Introduction to Aerospace Engineering

Lecture slides

Introduction to Aerospace Engineering

Aerodynamics 1&2

Prof. H. Bijl ir. N. Timmer

1 & 2.

Fundamentals of aerodynamics

Anderson chapters 2.1 – 2.5 and 4.1 – 4.4

Leonard Euler

1707-1783

Daniel Bernoulli

1700-1782

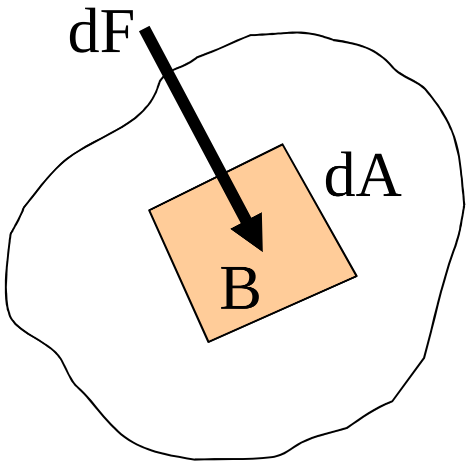
Subjects

Fundamental quantities

- pressure
- density
- temperature
- velocity

Equation of state

Pressure is the normal force per unit area on a surface



dA = element surface around B

dF = force on 1 side of dA

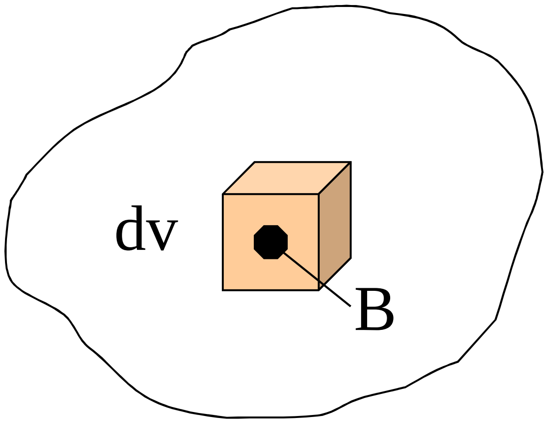
The pressure in point B is:

$$p = \lim \left(\frac{dF}{dA} \right) \quad , dA \rightarrow 0$$

Pressure is force per unit area

Dimension of pressure = $[N/m^2]$

Density of a substance is its mass per volume



dv = element volume around B

dm = mass of gas in dv

The density in point B is:

$$\rho = \lim\left(\frac{dm}{dv}\right) \quad ,dv \to 0$$

Density is mass per volume

Dimension of density = [kg/m^3]

Temperature is a measure of the average kinetic energy of the particles in the gas

Temperature is a measure of the average kinetic energy of the particles in the gas:

$$KE = \frac{3}{2} kT,$$

with k = Boltzmann constant ($1.38 \text{x} 10^{-23}$ J/K)

Dimension of temperature : K (Kelvin)
°C (degree Celsius),

with $\text{K} = 273.15 + \text{C}$

Relation among pressure, density and temperature is the equation of state

A ***perfect gas*** is a gas in which intermolecular forces are negligible

The equation of state for a perfect gas is:

$$\boxed{p = \rho R T}$$

With gas constant *R*:

$$\boxed{R = 287{,}05 \ \ \mathrm{J/(kg)(K)}}$$

The equation of state for a non-perfect or actual gas

For an actual gas the equation of state is approximated by the Berthelot equation:

$$\frac{p}{\rho RT} = 1 + \frac{ap}{T} - \frac{bp}{T^3}$$

The difference with the equation of state for a perfect gas becomes smaller as p decreases or T increases

(The distance between the molecules Inc r e a s e s)

The *Benedict-Webb-Rubin* (BWR) equation of state [2,3] within temperature range from -30 to 150°C, for densities up to 900kg/m3, and maximum pressure of 200bar:

$$p = \frac{RT}{v} + \left(B_0 RT - A_0 - \frac{C_0}{T^2} \right)\left(\frac{1}{v}\right)^2 + \left(bRT - a\right)\left(\frac{1}{v}\right)^3 + a\alpha\left(\frac{1}{v}\right)^6 + \frac{c}{T^2}\left(\frac{1}{v}\right)^3 \left[1 + \gamma\left(\frac{1}{v}\right)^2\right] \exp\left[-\gamma\left(\frac{1}{v}\right)^2\right]$$

where A_0, B_0, C_0, a, b, c, a, γ — gas constants for the equation

A_0 = 0.313 kg m^5 s^{-2} mol^{-2}
B_0 = 5.1953×10^{-5} m^3 mol^{-1}
C_0 = 1.289×10^4 kg m^5 K^2 s^{-2} mol^{-2}
a = 1.109 ×10^{-5} kg m^8 s^{-2} mol^{-3}
b = 3.775×10^{-9} m^6 mol^{-2}
c = 1.398 kg m^8 K^2 s^{-2} mol^{-3}
a = 9.377×10^{-14} m^9 mol^{-3}
γ = 5.301×10^{-9} m^6 mol^{-2}

Ex 2.1 Compute the temperature in a point on a wing of a Boeing 747, where pressure and density are given to be: 0.7×10^5 N/m^2 and 0.91 kg/m^3

$$T = 268.0 \text{ K}$$

$$= -5.2\ ^{o}\text{C}$$

Pressure, density and temperature under standard conditions

$$P_s = 1.01325 * 10^5 \text{ N/m}^2$$

$$\rho_s = 1.225 \text{ kg/m}^3$$

$$T_s = 288.15 \text{ K}$$

Ex. Compute the total weight of air in a room of 30 m x 15 m x 5 m under standard atmospheric conditions at sea level.

Volume=2250 m^2

ρ=1.225 kg/m^3

Weight = 2756 kg

Definition of specific volume

$$v = \frac{1}{\rho}$$

$$pv = RT$$

Ex 2.3 Compute the density and specific volume of air in a wind tunnel at P = 0.3 atm, and -100^{O} C.

$$\rho = \frac{p}{RT}$$

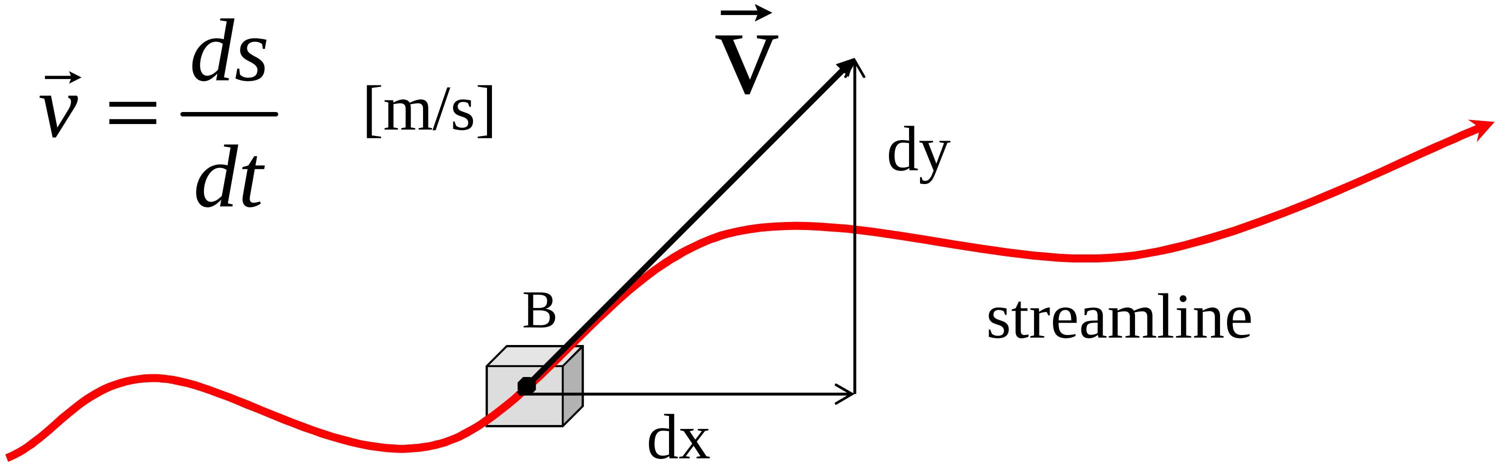
ρ= $0.3X10^5/(287.05 \times 173.15)$

= 0.60 kg/m^3

v = 1.66 m^3/kg

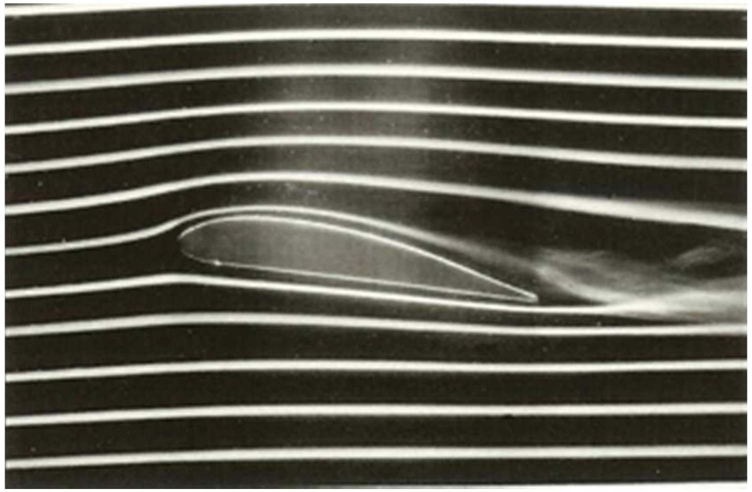
Velocity and streamlines

$$\vec{v} = \frac{ds}{dt} \quad [\text{m/s}]$$



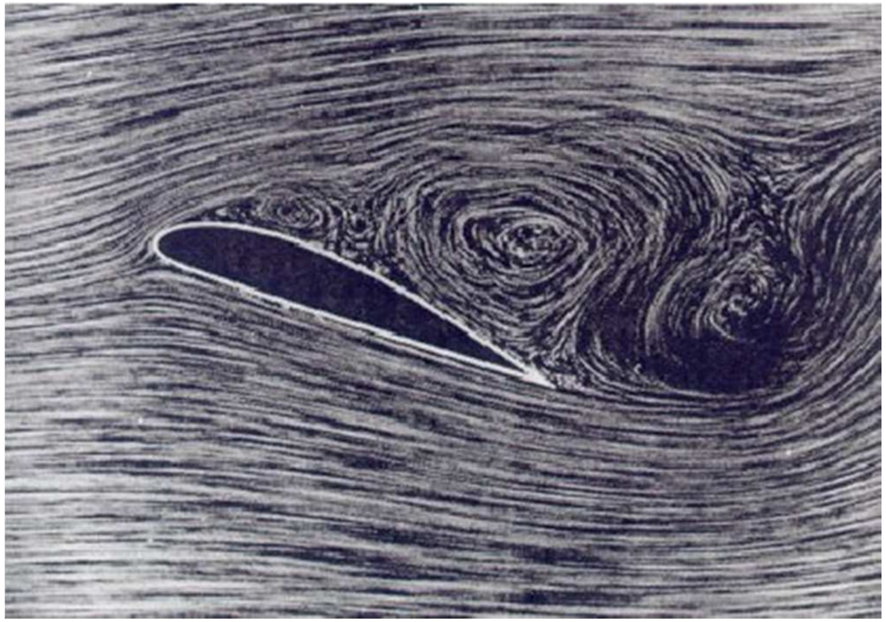
The velocity in point B is the velocity of an infinitesimally small fluid element as it sweeps through B

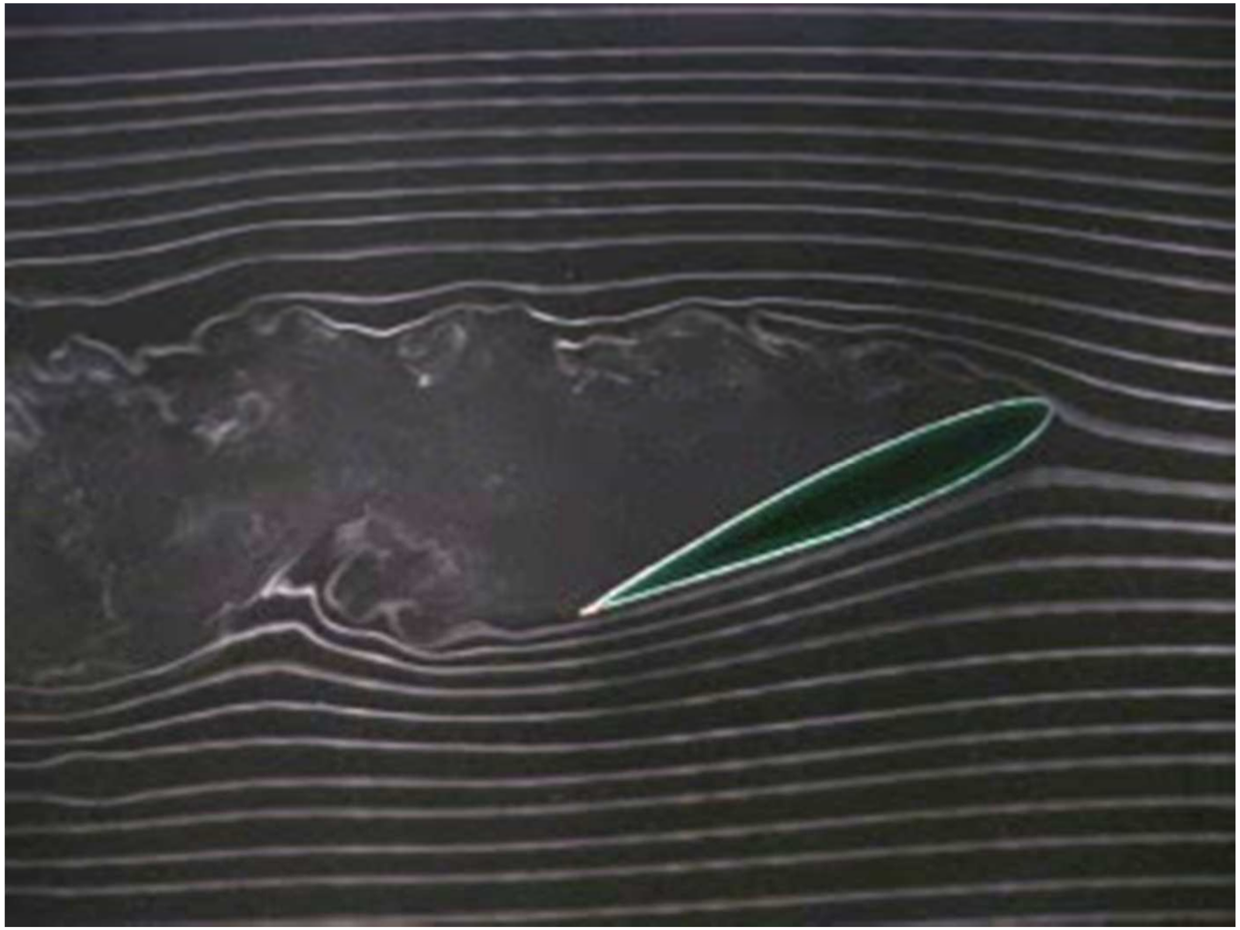
Visualisation of the velocity and streamlines



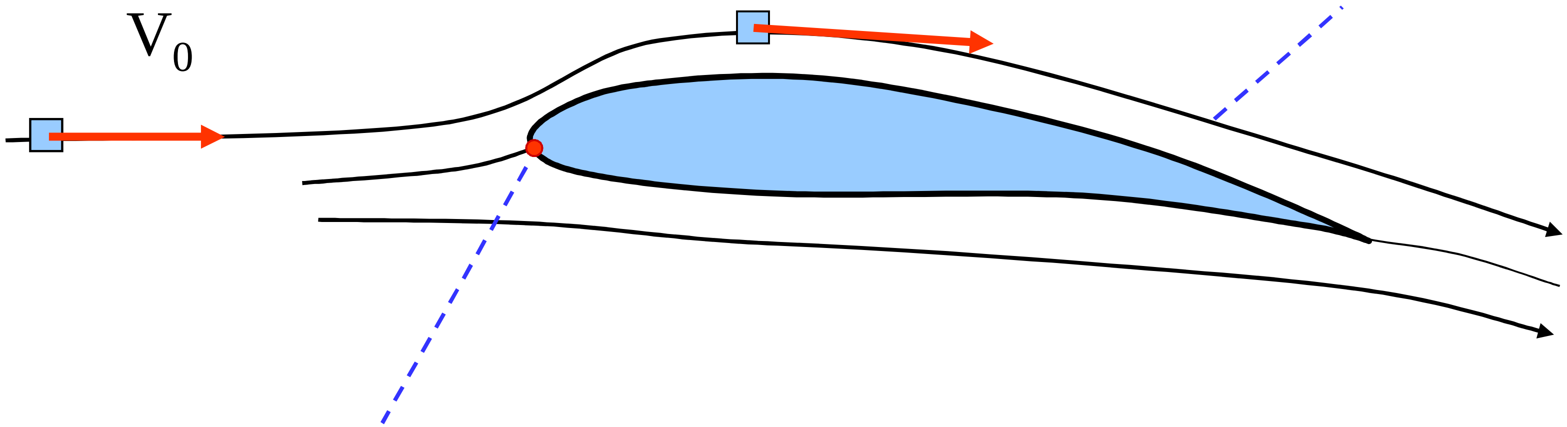
Smoke traces in air

Aluminum Particles in water





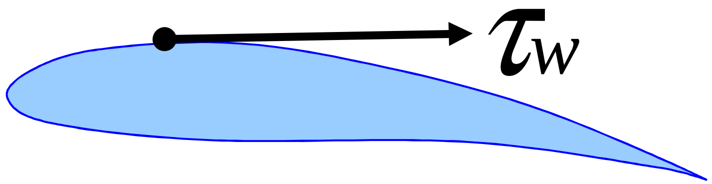
Velocity and streamlines



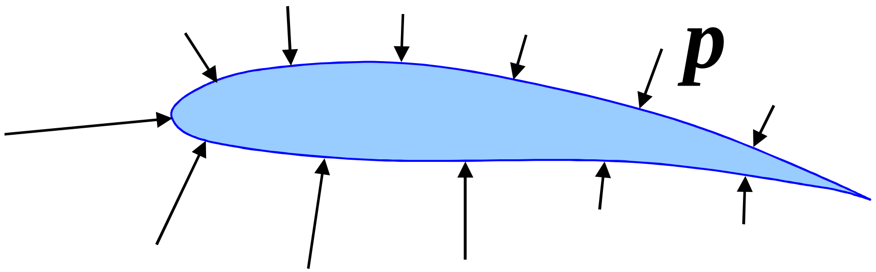
Stagnation point (stuwpunt, $V = 0$)

Aerodynamic forces

Shear stress or friction force



Pressure distribution



Homework:

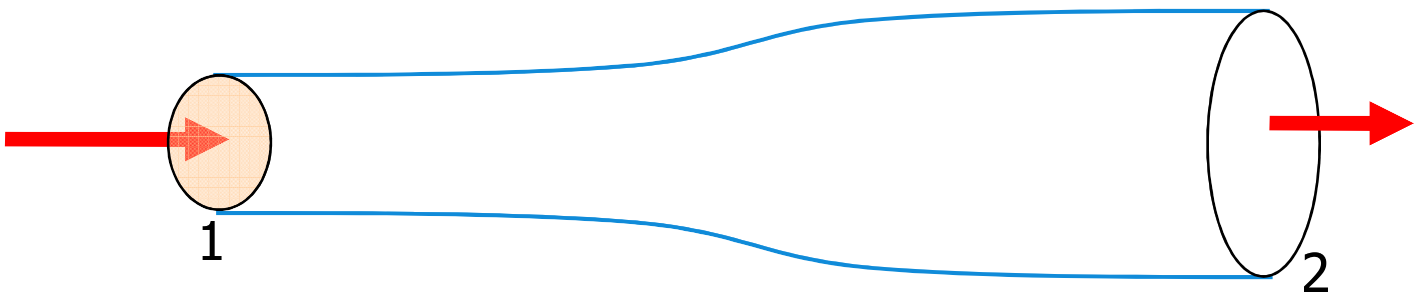
Problems 2.1, 2.7 and 2.8 from

Anderson, page 102 (sixth edition)

Fundamental equations

- Continuity equation: Conservation of mass
- Momentum equation (Euler eqn.): Conservation of momentum
- Bernouilli's law for incompressible flow

Continuity equation: Time rate of change of the mass of a material region is zero (mass is conserved)



For 1-directional incompressible flow: $\dot{m}_{in} = \dot{m}_{out}$

$$\dot{m} = \frac{dm}{dt} \qquad\qquad dm = \rho\,(Qdt),\;\; Q = AV$$

$$\dot{m} = \frac{dm}{dt} = \frac{\rho AVdt}{dt} = \rho AV$$

$$\boxed{\begin{aligned} \rho AV &= \text{constant} \\ \rho_1 A_1 V_1 &= \rho_2 A_2 V_2 \end{aligned}}$$

Continuity equation for 1-dimensional <u>incompressible</u> flow

(assumption: ρ and V uniformly distributed over A, or:

ρ and V are *mean* values)

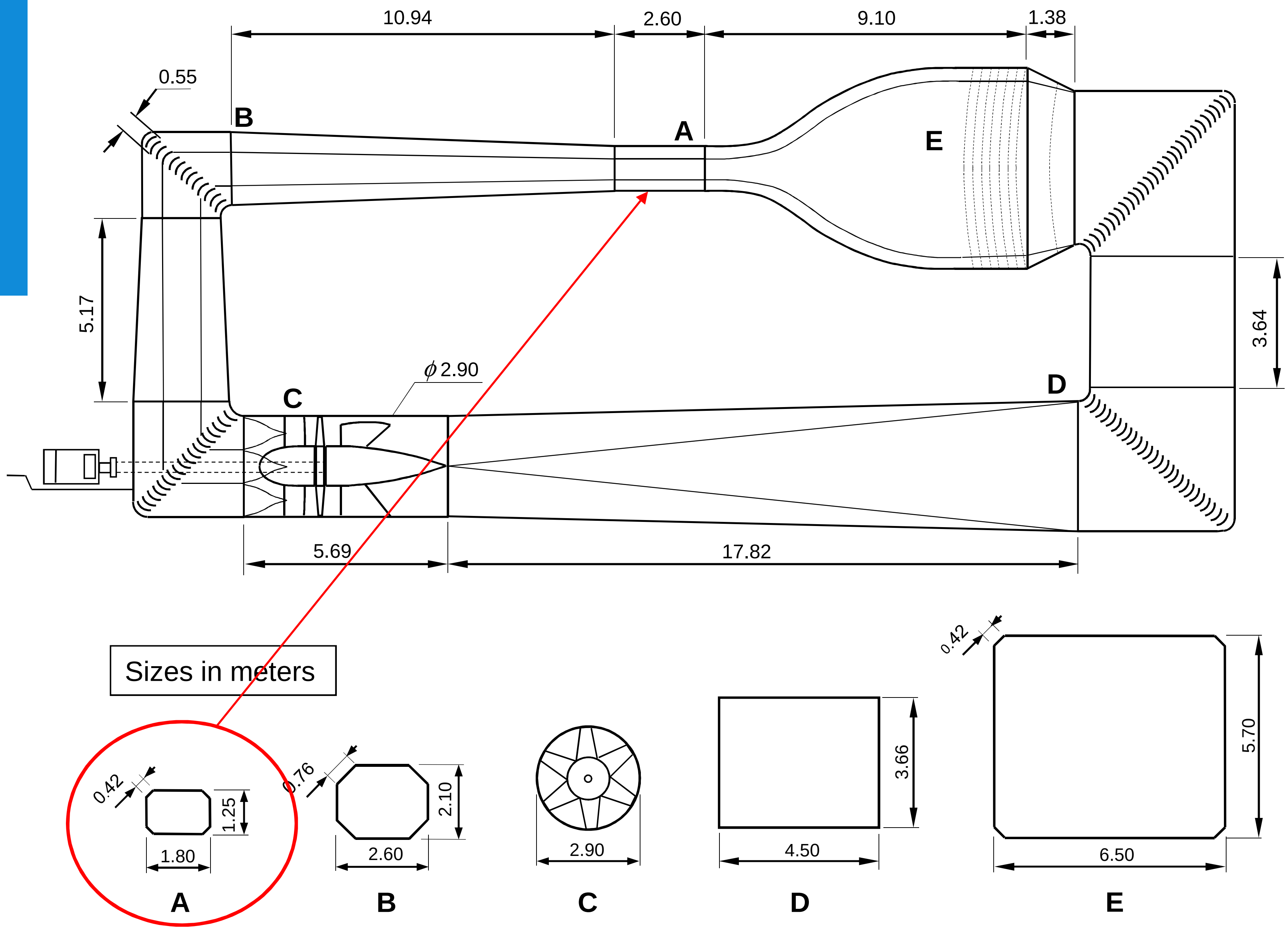
Example 4.1: Consider a convergent duct with an inlet area A_1 = 5 [m^2]. Air enters this duct with a velocity V_1 = 10 [m/s] and leaves the duct exit with a velocity V_2 = 30 [m/s]. What is the area of the duct exit?

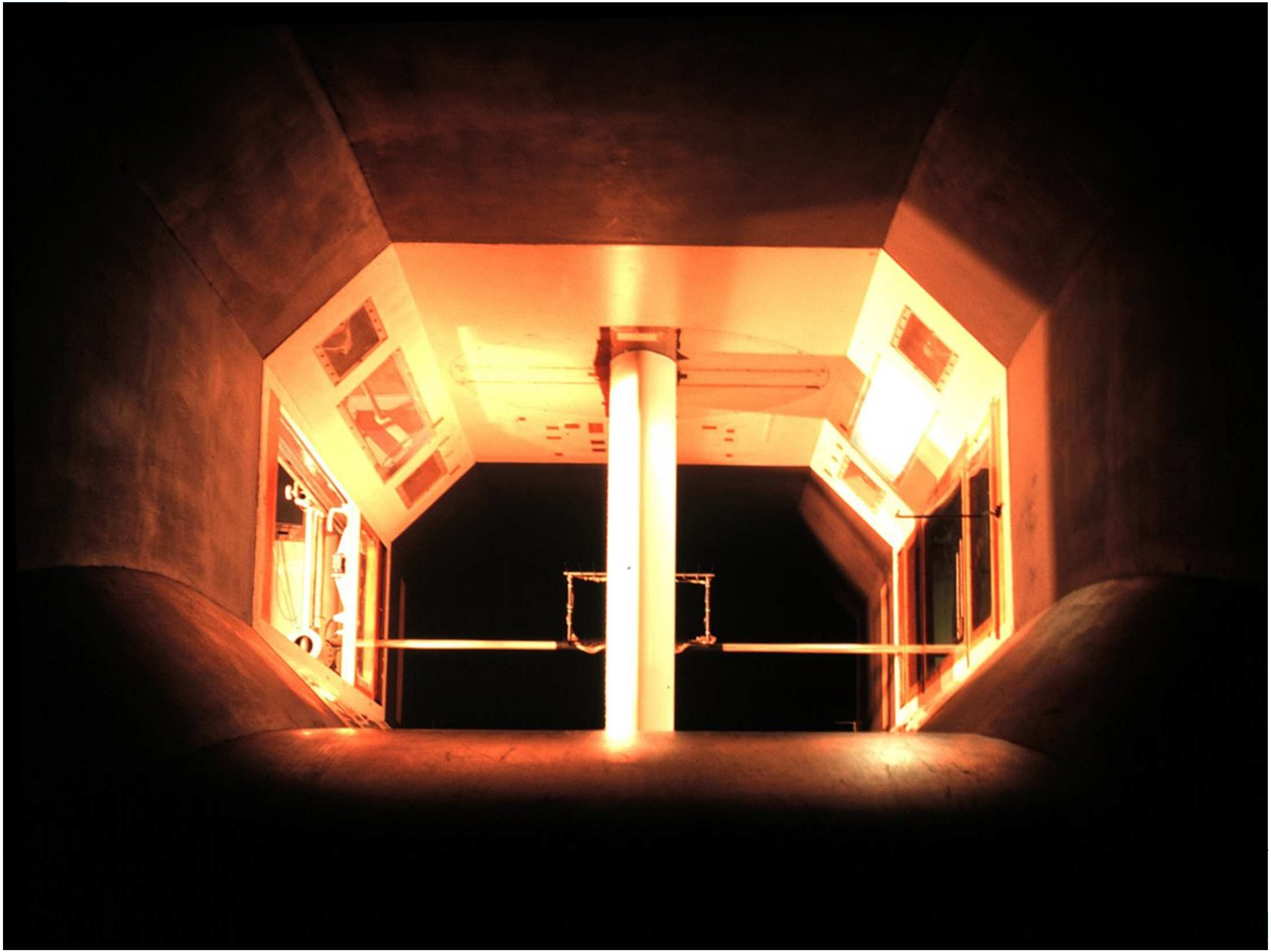
Assume a steady, incompressible 1-directional flow

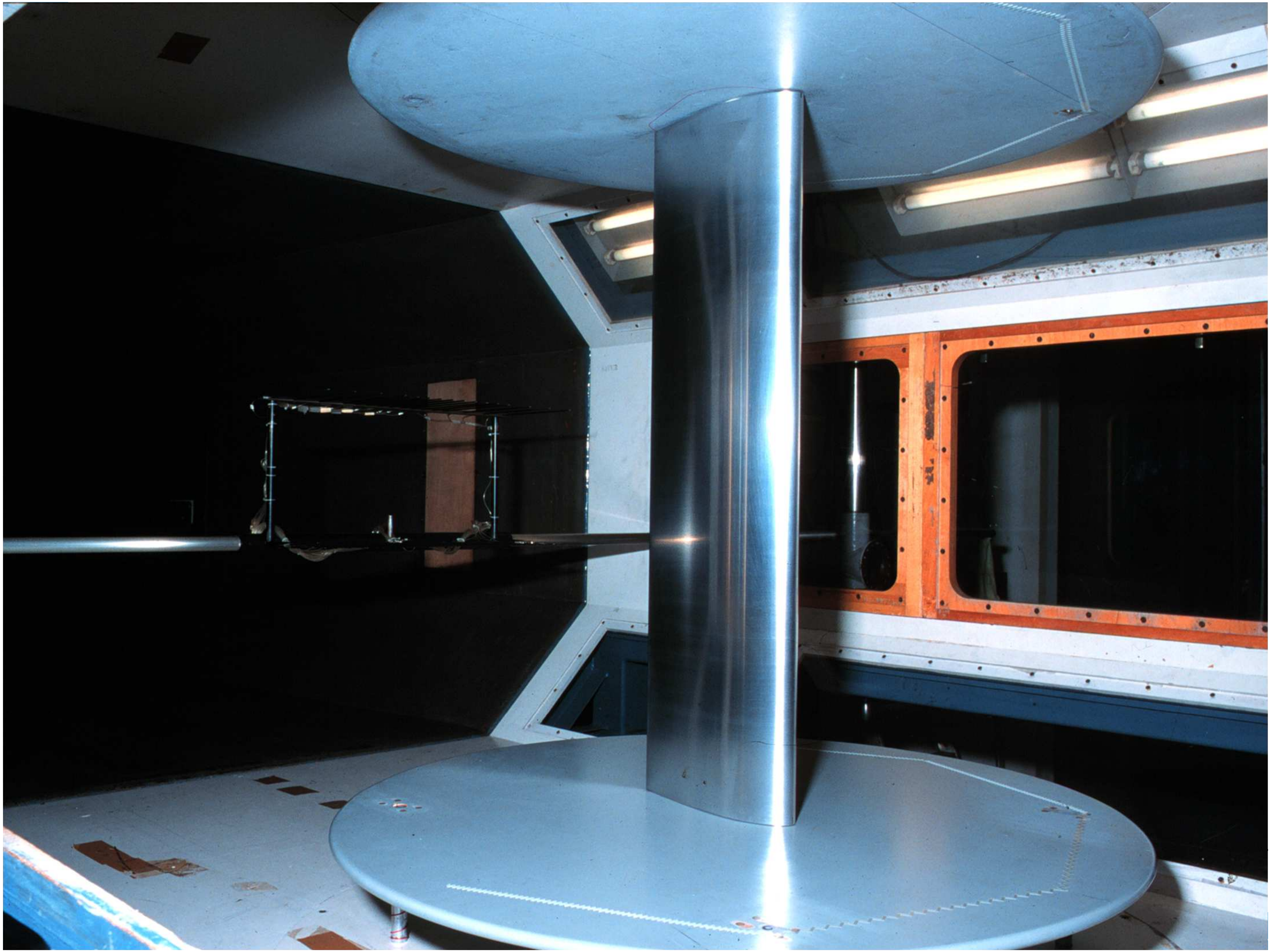
$$\rho A_1 V_1 = \rho A_2 V_2$$

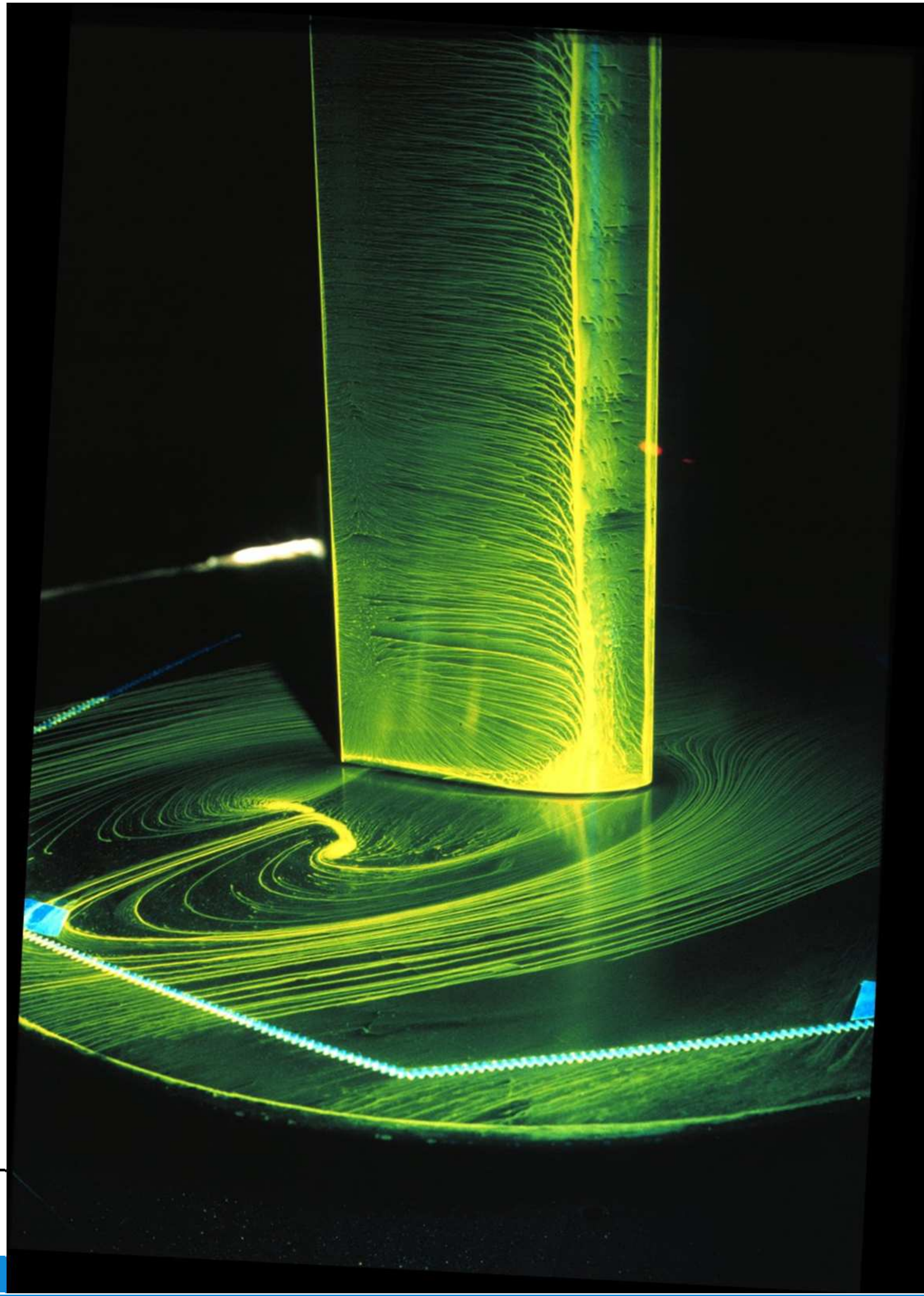
$$A_2 = \frac{\rho A_1 V_1}{\rho V_2} = A_1 \frac{V_1}{V_2} = 5 * \frac{10}{30} = 1.67 \ [\mathrm{m}^2]$$

Application of the continuity law:………………. e.g. wind tunnels

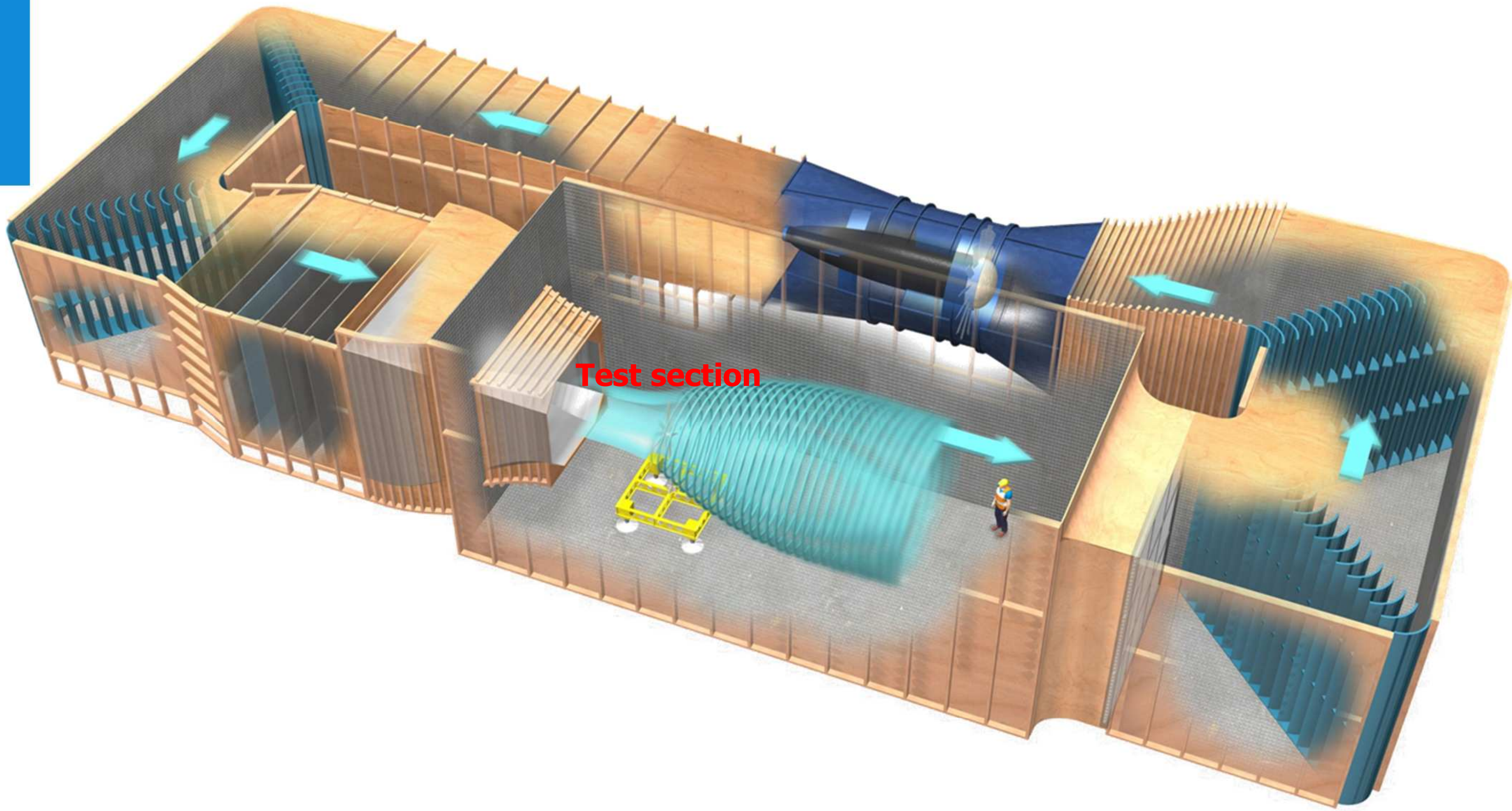

10.94
2.60
9.10
1.38
0.55
B
A
E
5.17
3.64
ϕ 2.90
C
D
5.69
17.82
Sizes in meters
0.42
1.25
1.80
A
0.76
2.10
2.60
B
2.90
C
3.66
4.50
D
0.42
5.70
6.50
E

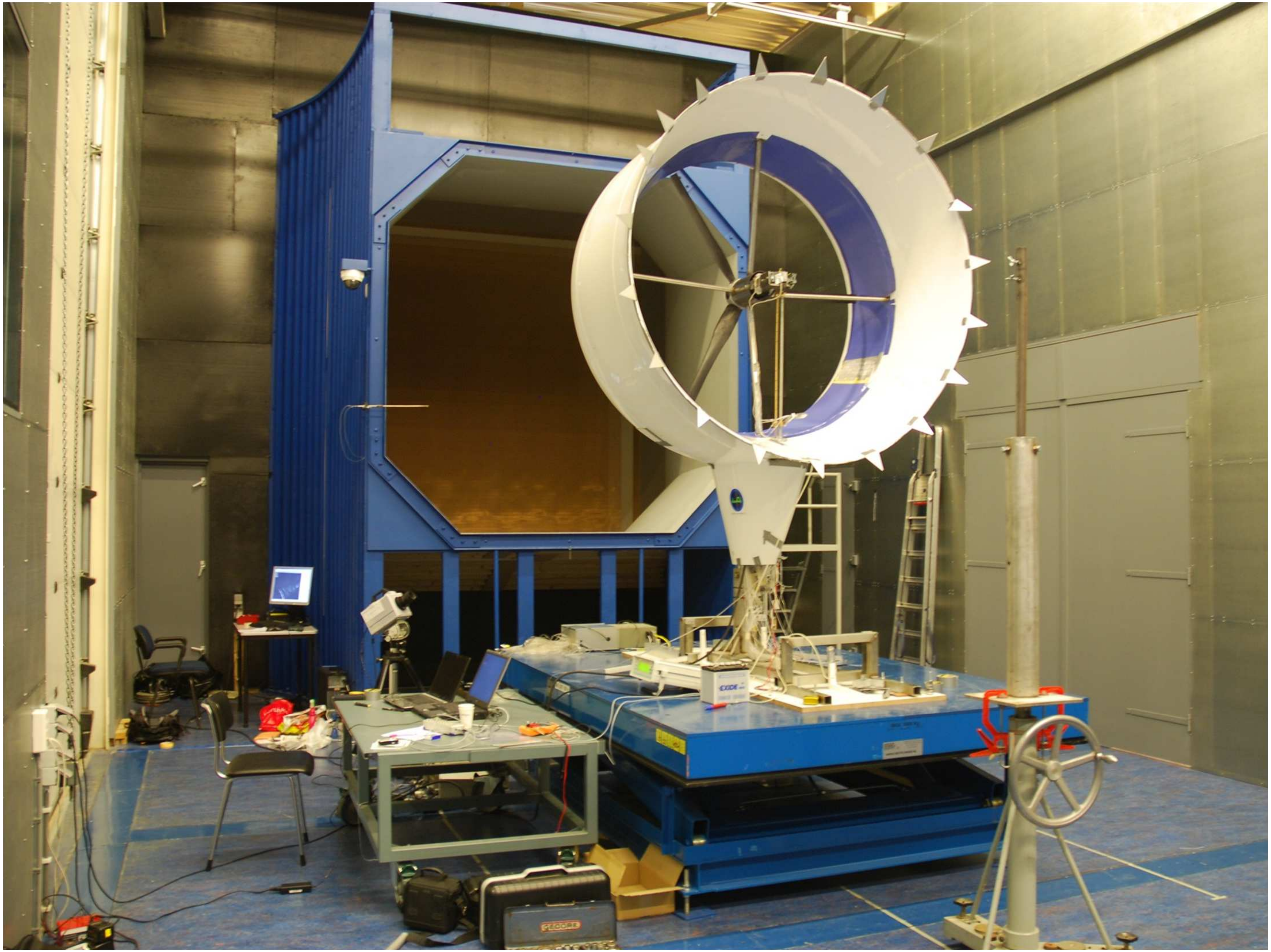






Circuit layout of Delft University Open-jet Facility

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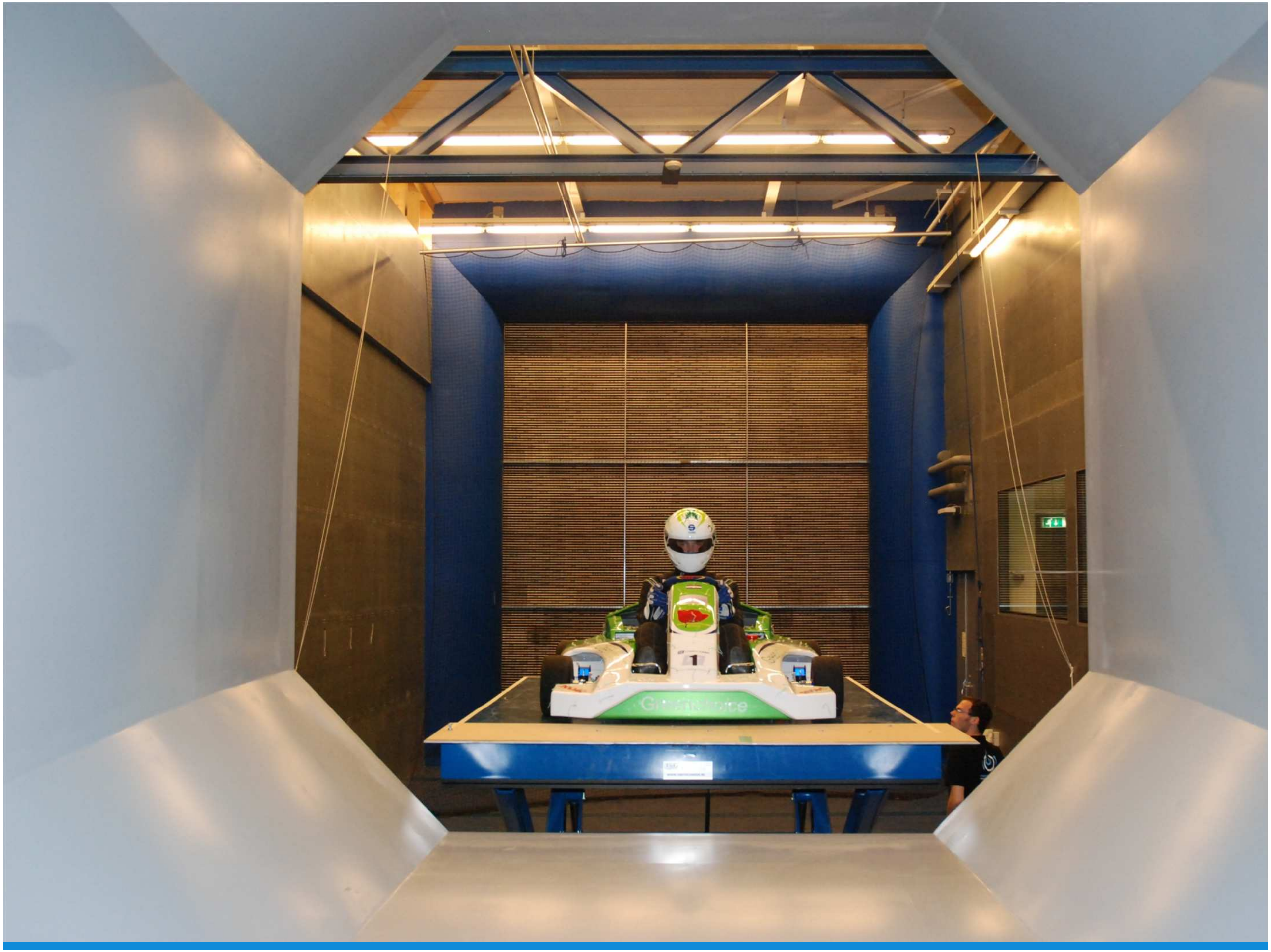
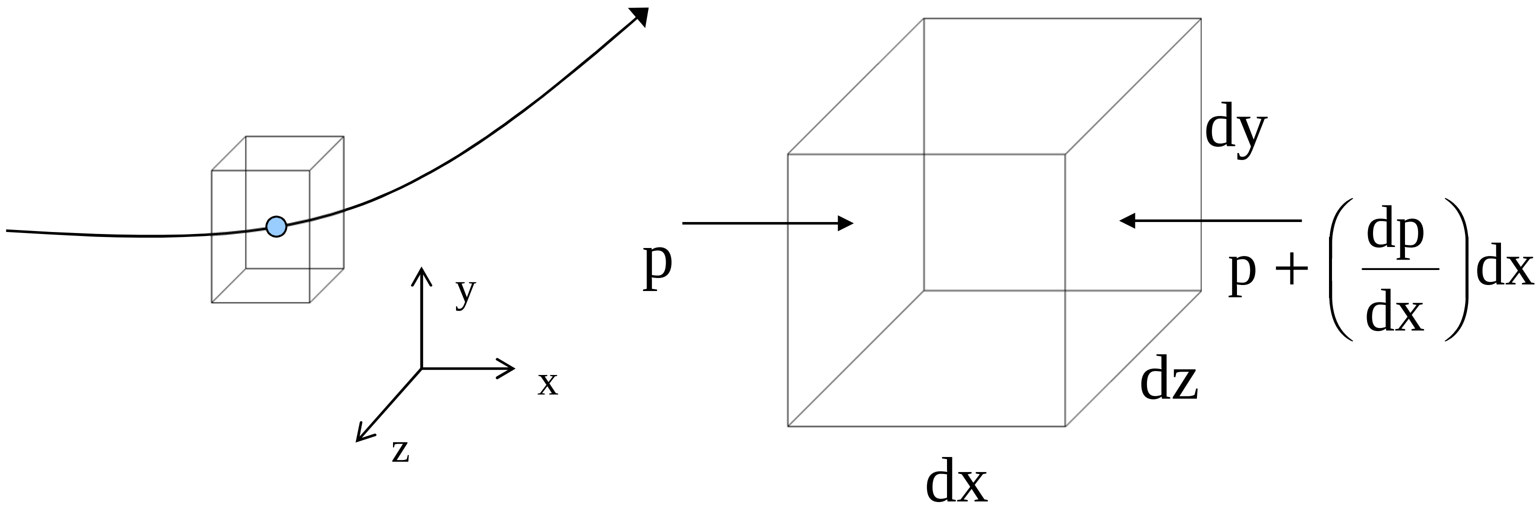


Euler equation

Newton's Second Law:

F = m.a (Force = mass * acceleration)

Applied to a flowing gas :



Euler equation, left hand part of F=m*a

In reality 3 forces act on this element:

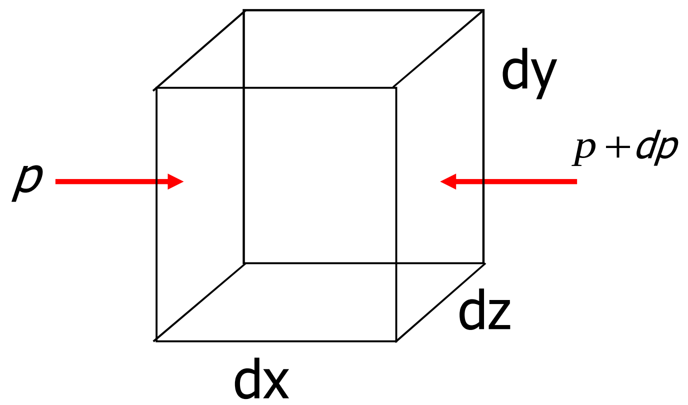
- Pressure force
- Friction force
- Gravity force

In the derivation we neglect 2 forces:

- Neglect the gravity force (small)
- Neglect the viscosity → No friction forces

Euler equation, left hand part (F=m*a)

The force in **x-direction**:



$$F = pdydz - \left(p + \frac{dp}{dx}dx \right) dydz$$

Hence: $$F = -\frac{dp}{dx}dxdydz$$ = force on fluid element due to pressure

Euler equation, right hand part (F=m*a) and resulting equation

The mass (**m**) of the fluid element is : m = ρ.Vol = ρ*dxdydz*

Acceleration (**a**) of the fluid element is: $a = \frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = \frac{dV}{dx}V$

Substitution in F = m*a ⇨ $-\frac{dp}{dx}(dxdydz) = \rho(dxdydz)V\frac{dV}{dx}$

Or: $dp = -\rho V dV$

Resulting equation:

$$dp = -\rho V dV$$

1. It is a relation between Force and Momentum
2. Also called the <u>momentum equation</u>

Euler equation

Euler (1707 - 1783)

Swiss Mathematician

$e^{i\pi} + 1 = 0 \qquad e^{i\phi} = \cos\phi + i\sin\phi$

Euler equation

$$\boxed{dp = -\rho V dV}$$

Keep in mind for this equation:

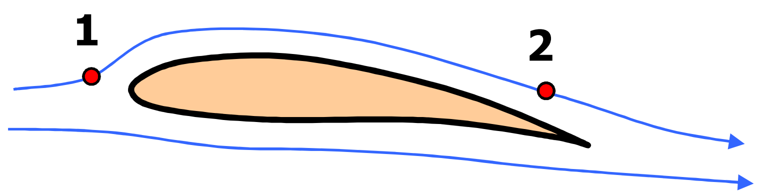
- Gravity forces are neglected
- Viscosity is neglected (inviscid flow)
- Steady flow
- Flow may be compressible!

The Euler equation is a *Differential equation*

Bernoulli's law for inviscid and incompressible flow, gravity forces neglected

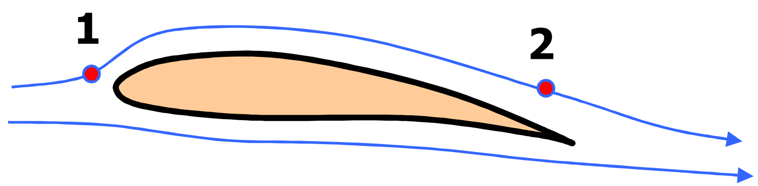
Integrate Euler equation (dp + ρVdV = 0) along streamline between point 1 and 2:

$$\int_{p_1}^{p_2} dp + \int_{V_1}^{V_2} \rho V dV = 0$$



Bernoulli's law for inviscid and incompressible flow, gravity forces neglected

$$\int_{p_1}^{p_2} dp + \int_{V_1}^{V_2} \rho V dV = 0 \rightarrow (p_2 - p_1) + \rho\left(\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2\right) = 0$$



Bernoulli's law for inviscid and incompressible flow, gravity forces neglected

$$p_1 + \frac{1}{2}\rho V_1^{\,2} = p_2 + \frac{1}{2}\rho V_2^{\,2}$$

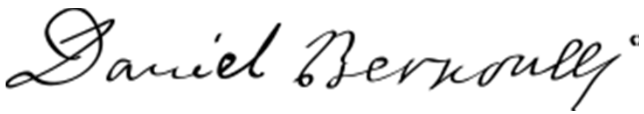
$$\underbrace{\boxed{p}}_{\text{Static pressure}} + \underbrace{\boxed{\frac{1}{2}\rho V^2}}_{\text{Dynamic pressure}} = \underbrace{p_t}_{\text{Total pressure}}$$

constant along a streamline

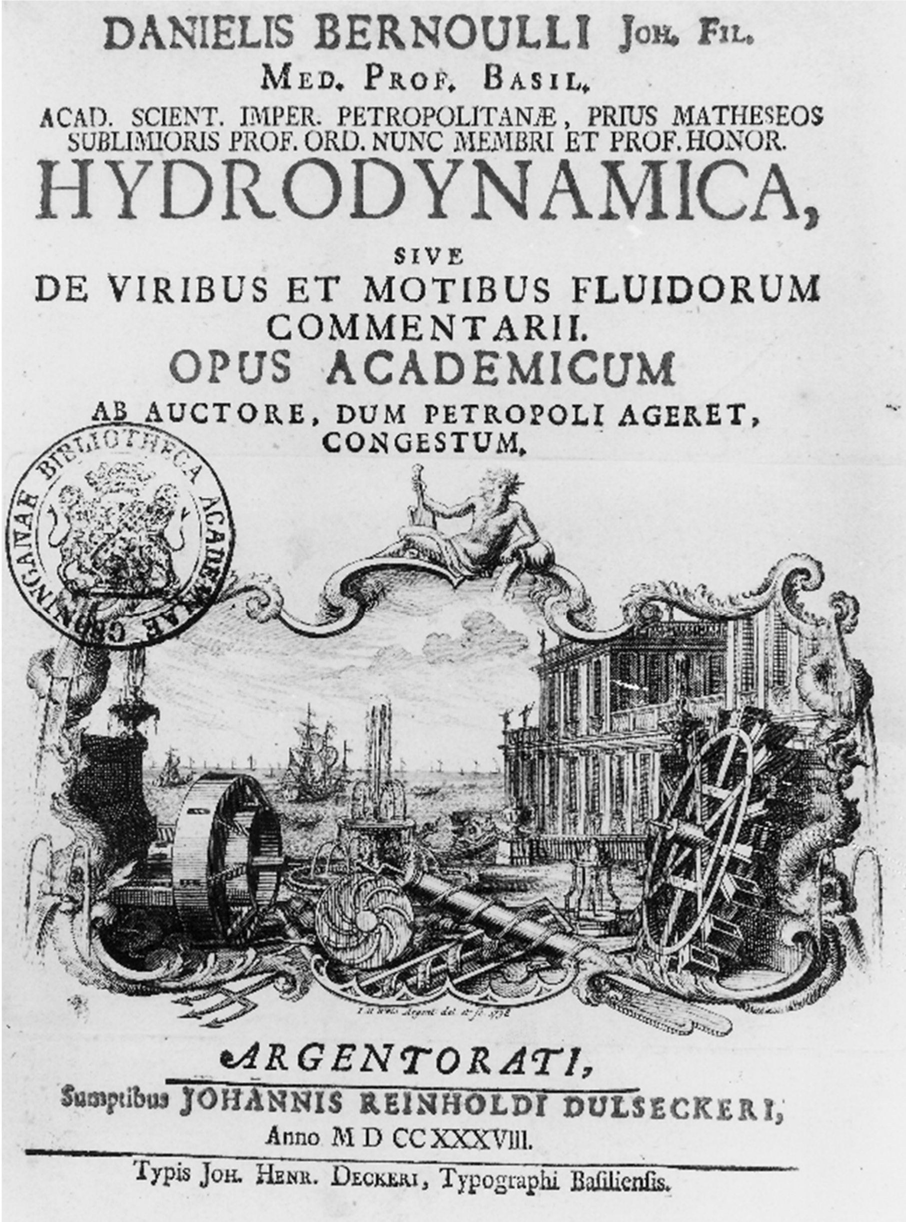
Bernoulli's law for inviscid and incompressible flow, gravity forces neglected

Bernoulli's law:

$$p + \frac{1}{2}\rho V^2 = \text{constant along a streamline}$$



Daniel Bernoulli *(1700-1782)*
Dutch born (Groningen, 29 jan. 1700),
Swiss Mathematician



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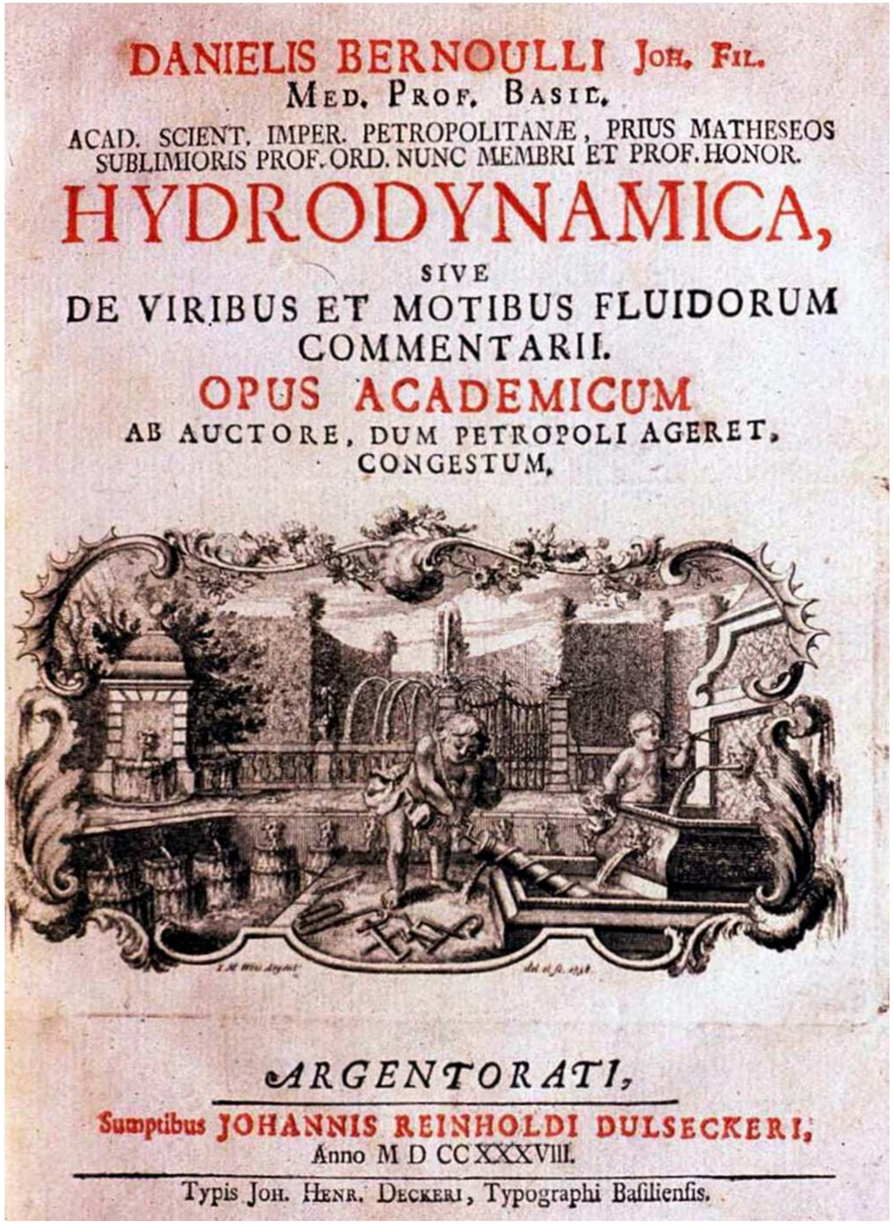

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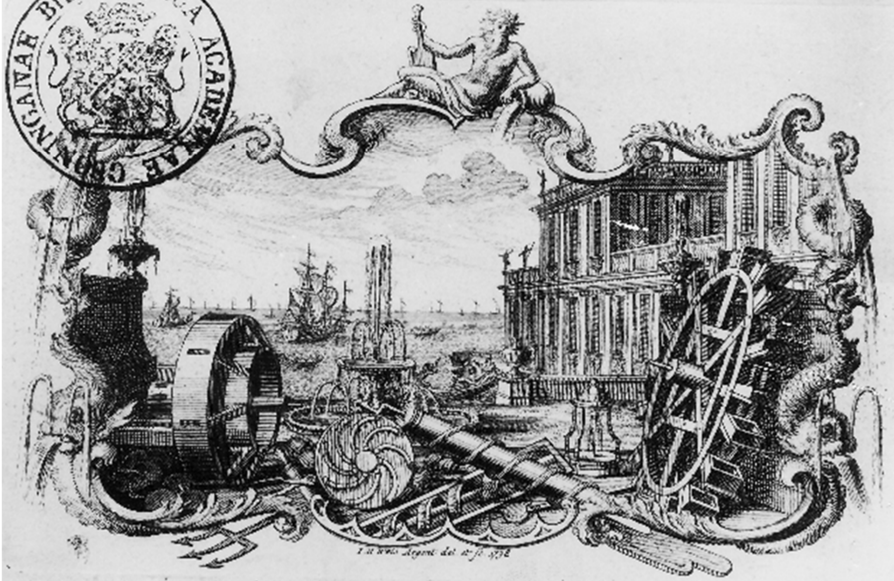

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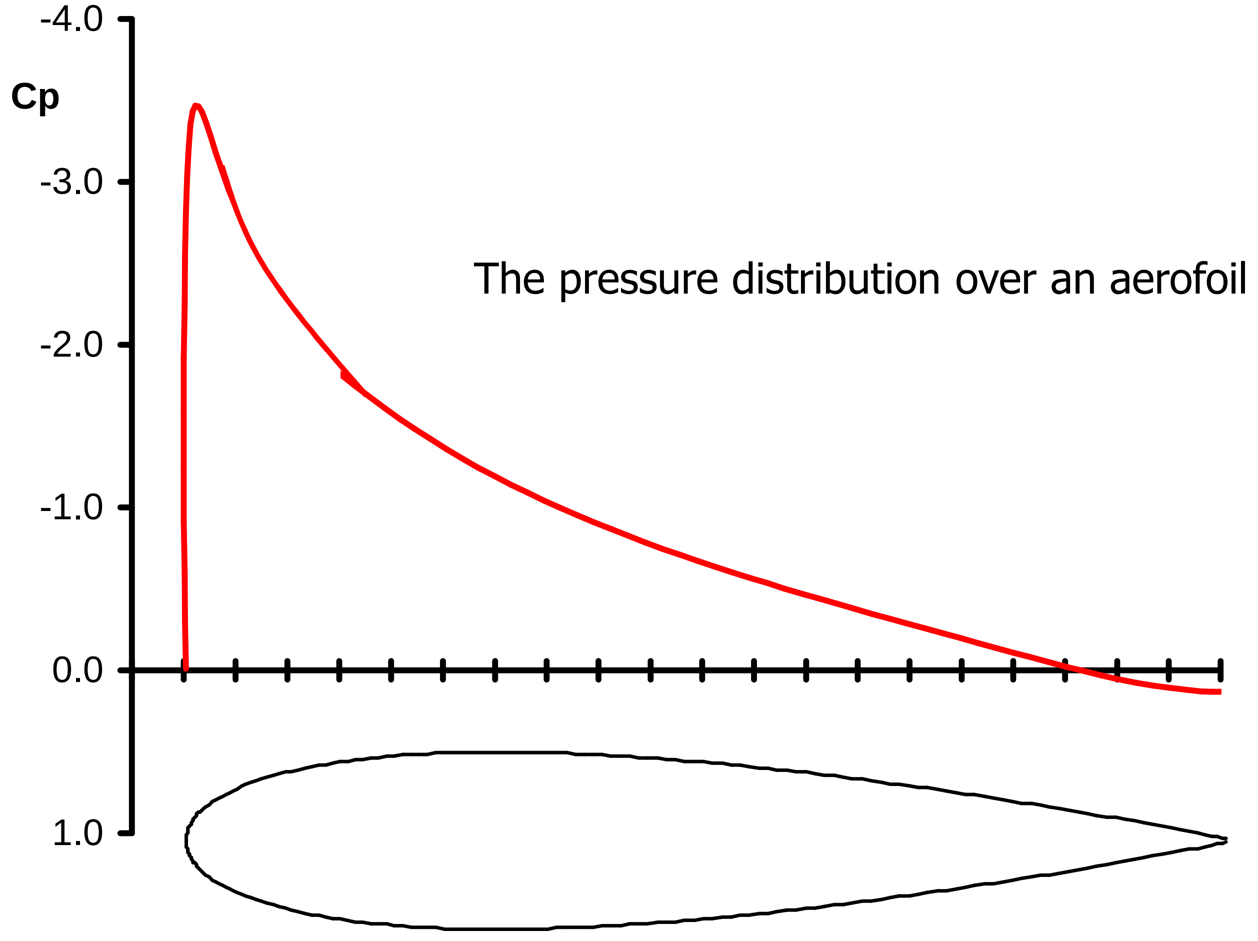
Application of Bernoulli's principle



Remember

- Bernoulli equation only for inviscid = frictionless, incompressible flow
- Bernoulli equation is valid along a streamline
- For compressible flow => Euler equation !
- Momentum equation, Euler equation and Bernoulli equation is in fact **F = m· a** (Newton) applied to fluid dynamics**.**

Ex 4.4. Consider the same convergent duct as in example 4.1. If the air pressure and temperature at the inlet are p_1 = 1.2 x 10^5 N/m^2 and T_1=330K, respectively, calculate the pressure at the exit.

Example: Consider the same convergent duct as in the previous example. If the air pressure and temperature at the inlet are p_1 = 1.2 x 10^5 [N/m^2] and T_1=330 [K] respectively, calculate the pressure at the exit

- **Assume inviscid, incompressible flow**
- **Neglect the gravity forces**
- **On a streamline in the duct: Apply Bernoulli's law!**

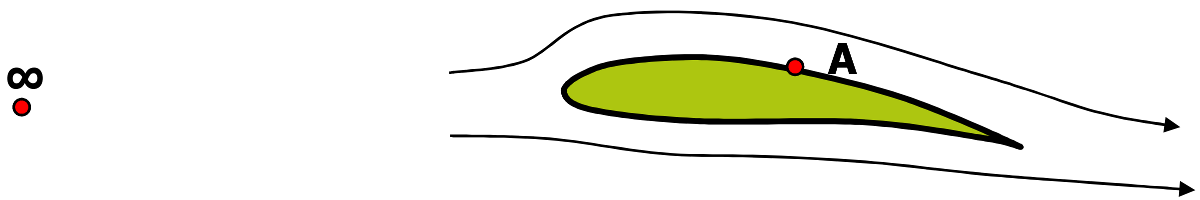
$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \rightarrow p_2 = p_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$

$$p_1 = \rho R T_1 \rightarrow \rho = \frac{p_1}{RT_1} = \frac{1.2*10^5}{\left(287.15*330\right)} = 1.266 \text{ [kg/m}^3\text{]}$$

$$p_2 = p_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) = 1.2*10^5 + \frac{1}{2}*1.266*\left(10^2 - 30^2\right) = 1.19*10^5 \text{ [N/m}^2\text{]}$$

Example: Consider an airfoil in a flow of air, where far ahead (upstream) of the airfoil, the pressure, velocity and density are: 1.01×10^5 [N/m^2], 150 [km/h] and 1.225 [kg/m^3] respectively. At a given point A on the airfoil the pressure is 9.95×10^4 [N/m^2]. What is the velocity in point A?

- Assume inviscid, incompressible flow
- Neglect the gravity forces
- On a streamline near the airfoil: Apply Bernoulli's law!



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Assume that point A and ∞ are on the same streamline. Bernoulli then gives:

$$p_\infty + \frac{1}{2}\rho_\infty V_\infty^{\,2} = p_A + \frac{1}{2}\rho_\infty V_A^{\,2} \rightarrow V_A = \sqrt{\frac{\left(p_\infty - p_A\right) + \frac{1}{2}\rho_\infty V_\infty^{\,2}}{\frac{1}{2}\rho_\infty}}$$

$$V_A = \sqrt{\frac{2\left(p_\infty - p_A\right)}{\rho_\infty} + V_\infty^{\,2}} = \sqrt{\frac{2\left(1.01*10^5 - 9.95*10^4\right)}{1.225} + \left(\frac{150}{3.6}\right)^2} = 64.7 \text{ [m/s]}$$

- Problems 4.3 and 4.5