Introduction to Aerospace Engineering

Lecture slides

Introduction to Aerospace Engineering

Aerodynamics 5 & 6

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5.

Compressibility continued

Anderson 4.13 – 4.14

Ernst Mach

1838-1916

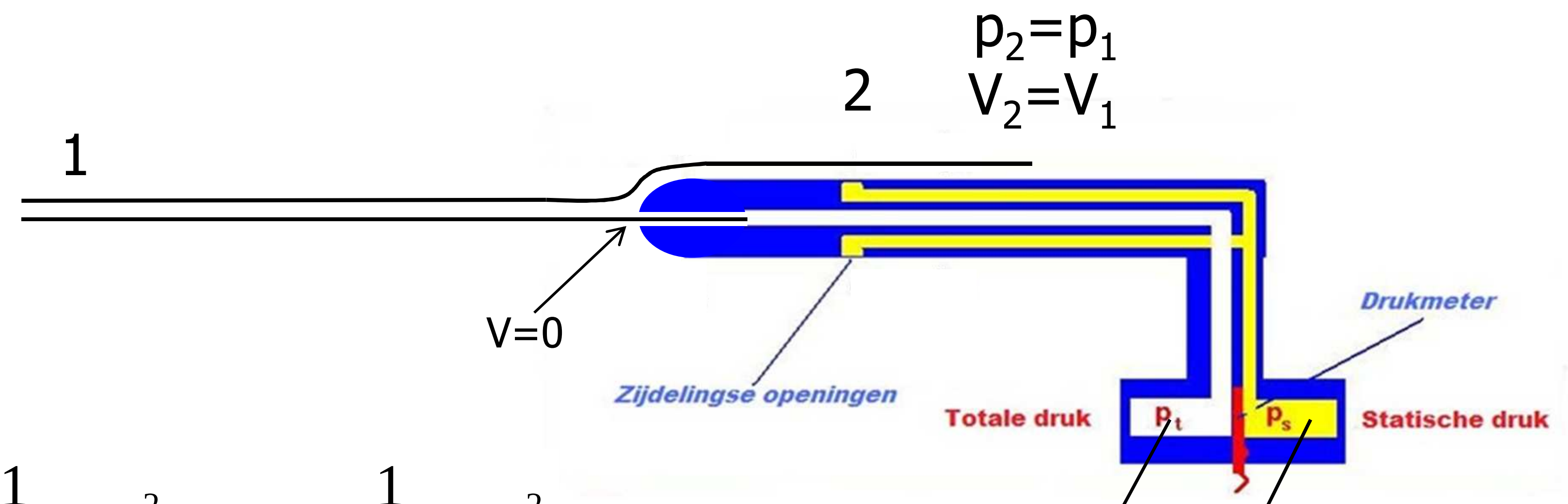
Summary of equations

For steady, frictionless, incompressible flow:

Continuity equation $A_1 V_1 = A_2 V_2$

Bernoulli's equation $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$

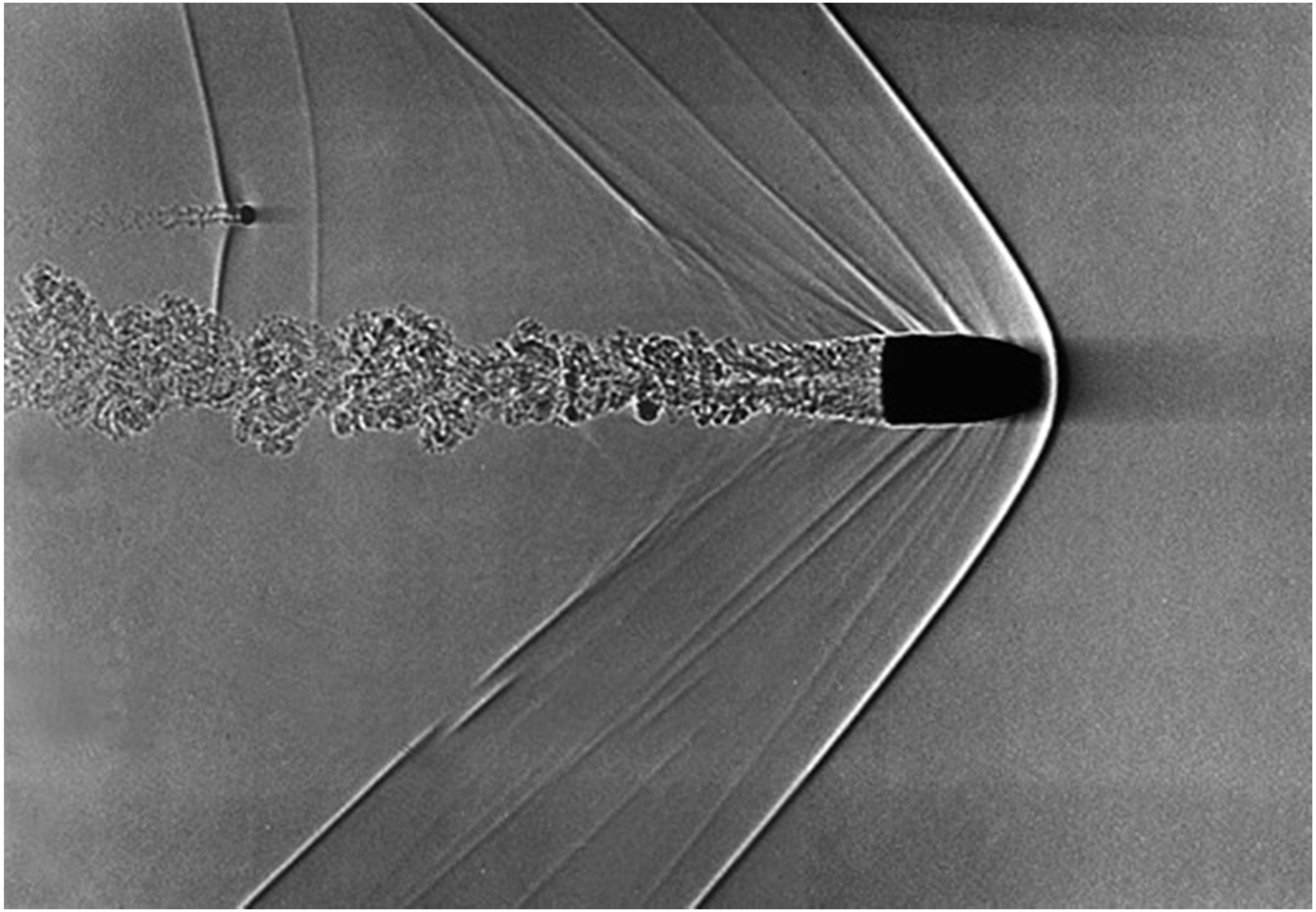
Pitot tube to measure velocity

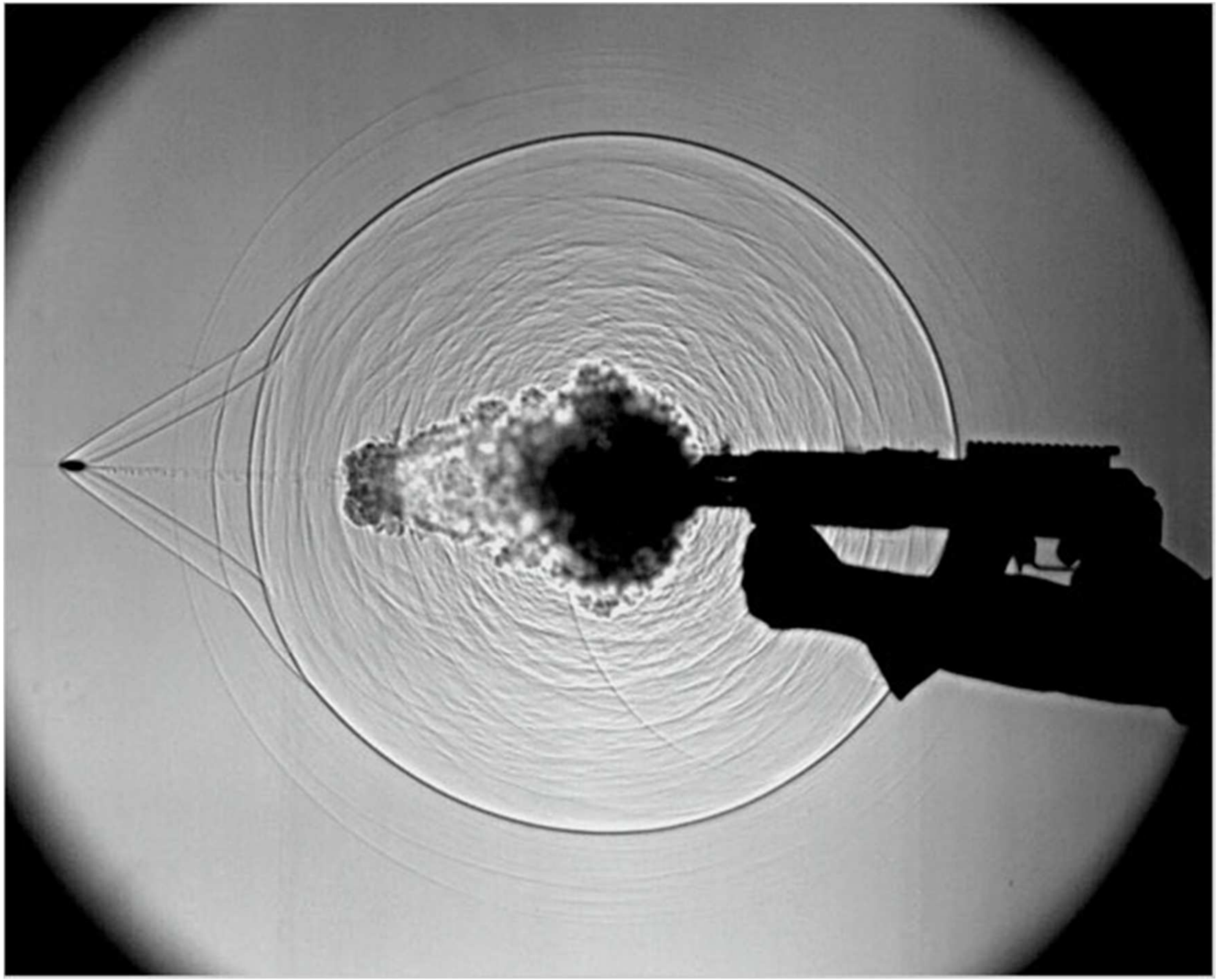


$$p_1 + \frac{1}{2}\rho V_1^{\,2} = p_2 + \frac{1}{2}\rho V_2^{\,2}$$

$Pt_1=Pt_2=Pt$

$$p_t - p = \frac{1}{2}\rho V_1^{\,2}$$

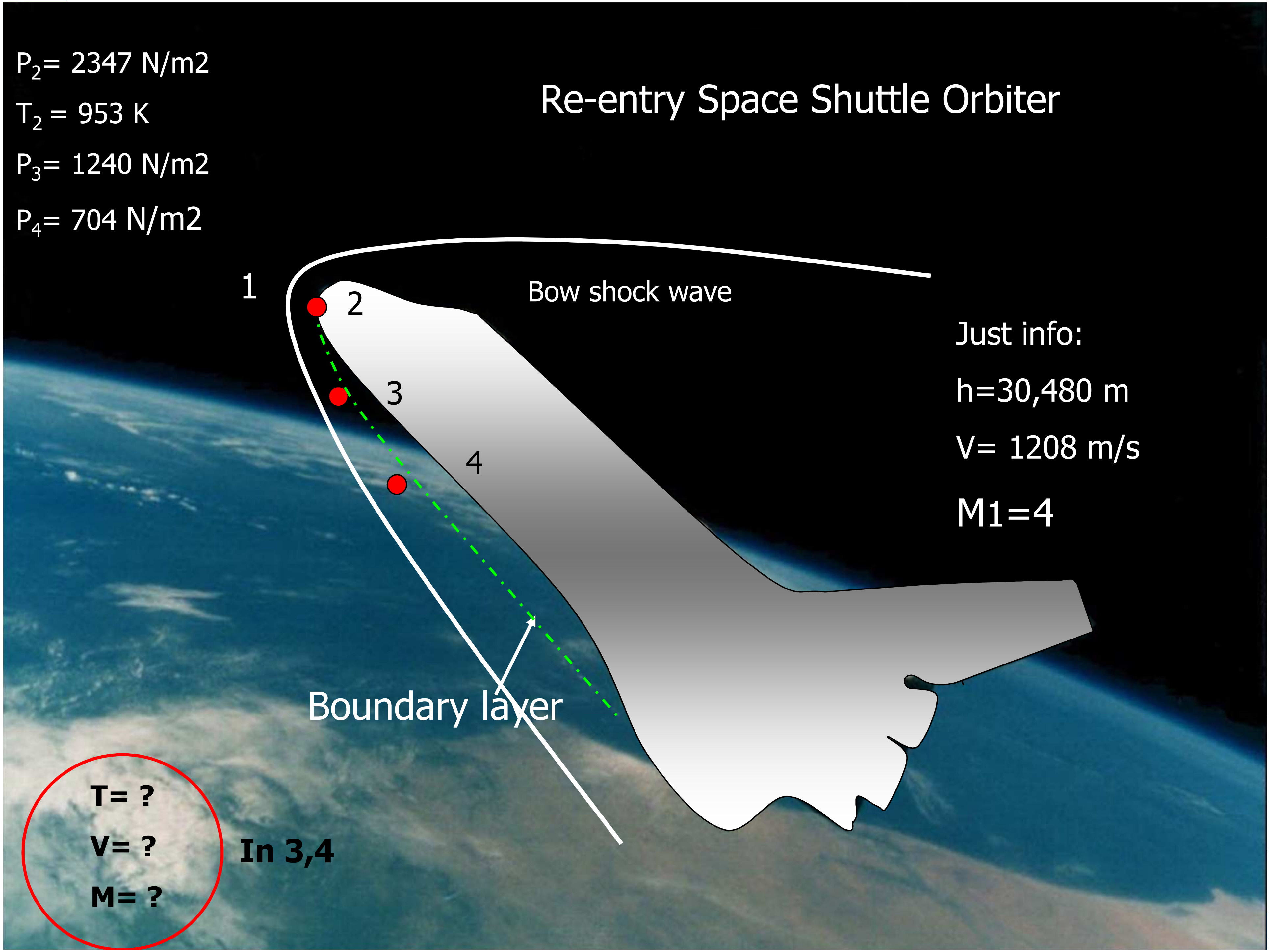




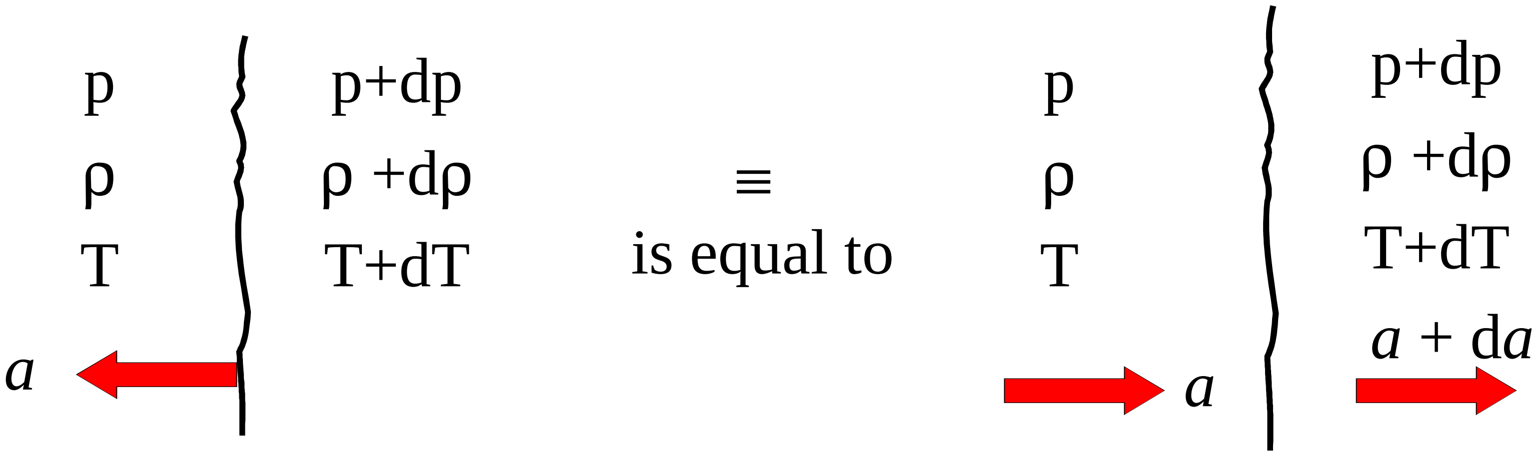
For steady, isentropic compressible flow (adiabatic and frictionless) :

Continuity equation	$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
Isentropic relations	$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$
Energy equation	$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$
Equation of state	$P = \rho R T$

Example : Re-entry Space Shuttle.


Re-entry Space Shuttle Orbiter
P2= 2347 N/m2
T2 = 953 K
P3= 1240 N/m2
P4= 704 N/m2
1
2
3
4
Bow shock wave
Just info:
h=30,480 m
V= 1208 m/s
M1=4
Boundary layer
T= ?
V= ?
M= ?
In 3,4

Speed of sound



<u>moving</u> sound wave with speed a
into a stagnant gas

<u>motionless</u> sound wave

$$\underbrace{\hspace{6em}}_{\text{static observer}} \qquad \underbrace{\hspace{6em}}_{\text{observer moving with sound wave}}$$

Speed of sound

Apply the continuity equation:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \Rightarrow \quad \rho A_1 a = (\rho + d\rho)\, A_2\, (a+da)$$

1-dimensional flow $\Rightarrow A_1 = A_2 = A = \text{constant}$

Thus: $\rho a = (\rho + d\rho)(a + da) \Rightarrow \quad \cancel{\rho a} = \cancel{\rho a} + a d\rho + \rho da + \cancel{d\rho da}$

$$\Rightarrow \qquad a = -\rho \frac{da}{d\rho}$$

$\boxed{\text{small, ignore}}$

Speed of sound

Now apply momentum equation (Euler equation) $\Rightarrow$

$dp = -\rho V dV \Rightarrow$ $$dp = -\rho a da \Rightarrow da = -\frac{1}{\rho a} dp$$

Substituting this into $$a = -\rho \frac{da}{d\rho}$$

We obtain $\Rightarrow$ $$a = +\rho \frac{1}{\rho a} \frac{dp}{d\rho} \Rightarrow$$

$$\boxed{a^2 = \frac{dp}{d\rho}}$$

Going through the sound wave there is no heat addition, friction is negligible $\Rightarrow$

$$\boxed{a = \sqrt{\left(\frac{dp}{d\rho}\right)_{isentropic}}}$$

Speed of sound

Now we may apply the isentropic relations.

We want to rewrite $\dfrac{dp}{d\rho}$

With isentropic relations we find $a = \sqrt{\gamma \dfrac{p}{\rho}}$

Use the equation of state: $\dfrac{p}{\rho} = RT \Rightarrow$

$$\boxed{a = \sqrt{\gamma RT}}$$

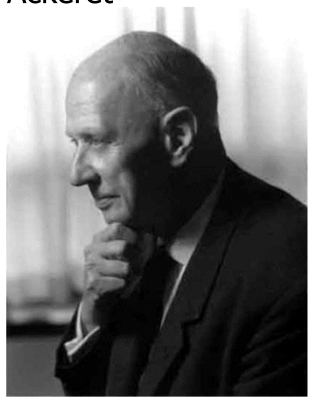
Speed of sound in a perfect gas depends only on T !

$$a = \sqrt{\gamma RT}$$

Mach

$$M = \frac{V}{a}$$

Ackeret



In honor of Ernst Mach the name “Mach number” was introduced in 1929 by Jacob Ackeret

Speed of sound

The Mach number is: $$M = \frac{V}{a}$$

$M < 1$	subsonic
$M = 1$	sonic
M around 1	transonic
$M > 1$	supersonic
$M > 5$	hypersonic

All velocity ranges have their own specific phenomena!

Equations for a perfect gas

- We defined: $h = e + pv$
- We found : $h = c_p T$ and $e = c_v T$
- With the eq. of state: $pv = RT$
- we find: $c_p T = c_v T + RT$
- hence: $c_p = c_v + R \;\; => c_p - c_v = R$

We defined: $\gamma = \dfrac{c_p}{c_v}$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

Isentropic flow relations, second form

Energy equation:

$$c_p T + \frac{1}{2}V^2 = constant$$

$$c_p T_1 + \frac{1}{2}V_1^2 = c_p T_o + \frac{1}{2}V_o^2$$

Assume index 0 = stagnation point $\Rightarrow V_o = 0 \Rightarrow$

$$c_p T_1 + \frac{1}{2}V_1^2 = c_p T_o \Rightarrow \quad \frac{T_o}{T_1} = 1 + \frac{V_1^2}{2c_p T_1}$$

Substitute: $C_p = \frac{\gamma R}{\gamma - 1}$ (from $C_p - C_v = R$)

$$\frac{T_o}{T_1} = 1 + \frac{\gamma - 1}{2}\frac{V_1^2}{\gamma R T_1}$$

TUDelft

Isentropic flow relations, second form

with $a_1^2 = \gamma R T$ $$\frac{T_o}{T_1} = 1 + \frac{\gamma - 1}{2}\frac{V_1^2}{a_1^2} = 1 + \frac{\gamma - 1}{2} M_1^2$$

Bring flow isentropically to rest $\Rightarrow$ Isentropic relation can be used $\Rightarrow$

$$\frac{p_o}{p_1} = \left(\frac{\rho_o}{\rho_1}\right)^{\gamma} = \left(\frac{T_o}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{\rho_o}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{\gamma-1}}$$

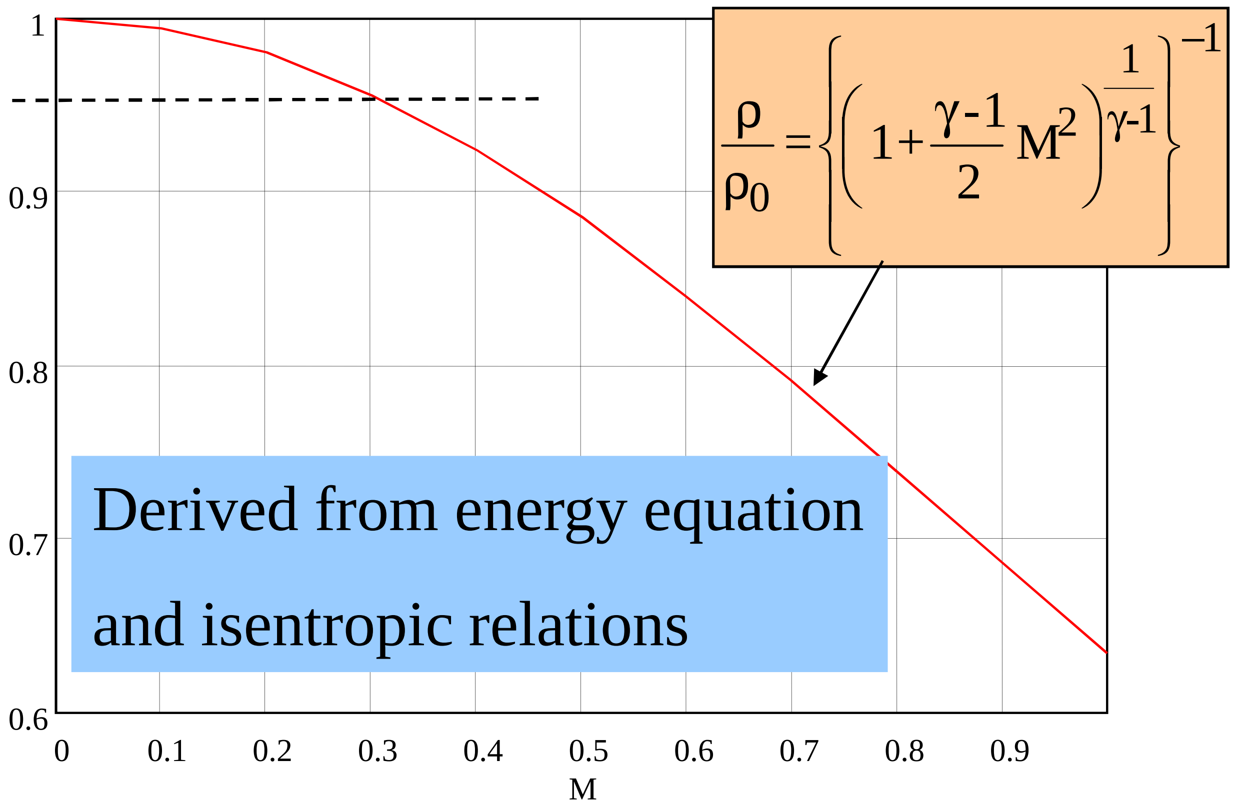
Second form of isentropic relations

$$\frac{T_o}{T_1} = 1 + \frac{\gamma\text{-}1}{2}\, M_1^2$$

$$\frac{\rho_o}{\rho_1} = \left(1 + \frac{\gamma - 1}{2}\, M_1^2\right)^{\frac{1}{\gamma\text{-}1}}$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma\text{-}1}{2}\, M_1^2\right)^{\frac{\gamma}{\gamma\text{-}1}}$$

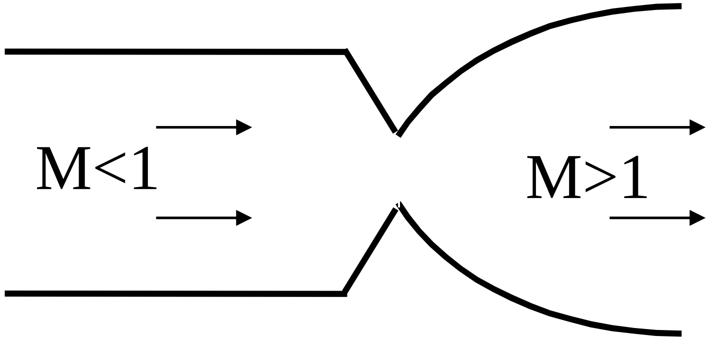
Compressibility



For M < 0.3 the change in density is less than 5 %

Thus : **for M < 0.3 the flow can be treated as incompressible**

Supersonic wind tunnel and rocket engine



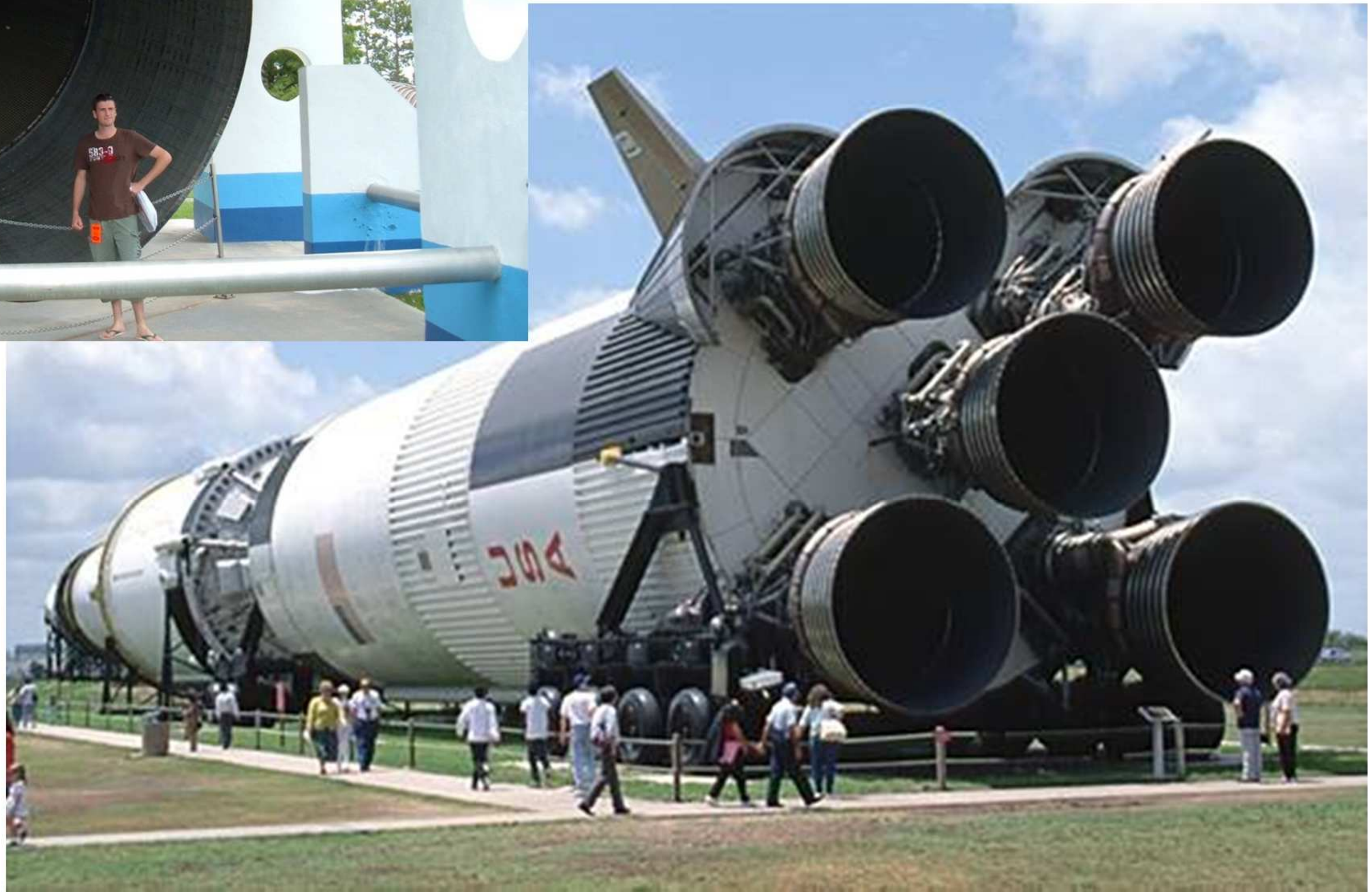
Rocket motor must be LIGHT

Thus the nozzle is short

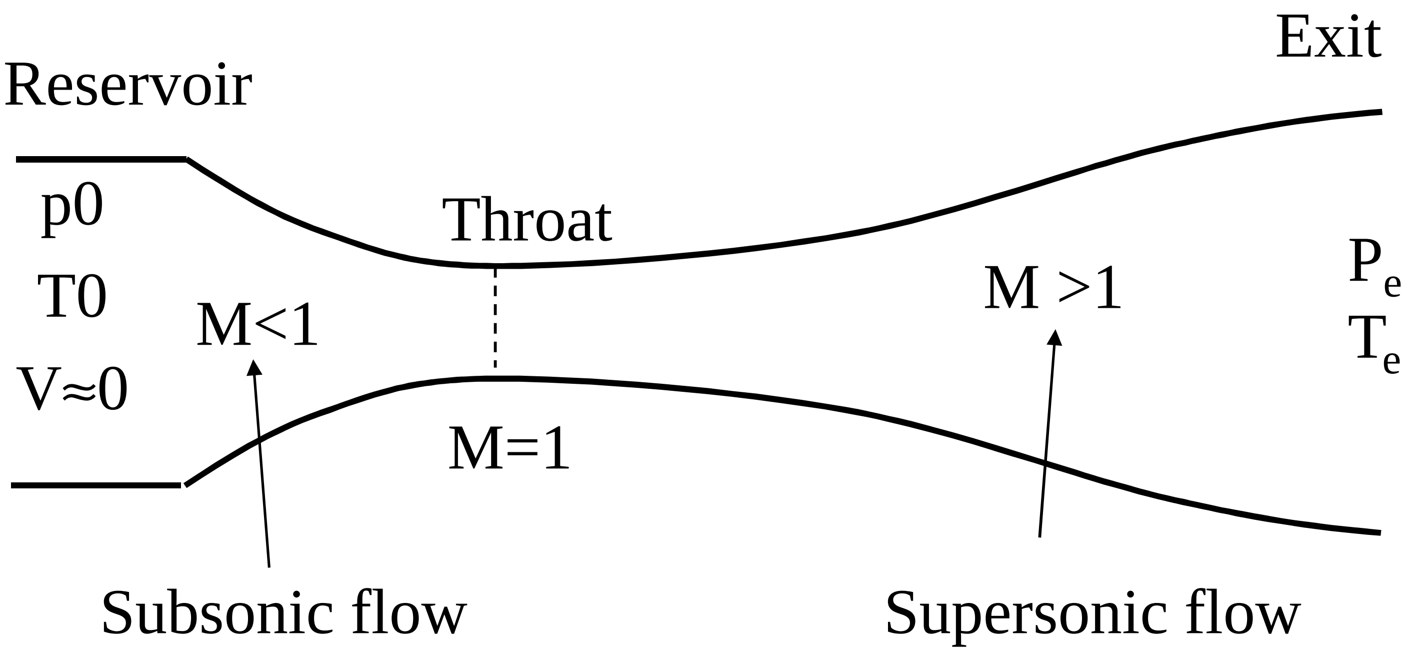
Exhaust of Saturn V

that powered the Apollo space missions


363'
125 tons to Earth orbit
333' 7"
84 tons to Earth orbit
224'
184' 2"
109'
95' 3"
83'
20 tons to Earth orbit
3.75 tons to Earth orbit
1.6 tons to Earth orbit
sub-orbital
Mercury Redstone
Mercury Atlas
Gemini Titan
Saturn IB
Saturn V
Saturn V w/ Skylab
space shuttle


USA

Supersonic Wind Tunnel



For both the wind tunnel and a rocket motor we can derive an area velocity relation using the continuity and the Euler equation

Supersonic tunnel and rocket engine

$$\rho VA = \text{const} \qquad \ln\rho + \ln A + \ln V = \ln(const)$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad dp = -\rho V dV$$

$$\frac{-d\rho V dV}{dp} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad \frac{d\rho}{dp} = \frac{1}{dp/d\rho} = \frac{1}{a^2}$$

$$\frac{-VdV}{a^2} + \frac{dA}{A} + \frac{dV}{V} = 0$$

Supersonic wind tunnel: area velocity relation

Area-Velocity Relation

$$\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$$

Conclusions:

Case A. Subsonic flow if dV > 0 then
dA < 0 and vice versa.

Case B. Supersonic flow if dV > 0 then
dA > 0 and vice versa.

Case C. If the flow is sonic (M=1) then ....

Supersonic windtunnel: M=1 in the throat

- $\frac{dV}{V} = \left(\frac{1}{M^2 - 1} \right) \frac{dA}{A} = \frac{1}{0} \frac{dA}{A}!$

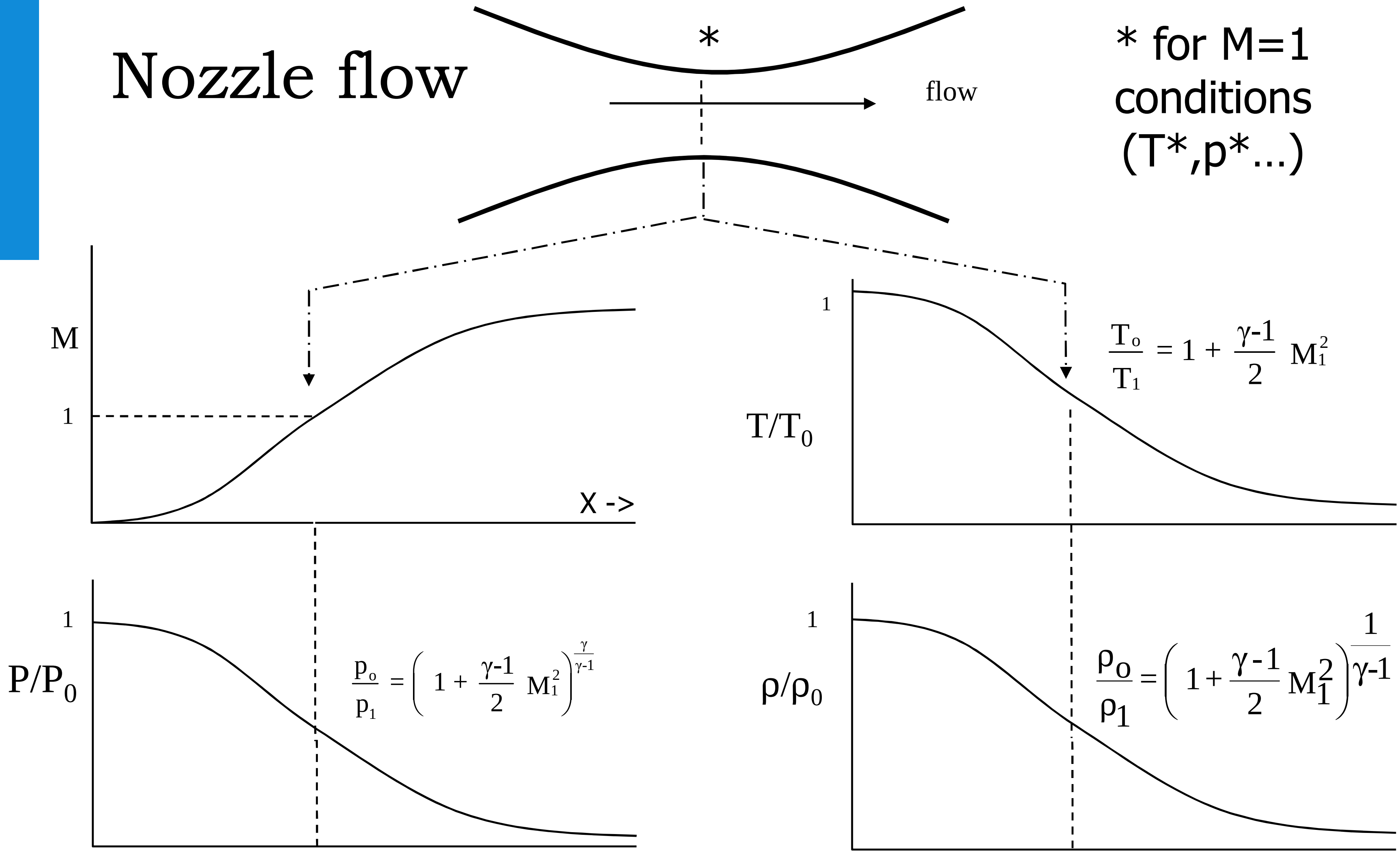
- At first glance we see $\frac{dV}{V} = \infty$.
  But this is not possible on physical basis.

- Then we must have (for finite $\frac{dV}{V}$): $\frac{dA}{A} = 0 \Rightarrow \frac{dV}{V} = \frac{0}{0} = FINITE!$

- If $\frac{dA}{A} = 0 \Rightarrow$ $\boxed{\text{In the THROAT : M = 1 !}}$

Nozzle flow

* for M=1 conditions (T*,p*…)



Example 4.9: A supersonic wind tunnel is considered.

The air temperature and pressure in the reservoir of the tunnel are:
T_0 = 1000 K and p_0 = 10 atm.

The static temperatures at the throat and exit are:
T^* = 833 K and T_e = 300 K.

The mass flow through the tunnel is 0.5 kg/s.
For air, c_p = 1008 J/(kg K). Calculate:

a) Velocity at the throat V^*
b) Velocity at the exit V_e
c) Area of the throat A^*
d) Area of the exit A_e

6.

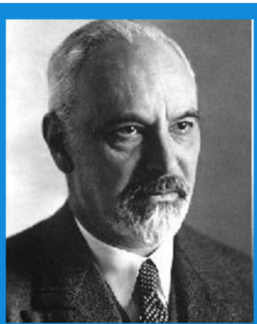
Viscous flows

Anderson 4.15 - 4.16

Osborne Reynolds

1842-1912

Ludwig Prandtl



1874-1953

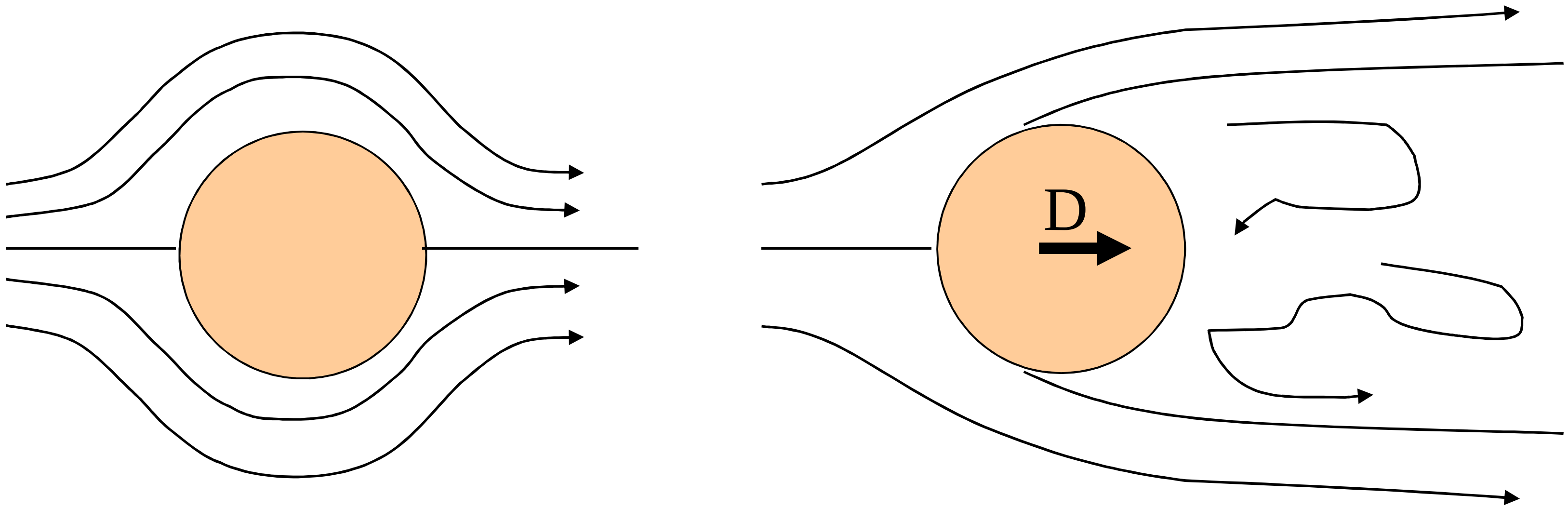
Subjects lecture 6

- Viscous flows
- Laminar boundary layers
    - calculation of boundary layer thickness
    - calculation of skin friction drag

Viscous flow

Up till now we have only dealt with frictionless flow.

What is the effect of friction ? ….



Inviscid flow (No friction)
NO DRAG

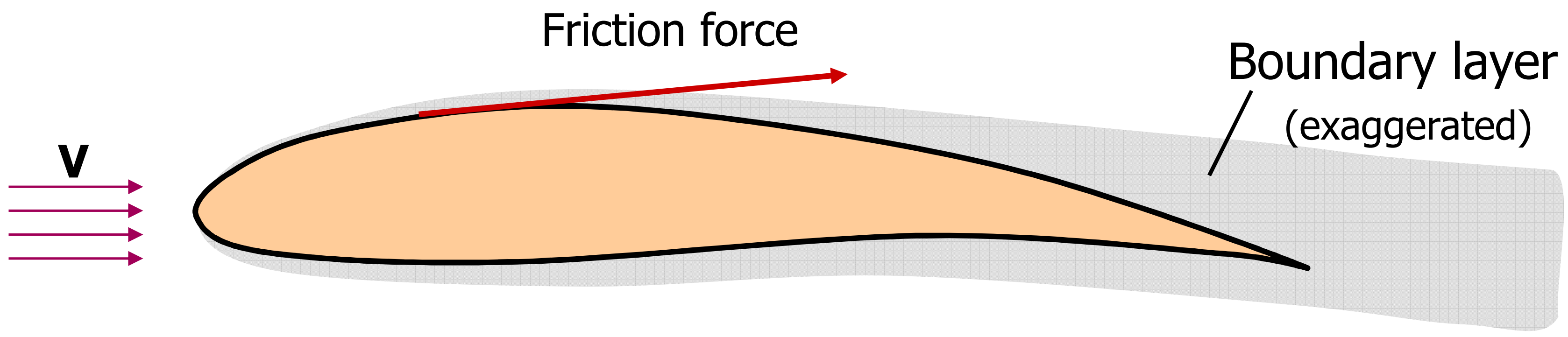
Viscous flow (friction)
FINITE DRAG

Viscous flow

In real life the flow at the surface adheres to the surface because of friction between the gas and the solid material:

- Right at the surface the velocity is zero

Boundary layer



➢ In the vicinity of the surface there is a thin region of retarded flow: the boundary layer

➢ The pressure through the boundary layer in a direction perpendicular to the surface is constant

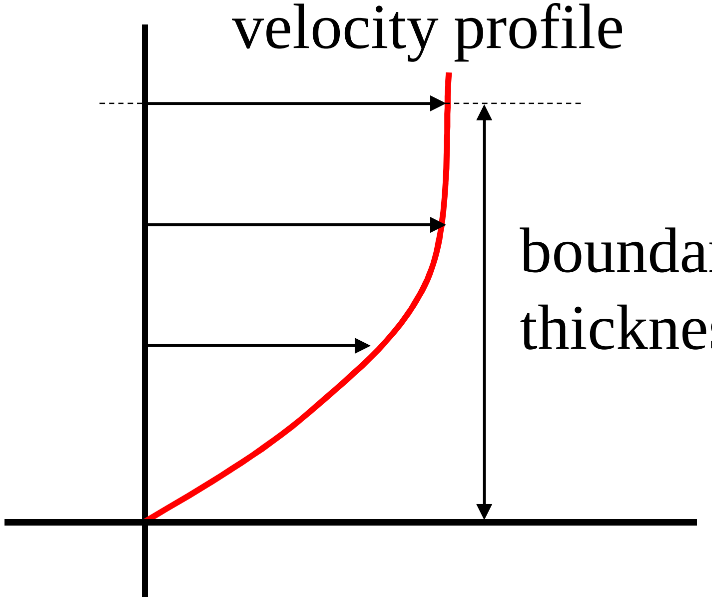
Viscous flow

Inside the boundary layer

Bernoulli's law is not valid!!!!!!

Viscous flows

Boundary layer velocity profile



Shear stress can be written as :

$$\tau_w = \mu \left(\frac{dU}{dy} \right)_{y=0}$$

shear stress, τ_w
$\Rightarrow$ skin friction drag

μ = absolute viscosity coefficient or viscosity
Air at standard sea level : $\mu = 1.789 * 10^{-5}$ kg/ms)

Viscous flow

$$\tau = \mu \frac{dU}{dy}$$

viscosity or" dynamic viscosity"

Fluids for which the shearing stress is <u>linearly</u> related to the rate of shearing strain are called:

<u>NEWTONIAN FLUIDS.</u>

(Most common fluids (liquids & gases) like are are NEWTONIAN!).

Often viscosity appears as $\nu = \frac{\mu}{\rho}$ = "KINEMATIC VISCOSITY"

Reynolds number

Characteristic velocity

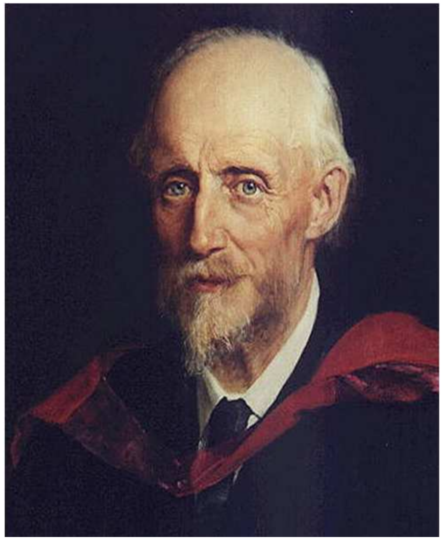
Characteristic length

$$\mathrm{Re} = \frac{V \cdot L}{\nu}$$

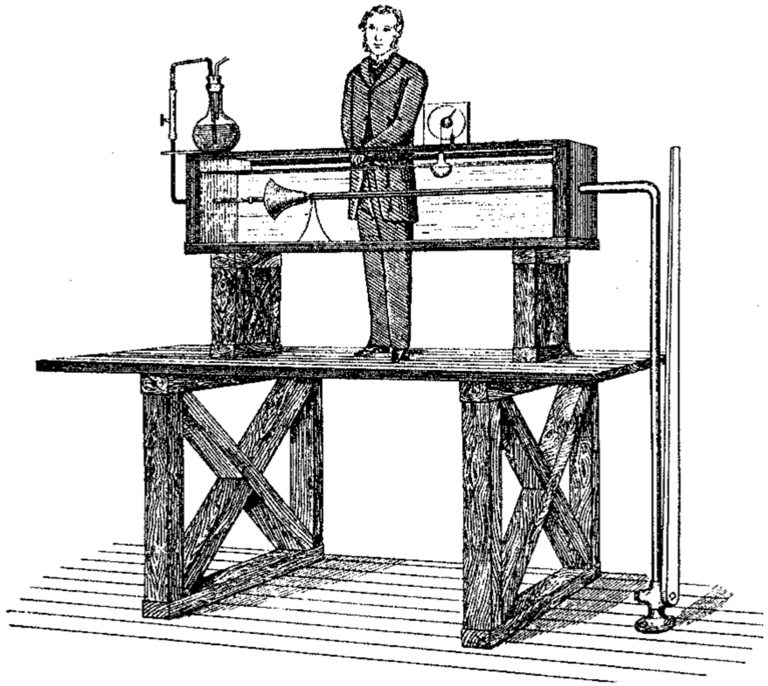
(kinematic) viscosity

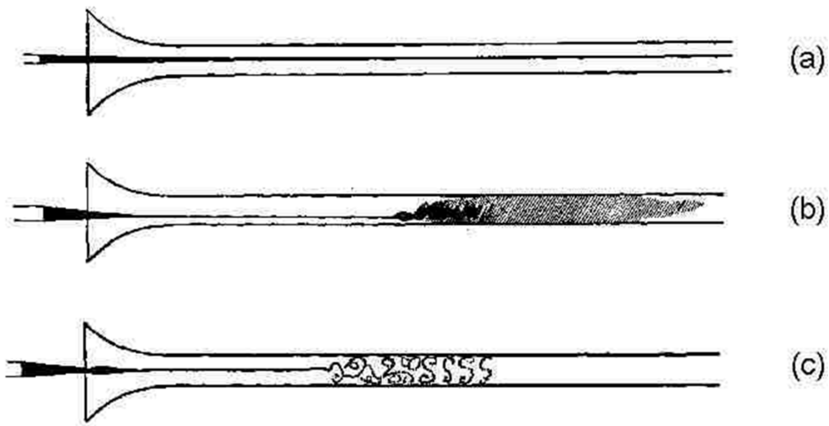
Osborne Reynolds

1842-1912

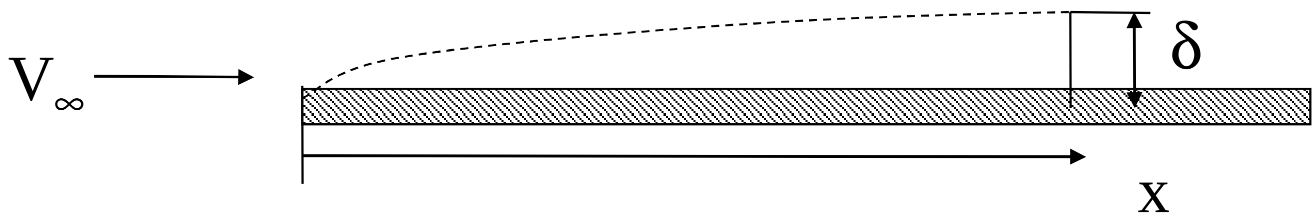


Sail planes	: Re ≈ 1•10^6, L=wing chord
Passenger jets	: Re ≈ 50•10^6 Idem
Lower leg athlete	: Re ≈ 1•10^5 L=Diameter leg




(a)
(b)
(c)

Viscous flows, some definitions



Reynolds number :

$$\boxed{Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty},}$$

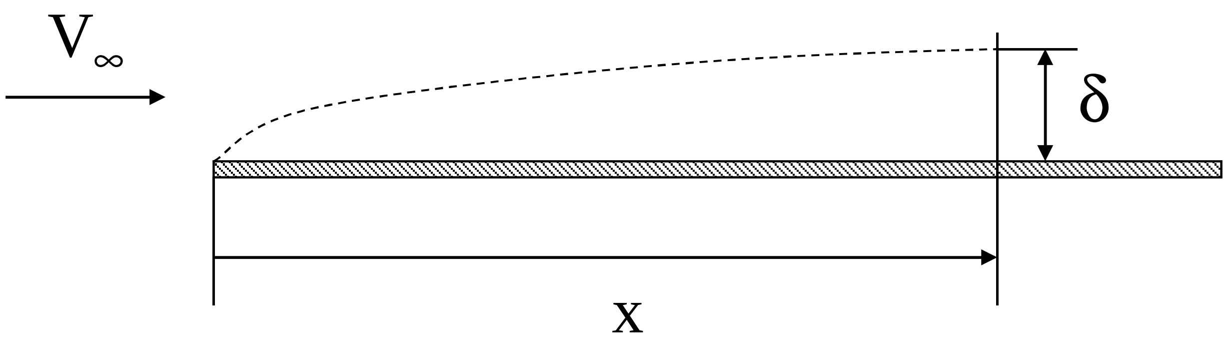
dimensionless, and varies linearly with x

Laminar flow : streamlines are smooth and regular and a fluid element moves smoothly along a streamline

Turbulent flow : streamlines break up and a fluid element moves in a random irregular way

Laminar boundary layer, boundary layer thickness

Consider flat plate flow. What is boundary layer thickness δ and skin friction drag D_f at location x?

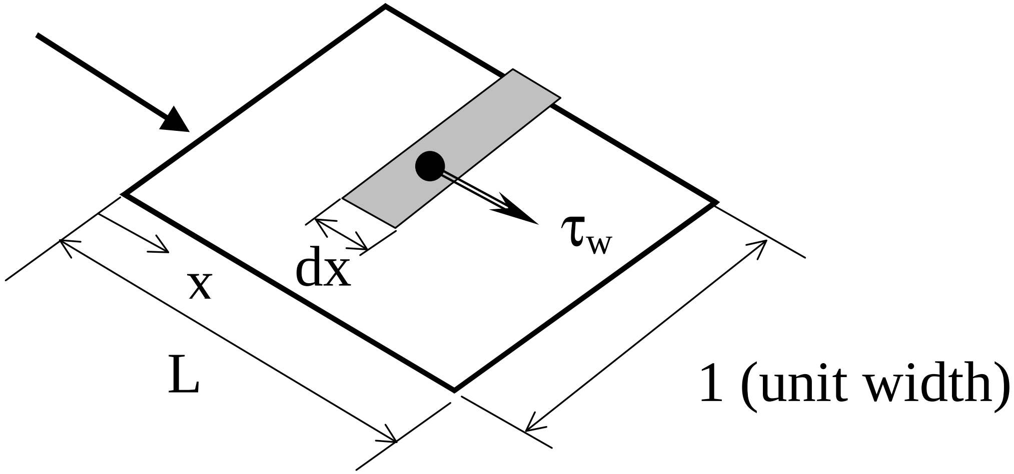


From **laminar** boundary layer theory :

$$\boxed{\delta = \frac{5.2\ x}{\sqrt{Re_x}}}$$

Thus δ is proportional to : $\sqrt{x}$ (parabolically)

Laminar boundary layer, skin friction drag



Total force = total pressure force + total friction force

Total friction force on element dx is: $\tau_W(x)\cdot dx\cdot 1=\tau_W(x)dx$

Total skin friction drag is: $D_f = \int_o^L \tau_w \, dx$

Laminar boundary layer, skin friction drag

For the skin friction coefficient we find from laminar boundary layer theory :

$$c_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} = \frac{0.664}{\sqrt{Re_x}}$$

Thus c_{f_x} and τ_w decrease as $\sqrt{x}$.

The skin friction at the beginning of the plate is larger than near the trailing edge.

To calculate the total aerodynamic force we must **integrate**!

Laminar boundary layer, skin friction drag

$$D_f = \int_o^L c_{f_x} \cdot q_\infty \, dx = 0.664\, q_\infty \int_o^L \frac{dx}{\sqrt{Re_x}} = \frac{0.664\ q_\infty}{\sqrt{V_\infty/\nu}} \int_o^L \frac{dx}{\sqrt{x}}$$

$$\int \frac{dx}{\sqrt{x}} = \int x^{-1/2} = 2\sqrt{x}$$

$$D_f = \frac{0.664\, q_\infty}{\sqrt{V_\infty/\nu_\infty}} 2\sqrt{L} = \frac{1.328 q_\infty L}{\sqrt{V_\infty L/\nu_\infty}}$$

Define total skin friction drag coefficient as $C_f = \dfrac{D_f}{q_\infty S}$

$$C_f = \frac{1.328}{\sqrt{Re_L}} \frac{L}{S} = \frac{1.328.L}{\sqrt{Re_L}\ L \cdot 1} \Rightarrow C_f = \frac{1.328}{\sqrt{Re_L}}$$

The skin friction coefficient for a laminar boundary layer

$$C_f = \frac{1.328}{\sqrt{Re_L}}$$

Only for low speed incompressible flow
(and reasonably accurate for high speed subsonic flow).

Re_L = Reynolds nr. based on length L.