Introduction to Aerospace Engineering

Lecture slides

Introduction to Aerospace Engineering

Aerodynamics 11&12

Prof. H. Bijl   ir. N. Timmer

11 & 12.

Airfoils and finite wings

Anderson 5.9 – end of chapter 5 excl. 5.19

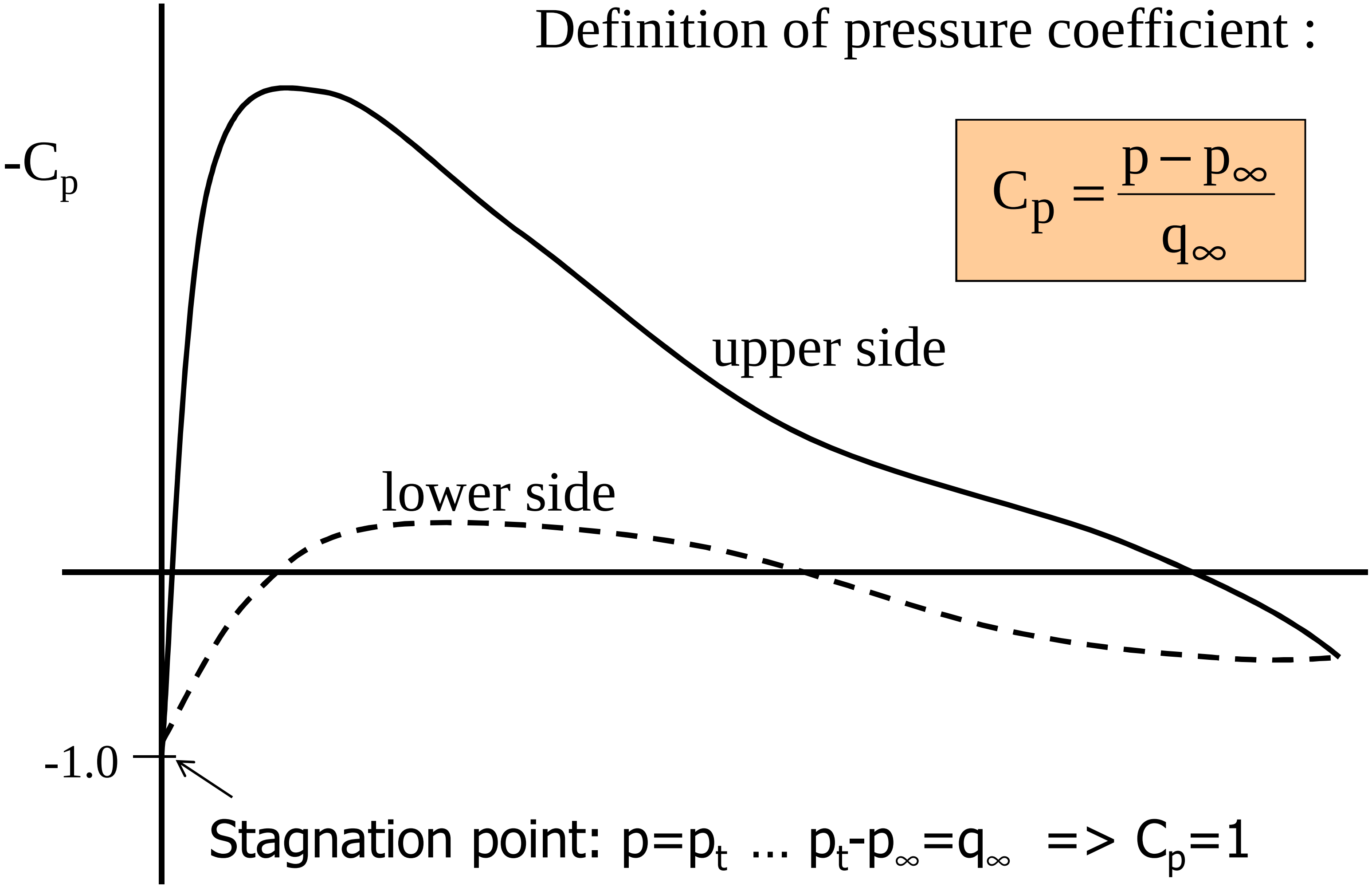
TUDelft

Topics lecture 11 & 12

- Pressure distributions and lift
- Finite wings
- Swept wings

Pressure coefficient

Typical example



Definition of pressure coefficient :

$$C_p = \frac{p - p_\infty}{q_\infty}$$

Stagnation point: $p=p_t$ … $p_t - p_\infty = q_\infty$ => $C_p=1$

Example 5.6

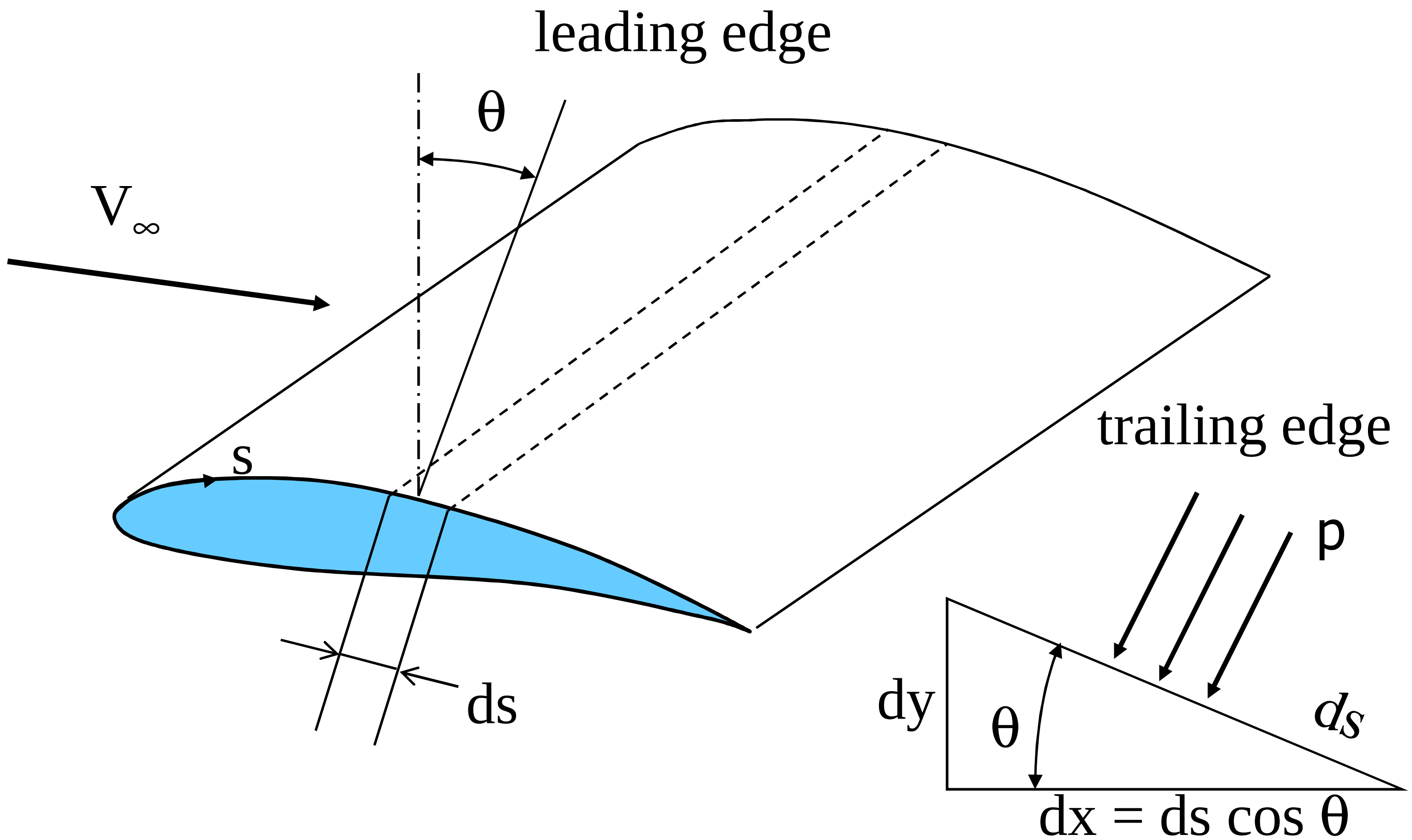
- The pressure on a point on the wing of an airplane is 7.58×10^4 N/m^2. The airplane is flying with a velocity of 70 m/s at conditions associated with standard altitude of 2000m. Calculate the pressure coefficient at this point on the wing

2000 m: $p_\infty = 7.95.10^4$ N/m^2 $\rho_\infty = 1.0066$ kg/m^3

$$C_p = \frac{p - p_\infty}{q_\infty}$$

$$C_p = -1.50$$

Obtaining lift from pressure distribution



Obtaining lift from pressure distribution

Normal force per meter span: $N = \int_{LE}^{TE} p_l \cos\theta ds - \int_{LE}^{TE} p_u \cos\theta ds$

with $\mathrm{ds}\cos\theta = \mathrm{dx}$ $\qquad N = \int_0^c p_l dx - \int_0^c p_u dx$

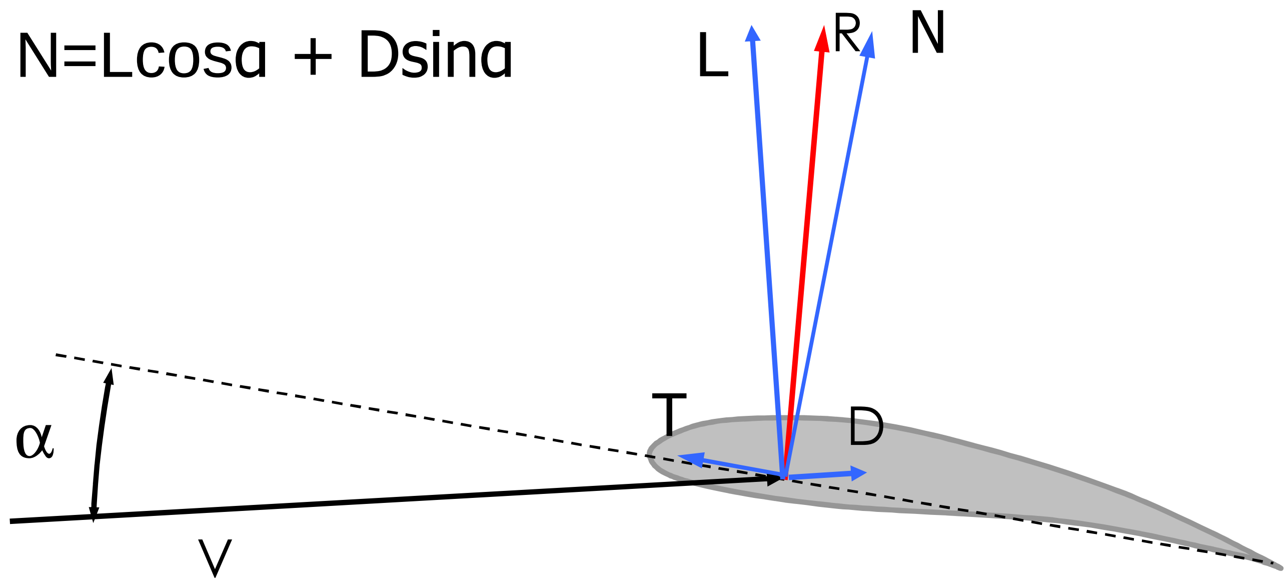
Write dimensionless force coefficient : $C_n = \frac{N}{\frac{1}{2}\rho V_\infty^2 c} = \frac{N}{q_\infty c}$

$$C_n = \int_0^1 \frac{p_l - p_\infty}{q_\infty} d\left(\frac{x}{c}\right) - \int_0^1 \frac{p_u - p_\infty}{q_\infty} d\left(\frac{x}{c}\right)$$

$$C_n = \int_0^1 \left(C_{p_l} - C_{p_u}\right) d\left(\frac{x}{c}\right)$$

T=Lsinα - Dcosα

N=Lcosα + Dsinα



α = angle of attack

Obtaining lift from normal force coefficient

$$L = N\cos\alpha - T\sin\alpha \qquad c_l = c_n\cos\alpha - c_t\sin\alpha$$

$$\frac{L}{q_\infty c} = \frac{N}{q_\infty c}\cos\alpha - \frac{T}{q_\infty c}\sin\alpha$$

For small angle of attack $\alpha \le 5^o: \quad \cos\alpha \approx 1, \quad \sin\alpha \approx 0$

$$C_l \approx \frac{1}{c}\int_0^1 \left(C_{p_l} - C_{p_u}\right) d(x)$$

Example 5.11

Consider an airfoil with chord length c and the running distance x measured along the chord. The leading edge is located at x/c=0 and the trailing edge at x/c=1. The pressure coefficient variations over the upper and lower surfaces are given as

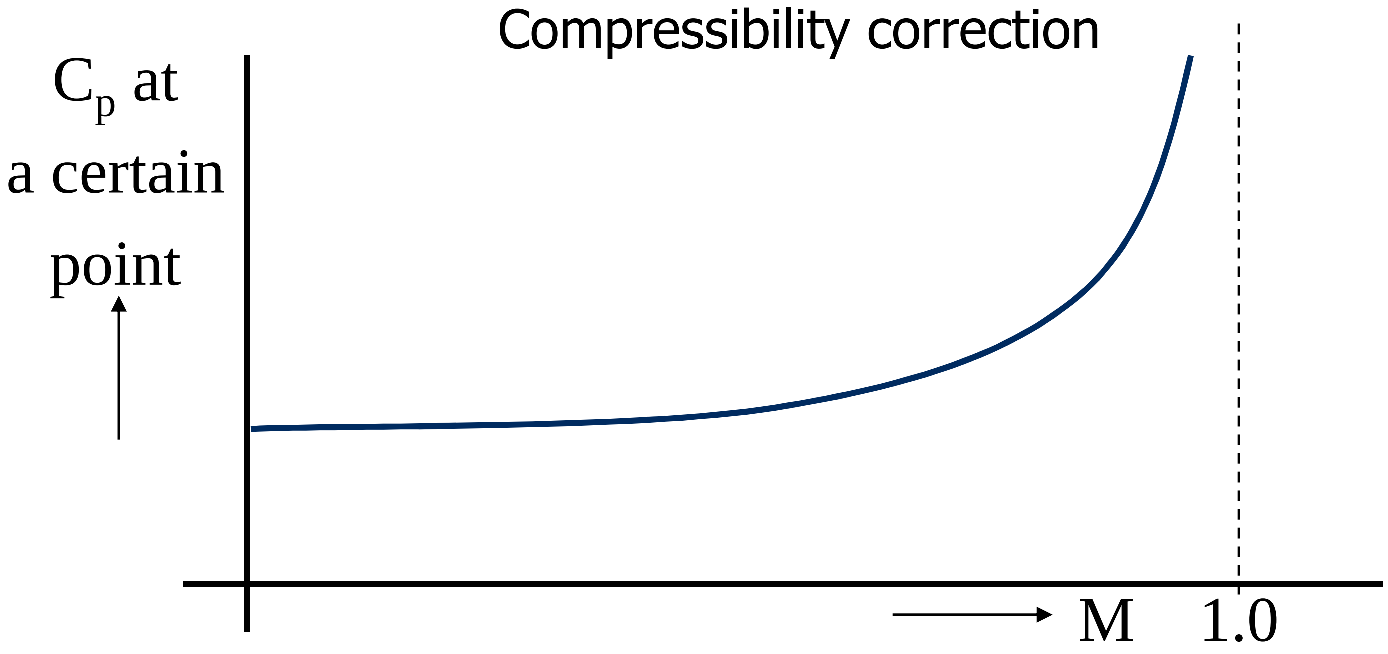
$$C_{p,u} = 1 - 300\left(\frac{x}{c}\right)^2 \qquad \text{for } 0 \le \left(\frac{x}{c}\right) \le 0.1$$

$$C_{p,u} = -2.2277 + 2.2777\left(\frac{x}{c}\right) \qquad \text{for } 0.1 \le \left(\frac{x}{c}\right) \le 1.0$$

$$C_{p,l} = 1 - 0.95\left(\frac{x}{c}\right) \qquad \text{for } 0 \le \left(\frac{x}{c}\right) \le 1.0$$

Calculate the normal force coefficient.

Compressibility correction of the pressure coefficient



Approximate theoretical correction (valid for 0 < M < 0.7) :

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}}$$

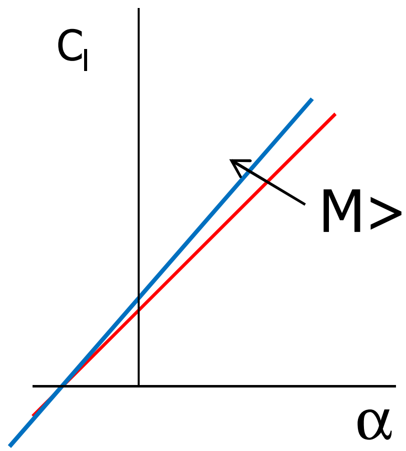
Prandtl-Glauert Rule

Compressibility correction for lift coefficient

$$C_l \approx \frac{1}{c}\int_0^c \frac{\left(C_{p_l} - C_{p_u}\right)_o d\left(x\right)}{\sqrt{1-M_\infty^2}} =$$

$$C_{l,0} \equiv \frac{1}{c}\int_0^c \left(C_{p_l} - C_{p_u}\right)_o d\left(x\right)$$

$$C_l = \frac{C_{l,0}}{\sqrt{1-M_\infty^2}}$$

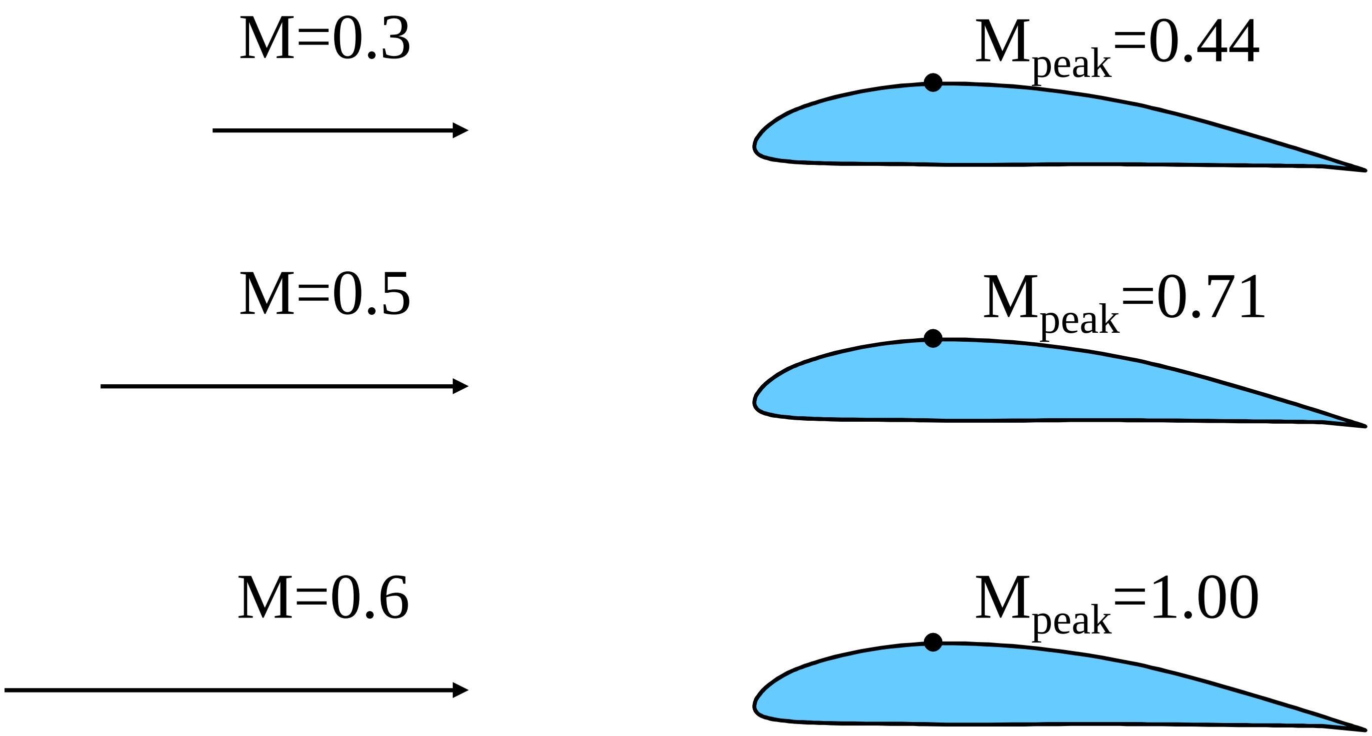


Example 5.12

- Consider an NACA 4412 airfoil at an angle of attack of 4^{o}. If the free-stream Mach number is 0.7, what is the lift coefficient?

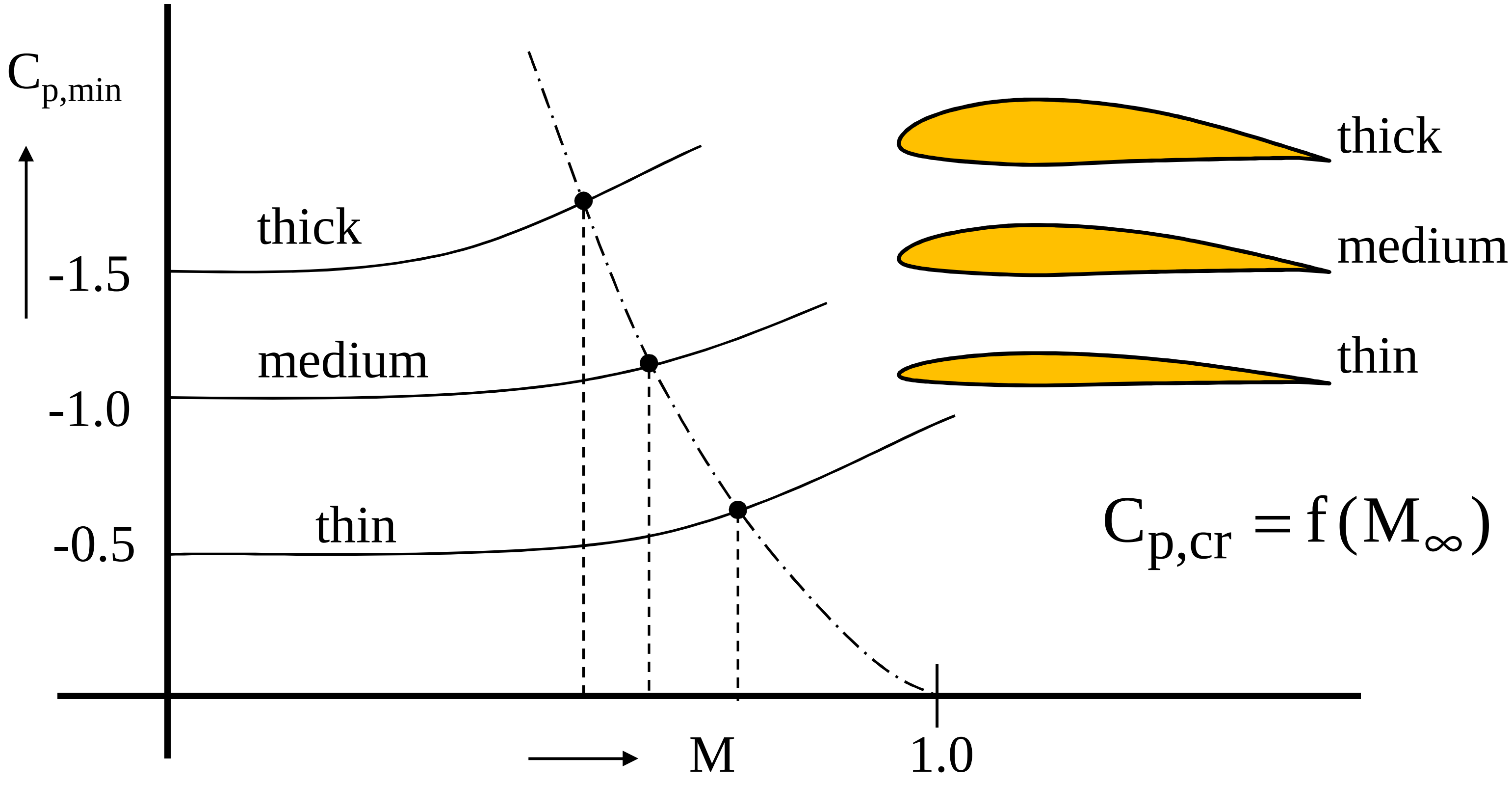
- - From App. D for angle of attack of 4^{o}, $c_l = 0.83$.However, these data were obtained at low speeds.

Critical Mach number and critical pressure coefficient



Critical Mach number for the airfoil

Critical pressure coefficient



Thicker airfoil reaches critical pressure coefficient at a lower value of M_∞

Critical pressure coefficient

Definition of pressure coefficient : $C_p = \frac{p - p_\infty}{q_\infty} = \frac{p_\infty}{q_\infty}\left(\frac{p}{p_\infty} - 1\right)$

Dynamic pressure : $q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^2 = \tfrac{1}{2}\frac{\rho_\infty}{\gamma p_\infty}\gamma p_\infty V_\infty^2$

$$= \tfrac{1}{2}\frac{V_\infty^2}{\gamma p_\infty / \rho_\infty}\gamma p_\infty$$

We have found before : $a_\infty^2 = \frac{\gamma p_\infty}{\rho_\infty}$ thus : $q_\infty = \tfrac{1}{2}\frac{V_\infty^2}{a_\infty^2}\gamma p_\infty = \frac{\gamma}{2} p_\infty M_\infty^2$

For isentropic flow we found : $\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$

and also : $\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma - 1}}$

Dividing these equations we find :...

Critical pressure coefficient

$$\frac{p}{p_\infty} = \left(\frac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)M^2} \right)^{\frac{\gamma}{\gamma-1}}$$

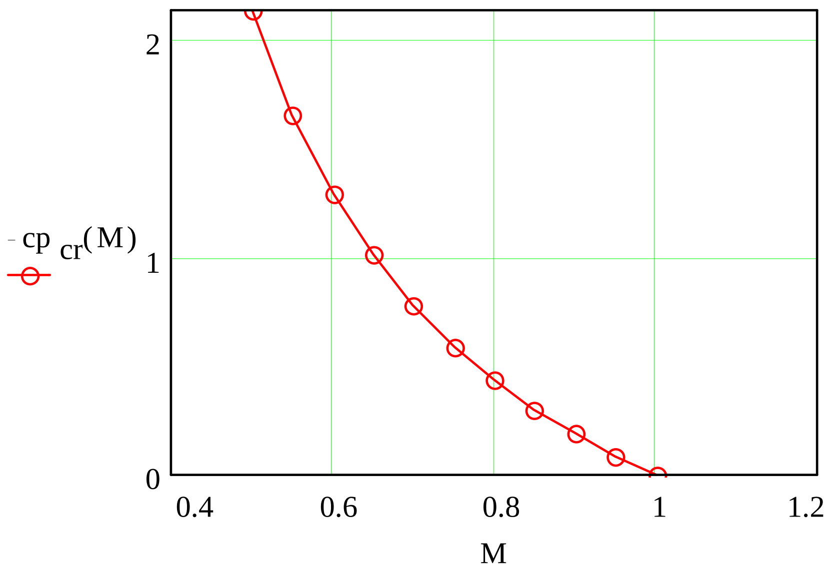
Substitution in Cp-equation gives :

$$C_p = \frac{p_\infty}{q_\infty}\left(\frac{p}{p_\infty}-1\right) = \frac{\cancel{p_\infty}}{\frac{1}{2}\gamma \cancel{p_\infty} M_\infty^2}\left[\left(\frac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)M^2} \right)^{\frac{\gamma}{\gamma-1}} -1\right]$$

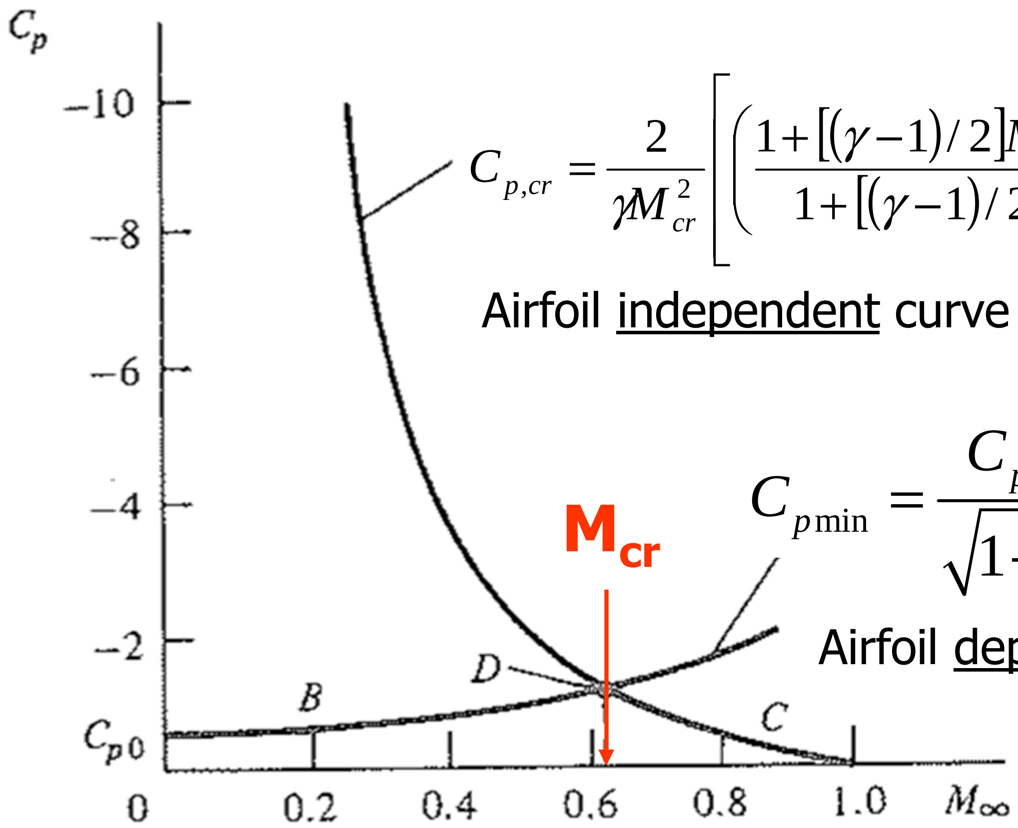
Critical pressure coefficient

The critical pressure coefficient is found when M=1 is reached :

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$



The intersection of this curve with the airfoil $C_{P,min}$ will give the value of the M_{cr} **specific** for the airfoil under consideration.



$$C_{p,cr} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+[(\gamma-1)/2]M_{cr}^2}{1+[(\gamma-1)/2]}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

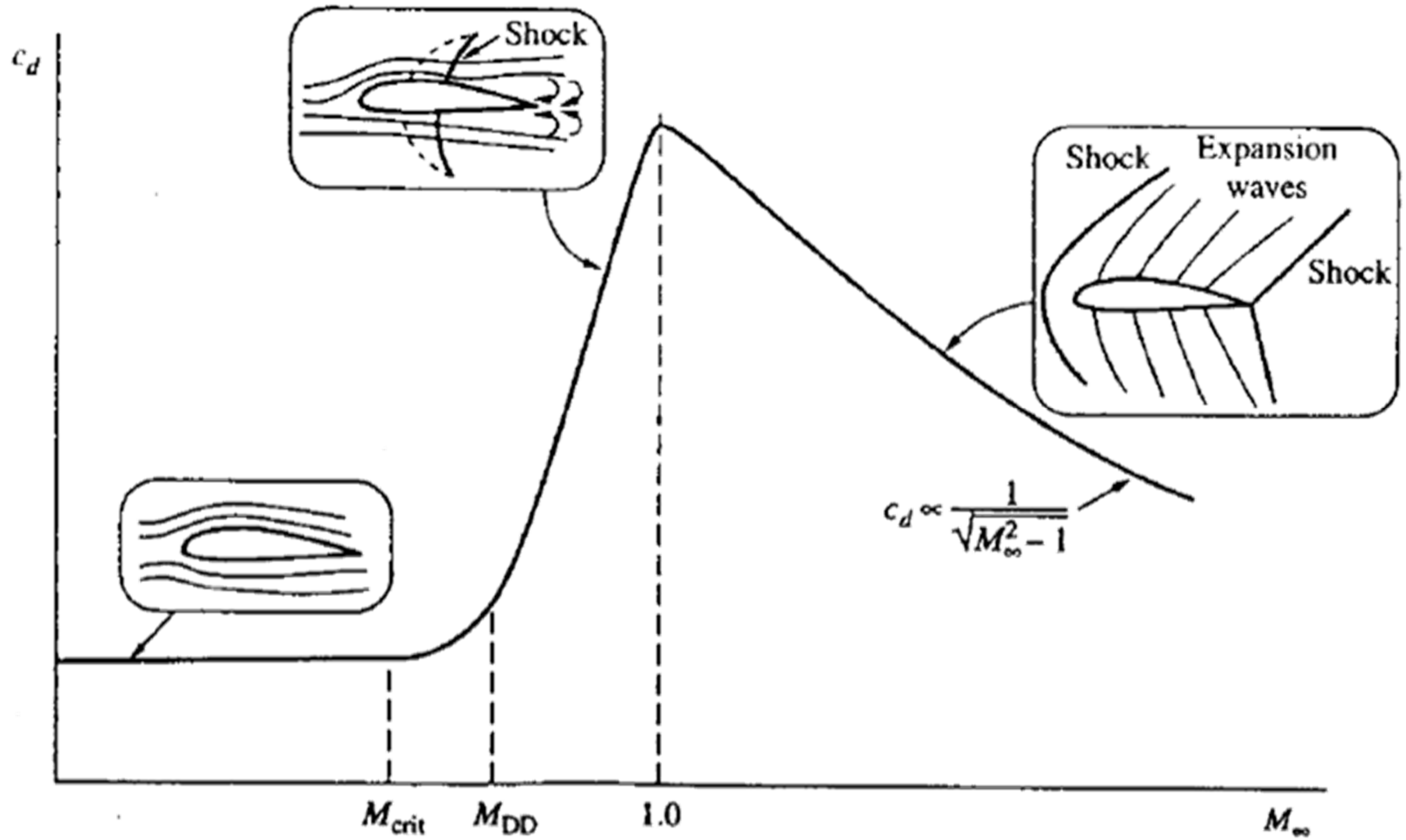
Airfoil independent curve

$$C_{p\min} = \frac{C_{p,0\min}}{\sqrt{1-M_\infty^2}}$$

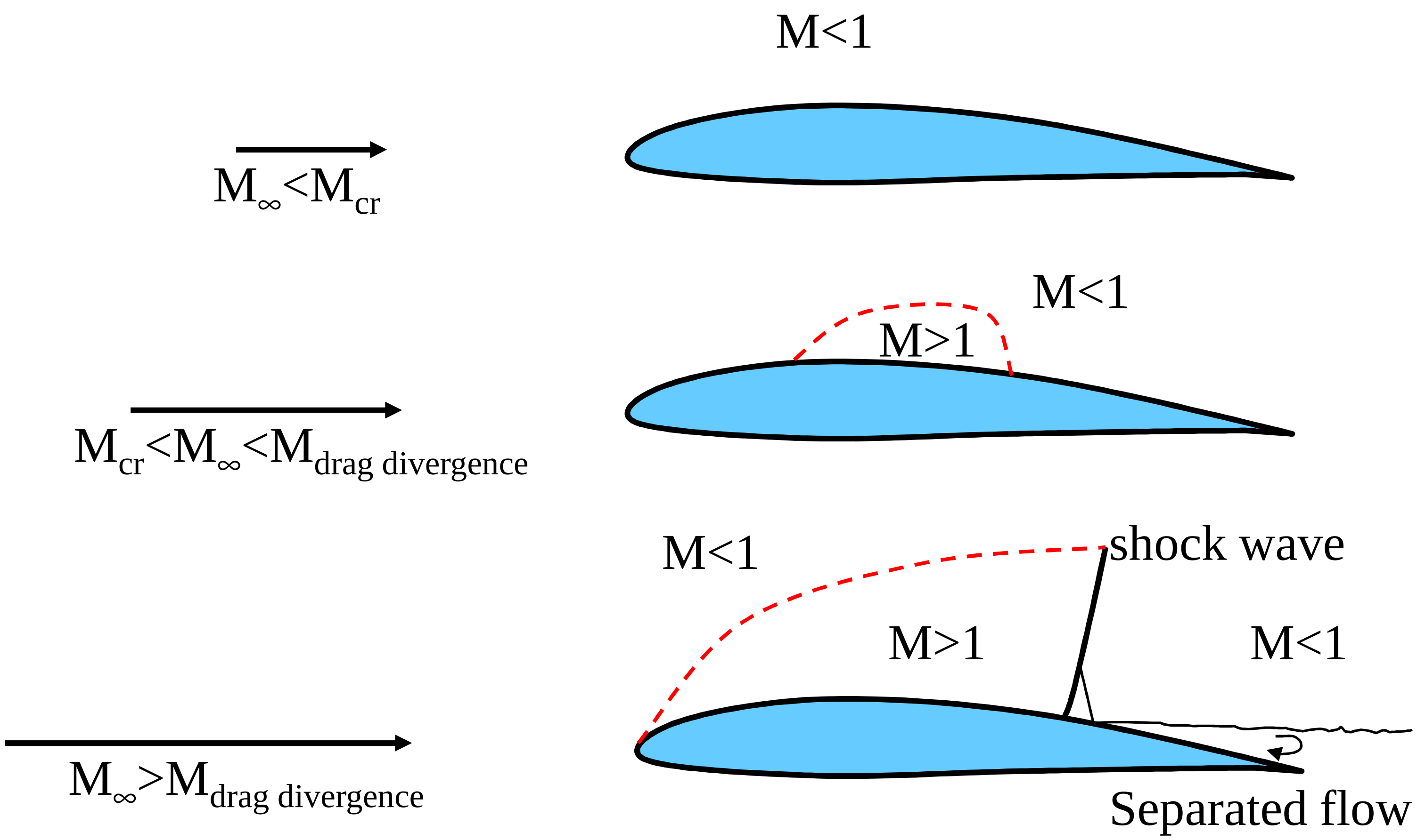
Airfoil dependent curve

TUDelft

Drag divergence

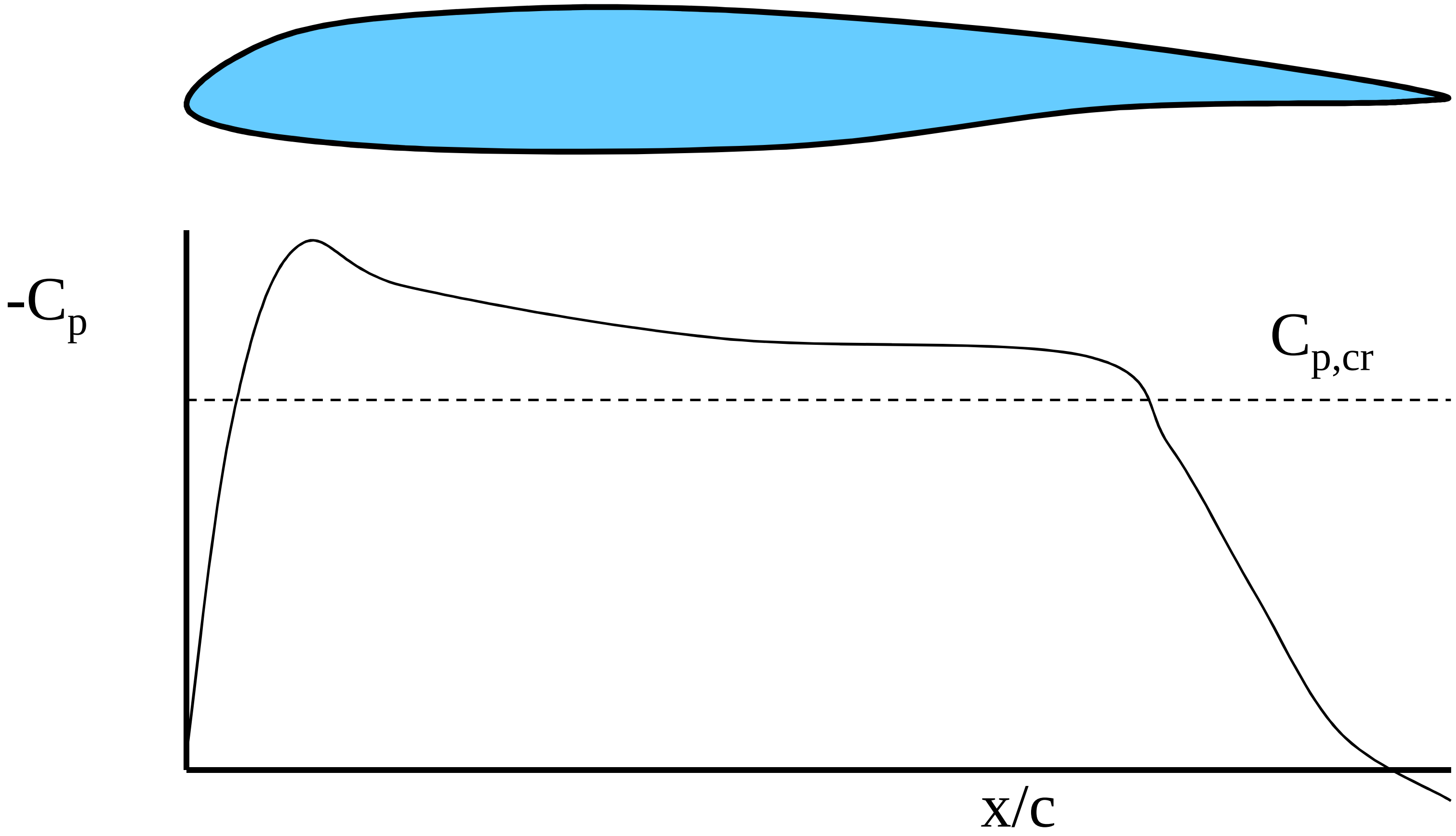


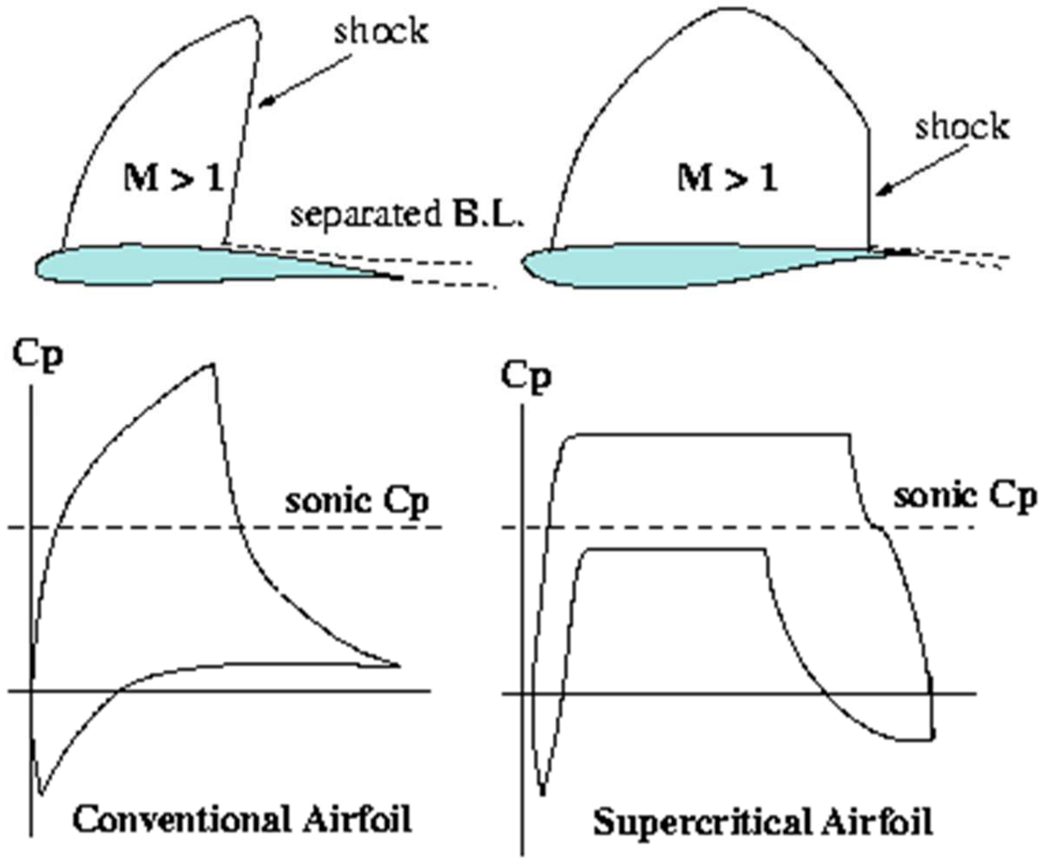
Drag divergence



Drag divergence

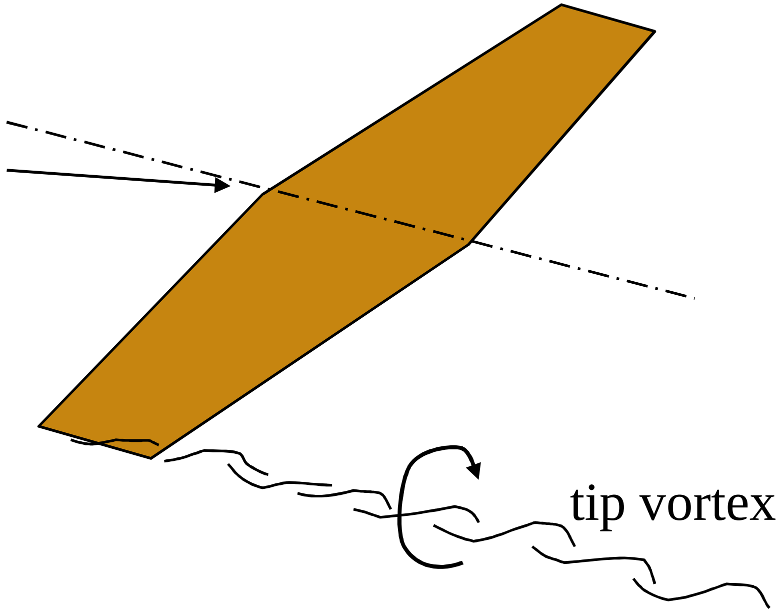
Development of supercritical airfoils

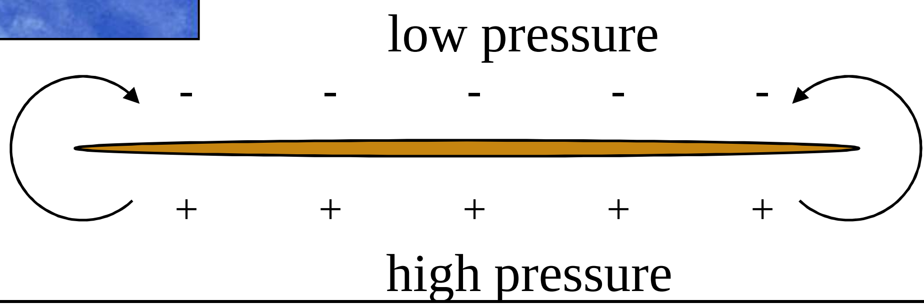



shock
M > 1
separated B.L.
M > 1
shock
Cp
sonic Cp
Conventional Airfoil
Cp
sonic Cp
Supercritical Airfoil

Finite Wings

Characteristics of finite wings

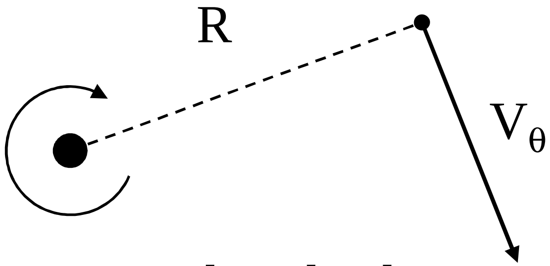


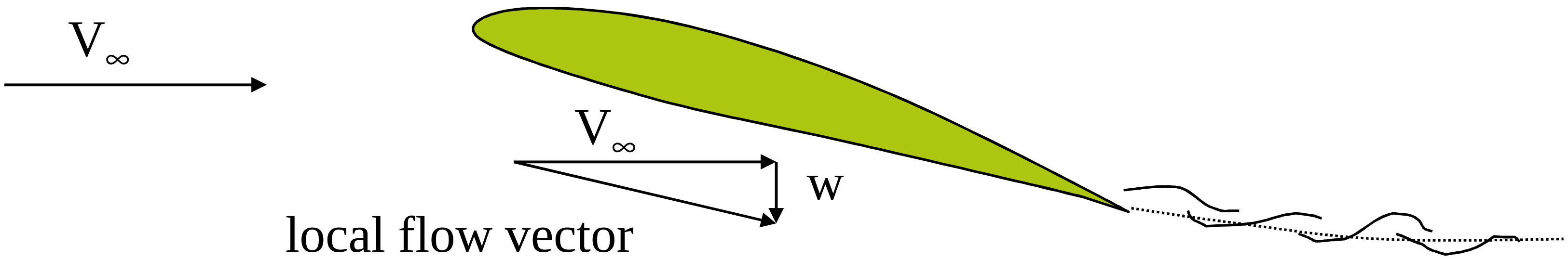


Flow around the wing tip generates tip vortices

Tip vortices

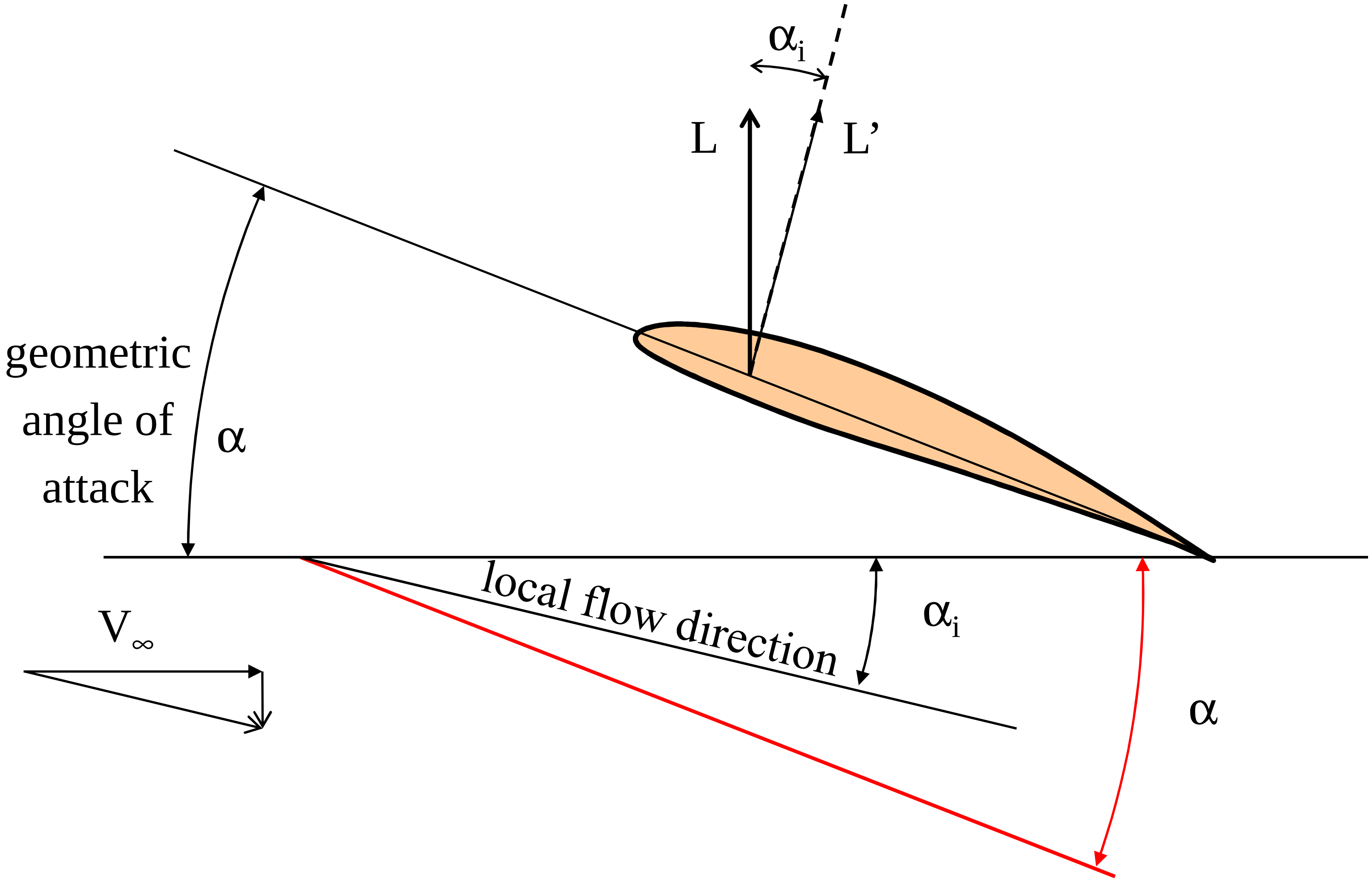
Cross section of tip vortex





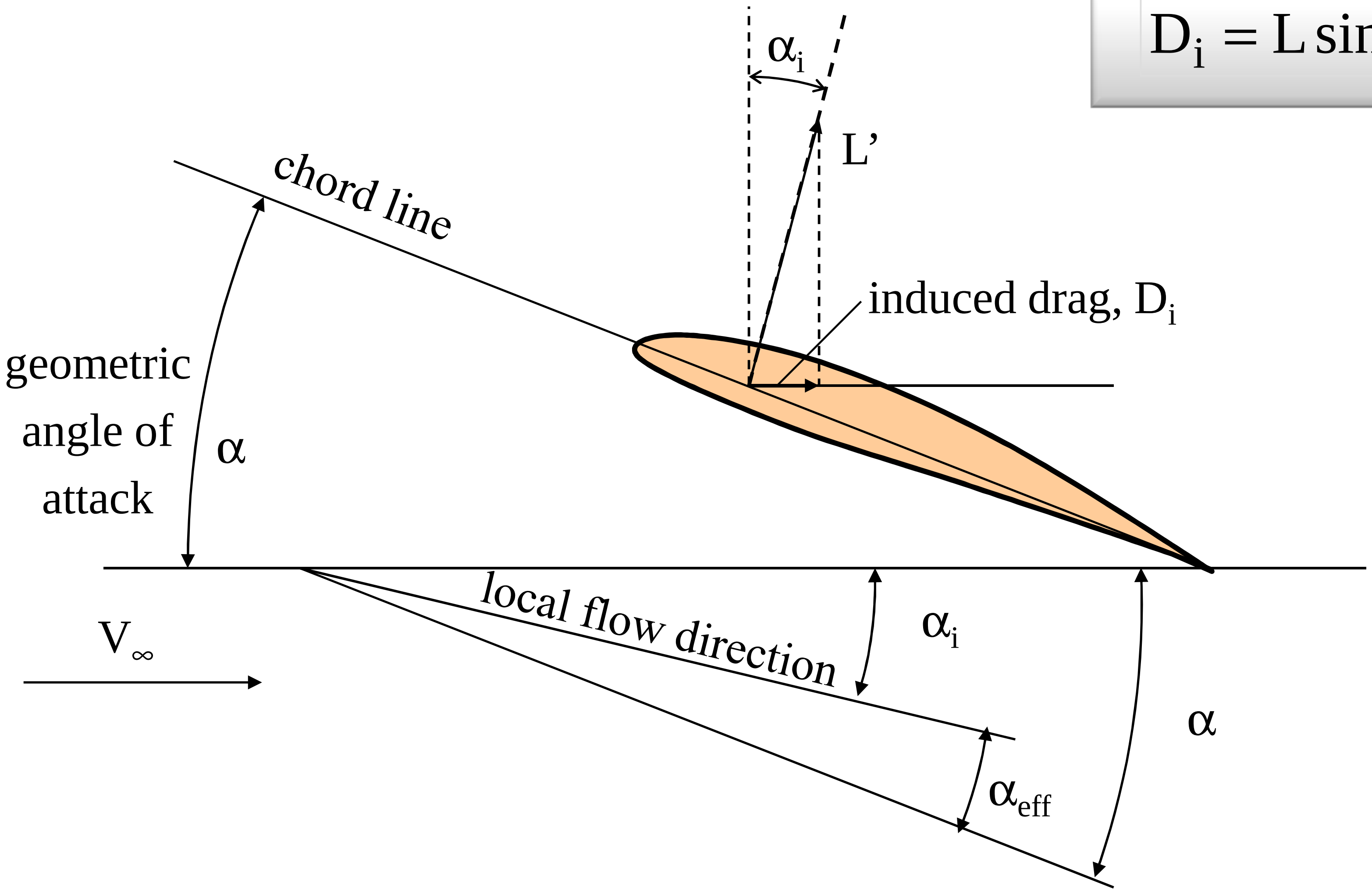
The trailing tip vortex causes downwash, w

Induced drag



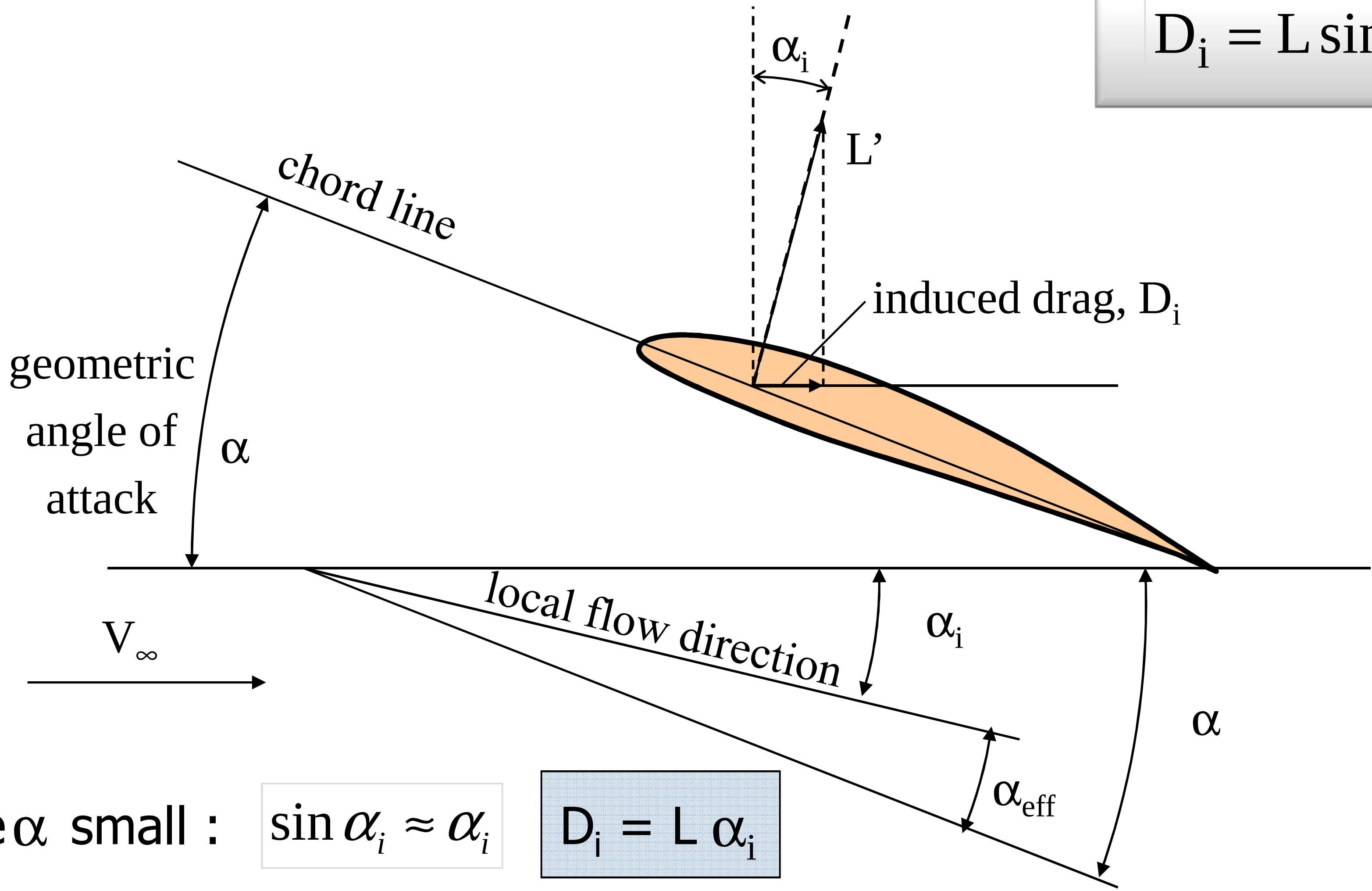
Induced drag

$$D_i = L \sin \alpha_i$$



Induced drag

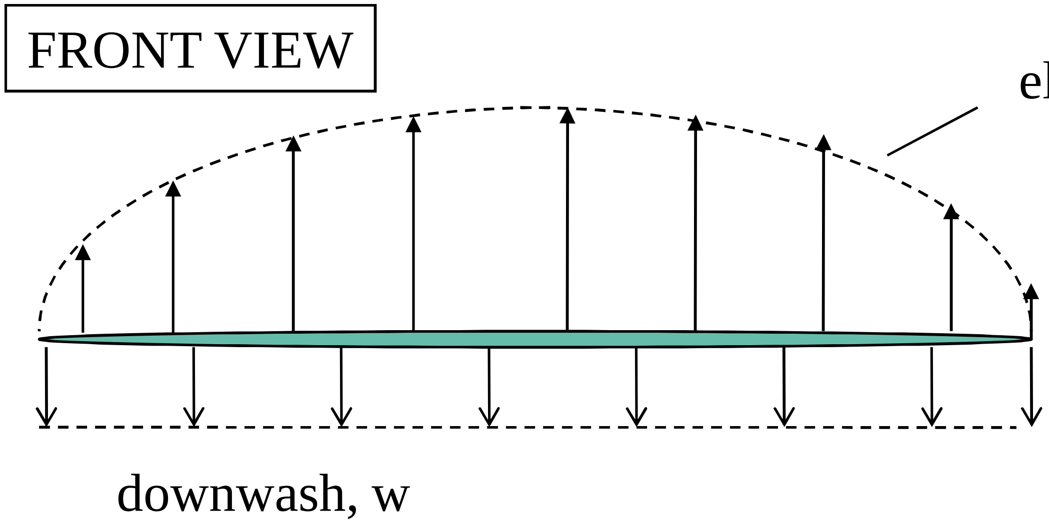
$$D_i = L \sin \alpha_i$$



Since α small : $\sin \alpha_i \approx \alpha_i$ $\quad D_i = L\,\alpha_i$

Induced drag

Example : **elliptical lift distribution**



Elliptical lift distribution results in constant downwash and therefore constant induced angle of attack

From incompressible flow theory : $\boxed{\alpha_i = \dfrac{C_L}{\pi A}}$

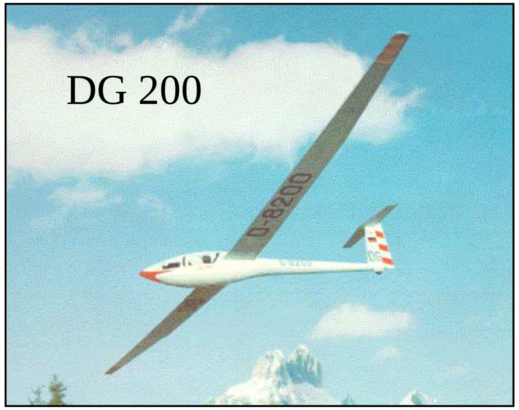
where : $A = \dfrac{b^2}{S}$ (Aspect ratio)

Thus : $\boxed{C_{D_i} = \dfrac{C_L^2}{\pi A}}$


Airplane-Pictures.net
Copyright © Emanuel Linert
Airplane-Pictures.net

Effect of aspect ratio on induced drag

High aspect ratio : Low Induced Drag



Low aspect ratio : High Induced Drag

Span efficiency factor

$$C_{D_i} = \frac{C_L^2}{\pi A e}$$

span efficiency factor (Oswald factor)

Elliptical loading	: e = 1 ; <u>minimum</u> induced drag
Non-elliptical loading	: e < 1 ; higher induced drag

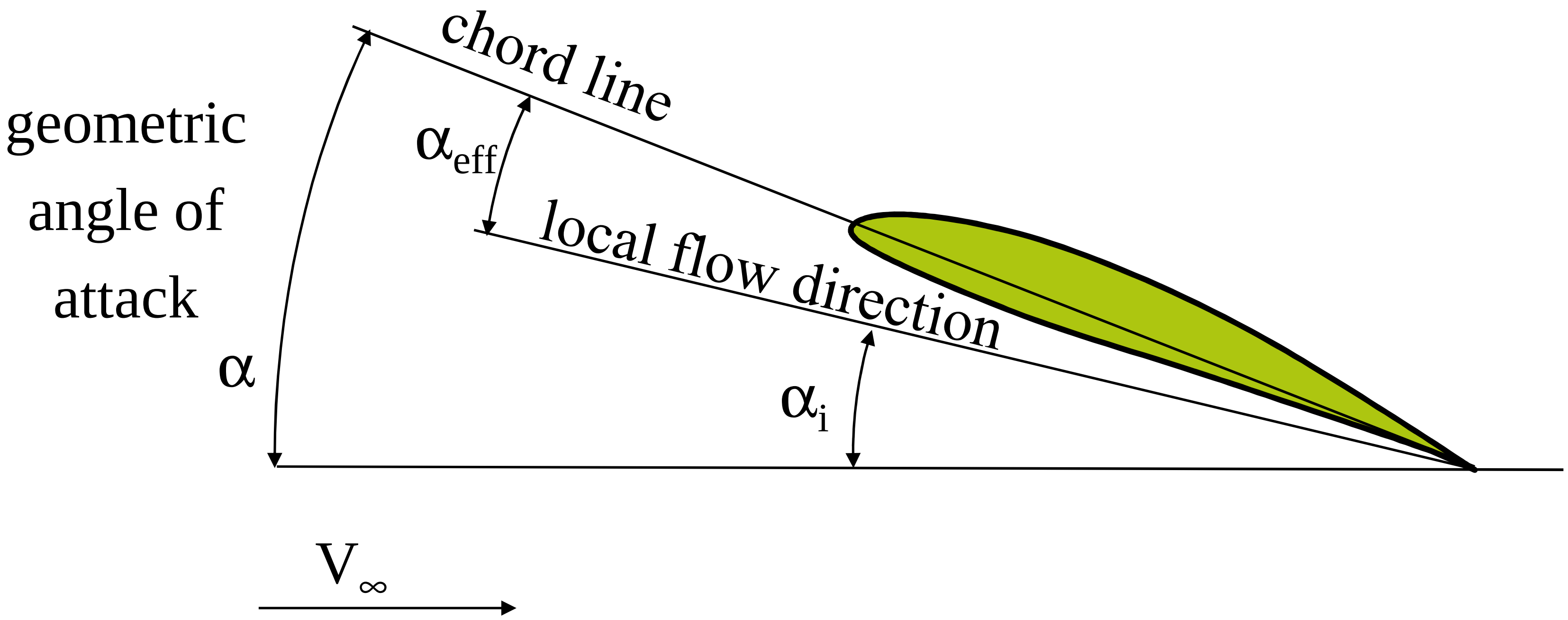
The total drag of the wing can now be written as :

$$C_D = \underbrace{C_{D_p}}_{\text{Profile drag}} + \underbrace{\frac{C_L^2}{\pi A e}}_{\text{Induced drag}}$$

Example 5.19

- Consider a flying wing with wing area of 206 m^2, an aspect ratio of 10, a span effectiveness factor of 0.95, and an NACA 4412 airfoil. The weight of the airplane is 7.5×10^5 N. If the density altitude is 3 km and the flight velocity is 100 m/s, calculate the total drag on the aircraft.

Lift curve slope



The effect of a finite wing is to **reduce the wing lift curve slope**

Lift curve slope

The induced angle of attack reduces

The local effective angle of attack : $\alpha_{eff} = \alpha - \alpha_i$

For a wing of a general plan form we may write :

$$\alpha_i = \frac{C_L}{\pi A e_1}$$

Angle is in radians!

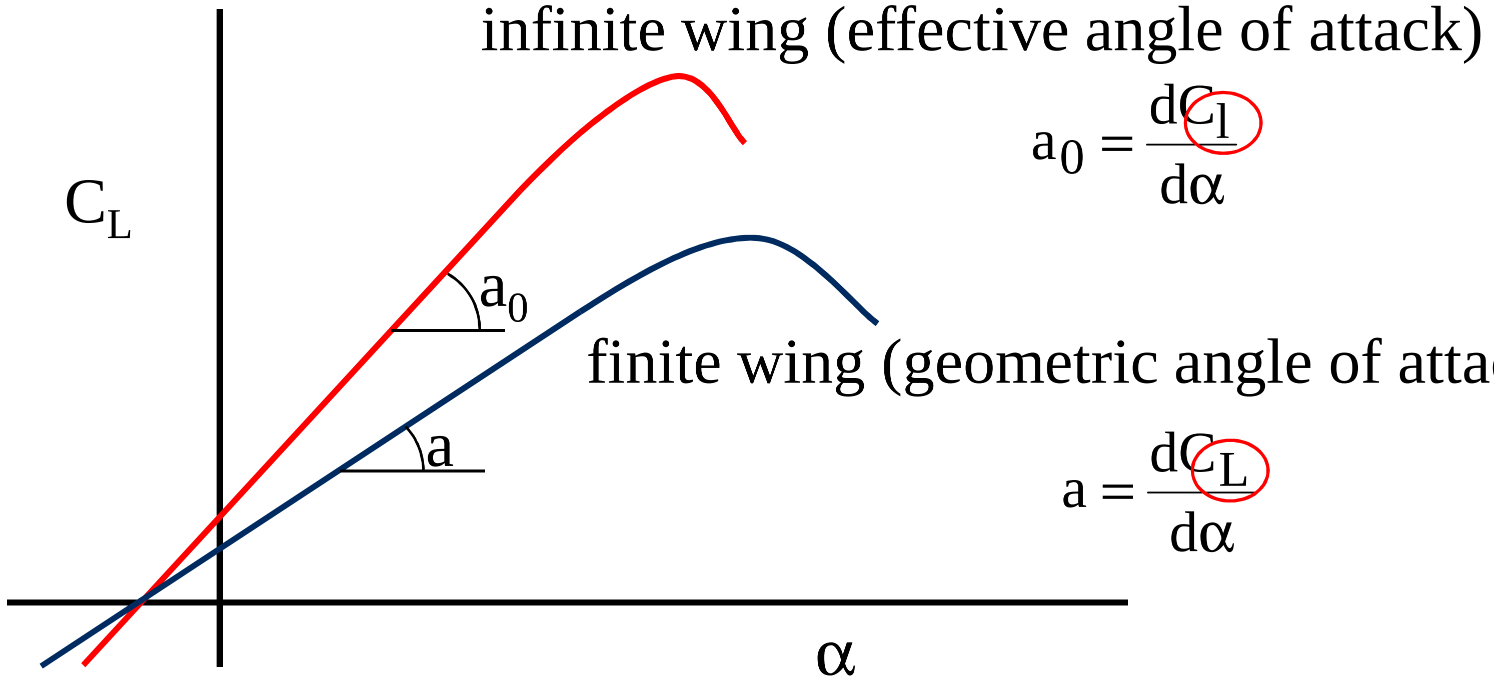
e_1 is span effectiveness factor

(theoretically different from e but in practice more or less the same)

$$\boxed{\alpha_i = \frac{57.3 C_L}{\pi A e_1}}$$

For angle in degrees

Lift curve slope



$$a_0 = \frac{dC_l}{d\alpha}$$

$$a = \frac{dC_L}{d\alpha}$$

From thin airfoil theory we find :

$$\boxed{a_0 = 2\pi}$$

Lift curve slope

Effective local angle of attack $\alpha_{eff} = \alpha - \alpha_i$

For an arbitrary loading distribution wing we find :

$\alpha_i = \frac{C_L}{\pi A e_1}$ $\frac{dC_L}{d(\alpha - \alpha_i)} = a_0$ Integrate :

$$C_L = a_0(\alpha - \alpha_i) + \text{const}, \qquad C_L = a_0\left(\alpha - \frac{C_L}{\pi A e_1}\right) + \text{const}$$

$$C_L = \frac{a_0 \alpha}{1 + \frac{a_0}{\pi A e_1}} + \frac{\text{const}}{1 + \frac{a_0}{\pi A e_1}}$$

Lift curve slope

Differentiating this equation results in : $$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi A e_1}}$$

Example :

Infinite wing : A=∞ then $\frac{dC_L}{d\alpha} = 2\pi$

Finite wings : A=12 (Fokker 50) then $\frac{dC_L}{d\alpha} = 2\pi \cdot 0.857 = 1.71\pi$

A=5 then $\frac{dC_L}{d\alpha} = 2\pi \cdot 0.714 = 1.43\pi$

$$a = \frac{dC_L}{d\alpha} = \frac{a_0}{1 + \dfrac{a_0}{\pi A e_1}}$$

$$a_0 = \frac{dC_l}{d\alpha}$$

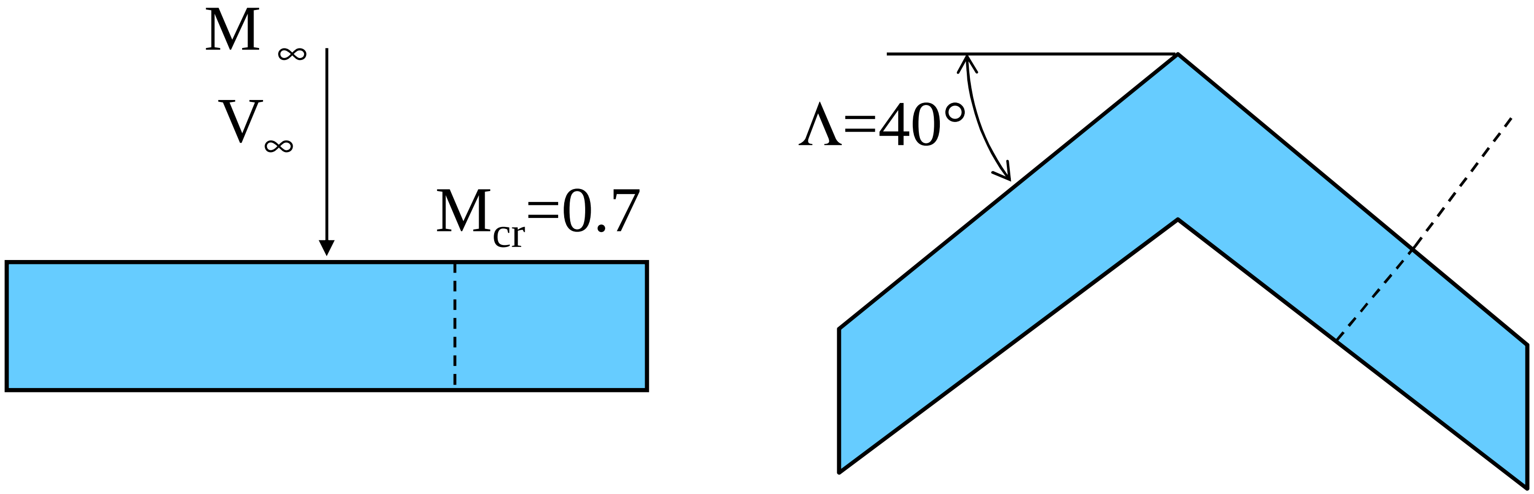
$$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \dfrac{57.3 a_0}{\pi A e_1}}$$

a_0 per degree

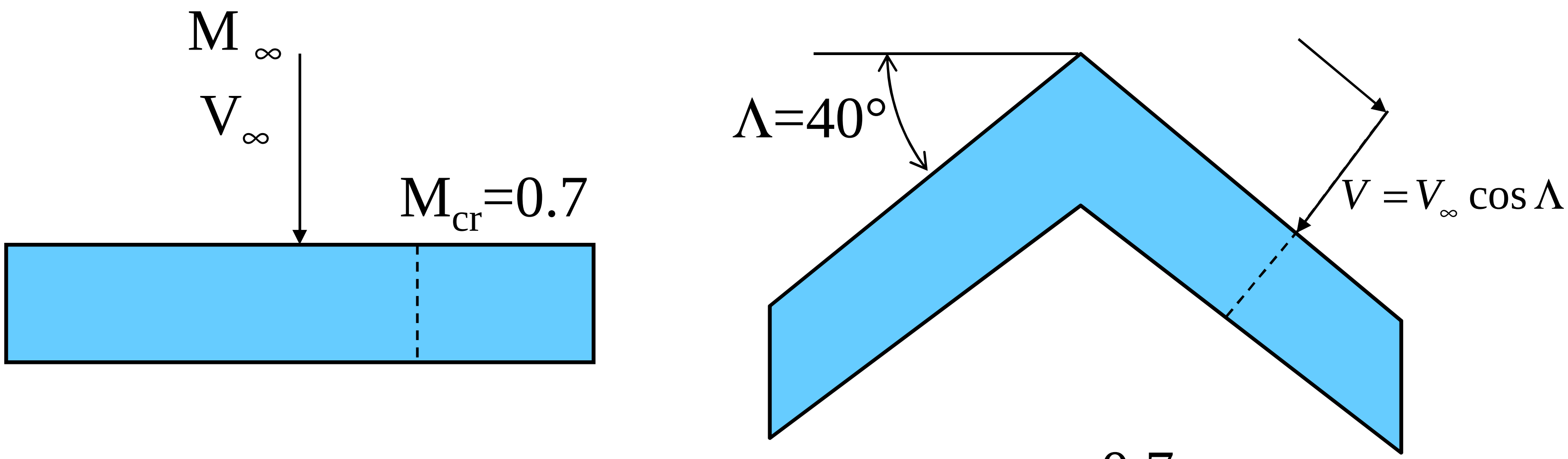
Example 5.21

- Consider a wing with an aspect ratio of 10 and NACA 23012 airfoil section. Assume Re ≈ 5 x 10^6. the span efficiency factor is $e=e_1=0.95$. If the wing is at 4^o angle of attack, calculate C_L and C_D.

Swept wings



Swept wings

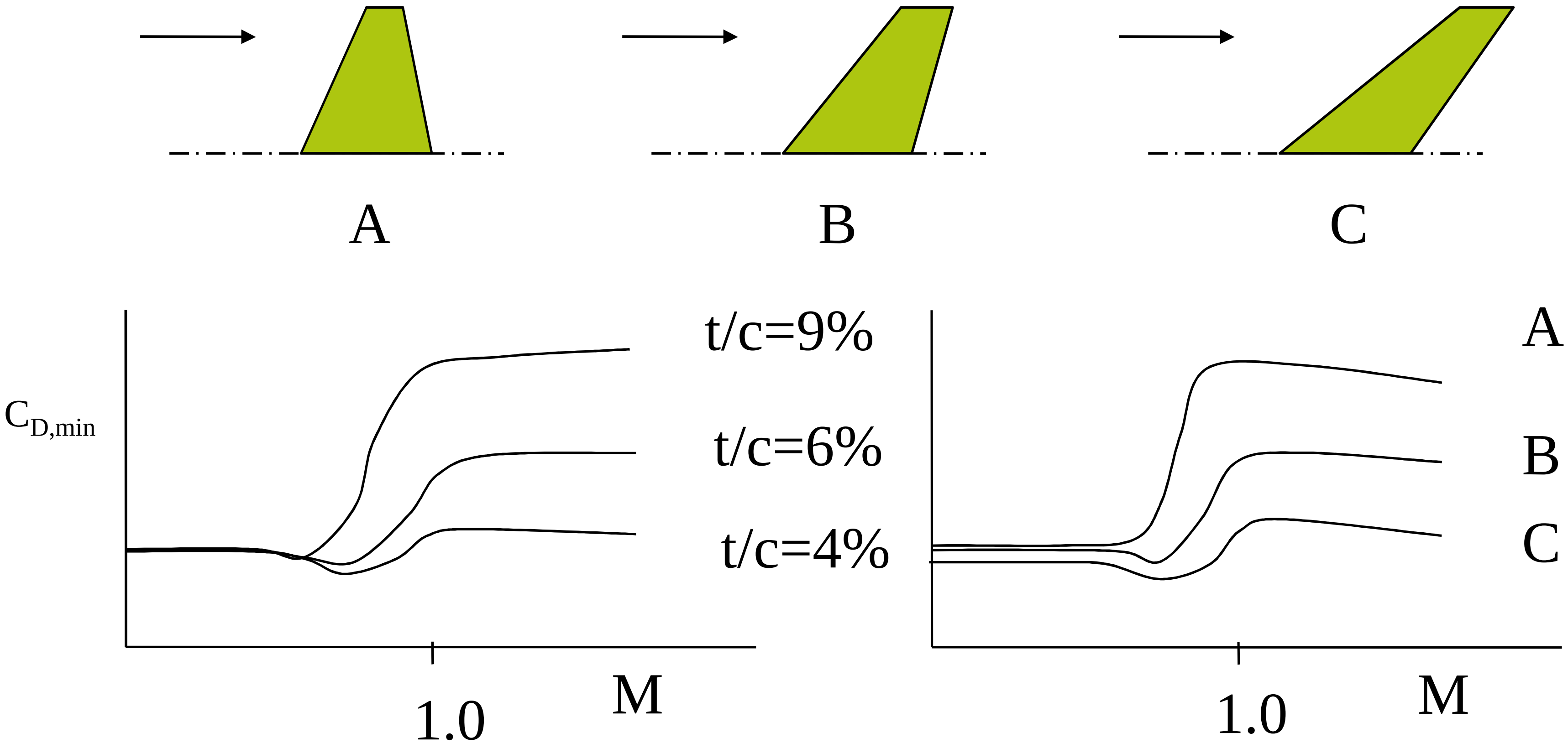


$$M_{cr} \text{ for swept wing } = \frac{0.7}{\cos\Lambda} = 0.91$$

By sweeping the wings of **subsonic** aircraft, the drag divergence is delayed to higher Mach numbers

Swept wings

Effect of wing thickness and sweepback angle on minimum wing drag coefficient

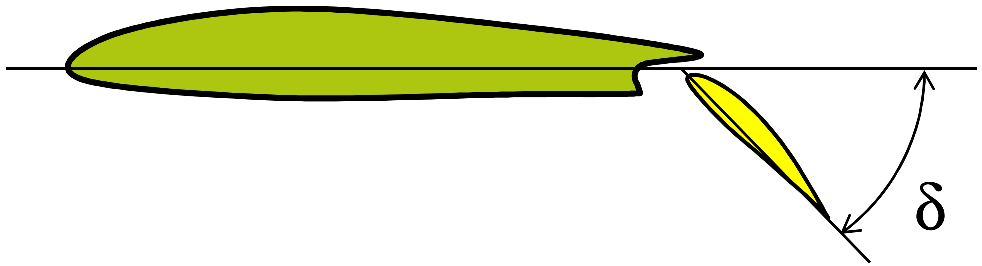


Total drag

$$C_D = C_{D_{profile}} + \frac{C_L^2}{\pi A e}$$

$$C_{D_{profile}} = C_{D_f} + C_{D_{pressure}} + C_{D_w}$$

Flaps

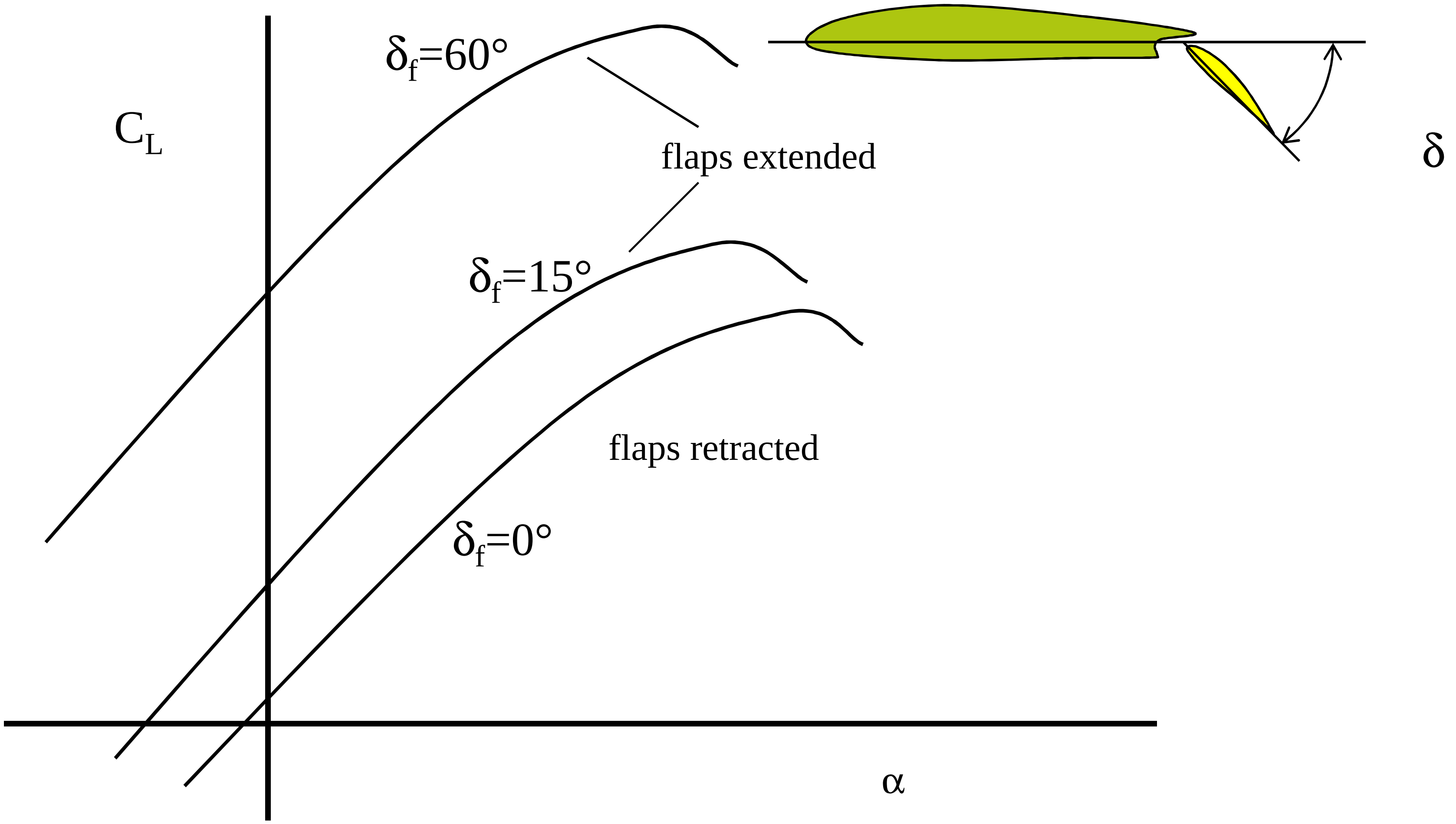


$$W = L = C_L \tfrac{1}{2}\rho V_\infty^2 S$$

Thus : $V_\infty = \sqrt{\dfrac{2W}{\rho_\infty S C_L}}$ and : $\boxed{V_{stall} = \sqrt{\dfrac{2W}{\rho_\infty S C_{L_{max}}}}}$

The landing speed is **decreased** when the maximum lift coefficient is **increased**

Flaps


C_L
δ_f=60°
flaps extended
δ_f=15°
flaps retracted
δ_f=0°
α
δ

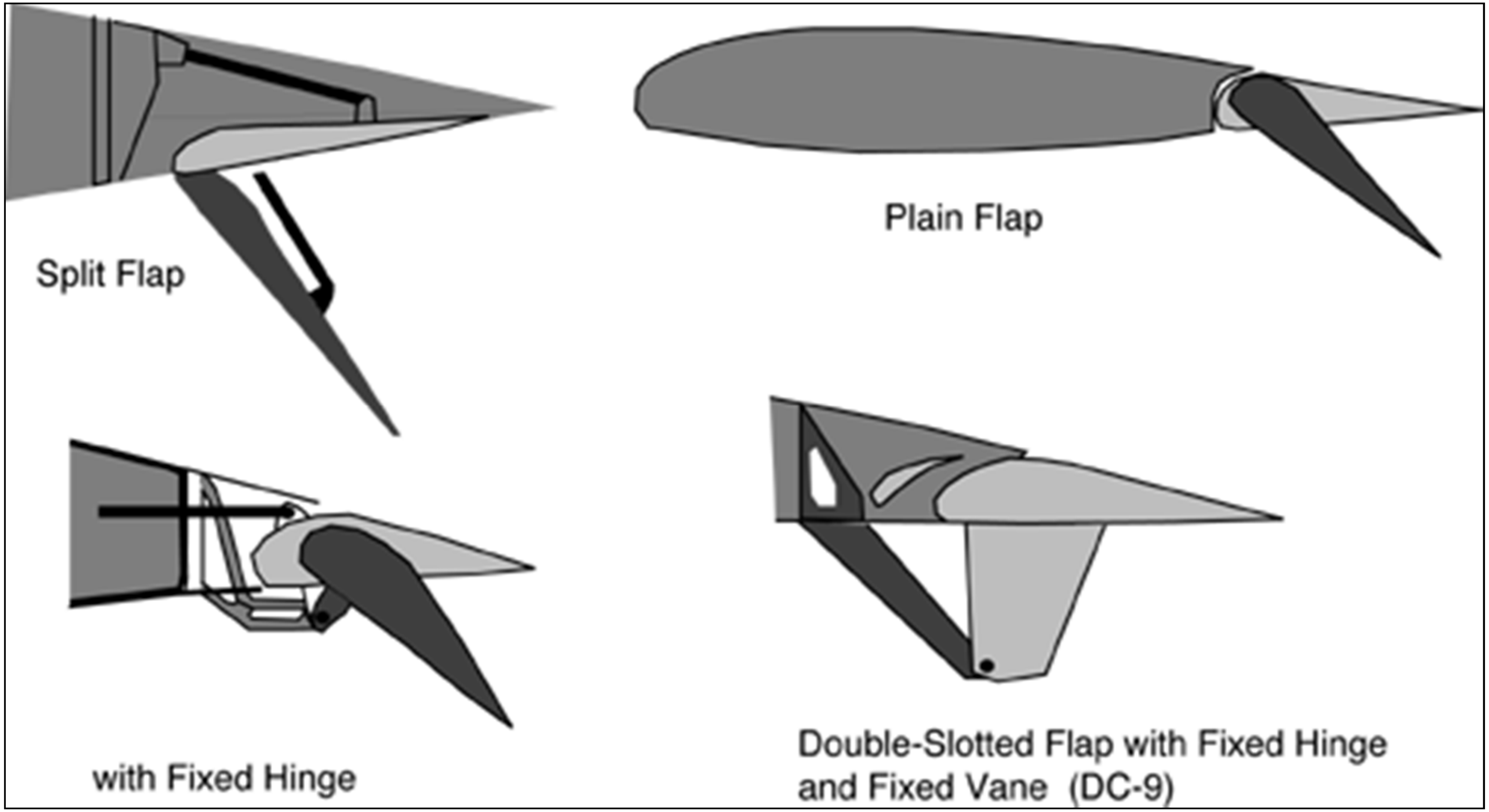
Flaps

Typical example

Flaps and slats

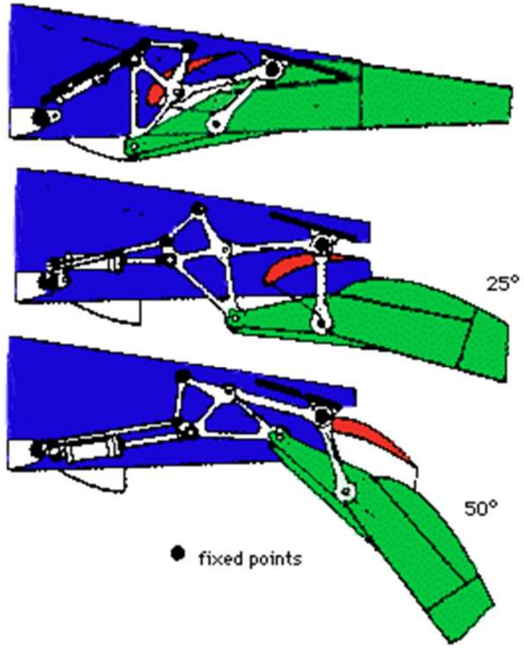
Flaps

Flaps



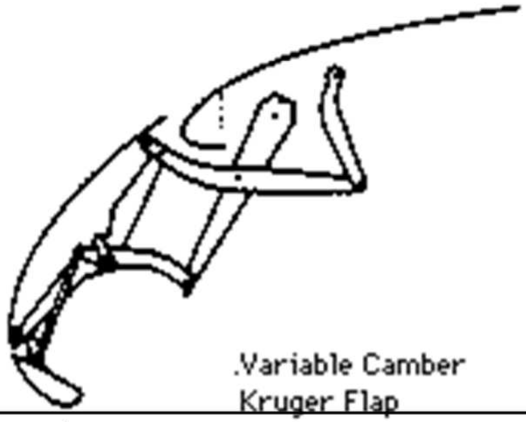
TUDelft

Flaps

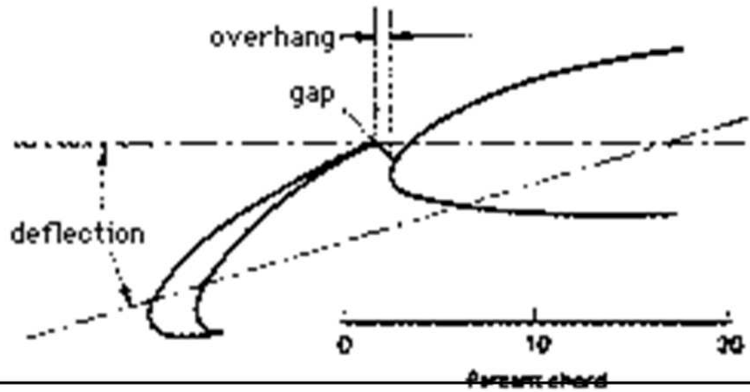


25°
50°
fixed points

Nose Flap
Fixed Slot
Kruger Flap
Slat



Variable Camber
Kruger Flap



overhang
gap
deflection
0
10
20