Introduction to Aerospace Engineering

Lecture slides

Intro to Aerospace Engineering

AE1101ab-3-4 The Standard Atmosphere

Prof.dr.ir. Jacco Hoekstra

http://natrium42.com/halo/flight2/

(h = 30 489 m, 100 020 ft)

30 km hoogte =
hoeveel %
vd atmosfeer onder je

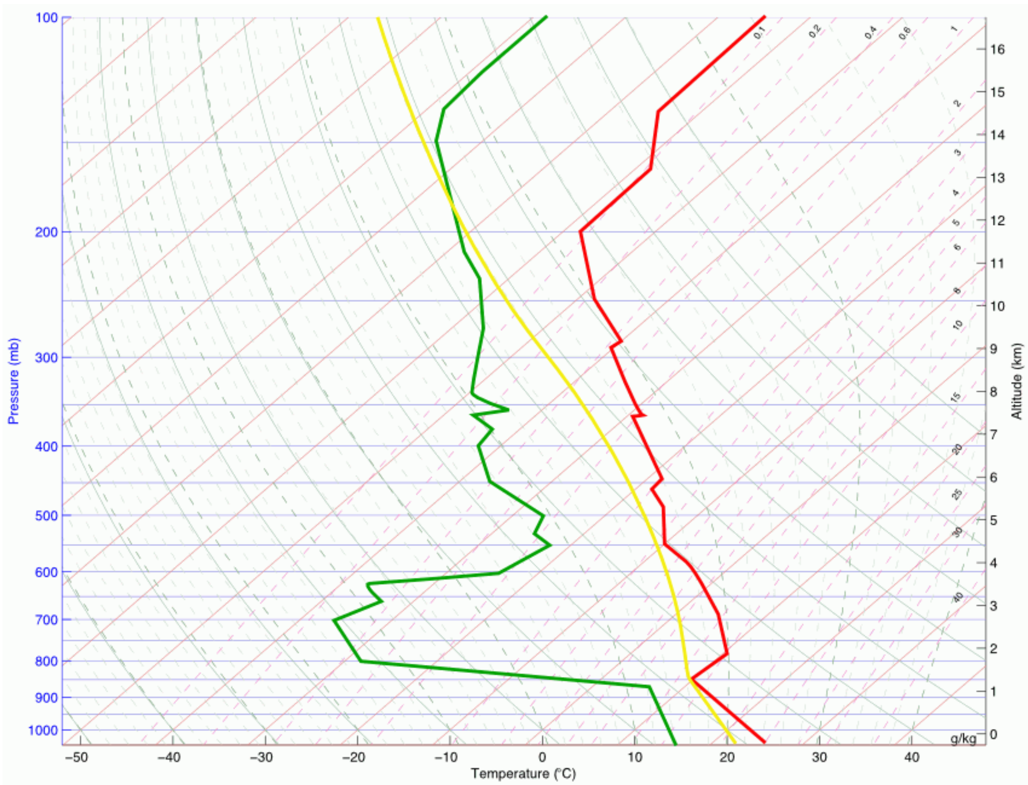
Link to video at 30 km

Tim Zaman's
(student) project


www.hollandshoogte.nl

Joe Kittinger: jump from 100,000 ft
“Jump from space” ?

Why a standard atmosphere?



Real atmosphere

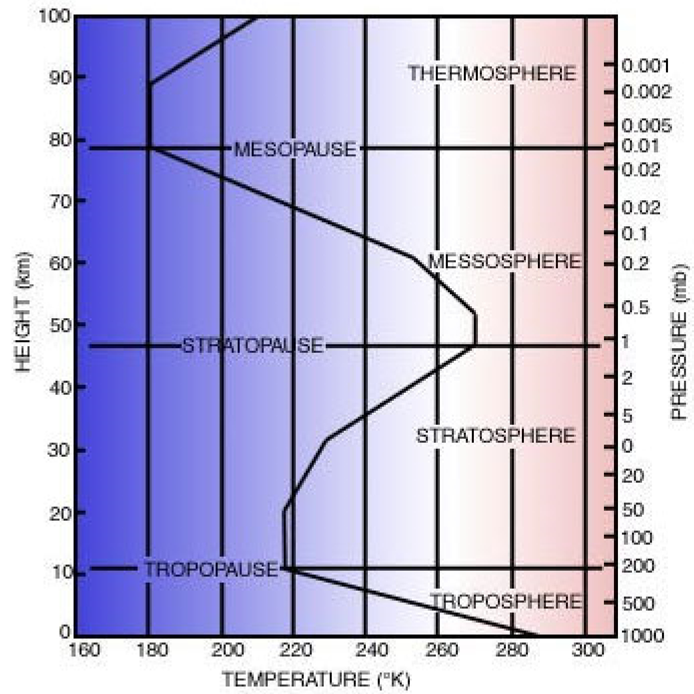
ISA is reference atmosphere for:

- Meaningful aircraft performance specification
- Pressure altitude definition & EAS/IAS/TAS definition
- Model atmosphere for simulation & analysis

International Standard Atmosphere (ISA)

Layers in the ISA

Layer	Level Name	Base Geopotential Height *h* (in km)	Base Geometric Height *z* (in km)	Lapse Rate (in °C/km)	Base Temperature *T* (in °C)	Base Atmospheric Pressure *p* (in Pa)
0	Troposphere	0.0	0.0	−6.5	+15.0	101,325
1	Tropopause	11.000	11.019	+0.0	−56.5	22,632
2	Stratosphere	20.000	20.063	+1.0	−56.5	5,474.9
3	Stratosphere	32.000	32.162	+2.8	−44.5	868.02
4	Stratopause	47.000	47.350	+0.0	−2.5	110.91
5	Mesosphere	51.000	51.413	−2.8	−2.5	66.939
6	Mesosphere	71.000	71.802	−2.0	−58.5	3.9564
7	Mesopause	84.852	86.000	—	−86.2	0.3734

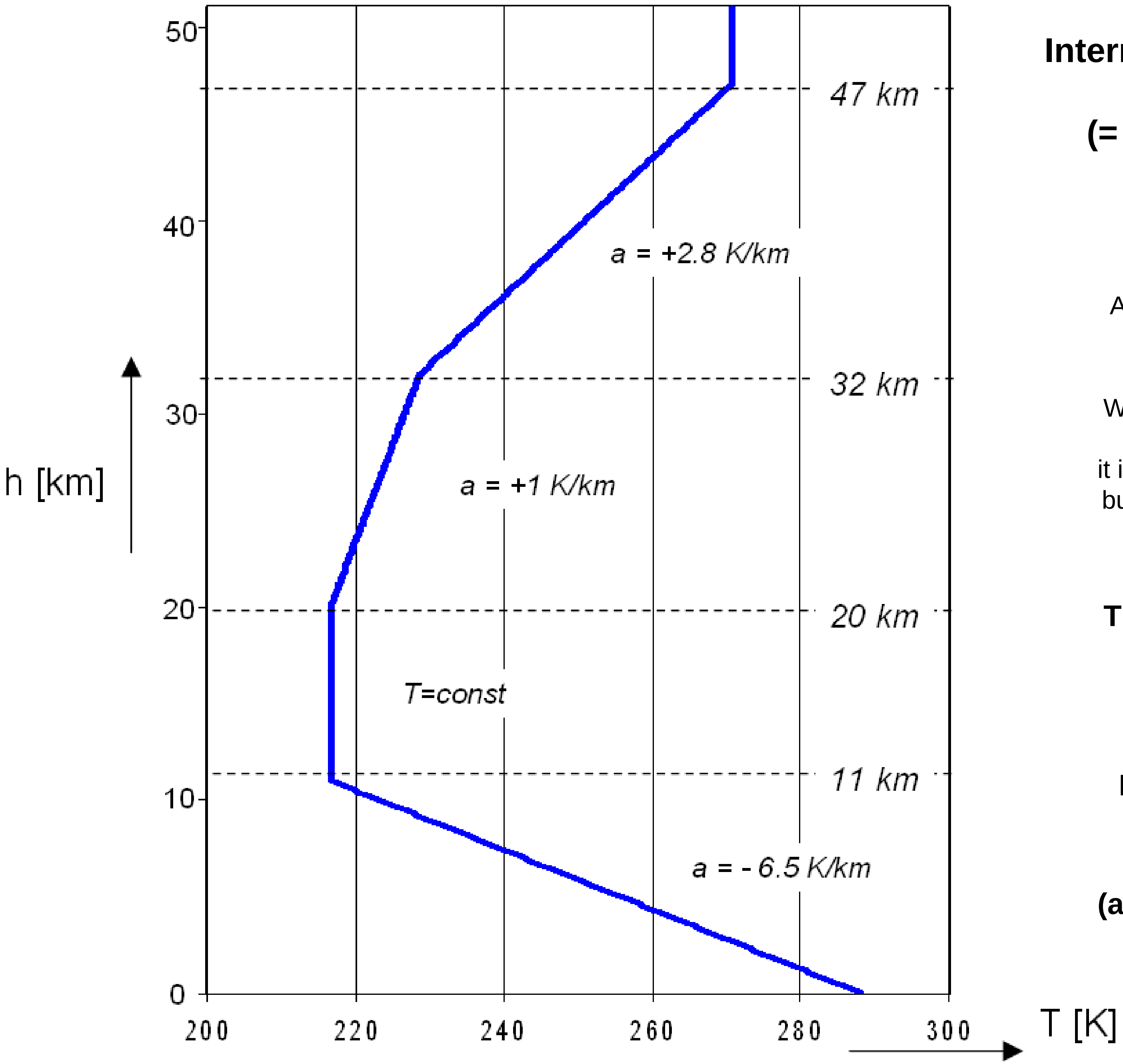


$$T = T_0 + a(h - h_0)$$

$$a = lapse\ rate$$

$$p = \rho R T$$

$$dp = -\rho\ g\ dh$$



International Standard Atmosphere (= ICAO Std Atm)

Use this instead of Anderson page 112!)

Watch out when using lapse rate *a*, it is often given per km, but you should use SI-units, so per m!

T = 288.15 K (15 ºC)
p_0 = 101325 Pa
ρ = 1.225 kg/m³

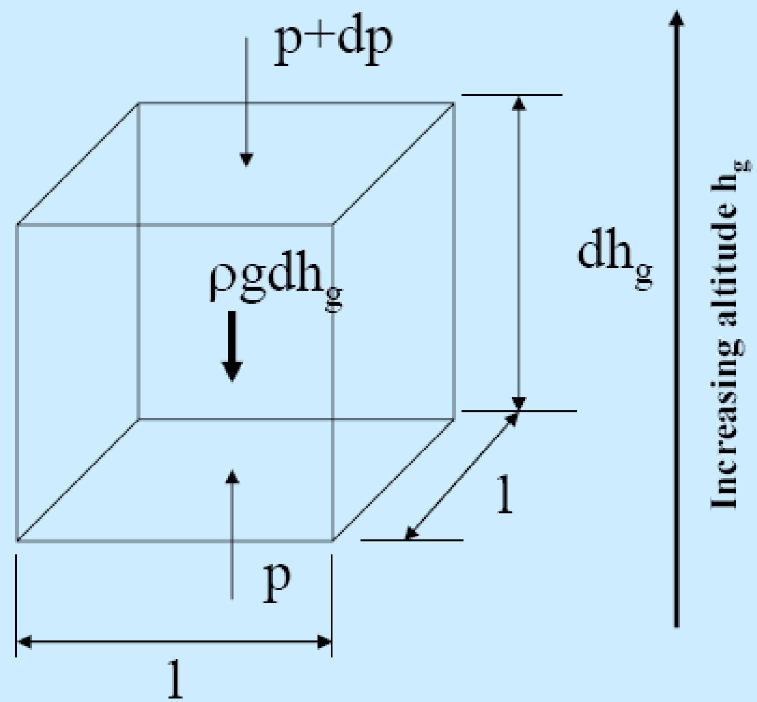
R = 287.05 J/kg K
g_0= 9.81 m/s²

(and no water vapour)
M = 28.97 g/mol

What do we need to define a standard atmosphere?

- Physically correct:
  - Pressure increases due to gravity
  - Gas law
- Two laws, while three variables define state:
  - Pressure
  - Temperature
  - Density
- So by defining one state variable, we define the entire atmosphere by applying the two laws of nature

Hydrostatic equation /geopotential altitude



Equilibrium of forces :

$$p = p + dp + \rho g \, dh_g \quad \Rightarrow$$

real g — *geometric altitude*

$$dp = -\rho g \, dh_g \qquad (1)$$

$$dp = -\rho g_0 \, dh \qquad (2)$$

g at sea level

geopotential altitude

This is the one we normally use. Difference is small e.g. 63500 ft vs 63307 ft = 0,3%

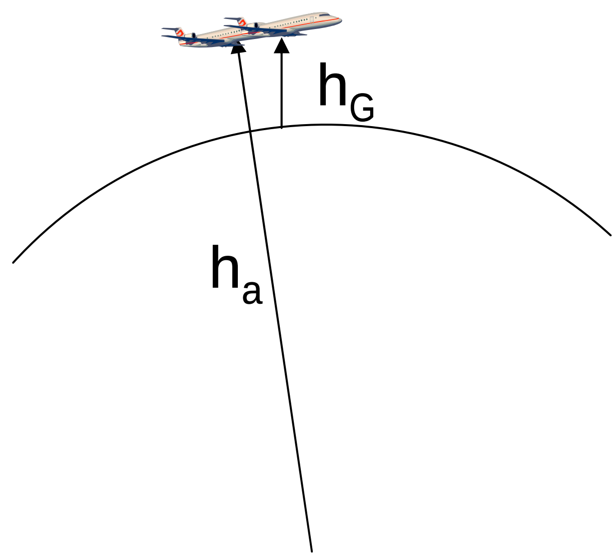
Absolute altitude & geometric altitude

Geometric altitude: real altitude with sea level = 0

Absolute altitude: distance to centre of earth

$$h_G = h_a + R_{earth}$$

$$R_{earth} = 6357 \text{ km}$$



Relation geopotential & geometric altitude

$$g = g_0 \left(\frac{r}{h_a}\right)^2 = g_0 \left(\frac{r}{r + h_G}\right)^2 \quad (3.1)$$

$$dp = -\rho g \, dh_G = -\rho g_0 \, dh \longrightarrow 1 = \frac{g_0}{g} \frac{dh}{dh_G}$$

$$dh = \frac{g}{g_0} dh_G \quad (3.4)$$

Eq. (3.1) into (3.4):

$$dh = \frac{r^2}{(r + h_G)^2} dh_G \quad (3.5)$$

$$\int_0^h dh = \int_0^{h_G} \frac{r^2}{(r + h_G)^2} dh_G = r^2 \int_0^{h_G} \frac{dh_G}{(r + h_G)^2}$$

$$h = r^2 \left(\frac{-1}{r + h_G}\right)_0^{h_G} = r^2 \left(\frac{-1}{r + h_G} + \frac{1}{r}\right) = r^2 \left(\frac{-r + r + h_G}{(r + h_G)r}\right)$$

Thus,

$$\boxed{h = \frac{r}{r + h_G} h_G} \quad (3.6)$$

International Standard Atmosphere (ISA) Layer with T gr

As an exercise:
try to making an Excel sheet
with a table for steps of 100 m

At sea level :

$p_s = 1.01325 * 10^5 \text{ N/m}^2$

$\rho_s = 1.225 \text{ kg/m}^3$

$T_s = 288.15 \text{ K}$

Temperature in the troposphere (lower part) :

$$\frac{dT}{dh} = -0.0065 \text{ K/m}$$

When T=T(h) is known as a function of the altitude the pressure and the density can be derived as a function of altitude

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/aR}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-((g_0/aR)+1)}$$

R=gas constant

Layer with constant temperature T (11 km -20 km)

Use values at 11 km as base 1 for this formulae

$$\frac{p}{p_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$

On exam you should be able to derive all ISA formulae!