Introduction to Aerospace Engineering

Lecture slides
Introduction Aerospace Engineering
Flight and Mechanics

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5. & 6

Climbing and descending flight
Contents

1. Summary horizontal flight
2. Introduction climbing and descending flight
3. Equations of motion straight and symmetric flight
4. Gliding flight
5. Climbing flight
6. Example calculations
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1. **Summary horizontal flight**
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Equations of motion
General equations of motion in symmetric flight (1)

Free Body Diagram

Kinetic Diagram

\[ L \]
\[ T \]
\[ X_b \]
\[ V \]

\[ \alpha \]
\[ \gamma \]
\[ \theta \]

\[ mV \frac{d\gamma}{dt} \]
\[ m \frac{dV}{dt} \]
Equations of motion
Horizontal, steady, symmetric flight

\[ F = m \ddot{a} \]

\[ V : T \cos \alpha_T - D - W \sin \gamma = m \frac{dV}{dt} = 0 \]

\[ L - W \cos \gamma + T \sin \alpha_T = mV \frac{dV}{dt} = 0 \]

Note: Clearly this is the most simple form of the equations of motion
Summary - Jet

- **Max thrust**

- **Airspeed**

- **Force**

  - Min. speed $C_{L,max}$
  - Max. Endurance $(C_L / C_D)_{max}$
  - Max. Range $(C_L/C_D^2)_{max}$
  - Max. Speed (depends on $T_{max}$)

- **Drag**
Summary – Propeller Aircraft

- Power required
- Power available

- Min. speed $C_{L,\text{max}}$
- Max. Endurance $(C_L^3 / C_D^2)_{\text{max}}$
- Max. Range $(C_L/C_D)_{\text{max}}$
- Max. Speed (depends on $P_{\text{amax}}$)

Airspeed

Power

$\text{Power available}$
Summary – Horizontal flight

\[
L = W
\]

\[
T = D
\]

Minimum airspeed

\[
C_{L,max}
\]

\[
V_{\min} = \sqrt{\frac{W}{S \rho} \frac{1}{C_{L,max}}}
\]

Maximum airspeed

\[
T_{\max} = D \text{ (Jet)}
\]

\[
P_{a,max} = P_r \text{ (Prop)}
\]

\[
C_L = \ldots
\]

\[
V_{\max} = \sqrt{\frac{W}{S \rho} \frac{1}{C_L}}
\]

Maximum range

\[
\left( \frac{V}{F} \right)_{\max}
\]

\[
\left( \frac{C_L}{C_D} \right)_{\max}^{\text{jet}}
\]

\[
C_L = \sqrt{\frac{1}{3} C_{D0} \pi A e}
\]

\[
V = \sqrt{\frac{W}{S \rho} \frac{1}{C_L}}
\]

\[
\left( \frac{C_L}{C_D} \right)_{\max}^{\text{prop}}
\]

\[
C_L = \sqrt{C_{D0} \pi A e}
\]
Summary – Horizontal flight

\[ L = W \]
\[ T = D \]

Maximum endurance

\[
\left( \frac{C_L}{C_D} \right)_{\text{max}} \rightarrow C_L = \sqrt{C_{D_0} \pi A e}
\]

\[
(F)_{\text{min}} \quad \text{jet} \quad \left( \frac{C_L}{C_D} \right)_{\text{max}} \rightarrow C_L = \sqrt{3C_{D_0} \pi A e}
\]

\[
(F)_{\text{min}} \quad \text{prop} \quad \left( \frac{C_L}{C_D^2} \right)_{\text{max}} \rightarrow V = \sqrt{\frac{W}{S \rho C_L}}
\]
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Introduction

- How fast can an aircraft climb?
- How steep can an aircraft climb?
- How far can an aircraft glide following an engine failure?
Rate of climb versus flight path angle

\[ V \sin \gamma = RC \]

Example case 1

\[ V_1 \]

\[ \gamma_1 \]

\[ RC_1 \]

Example case 2

\[ V_2 \]

\[ \gamma_2 \]

\[ RC_2 \]

\[ \gamma_1 > \gamma_2 \] however, \[ RC_1 < RC_2 \]!

So do not confuse the flight path angle with the rate of climb
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General equations of motion in symmetric flight (1)

Free Body Diagram

Kinetic Diagram

\[ mV \frac{d\gamma}{dt} \]

\[ m \frac{dV}{dt} \]
Equations of motion

General equations of motion in symmetric flight (2)

\[ \ddot{F} = m\ddot{a} \]

\[ V : T \cos \alpha_T - D - W \sin \gamma = m \frac{dV}{dt} \]

\[ \perp V : L - W \cos \gamma + T \sin \alpha_T = mV \frac{d\gamma}{dt} \]

Simplification:
The aircraft is performing a **straight** and **symmetric** flight.

Assumption:
The thrust is assumed to be in the direction of flight \((\alpha_T = 0)\).
Equations of motion

definitions

- **Straight flight**: flight in which the centre of gravity of the aircraft travels along a straight line \( \frac{d\gamma}{dt} = 0 \)

- **Steady flight**: Flight in which the forces and moments acting on the aircraft do not vary in time, neither in magnitude, nor in direction \( \frac{dV}{dt} = 0 \)

- **Horizontal flight**: The aircraft remains at a constant altitude \( \gamma = 0 \)

- **Symmetric flight**: flight in which both the angle of sideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth \( \beta = 0 \) and the aircraft is not turning)
Equations of motion

Straight, symmetric flight

\[ \ddot{F} = m\ddot{a} \]

\[ \dot{V} : T \cos \alpha_T - D - W \sin \gamma = m \frac{dV}{dt} = 0 \]

\[ \perp V : L - W \cos \gamma + T \sin \alpha_T = mV \frac{d\gamma}{dt} = 0 \]

\[ \dot{V} : T - D - W \sin \gamma = \frac{W}{g} \frac{dV}{dt} \]

\[ \perp V : L = W \]
Power equation

Straight, symmetric flight

\[ V : T - D - W \sin \gamma = \frac{W}{g} \frac{dV}{dt} \]
\[ \perp V : L = W \]

Multiply first equation with airspeed

\[ \frac{P_a - P_r}{W} = V \sin \gamma + \frac{1}{2g} \frac{dV^2}{dt} \quad (= \text{potential + kinetic energy increase}) \]
\[ L = W \]

So, excess power \((P_a - P_r)\) can be used to climb or to increase airspeed
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Gliding flight

\[ T = P_a = 0 \]

**Question:**  
How far can a Boeing 747 glide from 10km altitude?

**Answer:**  
Approximately 180 km!
Gliding flight

Power equation

\[ T = P_a = 0 \]

\[ -P_r = W \frac{dH}{dt} + \frac{W}{2g} \frac{dV^2}{dt} \]

What if the pilot tries to fly a horizontal path?

\[ -P_r = \frac{W}{2g} \frac{dV^2}{dt} \Rightarrow \text{aircraft decelerates} \]

Clearly this is not an option... glide at constant airspeed:

\[ -P_r = W \frac{dH}{dt} (= WV \sin \gamma) \]
Best gliding performance

1. $RC_{\text{min}}$ (long time)
2. $\gamma_{\text{min}}$ (long distance)

Option 1

$$
-P_r = WV \sin \gamma \\
RC_{\text{min}} \Rightarrow P_r,_{\text{min}}
$$

Option 2

$$
\frac{-DV}{WV} = \sin \gamma \\
\gamma_{\text{min}} \Rightarrow D_{\text{min}}
$$

Conclusion:
1. To glide as far as possible, one must glide at the condition for minimum drag (e.g. useful for engine failure)
2. To glide as long (time wise) as possible, one must glide at the condition for minimum power required (e.g. useful for gliders)

Note: This is not treated in 'introduction to flight'
Glide as far as possible

\[ \gamma = \arcsin\left(-\frac{D}{W}\right) = \arcsin\left(-\frac{D}{L}\right) \]

\[ \gamma = \arcsin\left(-\frac{C_D}{C_L}\right) \]

\[ \left(\frac{C_L}{C_D}\right)_{\text{max}} \Rightarrow C_L = \sqrt{C_{D0}\pi Ae} \quad \Rightarrow V = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}} \]

**Question:** Does the aircraft weight influence the minimum glide angle

a. Yes
b. No
c. ?
Glide as far as possible

\[ \gamma = \arcsin \left( \frac{-D}{W} \right) = \arcsin \left( \frac{-D}{L} \right) \]

\[ \gamma = \arcsin \left( \frac{-C_D}{C_L} \right) \]

\[ \left( \frac{C_L}{C_D} \right)_{\text{max}} \Rightarrow C_L = \sqrt{C_{D_0} \pi A e} \Rightarrow V = \sqrt{\frac{W}{S} \frac{2}{\rho C_l}} \]

**Question:** Does the aircraft weight influence the minimum glide angle

a. Yes
b. No
c. ?
Glide as far as possible

Strangely enough, the glide angle is independent of the aircraft weight!

\[ W_1 > W_2 \]
\[ \gamma_1 = \gamma_2 \text{ and } V_1 < V_2 \]
Story: We have lost both engines
Story: We have lost both engines

Question: Could the aircraft have made it to Dobbins Air Force Base if the pilots had decided to glide there?

\[
\begin{align*}
W &= 90,000 \text{ lb} \ (= 400500 \text{ N}) \\
S &= 1001 \text{ sq ft} \ (= 93 \text{ m}^2) \\
b &= 93.3 \text{ ft} \ (= 28.4 \text{ m}) \\
C_{D_0} &= 0.02 \\
e &= 0.85 \\
\end{align*}
\]

Distance to Dobbins AFB: 20 nautical miles \ (= 36 \text{ km})
Altitude: 7000 ft \ (= 2134 \text{ m})
Story: *We have lost both engines*

Answer: Yes!

Maximum distance when $C_L/C_D$ is max. So:

\[
(A = \frac{b^2}{S} = \frac{28.4^2}{93} = 8.7)
\]

\[
C_L = \sqrt{C_{D_0} \pi Ae} = \sqrt{0.02 \cdot \pi \cdot 8.7 \cdot 0.85} = 0.68
\]

\[
C_D = C_{D_0} + \frac{C_L^2}{\pi Ae} = 0.02 + \frac{0.68^2}{\pi \cdot 8.7 \cdot 0.85} = 0.04
\]

Now the glide angle and distance can be calculated:

\[
\gamma = \arcsin \left( -\frac{C_D}{C_L} \right) = \arcsin \left( -\frac{0.04}{0.68} \right) = -3.4 \text{ [deg]}
\]

distance = $h / \tan \gamma = 2134 / \tan 3.4 = 36 \text{ km}$
Glide as long as possible

Minimum rate of descent

\[-P_r = WV \sin \gamma\]

\[P_r = DV\]

\[D = \frac{C_D}{C_L} W\]

\[V = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}}\]

\[-P_{r,\text{min}} \Rightarrow \left( \frac{C_L^3}{C_D^2} \right)_{\text{max}} \Rightarrow C_L = \sqrt{3C_{D_0} \pi Ae}\]
Summary gliding flight

Minimum glide angle

Minimum rate of descent

Minimum glide angle
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Climbing performance

\[ \frac{P_a - P_r}{W} = V \sin \gamma + \frac{1}{2g} \frac{dV^2}{dt} \]  
\[ (\text{potential + kinetic energy increase}) \]

\[ L = W \]

Excess power

\[ P_a \] (at a certain setting of \( \Gamma \))

\[ V_{\text{min}} \]

\[ V_{\text{max, (hor,steady)}} \]
Climbing performance

Steady flight

- Maximum rate of climb (propeller)
- Steepest climb (jet)
Climbing performance

Steady flight

- Maximum rate of climb (propeller)
- Steepest climb (jet)
Maximum rate of climb

Steady flight

Maximum rate of climb $\rightarrow$ maximum excess power $(P_a - P_r)_{\text{max}}$

\[
\frac{P_a - P_r}{W} = V \sin \gamma \left( = RC \right)
\]

$L = W$
Solutions

Maximum rate of climb propeller aircraft

Assumption: $P_a$ is independent of airspeed

$$\pi \frac{C_L^3}{C_D^2} = 3$$

This corresponds to the following condition:

$$C_L = \sqrt{3C_D \pi A e}$$
Climbing performance

Steady flight

- Maximum rate of climb (propeller)
- Steepest climb (jet)
Steepest climb
Jet aircraft

\[
\frac{T - D}{W} = \sin \gamma \\
L = W
\]

Maximum excess thrust
Solutions
Steepest climb for jet aircraft

Assumption: $T$ is independent of airspeed

$\gamma_{\text{max}}$ at $D_{\text{min}}$
This corresponds to the following condition:

$$\left( \frac{C_L}{C_D} \right)_{\text{max}}$$

$$\Rightarrow C_L = \sqrt{C_{D_0} \pi A e}$$
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Example 1

Climbing performance of the Beach King Air

Two engine propeller aircraft

\[ C_D = C_{D0} + kC_L^2 \]
\[ C_{D0} = 0.02 \]
\[ k = 0.04 \]
\[ W = 60 \ [kN] \]
\[ S = 28.2 \ [m^2] \]

Maximum power available (741 kW) can be assumed independent of airspeed. The aircraft is performing a steady symmetrical climb.

**Question:** What is the maximum rate of climb of this aircraft at sea-level \((\rho = 1.225 \ [kg/m^3])\) and what is the corresponding airspeed?
Example 1

Solution

Power equation:
\[ \frac{P_a - P_r}{W} = V \sin \gamma + \frac{1}{2g} \frac{dV^2}{dt} \]

steady flight
\[ \frac{P_a - P_r}{W} = V \sin \gamma = RC \]

Maximum power available is independent of airspeed (741 kW)
RC_{\text{max}} \text{ at } P_{r,\text{min}}
\[ P_r = DV \]
\[ D = \frac{C_D}{C_L} W \]
\[ P_r = W \frac{\sqrt{W 2 C_D^2}}{S \rho C_L} \]

\[ P_{r,\text{min}} \Rightarrow \left( \frac{C_L^3}{C_D^2} \right)_{\text{max}} \Rightarrow C_L = \sqrt{3 \frac{C_{\rho_0}}{k}} = \sqrt{3 \frac{0.02}{0.04}} = 1.22 \]

\[ L = W \Rightarrow V = \frac{W 2 \frac{1}{S \rho C_L}}{\sqrt{28.2 \frac{1}{2}} 1.225 1.22} = 53 \text{ [m/s]} (=190 \text{ km/h}) \]

\[ C_D = C_{\rho_0} + kC_L^2 = 0.02 + 0.04 \cdot 0.41^2 = 0.08 \]

\[ P_r = DV = C_D \frac{1}{2} \rho V^3 S = 0.08 \cdot \frac{1}{2} \cdot 1.225 \cdot 53^3 \cdot 28.2 = 206 \text{ [kW]} \]

\[ RC = \frac{P_a - P_r}{W} = \frac{741000 - 206000}{60000} = 8.9 \text{ [m/s]} \]