Introduction to Aerospace Engineering

Lecture slides

Introduction Aerospace Engineering
Flight Mechanics

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1.& 2.

Flight Mechanics

Question

A Boeing 747 runs out of fuel at 10 km altitude

Suddenly all engines stop…

How far will this aircraft be able to glide?

a) 180 [m]
b) 1800 [m]
c) 18000 [m]
d) 180000 [m]


msby
Norden
Aurich
Wilhelmshaven
Groningen
Oldenburg
(Oldenburg)
Emmen
Norwich
orough
Lowestoft
Alkmaar
Zwolle
Lelystad
Amsterdam
Almere
Deventer
Cambridge
Den Haag
Utrecht
Apeldoorn
Enschede
Osnabrück
Bielefeld
Ipswich
Colchester
Westland
Zoetermeer
Arnhem
Münster
Chelmsford
Rotterdam
Nijmegen
's-Hertogenbosch
Basildon
Breda
Herne
Hamm
Southend-on-sea
Tilburg
Duisburg
Dortmund
Gillingham
Eindhoven
Essen
Brugge
Antwerpen
Wuppertal
Dunkerque
Gent
Düsseldorf
Solingen
Calais
Geleen
Köln
Heerlen
Siegen
hton
Brussel
Bruxelles
Aachen
Roubaix
Bonn
Lille
Liège
Namur
Charleroi
Koblenz
50 km
50 mi
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Flight mechanics

Key questions

- What is the performance of a given aircraft; i.e. how far, high, fast, slow can it fly?
- How long can an aircraft remain airborne following an engine failure and how far can it glide?
- How is aircraft morphology related to aircraft performance?

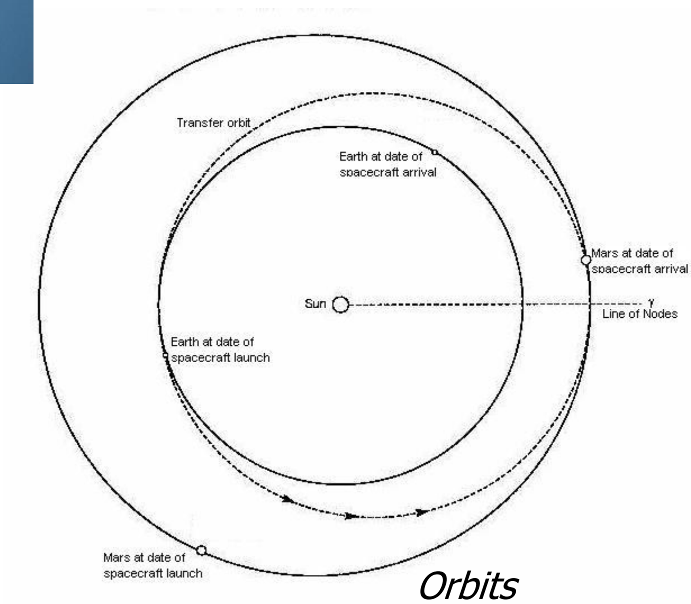
Flight Mechanics

The performance of the complete vehicle is analyzed

Launchers

Gliding flight

Low speed

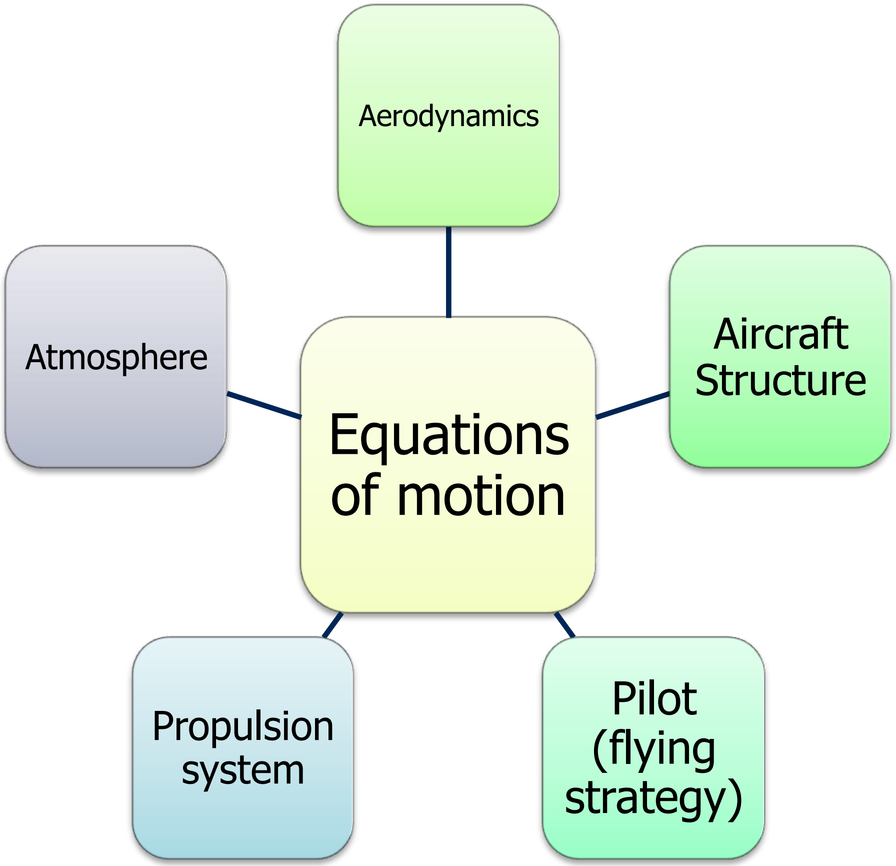


Orbits

High altitude

Steep climbs

Flight Mechanics



Contents

Contents

Flight mechanics

Hours 1 & 2 :	Introduction, general equations of motion
Hours 3 & 4 :	Horizontal flight performance
Hours 5 & 6 :	Climbing and descending flight
Hours 7 & 8 :	Flight envelope
Hours 9 & 10:	Example questions and solutions

What do you need to learn?

- **Most important!!! Lecture sheets (blackboard)**
- Introduction to flight: paragraphs; 6.1, 6.2, 9.1 9.2, 9.4, 9.6
- Better book to consult for more information: Ruijgrok, "*Elements of Airplane Performance*" (only 14 euro's at VSV)
- Practice questions (blackboard)

Contents

Equations of motion

Overview

- Newton's laws
- Coordinate systems
- Assumptions
- Equations of motion

Equations of motion

Newton's laws

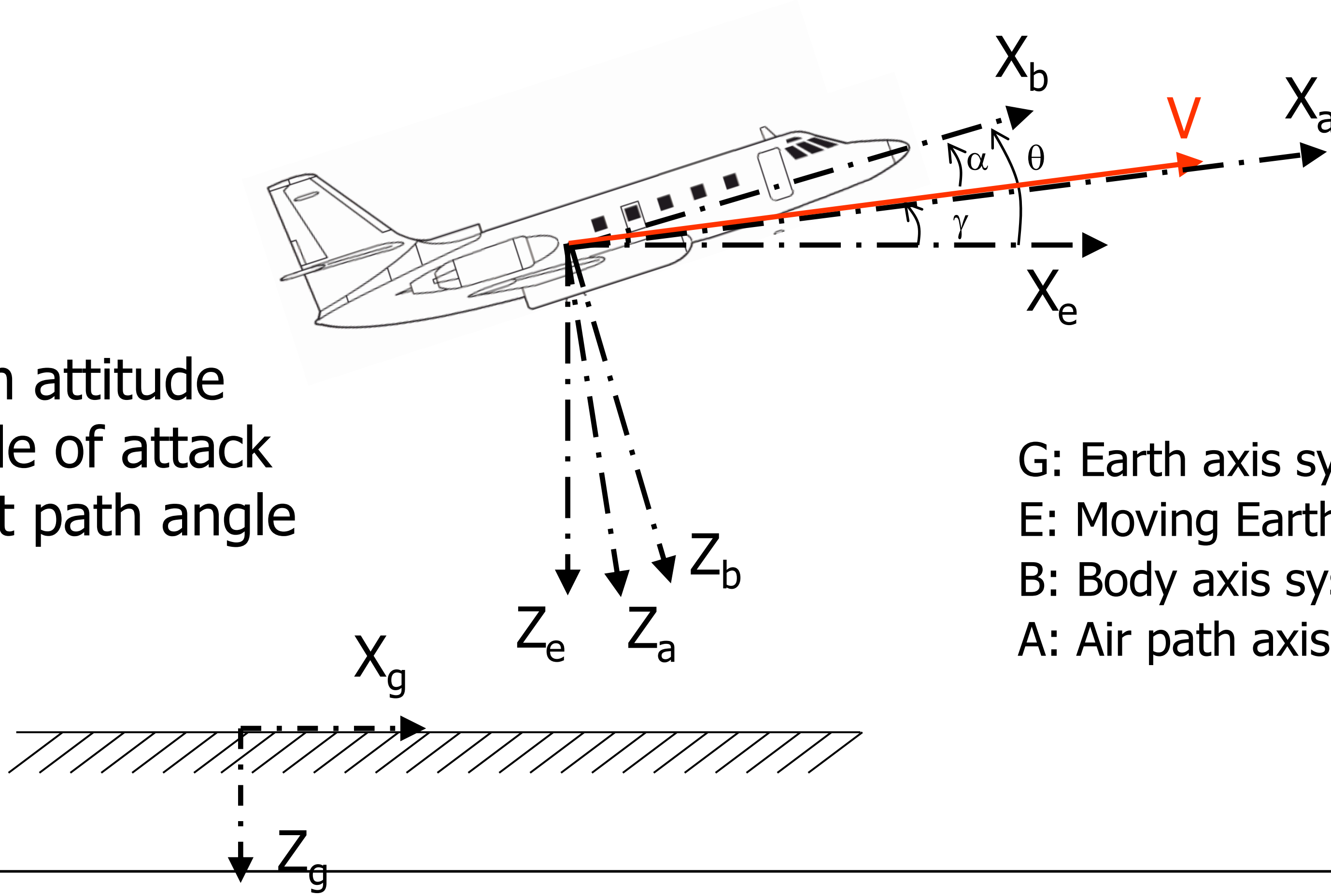
Newton's laws only hold with respect to a frame of reference which is in **absolute rest**. This is called an **inertial frame of reference**

Coordinate systems **translating uniformly** to the frame of reference in absolute rest are also **inertial frames of reference**

A **rotating** frame of reference is **not** an inertial frame of reference

Equations of motion

Coordinate systems

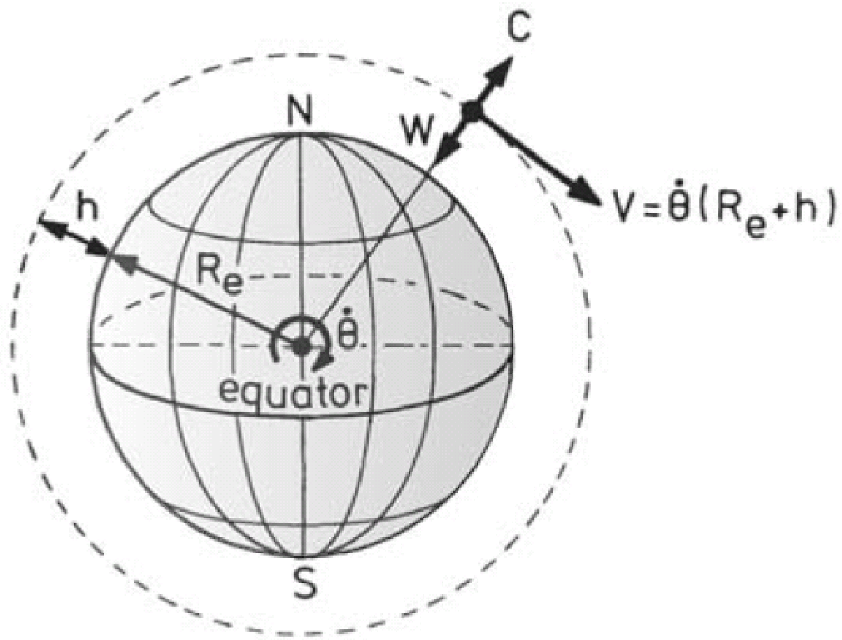


Angles:
θ - pitch attitude
α - angle of attack
γ - flight path angle

G: Earth axis system
E: Moving Earth axis system
B: Body axis system
A: Air path axis system

Assumptions

Assumption 1: the earth is flat



Centrifugal force

$$C = \frac{W}{g}\frac{V^2}{R_e + h}$$

$$\frac{C}{W} = \frac{V^2}{(R_e + h)g}$$

Example

$V = 100$ [m/s]

$R_e = 6371$ [km]

$g = 9.80665$ [m/s^2]

$h = 0$ [m]

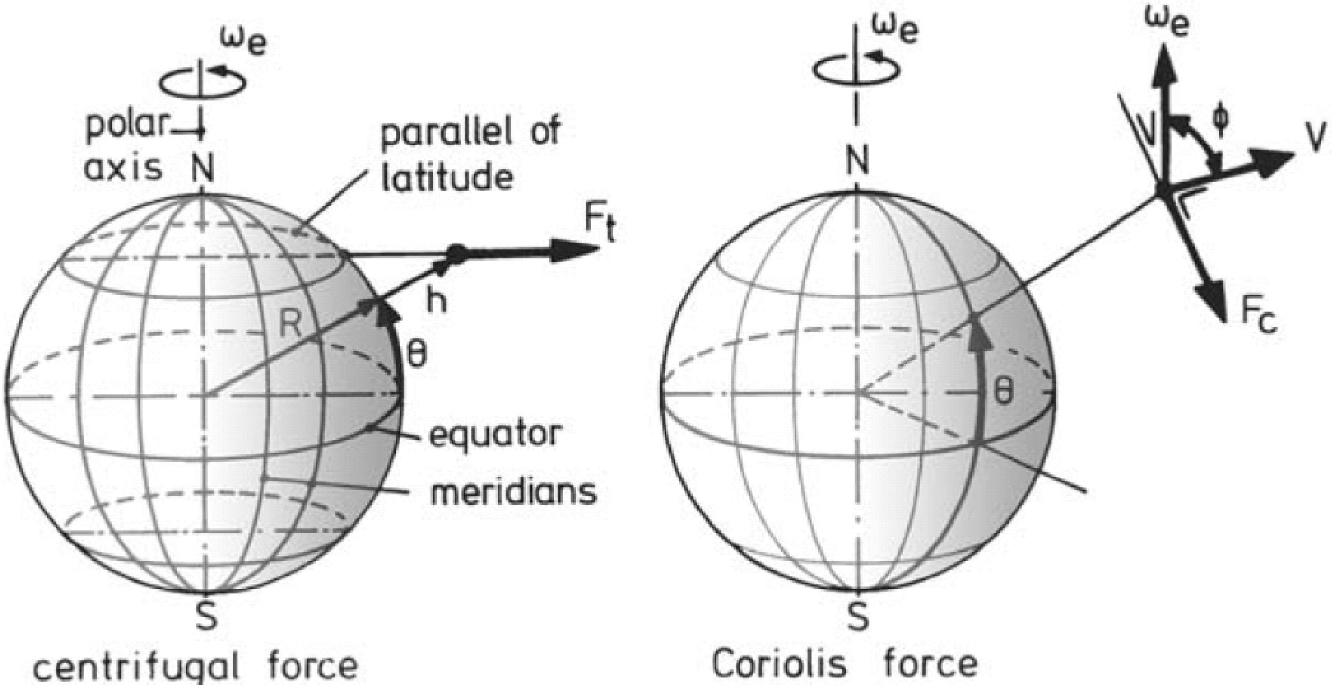
$$\frac{C}{W} = \frac{100^2}{(6371000 + 0)\,9.80665} = 0.00016$$

(0.016%)

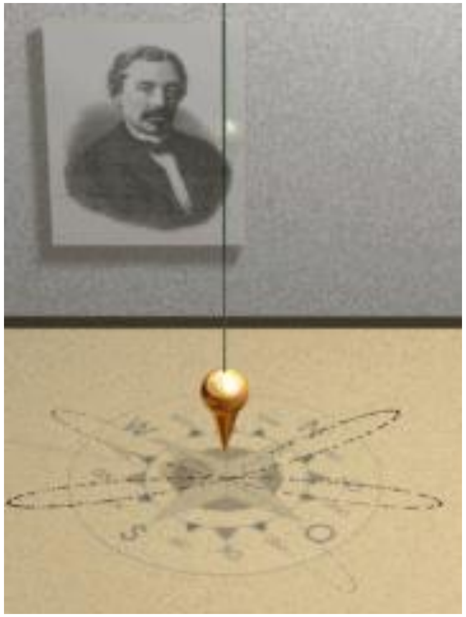
√ Valid assumption

Assumptions

Assumption 2: the earth is non-rotating



Foucault - story



Assumptions

Assumption 3: Gravity is constant

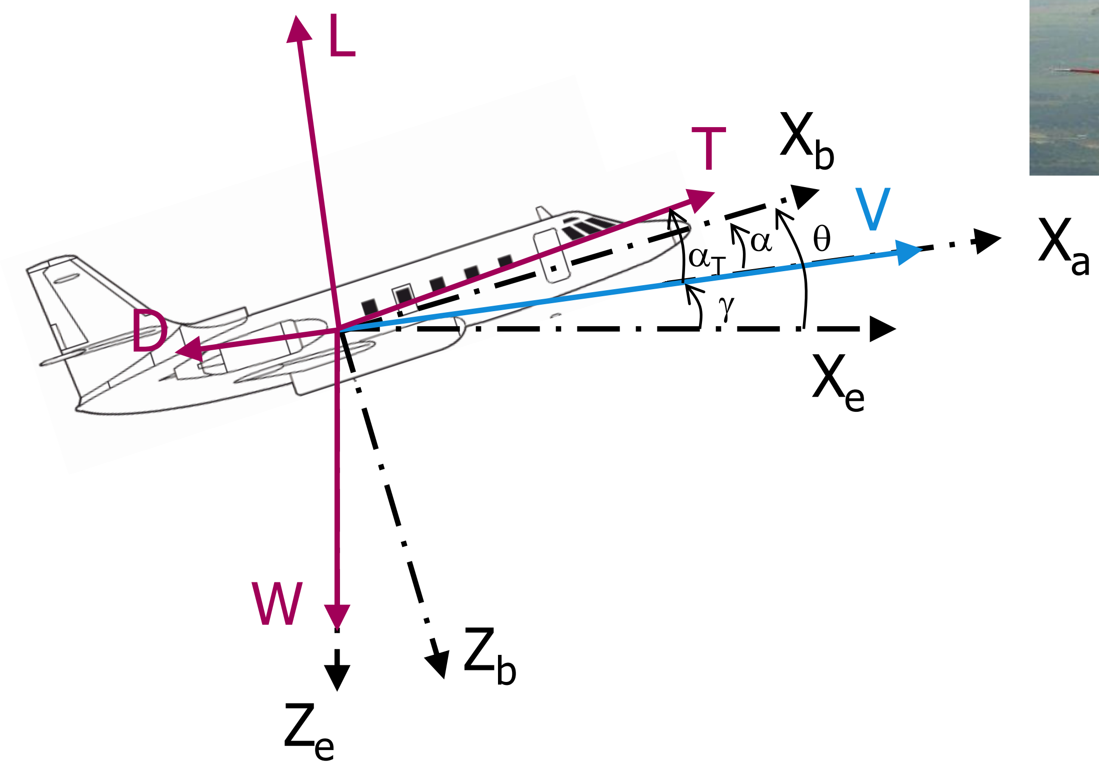
$$F = \frac{\mu M_1 M_2}{R^2}$$ (Newton's law of gravitation)

At maximum altitude for atmospheric flight (60 – 80 km), g is very close to g at sea level

Note: these assumptions are fine for flight mechanics but not for orbital mechanics

Equations of motion

Free Body Diagram - Forces

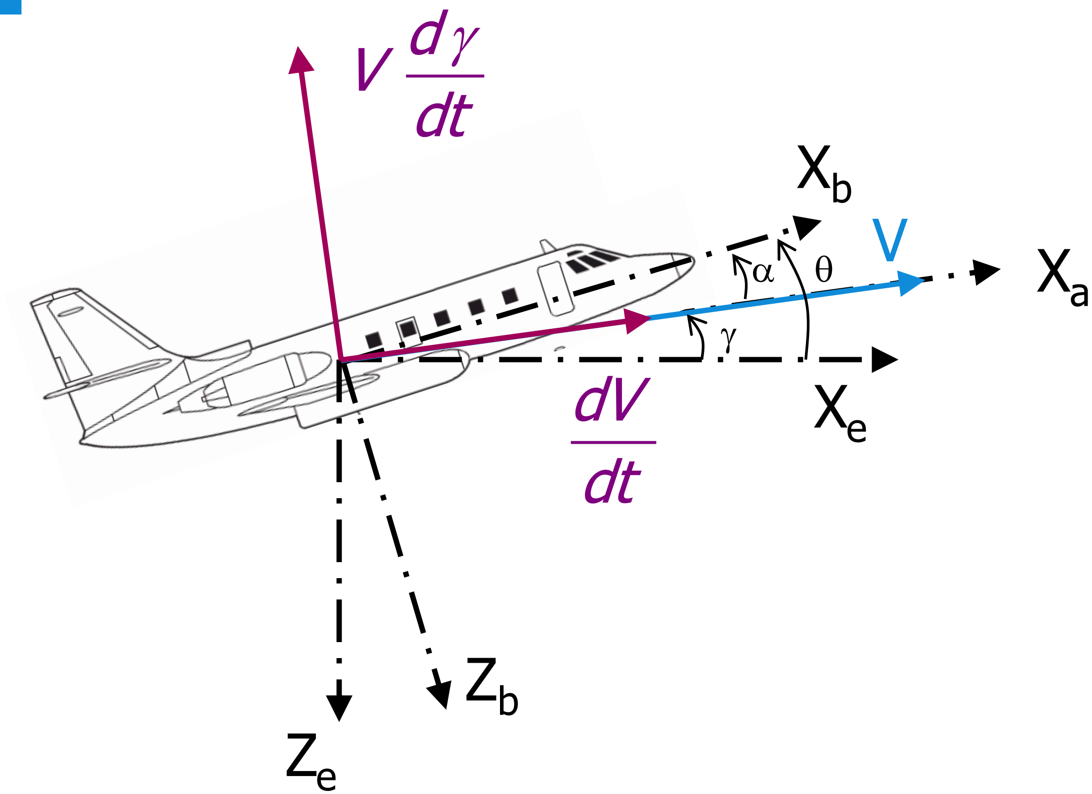


<u>Important:</u>

- Lift vector perpendicular to airspeed
- Drag parallel to airspeed
- Thrust not necessarily in direction of X_b

Equations of motion

Kinetic diagram - accelerations



Equations of motion

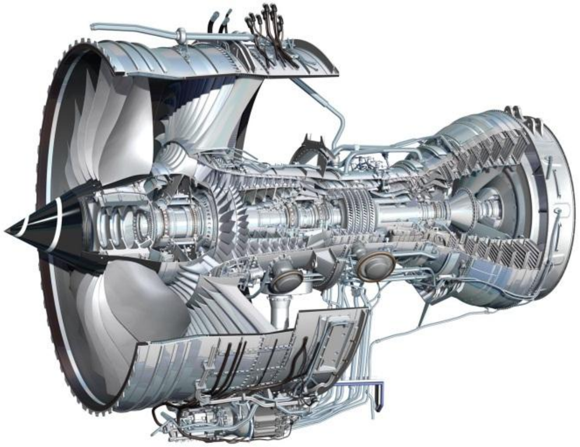
$$\sum F_{//V} : \frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\sum F_{\perp V} : \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

Contents

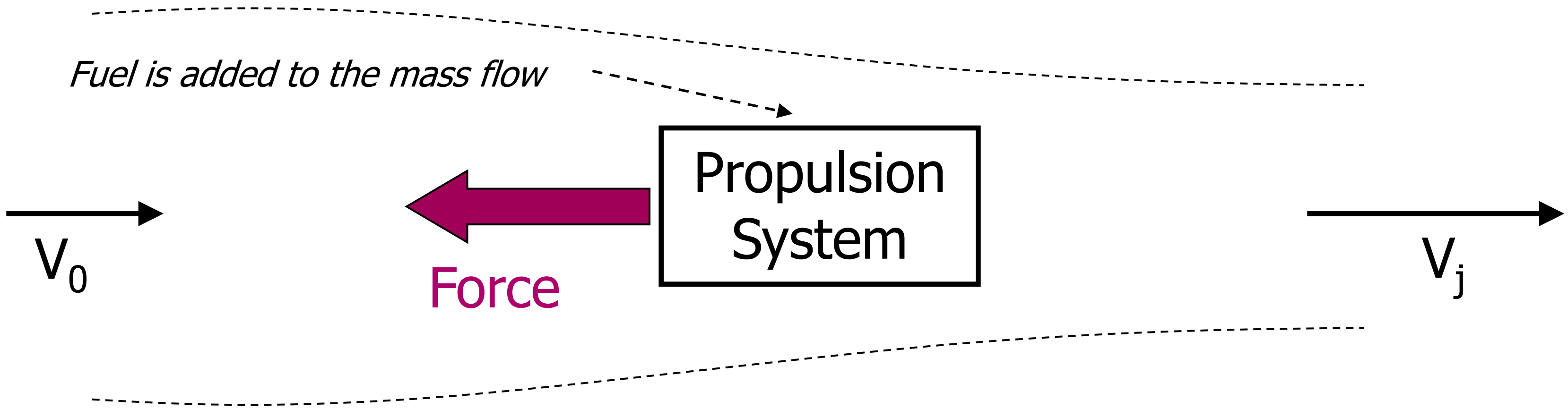
Propulsion
Overview

1. Fundamental equations and definitions
2. Propeller or Jet engine?
3. Pure Jet engine
4. Propeller



Propulsion

Fundamental equations



Momentum equation

$$F = \Delta I = I_{out} - I_{in}$$

$$T = \left(m + m_f\right)V_j - mV_0$$

$$T \approx m\left(V_j - V_0\right)$$

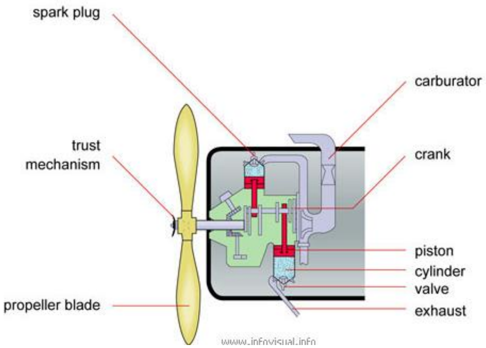
Conclusion

A force is created by acceleration of mass. Two fundamental options:

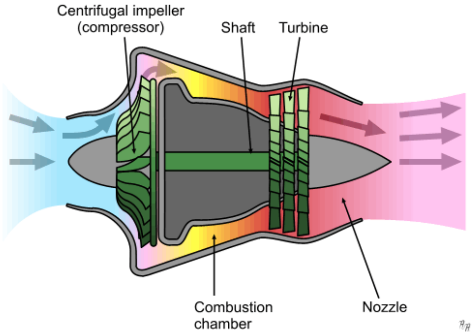
1. Give a small amount of mass a large acceleration
2. Give a large amount of mass a small acceleration

Fundamental equations

- A small acceleration to a large mass:

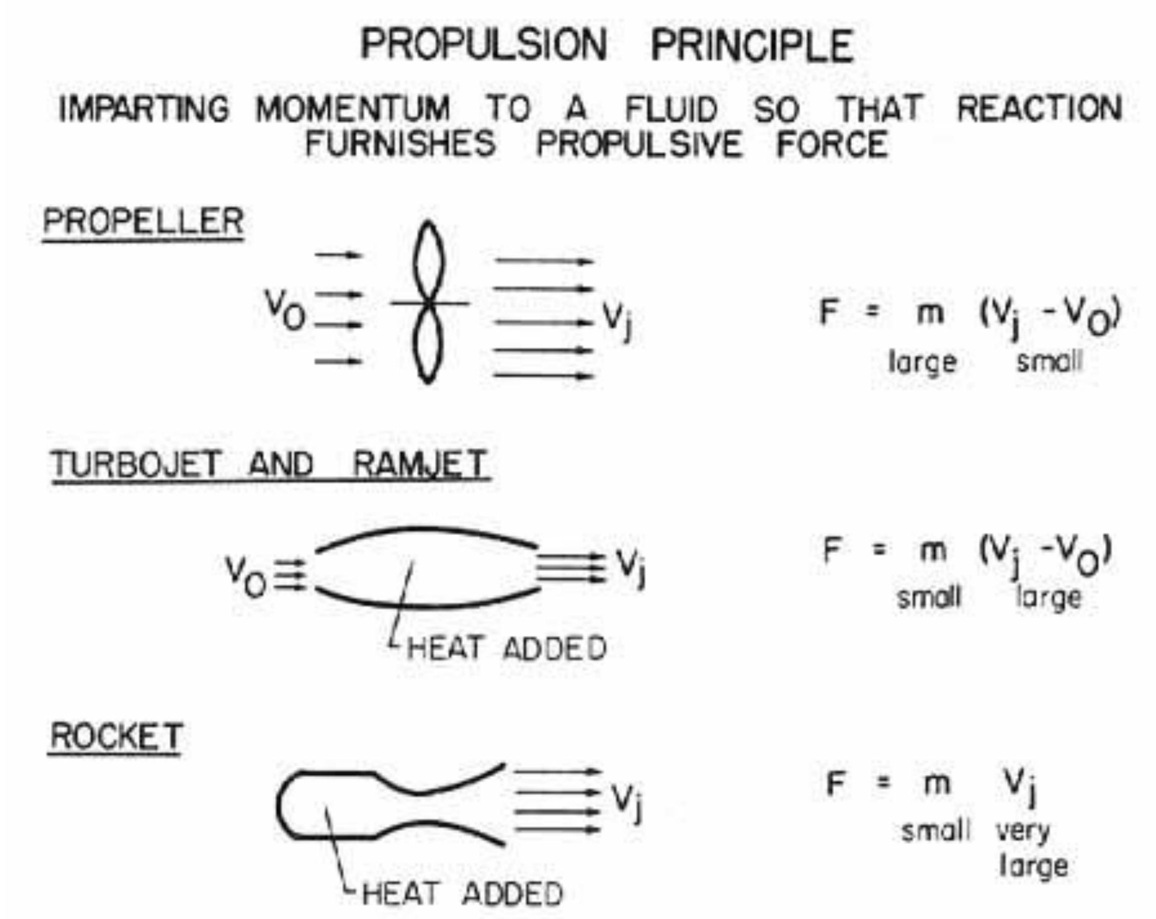


- A large acceleration to a small mass:



- The mass can be taken from the surrounding air (airbreathing engine) but it can also be taken along

Fundamental equations



Fundamental equations

- **Which propulsion type is the best?**
  - Propeller
  - Jet
  - Turbofan
  - Ramjet
  - Rocket

- Before we can answer this question we must define what is efficient

Useful definitions

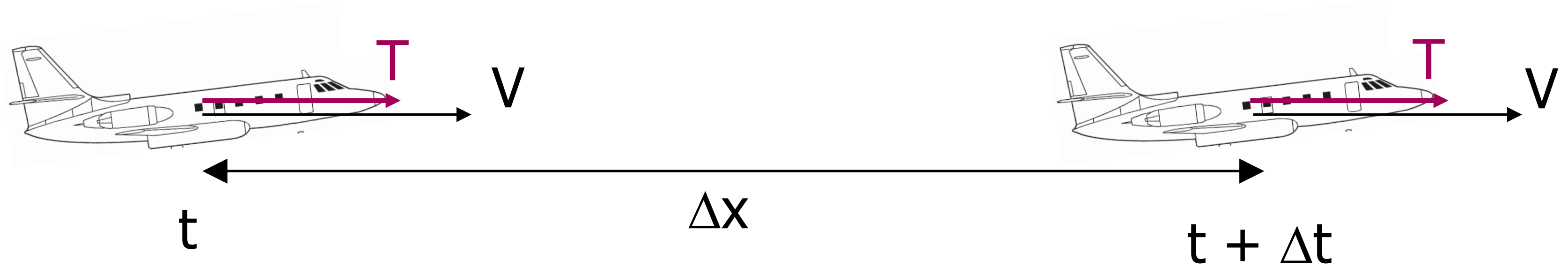
- Power Available
- Jet Power
- Thermal Power

↓

- Total efficiency
- Propulsive efficiency
- Thermal efficiency

Useful definitions

Power available



Work (energy)

$$W = F\,\Delta x$$

$$W = T\left(x_2 - x_1\right)$$

Power (energy per second)

$$P = \frac{W}{\Delta t}$$

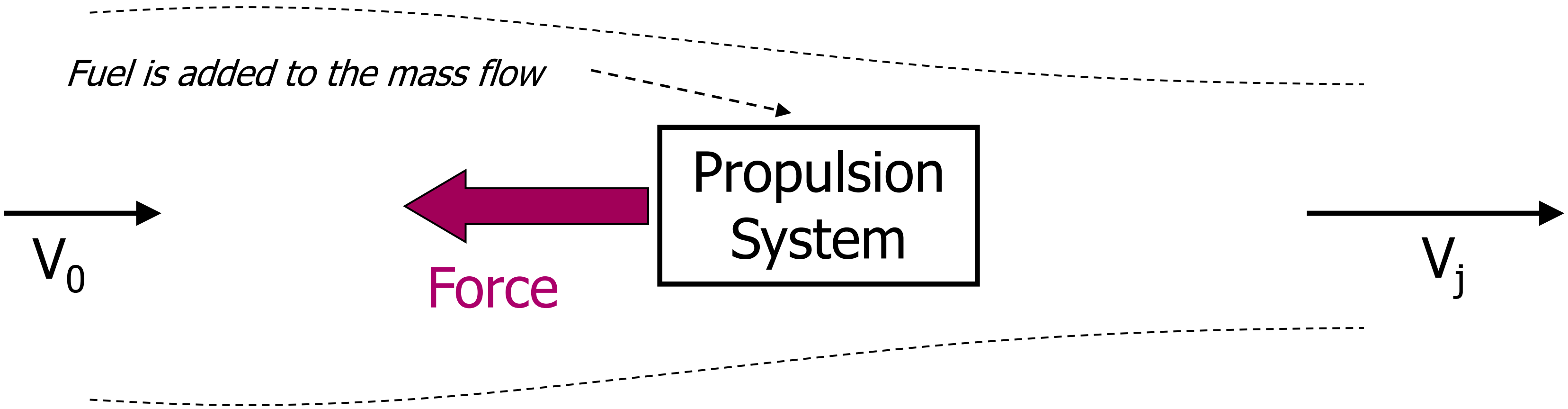
$$P = \frac{T\left(x_2 - x_1\right)}{\Delta t} = T\,\frac{\Delta x}{\Delta t} = TV$$

$$\boxed{P_a = TV}$$

Useful definitions

Jet power

Jet power is defined as the *increase in kinetic energy of the flow*



$$P_j = \tfrac{1}{2} m V_j^2 - \tfrac{1}{2} m V^2$$

Useful definitions

Thermal power

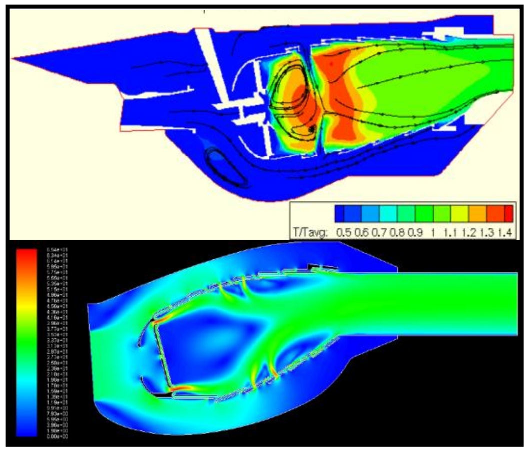
Thermal power (Q) is the heat energy supplied to the process (burning fuel)

$$Q = m_f H$$

$$Q = \frac{FH}{g}$$

$$Q = m q_{in}$$

$$\boxed{Q = \frac{FH}{g}}$$



Variables:
m_f mass flow of fuel [kg/s]
F fuel flow [N/s]
H heating value [J/kg] (constant)
m mass flow of air [kg/s]
q_{in} heat per kg air [J/kg]

Useful definitions

Total Efficiency

- Total efficiency is defined as the ratio of Power available (energy for transportation) over Thermal power (fuel required)

$$\eta_{tot} = \frac{P_a}{Q}$$

$$\eta_{tot} = \frac{P_a}{P_j}\frac{P_j}{Q} = \eta_j \eta_{th}$$

- It is also the multiplication of propulsive efficiency and thermal efficiency

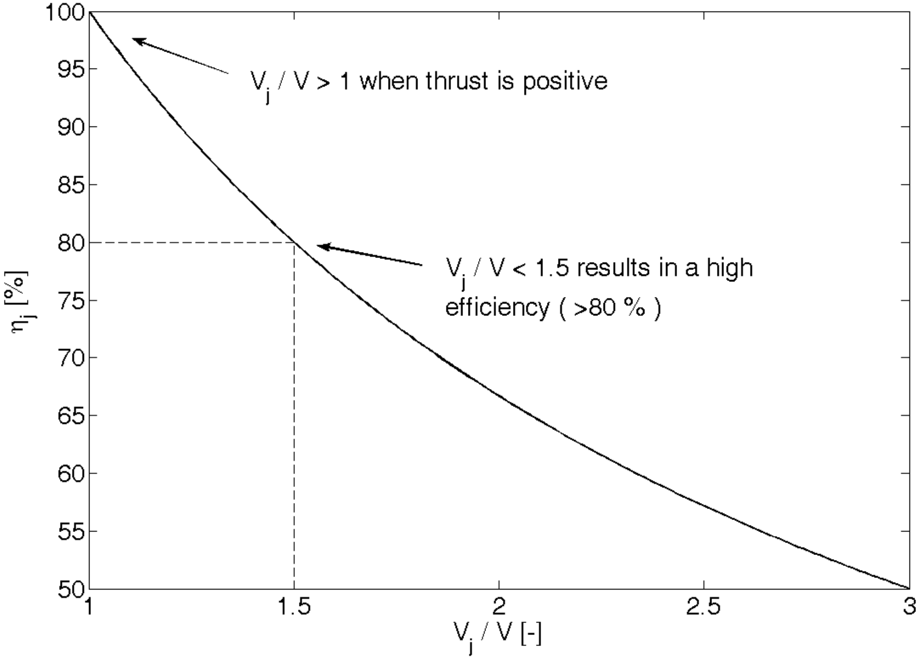
Useful definitions

Propulsive efficiency

$$\eta_j = \frac{P_a}{P_j} = \frac{2}{1 + \frac{V_j}{V}}$$

<u>Conclusions:</u>

- The jet velocity V_j must be larger than V in order to create thrust
- Therefore propulsive efficiency must be smaller than 100%!



Useful definitions

Jet or propeller?

$$T = 100 \text{ [N]}$$

$$T = m\left(V_j - V\right)$$

Option 1:

$$T = 1\left(200 - 100\right) = 100 \text{ [N]}$$

Option 2:

$$T = 1\left(300 - 200\right) = 100 \text{ [N]}$$

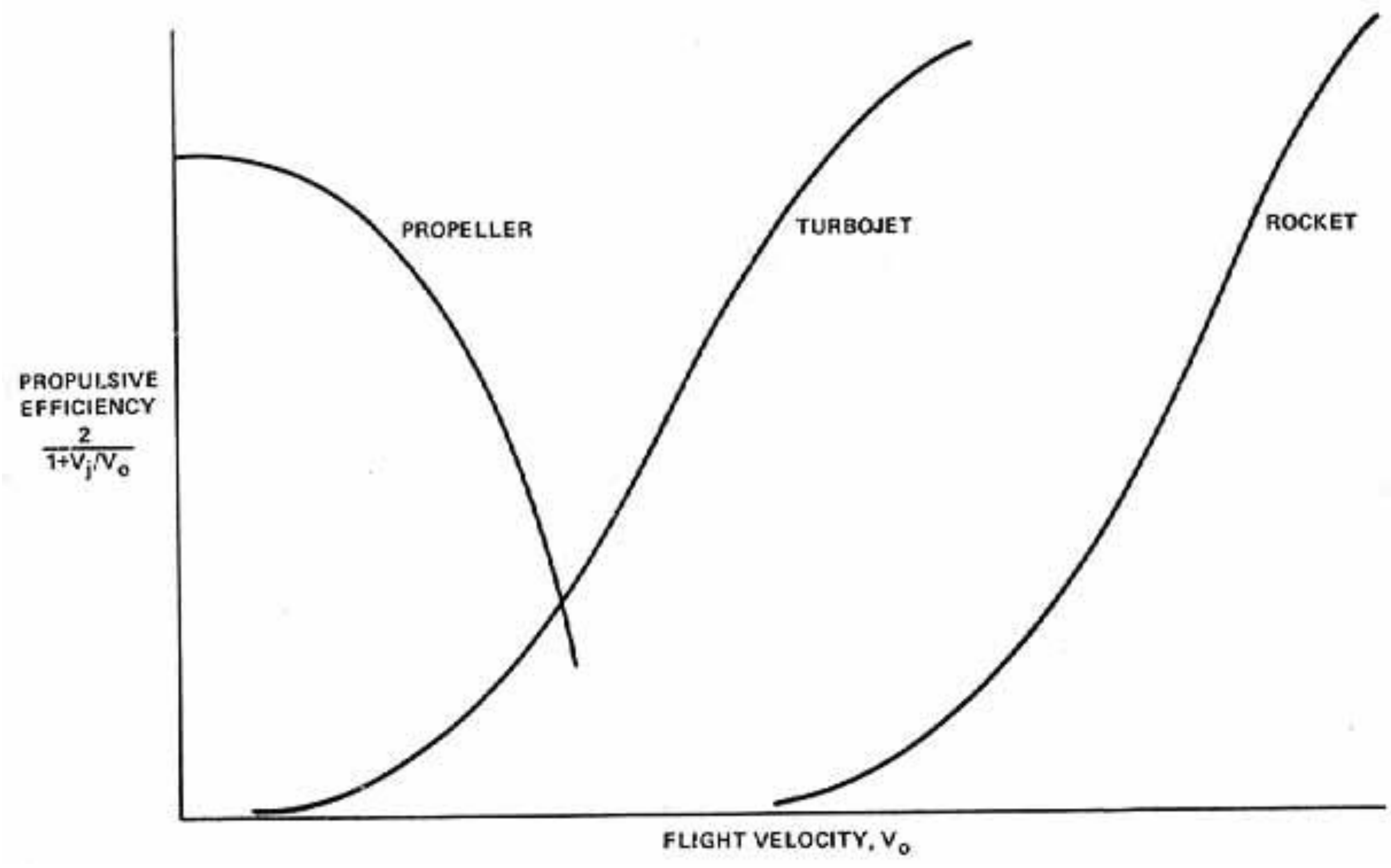
$$\eta_{j,1} = \frac{2}{1 + \frac{V_j}{V}} = \frac{2}{1 + \frac{200}{100}} = 66\%$$

$$\eta_{j,2} = \frac{2}{1 + \frac{V_j}{V}} = \frac{2}{1 + \frac{300}{200}} = 80\%$$

- At large airspeeds it becomes more efficient to give a large acceleration to air
- At low airspeeds it is more efficient to give a large amount of air a small acceleration

Useful definitions

Jet or propeller?



Useful definitions

Summary

Thrust

$$T = m\left(V_j - V\right)$$

Power

$$P_a = TV$$

$$P_j = \tfrac{1}{2} m V_j^2 - \tfrac{1}{2} m V^2$$

$$Q = \frac{FH}{g}$$

Efficiency

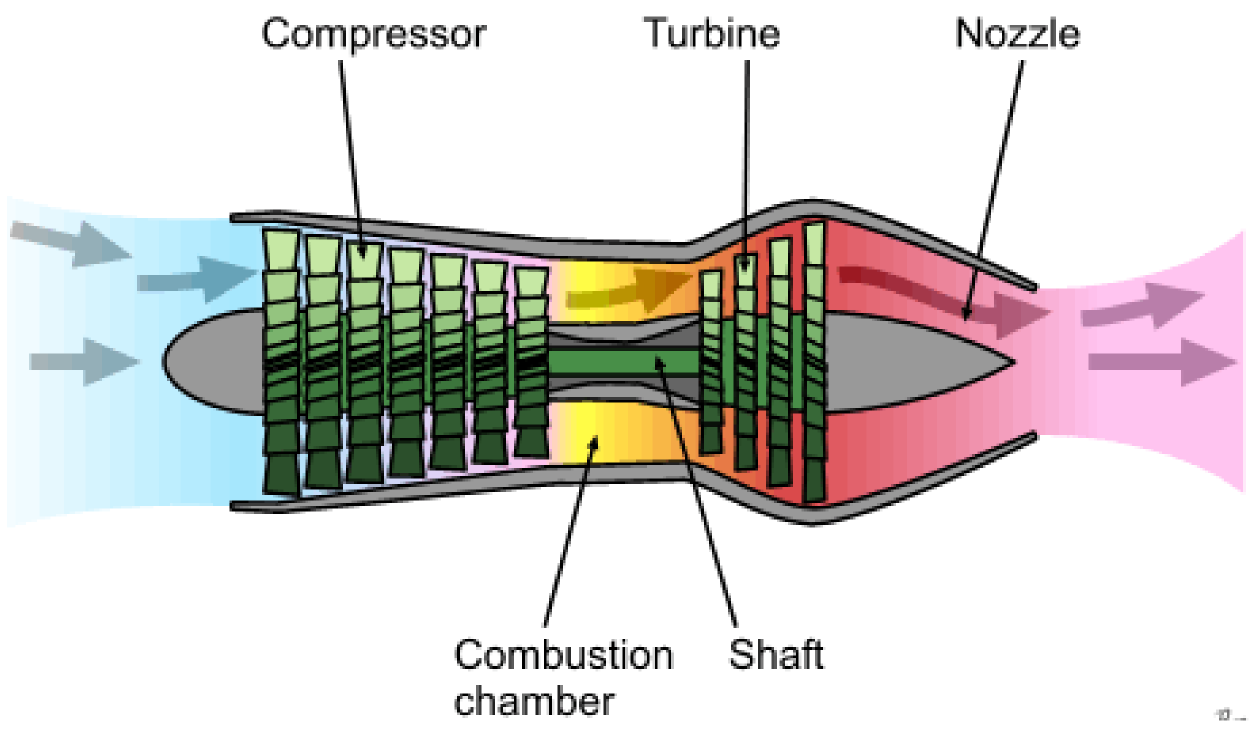
$$\eta_{tot} = \frac{P_a}{Q} = \eta_j \cdot \eta_{th}$$

$$\eta_j = \frac{P_a}{P_j} = \frac{2}{1 + \frac{V_j}{V}}$$

$$\eta_{th} = \frac{P_j}{Q}$$

Try to understand what these equations mean. Then you will be able to derive them. Do not memorize them

Working principle jet engine



See also, Physics 1 – lecture 7 (Chapter 9 of Cengel & Boles)

Working principle jet engine

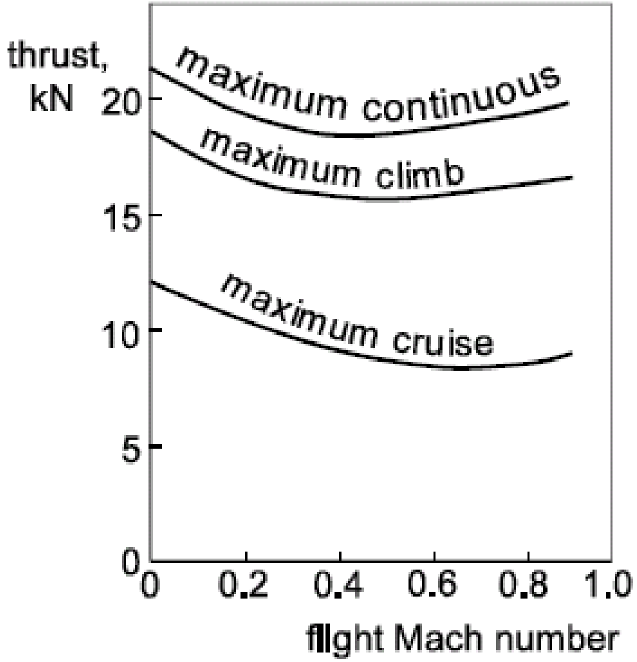
- The compression at the intake depends on the flight velocity (kinetic energy)
- The compression ratio (p_2 / p_1) at the intake is relatively **low** (in the order of 1 ~ 2)
- Compression ratio in the compressor is relatively **high** (in the order of 30 – 40)
- Jet velocity is very high

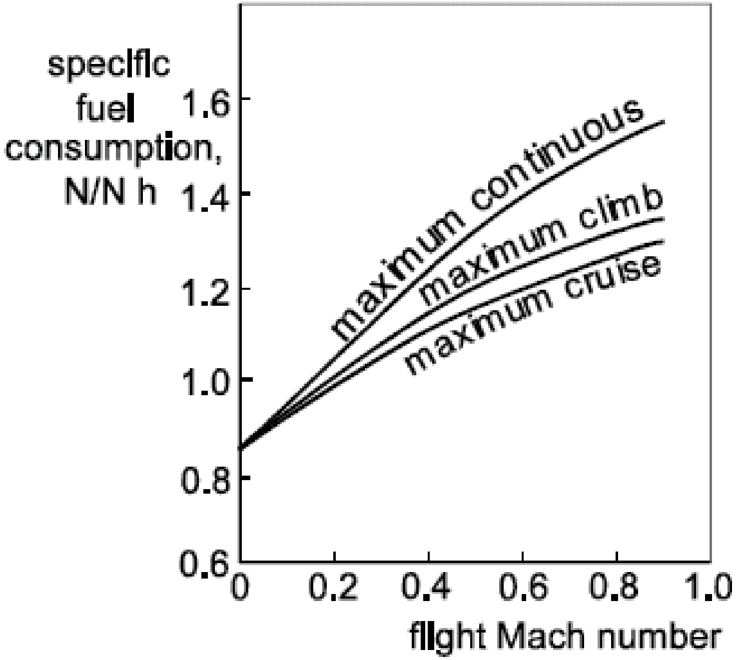
$$T = m\left(V_j - V_0\right)$$

Propulsive force

Typical analytical assumption – Jet

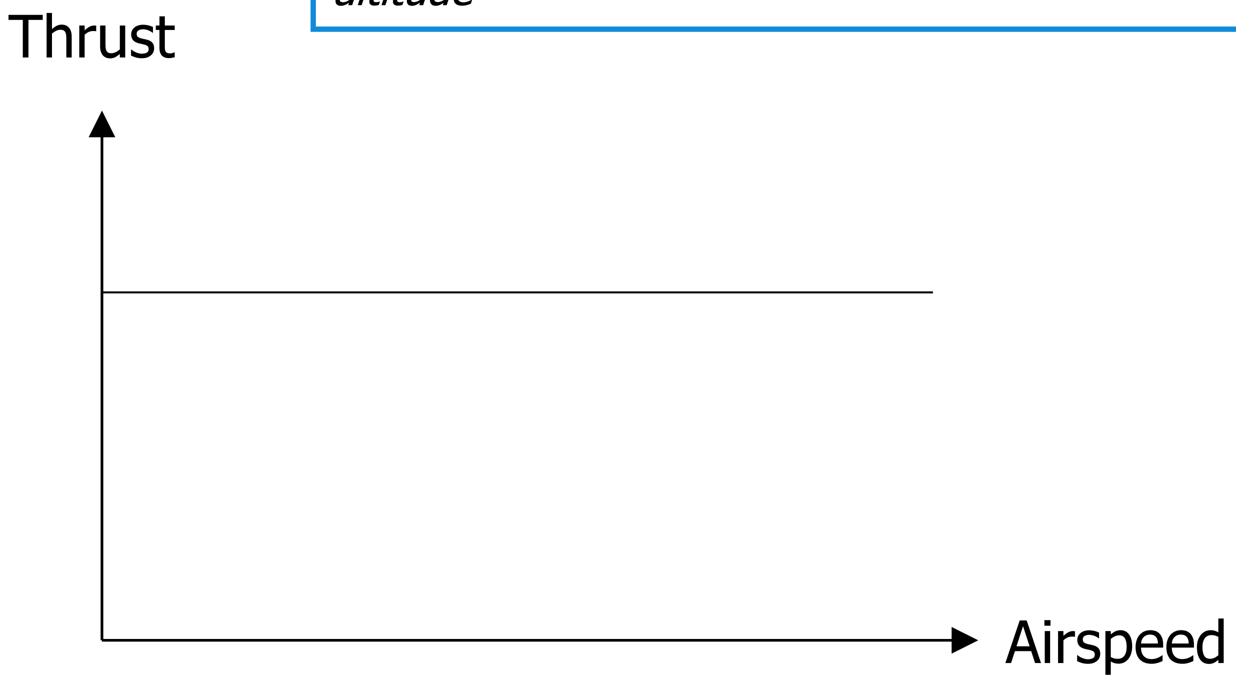
Thrust is assumed to be independent of airspeed





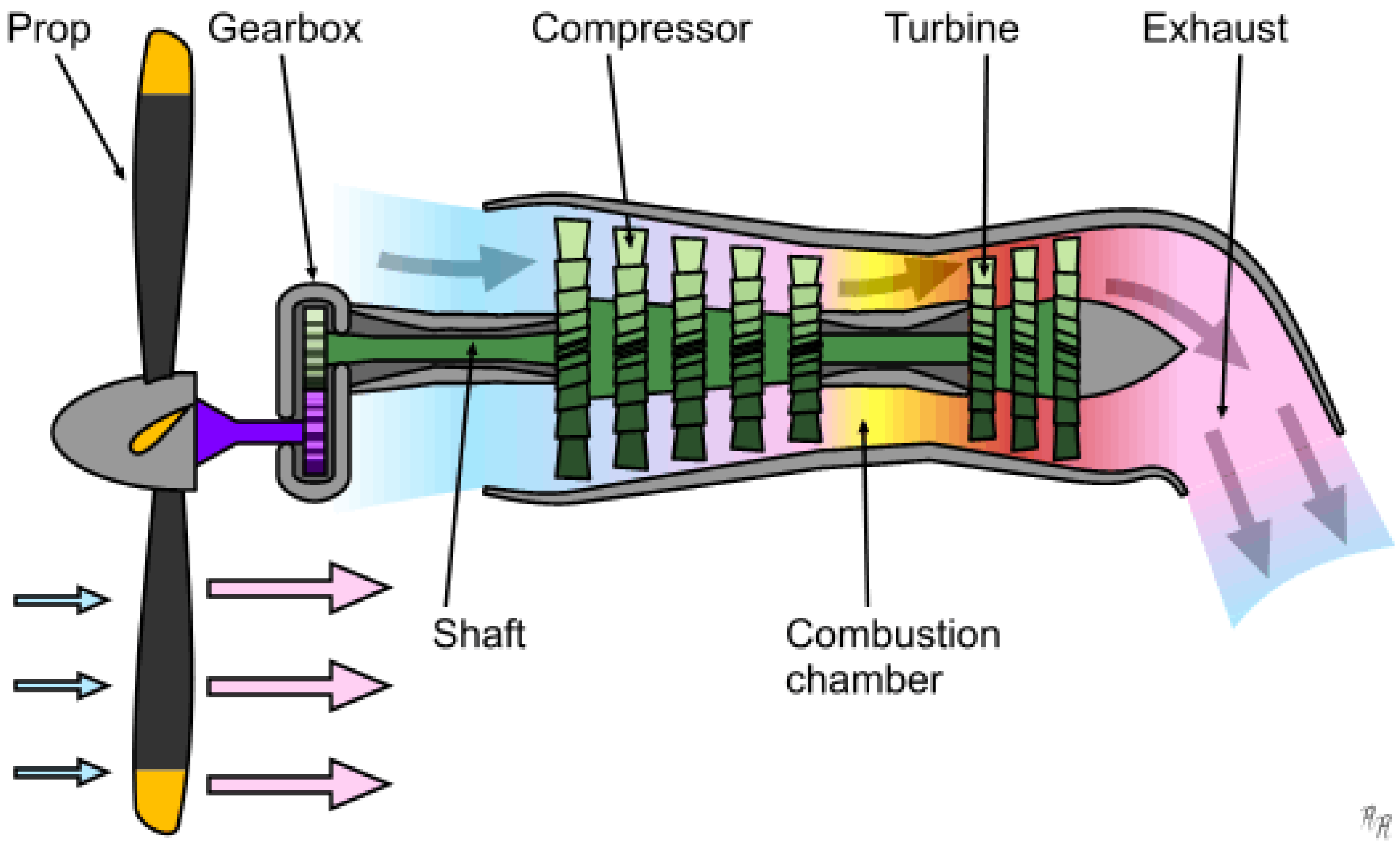
Jet engine - simplified

*For basic flight mechanics applications, **thrust** of a **turbojet** can be assumed to be **constant with airspeed** for a given flight altitude*



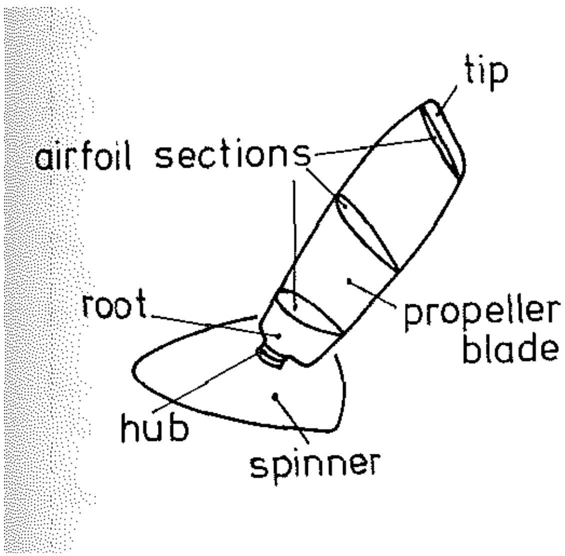
Working principle turboprop

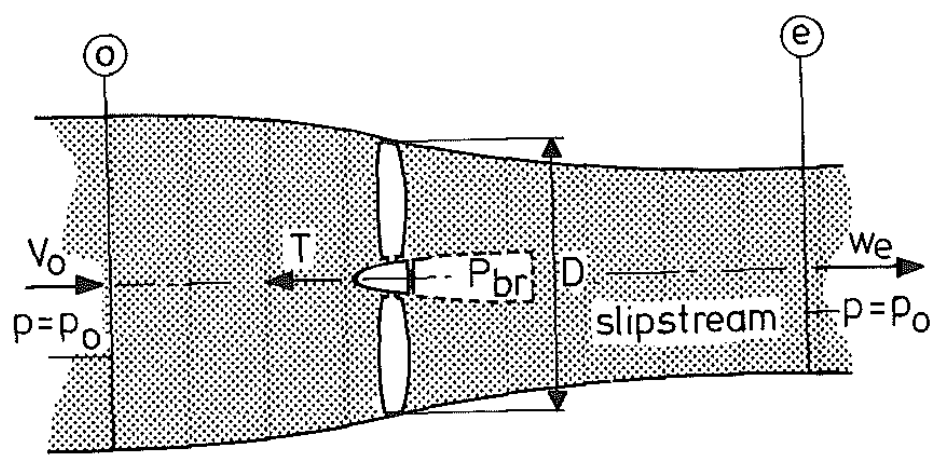
Schematic



Working principle turboprop

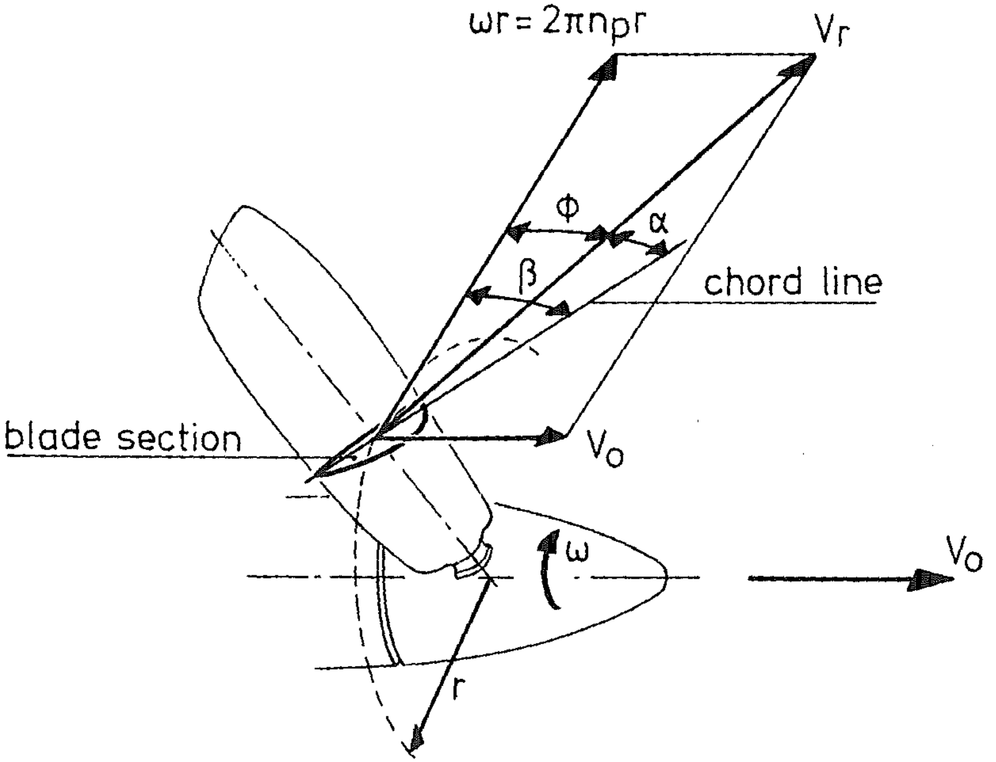
- Shaft power (P_{br}) is provided by gasturbine
- Propeller accelerates air





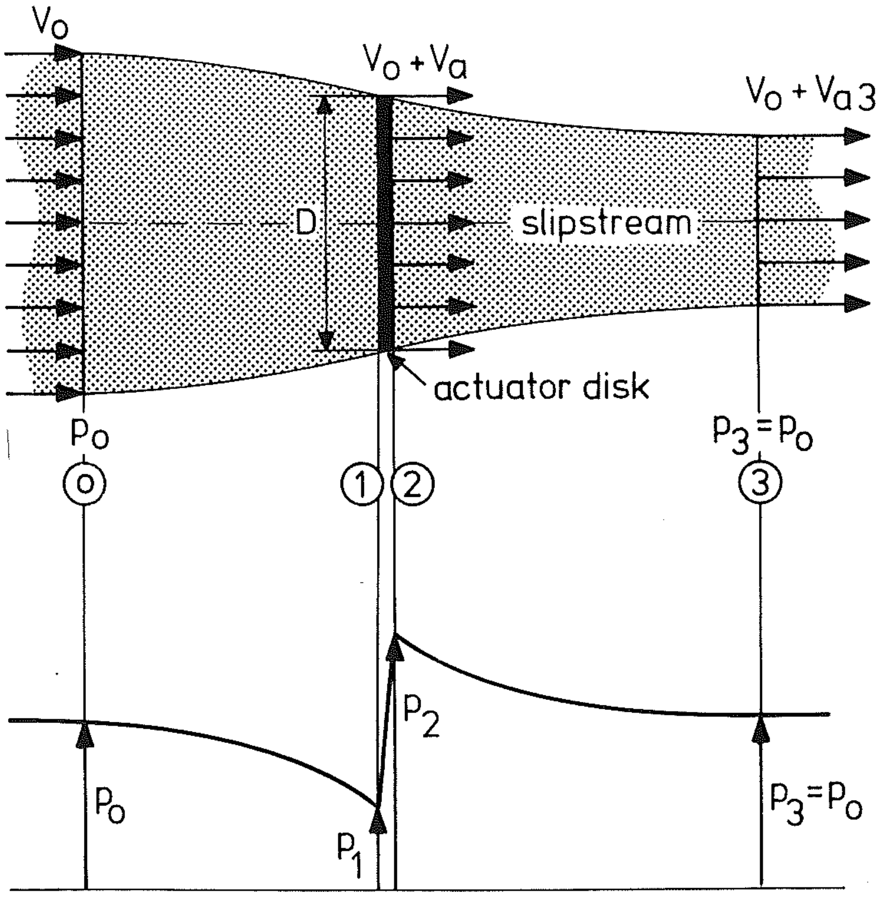
Working principle propeller

Motion of a propeller blade section



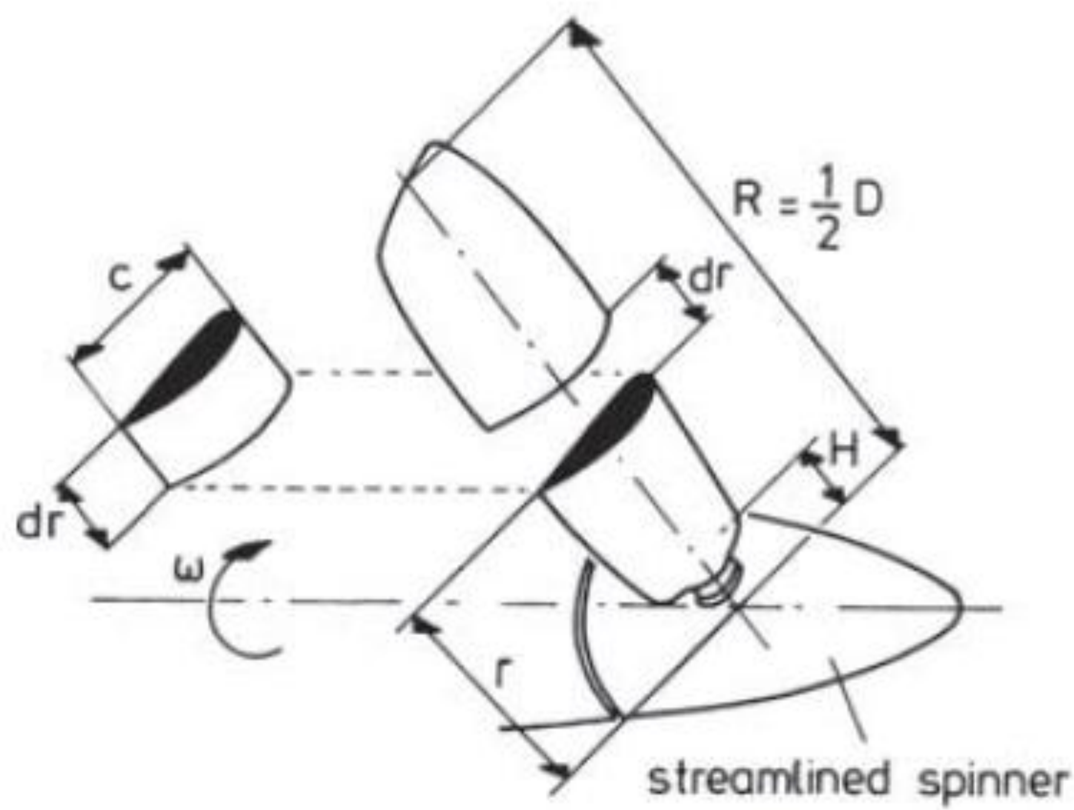
Working principle turboprop

Induced velocity



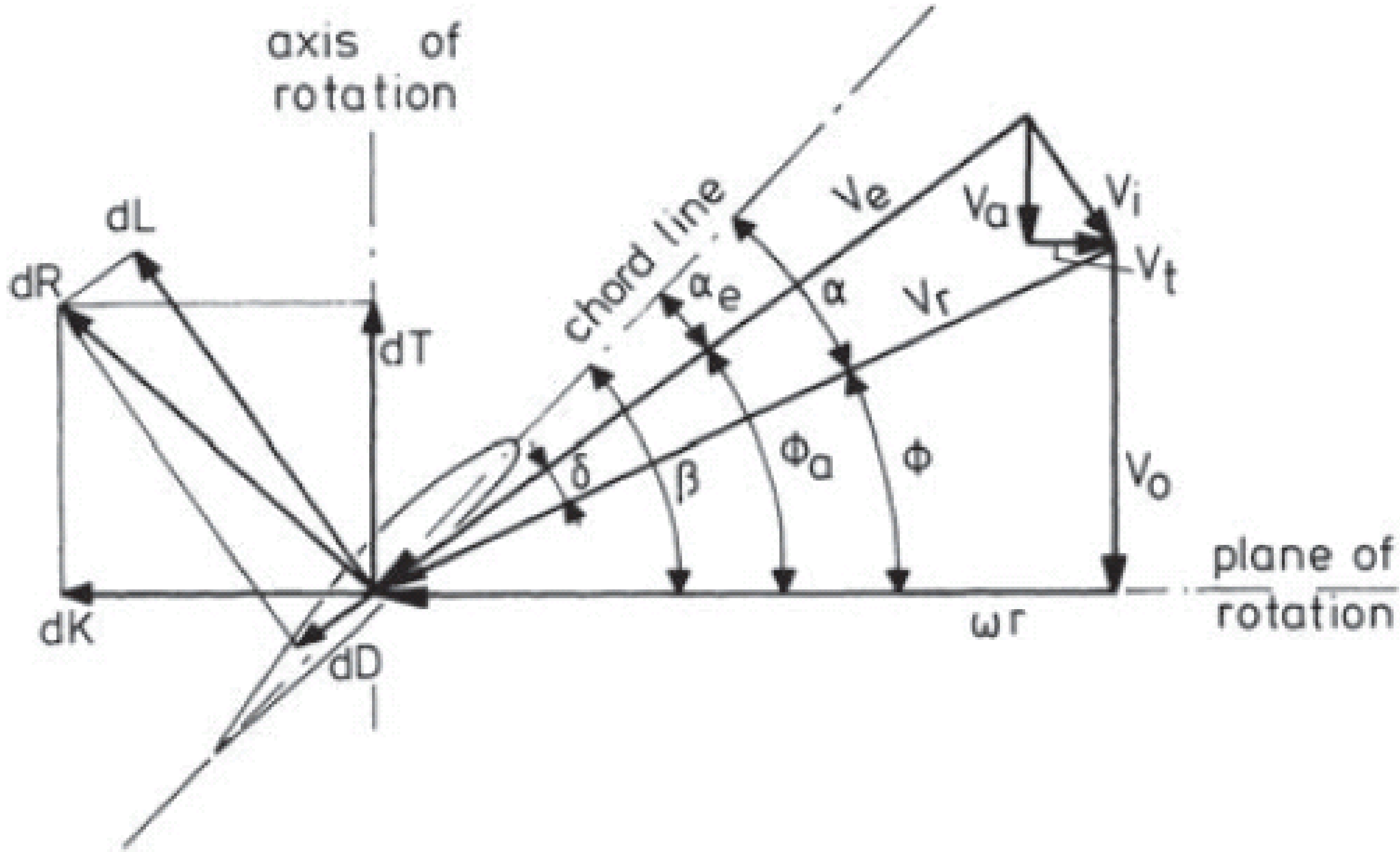
Working principle propeller

Actuator disk theory – blade element theory



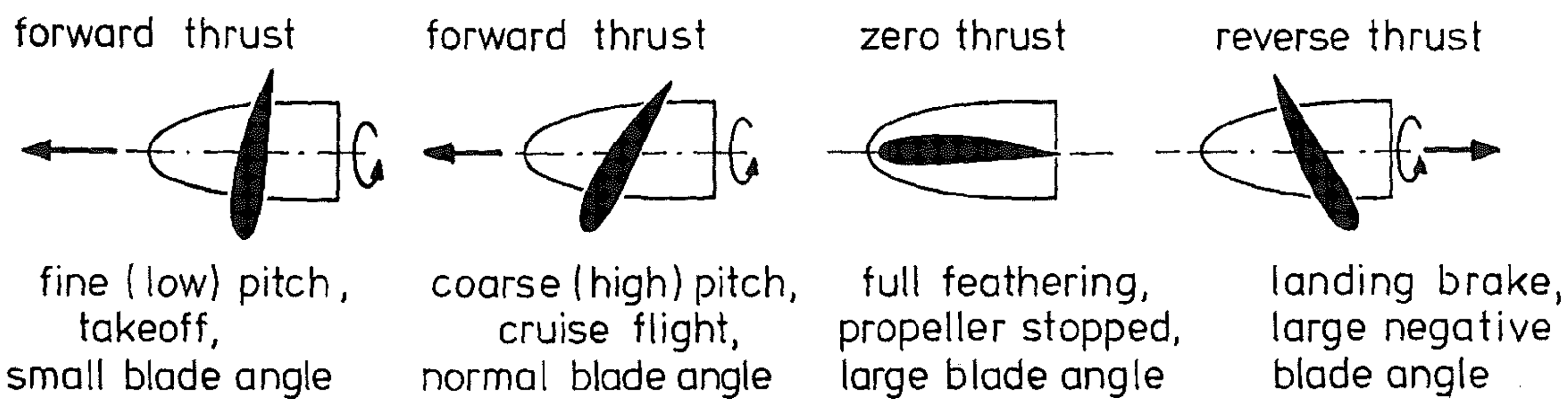
Working principle propeller

Local velocities at a blade section



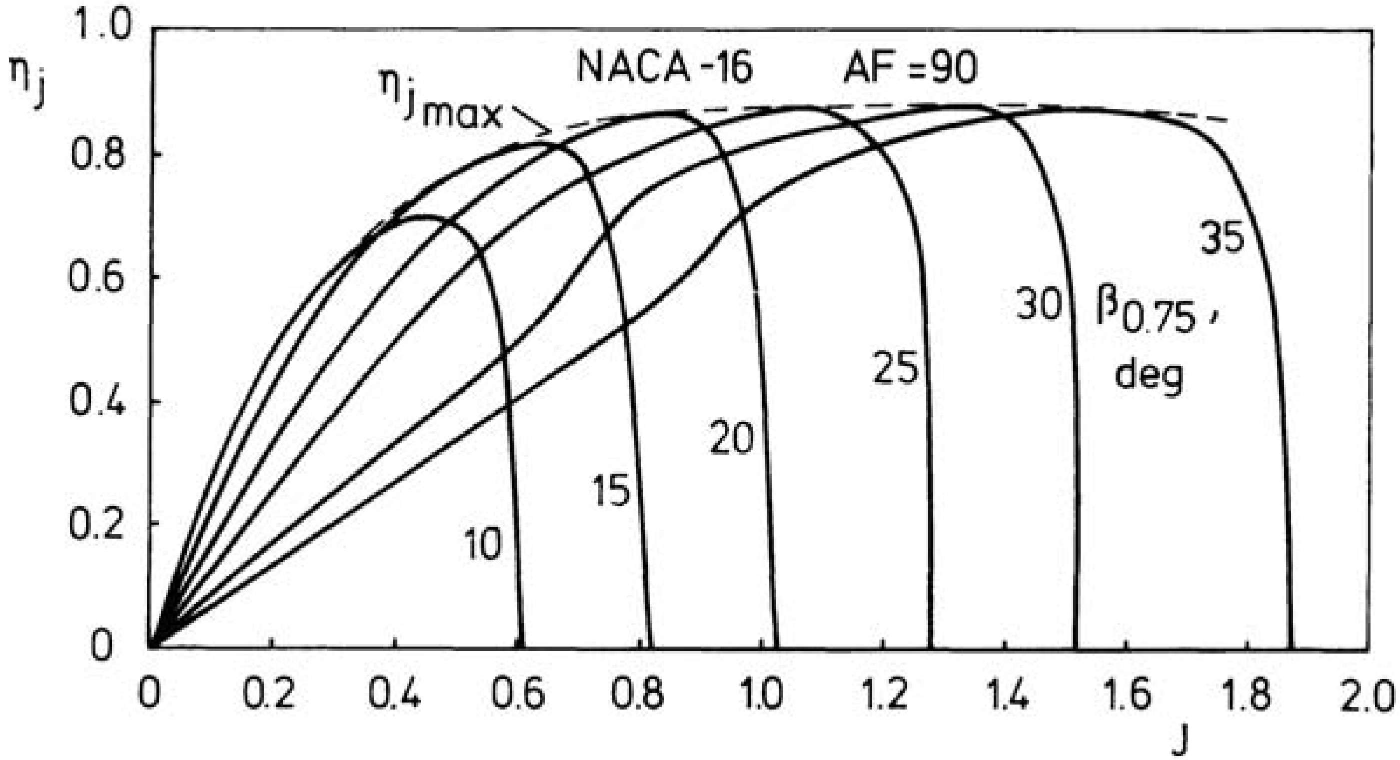
Working principle propeller

Variable blade pitch



Working principle propeller

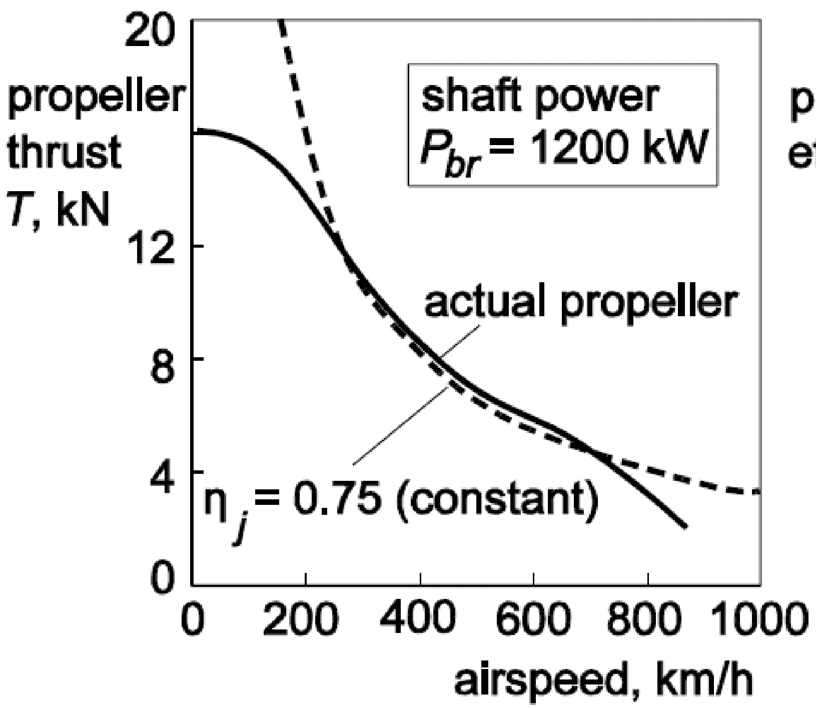
Variable blade pitch

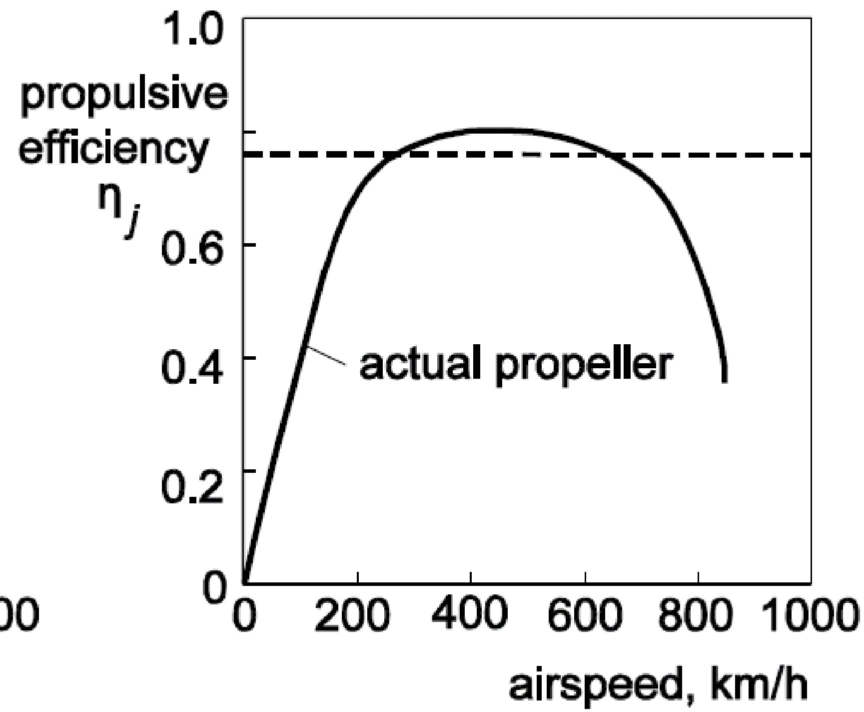


Propulsive force

Typical analytical assumption – Propeller

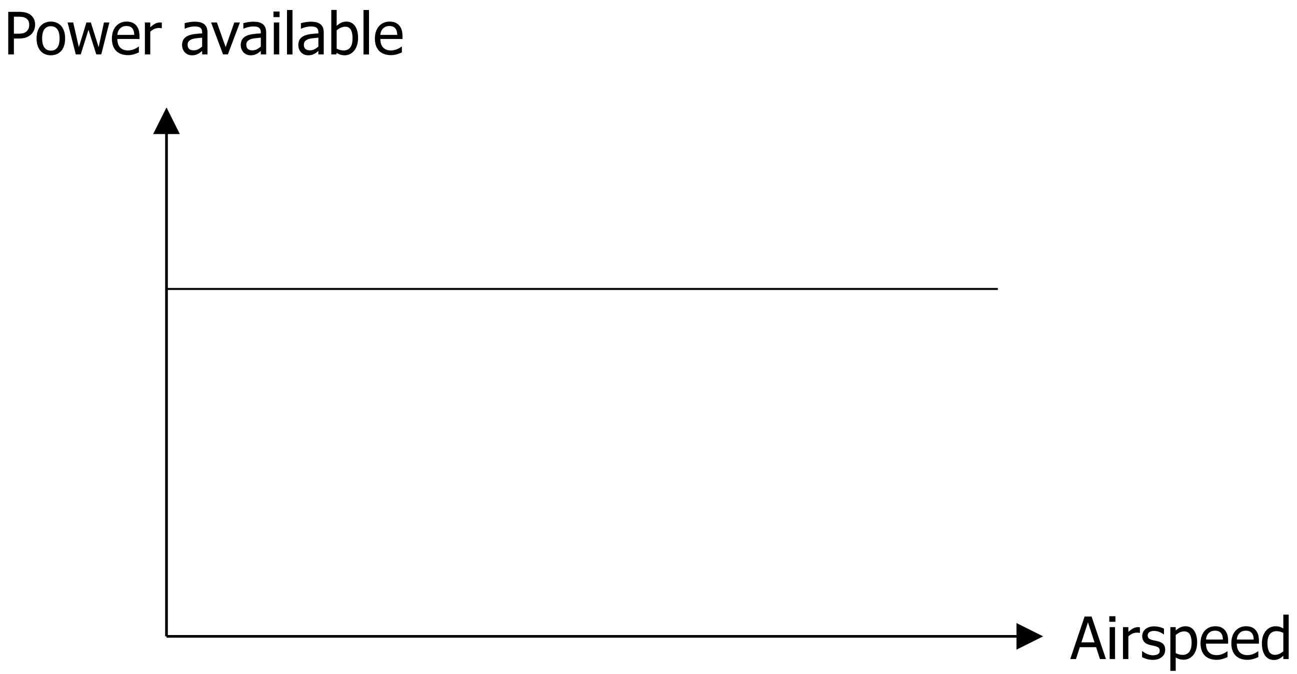
P_a is assumed to be constant and independent of airspeed





Propeller - simplified

For basic flight mechanics applications, ***power available*** *of a* ***propeller aircraft*** *can be assumed to be* ***constant with airspeed*** *for a given flight altitude*



Contents

Aerodynamics

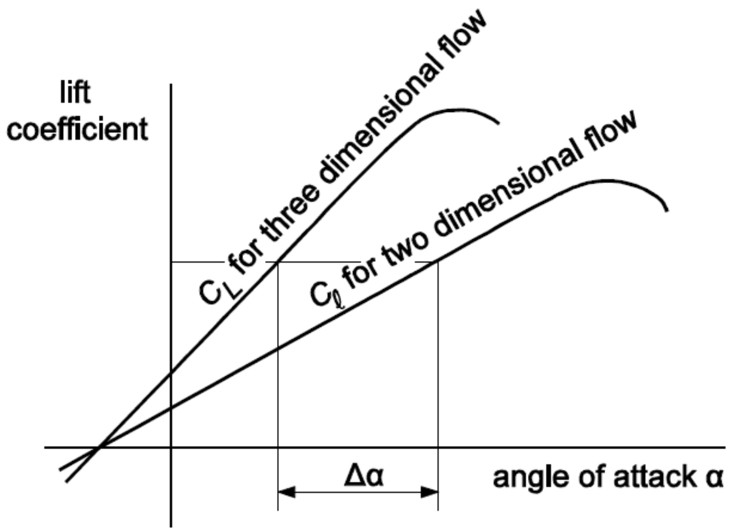
Lift

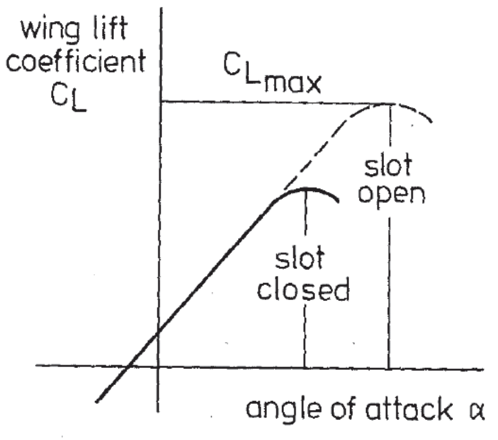
$$L = W$$

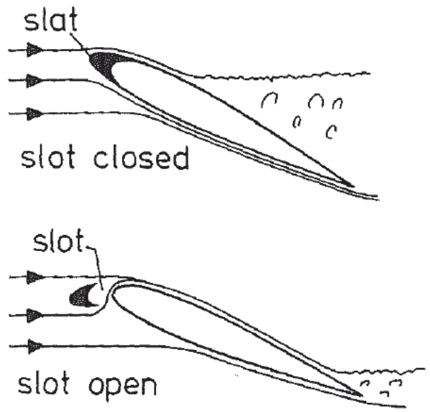
$$C_L \tfrac{1}{2} \rho V^2 S = W$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$V_{min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{max}}}}$$

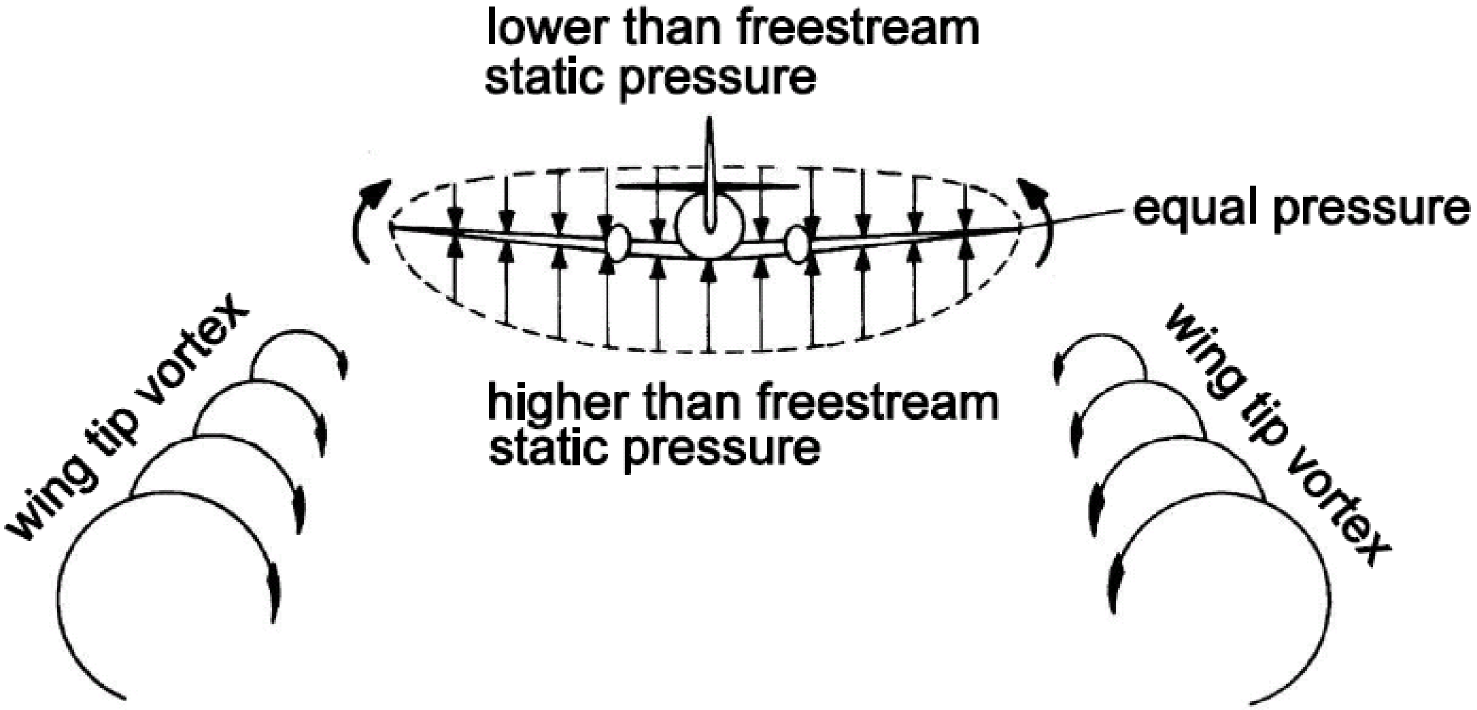






Aerodynamic forces

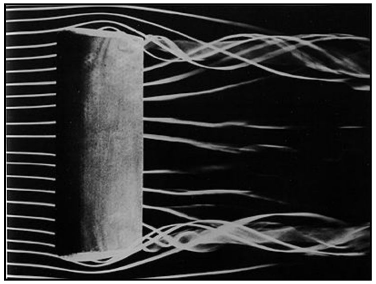
Drag polar



Aerodynamic forces

Drag polar

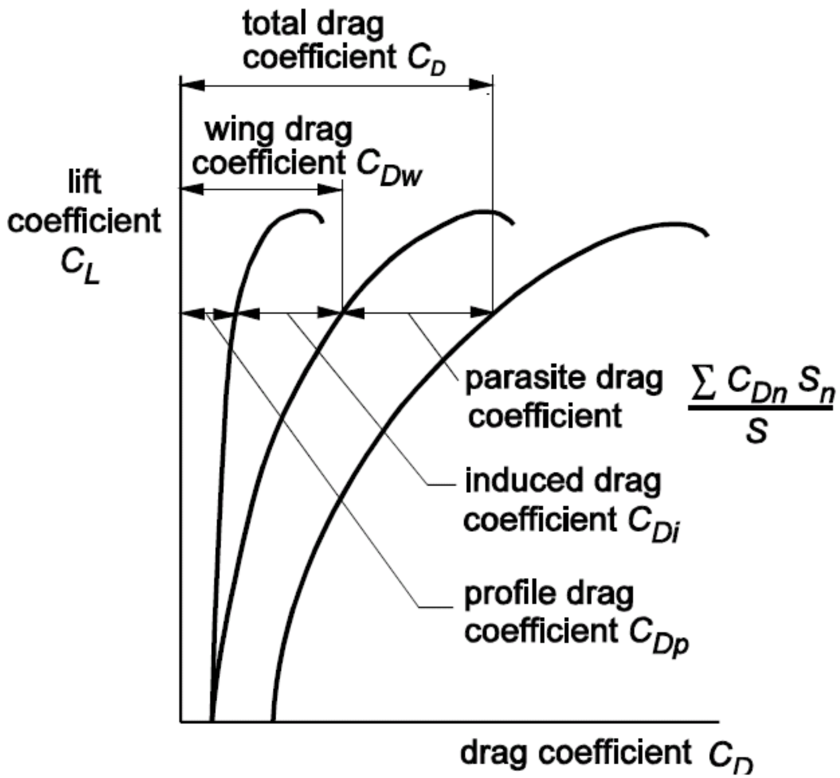
$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

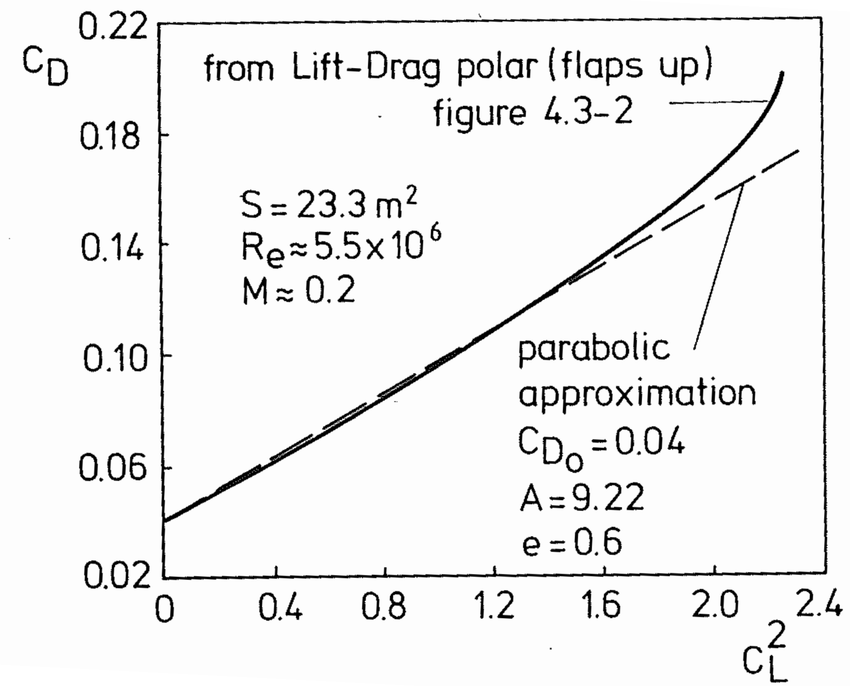


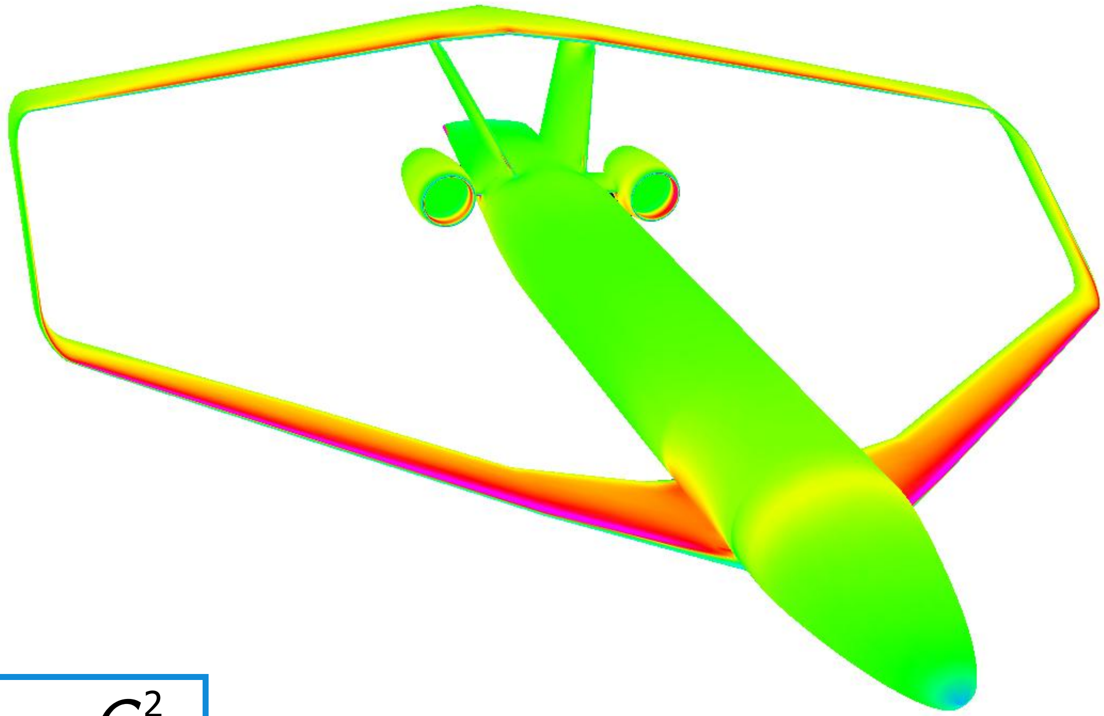
Aerodynamic forces

Lift – Drag polar

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$







$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Aerodynamic forces

Drag as a function of airspeed

Lift

$$L = W$$

$$C_L \tfrac{1}{2} \rho V^2 S = W$$

$$C_L = \frac{W}{S} \frac{2}{\rho} \frac{1}{V^2}$$

Drag

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

$$D = C_D \tfrac{1}{2} \rho V^2 S$$

$$D = C_{D_0} \tfrac{1}{2} \rho V^2 S + \frac{C_L^2}{\pi A e} \tfrac{1}{2} \rho V^2 S$$

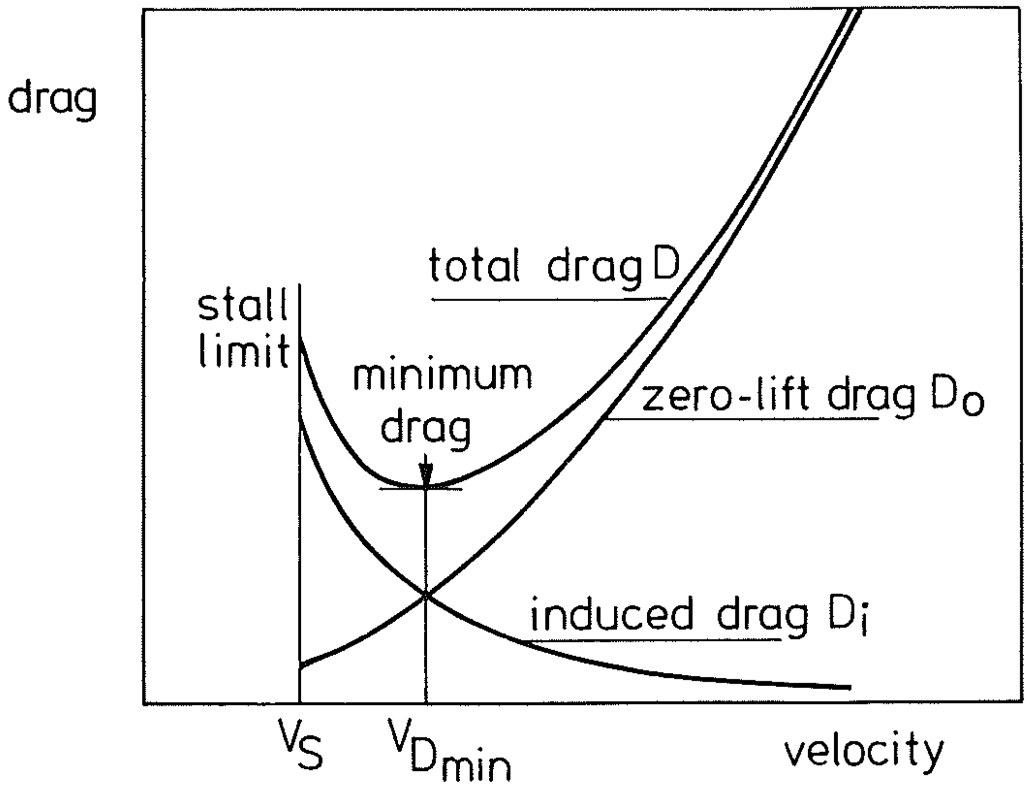
$$D = C_{D_0} \tfrac{1}{2} \rho V^2 S + \frac{W^2}{S^2} \frac{4}{\rho^2} \frac{1}{V^4} \frac{1}{\pi A e} \tfrac{1}{2} \rho V^2 S$$

$$\boxed{D = C_{D_0} \tfrac{1}{2} \rho V^2 S + \frac{W^2}{\pi A e \tfrac{1}{2} \rho V^2 S}}$$

So, one part of the drag decreases (!) with airspeed ($1/V^2$) and one part increases with airspeed (V^2)

Aerodynamic forces

Drag as a function of airspeed



Aircraft are quite unique in the sense that drag increases when airspeed decreases!

$$D = D_0 + D_i$$

$$D_0 = C_{D_0} \tfrac{1}{2} \rho V^2 S$$

$$D_i = \frac{W^2}{\pi A e \tfrac{1}{2} \rho V^2 S}$$

Aerodynamic forces

Consequences for aircraft design

- D_i predominant factor at low airspeeds
  → Large S; low wing loading (W/S) required
- D_0 predominant factor at high airspeeds
  → Small parasite drag C_{D0} and small S; high wing loading (W/S)

Very low speed aircraft (bicycle plane)

high speed aircraft (F104 Starfighter)

Contents

Summary

- You should be able to derive the equations of motion for 2 dimensional flight

$$\sum F_{//V} : \frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\sum F_{\perp V} : \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

- The thrust (T) of a jet aircraft can be assumed independent of airspeed
- The power available (P_a) of a propeller aircraft can be assumed independent of airspeed
- The complete aircraft aerodynamics can be represented by 1 equation; the lift drag polar

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Questions

Contents

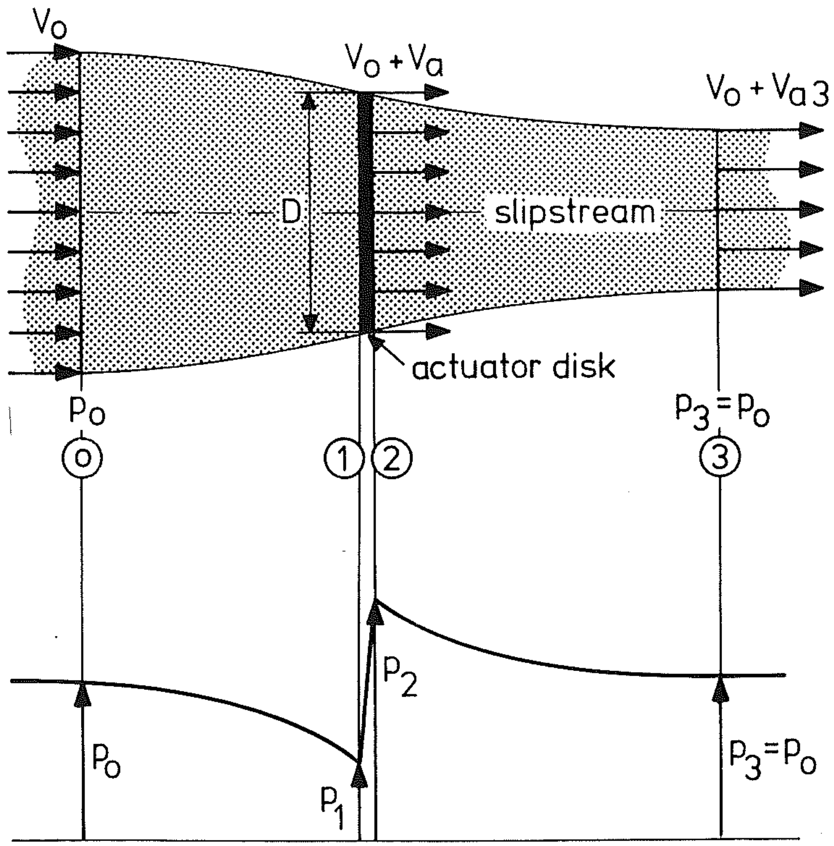
Additional material

Actuator disk theory

Extra information which may be useful for the propeller practical in the AE1100 project is included in the next couple of sheets. You do not have to learn this for the exam.

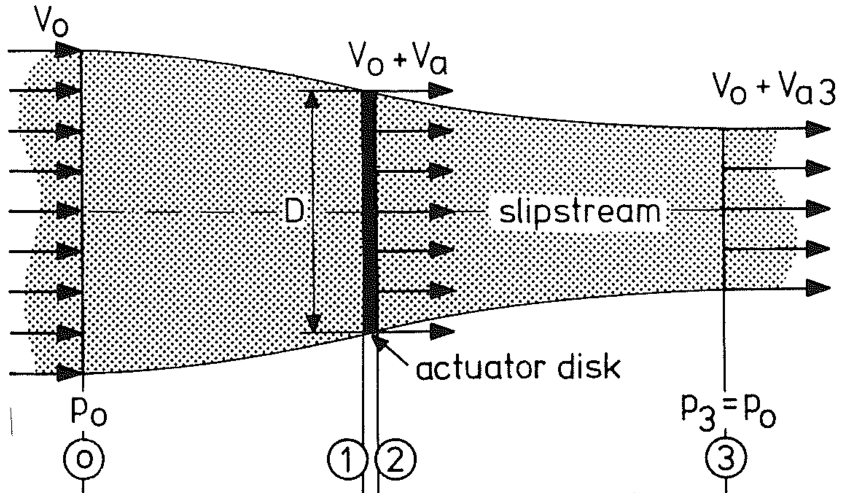
Working principle turboprop

Actuator disk theory



Working principle propeller

Actuator disk theory



Bernoulli

$$p_0 + \tfrac{1}{2}\rho V_0^2 = p_1 + \tfrac{1}{2}\rho\left(V_0 + V_a\right)^2$$

$$p_2 + \tfrac{1}{2}\rho\left(V_0 + V_a\right)^2 = p_0 + \tfrac{1}{2}\rho\left(V_0 + V_{a3}\right)^2$$

$$T = \pi R^2 \left(p_2 - p_1\right)$$

$$T = \pi R^2 \left(\tfrac{1}{2}\rho\left(V_0 + V_{a3}\right)^2 - \tfrac{1}{2}\rho V_0^2\right)$$

$$T = \pi R^2 \rho V_{a3}\left(V_0 + \tfrac{1}{2}V_{a3}\right)$$

Momentum equation

$$T = m\left(V_j - V_0\right)$$

$$T = m\left(\left(V_0 + V_{a3}\right) - V_0\right)$$

$$T = m V_{a3}$$

$$T = \pi R^2 \rho V_{a3}\left(V_0 + V_a\right)$$

$$\Rightarrow \quad \boxed{V_a = \tfrac{1}{2}V_{a3}}$$

Working principle propeller

Actuator disk theory – propulsive efficiency

Shaft power P_{br} may be expressed as the increase in kinetic energy of the air mass flow

$$P_{br} = \tfrac{1}{2} m\left(V_j^2 - V_0^2\right)$$

$$P_{br} = \tfrac{1}{2} \rho \pi R^2 \left(V_0 + V_a\right)\left[\left(V_0 + V_{a3}\right)^2 - V_0^2\right]$$

$$P_{br} = \rho \pi R^2 \left(V_0 + V_a\right)^2 V_{a3}$$

$$\eta_j = \frac{T V_0}{P_{br}} = \frac{V_0}{V_0 + V_a} = \frac{1}{1 + \dfrac{V_a}{V_0}}$$

$$\eta_j = \frac{2}{1 + \sqrt{1 + \dfrac{T}{\rho \pi R^2 V_0^2}}}$$

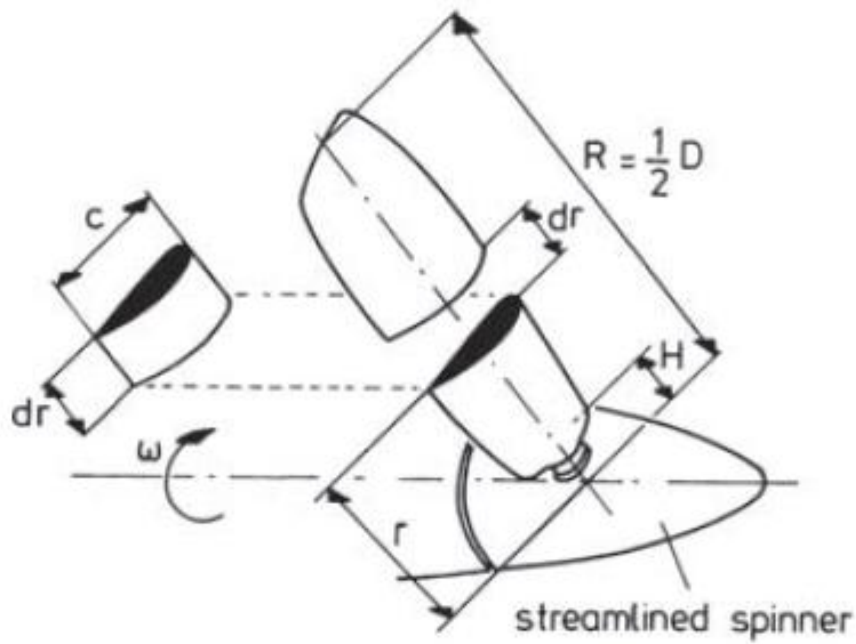
This efficiency is a theoretical upper limit

Assumptions:

- No rotational kinetic energy in slipstream
- Axial velocity is uniform over the disk

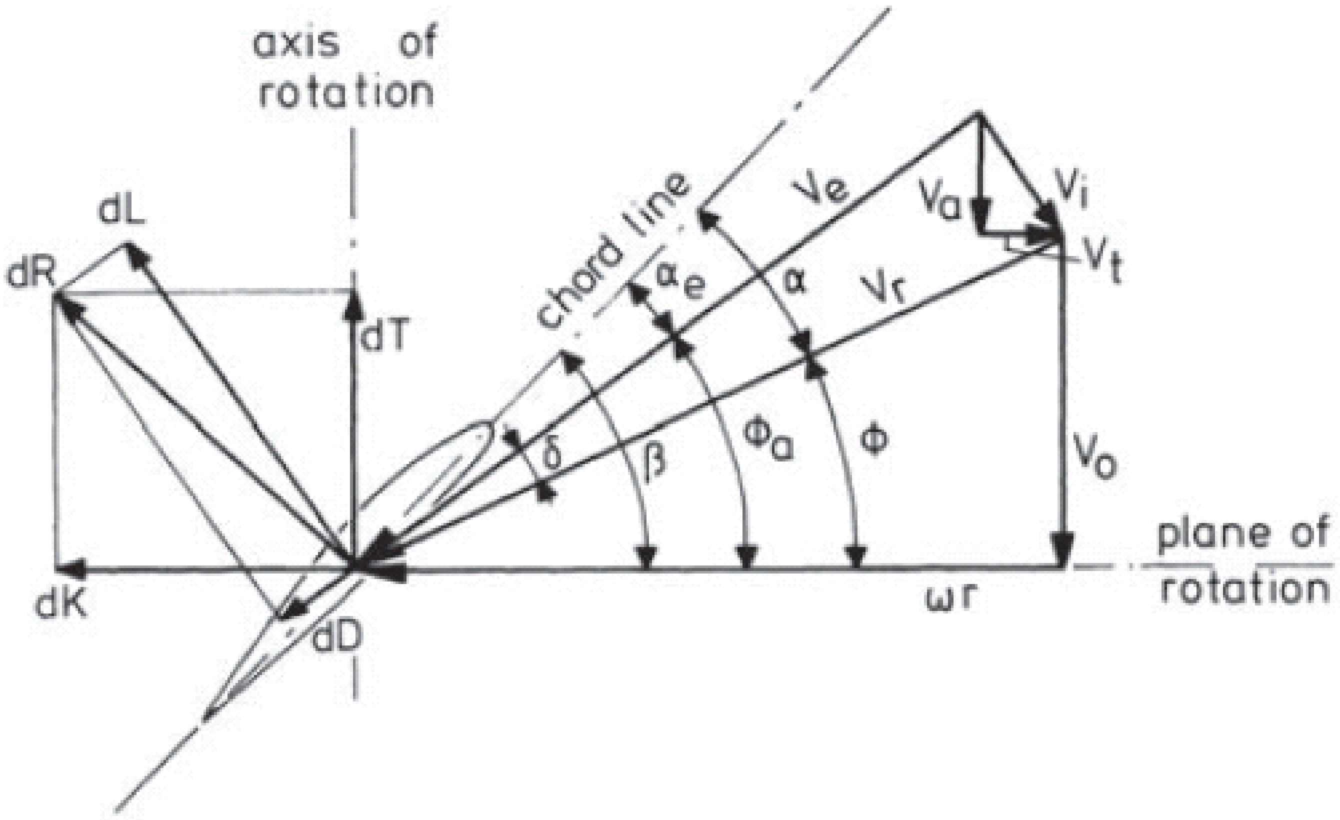
Working principle propeller

Actuator disk theory – blade element theory



Working principle propeller

Actuator disk theory – blade element theory



Blade element theory

How to calculate the thrust and torque of a prop