Introduction to Aerospace Engineering

Lecture slides

Introduction Aerospace Engineering

Flight and Mechanics

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5.&6

Climbing and descending flight

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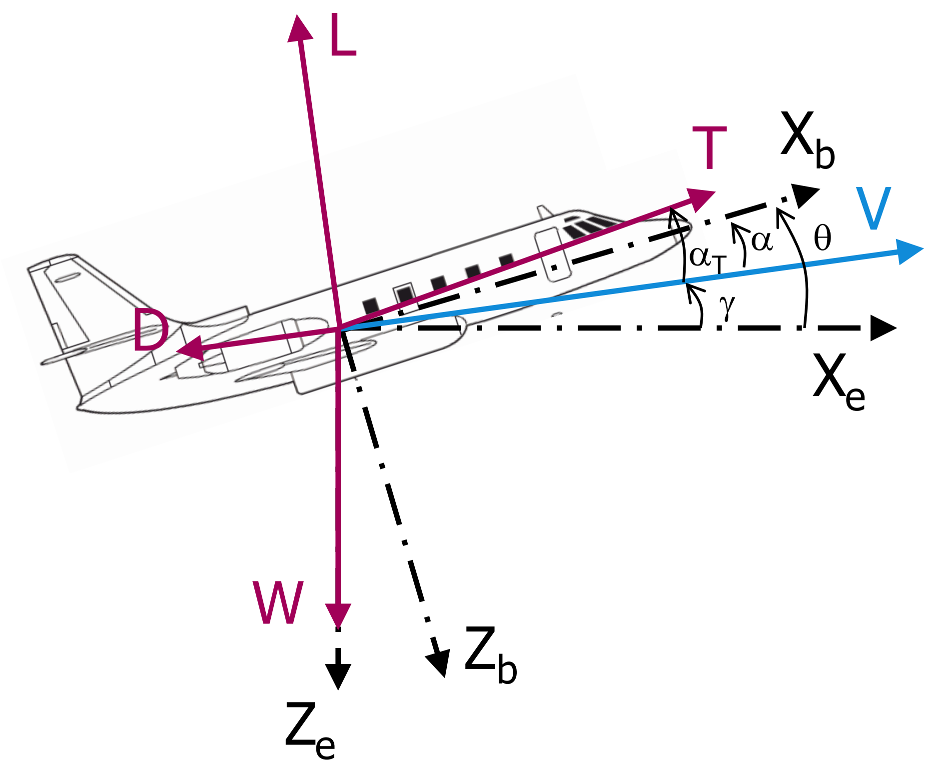
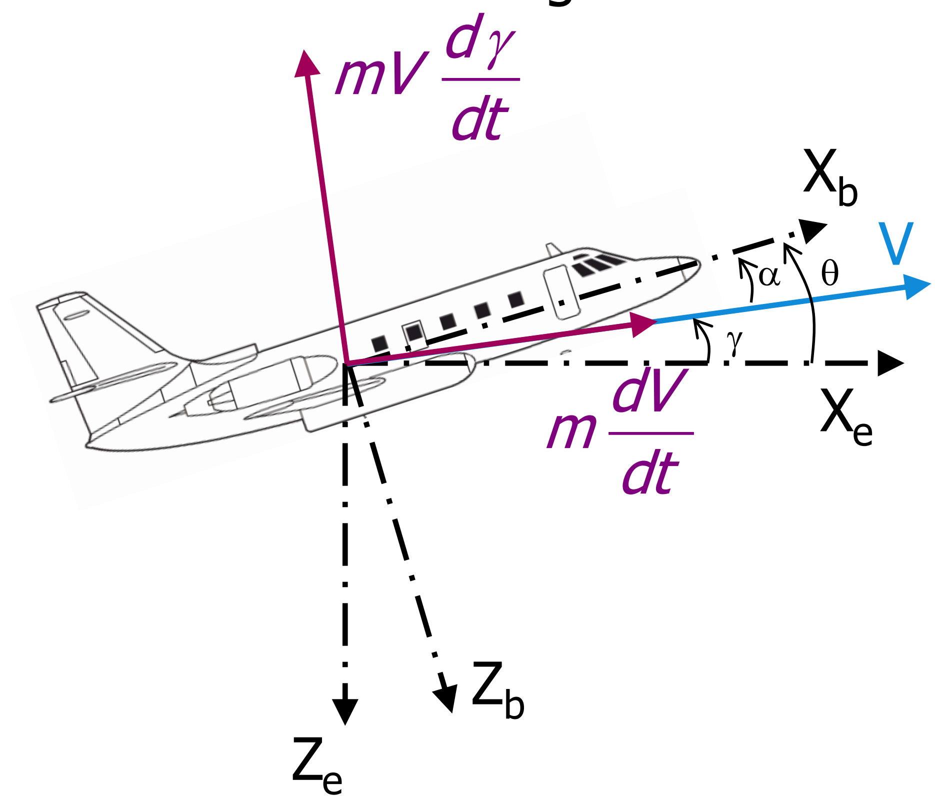
Equations of motion

General equations of motion in symmetric flight (1)

Free Body Diagram



Kinetic Diagram



Equations of motion

Horizontal, steady, symmetric flight

$$\vec{F} = m\vec{a}$$

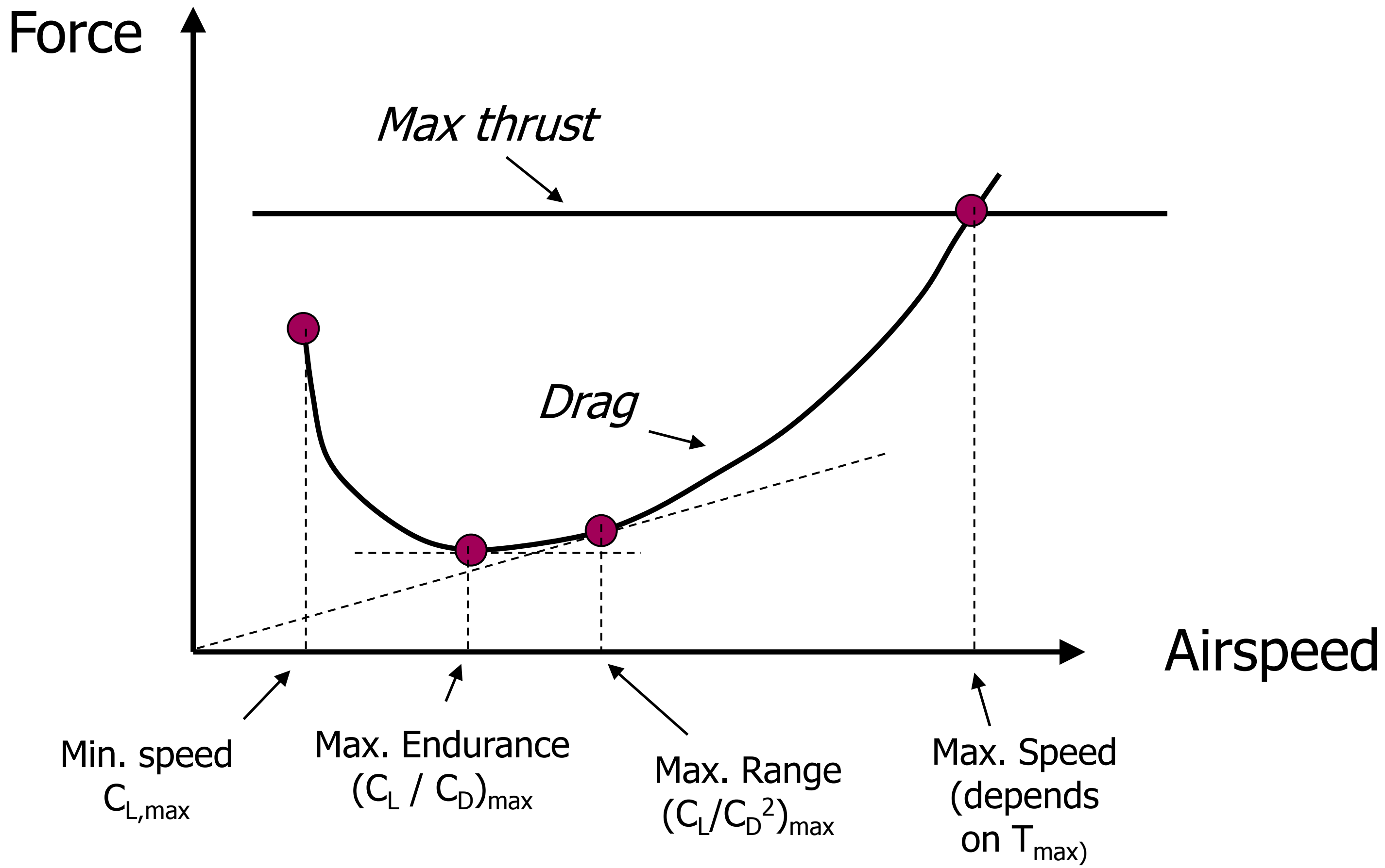
$$\parallel V: T\,\overbrace{\cancel{\cos\alpha_T}}^{=1} - D - W\,\overbrace{\cancel{\sin\gamma}}^{=0} = m\overbrace{\cancel{\frac{dV}{dt}}}^{=0}$$

$$\perp V: L - W\,\overbrace{\cancel{\cos\gamma}}^{=1} + T\,\overbrace{\cancel{\sin\alpha_T}}^{=0} = mV\overbrace{\cancel{\frac{d\gamma}{dt}}}^{=0}$$

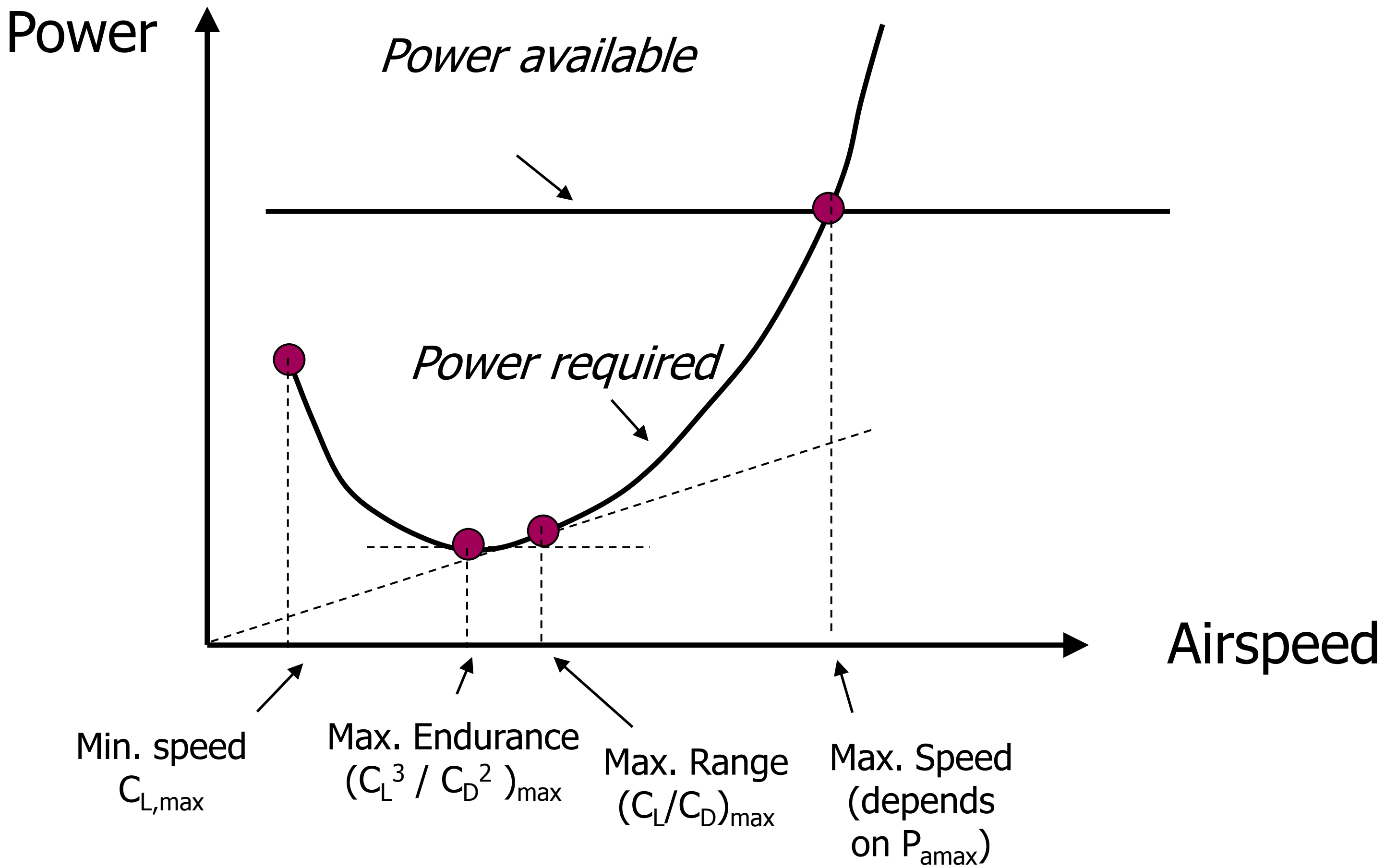
$$\boxed{\begin{aligned}\parallel V&: T = D\\ \perp V&: L = W\end{aligned}}$$

Note: Clearly this is the most simple form of the equations of motion

Summary - Jet


Force
Max thrust
Drag
Airspeed
Min. speed
$C_{L,max}$
Max. Endurance
$(C_L / C_D)_{max}$
Max. Range
$(C_L/C_D^2)_{max}$
Max. Speed
(depends
on T_{max})

Summary – Propeller Aircraft



Summary – Horizontal flight

$$\boxed{\begin{aligned} L &= W \\ T &= D \end{aligned}}$$

Minimum airspeed

$$C_{L,\max} \longrightarrow V_{\min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L,\max}}}$$

Maximum airspeed

$$\begin{aligned} T_{\max} &= D \text{ (Jet)} \\ P_{a,\max} &= P_r \text{ (Prop)} \end{aligned} \longrightarrow C_L = \ldots \longrightarrow V_{\max} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

Maximum range

$$\left(\frac{V}{F}\right)_{\max}$$

jet:

$$\left(\frac{C_L}{C_D^2}\right)_{\max} \longrightarrow C_L = \sqrt{\tfrac{1}{3} C_{D_0} \pi A e}$$

prop:

$$\left(\frac{C_L}{C_D}\right)_{\max} \longrightarrow C_L = \sqrt{C_{D_0} \pi A e}$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

Summary – Horizontal flight

$$\boxed{\begin{aligned} L &= W \\ T &= D \end{aligned}}$$

<u>Maximum endurance</u>

$$(F)_{\min} \xrightarrow{\text{jet}} \left(\frac{C_L}{C_D}\right)_{\max} \longrightarrow C_L = \sqrt{C_{D_0}\pi A e} \longrightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$(F)_{\min} \xrightarrow{\text{prop}} \left(\frac{C_L^3}{C_D^2}\right)_{\max} \longrightarrow C_L = \sqrt{3C_{D_0}\pi A e} \longrightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

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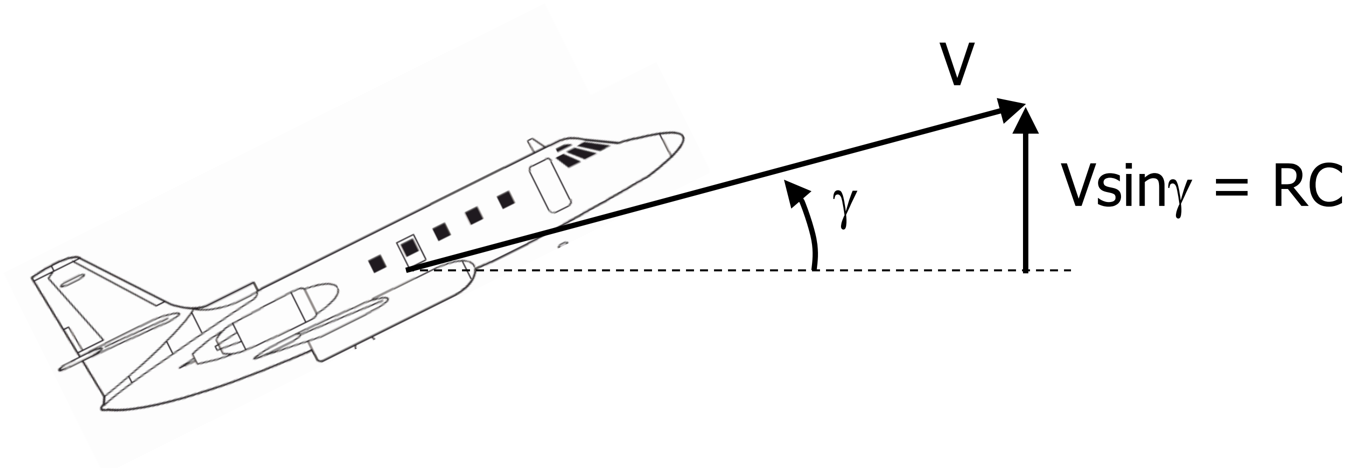
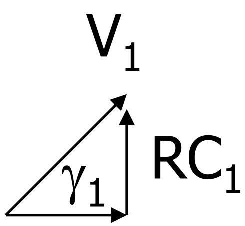
Introduction

- How fast can an aircraft climb?
- How steep can an aircraft climb?
- How far can an aircraft glide following an engine failure?

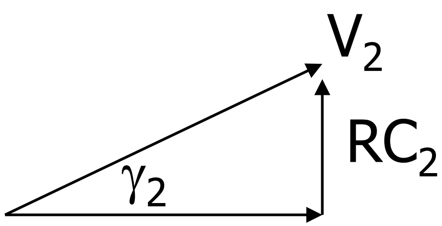
Rate of climb versus flight path angle



Example case 1



Example case 2



$\gamma_1 > \gamma_2$ however, $RC_1 < RC_2$!

So do not confuse the flight path angle with the rate of climb

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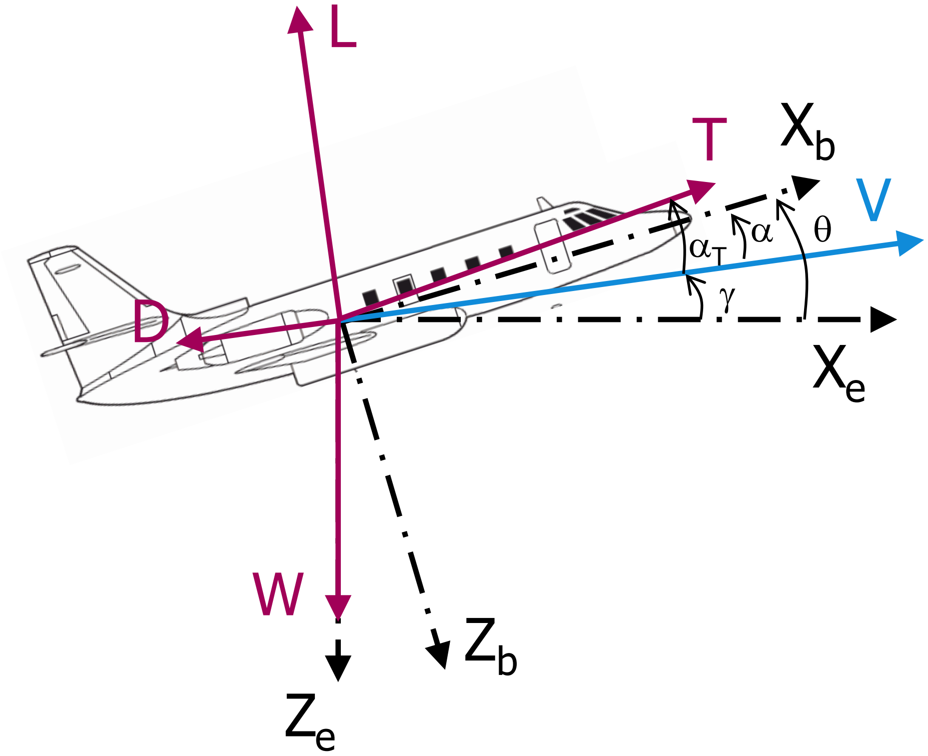
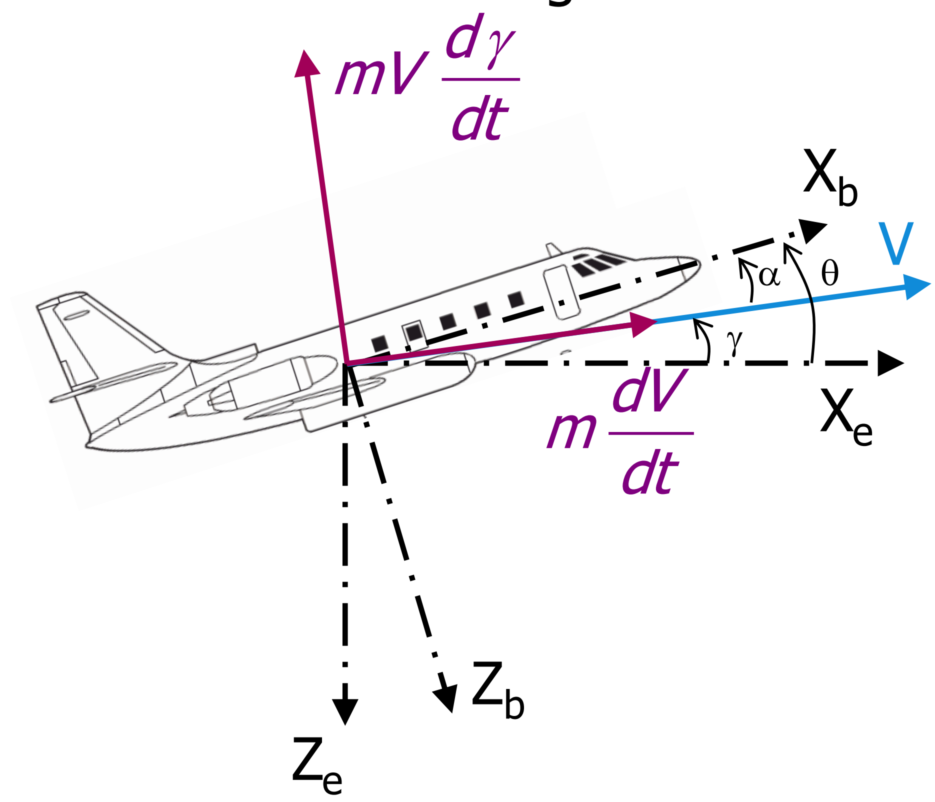
Equations of motion

General equations of motion in symmetric flight (1)

Free Body Diagram



Kinetic Diagram



Equations of motion

General equations of motion in symmetric flight (2)

$$\vec{F} = m\vec{a}$$

$$\square V: T\cos\alpha_T - D - W\sin\gamma = m\frac{dV}{dt}$$

$$\perp V: L - W\cos\gamma + T\sin\alpha_T = mV\frac{d\gamma}{dt}$$

<u>Simplification:</u>
The aircraft is performing a **straight** and **symmetric** flight.

<u>Assumption:</u>
The thrust is assumed to be in the direction of flight ($\alpha_T = 0$).

Equations of motion

definitions

- **Straight flight**: flight in which the centre of gravity of the aircraft travels along a straight line ($d\gamma/dt = 0$)
- **Steady flight**: Flight in which the forces and moments acting on the aircraft do not vary in time, neither in magnitude, nor in direction ($dV/dt = 0$)
- **Horizontal flight**: The aircraft remains at a constant altitude ($\gamma = 0$)
- **Symmetric flight**: flight in which both the angle of sideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth ($\beta = 0$ and the aircraft is not turning)

Equations of motion

Straight, symmetric flight

$$\vec{F} = m\vec{a}$$

$$\square V : T \cos \overbrace{\cancel{\alpha_T}}^{=1} - D - W \sin\gamma = m \frac{dV}{dt}$$

$$\perp V : L - W \overbrace{\cancel{\cos\gamma}}^{\cong 1} + T \overbrace{\cancel{\sin\alpha_T}}^{=0} = mV \overbrace{\cancel{\frac{d\gamma}{dt}}}^{=0}$$

$$\square V : T - D - W \sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\perp V : L = W$$

Power equation

Straight, symmetric flight

$$\square V : T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\perp V : L = W$$

Multiply first equation with airspeed

$$\frac{P_a - P_r}{W} = V\sin\gamma + \frac{1}{2g}\frac{dV^2}{dt} \left(= \text{potential} + \text{kinetic energy increase}\right)$$

$$L = W$$

So, excess power (P_a – P_r) can be used to climb or to increase airspeed

Contents

Gliding flight

$$\boxed{T = P_a = 0}$$

Question:
How far can a Boeing 747 glide from 10km altitude?

Answer:
Approximately 180 km!

Norden
Aurich
Wilhelmshaven
Groningen
Oldenburg (Oldenburg)
Emmen
Norwich
Lowestoft
Alkmaar
Zwolle
Lelystad
Amsterdam
Almere
Deventer
Den Haag
Utrecht
Apeldoorn
Enschede
Osnabrück
Bielefeld
Münster
Cambridge
Ipswich
Colchester
Westland
Zoetermeer
Arnhem
Chelmsford
Rotterdam
Nijmegen
's-Hertogenbosch
Basildon
Breda
Hamm
Southend-on-sea
Tilburg
Duisburg
Dortmund
Gillingham
Eindhoven
Essen
Brugge
Antwerpen
Wuppertal
Gent
Düsseldorf
Solingen
Dunkerque
Calais
Geleen
Köln
Heerlen
Siegen
Brussel
Bruxelles
Aachen
Bonn
Roubaix
Lille
Liège
Namur
Charleroi
Koblenz
50 km
50 mi
©2009 Google

Gliding flight

Power equation

$$\boxed{\begin{aligned} &T = P_a = 0 \\ &-P_r = W\frac{dH}{dt} + \frac{W}{2g}\frac{dV^2}{dt} \end{aligned}}$$

What if the pilot tries to fly a horizontal path?

$$-P_r = \frac{W}{2g}\frac{dV^2}{dt} \Rightarrow \text{ aircraft decelerates}$$

Clearly this is not an option… glide at constant airspeed:

$$\boxed{-P_r = W\frac{dH}{dt}\left(= WV\sin\gamma\right)}$$

Best gliding performance

1. RC_{min} (long time)
2. γ_{min} (long distance)

<u>Option 1</u>

$$\boxed{\begin{array}{l} -P_r = WV\sin\gamma \\ RC_{min} \Rightarrow P_{r,min} \end{array}}$$

<u>Option 2</u>

$$\boxed{\begin{array}{l} \dfrac{-DV}{WV} = \sin\gamma \\ \gamma_{min} \Rightarrow D_{min} \end{array}}$$

<u>Conclusion:</u>

1. To glide as **far** as possible, one must glide at the condition for minimum drag *(e.g. useful for engine failure)*
2. To glide as **long** (time wise) as possible, one must glide at the condition for minimum power required *(e.g. useful for gliders)*

Note: This is not treated in 'introduction to flight'

Glide as far as possible

$$\gamma = \arcsin\left(-D/W\right) = \arcsin\left(-D/L\right)$$

$$\gamma = \arcsin\left(-C_D/C_L\right)$$

$$\left(\frac{C_L}{C_D}\right)_{\max} \Rightarrow C_L = \sqrt{C_{D_0}\pi Ae} \quad \Rightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

Question: Does the aircraft weight influence the minimum glide angle

a. Yes
b. No
c. ?

Glide as far as possible

$$\gamma = \arcsin\left(-D/_W\right) = \arcsin\left(-D/_L\right)$$

$$\gamma = \arcsin\left(-C_D/_{C_L}\right)$$

$$\left(\frac{C_L}{C_D}\right)_{\max} \Rightarrow C_L = \sqrt{C_{D_0}\pi Ae} \quad \Rightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

Question: Does the aircraft weight influence the minimum glide angle

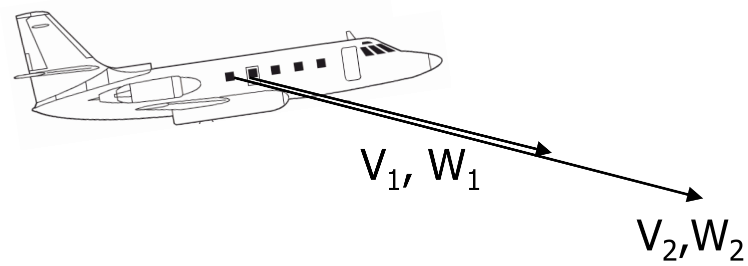
a. Yes
b. No
c. ?

Glide as far as possible

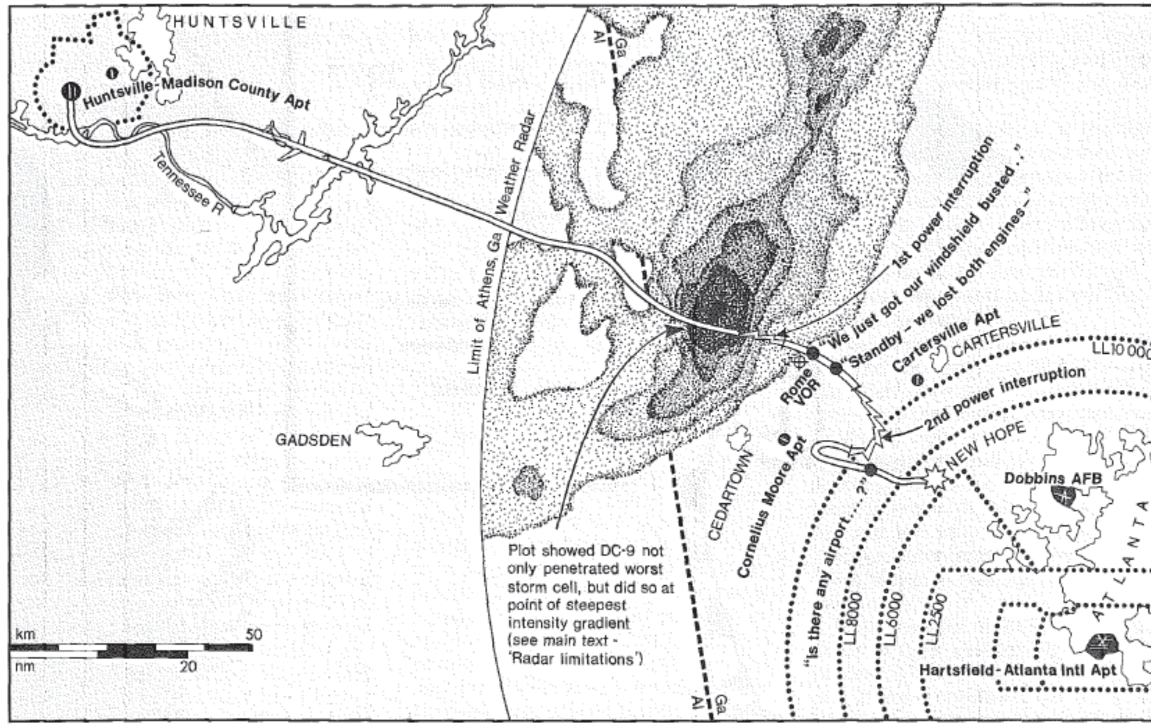
Strangely enough, the glide angle is independent of the aircraft weight!

$$W_1 > W_2$$

$$\gamma_1 = \gamma_2 \text{ and } V_1 < V_2$$



Story: *We have lost both engines*



Larger scale map of northern Alabama and Georgia, showing track flown by N1335U from Huntsville to accident site, as determined by investigators from radar plots and eyewitness sightings. Some published accounts of this accident have speculated that the seemingly inexplicable turn back towards the west might have been the result of the crew's sighting of Cornelius Moore Airport through breaks in the rain and cloud as they descended. Loss of visual contact, or a sudden realisation of the airport's unsuitability have similarly been held as the reason for the further course reversal back towards the southeast. The second interruption to the DC-9's electrical power at this time, however, obliterated any evidence there might have been on the CVR to support this theory. The stippling shows areas of storm activity recorded by National Weather Service radar at the time of the aircraft's total loss of engine power, its density indicating the estimated intensity of precipitation. (Matthew Tesch, with acknowledgement to NTSB)

Story: *We have lost both engines*

Question: Could the aircraft have made it to Dobbins Air Force Base if the pilots had decided to glide there?

W = 90,000 lb (= 400500 N)
S = 1001 sq ft (=93 m^2)
b = 93.3 ft (=28.4 m)
C_{D0} = 0.02
e = 0.85

Distance to Dobbins AFB: 20 nautical miles (=36 km)
Altitude: 7000 ft (=2134 m)

Story: *We have lost both engines*

Answer: Yes!

Maximum distance when C_L/C_D is max. So:

$$\left(A = \frac{b^2}{S} = \frac{28.4^2}{93} = 8.7\right)$$

$$C_L = \sqrt{C_{D_0} \pi A e} = \sqrt{0.02 \cdot \pi \cdot 8.7 \cdot 0.85} = 0.68$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} = 0.02 + \frac{0.68^2}{\pi \cdot 8.7 \cdot 0.85} = 0.04$$

Now the glide angle and distance can be calculated:

$$\gamma = \arcsin\left(-\frac{C_D}{C_L}\right) = \arcsin\left(-\frac{0.04}{0.68}\right) = -3.4 \text{ [deg]}$$

$$\text{distance} = h / \tan\gamma = 2134 / \tan 3.4 = 36 \text{ km}$$

Glide as long as possible

Minimum rate of descent

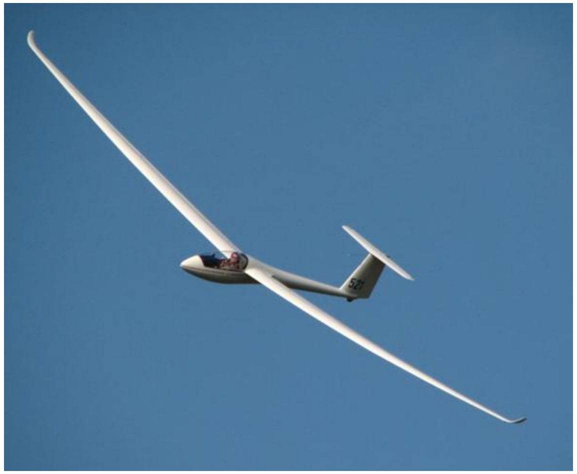
$$\boxed{-P_r = WV\sin\gamma}$$

$$P_r = DV$$

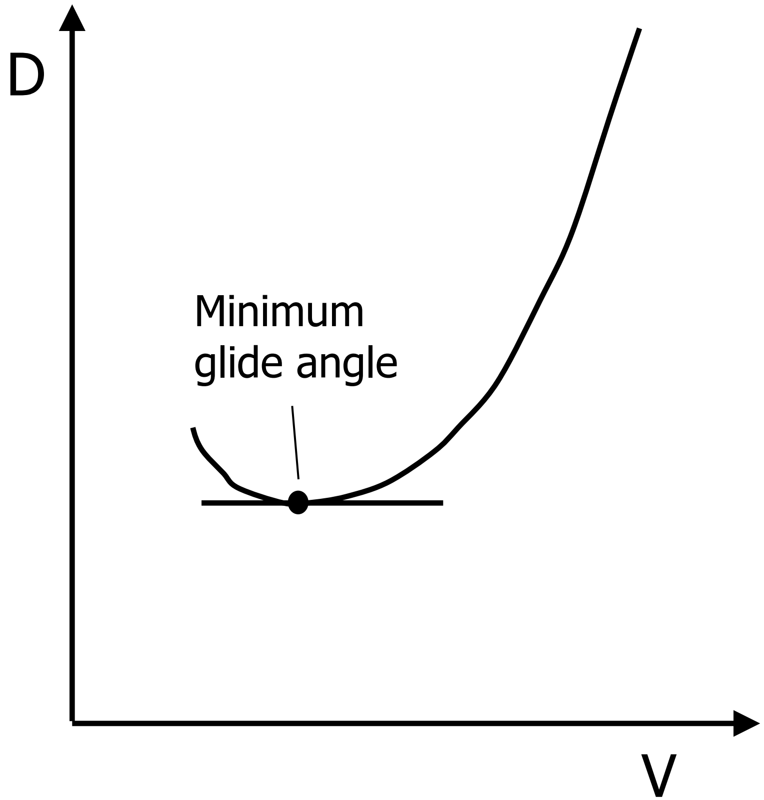
$$D = \frac{C_D}{C_L}W \Rightarrow P_r = W\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

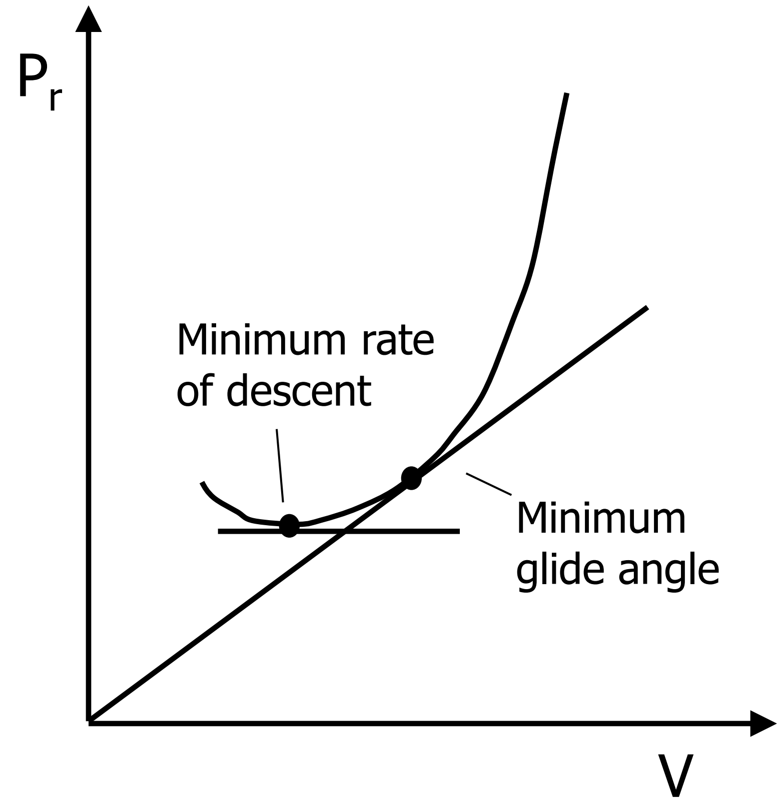
$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$P_{r,\min} \Rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{\max} \Rightarrow C_L = \sqrt{3C_{D_0}\pi Ae}$$



Summary gliding flight





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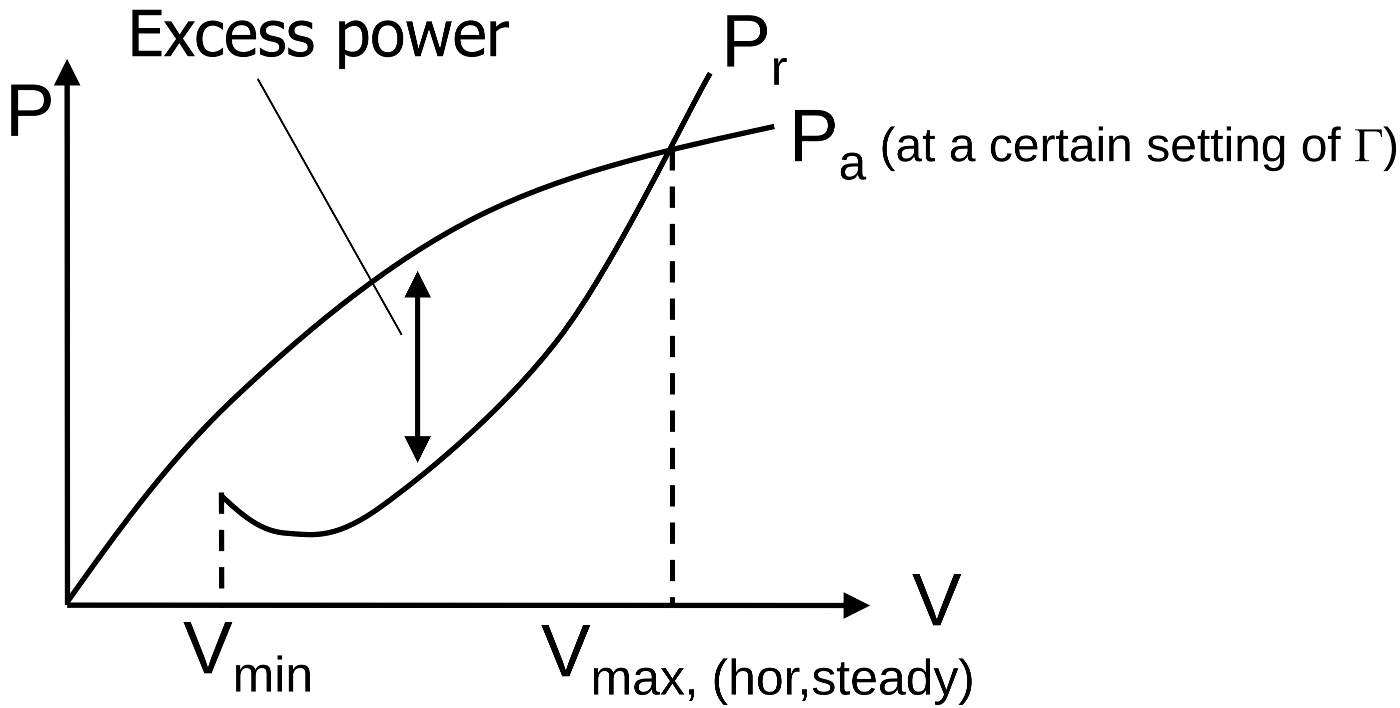
Climbing performance

$$\frac{P_a - P_r}{W} = V\sin\gamma + \frac{1}{2g}\frac{dV^2}{dt} \left(= \text{potential} + \text{kinetic energy increase}\right)$$

$$L = W$$



Climbing performance

Steady flight

- Maximum rate of climb (propeller)
- Steepest climb (jet)

Climbing performance

Steady flight

- **Maximum rate of climb (propeller)**
- Steepest climb (jet)

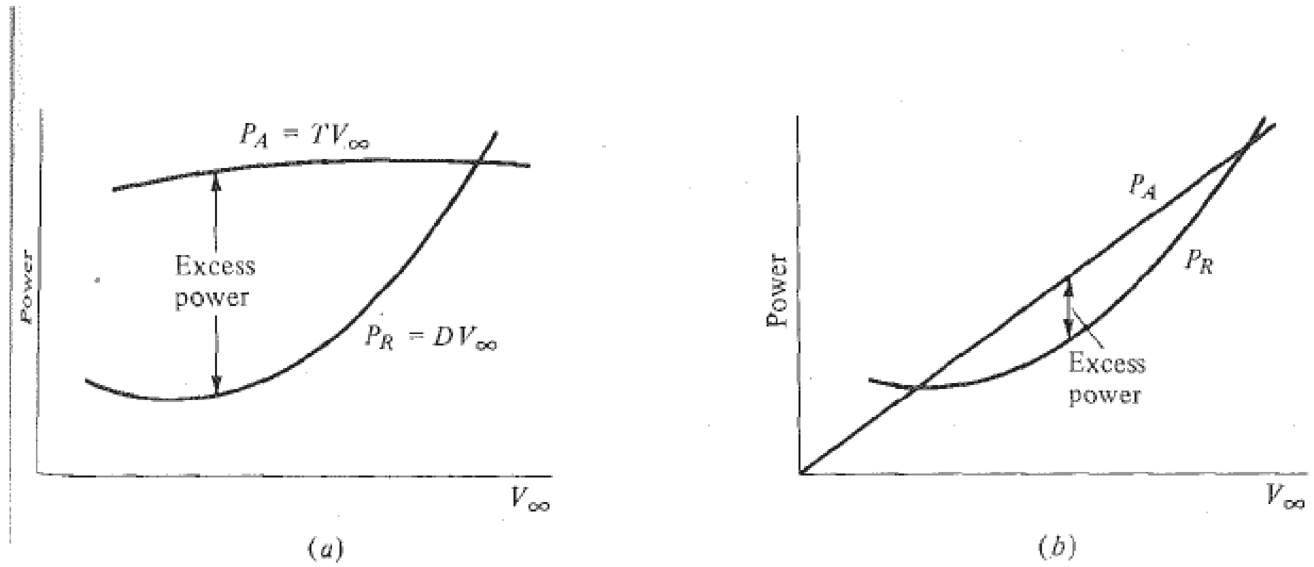
Maximum rate of climb

Steady flight

Maximum rate of climb → maximum excess power $(P_a - P_r)_{max}$

$$\frac{P_a - P_r}{W} = V \sin\gamma \left(= RC\right)$$

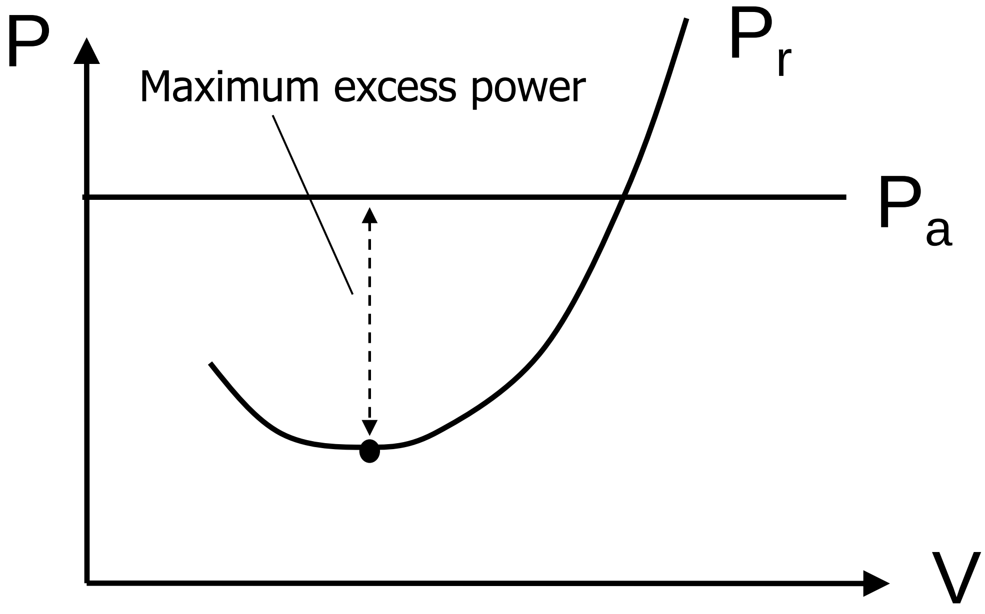
$$L = W$$



Solutions

Maximum rate of climb propeller aircraft

Assumption: P_a is independent of airspeed



RC_{max} at $P_{r,min}$
This corresponds to the following condition:

$$\left(\frac{C_L^3}{C_D^2}\right)_{max}$$

$$\Rightarrow C_L = \sqrt{3C_{D_0}\pi Ae}$$

Climbing performance

Steady flight

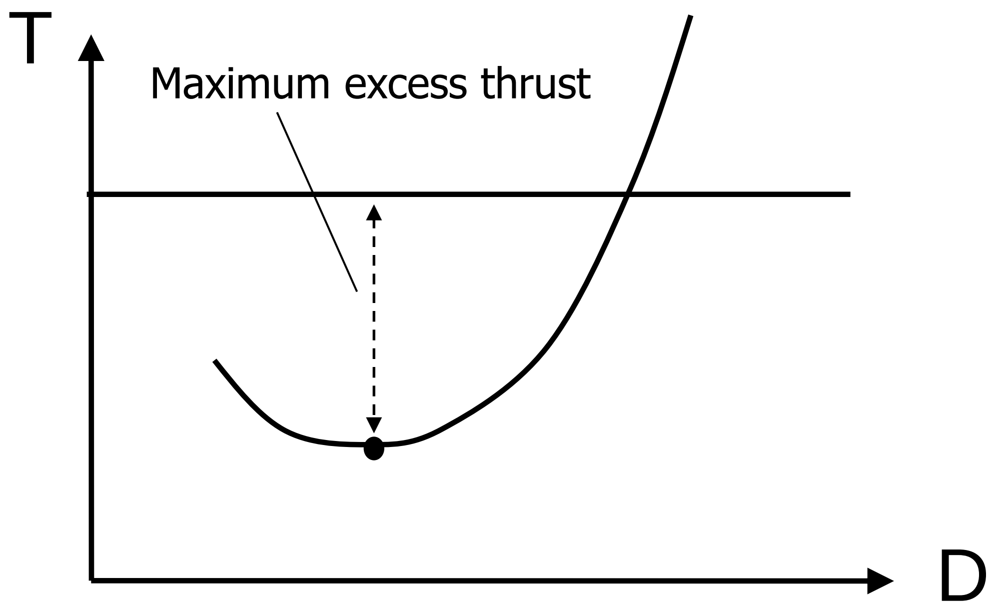
- Maximum rate of climb (propeller)
- **Steepest climb (jet)**

Steepest climb

Jet aircraft

$$\frac{T-D}{W} = \sin\gamma$$

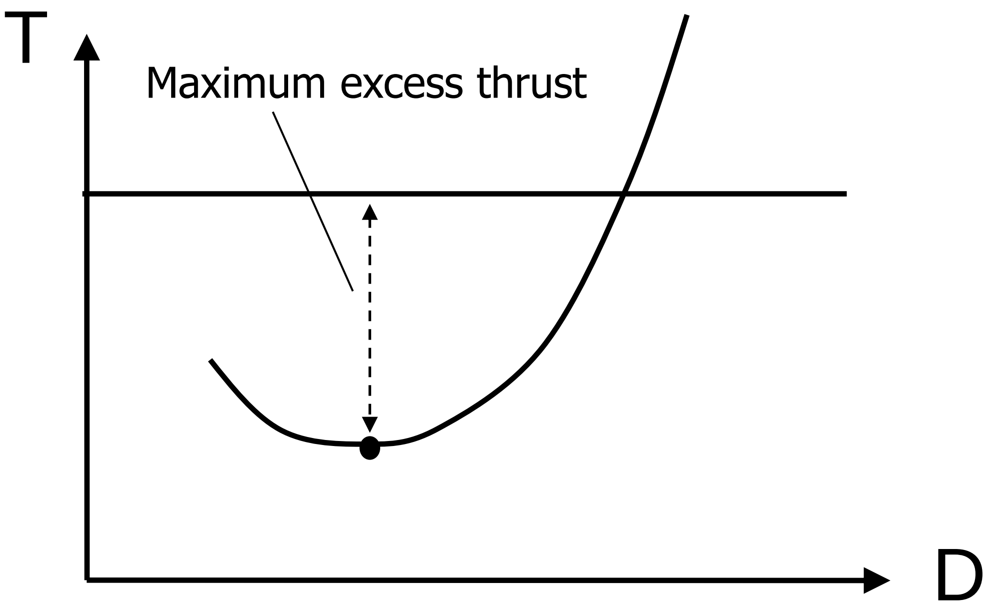
$$L = W$$



Solutions

Steepest climb for jet aircraft

Assumption: T is independent of airspeed



γ_{max} at D_{min}

This corresponds to the following condition:

$$\boxed{\left(\frac{C_L}{C_D}\right)_{max} \Rightarrow C_L = \sqrt{C_{D_0} \pi A e}}$$

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Example 1

Climbing performance of the Beach King Air

Two engine propeller aircraft

$C_D = C_{D0} + kC_L^2$
$C_{D0} = 0.02$
$k = 0.04$
$W = 60$ [kN]
$S = 28.2$ [m^2]

Maximum power available (741 kW) can be assumed independent of airspeed. The aircraft is performing a steady symmetrical climb

Question: *What is the maximum rate of climb of this aircraft at sea-level (ρ = 1.225 [kg/m^3] and what is the corresponding airspeed*

Example 1

Solution

Power equation:

$$\frac{P_a - P_r}{W} = V\sin\gamma + \frac{1}{2g}\frac{dV^2}{dt}$$

steady flight

$$\frac{P_a - P_r}{W} = V\sin\gamma = RC$$

Maximum power available is independent of airspeed (741 kW)

RC_{max} at $P_{r,min}$

$$\left.\begin{aligned} P_r &= DV \\ D &= \frac{C_D}{C_L}W \end{aligned}\right\} P_r = W\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

$$P_{r,\min} \Rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{\max} \Rightarrow C_L = \sqrt{3\frac{C_{D_0}}{k}} = \sqrt{3\frac{0.02}{0.04}} = 1.22$$

$$L = W \Rightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{60000}{28.2}\frac{2}{1.225}\frac{1}{1.22}} = 53 \text{ [m/s] (=190 km/h)}$$

$$C_D = C_{D_0} + kC_L^2 = 0.02 + 0.04 \cdot 0.41^2 = 0.08$$

$$P_r = DV = C_D \tfrac{1}{2}\rho V^3 S = 0.08 \cdot \tfrac{1}{2} \cdot 1.225 \cdot 53^3 \cdot 28.2 = 206 \text{ [kW]}$$

$$RC = \frac{P_a - P_r}{W} = \frac{741000 - 206000}{60000} = 8.9 \text{ [m/s]}$$