# Introduction to Aerospace Engineering

## Lecture slides

# Introduction Aerospace Engineering

## Flight Mechanics

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# 3.&4

*Horizontal flight performance*

# Flight mechanics

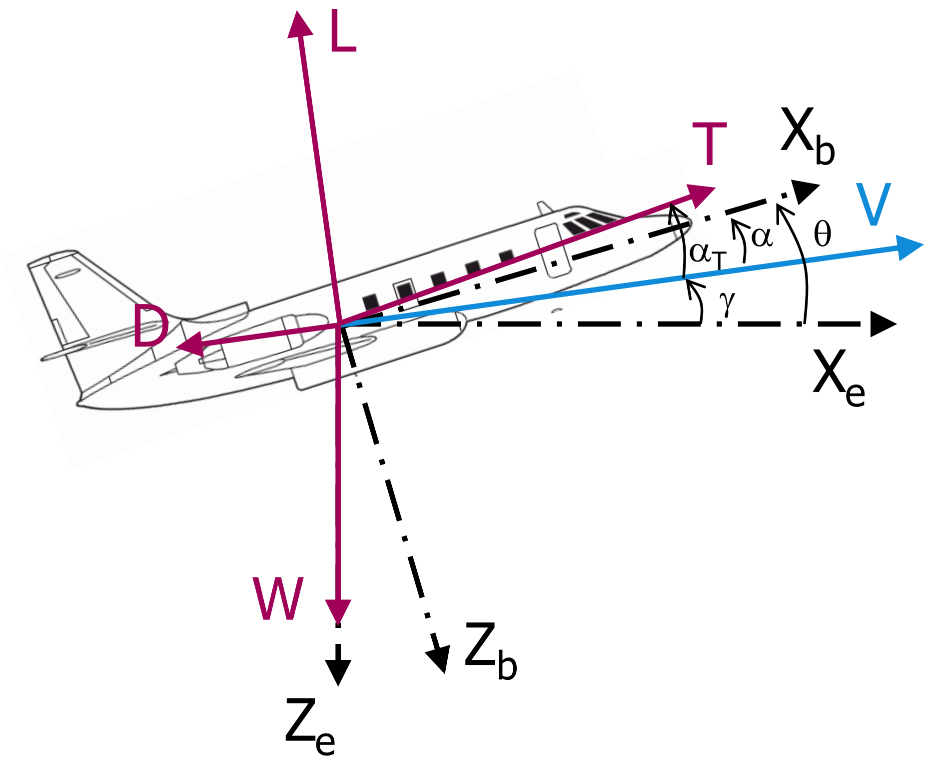
| | |
|---|---|
| Hours 1 & 2 : | Introduction, general equations of motion |
| Hours 3 & 4 : | Horizontal flight performance |
| Hours 5 & 6 : | Climbing and descending flight |
| Hours 7 & 8 : | Flight envelope |
| Hours 9 & 10: | Example questions and solutions |

# Contents

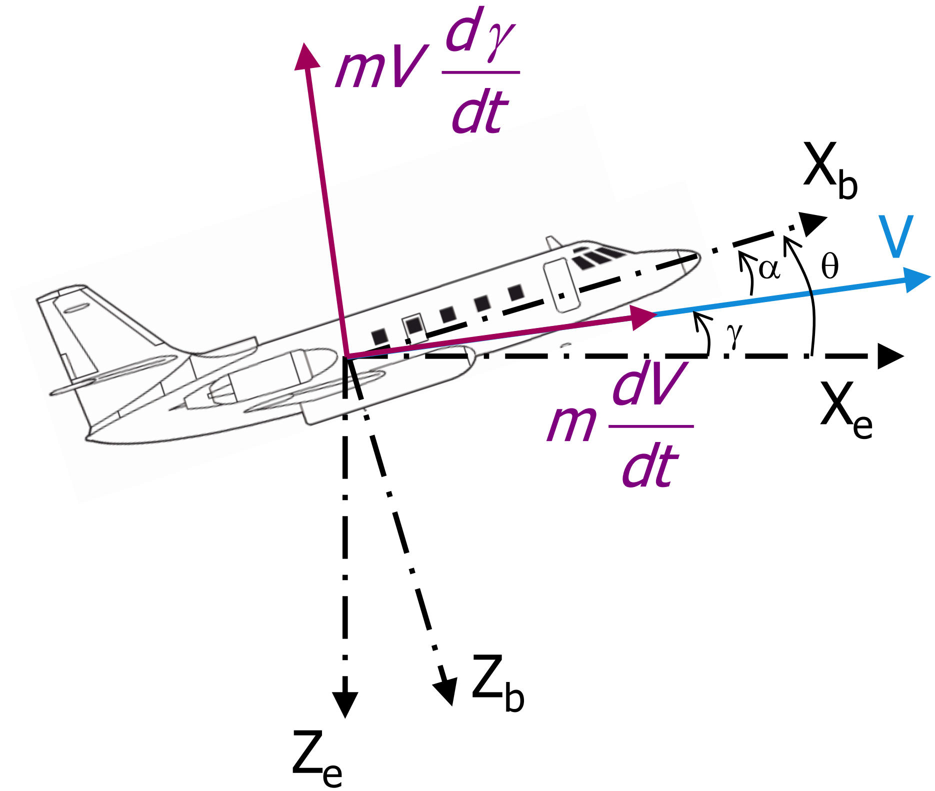
# Equations of motion

## General equations of motion in symmetric flight (1)

Free Body Diagram




Kinetic Diagram



# Equations of motion

## General equations of motion in symmetric flight (2)

$$\vec{F} = m\vec{a}$$

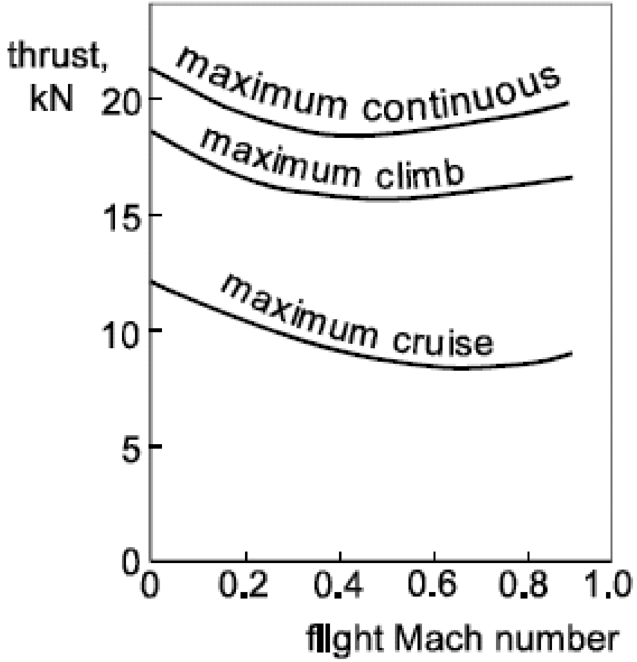
$$\parallel V : T\cos\alpha_T - D - W\sin\gamma = m\frac{dV}{dt}$$

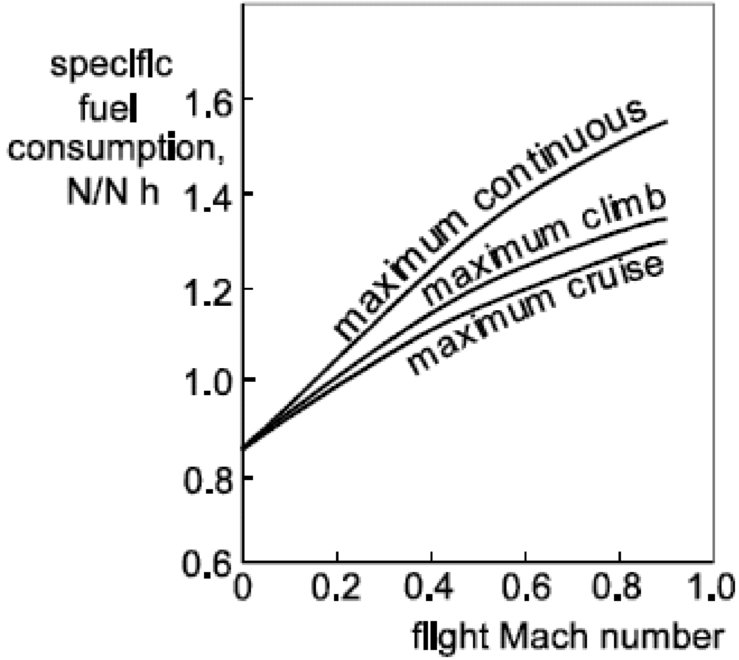
$$\perp V : L - W\cos\gamma + T\sin\alpha_T = mV\frac{d\gamma}{dt}$$

# Propulsive force

## Typical analytical assumption – Jet

Thrust is assumed to be independent of airspeed

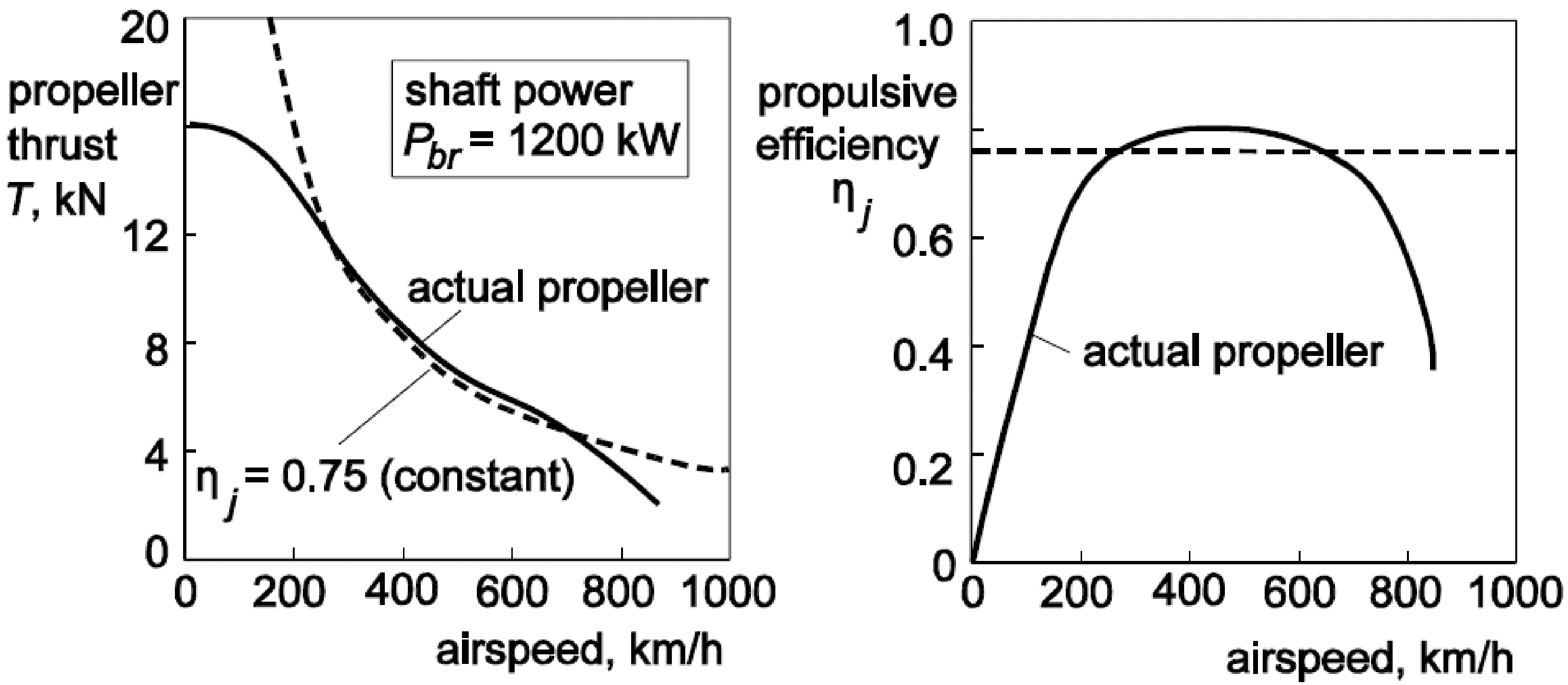


# Propulsive force

## Typical analytical assumption – Propeller

$P_a$ is assumed to be constant and independent of airspeed

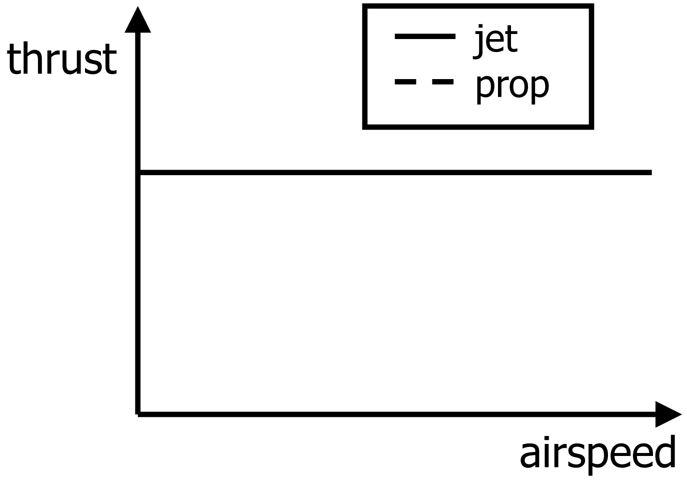


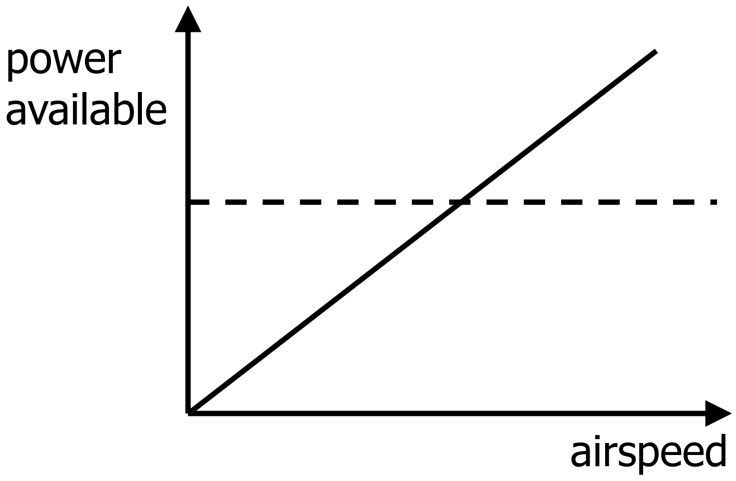
# Propulsive force

## Analytical assumptions - summary

**Jet**: Thrust is (assumed) independent of airspeed

**Propeller**: Power available is (assumed) independent of airspeed



# Summary previous lecture

- You should be able to derive the equations of motion for 2 dimensional flight
- The thrust of a jet aircraft can be assumed independent of airspeed
- The power available of a propeller aircraft can be assumed independent of airspeed
- The complete aircraft aerodynamics can be represented by 1 equation; the lift drag polar

$$\sum F_{//V}: \frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\sum F_{\perp V}: \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

# Contents

# Contents

# Introduction

## Horizontal flight performance

- How fast can an aircraft fly?
- How slow can a given aircraft fly?
- At what speed should be flown to be able to fly as far as possible? (Interesting for airliners)
- At what speed should be flown to stay in the air as long as possible? (Search and rescue, military purposes)

# Introduction
## What to learn?

**Most important are the lecture sheets!!!**

<u>Background material:</u>
Introduction to Flight (Anderson)
Par. 6.1 – 6.2
Par. 6.3 – 6.6
Par. 6.12 – 6.14

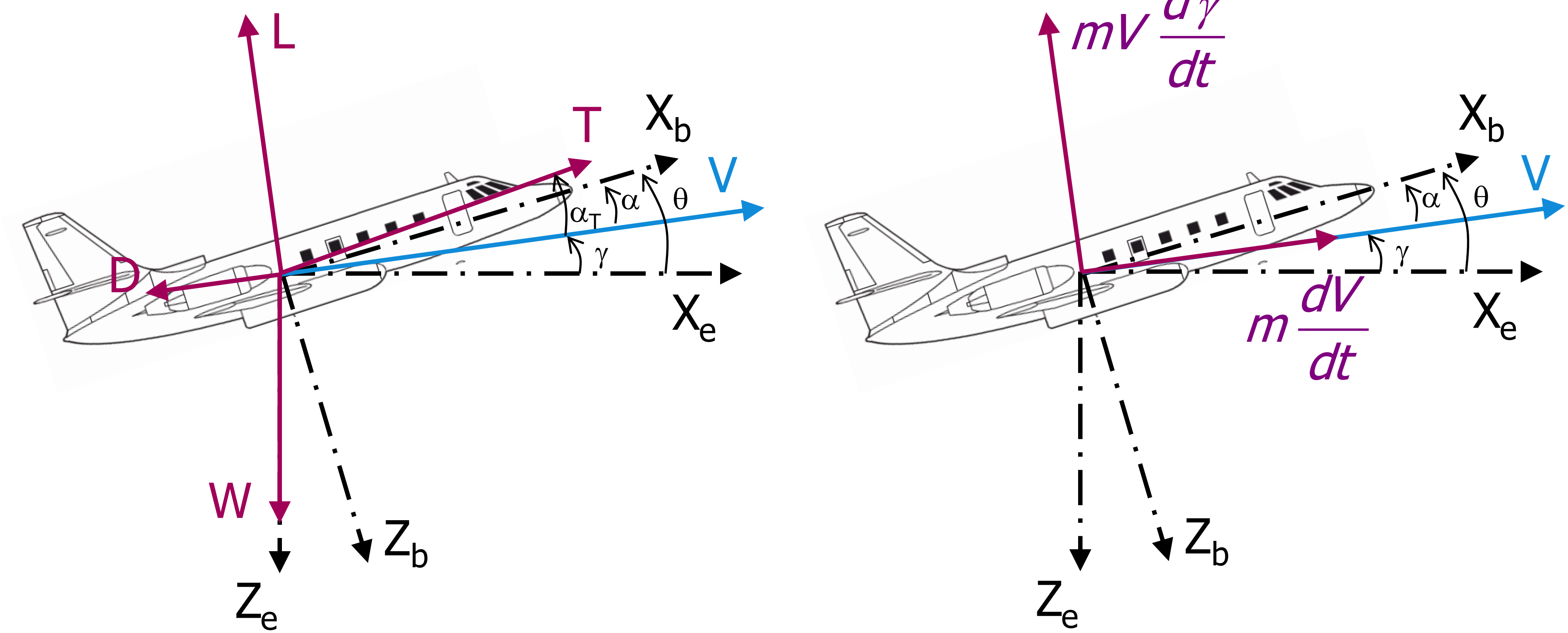
# Contents

# Equations of motion

## General equations of motion in symmetric flight (1)

Free Body Diagram

Kinetic Diagram



# Equations of motion

## General equations of motion in symmetric flight (2)

$$\vec{F} = m\vec{a}$$

$$\Box V : T\cos\alpha_T - D - W\sin\gamma = m\frac{dV}{dt}$$

$$\perp V : L - W\cos\gamma + T\sin\alpha_T = mV\frac{d\gamma}{dt}$$

Flight Condition:

The aircraft is performing a **horizontal** (**straight**) and **steady** flight.

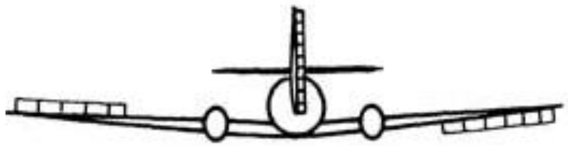
Assumption:

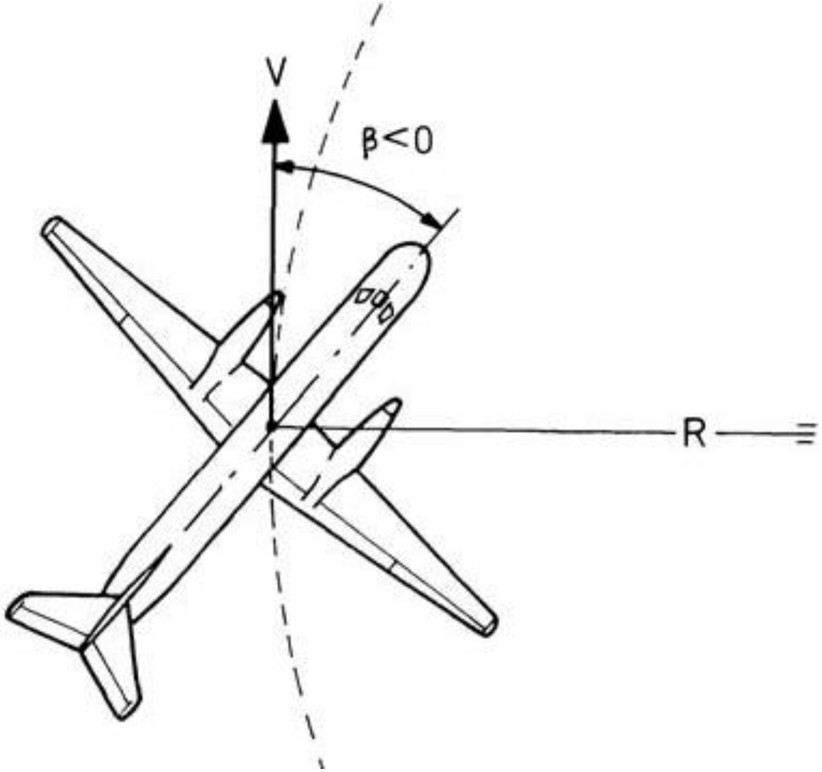
The thrust is assumed to be in the direction of flight ($\alpha_T = 0$).

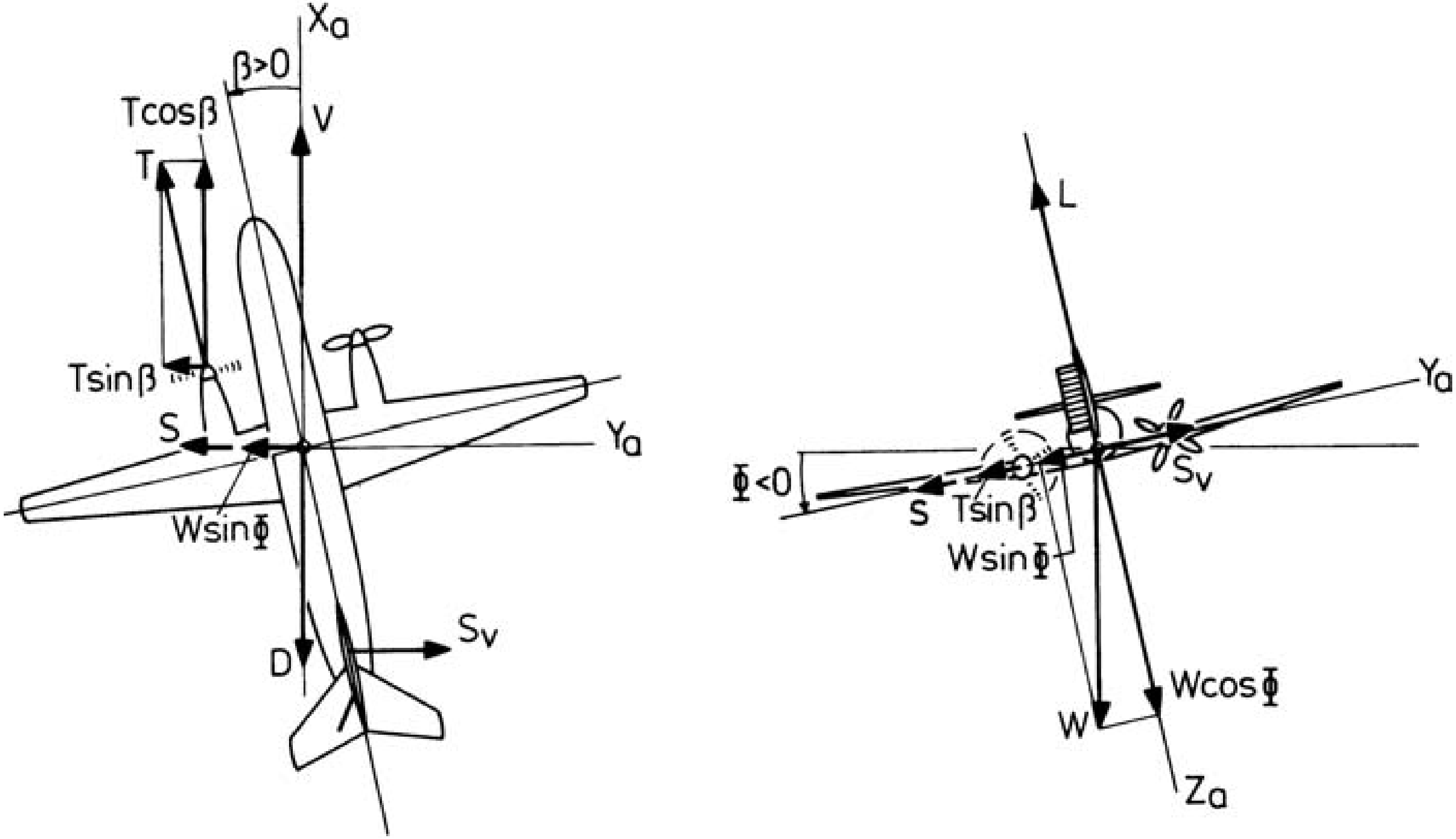
# Equations of motion

## definitions

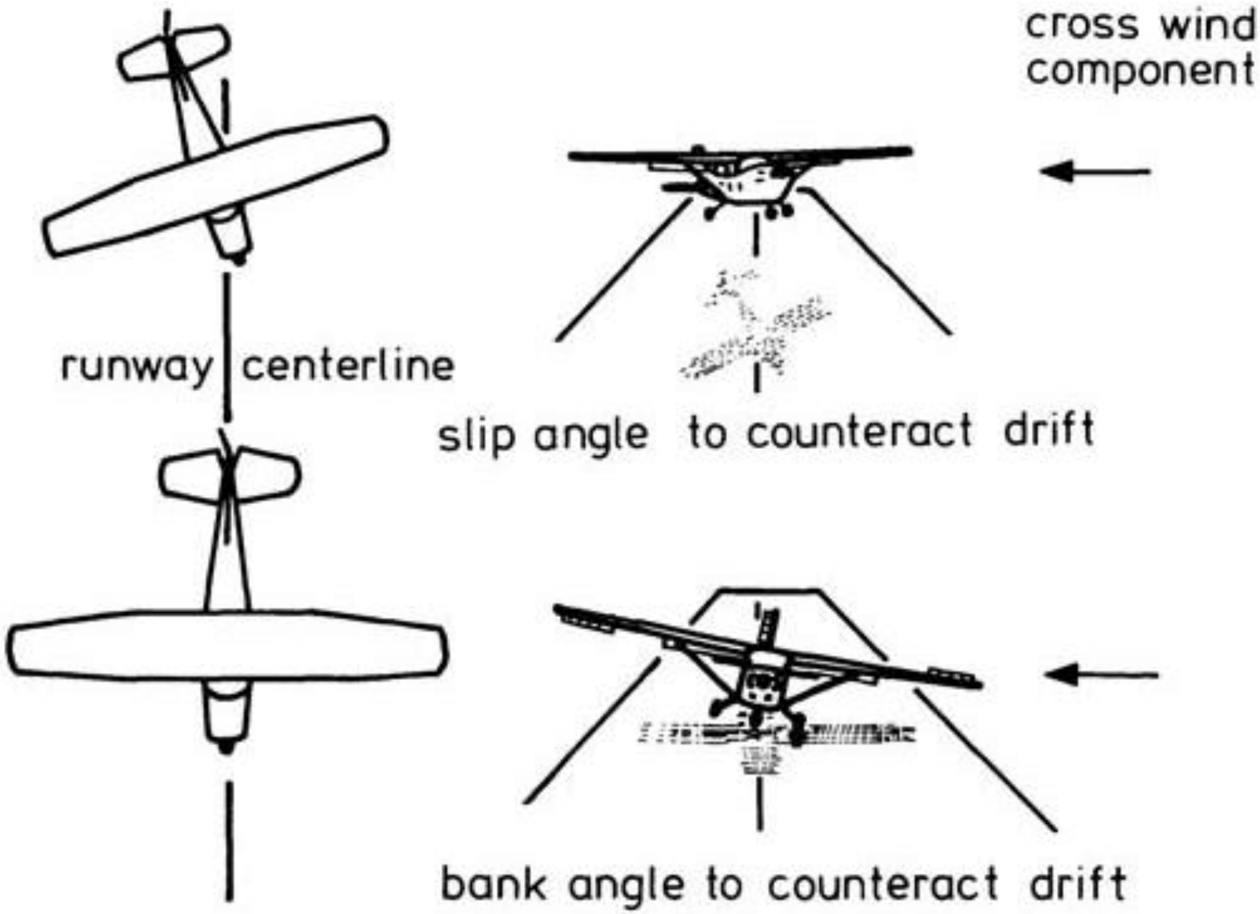
- **Straight flight**: flight in which the centre of gravity of the aircraft travels along a straight line ($d\gamma/dt = 0$)
- **Steady flight**: Flight in which the forces and moments acting on the aircraft do not vary in time, neither in magnitude, nor in direction ($dV/dt = 0$)
- **Horizontal flight**: The aircraft remains at a constant altitude ($\gamma = 0$)
- **Symmetric flight**: flight in which both the angle of sideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth ($\beta = 0$ and the aircraft is not turning)




V
β<0
R



$X_a$
$\beta>0$
$T\cos\beta$
$V$
$T$
$T\sin\beta$
$S$
$Y_a$
$W\sin\Phi$
$S_v$
$D$
$L$
$Y_a$
$\Phi<0$
$S_v$
$S$
$T\sin\beta$
$W\sin\Phi$
$W\cos\Phi$
$W$
$Z_a$




http://www.youtube.com/watch?v=TCUHQ_-l6Qg
B52 landing in crosswind

# Equations of motion

## Horizontal, steady, symmetric flight

$$\vec{F} = m\vec{a}$$

$$\parallel V: T \overset{=1}{\cancel{\cos \alpha_T}} - D - W \overset{=0}{\cancel{\sin \gamma}} = m \overset{=0}{\cancel{\frac{dV}{dt}}}$$

$$\perp V: L - W \overset{=1}{\cancel{\cos \gamma}} + T \overset{=0}{\cancel{\sin \alpha_T}} = mV \overset{=0}{\cancel{\frac{d\gamma}{dt}}}$$

$$\Rightarrow \quad \begin{aligned} \parallel V &: T = D \\ \perp V &: L = W \end{aligned}$$

Note: Clearly this is the most simple form of the equations of motion

How do these forces (aerodynamic, propulsive, gravitational) vary with airspeed?

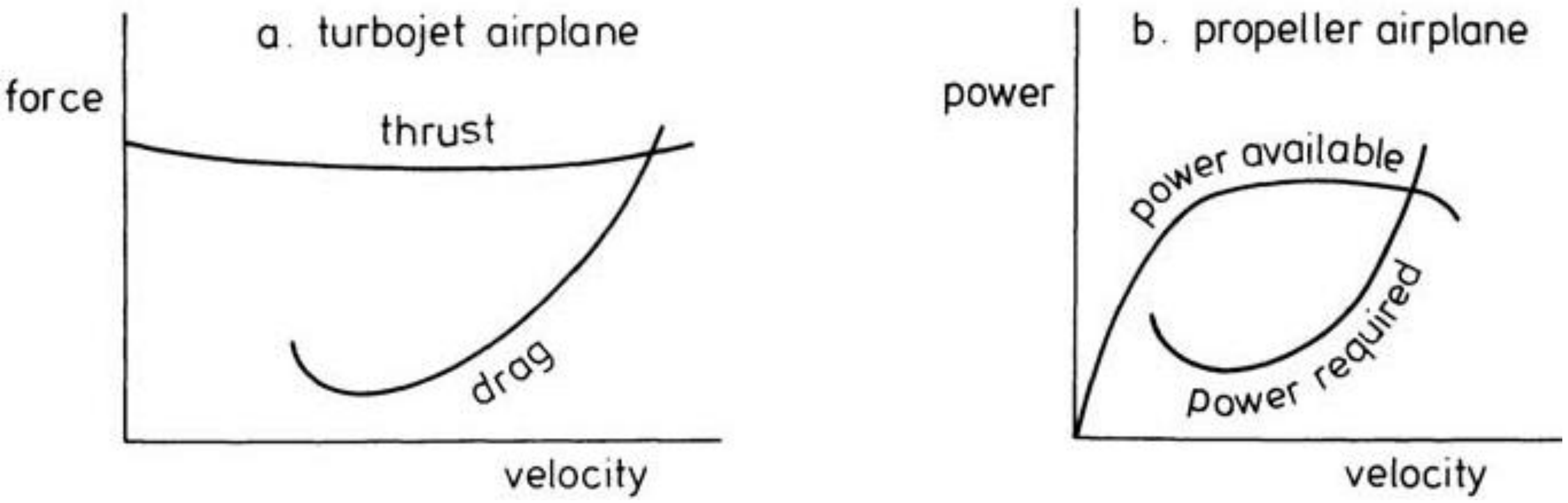
# Contents

# Performance diagram

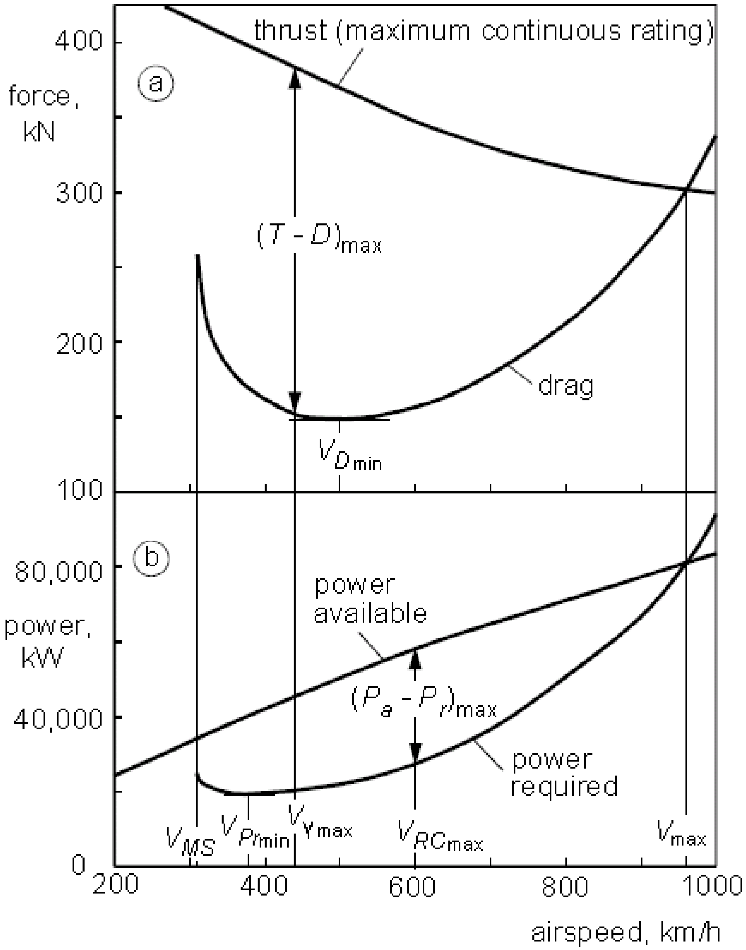
- For a given altitude, configuration and engine control setting, the drag or power required and thrust or power available are plotted against airspeed
- Also called *Pernaud* diagram
- Very useful to determine aircraft performance parameters

# Performance diagram

- Typical shape:

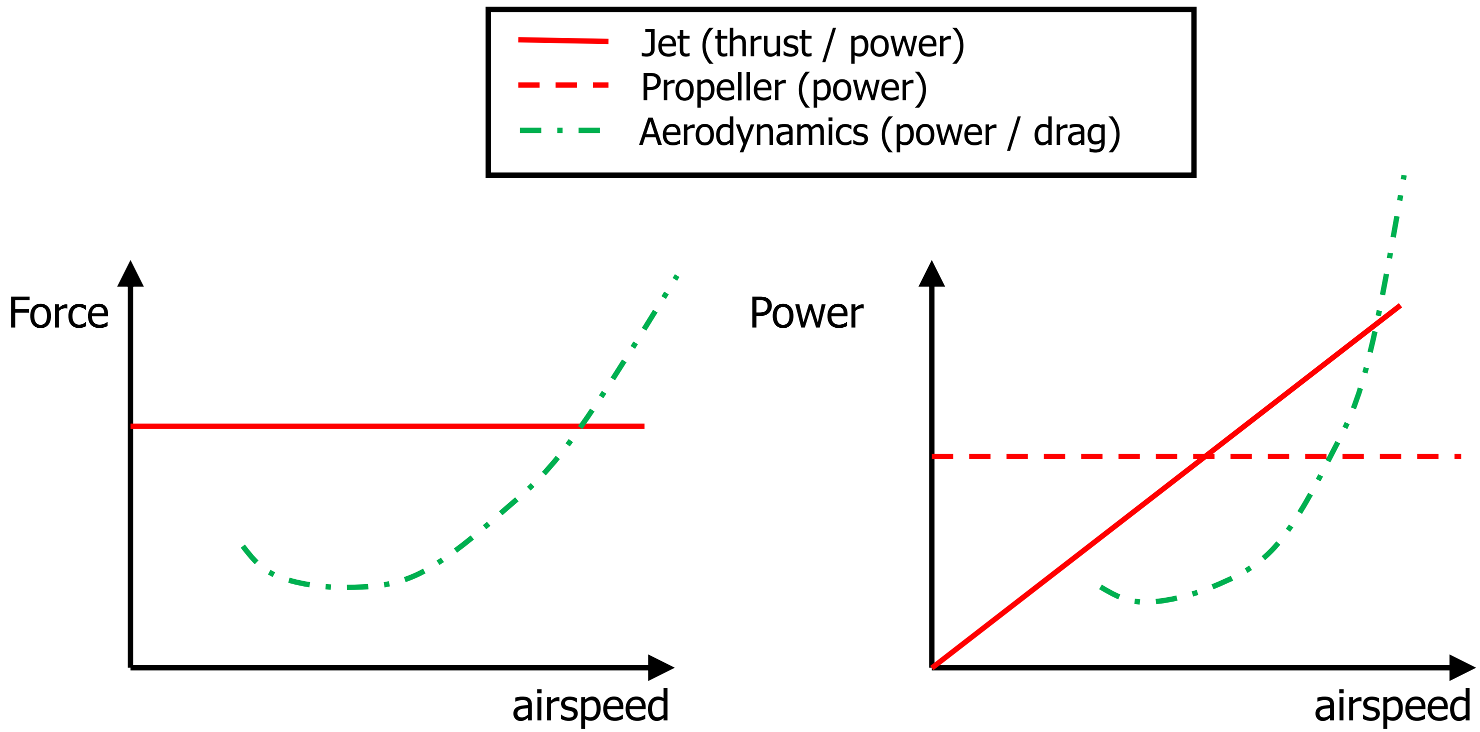


# Performance diagram




$$P_a = T \cdot V$$

$$P_r = D \cdot V$$


Jet (thrust / power)
Propeller (power)
Aerodynamics (power / drag)
Force
airspeed
Power
airspeed

# Contents

# Horizontal flight performance

- Minimum airspeed (How slow)
- Maximum airspeed (How fast)
- Range (How far)
- Endurance (How long)
- Speed stability / instability

Assumptions: **Horizontal steady symmetric** flight
( $L = W$, $T = D$, $P_a = P_r$ ), except for the speed stability section

# Contents

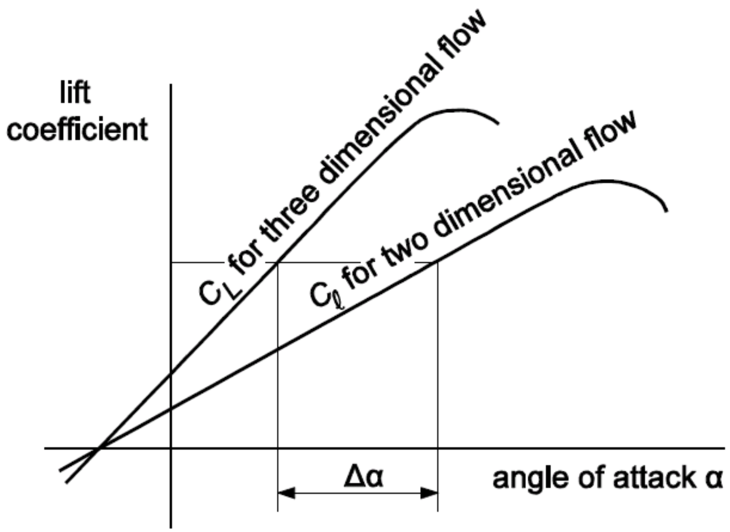
# Minimum airspeed

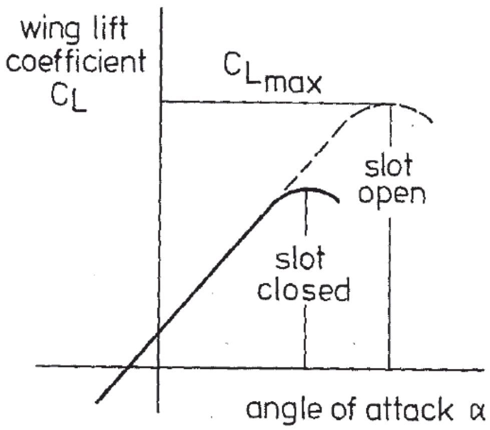
$$L = W$$

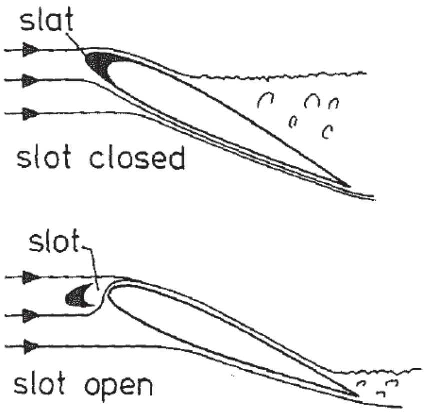
$$C_L \tfrac{1}{2} \rho V^2 S = W$$

$$V = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

$$V_{\min} = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_{L_{\max}}}}$$

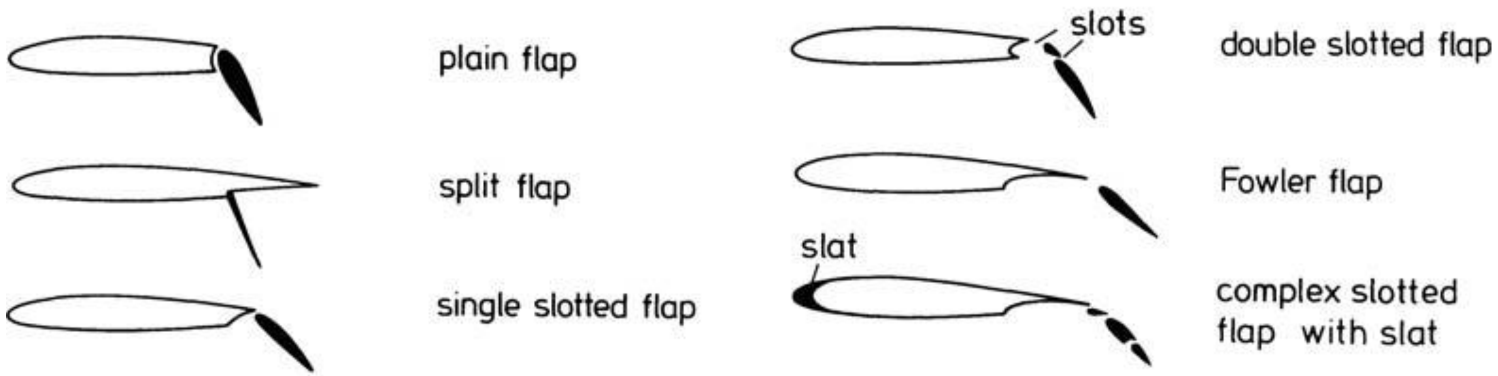


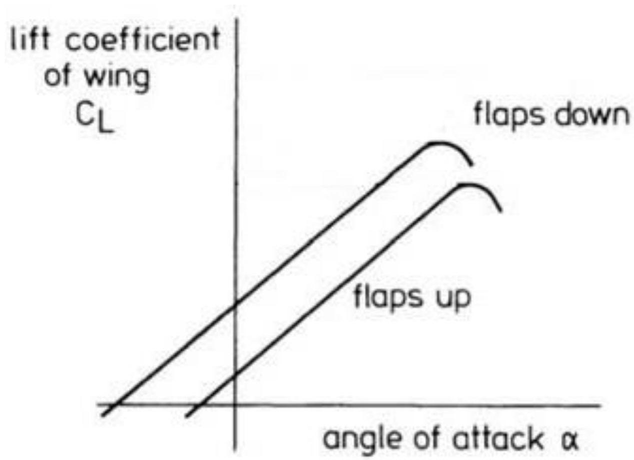




# Minimum airspeed

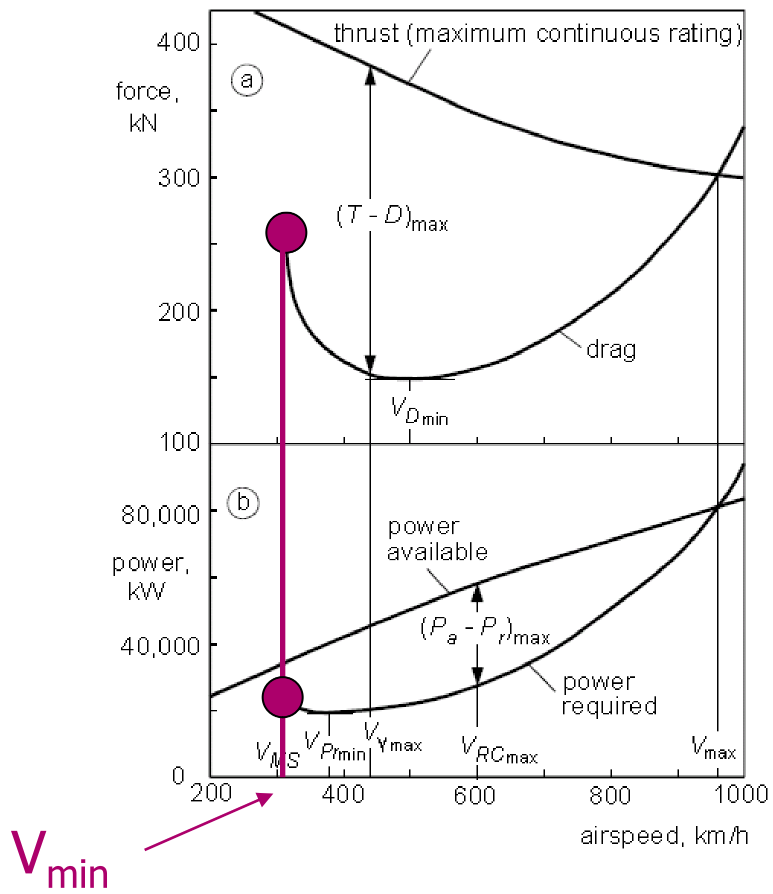
## Flaps



# Minimum airspeed

## Performance diagram



# Minimum airspeed

## Example calculation

- **Question**: An aircraft has a wing loading (W/S) 2400 N/m$^2$ and $C_{Lmax}$ = 1.4. Find the airspeed at which stall occurs (minimum airspeed) at (i) sea level ($\rho$ = 1.225 kg/m$^3$) and (ii) at 5000m ($\rho$ = 0.737 kg/m$^3$)

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- **Answer**:

$$L = W$$

$$C_L \tfrac{1}{2}\rho V^2 S = W$$

$$V_{min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L,max}}}$$

$$\text{(i)}V_{min} = \sqrt{2400\frac{2}{1.225}\frac{1}{1.4}} = 51.8 \text{ m/s}$$

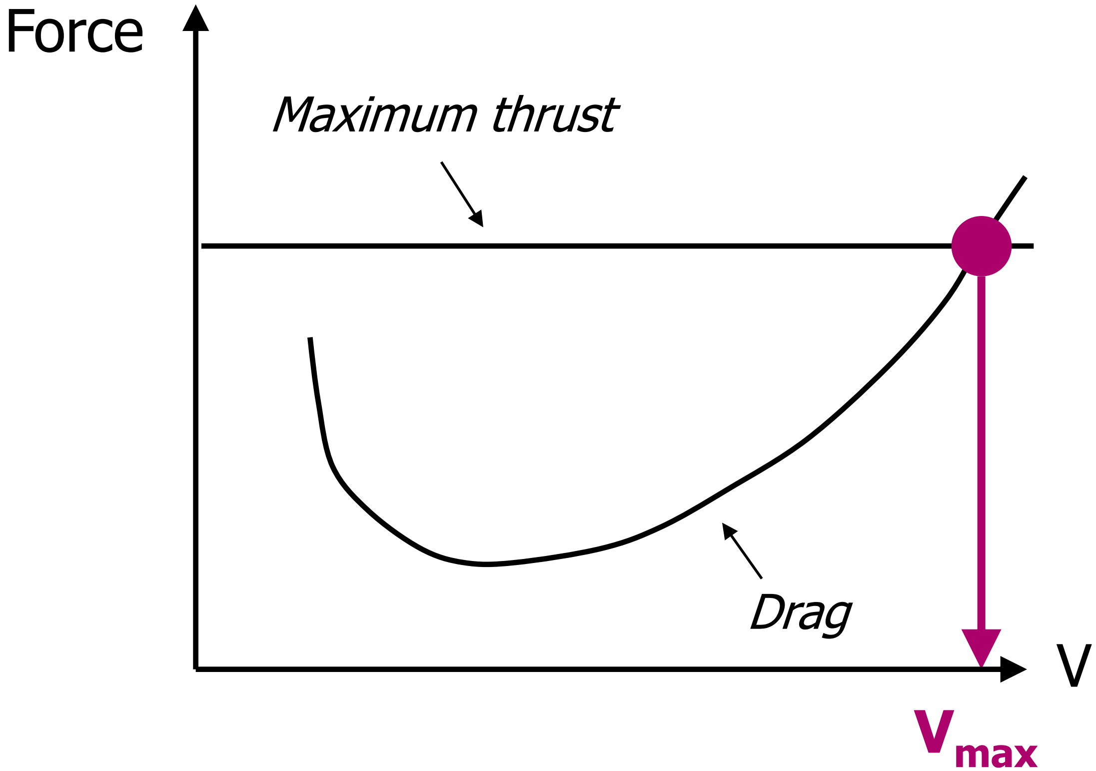
$$\text{(ii)}V_{min} = \sqrt{2400\frac{2}{0.737}\frac{1}{1.4}} = 66.8 \text{ m/s}$$

# Contents

# Maximum airspeed

## Jet aircraft

- Pilot applies maximum throttle (max. thrust or Power available)




**Solution**

$$T_{max} = D$$

$$L = W$$

$$T_{max} = D\frac{L}{L} = \frac{C_D}{C_L}W$$

$$C_D = C_{D_0} + k \cdot C_L^2$$

$$\frac{C_{D_0}}{C_L} + k \cdot C_L = \frac{T_{max}}{W}$$

$$kC_L^2 - \frac{T_{max}}{W}C_L + C_{D_0} = 0$$

$$\left(ax^2 + bx + c = 0\right)$$

$$C_L = \ldots$$

$$L = W \Rightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

# Maximum airspeed
## Example


The following data is known of the Cessna Citation II (subsonic jet)

Aircraft Weight : W = 60 kN,
Wing area : S = 30 $m^2$,
(Parabolic) Lift-Drag polar : $C_D = C_{Do} + kCL^2$; $C_{Do} = 0.022$, $k = 0.047$, $C_{Lmax} = 1.35$,
Maximum Thrust at 0 m ISA :T0 = 12 kN.
The aircraft is flying at an altitude of H = 0 m in the International Standard Atmosphere ($\rho_0 = 1.225$ kg/$m^3$)
Thrust is assumed to be independent of the airspeed

Calculate (1) the maximum airspeed of this aircraft when flying at H = 0 m and (2) the corresponding Mach number

# Maximum airspeed

## Solution

Steady symmetric horizontal flight:

$L = W, T = D$

With

$L = C_L \frac{1}{2}\rho V^2 S, \quad D = C_D \frac{1}{2}\rho V^2 S$

Thrust must be at maximum to achieve Vmax

Combining the above yields:

$$\frac{C_L}{C_D} = \frac{W}{T} = \frac{60000}{12000} = 5 \rightarrow C_D = \frac{C_L}{5}$$

Lift drag polar:

$$C_D = C_{D_0} + k \cdot C_L^2$$

$$C_L^2 - \frac{C_L}{5k} - \frac{C_{D_0}}{k} = 0$$

$$C_L^2 - \frac{C_L}{5 \cdot 0.047} - \frac{0.022}{0.047} = 0$$

$$C_L = 0.11$$

$$V_{max} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{60000}{30}\frac{2}{1.225}\frac{1}{0.11}} = 172.1 \text{ m/s}$$

Maximum Mach number?

$$M_{max} = \frac{V_{max}}{a}$$

$$a = \sqrt{\gamma R T} = \sqrt{1.4 \cdot 287.05 \cdot 288.15} = 340.3 \text{ m/s}$$

$$M_{max} = \frac{172.1}{340.3} = 0.5$$

# Contents

# Range

- How can we fly as far as possible for a given amount of fuel?
- How can we fly a given distance with the minimum amount of fuel?

Specific range is defined as the ratio V/F, where F equals the fuel flow

Dimensional analysis: $\left[\frac{V}{F}\right] = \frac{[V]}{[F]} = \frac{[m/s]}{[N/s]} = [m/N]$

Or, put in words, **specific range** is the **distance that can be flown for 1 Newton of fuel**. Clearly, this variable must be maximised

# Range

## Maximum range propeller aircraft

$$F \;\square\; c_P P_{br} \Leftrightarrow F = c_P \frac{P_a}{\eta_j}$$

$P_a = P_r$ (steady horizontal flight)

$$F = c_P \frac{P_r}{\eta_j} = c_P \frac{DV}{\eta_j}$$

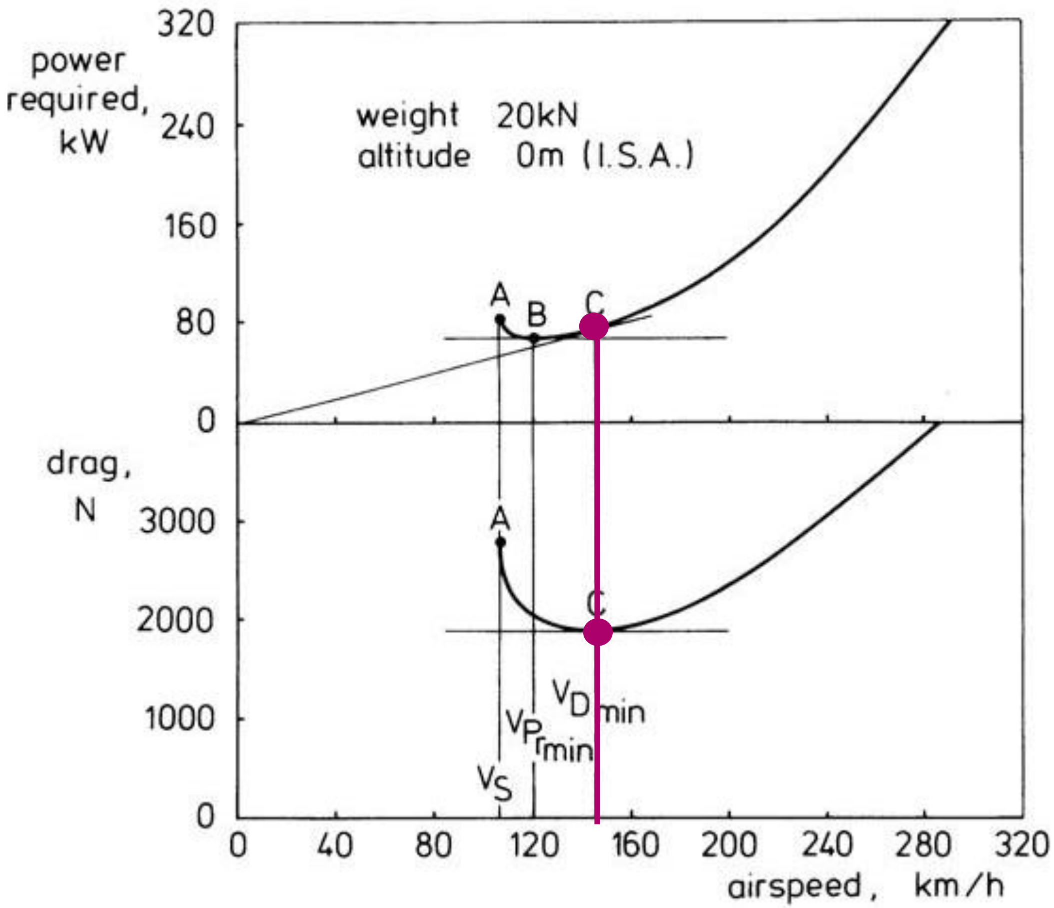
$$\frac{V}{F} = \frac{\eta_j}{c_P} \frac{1}{D}$$

$\eta_j$ and $c_P$ can be assumed to be constant as a function of airspeed (over the range of cruising speeds) for propeller aircraft

$$R_{\max} \Rightarrow \left(\frac{V}{F}\right)_{\max} \Rightarrow D_{\min} \Rightarrow \left(\frac{C_L}{C_D}\right)_{\max}$$

# Range

## Maximum range propeller aircraft



# Problem!

Result:
Prop: maximum range at maximum $C_L / C_D$

***Dear pilot, could you please fly at $C_L / C_D$ max?***

We must give an airspeed to the pilot

# Solution

$$\boxed{C_D = C_{D_0} + \frac{C_L^2}{\pi A e}}$$

Result:

Prop: maximum range at maximum $C_L$ / $C_D$

$$C_D = f\left(C_L\right) \quad \Leftrightarrow \quad y = f(x)$$

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = 0 \quad \Leftrightarrow \quad \frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{hg' - gh'}{h^2}$$

$$\frac{C_D \cdot 1 - C_L \cdot \dfrac{dC_D}{dC_L}}{C_D^2} = 0$$

$$\frac{dC_D}{dC_L} = \frac{C_D}{C_L}$$

$$\frac{2C_L}{\pi A e} = \frac{C_{D_0} + \dfrac{C_L^2}{\pi A e}}{C_L}$$

$$C_L = \sqrt{C_{D_0} \pi A e}$$

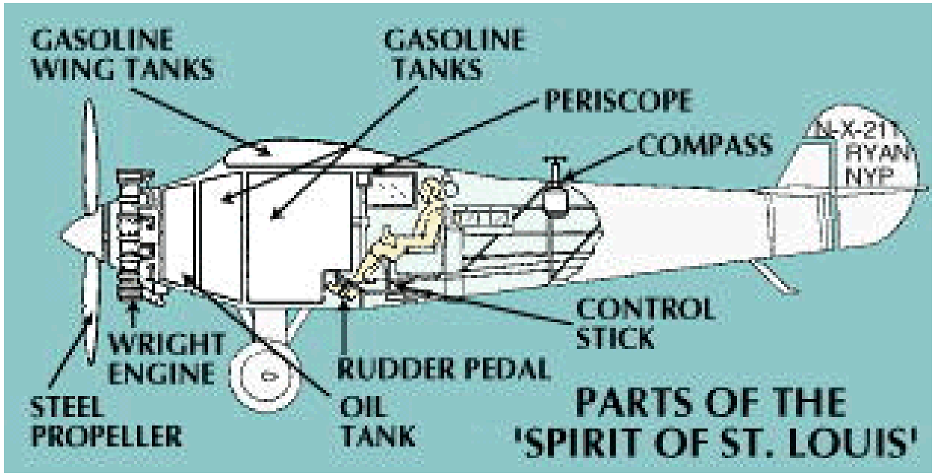
$$L = W$$

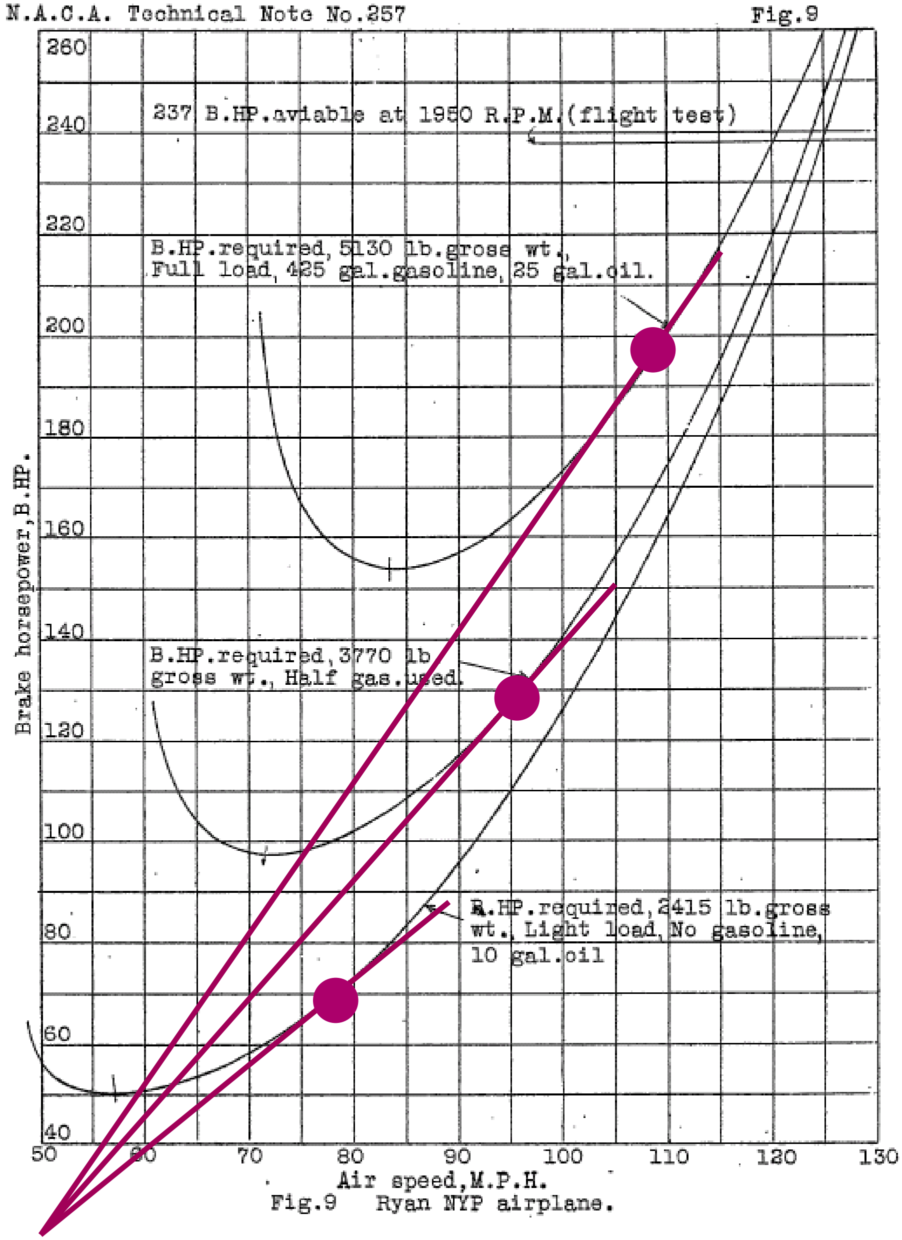
$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

# Range

## Example: Spirit of St. Louis

- Charles Lindbergh (1927)
- First solo, non stop flight across the Atlantic ocean






**Performance diagram**

Spirit of St Louis

(ref: NASA)

**Note**:

The $P_r$ curve shifts down when weight decreases. This means that Charles had to decrease airspeed during the flight

# Contents

# Range

## Jet aircraft

*So, at what speed should the pilot fly to maximise specific range?*

$$F = c_T T$$

Typically, $c_T$ can be considered to be constant

$$T = D$$

$$F = c_T T = c_T D$$

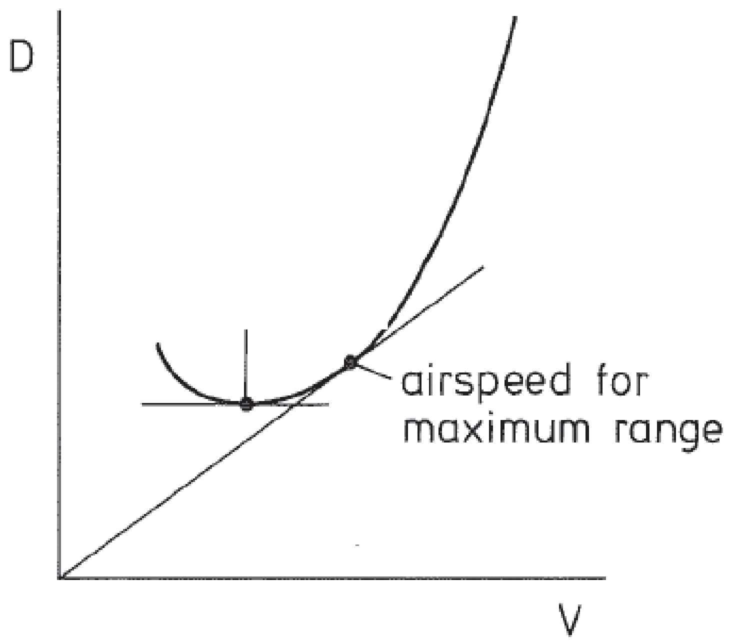
$$V/F = \frac{V}{c_T D}$$

$$\left(V/F\right)_{\text{max}} \Rightarrow \left(V/D\right)_{\text{max}} \Rightarrow \left(D/V\right)_{\text{min}}$$

# Range

## Jet aircraft

**Performance diagram (jet)**




$$\left(\frac{D}{V}\right)_{\max} = \left(\frac{D}{L} L \frac{1}{V}\right)_{\max}$$

$$L = W$$

$$\left(\frac{D}{V}\right)_{\max} = \left(\frac{C_D}{C_L} W \frac{1}{V}\right)_{\max}$$

$$C_L \tfrac{1}{2} \rho V^2 S = W \Leftrightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$\frac{D}{V} = \frac{C_D}{C_L} W \frac{1}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}} = \frac{W}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_L}{C_D^2}}}$$

$$\boxed{\left(\frac{D}{V}\right)_{\min} \Rightarrow \left(\frac{C_L}{C_D^2}\right)_{\max}}$$

# Problem!

Result:
Jet: maximum range at maximum $C_L / C_D^2$

***Dear pilot, could you please fly at $C_L / C_D^2$ max?***

We must give an airspeed to the pilot

# Solution

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Result:

Jet: maximum range at maximum $C_L / C_D^2$

$$C_D = f\left(C_L\right) \quad \Leftrightarrow \quad y = f(x)$$

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D^2}\right) = 0 \quad \Leftrightarrow \quad \frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{hg' - gh'}{h^2}$$

$$\frac{C_D^2 \cdot 1 - C_L \cdot 2C_D \dfrac{dC_D}{dC_L}}{C_D^4} = 0$$

$$\frac{dC_D}{dC_L} = \tfrac{1}{2}\frac{C_D}{C_L}$$

$$\frac{2C_L}{\pi A e} = \tfrac{1}{2}\frac{C_{D_0} + \dfrac{C_L^2}{\pi A e}}{C_L}$$

$$L = W$$

$$C_L = \sqrt{\tfrac{1}{3} C_{D_0} \pi A e}$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

# How environmentally (un)friendly are aircraft?

- Example: Boeing 747:
  - W = 3000 kN
  - S = 510 $m^2$
  - b = 60 m
  - $h_{cr}$ = 11000 m
  - $M_{cr}$ = 0.85


- Estimated:
  - $C_{D0}$ = 0.02; e = 0.85; $\eta_{tot}$ = 0.40

# How environmentally (un)friendly are aircraft really

- W/S = 5882 N/m$^2$; A = 7.06
- a = 295 m/s; V = 250 m/s (=900 km/hr)

$$C_L = \frac{W}{S}\frac{2}{\rho}\frac{1}{V^2} = 0.517$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} = 0.0342$$

$$\frac{C_L}{C_D} = 15.1$$

$$D = \frac{C_D}{C_L}W = 198.4 \text{ kN}$$

$$P_a = P_r = DV = 49591 \text{ kJ/s}$$

# How environmentally (un)friendly are aircraft really?

$$\eta_{tot} = \frac{P_a}{Q} = 0.40 \text{ estimated (in the order of efficiency power plant)}$$

$$F = \frac{g}{H}\frac{P_a}{\eta_{tot}} = 28.3 \text{ N/s}$$

$$H = 43000 \text{ kJ/kg}$$

$$\rho_{fuel} = 0.8 \text{ kg/m}^3$$

Fuel consumption: 12980 l/hr (in accordance with data manufacturer)

Every hour 900 km is travelled and 12980 l of fuel is used:

69 m/l

With 400 passengers: 27.7 pax km/l

# Comparison to a car

- Car driving 1:15 and 4 passengers: 60 pax km/l
- Conclusion: consumption of car is half the consumption of a 747 despite the speed ratio 9:1
- In practice ( seat occupation < 100% ) consumption per passenger km is similar for car and aviation traffic

# Contents

# Endurance

- Endurance: **time duration** spent in flight
- Do not confuse endurance with range!
- Solution: maximum endurance for **minimum fuel flow**
- $\mathbf{F_{min}}$


# Endurance

## Jet aircraft

$$c_T \equiv \frac{F}{T}$$

$$F = c_T T$$

Horizontal steady flight

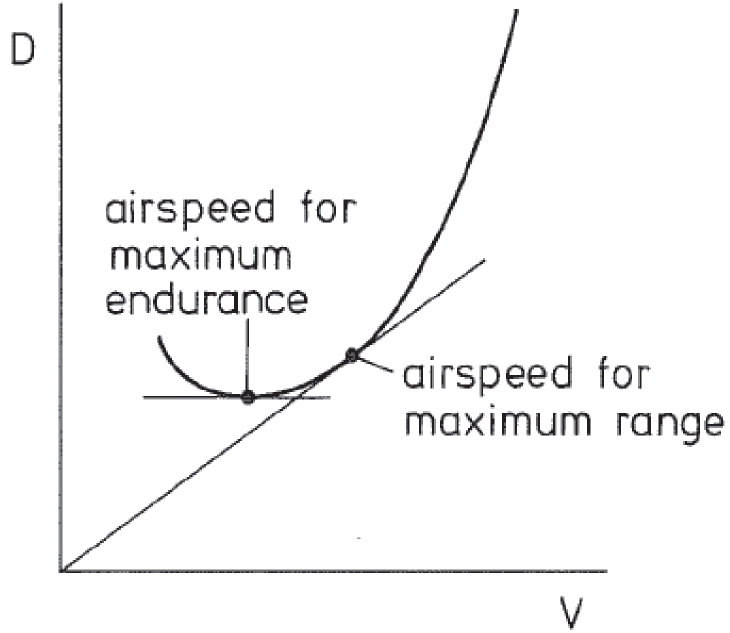
$$T = D$$

$$F = c_T D$$

Assumption: $c_T = \text{constant}$

$$F_{\min} \Rightarrow D_{\min} \Rightarrow \left(\frac{C_L}{C_D}\right)_{\max}$$

**Performance diagram (jet)**



# Endurance

## Propeller aircraft

$$c_P \equiv \frac{F}{P_{br}} \Leftrightarrow F = c_P \frac{P_a}{\eta_j}$$

$P_a = P_r$ (steady horizontal flight)

$$F = c_P \frac{P_r}{\eta_j} = c_P \frac{DV}{\eta_j}$$

$$F = c_P \frac{D}{\eta_j} \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}} =$$

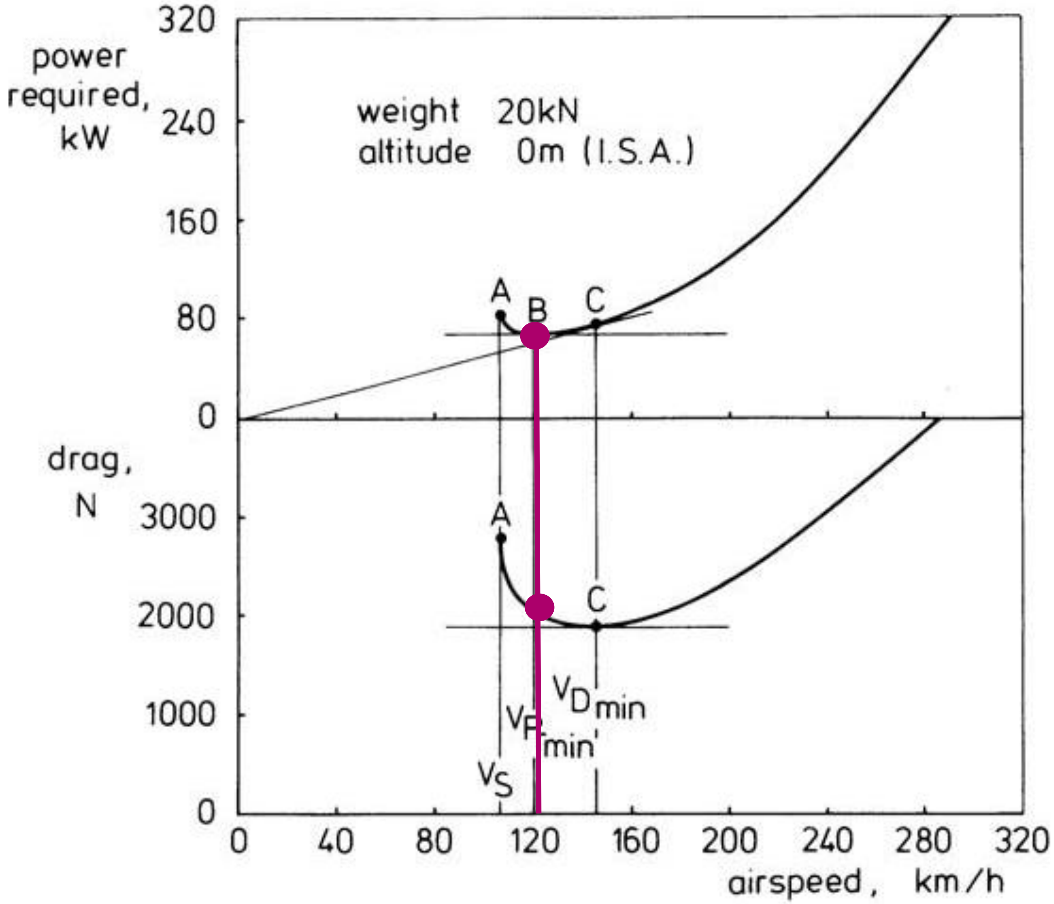
$$F = \frac{c_P}{\eta_j} \frac{C_D}{C_L} W \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}} = \frac{c_P}{\eta_j} \sqrt{\frac{W^3}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3}}$$

$\eta_j$ and $c_P$ can be assumed to be constant as a function of airspeed (over the range of cruising speeds) for propeller aircraft

$$E_{\max} \Rightarrow F_{\min} \Rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{\max} \Leftrightarrow (DV)_{\min} = P_{r,\min}$$

# Endurance

## Propeller aircraft




$P_r$ should be minimal!

# Problem!

<u>Result:</u>
Propeller: maximum endurance at maximum $C_L^3 / C_D^2$

***Dear pilot, could you please fly at $C_L^3 / C_D^2$ max?***

We must give an airspeed to the pilot

# Solution

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Result:

Prop: maximum endurance at maximum $C_L^3 / C_D^2$

$$C_D = f\left(C_L\right) \quad \Leftrightarrow \quad y = f(x)$$

$$\frac{d}{dC_L}\left(\frac{C_L^3}{C_D^2}\right) = 0 \quad \Leftrightarrow \quad \frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{hg' - gh'}{h^2}$$

$$\frac{C_D^2 \cdot 3C_L^2 - C_L^3 \cdot 2C_D \dfrac{dC_D}{dC_L}}{C_D^4} = 0$$

$$\frac{dC_D}{dC_L} = \tfrac{3}{2}\frac{C_D}{C_L}$$

$$\frac{2C_L}{\pi A e} = \tfrac{3}{2}\frac{C_{D_0} + \dfrac{C_L^2}{\pi A e}}{C_L}$$

$$L = W$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$C_L = \sqrt{3 C_{D_0} \pi A e}$$

# Contents

# Speed instability

- General equations of motion

$$\vec{F} = m\vec{a}$$

$$\parallel V: T\cos\alpha_T - D - W\sin\gamma = m\frac{dV}{dt}$$

$$\perp V: L - W\cos\gamma + T\sin\alpha_T = mV\frac{d\gamma}{dt}$$

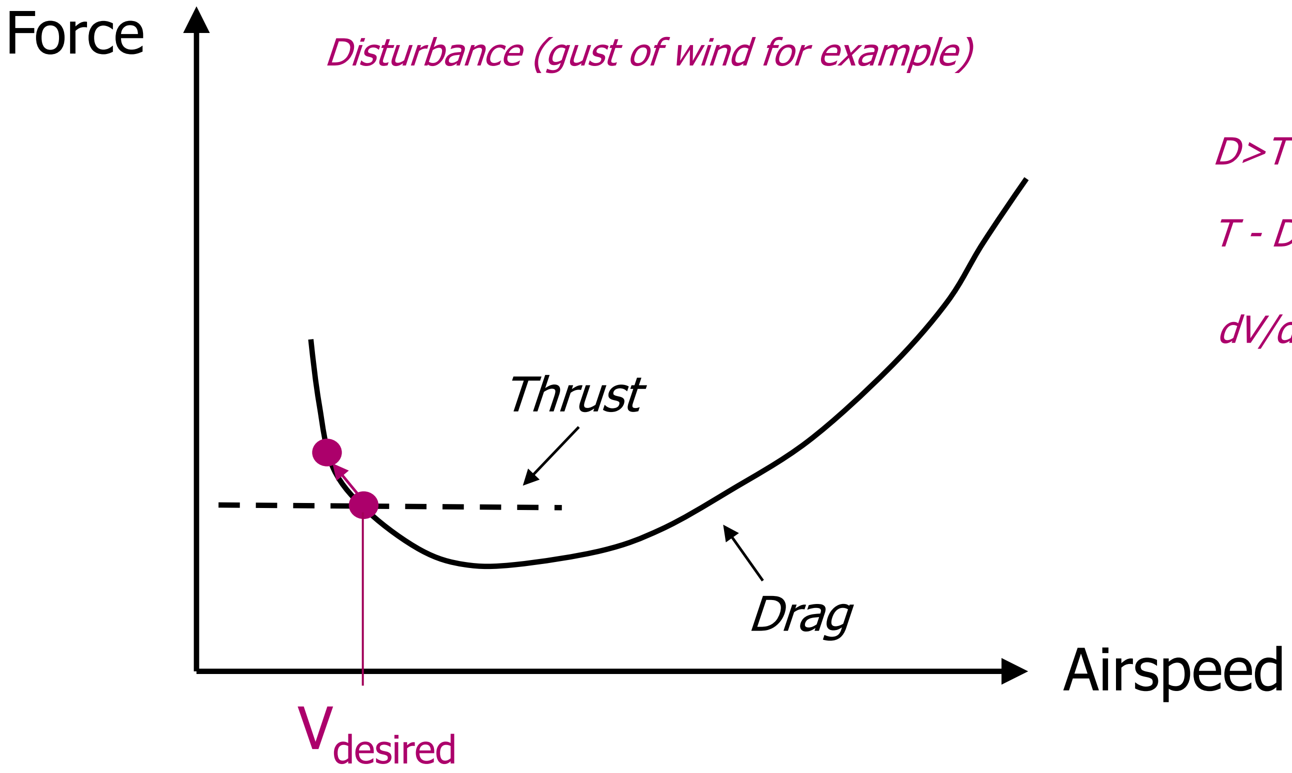
- Equations of motion **horizontal symmetric flight** (thrust parallel to airspeed

$$\vec{F} = m\vec{a}$$

$$\parallel V: T - D = m\frac{dV}{dt}$$

$$\perp V: L = W$$

# Speed instability



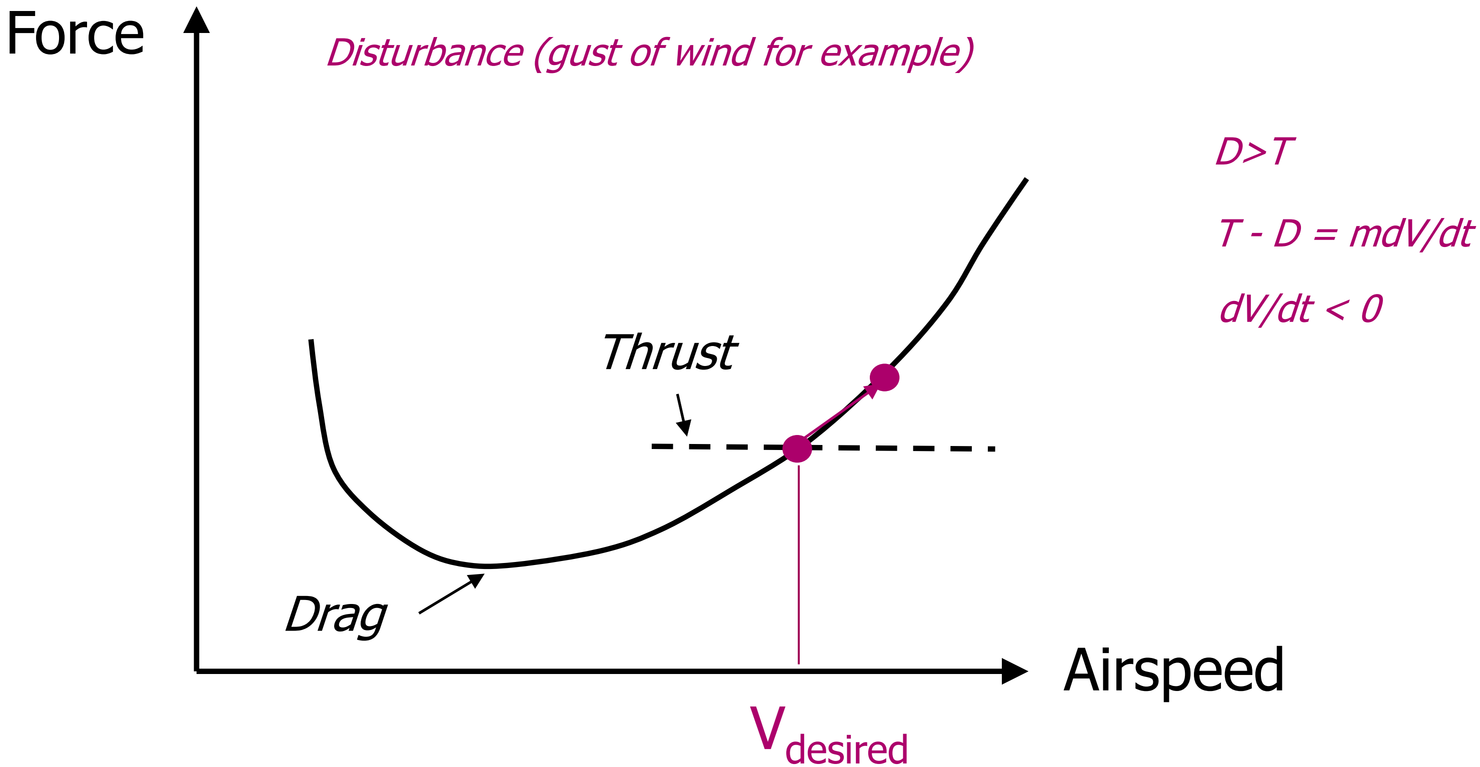

*D>T*

*T - D = mdV/dt*

*dV/dt < 0*

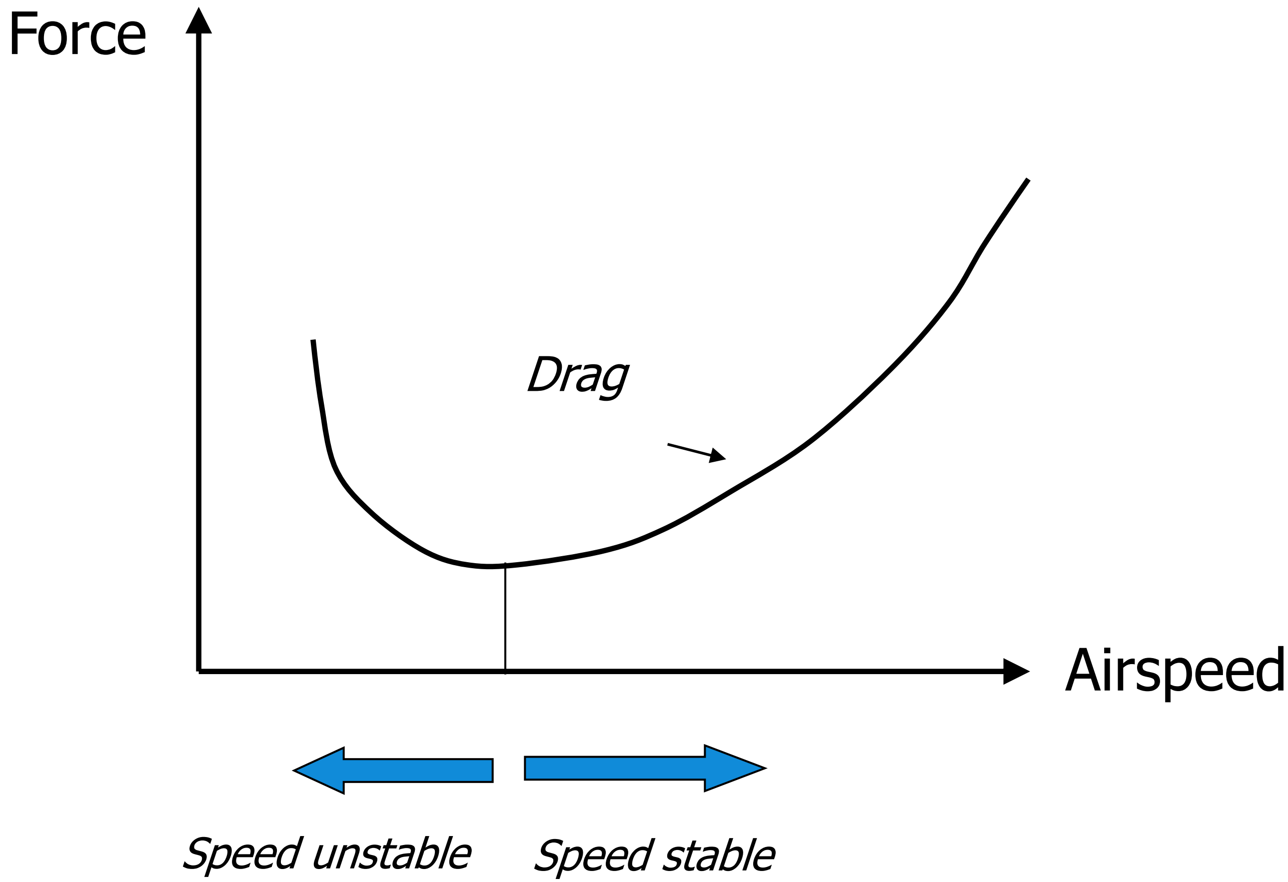
Conclusion: the aircraft is 'speed unstable' and will deviate from the original position following a disturbance. This makes it difficult for the pilot

# Speed stability



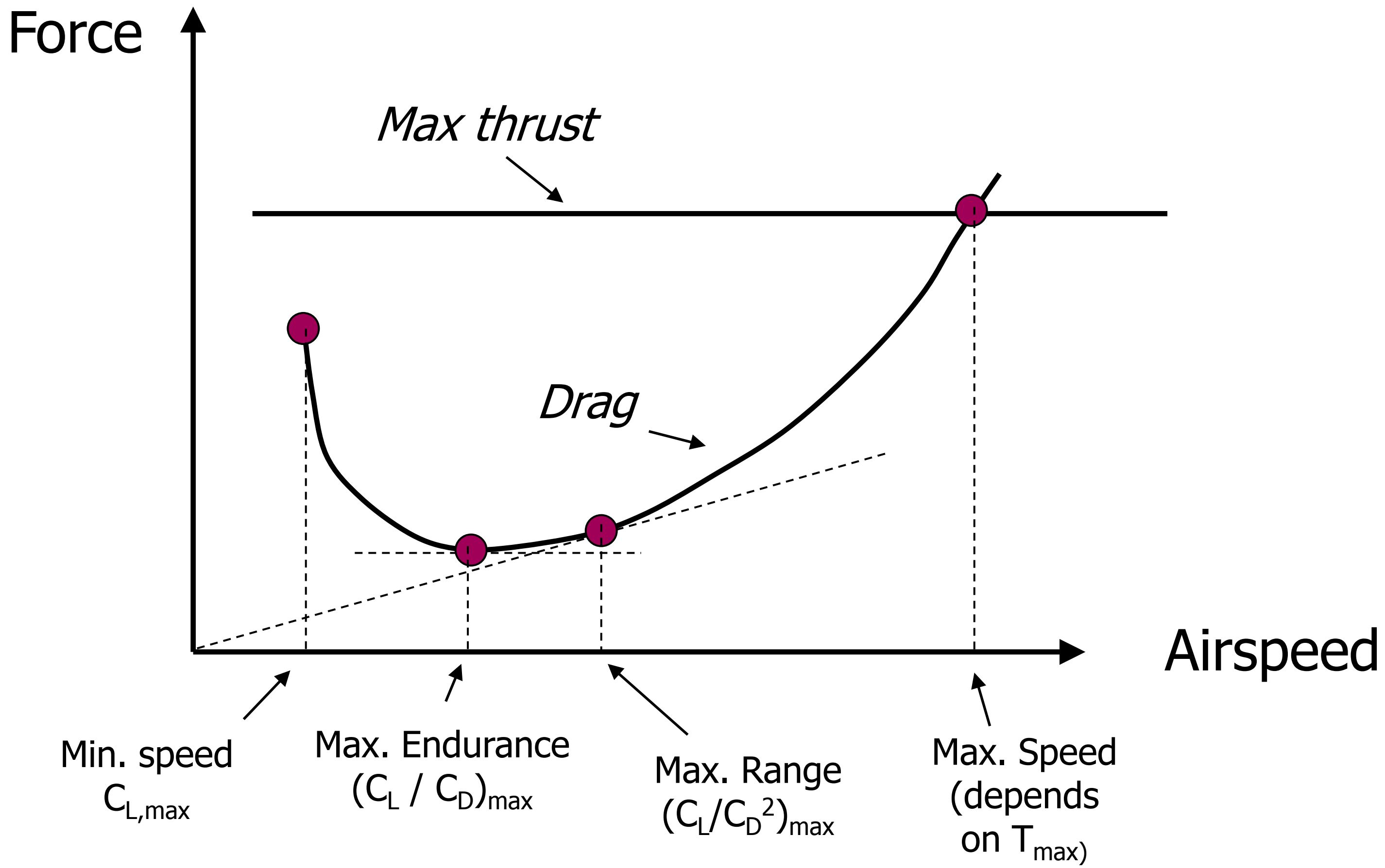

Conclusion: the aircraft is speed stable and the aircraft will automatically return to the original airspeed. Very nice for the pilot!

# Speed stability - instability



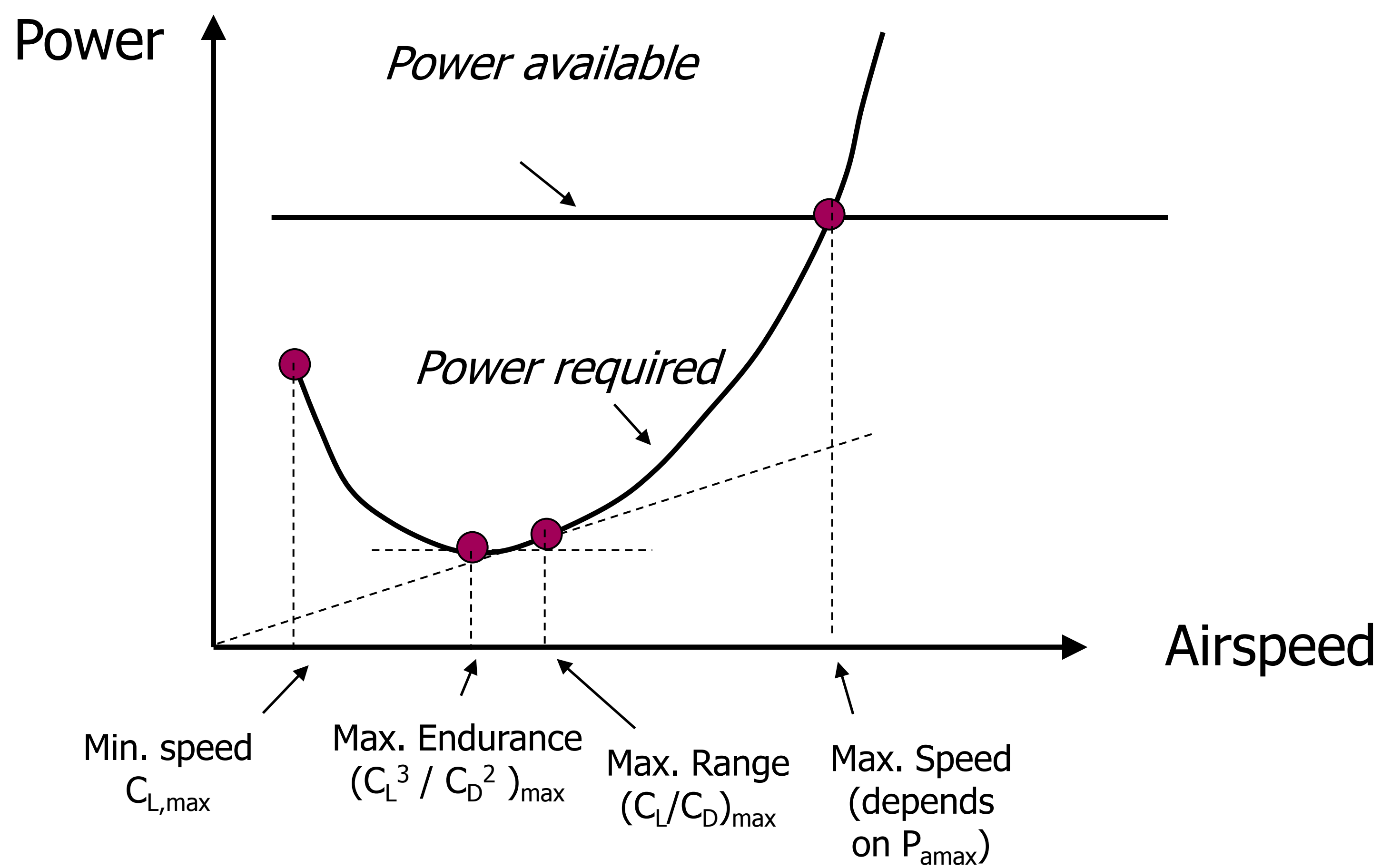
# Contents

# Summary - Jet



Force
Max thrust
Drag
Airspeed
Min. speed
$C_{L,max}$
Max. Endurance
$(C_L / C_D)_{max}$
Max. Range
$(C_L/C_D^2)_{max}$
Max. Speed
(depends
on $T_{max}$)

# Summary – Propeller Aircraft



# Summary – Horizontal flight

$$\boxed{\begin{aligned} L &= W \\ T &= D \end{aligned}}$$

<u>Minimum airspeed</u>

$$C_{L,\max} \longrightarrow V_{\min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L,\max}}}$$

<u>Maximum airspeed</u>

$$\left.\begin{aligned} T_{\max} &= D \text{ (Jet)} \\ P_{a,\max} &= P_r \text{ (Prop)} \end{aligned}\right. \longrightarrow C_L = \ldots \longrightarrow V_{\max} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

<u>Maximum range</u>

$$\left(\frac{V}{F}\right)_{\max} \xrightarrow{\text{jet}} \left(\frac{C_L}{C_D^2}\right)_{\max} \longrightarrow C_L = \sqrt{\tfrac{1}{3} C_{D_0} \pi A e}$$

$$\left(\frac{V}{F}\right)_{\max} \xrightarrow{\text{prop}} \left(\frac{C_L}{C_D}\right)_{\max} \longrightarrow C_L = \sqrt{C_{D_0} \pi A e}$$

$$\longrightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

# Summary – Horizontal flight

$$\boxed{\begin{aligned} L &= W \\ T &= D \end{aligned}}$$

<u>Maximum endurance</u>

$$\begin{tikzcd}
 & \left(\dfrac{C_L}{C_D}\right)_{\max} \arrow[r] & C_L = \sqrt{C_{D_0}\pi A e} \arrow[dr] & \\
(F)_{\min} \arrow[ur, "jet"] \arrow[dr, "prop"'] & & & V = \sqrt{\dfrac{W}{S}\dfrac{2}{\rho}\dfrac{1}{C_L}} \\
 & \left(\dfrac{C_L^3}{C_D^2}\right)_{\max} \arrow[r] & C_L = \sqrt{3C_{D_0}\pi A e} \arrow[ur] &
\end{tikzcd}$$

# Contents

# Example exam question (homework)

## Problem

The Gulfstream IV, indicated in Figure 1 is a twin-turbofan executive transport aircraft. Data for this aircraft are given below:

$S = 88.3$ [m$^2$]
$b = 23.7$ [m]
$A = 6.36$ [-]
$e = 0.67$ [-]
$C_{D0} = 0.015$ [-]
$C_D = C_{D0} + C_L^2 / \pi Ae$
$W = 311000$ [N]

This aircraft has two *Rolls Royce Tay* turbofan engines. Fuel consumption can be represented with the following equation: $F = c_T T$, where $c_T = 0.69$ [N/hr/N]. The thrust specific fuel consumption is assumed to be independent of airspeed and altitude

**Question**:

What is the maximum specific range of this aircraft when flying at 8000m ($\rho = 0.5252$ kg/m$^3$) ?

# Example exam question (homework)

## Solution (part 1)

Specific range = V/F (m/kg) = $V/c_T T$

Steady horizontal flight: T = D; L = W

$V/F = V/c_T D$

$c_T$ is a constant so maximum specific range when (D/V) is minimal

$$D = D\frac{L}{L} = \frac{C_D}{C_L}W$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$\frac{D}{V} = \frac{C_D}{C_L}W\frac{1}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}} = \frac{W}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_L}{C_D^2}}}$$

So $C_L / C_D^2$ should be maximal. This is the case when $C_L = \sqrt{(C_{D0}\pi Ae/3)}$

(you should derive this on the exam, see sheet 48)

$$C_L = \sqrt{\tfrac{1}{3}\cdot 0.015\pi\cdot 6.36\cdot 0.67} = 0.26$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi Ae} = 0.015 + \frac{0.26^2}{\pi\cdot 6.36\cdot 0.67} = 0.02$$

# Example exam question (homework)

## Solution (part 2)

From the lift coefficient, we can calculate the airspeed

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{311000}{88.3}\frac{2}{0.5252}\frac{1}{0.26}} = 227 \text{ [m/s]}$$

The aerodynamic drag can be calculated with the drag coefficient. This must be equal to the thrust

$$T = D = \frac{C_D}{C_L}W = \frac{0.02}{0.26}311000 = 23.9 \text{ [kN]}$$

Now, all parameters are known to calculate the maximum specific range for this altitude and aircraft weight

$$\frac{V}{F} = \frac{V}{c_T T} = \frac{227}{(0.69/3600)\cdot 23900} = 50 \text{ [m/N]}$$