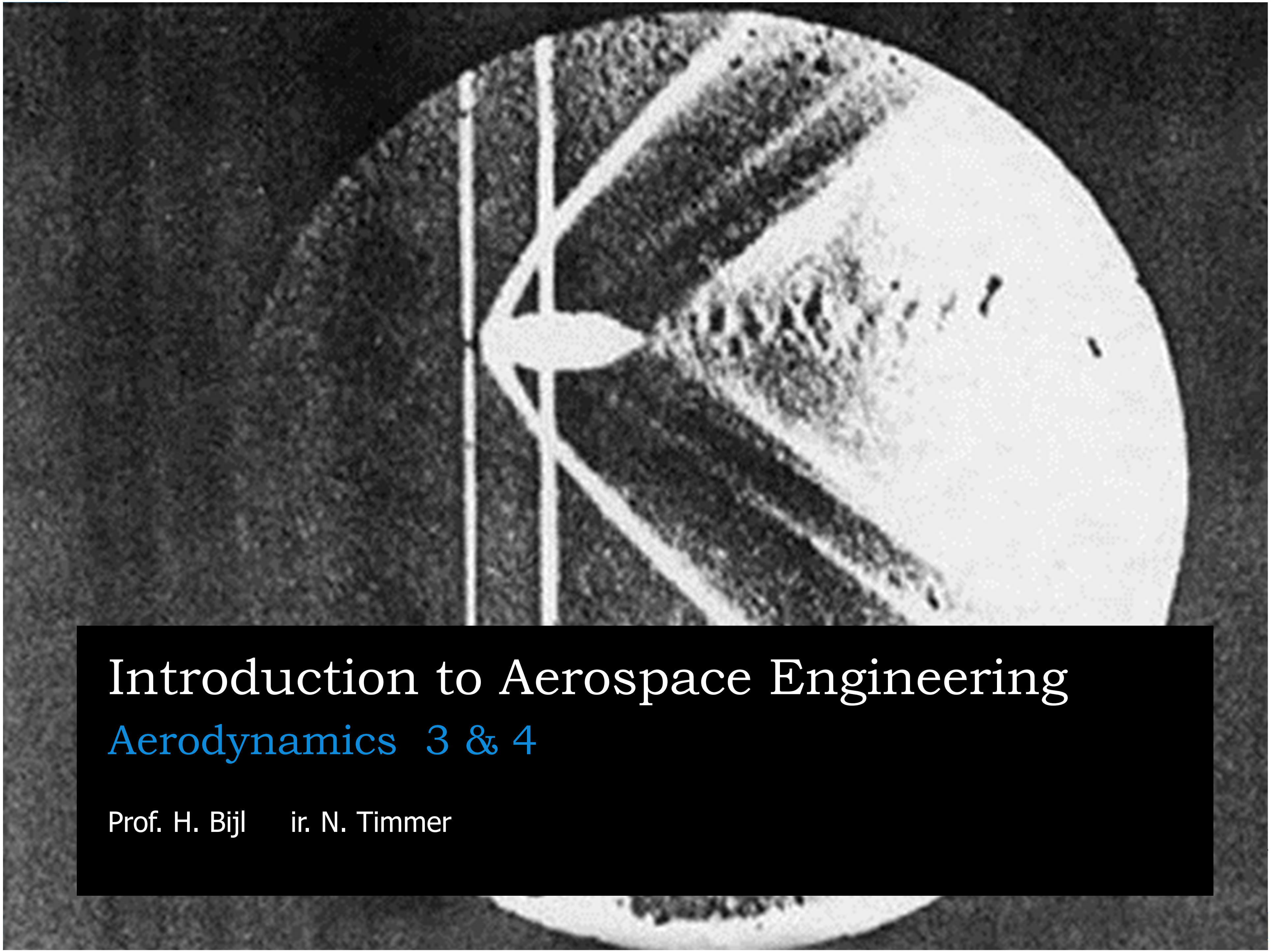
Introduction to Aerospace Engineering

Lecture slides



Introduction to Aerospace Engineering

Aerodynamics 3 & 4

Prof. H. Bijl   ir. N. Timmer

3 & 4.

Compressibility

Anderson 4.5-4.11.1

Ernst Mach

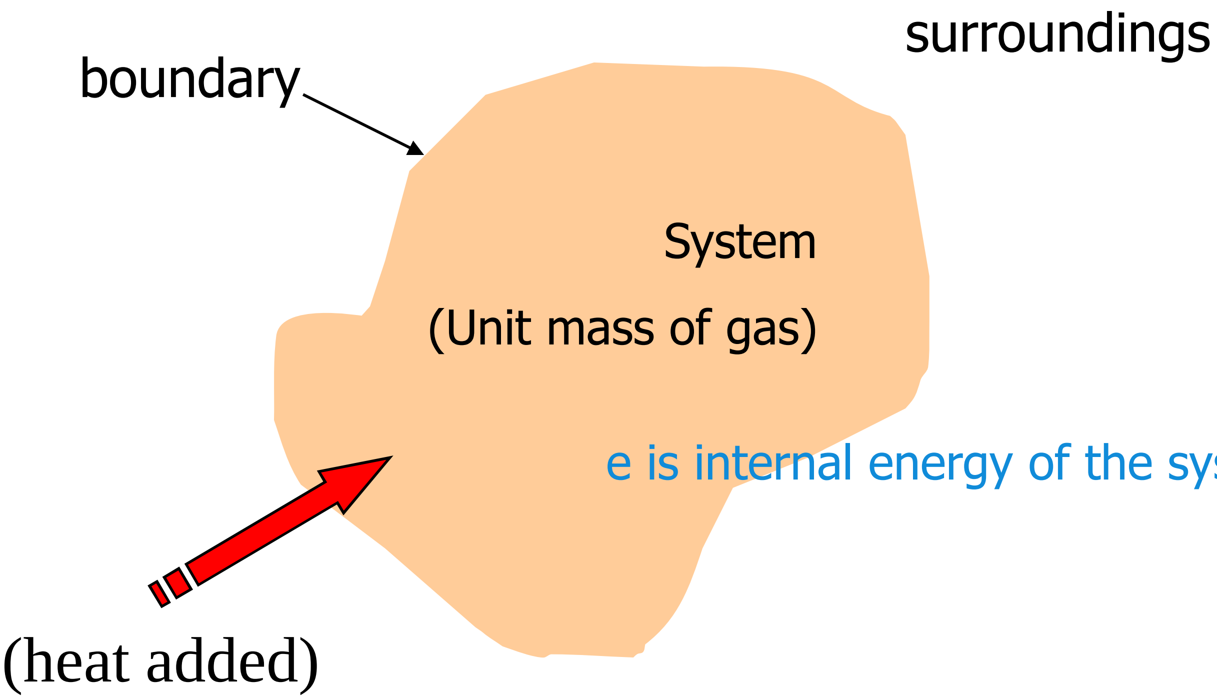
1838-1916

- Picture on front page:

  First picture ever of the shock wave around a bullet at supersonic speed presented by Mach at the Academy of Sciences in Vienna in 1887
  The two vertical lines are made by the trip wires that triggered the camera as the projectile passed by
  He employed an innovative approach called the shadowgraph (predecessor of the Schlieren technique) based on the diffraction of light that changes with air density and temperature

Subjects lecture 3

- Thermodynamics
  - First law of thermodynamics
  - Specific heat
  - Equations for a perfect gas
- Isentropic flow
- Energy equation



e is internal energy of the system

e can only change when:

- Heat is added or taken away from the system

  (δq is heat per unit mass)

- Work is done on, or by, the system

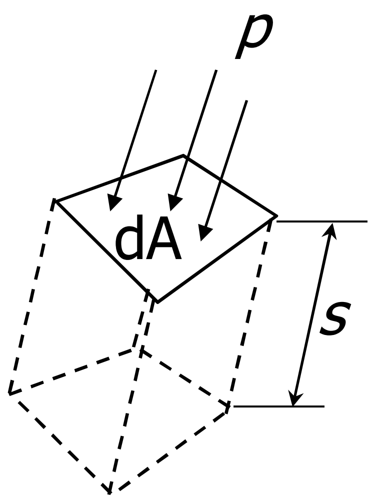
  (δw is work done per unit mass)

$$\boxed{de = \delta q + \delta w}$$

First law of thermodynamics

Different forms of first law

For the work at surface: work is *Pressure* x *Area* x *distance* = *p dA s*



$$\delta w = \int_A p dA s = p \int_A s dA = -p dv$$

For a reversible process : $\delta w = -p dv$

dv is an incremental change in the volume due to a displacement of the boundary of the system

hence : $$\boxed{de = \delta q - p dv}$$

Enthalpy

- Lets introduce another quantity of the flow: the specific enthalpy h

$$h=e+pv$$

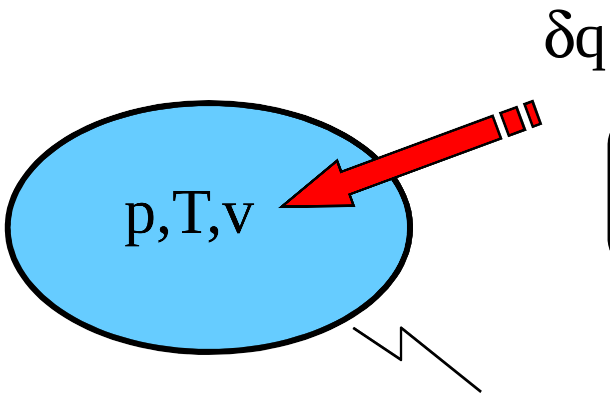
With differential: $dh=de+d(pv)$

$$dh=de+pdv+vdp$$

$$\boxed{dh=\delta q+vdp}$$

2 Types of Processes

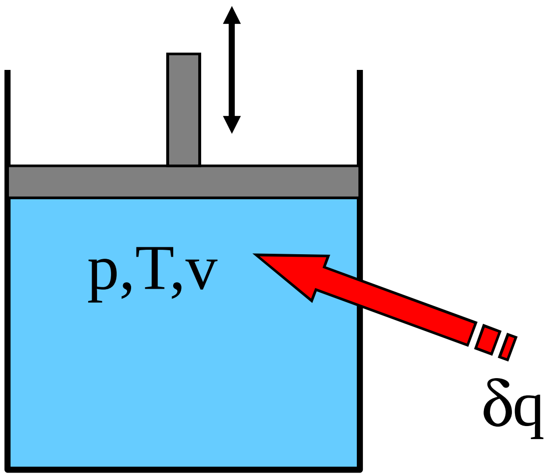
1st example



Constant Volume Process

rigid boundary = constant volume

2nd example



Constant Pressure Process

Piston moves just to keep pressure p constant

Specific Heat

Definition of Specific Heat :
(depends on type of process, thus=multivalued)

$$c \equiv \frac{\delta q}{dT}$$

The resulting value of dT when adding δq depends on the type of process!

$$c_v = \left(\frac{\delta q}{dT}\right)_{\text{at constant volume}}$$

$$c_p = \left(\frac{\delta q}{dT}\right)_{\text{at constant pressure}}$$

Now relate c_v and c_p to e, h etc.

Specific Heat

- For constant volume process $\Rightarrow dv = 0$ $\boxed{de = c_v\, dT}$

  This can be integrated to : $\boxed{e = c_v\, T}$

- For constant pressure process $\Rightarrow dp = 0$ $\boxed{dh = c_p\, dT}$

  This can be integrated to : $\boxed{h = c_p\, T}$ *(h=0 at T=0)*

Above relations also hold *in general* as long as the gas is a *perfect gas*.

Equations for a perfect gas

Hence for a *perfect gas* (no molecular forces) :

$$de = c_v dT$$
$$dh = c_p dT$$
$$e = c_v T$$
$$h = c_p T$$

For air: $c_v = \ \ 720$ J/(kg K)

$c_p = 1008$ J/(kg K)

(For any process with T < 600 K)

Isentropic flow

We introduce one more concept to bridge thermodynamics and compressible aerodynamics : Isentropic flow

<u>Definitions:</u>

- Adiabatic process : $\delta q = 0$
- Reversible process : no frictional or dissipative effects
- Isentropic Process : both adiabatic **<u>and</u>** reversible.

Note: Though the flow can be isentropic the TEMPERATURE might change from point to point!

Isentropic flow, ratio of specific heats

For isentropic flow (or a perfect gas) $\Rightarrow$ Several important relationships between p, T, ρ:

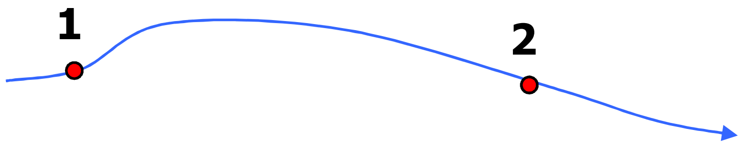
Since $\delta q = 0 \Rightarrow \delta q = de + pdv = 0 \Rightarrow$ $-pdv = de = c_v dT$

Since $\delta q = 0 \Rightarrow \delta q = dh - vdp = 0 \Rightarrow$ $vdp = dh = c_p dT$

$$\frac{-pdv}{vdp} = \frac{c_v}{c_p} \quad \Rightarrow \quad \frac{dp}{p} = -\left(\frac{c_p}{c_v}\right)\frac{dv}{v} \quad \Rightarrow \quad \frac{dp}{p} = -\gamma\frac{dv}{v}$$

Ratio of specific heats $\dfrac{c_p}{c_v} = \gamma = 1.4$ ← for air

Isentropic Flow



Now integrate between points 1 and 2 on a streamline ...

$$\int_1^2 \frac{dp}{p} = -\gamma \int_1^2 \frac{dv}{v} \Rightarrow \qquad [\ln p]_1^2 = -\gamma [\ln v]_1^2 \Rightarrow$$

$$\ln p_2 - \ln p_1 = -\gamma [\ln v_2 - \ln v_1]$$

Remember : $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ and $a \ln b = \ln(b^a)$

$$\ln\left(\frac{p_2}{p_1}\right) = -\gamma \ln \frac{v_2}{v_1} \Rightarrow \qquad \left(\frac{p_2}{p_1}\right) = \left(\frac{v_2}{v_1}\right)^{-\gamma}$$

Since the specific volume is $v_1 = \dfrac{1}{\rho_1}$ $\quad v_2 = \dfrac{1}{\rho_2}$ we find :

$$\frac{p_2}{p_1} = \left(\frac{\rho_1}{\rho_2}\right)^{-\gamma} \Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

for ISENTROPIC FLOW

Isentropic Flow

Now with the equation of state pv=RT or p/ρ=RT we may write :

$$\frac{p_2}{p_1}=\left(\frac{p_2}{RT_2}\frac{RT_1}{p_1}\right)^{\gamma}=\left(\frac{p_2}{p_1}\frac{T_1}{T_2}\right)^{\gamma}\Rightarrow\left(\frac{p_2}{p_1}\right)^{1-\gamma}=\left(\frac{T_1}{T_2}\right)^{\gamma}$$

$$\left(\frac{p_2}{p_1}\right)^{1-\gamma}=\left(\frac{T_2}{T_1}\right)^{-\gamma}\Rightarrow\boxed{\frac{p_2}{p_1}=\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

for ISENTROPIC FLOW

AGAIN: **Remember**
Isentropic flow relations are only relevant to **compressible** flow

Isentropic Flow Equations

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

Energy equation for frictionless adiabatic flow, derived from the 1st law of thermodynamics

Physical principle:

Energy can neither be created nor destroyed

Energy equation .... (ctnd)

One of the forms of the 1st law of thermodynamics:

$$\delta q = dh - vdp$$

For adiabatic flow $\delta q = 0 \;\Rightarrow$

$$dh - vdp = 0$$

Energy equation .... (ctnd)

$$dh - vdp = 0$$

For frictionless flow the Euler equation gives:

$$dp = - \rho VdV$$

With this the 1st law becomes:

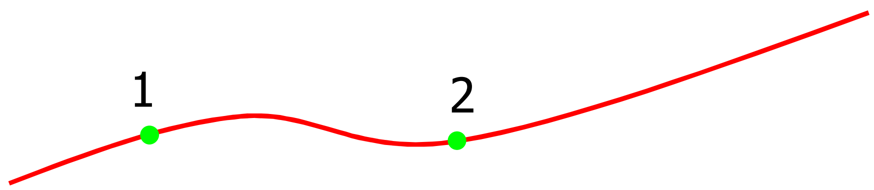
$$0 = dh + v \rho VdV$$

Energy equation .... (ctnd)

Since $v=1/\rho$ (= specific volume) we find:

$$dh + VdV = 0$$

Now integrate between two points along a streamline ...



Energy equation for frictionless, adiabatic flow

$$\int_{h_1}^{h_2} dh + \int_{V_1}^{V_2} V dV = 0 \Rightarrow \qquad h_2 - h_1 + \frac{1}{2}(V_2^2 - V_1^2) = 0 \Rightarrow$$

$$h_2 + \frac{1}{2}V_2^2 = h_1 + \frac{1}{2}V_1^2 \qquad \text{or}$$

$$\boxed{h + \frac{1}{2}V^2 = \text{constant}}$$

Since also $h = C_p T \Rightarrow$ $$\boxed{C_p T + \frac{1}{2}V^2 = \text{constant}}$$

Summary of equations

For steady, frictionless, incompressible flow:

Continuity equation $A_1 V_1 = A_2 V_2$

Bernoulli's equation $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$

For steady, isentropic compressible flow (adiabatic and frictionless) :

Continuity equation $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

Isentropic relations $\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$

Energy equation $c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$

Equation of state $P_1 = \rho_1 R T_1 \quad P_2 = \rho_2 R T_2$

Example 4.9: A supersonic wind tunnel is considered.

The air temperature and pressure in the reservoir of the tunnel are:
T_0 = 1000 K and p_0 = 10 atm.

The static temperatures at the throat and exit are:
T^* = 833 K and T_e = 300 K.

The mass flow through the tunnel is 0.5 kg/s.
For air, c_p = 1008 J/(kg K). Calculate:

a) Velocity at the throat V^*
b) Velocity at the exit V_e
c) Area of the throat A^*
d) Area of the exit A_e

Subjects lecture 4

- Speed of sound
- Measurement of air speed for:
  - incompressible, frictionless flow
  - subsonic, high speed adiabatic, frictionless flow

Summary of equations

For steady, frictionless, incompressible flow:

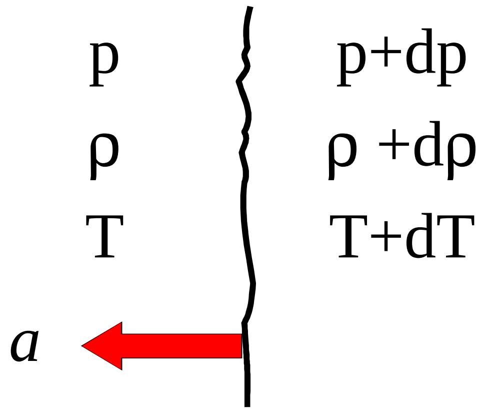
$$A_1 V_1 = A_2 V_2$$ Continuity equation

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$ Bernoulli's equation

For steady, isentropic (adiabatic and frictionless) compressible flow:

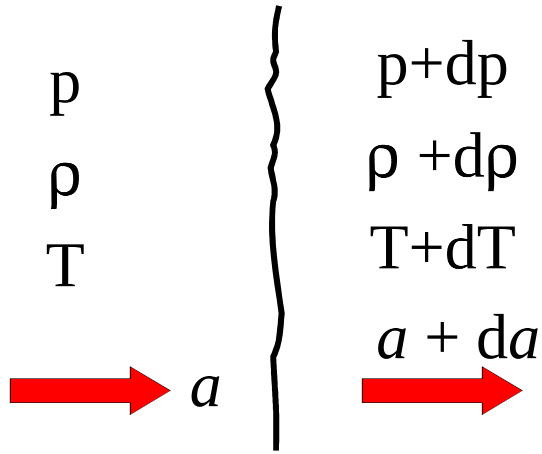
$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$	Continuity equation
$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$	Isentropic relations
$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$	Energy equation
$P = \rho R T$	Equation of state

Speed of sound



≡

is equal to



<u>moving</u> sound wave with speed *a* into a stagnant gas

<u>motionless</u> sound wave

static observer

observer moving with sound wave

Speed of sound

Apply the continuity equation:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \Rightarrow \quad \rho A_1 a = (\rho + d\rho)\, A_2\, (a+da)$$

1-dimensional flow $\Rightarrow A_1 = A_2 = A = \text{constant}$

Thus: $\rho a = (\rho + d\rho)(a + da) \Rightarrow \quad \cancel{\rho a} = \cancel{\rho a} + a d\rho + \rho da + \cancel{d\rho da}$

$$\Rightarrow \qquad a = -\rho \frac{da}{d\rho}$$

small, ignore

Speed of sound

Now apply momentum equation (Euler equation) $\Rightarrow$

$dp = -\rho V dV \Rightarrow$ $\quad dp = -\rho a da \;\Rightarrow\; da = -\dfrac{1}{\rho a} dp$

Substituting this into $\quad a = -\rho \dfrac{da}{d\rho}$

We obtain $\Rightarrow$ $\quad a = +\rho \dfrac{1}{\rho a} \dfrac{dp}{d\rho} \quad \Rightarrow$

$$\boxed{a^2 = \frac{dp}{d\rho}}$$

Going through the sound wave there is no heat addition, friction is negligible $\Rightarrow$

$$\boxed{a = \sqrt{\left(\frac{dp}{d\rho}\right)_{isentropic}}}$$

Speed of sound

Now we may apply the isentropic relations.

We want to rewrite $\dfrac{dp}{d\rho}$

With isentropic relations we find $a = \sqrt{\gamma \dfrac{p}{\rho}}$

Use the equation of state: $\dfrac{p}{\rho} = RT \Rightarrow$

$$\boxed{a = \sqrt{\gamma RT}}$$

Speed of sound in a perfect gas depends only on T !

Speed of sound

The Mach number is: $$M = \frac{V}{a}$$

M < 1	subsonic
M = 1	sonic
M around 1	transonic
M > 1	supersonic
M > 5	hypersonic

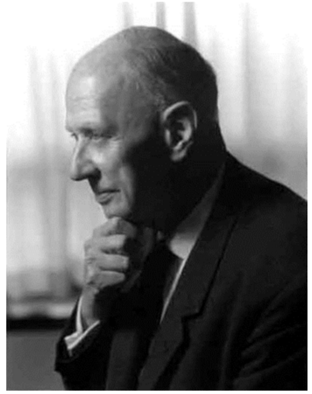
All velocity ranges have their own specific phenomena!

$$a = \sqrt{\gamma RT}$$

Mach

$$M = \frac{V}{a}$$

Ackeret



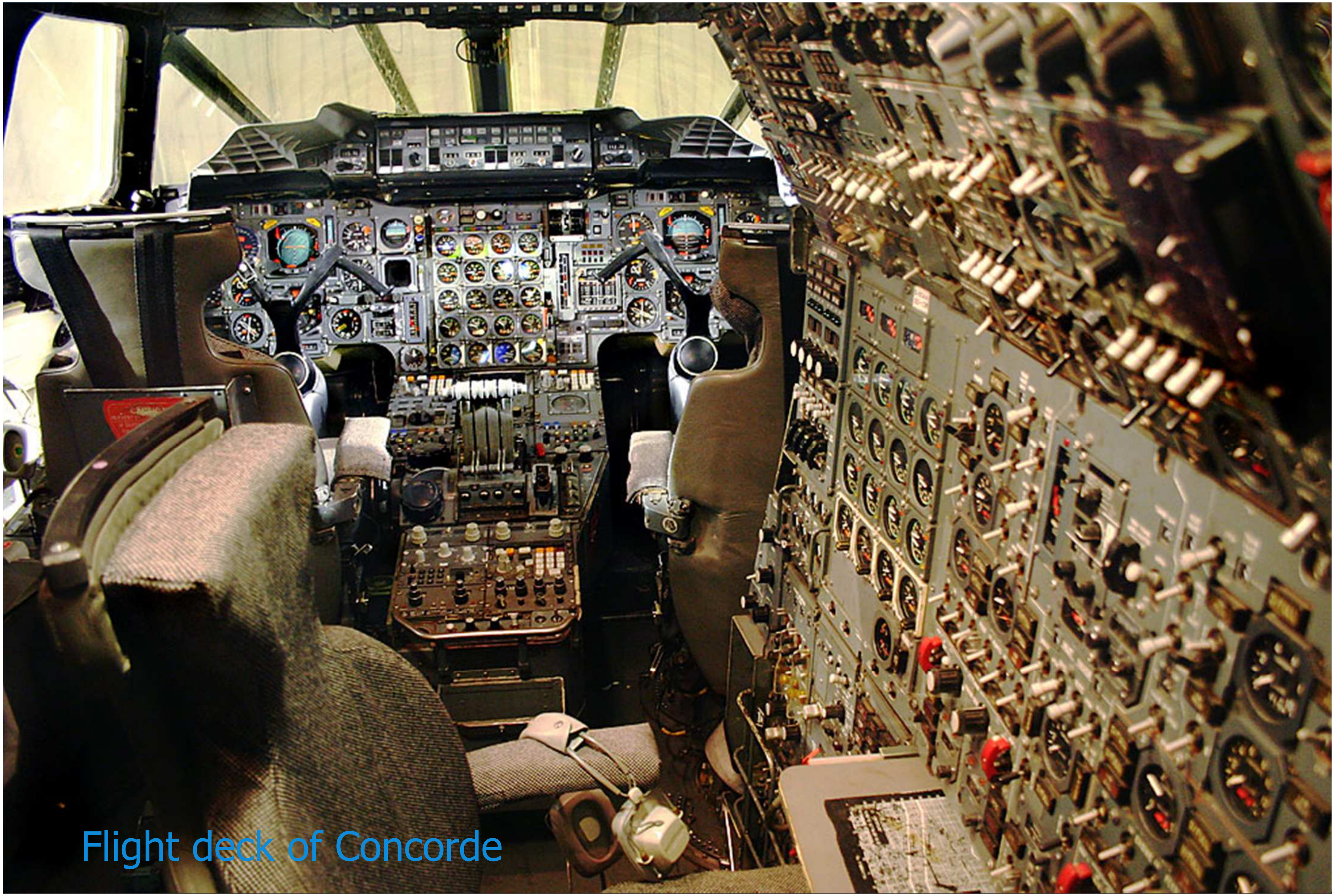
In honor of Ernst Mach the name “Mach number” was introduced in 1929 by Jacob Ackeret

A380: 2005 – present   Max # pass. 853 Mach=0.85

A380 flight deck


A380
A380 AIRBUS

BAC-Sud Aviation CONCORDE



Flight deck of Concorde


SORTIE EXIT
ATTENTION
ICI
ENREGISTREUR
DE VOL
CAUTION
FLIGHT
RECORDER
HERE

Tupolev TU-144


AIR FRANCE
to Copyright © Peter Unmuth - VAP
AIRLI

Equations for a perfect gas

- We defined: $h=e+pv$
- We found : $h=c_pT$ *and* $e=c_vT$
- With the eq. of state: $pv=RT$
- we find: $c_pT=c_vT+RT$
- hence: $c_p=c_v+R \;=>\; c_p\text{-}c_v=R$

We defined: $\gamma=\frac{c_p}{c_v}$

$$c_p=\frac{\gamma R}{\gamma-1}$$

Measurement of airspeed / flow from a reservoir : V=0

Energy equation: $$c_p T + \frac{1}{2} V^2 = constant$$

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_o + \frac{1}{2} V_o^2$$

Assume index 0 = stagnation point $\Rightarrow$ $V_o = 0 \Rightarrow$

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_o \Rightarrow \quad \frac{T_o}{T_1} = 1 + \frac{V_1^2}{2 c_p T_1}$$

Substitute: $C_p = \dfrac{\gamma R}{\gamma - 1}$ $\boxed{\text{(from } C_p - C_v = R\text{)}}$

$$\frac{T_o}{T_1} = 1 + \frac{\gamma - 1}{2} \frac{V_1^2}{\gamma R T_1}$$

Measurement of airspeed / flow from a reservoir : V=0

With $a_1^2 = \gamma RT$

$$\frac{T_o}{T_1} = 1 + \frac{\gamma - 1}{2}\frac{V_1^2}{a_1^2} = 1 + \frac{\gamma - 1}{2} M_1^2$$

Bring flow isentropically to rest $\Rightarrow$ Isentropic relation can be used $\Rightarrow$

$$\frac{p_o}{p_1} = \left(\frac{\rho_o}{\rho_1}\right)^{\gamma} = \left(\frac{T_o}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{\gamma-1}}$$

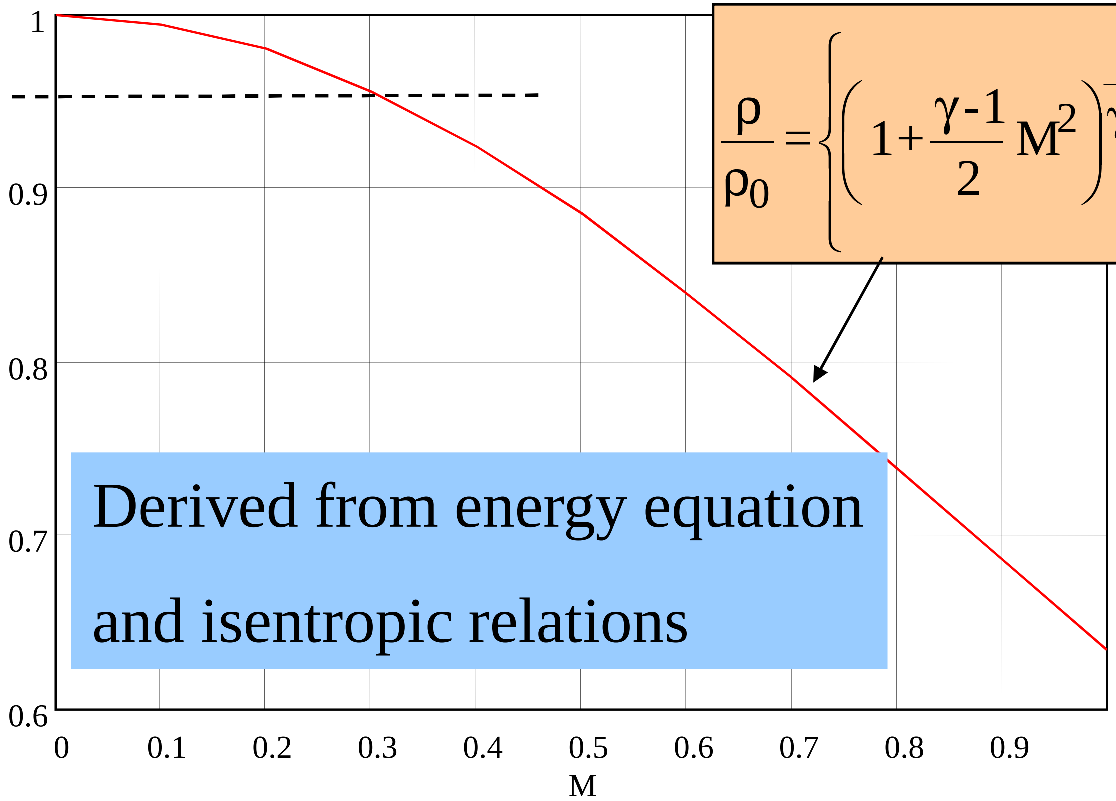
Second form of isentropic relations

$$\frac{T_o}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{1}{\gamma-1}}$$

Compressibility



$$\frac{\rho}{\rho_0} = \left\{ \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma - 1}} \right\}^{-1}$$

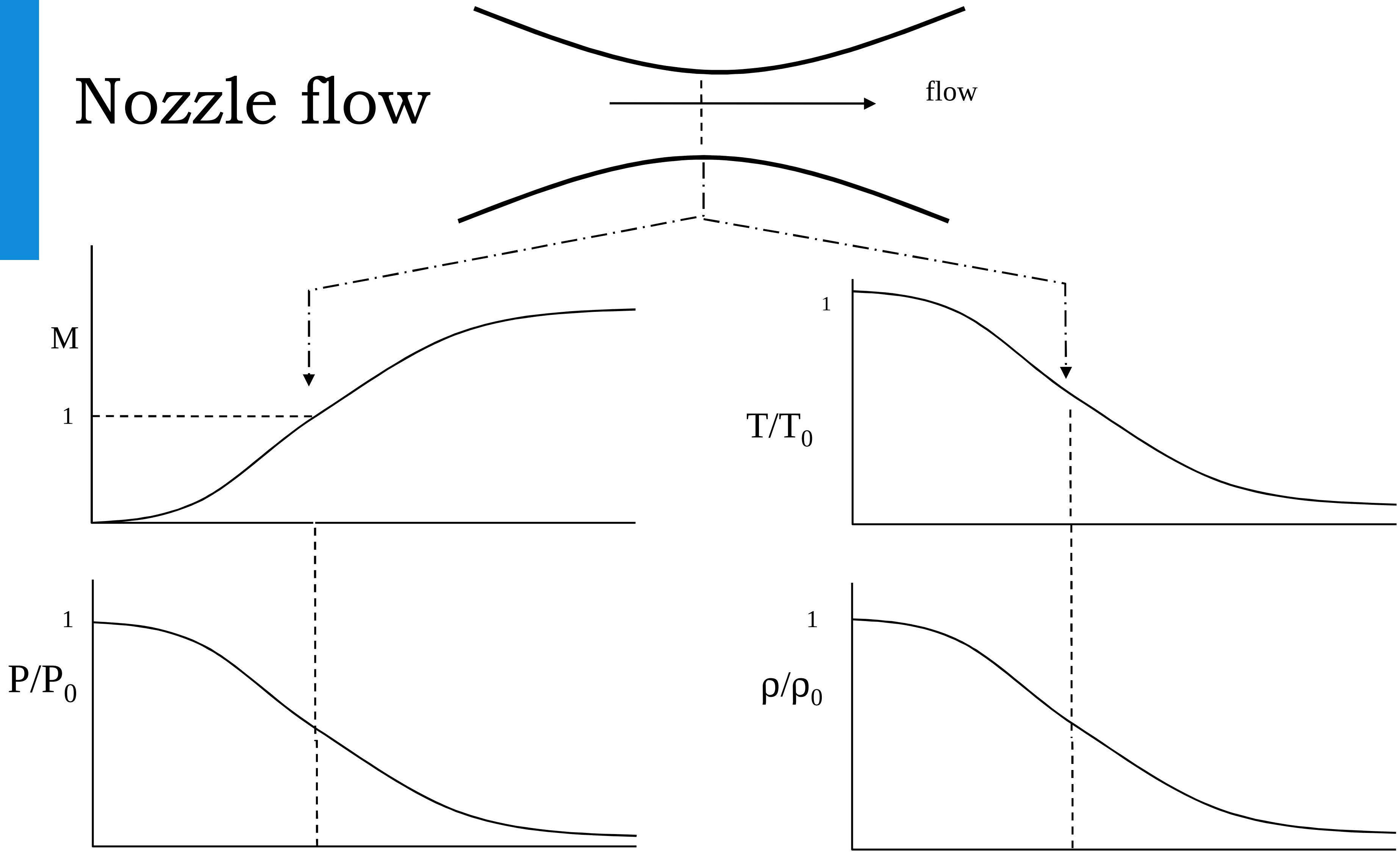
Derived from energy equation

and isentropic relations

For M < 0.3 the change in density is less than 5 %

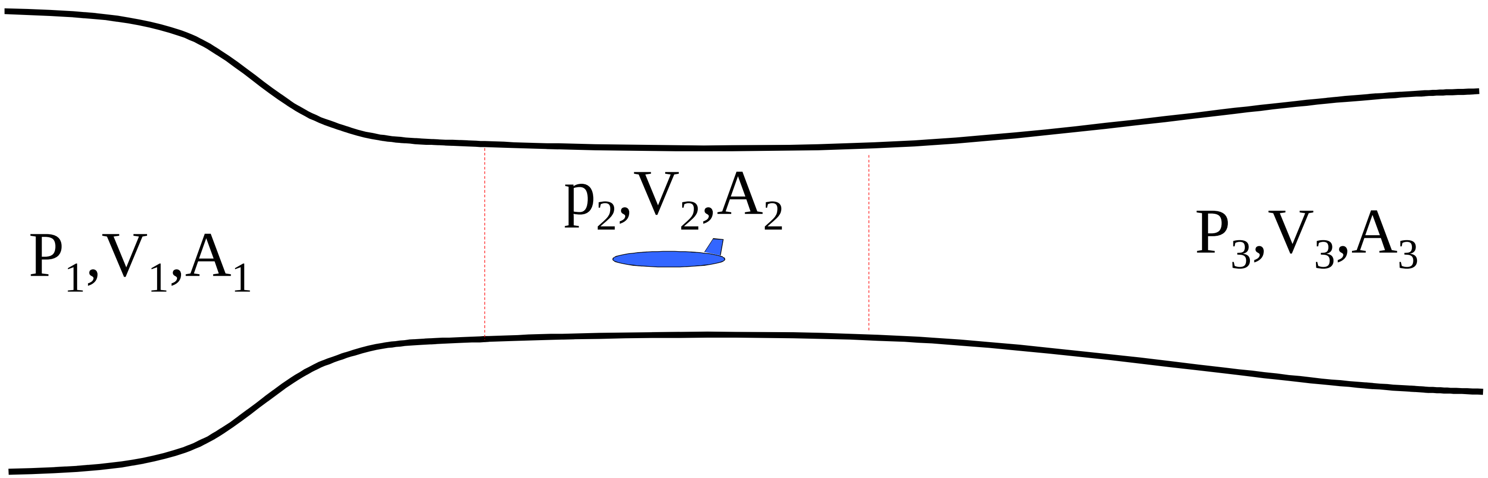
Thus : **for M < 0.3 the flow can be treated as incompressible**

Nozzle flow



Measurement of airspeed, subsonic low speed (=incompressible), frictionless flow

Subsonic nozzle:



P_{total} : $V = 0$

What is value of the air speed V_2?

Measurement of airspeed, incompressible, frictionless flow

Bernoulli: $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \Rightarrow \quad \frac{1}{2}\rho V_2^2 = p_1 - p_2 + \frac{1}{2}\rho V_1^2$

Continuity $\Rightarrow$ $V_1A_1 = V_2A_2 \Rightarrow$ $V_1 = \frac{A_2}{A_1}V_2$ substitute $\Rightarrow$

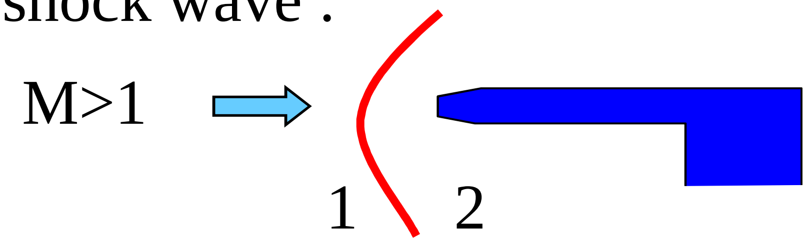
$$\frac{1}{2}\rho V_2^2 = p_1 - p_2 + \frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 V_2^2 \quad \Rightarrow \quad \left\{\frac{1}{2}\rho - \frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2\right\}V_2^2 = p_1 - p_2$$

$$\Rightarrow \quad V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}}$$

When pressure difference p_1-p_2 is measured, V_2 is known

Measurement of airspeed, supersonic flow

$M > 1 \;\Rightarrow$ Formation of a shock wave :



Across the shock wave:

$M_1 > M_2$

$p_1 < p_2$

$V_1 > V_2$

$T_1 < T_2$

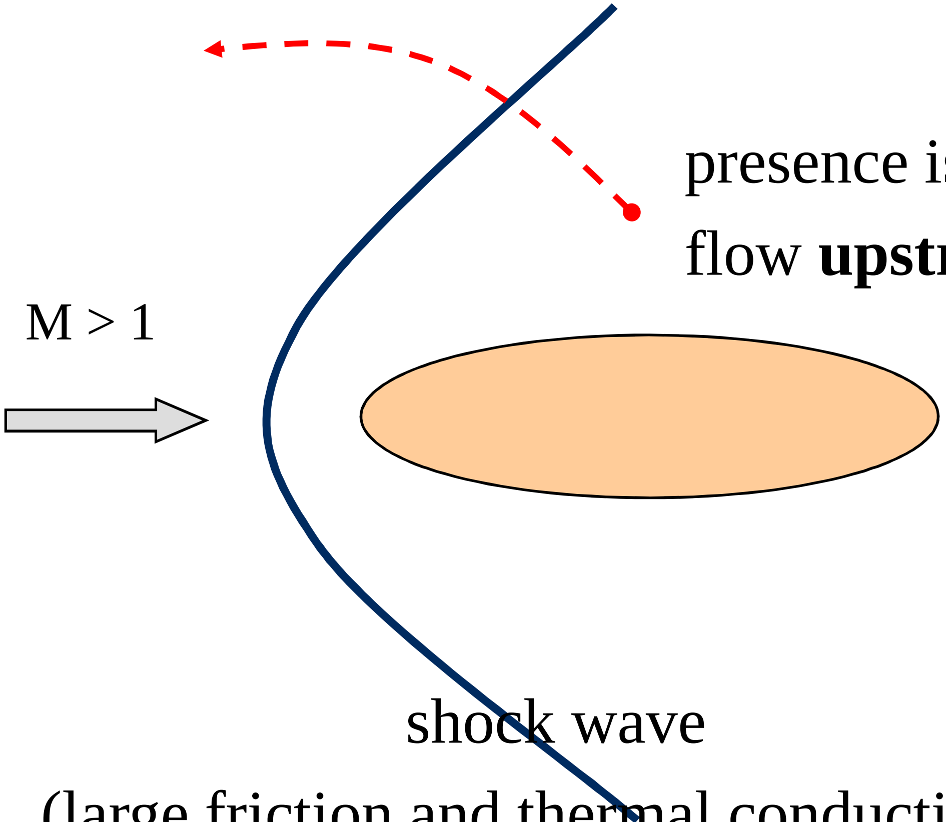
$p_{t1} > p_{t2}$

$T_{t1} = T_{t2}$ (for perfect gas)

Flow through shock-wave is :
non-isentropic

Because the flow is non-isentropic a special shock wave theory must be developed to relate the pitot tube measurement to M.
(This is beyond the scope of our lecture!).

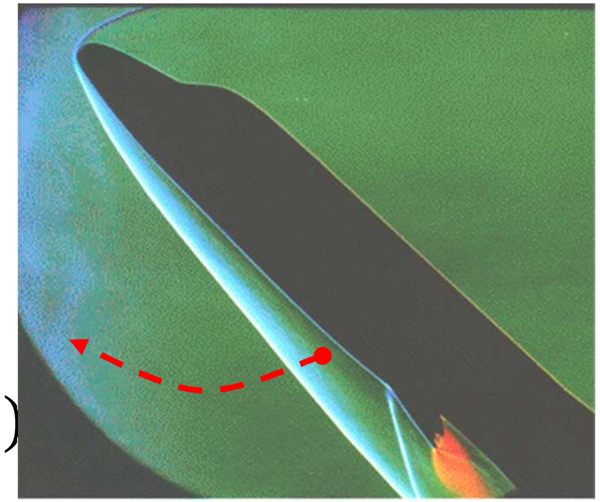
Measurement of airspeed, supersonic flow



presence is **not** communicated to the flow **upstream**

shock wave

(large friction and thermal conduction !)



Opgave 1

In de verbrandingskamer van een raketmotor worden kerosine en zuurstof verbrand. Dit resulteert in een heet gasmengsel onder hoge druk in de verbrandingskamer met de volgende condities en eigenschappen:

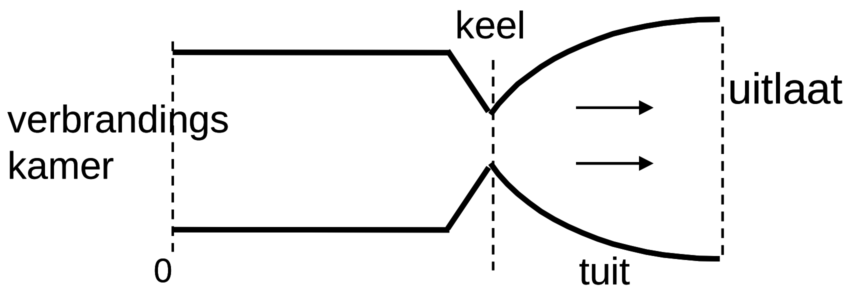
- T_0 = 2500 K
- p_0 = 18 atm
- R = 378 J/kg K
- γ = 1.26

De druk in de uitlaat van de tuit van de raketmotor is 1 atm, en de oppervlakte van de keel is gelijk aan 0.08 m^2.

Neem isentrope stroming aan.

Gevraagd:

- Bereken de snelheid in de uitlaat van de raketmotor.
- Bereken de massastroom door de tuit van de raketmotor.



Problem 1

In the combustion chamber of a rocket engine, kerosene is burned, resulting in a hot, high-pressure gas mixture with the following properties:

- T_0 = 2500 K
- p_0 = 18 atm
- R = 378 J/kg K
- γ = 1.26

This gas flows from the combustion chamber through the rocket nozzle. The pressure at the exit of the rocket nozzle is: 1 atm. and the area of the throat of the rocket nozzle is: 0.08 m^2.

Assume isentropic flow.

- Compute the velocity at the exit of the rocket nozzle.
- Compute the mass flow through the rocket nozzle.