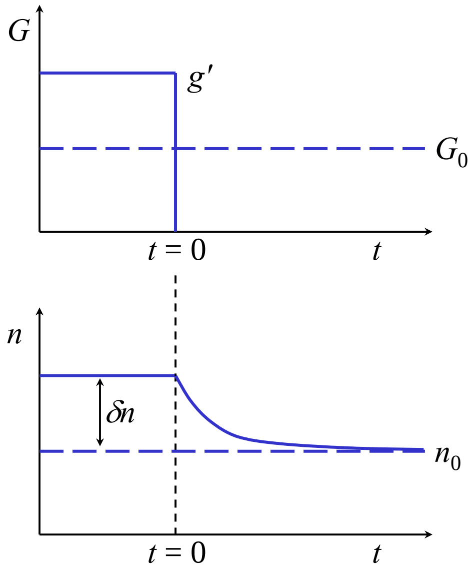
$$\frac{dn(t)}{dt} = g_{\text{thermal}} - \alpha_r n(t)p(t) = \alpha_r \left[n_i^2 - n(t)p(t) \right]$$

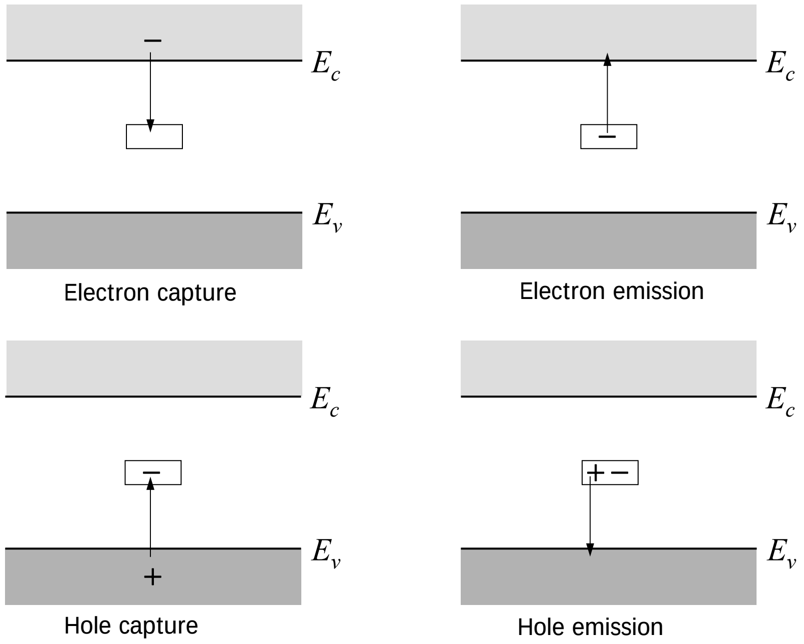
Low-level injection, p-type: $n_0 << \delta n << p_0$

$$\delta n(t) = \delta n(0)e^{-\alpha_r p_0 t} = \delta n(0)e^{-t/\tau_{n0}}$$

$$\tau_{n0} = \frac{1}{\alpha_r p_0}$$

$$R_n = R_p = \frac{\delta n(t)}{\tau_{n0}}$$





Recombination via states in the gap

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')} \equiv R$$

$$n' = N_C \exp\left[\frac{-(E_C - E_t)}{kT}\right]$$

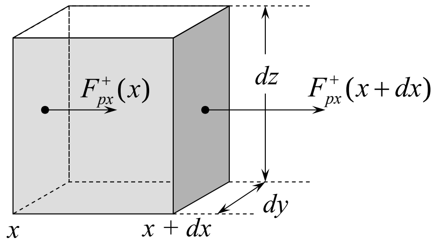
$$p' = N_V \exp\left[\frac{-(E_t - E_C)}{kT}\right]$$

Recombination via states in the gap

"Low-level" injection for n-type: $R = C_p N_t \delta p = \dfrac{\delta p}{\tau_{pt}}$

So: $$\tau_{pt} = \frac{1}{C_p N_t}$$

Here, C_p is proportional to the capture cross section for holes and N_t is the density of states in the gap (trap density).



$$\frac{\partial p}{\partial t} dxdydz = \left[F_{px}^{+}(x) - F_{px}^{+}(x+dx)\right] dydz = -\frac{\partial F_{px}^{+}}{\partial x} dxdydz$$

$$\boxed{\frac{\partial p}{\partial t} = -\frac{\partial F_{p}^{+}}{\partial x} + g_p - \frac{p}{\tau_{pt}}}$$

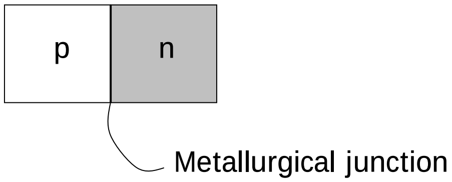
→ Defines also the lifetime of excess holes

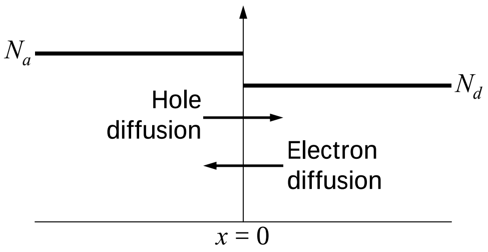
Ambipolar transport equation

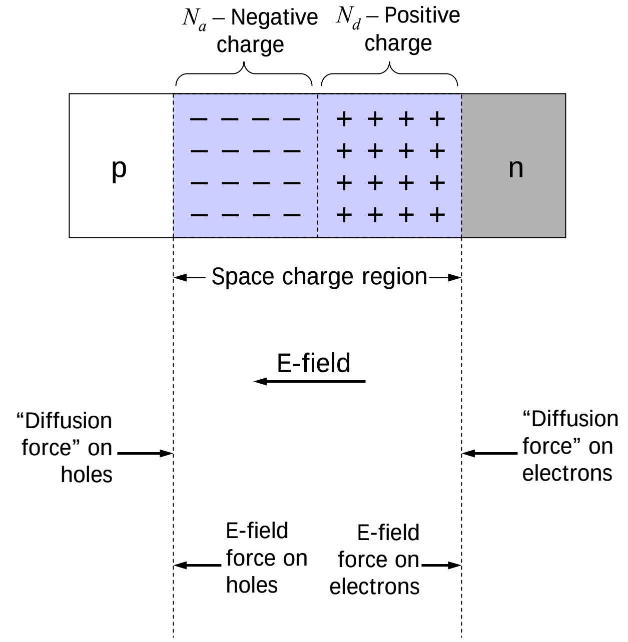
$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p)}{\delta x} + g_p - \frac{p}{\tau_{pt}} = \frac{\partial(\delta p)}{\partial t}$$

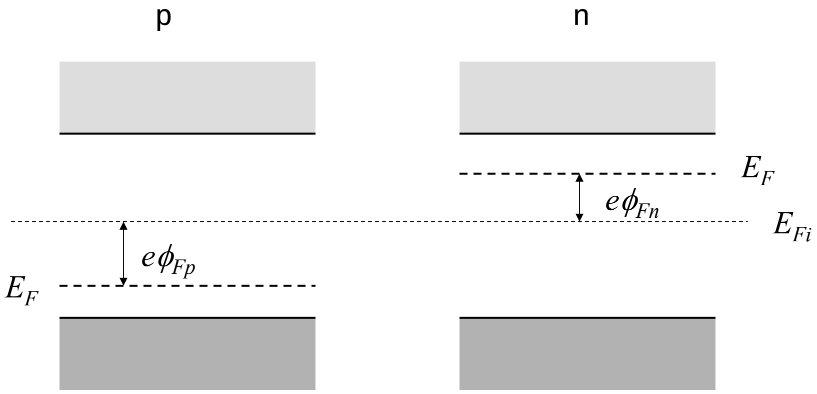
$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial(\delta n)}{\delta x} + g_n - \frac{n}{\tau_{nt}} = \frac{\partial(\delta n)}{\partial t}$$

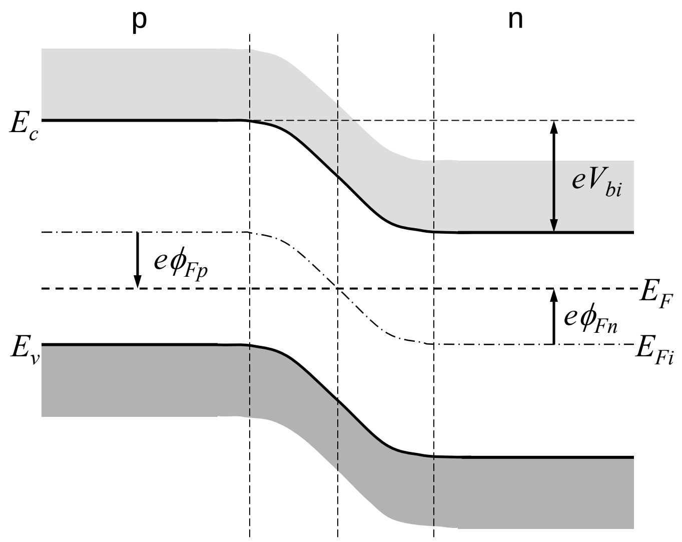
Important: the charge-carrier transport is determined by the minority charge carriers!!! Under the condition of low-level injection.



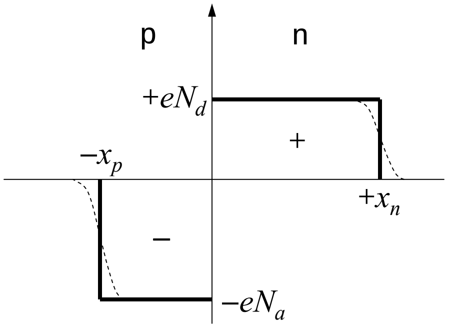



N_a – Negative charge
N_d – Positive charge
p
n
Space charge region
E-field
"Diffusion force" on holes
"Diffusion force" on electrons
E-field force on holes
E-field force on electrons


p
n
E_F
$e\phi_{Fn}$
E_{Fi}
$e\phi_{Fp}$
E_F



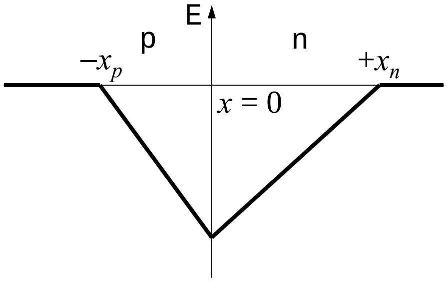
Abrupt depletion approximation



$$\boxed{\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_s} = -\frac{d\,\mathrm{E}(x)}{dx}}$$

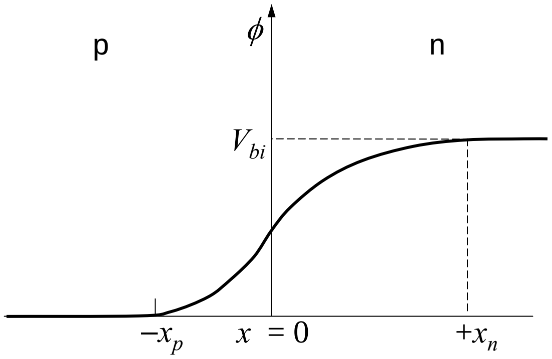
Poisson equation

Calculation electric field



$$\mathrm{E} = \frac{-eN_a}{\varepsilon_s}(x + x_p) \qquad -x_p \le x \le 0$$

...and the potential



$$\phi(x) = \frac{eN_a}{2\varepsilon_s}\left(x + x_p\right)^2 \qquad \left(-x_p \le x \le 0\right)$$

Calculation V_{bi}

n-side: $n_0 = n_i \exp\left[(E_F - E_{Fi})/kT\right] = n_i \exp\left[-e\phi_{Fn}/kT\right] \approx N_d$

p-side: $p_0 = n_i \exp\left[(E_{Fi} - E_F)/kT\right] = n_i \exp\left[-e\phi_{Fp}/kT\right] \approx N_a$

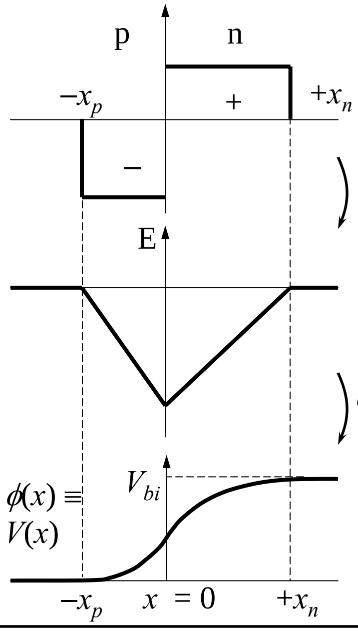
Then the potentials ϕ_{Fn} and ϕ_{Fp} become: $\phi_{Fn} = \frac{kT}{e}\ln\left(\frac{N_d}{n_i}\right)$

$$\phi_{Fp} = \frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right)$$

...and we find for the built-in potential difference:

$$V_{bi} = \phi_{Fn} + \phi_{Fp} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = \frac{e}{2\varepsilon_s}\left(N_d x_n^2 + N_a x_p^2\right)$$

Relation band diagram ⇔ electrostatic quantities



$$\boxed{\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_s} = -\frac{d\,\mathrm{E}(x)}{dx}}$$

$$\mathrm{E} = \int_{-x_p}^{+x_n} \frac{\rho(x)}{\varepsilon_s}\,dx$$

$$\phi(x) = V(x) = -\int_{-x_p}^{+x_n} \mathrm{E}(x)\,dx$$

Calculation depletion width

Depletion width follow from charge neutrality: $N_a x_p = N_d x_n$

...... and from: $W = x_n + x_p$

Then we obtain:

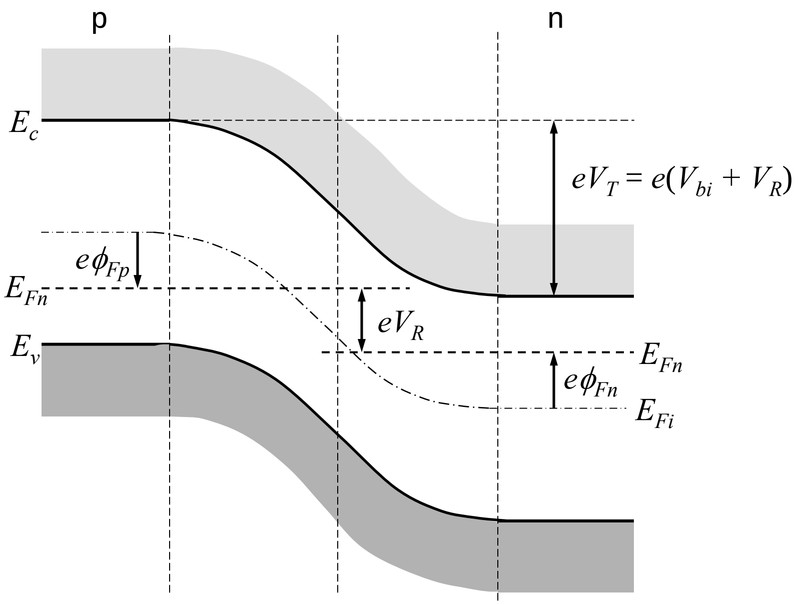
$$W = \left\{ \frac{2\varepsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

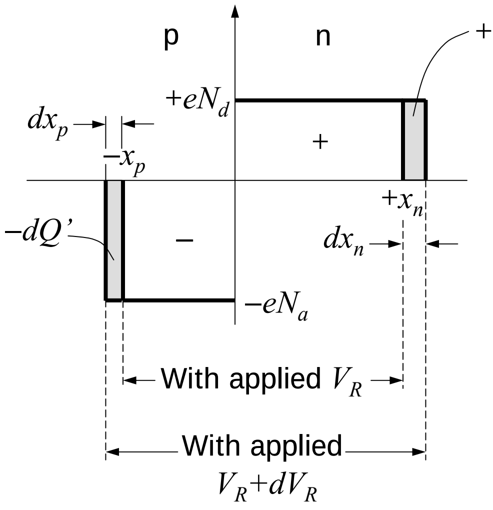
Applying a reverse bias

- Increase the barrier for electron injection from the n-type region into the p-type region and vice versa.
- Hardly any currents are flowing ⇒ electrostatic approximation, so:

$$V_{\text{total}} = V_{bi} + V_R$$

$$W = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$



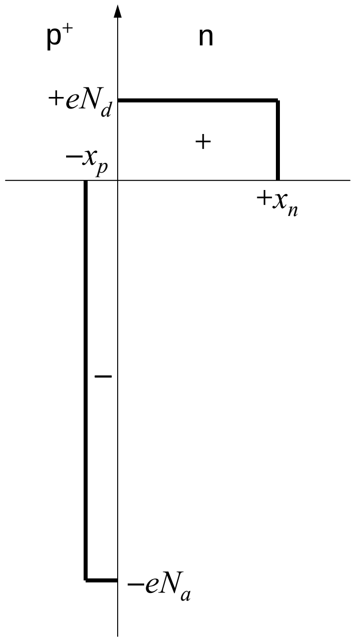


$$\boxed{C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}}$$

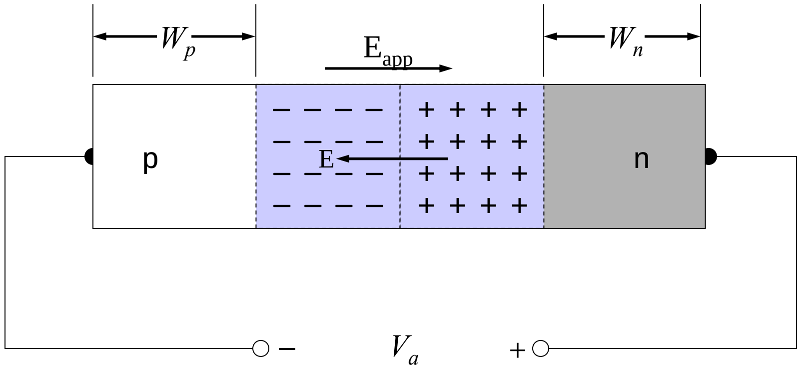
$$x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]}\; ; \qquad C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}$$

⇓

$$\boxed{C' = \sqrt{\frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}}\; ; \qquad C' = \frac{\varepsilon_s}{W}}$$



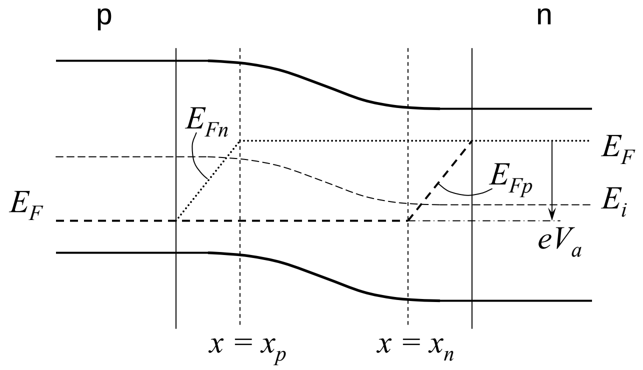
$$\boxed{\begin{aligned} W &\approx \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{eN_d}} \\ C' &\approx \sqrt{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}} \end{aligned}}$$



Assumptions for ideal *J*, *V* relation

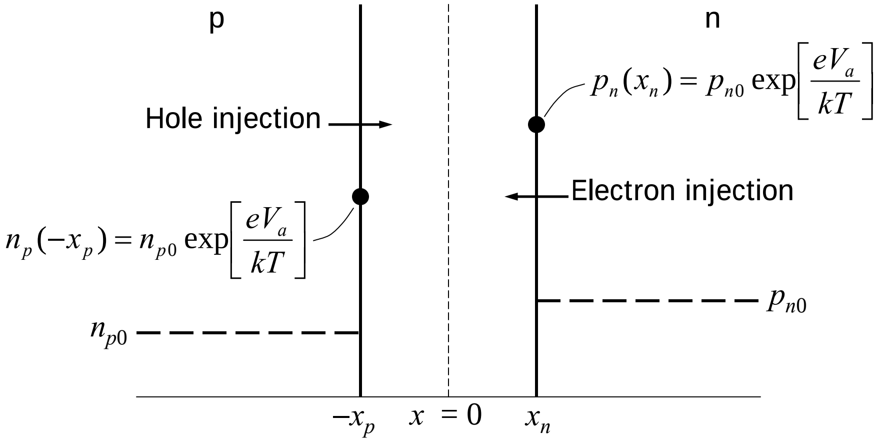
1. Abrupt boundaries between quasi-neutral region and depletion region.
2. Maxwell-Boltzmann approximation is valid.
3. Assume low-level injection.
4. In the structure:
   - The total current is constant
   - Individual electron and hole currents are continuous.
   - Individual electron and hole currents are constant in the depletion region.

Determination of the boundary conditions



$$p_n(x_n) = n_i \exp\left[\frac{E_i - E_{Fp}}{kT}\right] = n_i \exp\left[\frac{E_i - E_F}{kT}\right] \exp\left[\frac{eV_a}{kT}\right] = p_{n0} \exp\left[\frac{eV_a}{kT}\right]$$

$$n_p(x_p) = n_i \exp\left[\frac{E_{Fn} - E_i}{kT}\right] = n_i \exp\left[\frac{E_F - E_i}{kT}\right] \exp\left[\frac{eV_a}{kT}\right] = n_{p0} \exp\left[\frac{eV_a}{kT}\right]$$



Apply ambipolar transport equation:

$$D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \mu_p E \frac{\partial(\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t}$$

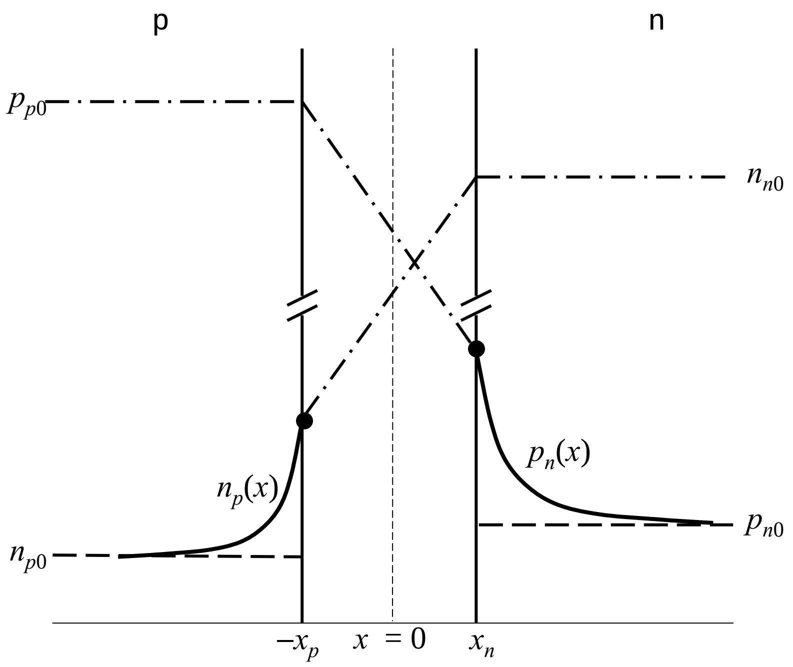
$$\Rightarrow \frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \quad (x > x_n)$$

Solve differential equation (try exponential function: $Ae^{-x/L} + Be^{x/L}$)

And apply the correct boundary conditions: $\delta p_n(x \to \infty) = 0$

$$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right)$$

$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right)$$



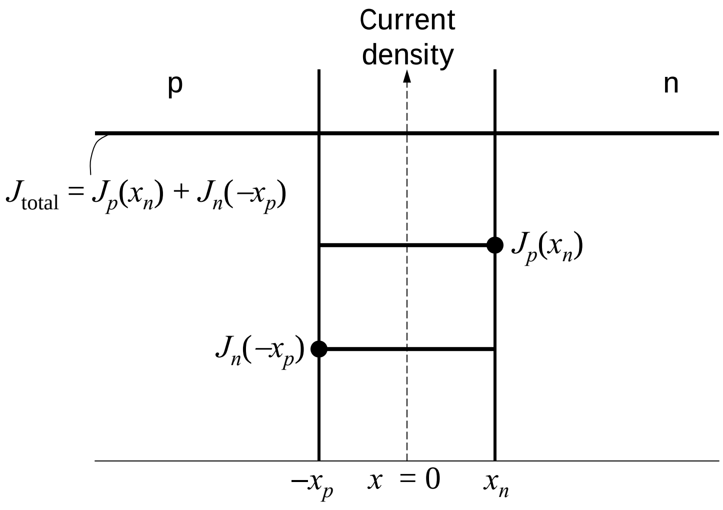
The current density through the junction

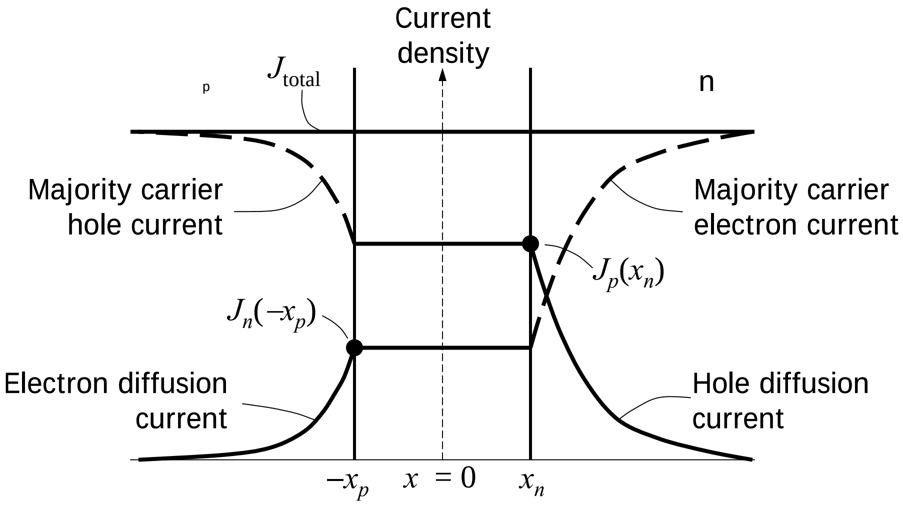
$$J_n(x_p) = eD_n \left.\frac{dn}{dx}\right|_{x=x_p} = eD_n \left.\frac{d(\delta n(x))}{dx}\right|_{x=x_p} = \frac{eD_n n_{p0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$

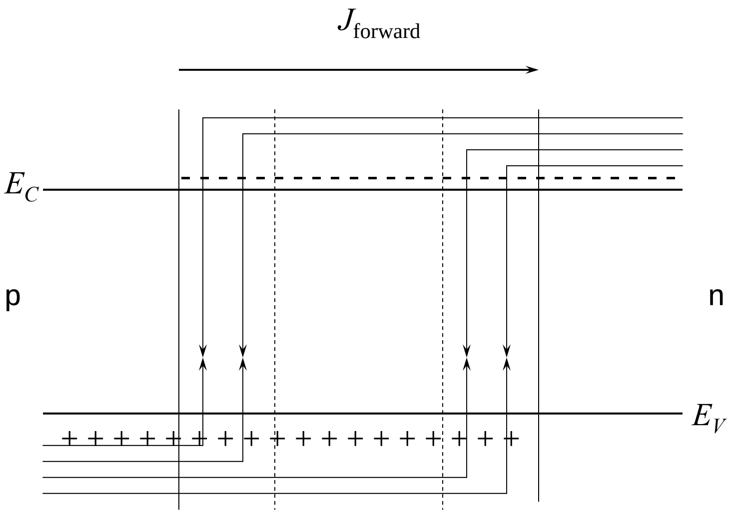
$$J_p(x_n) = -eD_p \left.\frac{dp}{dx}\right|_{x=x_n} = -eD_p \left.\frac{d(\delta p(x))}{dx}\right|_{x=x_n} = \frac{eD_p p_{n0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$

The total current is the addition of the electron and hole currents and is determined by the minority properties!!!

$$J = J_p(x_n) + J_n(x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$







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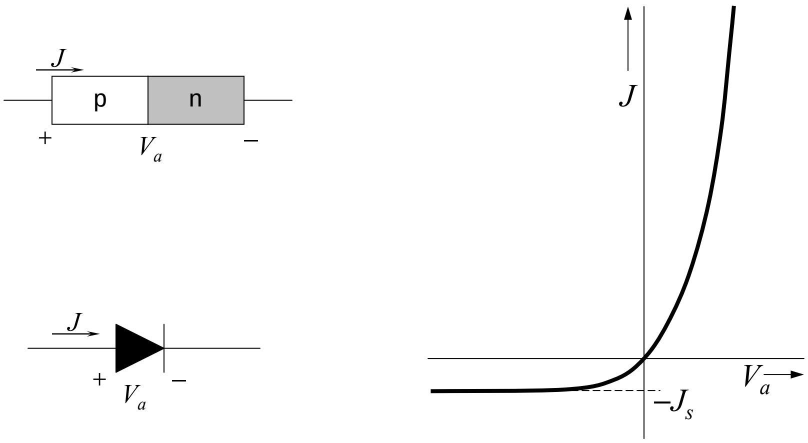
Diode equation

$$J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]$$



Generation-recombination current

Assumptions:

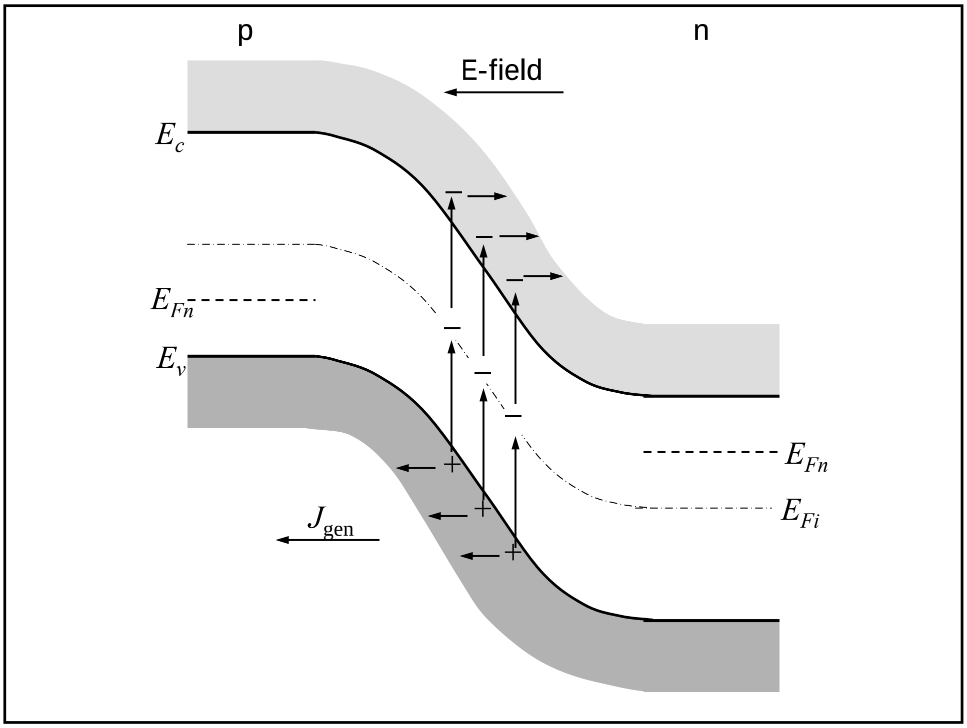
1. In the whole depletion region $n = p = 0$
2. Further: $\tau_0 = (\tau_{n0} + \tau_{p0})/2$, and use: $1/N_tC_{p,n} = t_{p0,n0}$
3. Trap energy level E_t in the middle of the gap → $n' = p' = n_i$

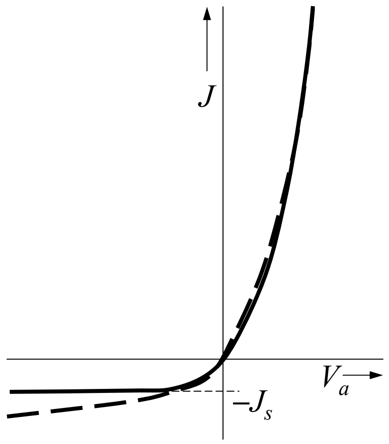
So: $$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')} = \frac{-n_i}{2\tau_0} \equiv -G$$

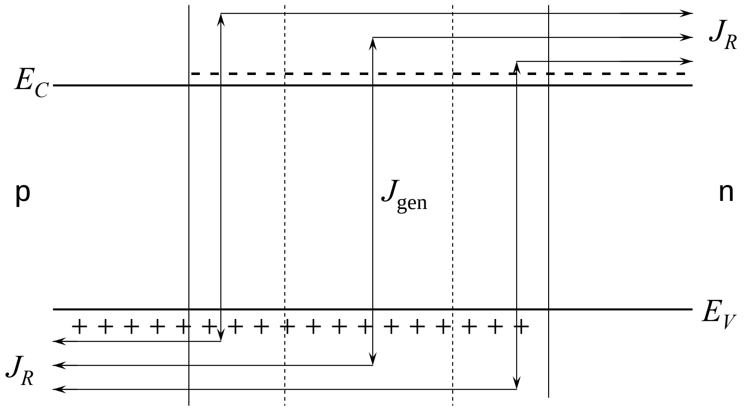
The generation current is all the current generated in the depletion region, thus:

$$J_{\text{gen}} = \int_0^W eG dx = \frac{en_iW}{2\tau_0} \Rightarrow J_{\text{reverse}} = J_s + J_{\text{gen}}$$

Note that J_{reverse} is dependent on V_a because W and thus J_{gen} is dependent on V_a!!!



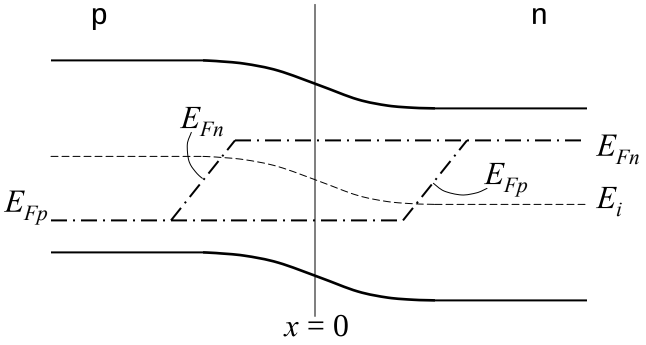

J
V_a
$-J_s$


J_R
E_C
p
J_{gen}
n
E_V
+ + + + + + + + + + + + + + + + + + +
J_R

Forward recombination current

$$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')} = \frac{np - n_i^2}{\tau_{p0}(n+n') + \tau_{n0}(p+p')}$$

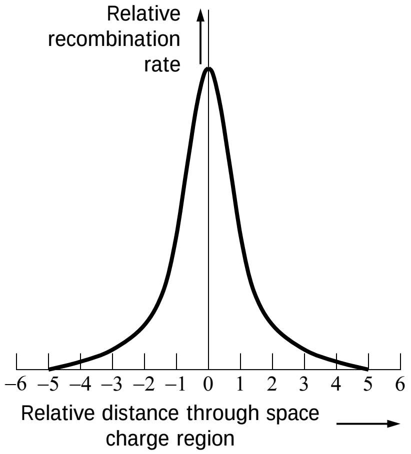
- Charge carriers injected in the depletion region under forward bias, can recombine in that region via states in the band gap.
- First determine where the recombination rate, R, is maximal.
- R is maximal when:
  1. The product np in the numerator is maximal and → if $x = 0$
  2. The trap energy level E_t (that determines n' and p') is close to E_{fi}.

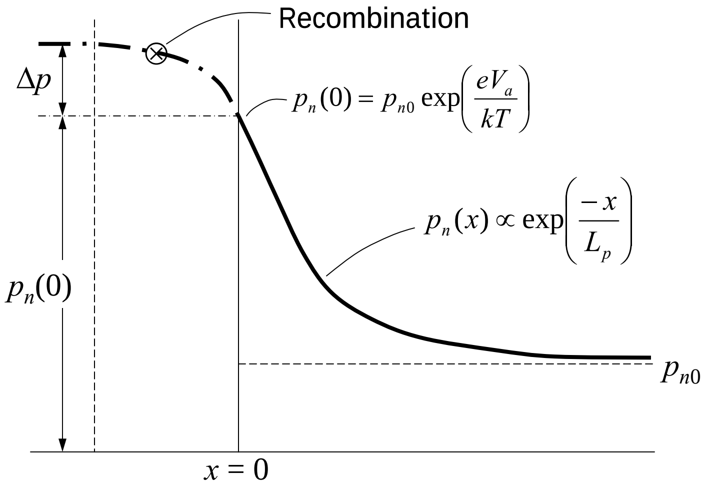


$$R = \frac{np - n_i^2}{\tau_{p0}(n+n') + \tau_{n0}(p+p')}$$

$$n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right]$$

$$p = n_i \exp\left[\frac{E_{Fi} - E_{Fp}}{kT}\right]$$


Relative recombination rate
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6
Relative distance through space charge region


Recombination
Δp
$p_n(0) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$
$p_n(x) \propto \exp\left(\frac{-x}{L_p}\right)$
$p_n(0)$
p_{n0}
$x = 0$

Forward recombination current

From the assumptions →

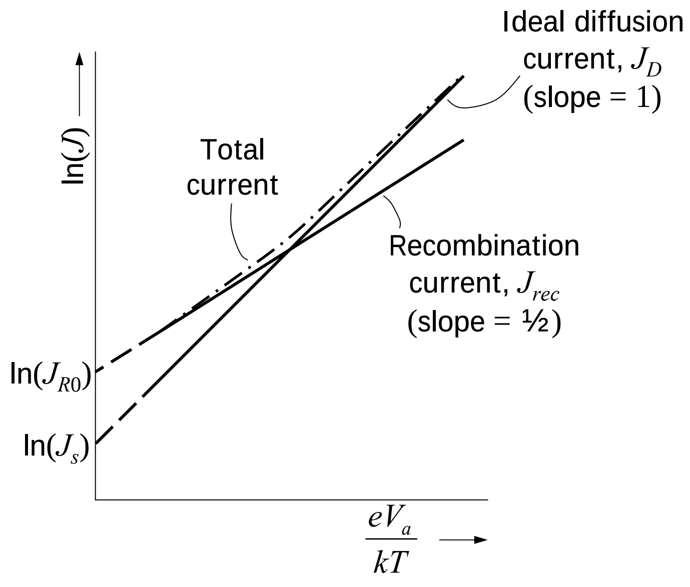
$$n = p = n_i \exp\left(\frac{eV_a}{\mathbf{2}kT}\right)$$

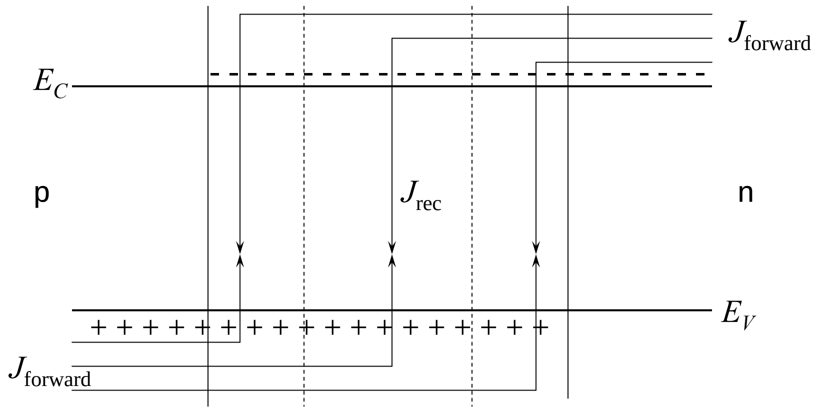
$$\left.\begin{aligned} n' &= n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right) \\ p' &= n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right) \end{aligned}\right\} \Rightarrow n' = p' = n_i$$

And thus the recombination and the current becomes ↓

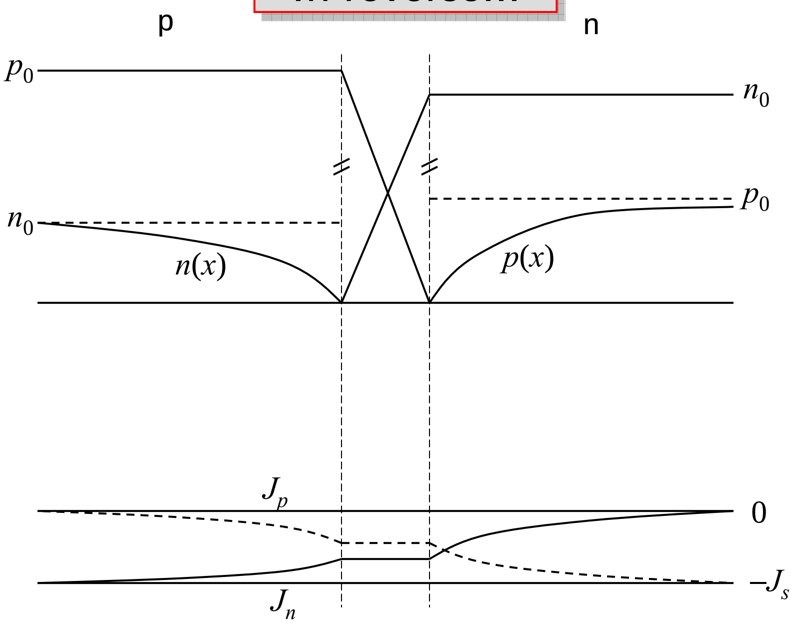
$$R_{\max} = \frac{n_i}{2\tau_0} \frac{\left[\exp\left(eV_a/kT\right) - 1\right]}{\left[\exp\left(eV_a/2kT\right) + 1\right]} \approx \frac{n_i}{2\tau_0} \exp\left(\frac{eV_a}{\mathbf{2}kT}\right)$$

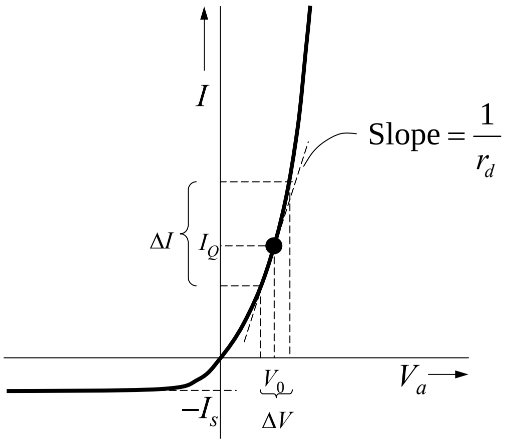
$$J_{\text{rec}} = \int_0^W eR\,dx \approx \frac{eWn_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right) = J_{r0} \exp\left(\frac{eV_a}{\mathbf{2}kT}\right)$$




J_{forward}
E_C
p
J_{rec}
n
E_V
+ + + + + + + + + + + + + + + + + + +
J_{forward}

In reverse...


p
n
p_0
n_0
p_0
n_0
$n(x)$
$p(x)$
J_p
0
$-J_s$
J_n



$$I_D = I_s\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$

$$r_d = \frac{V_t}{I_{DQ}}$$

Small-signal admittance

- In this case the *time* dependent transport equation for minority carriers should be applied: $E = 0$, $g' = 0$ and $\partial/\partial t \neq 0$.
- The bias across the depletion region is equal to the applied bias at any moment.
- The bias across the depletion region is in equilibrium with the minority carrier concentration on the depletion region edge at any moment.

Admittance

The admittance of the diode is:

$$Y = \frac{\hat{I}_p + \hat{I}_n}{\hat{V}_1} = \frac{1}{V_t}\left[I_{p0}\sqrt{1+j\omega\tau_{p0}} + I_{n0}\sqrt{1+j\omega\tau_{n0}}\right]$$

For small $\omega\tau_{p0} << 1$ and $\omega\tau_{n0} << 1$ we obtain the form $Y = g_d + j\omega C_d$, which in turn can be related to an equivalent circuit:

$$g_d = \frac{I_{p0} + I_{n0}}{V_t} \longleftarrow \boxed{\text{diffusion conductance}}$$

$$C_d = \frac{I_{p0}\tau_{p0} + I_{n0}\tau_{n0}}{2V_t} \longleftarrow \boxed{\text{diffusion capacitance}}$$

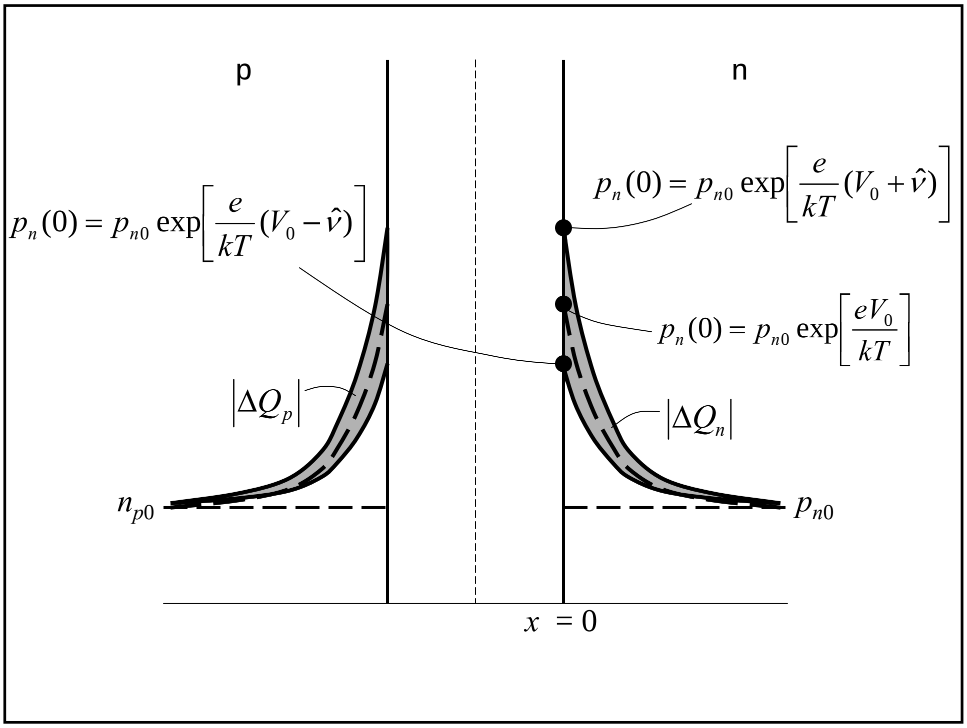
Diffusion capacitance

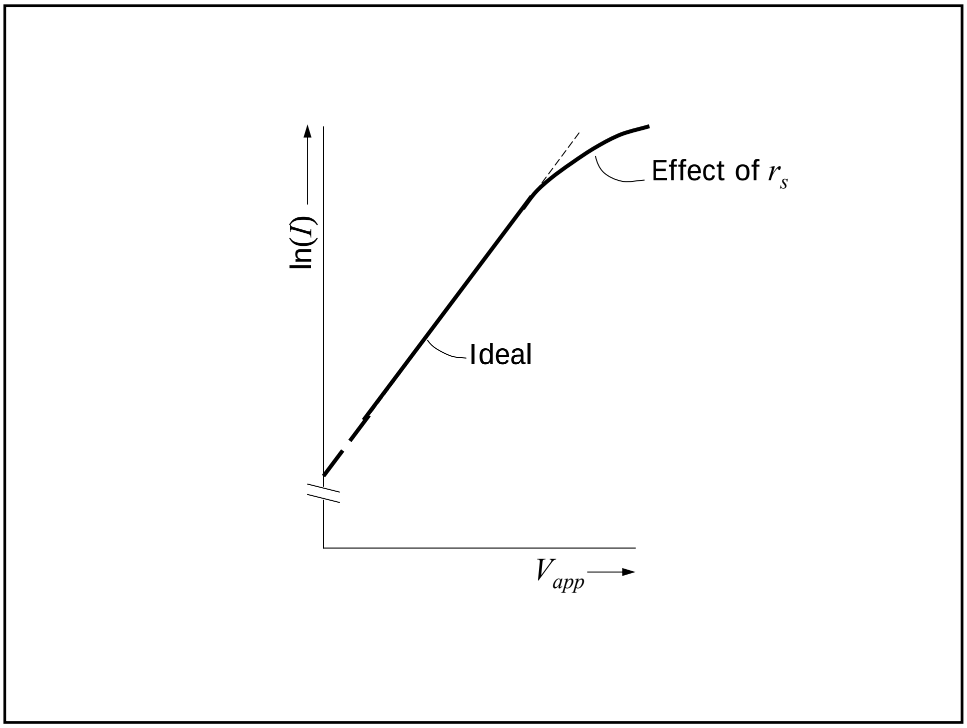
Minority carrier storage in n-region: $Q_p = e\int_{x_n}^{\infty}(p_n - p_{n0})dx$

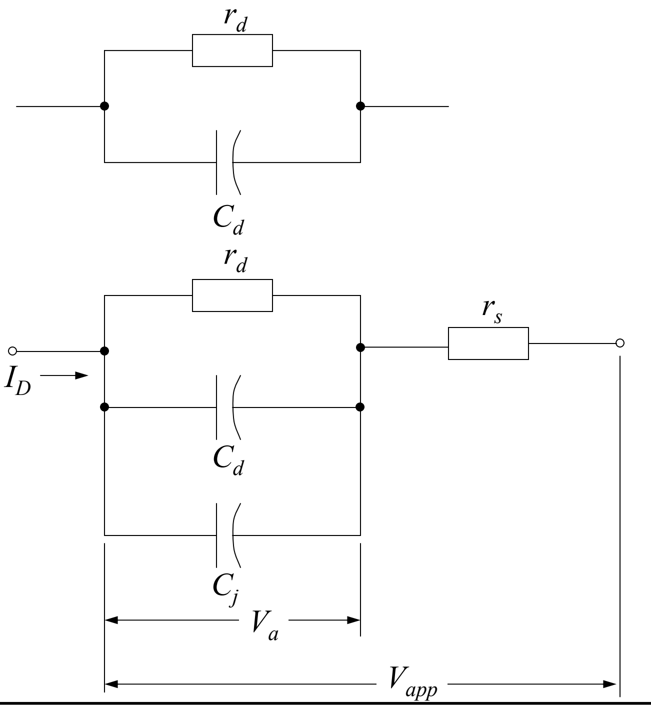
$$= e\int_{x_n}^{\infty} p_{n0}\left(e^{eV/kT} - 1\right)e^{-(x-x_n)/L_p}dx$$

$$= eL_p p_{n0}\left(e^{eV/kT} - 1\right)$$

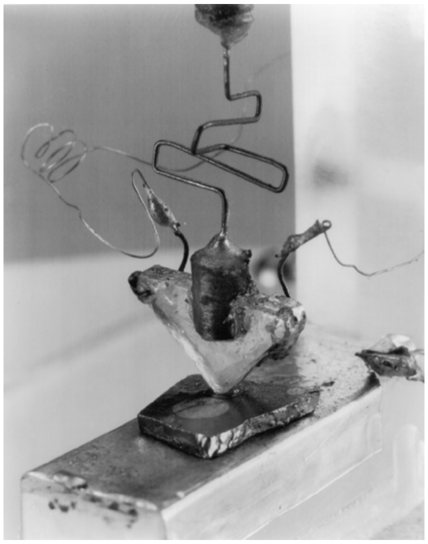
Diffusion capacitance becomes: $C_d = \frac{dQ_p}{dV} = e \cdot \left(\frac{1}{V_t}\right) \cdot L_p p_{n0} e^{eV/kT}$

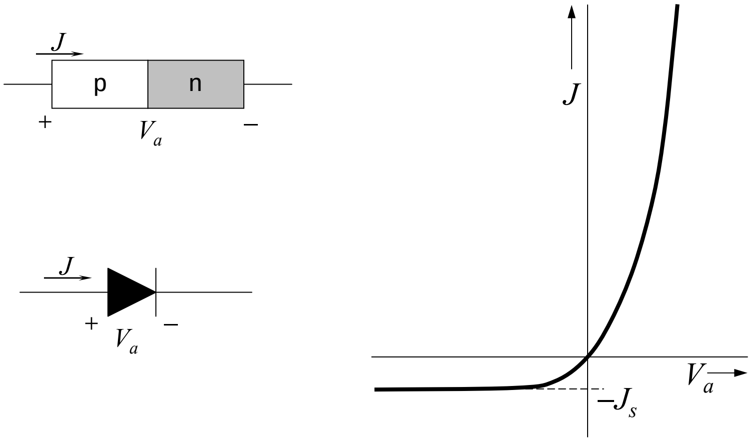

p
n
$p_n(0) = p_{n0} \exp\left[\frac{e}{kT}(V_0 + \hat{\nu})\right]$
$p_n(0) = p_{n0} \exp\left[\frac{e}{kT}(V_0 - \hat{\nu})\right]$
$p_n(0) = p_{n0} \exp\left[\frac{eV_0}{kT}\right]$
$|\Delta Q_p|$
$|\Delta Q_n|$
n_{p0}
p_{n0}
$x = 0$


ln(I)
Effect of r_s
Ideal
V_{app}



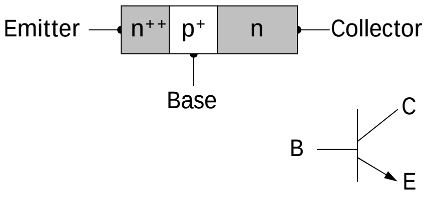
The transfer resistor...

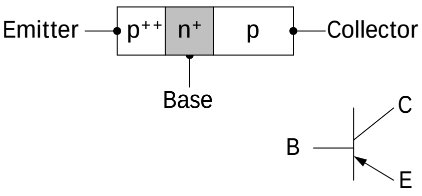


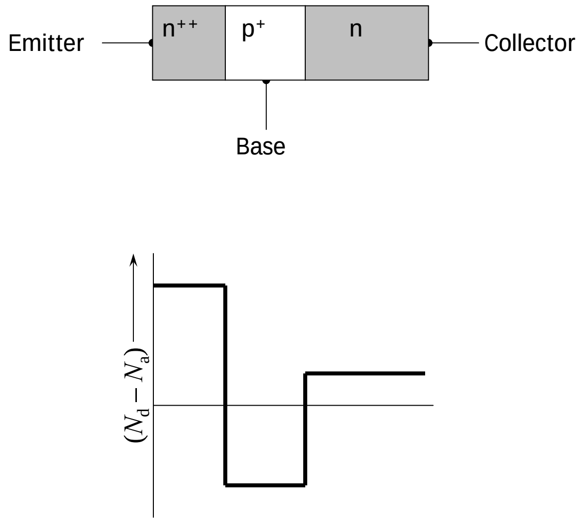


$$J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]$$







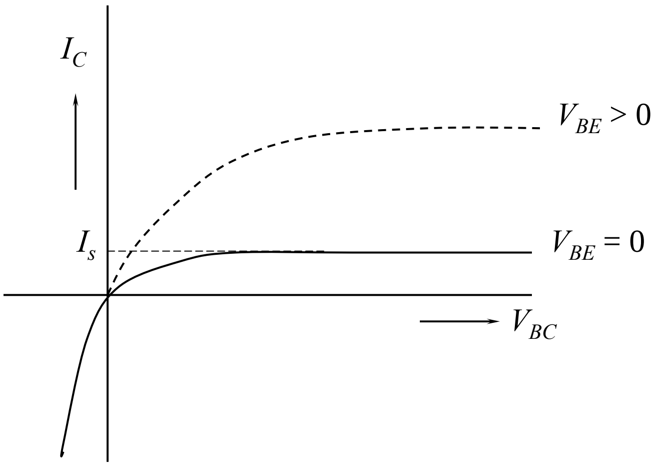
Characteristics in forward-active mode

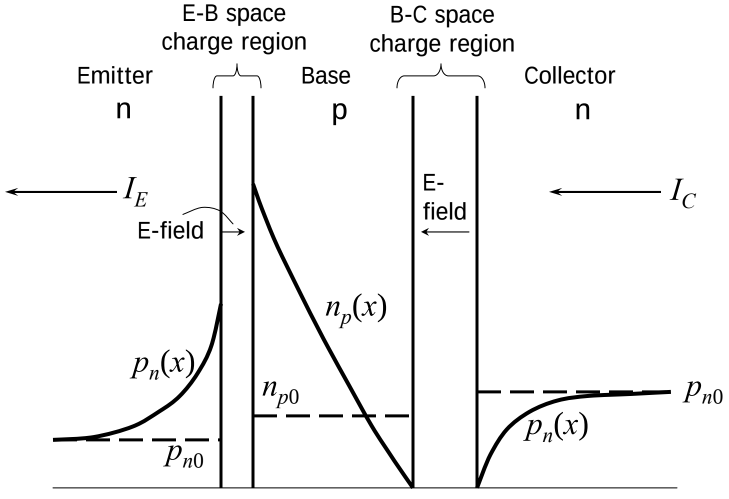
- Short base width, so the emitter-base junction is in principle a short diode
- Emitter-base junction is forward biased, the base-collector junction is reversed biased
- In an n-p-n transistor electrons are injected from the emitter into the base
- The electron concentration near the depletion region of the base-collector junction is zero → large electron-concentration gradient

Characteristics in forward-active mode

– Ideally, all electrons injected into base reach collector → collector current independent of base-collector bias and determined by the emitter-base bias

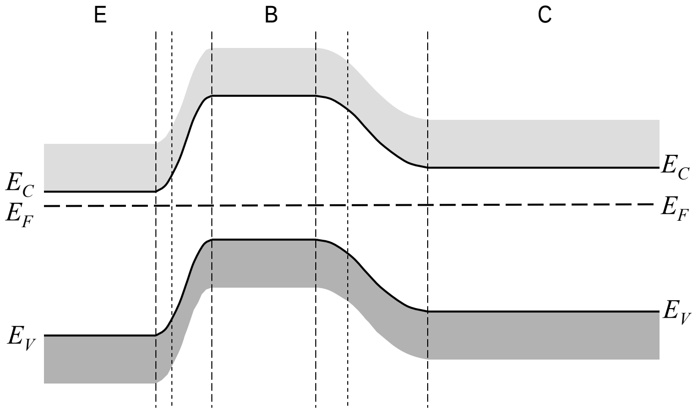
⇒Transistor action

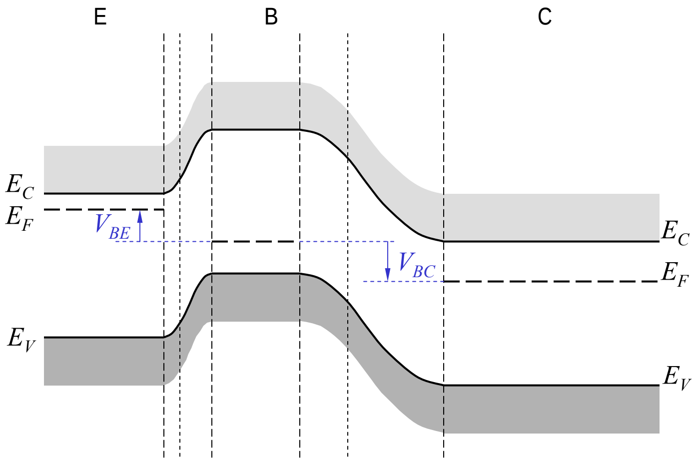


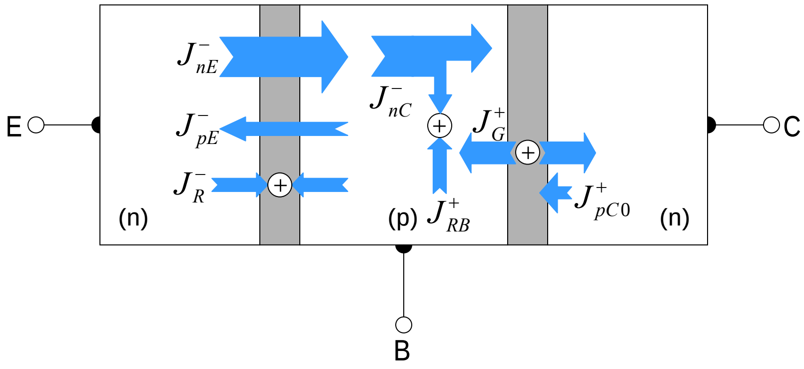


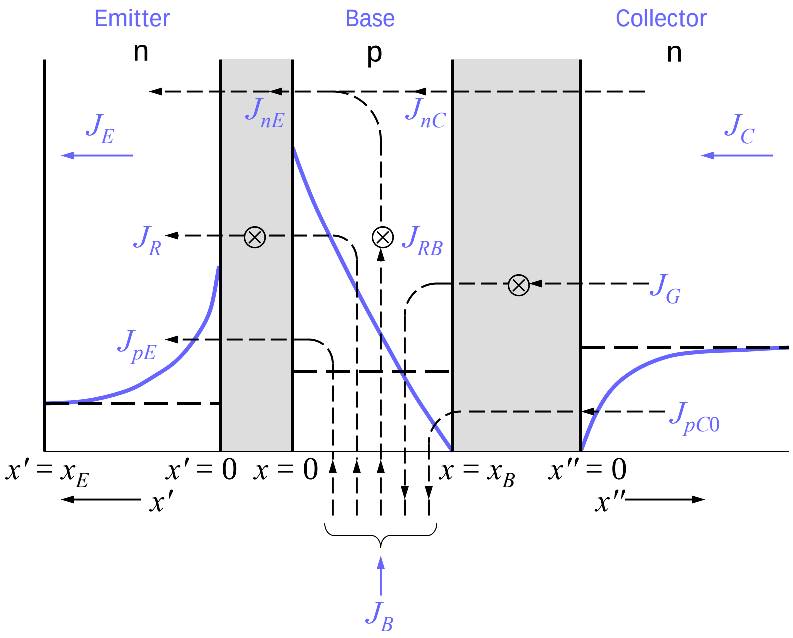
I_C onafhankelijk van V_{BC}

I_C bepaald door V_{BE}




E
B
C
E_C
E_F
V_{BE}
V_{BC}
E_C
E_F
E_V
E_V


J^-_{nE}
J^-_{nC}
E
J^-_{pE}
J^+_G
C
J^-_R
J^+_{pC0}
(n)
(p) J^+_{RB}
(n)
B



Current components of the transistor and current gain factor

$$J_C = J_{nC} + J_G + J_{pC0}$$

$$J_E = J_{nE} + J_R + J_{pE}$$

$$J_B = -J_E - J_C$$

The dc common base current gain factor is defined as: $\alpha_0 = \dfrac{I_C}{I_E}$

If we assume that the emitter-base area is equal to the base-collector area, we obtain:

$$\alpha_0 = \frac{J_C}{J_E} = \frac{J_{nC} + J_G + J_{pC0}}{J_{nE} + J_R + J_{pE}}$$

Small-signal common base current gain

We are interested in the collector current-change as a function of the emitter-current change:

$$\alpha = \left.\frac{\partial J_C}{\partial J_E}\right|_{V_{BC}} = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$$

$$= \boxed{\underbrace{\left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right)}_{\gamma}}\boxed{\underbrace{\left(\frac{J_{nC}}{J_{nE}}\right)}_{\alpha_T}}\boxed{\underbrace{\left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}\right)}_{\delta}}$$

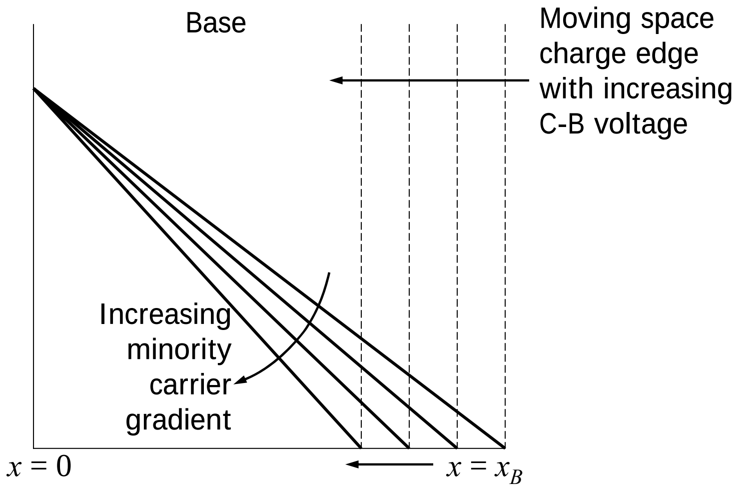
$$\alpha = \boxed{\underbrace{\left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right)}_{\gamma}}\boxed{\underbrace{\left(\frac{J_{nC}}{J_{nE}}\right)}_{\alpha_T}}\boxed{\underbrace{\left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}\right)}_{\delta}}$$

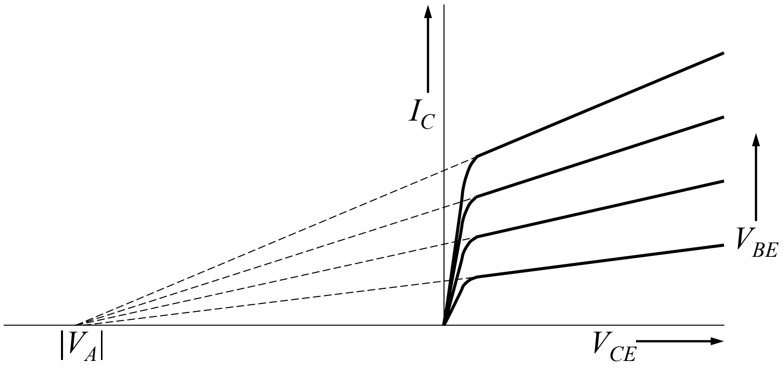
- Emitter efficiency injection factor, γ: ratio electron diffusion current / total diffusion current
- Base transport factor, α_T: determines the efficiency of charge transport across the base, in other words is a measure for the recombination losses in the base
- Recombination factor, δ: measure for the quality of the emitter-base junction and gives an indication about the recombination

Non-ideal effects

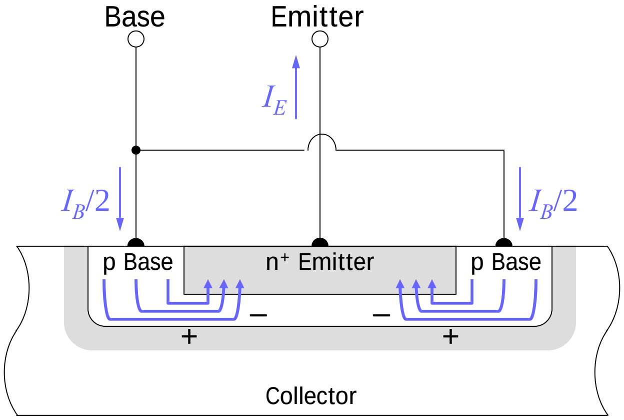
So far we assumed that:

- All regions are uniformly doped,
- Low-level injection is valid,
- The emitter- and base widths are constant,
- The band gap is constant in the whole device,
- The current densities are uniform and
- There is no breakdown

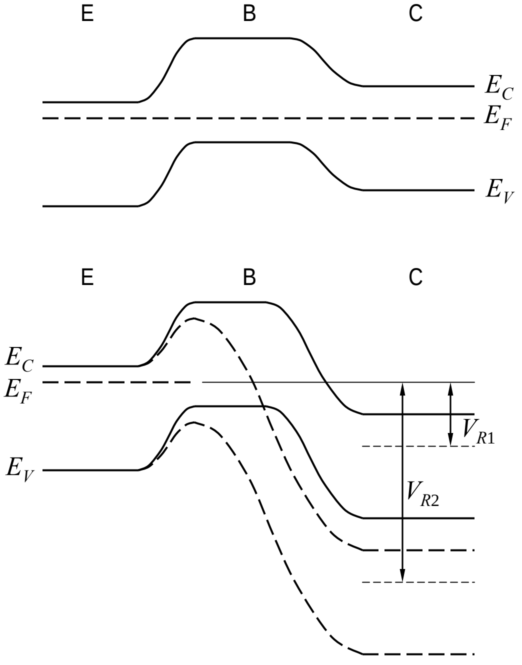


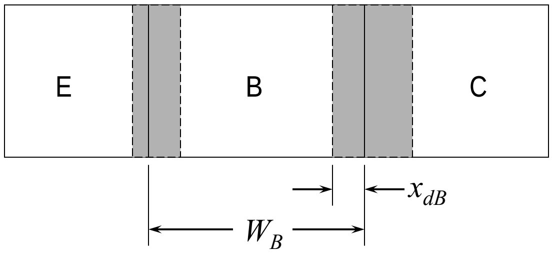


I_C
V_{BE}
$|V_A|$
V_{CE}



Base
Emitter
I_E
$I_B/2$
$I_B/2$
p Base
n⁺ Emitter
p Base
–
–
+
+
Collector



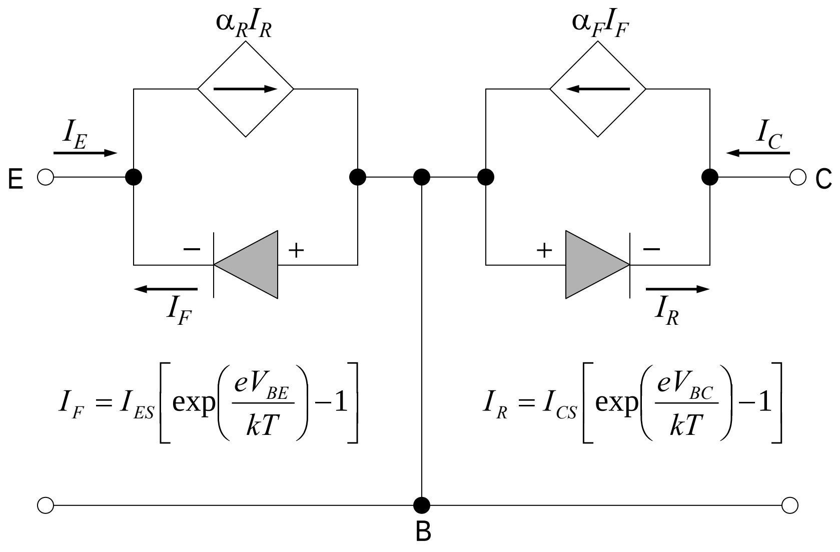


$$x_{dB} = W_B = \left\{ \frac{2\varepsilon_s (V_{bi} + V_{pt})}{e} \cdot \frac{N_C}{N_B} \cdot \frac{1}{N_C + N_B} \right\}^{1/2}$$

$$V_{pt} = \frac{eW_B^2}{2\varepsilon_s} \cdot \frac{N_B (N_C + N_B)}{N_C}$$

Ebers-Moll model

- Transistor represented by two interacting diodes with common base
- Forward current of one diode flows into the other reverse-biased diode. The emitter-base current is represented by I_F
- This current is a function of the emitter-base bias, V_{BE}
- In forward active mode a large part of the current injected into the base reaches the collector
- This fraction is represented by a current source with value $\alpha_F I_F$, in which α_F is the common-base current gain
- In the inverse mode the base-collector junction is forward biased and conducts a current I_R which is a function of the base-collector bias, V_{BC}
- Current across the base is now represented by a current source with value $\alpha_R I_R$



$$I_F = I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

$$I_R = I_{CS}\left[\exp\left(\frac{eV_{BC}}{kT}\right) - 1\right]$$

Ebers-Moll equations

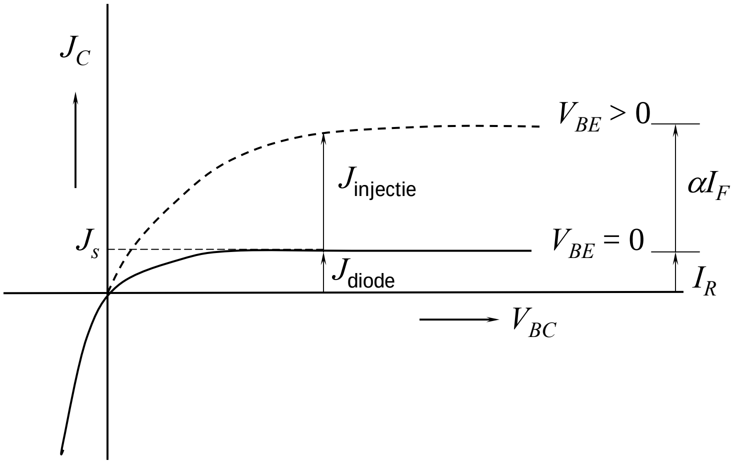
The Ebers-Moll equations follow from the equivalent circuit:

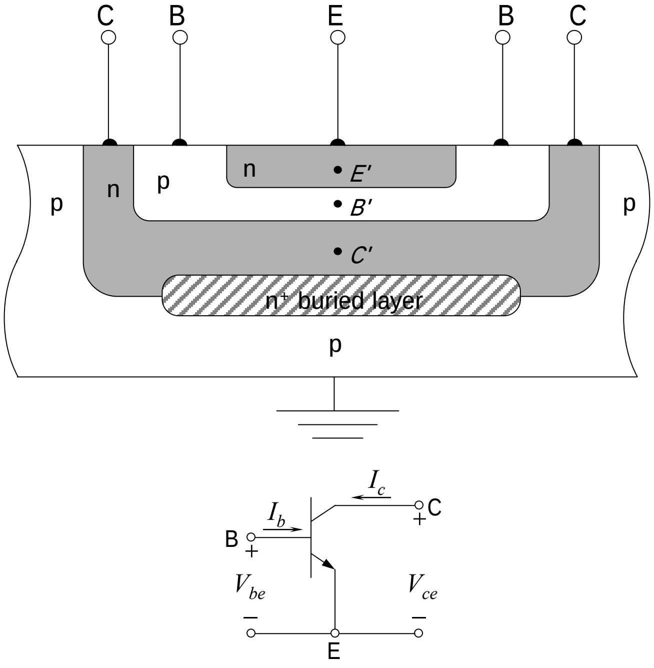
$$I_C = \alpha_F I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right)-1\right] - I_{CS}\left[\exp\left(\frac{eV_{BC}}{kT}\right)-1\right]$$

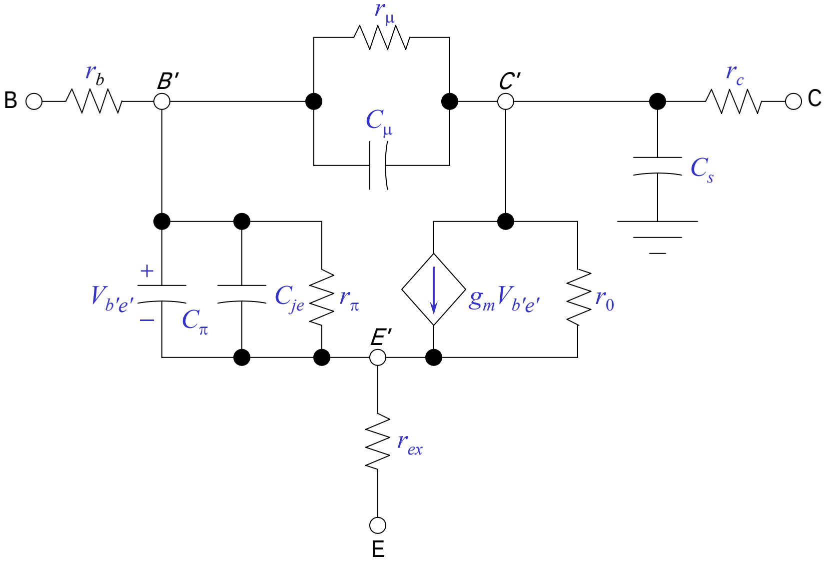
$$I_E = \alpha_R I_{CS}\left[\exp\left(\frac{eV_{BC}}{kT}\right)-1\right] - I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right)-1\right]$$

With I_{ES} and I_{CS} the reverse-bias currents of the emitter-base and the base-collector junction, respectively

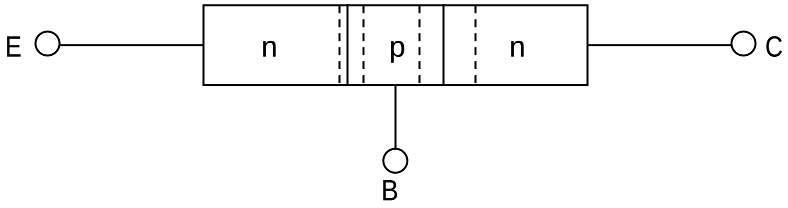
$(J_C\text{-}V_{CB})_{Vbe}$ characteristics




C
B
E
B
C
p
n
p
n
E′
B′
C′
p
n+ buried layer
p
I_c
C
+
I_b
B
+
V_{be}
–
V_{ce}
–
E


r_μ
B
r_b
B′
C′
r_c
C
C_μ
C_s
+
$V_{b'e'}$
–
C_π
C_{je}
r_π
$g_m V_{b'e'}$
r_0
E′
r_{ex}
E

Cut-off frequency



$$\Delta I_E \xrightarrow{\tau_{ec}} \Delta V_{BC}$$

Time required to travel from the emitter to the collector:

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

Charging time, τ_e, of the emitter-base junction capacitance:

$$\tau_e = r_e'\left(C_{je} + C_p\right) = \frac{kT}{e}\frac{1}{I_E}\left(C_{je} + C_p\right) \leftarrow$$

Emitter-base junction capacitance charging time

Time required to diffuse across the base:

$$\tau_b = \int_0^{\tau_b} dt = \int_0^{x_B} \frac{dx}{v(x)} = \int_0^{x_B} \frac{en_B(x)dx}{(-J_n)}$$

$$= \frac{x_B^2}{2D_n} \longleftarrow$$

Base transit time

Time required to cross the depletion region between the base and the collector:

$$\tau_d = \frac{x_{dc}}{v_s}$$

Collector depletion region transit time

with: x_{dc} the base-collector depletion-region width and v_s the saturation velocity at high electric fields

Charging time, τ_c, of the base-collector junction capacitance:

$$\tau_c = r_c\left(C_\mu + C_s\right)$$

Collector capacitance Charging time

with: C_μ the base-collector junction capacitance and C_s the collector-base capacitance

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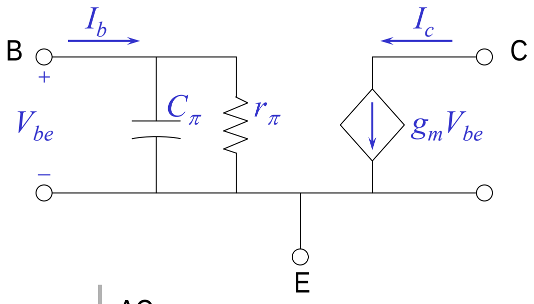


DC:

$$V_{be} = I_b r_\pi$$

$$I_c = g_m V_{be} = g_m r_\pi I_b$$

$$\frac{I_c}{I_b} = g_m r_\pi \quad (= \beta_0)$$

AC:

$$V_{be} = I_b \frac{r_\pi}{1 + j\omega r_\pi C_\pi}$$

$$I_c = g_m V_{be} = I_b \frac{g_m r_\pi}{1 + j\omega r_\pi C_\pi}$$

$$\frac{I_c}{I_b} = \frac{g_m r_\pi}{1 + j\omega r_\pi C_\pi} = \frac{\beta_0}{1 + j\left(f/f_\beta\right)}, \text{ with } f_\beta = \frac{1}{2\pi r_\pi C_\pi}$$

Definition cut-off frequency

$$f_\alpha \equiv \quad \alpha \to \frac{\alpha_0}{\sqrt{2}}$$

$$f_\beta \equiv \quad \beta \to \frac{\beta_0}{\sqrt{2}}$$

$$f_T \equiv \quad \beta \to 1$$

$$\alpha = \frac{\alpha_0}{1 + j\frac{f}{f_\alpha}}$$

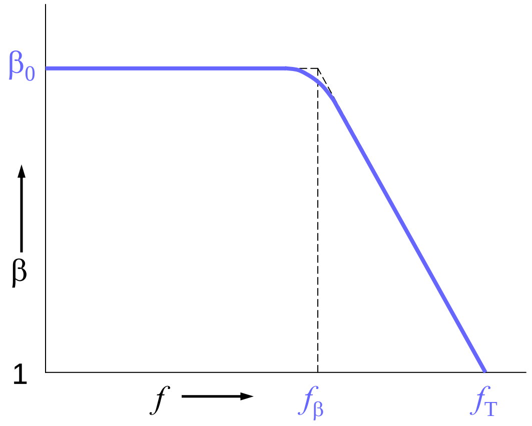
$$f_\alpha = \frac{1}{2\pi\tau_{ec}}$$

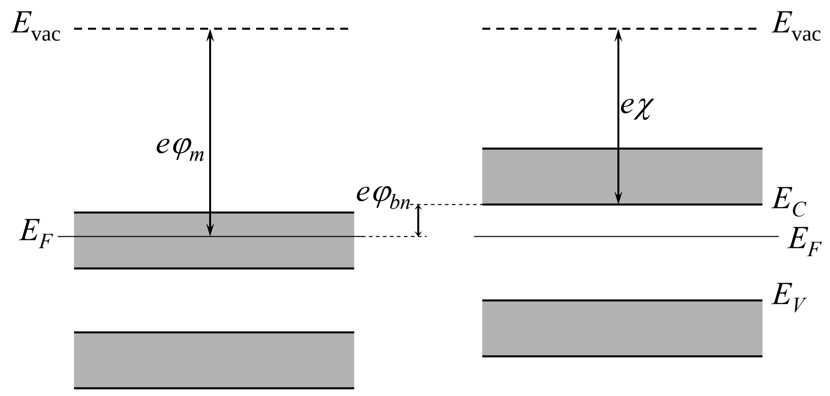
$$\beta = \frac{\alpha}{1-\alpha} = \frac{\alpha_0}{1-\alpha_0 + j\frac{f}{f_\alpha}}$$

$$|\beta| = \left|\frac{\alpha}{1-\alpha}\right| \cong \frac{f_\alpha}{f}$$

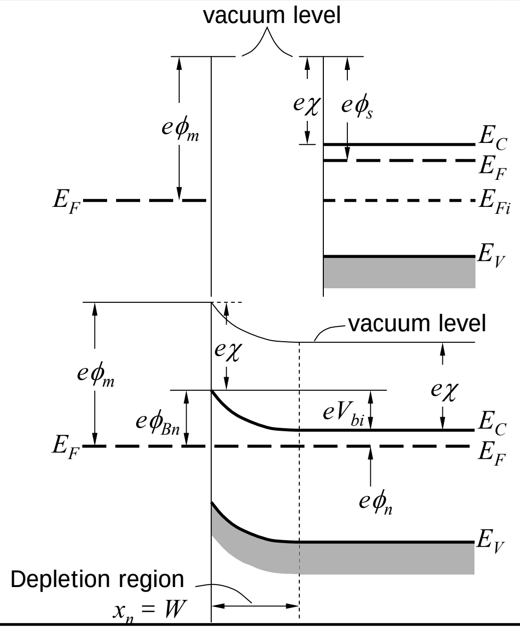
Definition f_T : $|\beta| = 1$ if $f = f_T$

thus : $\boxed{f_T = f_\alpha = \frac{1}{2\pi\tau_{ec}} \cong \beta_0 f_\beta}$





Metal/n-type semiconductor junction



$$\phi_{Bn} = (\phi_m - \chi)$$

$$V_{bi} = \phi_{Bn} - \phi_n$$

Ideal junction characteristics

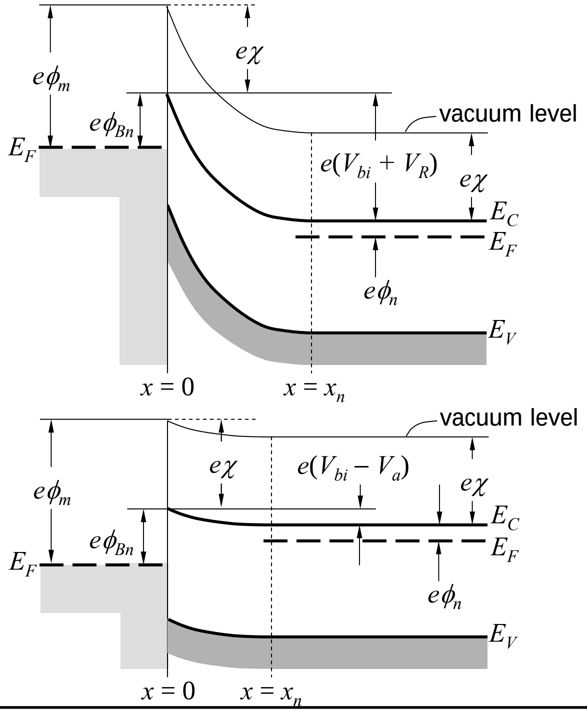
Poisson equation: $\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\varepsilon_s}$

Then follows simply: $W = x_n = \left[\frac{2\varepsilon_s\left(V_{bi} + V_R\right)}{eN_d}\right]^{1/2}$

$$C' = eN_d \frac{dx_n}{dV_R} = \left[\frac{e\varepsilon_s N_d}{2\left(V_{bi} + V_R\right)}\right]^{1/2} \Rightarrow \left(\frac{1}{C'}\right)^2 \propto V_R$$

$\Rightarrow$ from C' measurement V_{bi} as N_d can be determined

$\Rightarrow$ subsequently φ_n and thus φ_{bn} can be calculated



Current-voltage relation

$$J = J_{sT}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

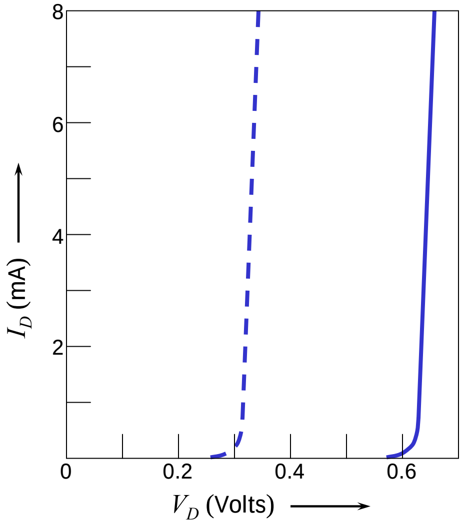
$$J_{sT} = A^* T^2 \exp\left(-\frac{e\phi_{Bn}}{kT}\right), \quad \text{with}: \ A^* \equiv \frac{4\pi m_n^* k^2}{h^3}$$

Compare with p-n diode, in which:

$$J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$$

Differences between Schottky and p-n diodes

- In a Schottky diode the current is determined by the majority carriers, in a p-n diode by the minority carriers.
- In a Schottky diode charge transport is determined by thermionic emission of majority carriers over a potential barrier, in a p-n diode by the diffusion of minority carriers.
- The saturation current of a Schottky diode is much larger than that of a p-n diode.
- A forward biased Schottky diode does not have diffusion capacitance → much faster device.



8
6
4
2
I_D (mA)
0
0.2
0.4
0.6
V_D (Volts)