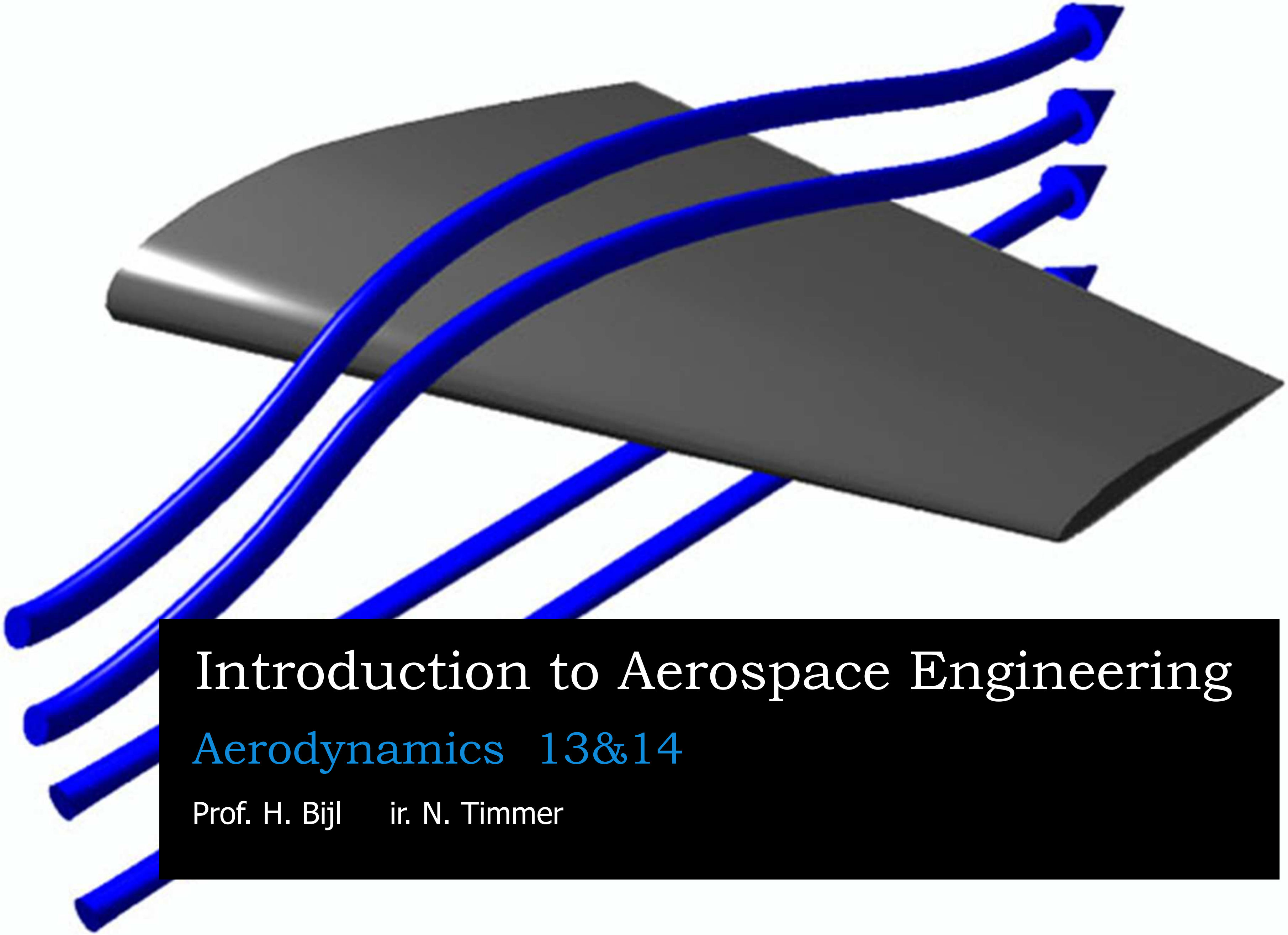
Introduction to Aerospace Engineering

Lecture slides



Introduction to Aerospace Engineering

Aerodynamics 13&14

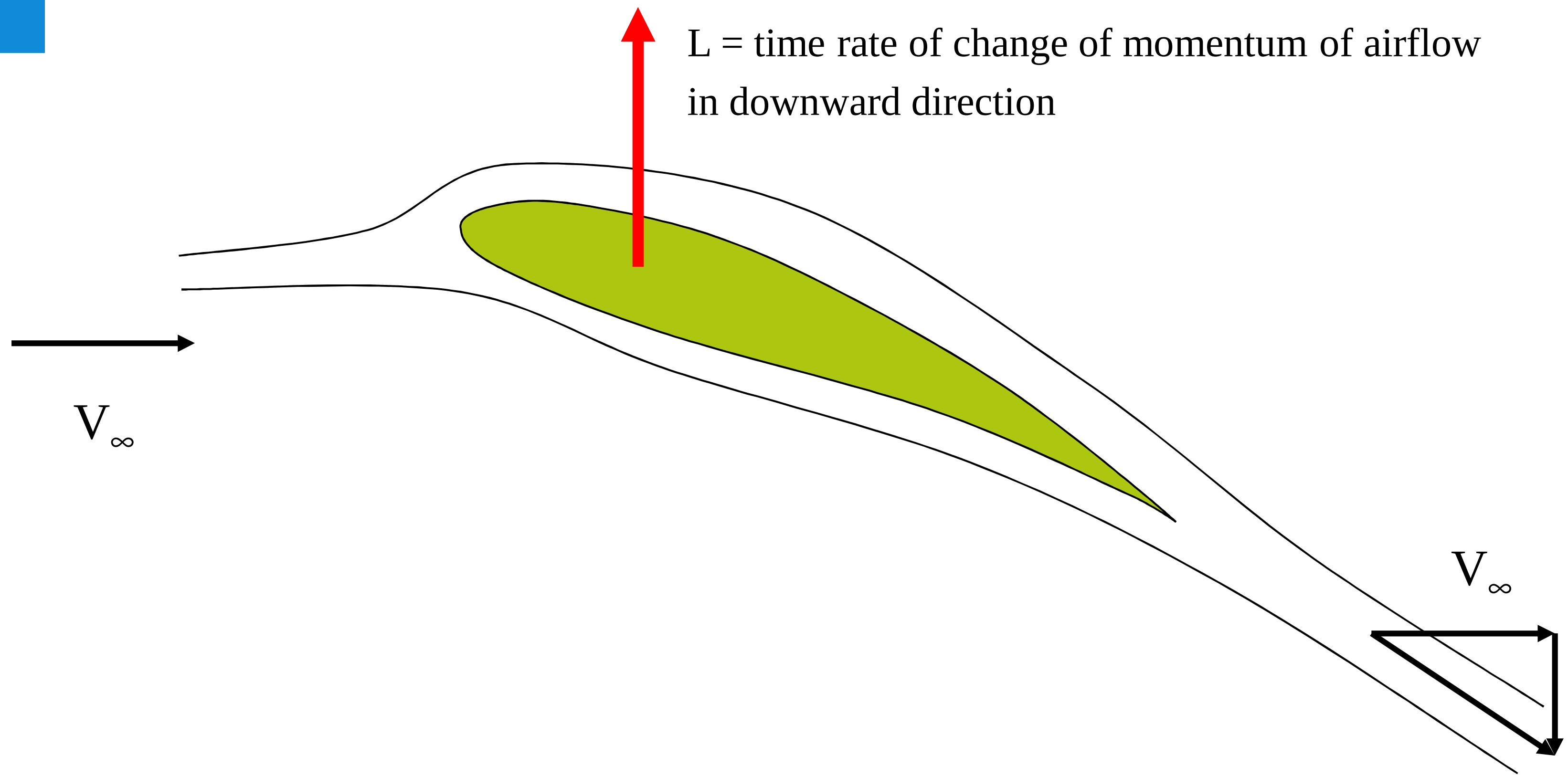
Prof. H. Bijl   ir. N. Timmer

13 & 14.

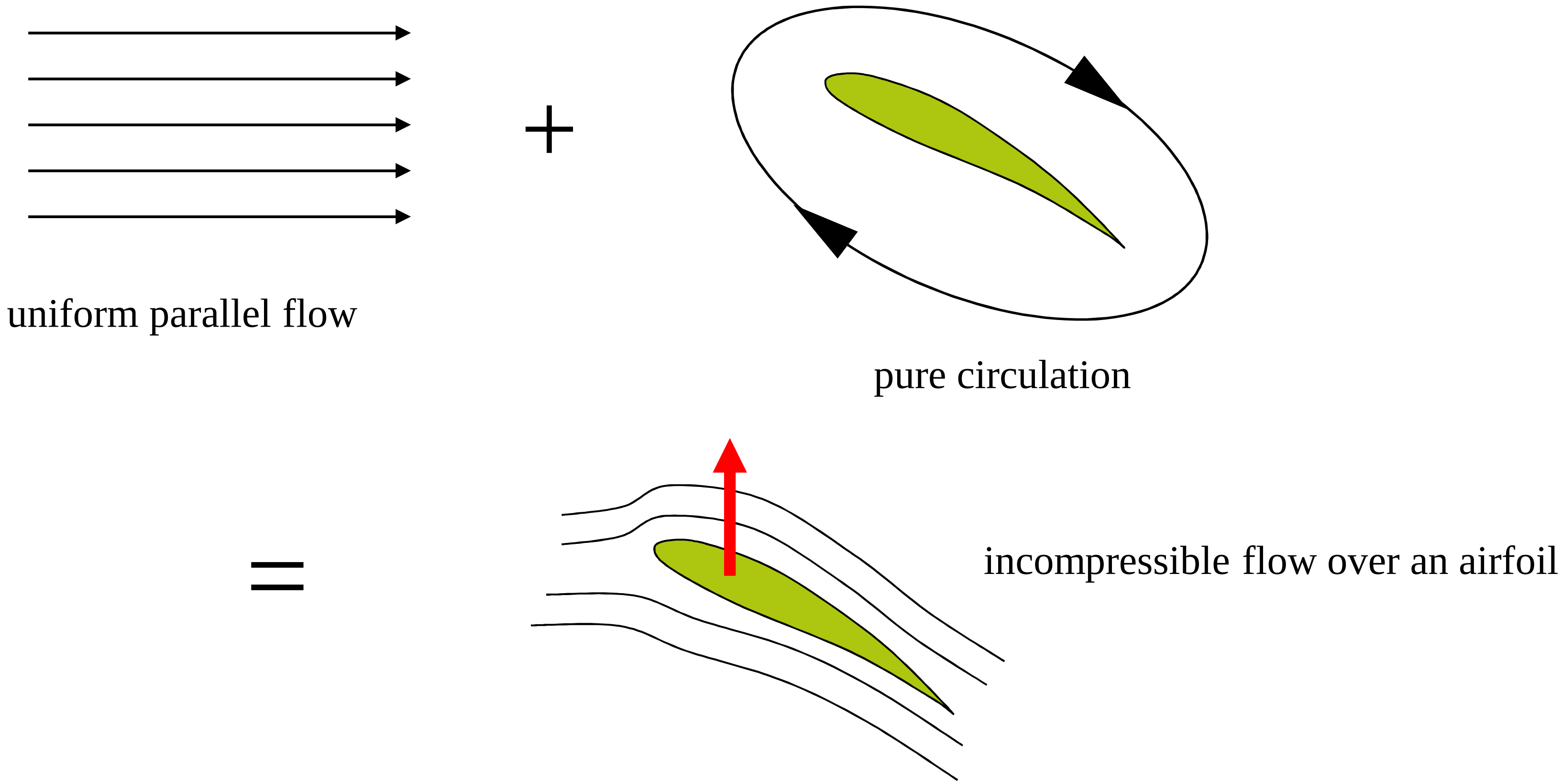
Repetition and Problems

Generation of lift

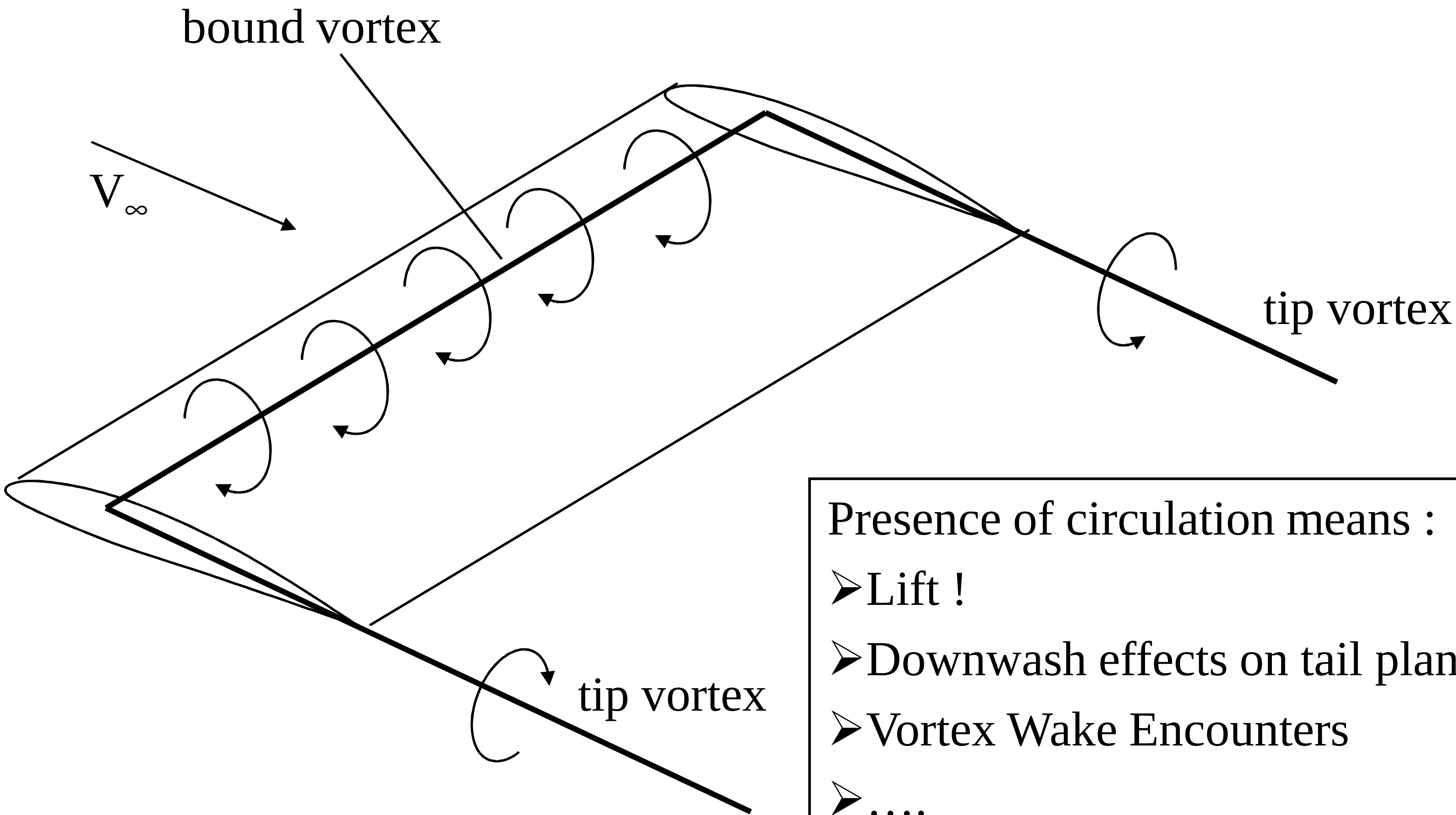
L = time rate of change of momentum of airflow in downward direction



Circulation theory of lift



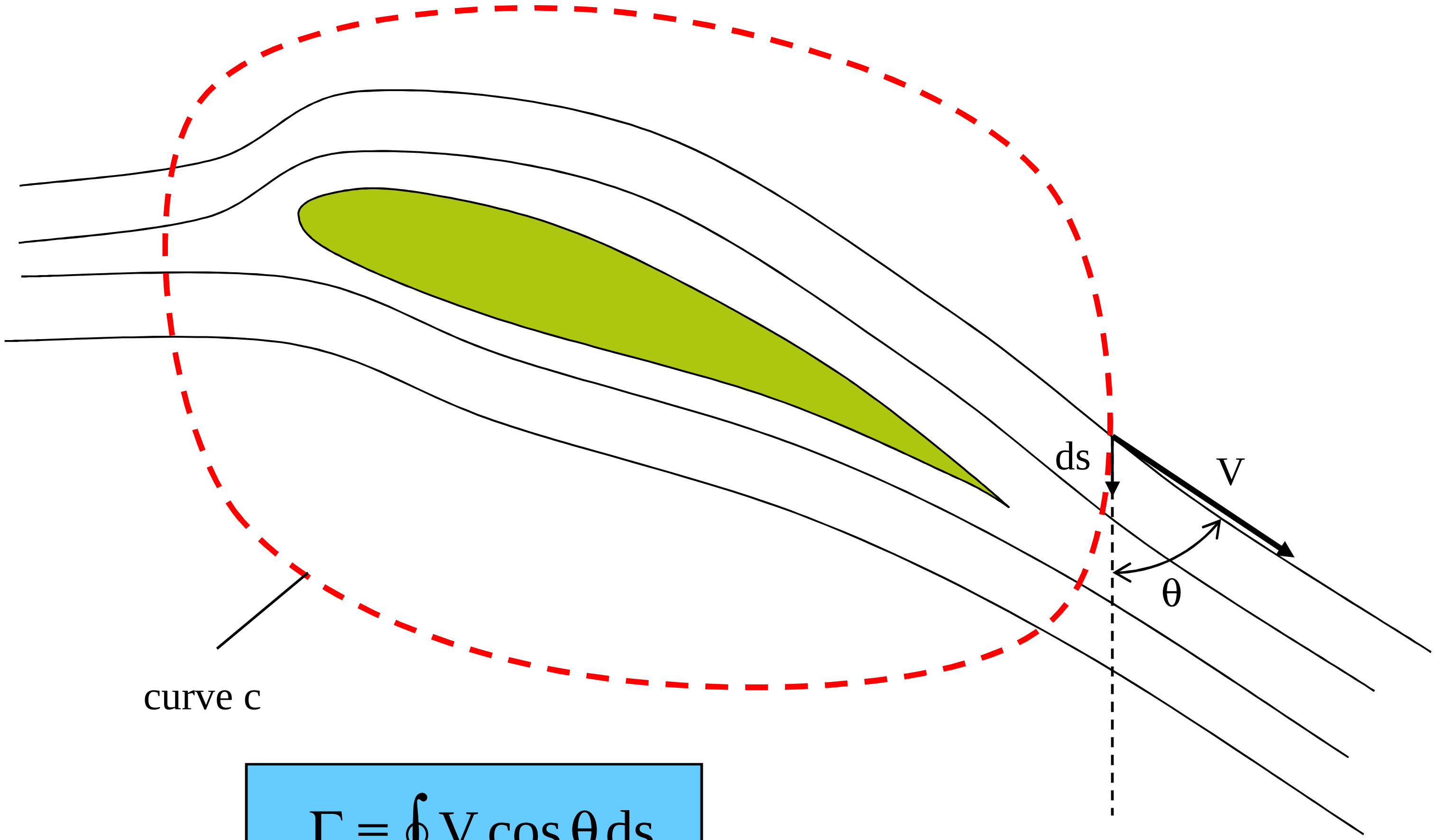
Circulation



Presence of circulation means :

- ➢Lift !
- ➢Downwash effects on tail plane
- ➢Vortex Wake Encounters
- ➢….

Circulation theory of lift



Circulation

$$\Gamma \equiv \oint_c V \cos\theta \, ds$$

Circulation theory of lift

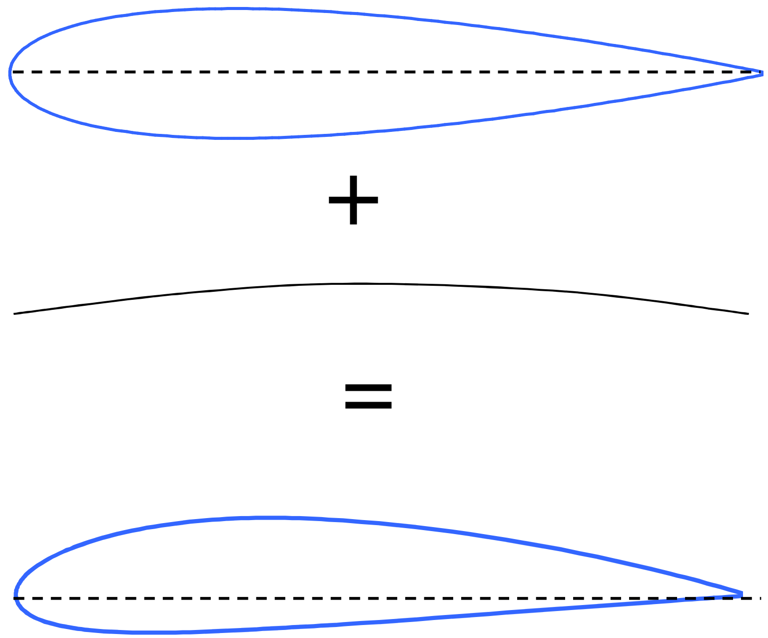
Circulation $$\boxed{\Gamma \equiv \oint_{c} V \cos\theta \, ds}$$

Kutta-Joukowsky Theorem : Lift per unit span :

$$\boxed{L = \rho_\infty V_\infty \Gamma_\infty}$$

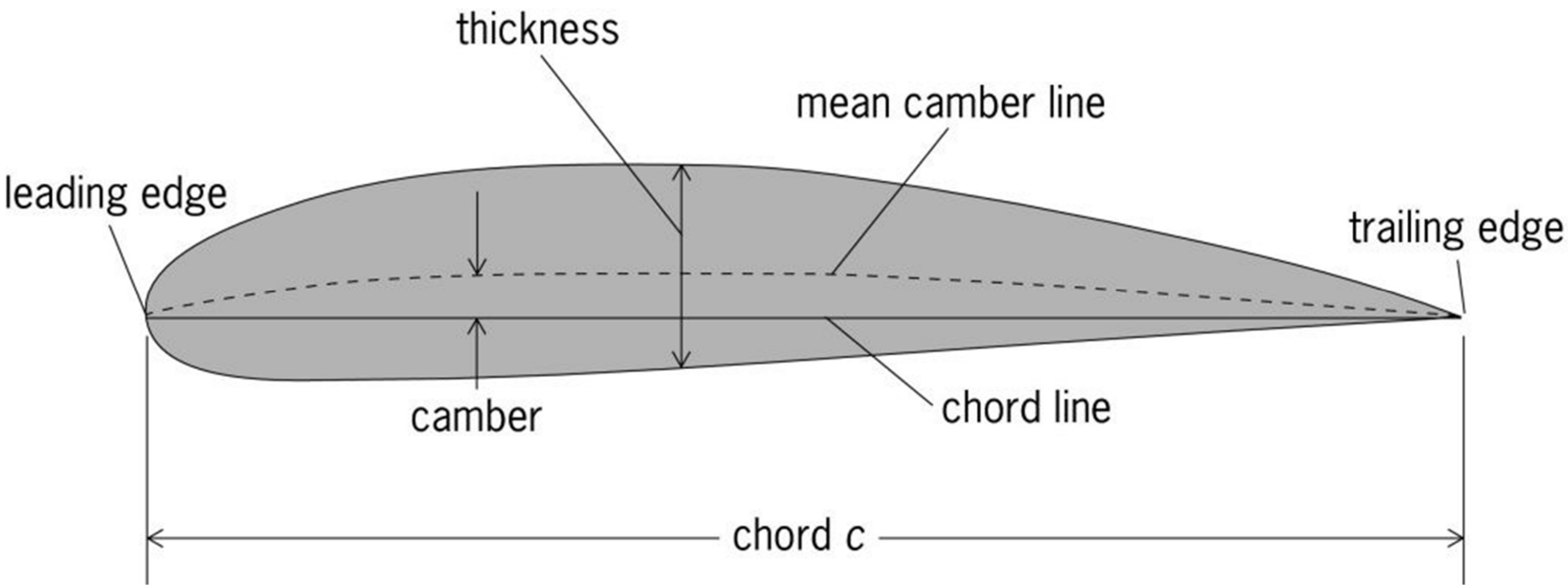
NACA airfoil development

- Systematic in the sense that different camber lines were combined with different (symmetric) thickness distributions



Welvingslijn /mean line / camber line

- General: Camber line is the mean of the upper and lower surface coordinates



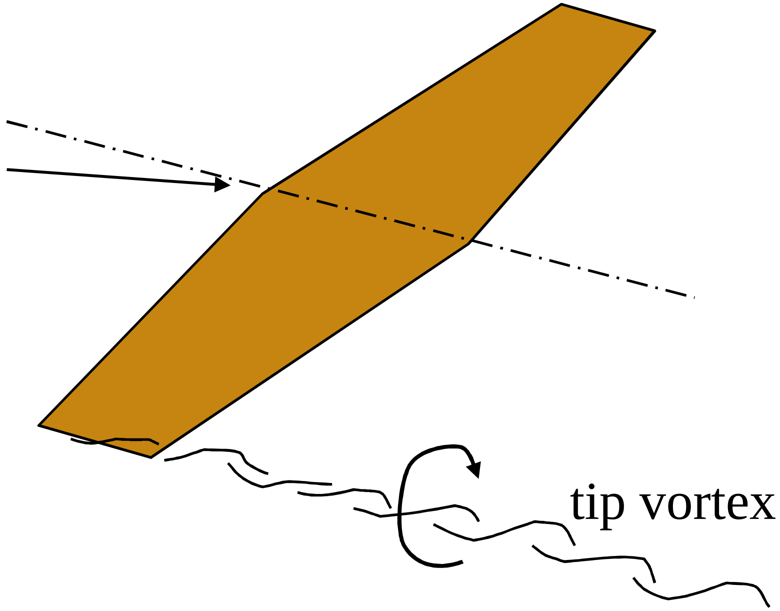
Aerodynamic centre of an airfoil:

The point on the chord where $\dfrac{dC_m}{d\alpha} = 0$

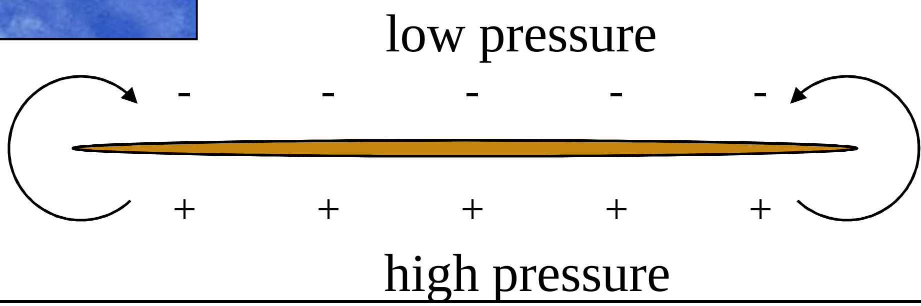
in general close to c/4 (fixed point)

Centre of pressure : point on chord where the forces cross (drukpunt)

Characteristics of finite wings

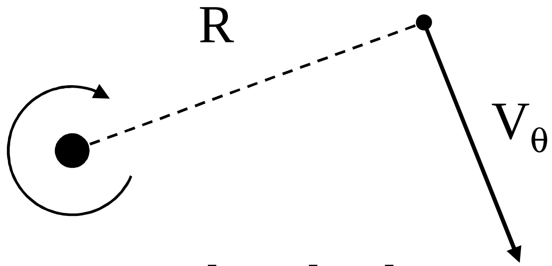


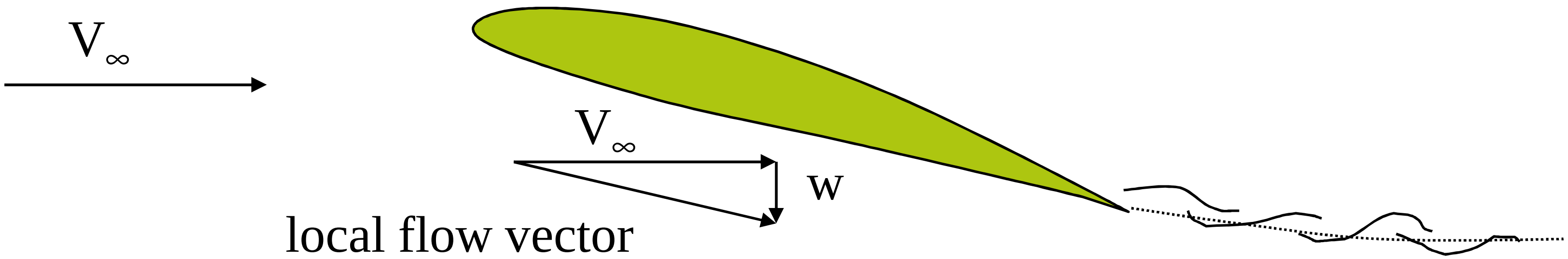
Flow around the wing tip generates tip vortices



Tip vortices

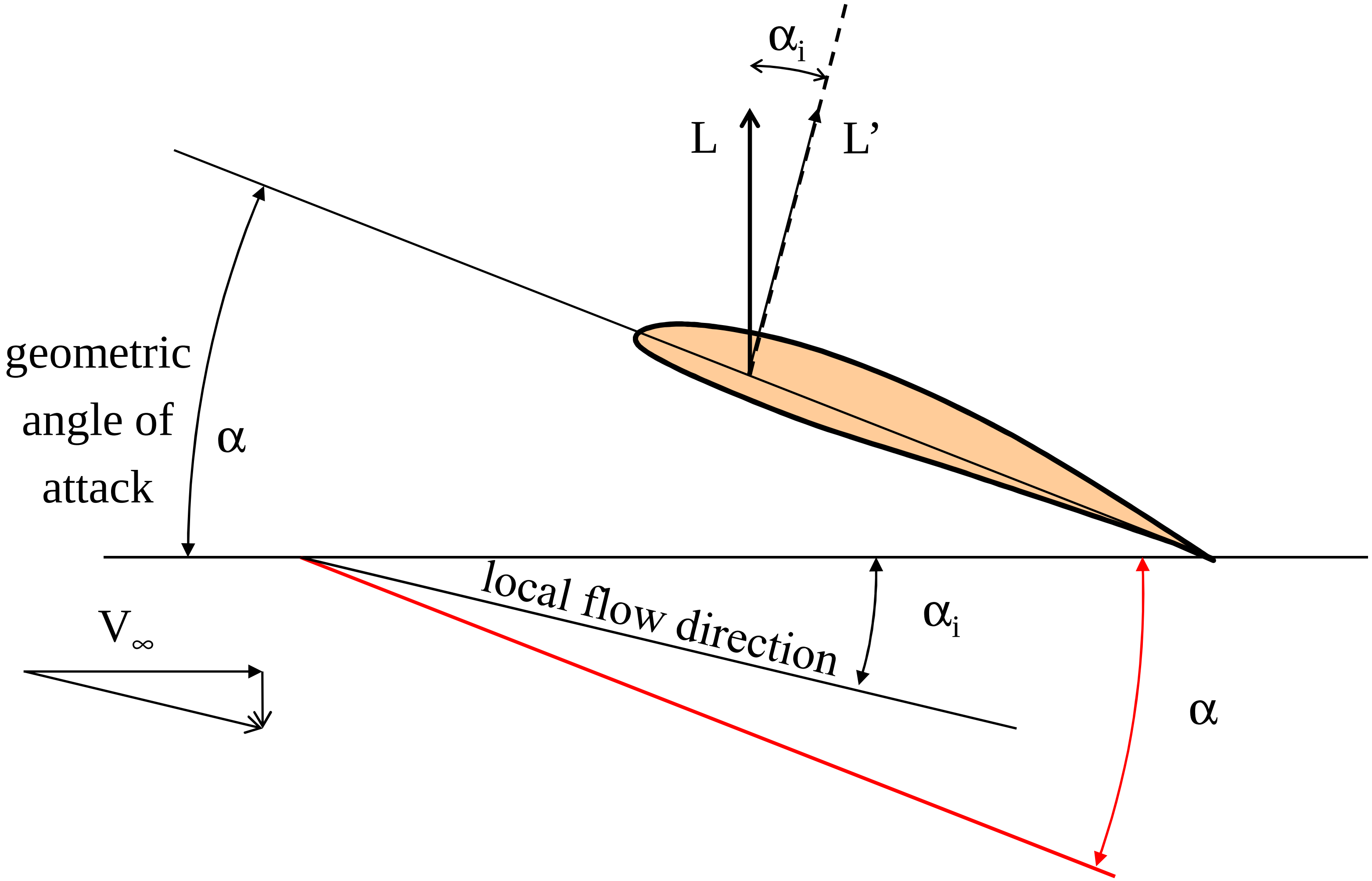
Cross section of tip vortex





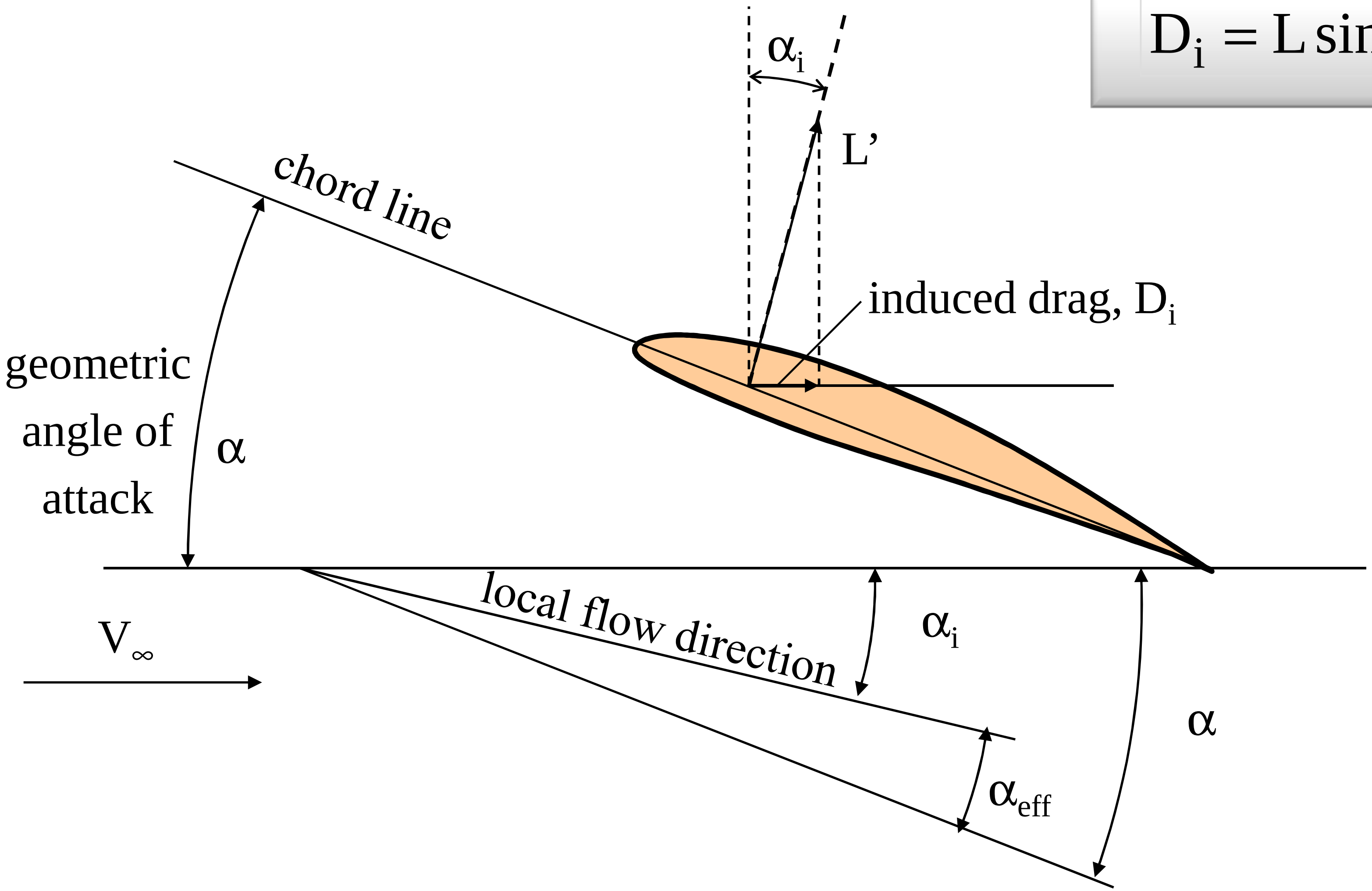
The trailing tip vortex causes downwash, w

Induced drag



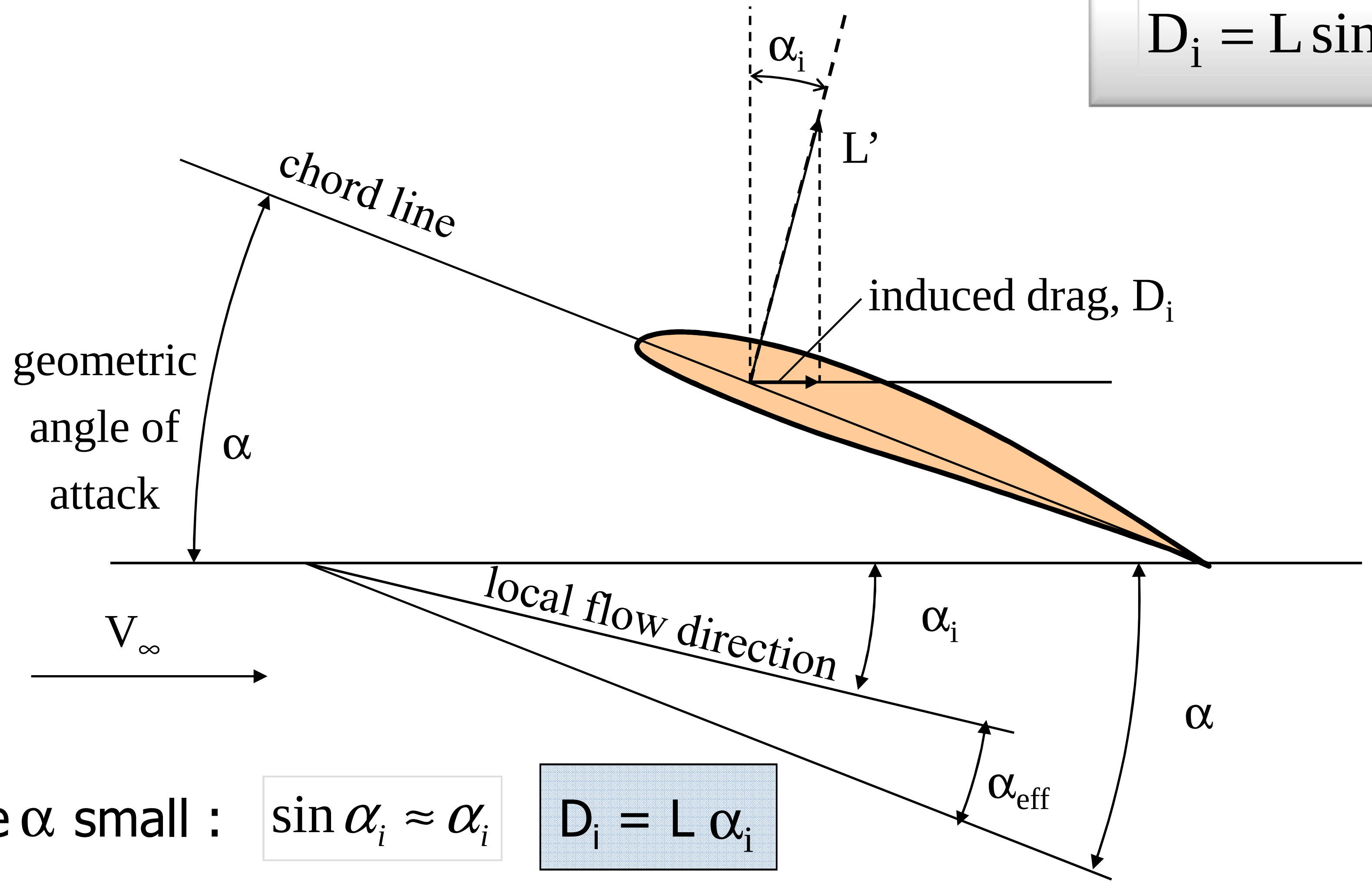
Induced drag

$$D_i = L \sin \alpha_i$$



Induced drag

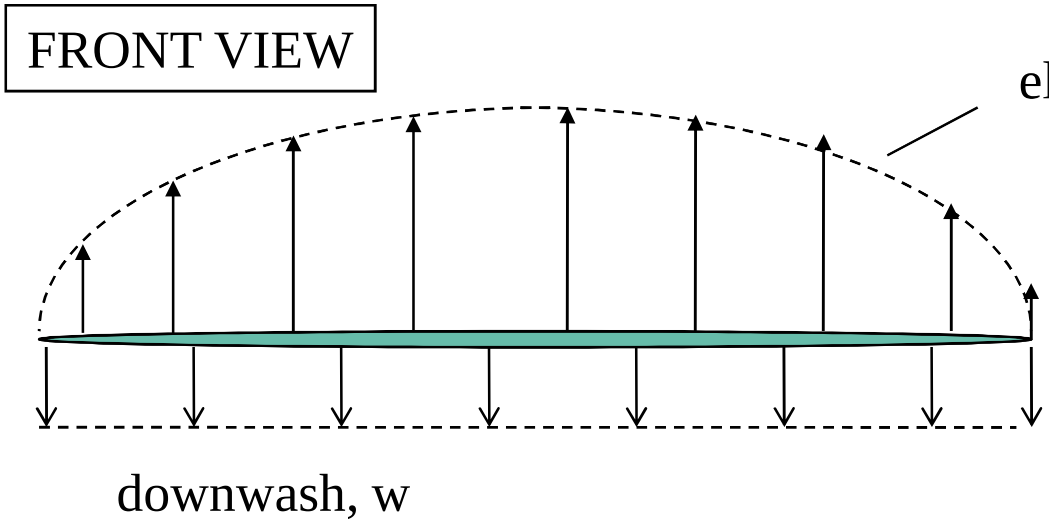
$$D_i = L \sin \alpha_i$$



Since α small : $\sin \alpha_i \approx \alpha_i$ $\quad D_i = L\,\alpha_i$

Induced drag

Example : **elliptical lift distribution**



Elliptical lift distribution results in constant downwash and therefore constant induced angle of attack

From incompressible flow theory : $\boxed{\alpha_i = \dfrac{C_L}{\pi A}}$

where : $A = \dfrac{b^2}{S}$ (Aspect ratio)

Thus : $\boxed{C_{D_i} = \dfrac{C_L^2}{\pi A}}$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Effect of aspect ratio on induced drag

High aspect ratio : Low Induced Drag

Low aspect ratio : High Induced Drag

Mirage 2000

Span efficiency factor

$$C_{D_i} = \frac{C_L^2}{\pi A e}$$

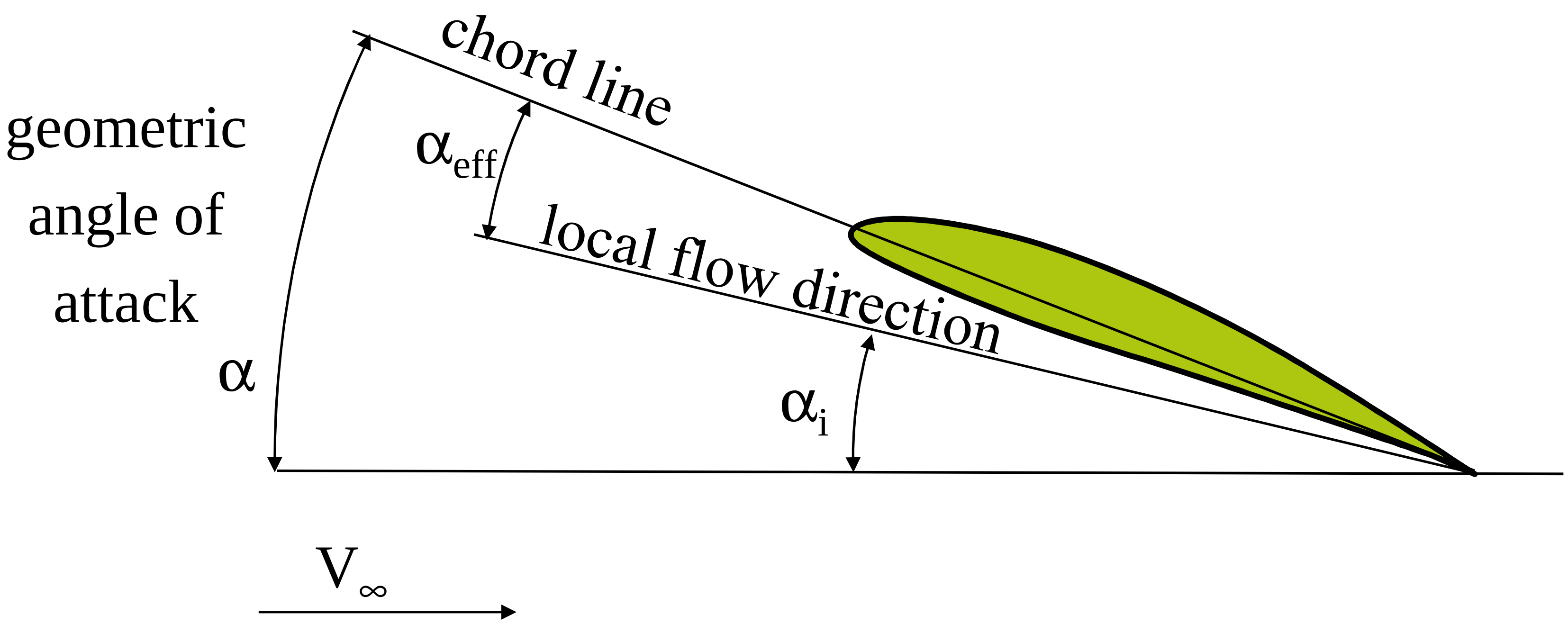
span efficiency factor (Oswald factor)

Elliptical loading	: e = 1 ; minimum induced drag
Non-elliptical loading	: e < 1 ; higher induced drag

The total drag of the wing can now be written as :

$$C_D = \underbrace{C_{D_p}}_{\text{Profile drag}} + \underbrace{\frac{C_L^2}{\pi A e}}_{\text{Induced drag}}$$

Lift curve slope



The effect of a finite wing is to **reduce the wing lift curve slope**

Lift curve slope

The induced angle of attack reduces the local effective angle of attack :

$$\alpha_{eff} = \alpha - \alpha_i$$

For a wing of a general plan form we may write :

$$\alpha_i = \frac{C_L}{\pi A e_1}$$

Angle is in radians!

e_1 is span effectiveness factor (theoretically different from e but in practice more or less the same)

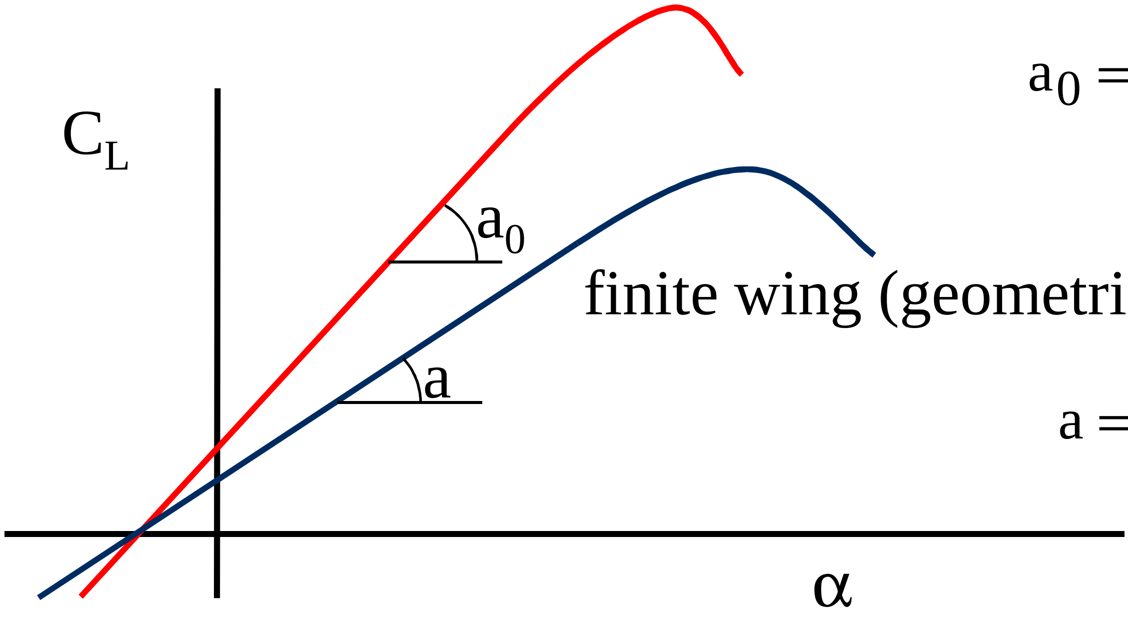
$$\boxed{\alpha_i = \frac{57.3 C_L}{\pi A e_1}}$$

For angle in degrees

Lift curve slope

infinite wing (effective angle of attack)

$$a_0 = \frac{dC_l}{d\alpha}$$



finite wing (geometric angle of attack)

$$a = \frac{dC_L}{d\alpha}$$

From thin airfoil theory we find :

$$\boxed{a_0 = 2\pi}$$

Lift curve slope

Effective local angle of attack $\alpha_{eff} = \alpha - \alpha_i$

For an arbitrary loading distribution wing we find :

$$\alpha_i = \frac{C_L}{\pi A e_1} \qquad \frac{dC_L}{d(\alpha - \alpha_i)} = a_0$$

Integrate :

$$C_L = a_0(\alpha - \alpha_i) + \text{const}, \qquad C_L = a_0\left(\alpha - \frac{C_L}{\pi A e_1}\right) + \text{const}$$

$$C_L = \frac{a_0 \alpha}{1 + \frac{a_0}{\pi A e_1}} + \frac{\text{const}}{1 + \frac{a_0}{\pi A e_1}}$$

TU Delft

Lift curve slope

Differentiating this equation results in : $$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi A e_1}}$$

Example :

Infinite wing : A=∞ then $$\frac{dC_L}{d\alpha} = 2\pi$$

Finite wings : A=12 (Fokker 50) then $$\frac{dC_L}{d\alpha} = 2\pi \cdot 0.857 = 1.71\pi$$

A=5 then $$\frac{dC_L}{d\alpha} = 2\pi \cdot 0.714 = 1.43\pi$$

$$a = \frac{dC_L}{d\alpha} = \frac{a_0}{1 + \dfrac{a_0}{\pi A e_1}}$$

$$a_0 = \frac{dC_l}{d\alpha}$$

$$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \dfrac{57.3 a_0}{\pi A e_1}}$$

a_0 per degree

Pressure, density and temperature under standard conditions

$$P_s = 1.01325 * 10^5 \ N/m^2$$

$$\rho_s = 1.225 \ kg/m^3$$

$$T_s = 288.15 \ K$$

Summary of equations

<u>For steady, frictionless, incompressible flow:</u>

$$A_1V_1 = A_2V_2$$ Continuity equation

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$ Bernoulli's equation

$$P = \rho RT$$ Equation of state

For steady, isentropic (adiabatic and frictionless) compressible flow:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Continuity equation

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

Isentropic relations

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$$

Energy equation

$$P = \rho R T$$

Equation of state

$$a = \sqrt{\gamma R T}$$

Second form of isentropic relations

$$\frac{T_o}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho_1} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{1}{\gamma-1}}$$

Viscous flows

- What is:
  - Laminar flow
  - Turbulent flow
  - Transition
  - Separation
  - Reynolds number
  - Critical Reynolds number

Lift and drag coefficients

$L = C_L \cdot q_\infty \cdot S$	$C_L = \dfrac{L}{q_\infty S}$
$D = C_D \cdot q_\infty \cdot S$	$C_D = \dfrac{D}{q_\infty S}$
$M = C_M \cdot q_\infty \cdot S \cdot c$	$C_M = \dfrac{M}{q_\infty S c}$

$C_L = f_1(\alpha, M_\infty, Re)$ $C_D = f_2(\alpha, M_\infty, Re)$ $C_M = f_3(\alpha, M_\infty, Re)$

Obtaining lift from pressure distribution

$$C_p = \frac{p - p_\infty}{q_\infty}$$

$$C_n = \int_0^1 \left(C_{p_l} - C_{p_u}\right) d\left(\frac{x}{c}\right)$$

$$C_l \approx \frac{1}{c}\int_0^1 \left(C_{p_l} - C_{p_u}\right) d\left(x\right)$$

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

Prandtl-Glauert formula (M<0.7)

Total drag and wing lift gradient

$$C_D = C_{D_{profile}} + \frac{C_L^2}{\pi A e}$$

$$C_{D_{profile}} = C_{D_f} + C_{D_{pressure}} + C_{D_w}$$

$$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi A e_1}}$$

Boundary layer thickness : $\boxed{\delta = \frac{5.2\ x}{\sqrt{Re_x}}}$

Total friction coefficient : $C_f = \frac{1.328}{\sqrt{Re_L}}$

Laminar boundary layer

Boundary layer thickness : $\boxed{\delta = \frac{0.37x}{Re_x^{0.2}}}$

Total friction coefficient : $C_f = \frac{0.074}{Re_L^{0.2}}$

Turbulent boundary layer

Problems

1

- A Boeing 727-300 flies at standard atmosferic sea level conditions.

  From this aircraft we know:

- The mass	: 68.000 kg
- The maximum lift coefficient (flaps in)	: 1.2
- The reference wind area	: 149 m^2
- The span	: 34.8 m
- The Oswald factor	: 0.82
- The zero-lift drag coefficient	: 0.0182
- The gravity constant	: 9.8 m/s^2

- a) Calculate the stall speed V_{stall} associated with the maximum lift coefficient
- b) Calculate the total drag force (N) of this aircraft at the stall speed
- c) What is the absolute value of the static pressure in the stagnation point on the wing leading edge.
- d) Calculate the speed of the aircraft at a Mach number of 0.3

2

Consider an isentropic flow over a wing airfoil. The pressure, speed and density of the free flowing air are: $1.013 \cdot 10^5$ N/m2, 500 km/hour, 1.225 kg/m3. In a certain point A on the wing the pressure is $7.167 \cdot 10^4$ N/m2.

- Calculate the speed in A.
- Calculate the Mach number in A.

3

Consider an airplane flying at a standard altitude of 5 km with a velocity of 270 m/s. At a point on the wing the velocity is 330 m/s. To=255.7 K, Po=5.405×10^4 N/m2, Cp=1008 J/kgK

- Calculate the pressure in this point