Introduction to Aerospace Engineering

Lecture slides

Intro to Aerospace Engineering

AE1101 Stability & Control

Prof.dr.ir. Jacco Hoekstra

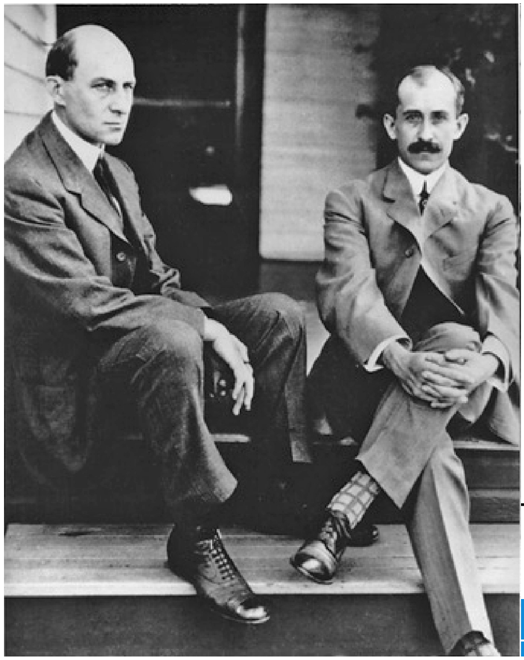
Stability & control
- Anderson 6.17, 7.1-7.11
- ....and some extra stuff

“When this one feature [balance and control] has been worked out, the age of flying machines will have arrived, for all other difficulties are of minor importance.”

Papers of Wilbur and Orville Wright

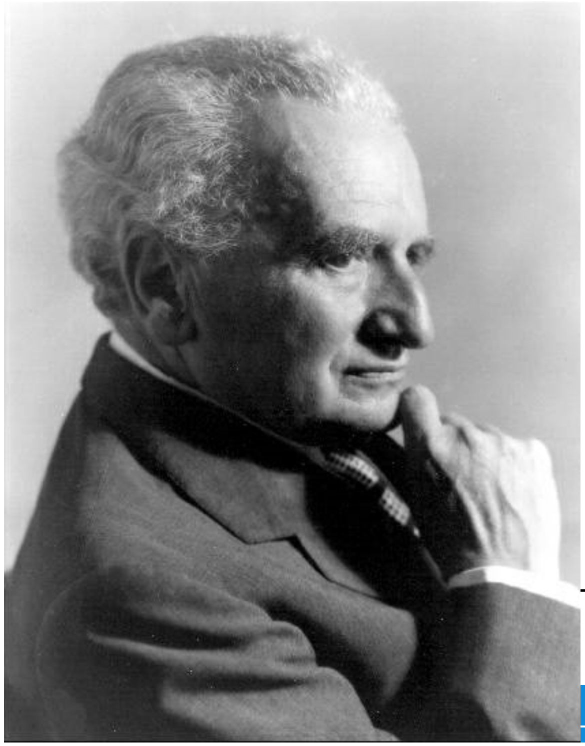
Wilbur

Orville



"A spin is like a love affair;
you don't notice how you get into it
and it is very hard to get out of"

Theodore von Kármán,
answering a question during a conference



Stability is not easy

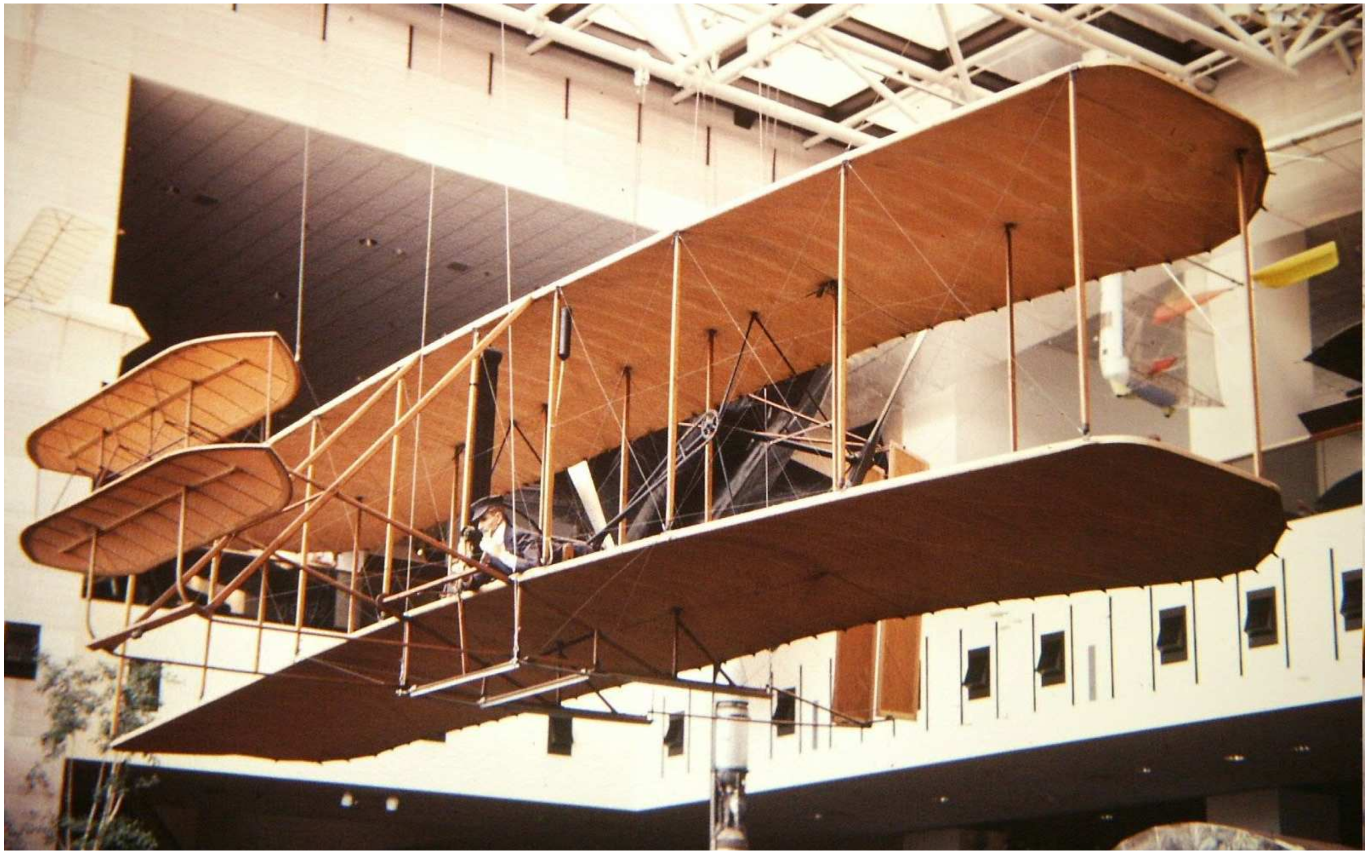
1.

Controls

Different approach pioneers

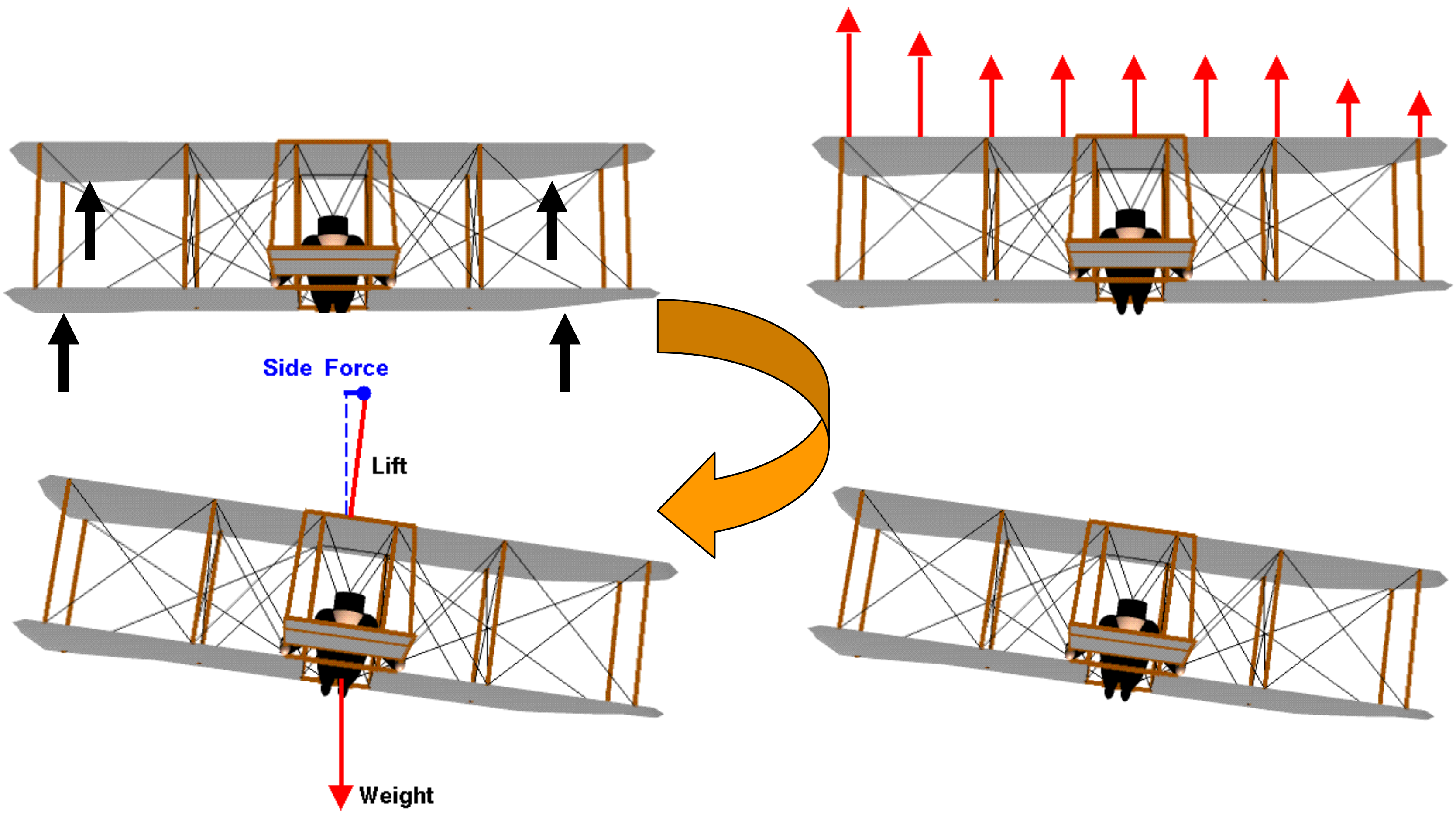
Europe: Voisin Farman I-bis at Brussels Air Museum
January 13, 1908: Grand Prix d'Aviation for circle > 1 km

Different approach pioneers



Wright Flyer I in Smithsonian Air & Space Museum Washington DC
First powered manned flight

Concept of Wing Warping



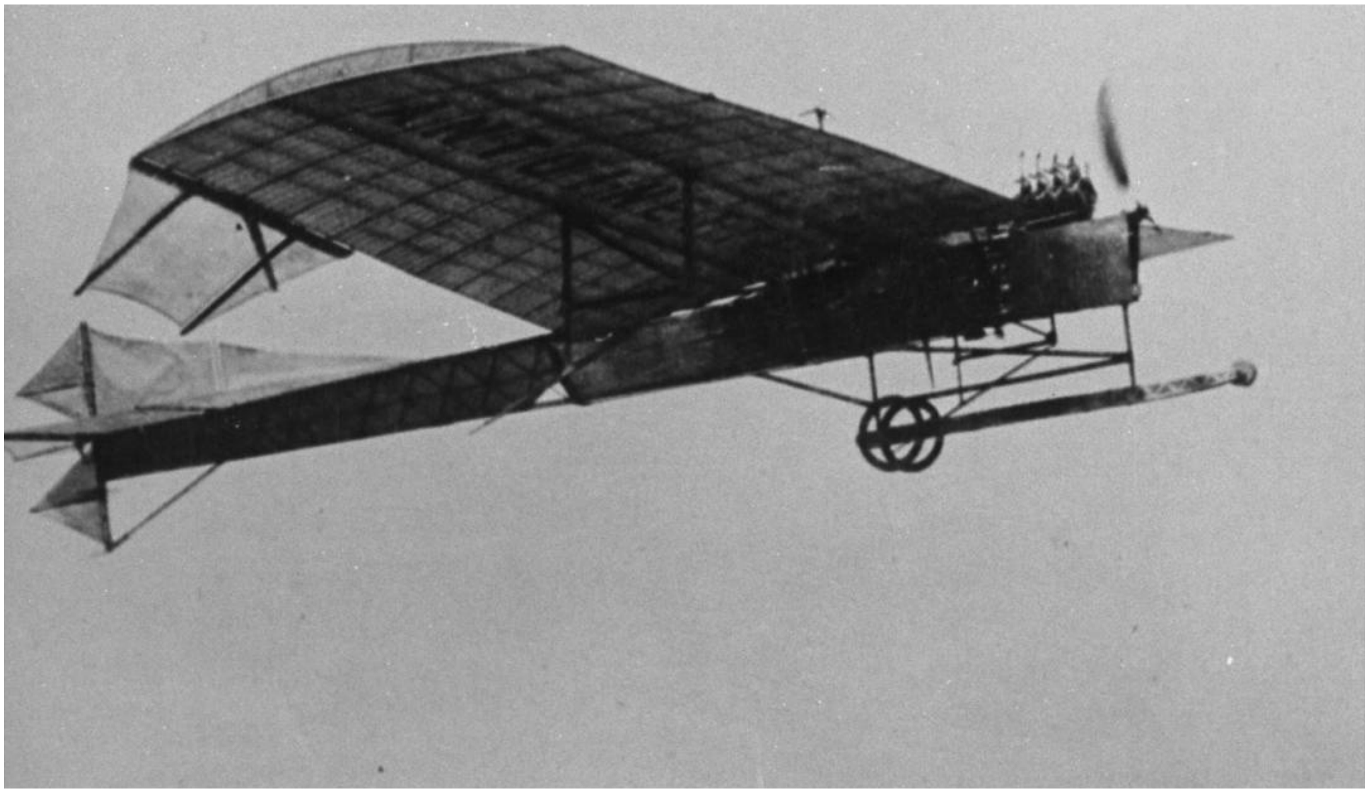
Wing warping for roll control

31 August 1911, Haarlem

1 September 1911, Haarlem

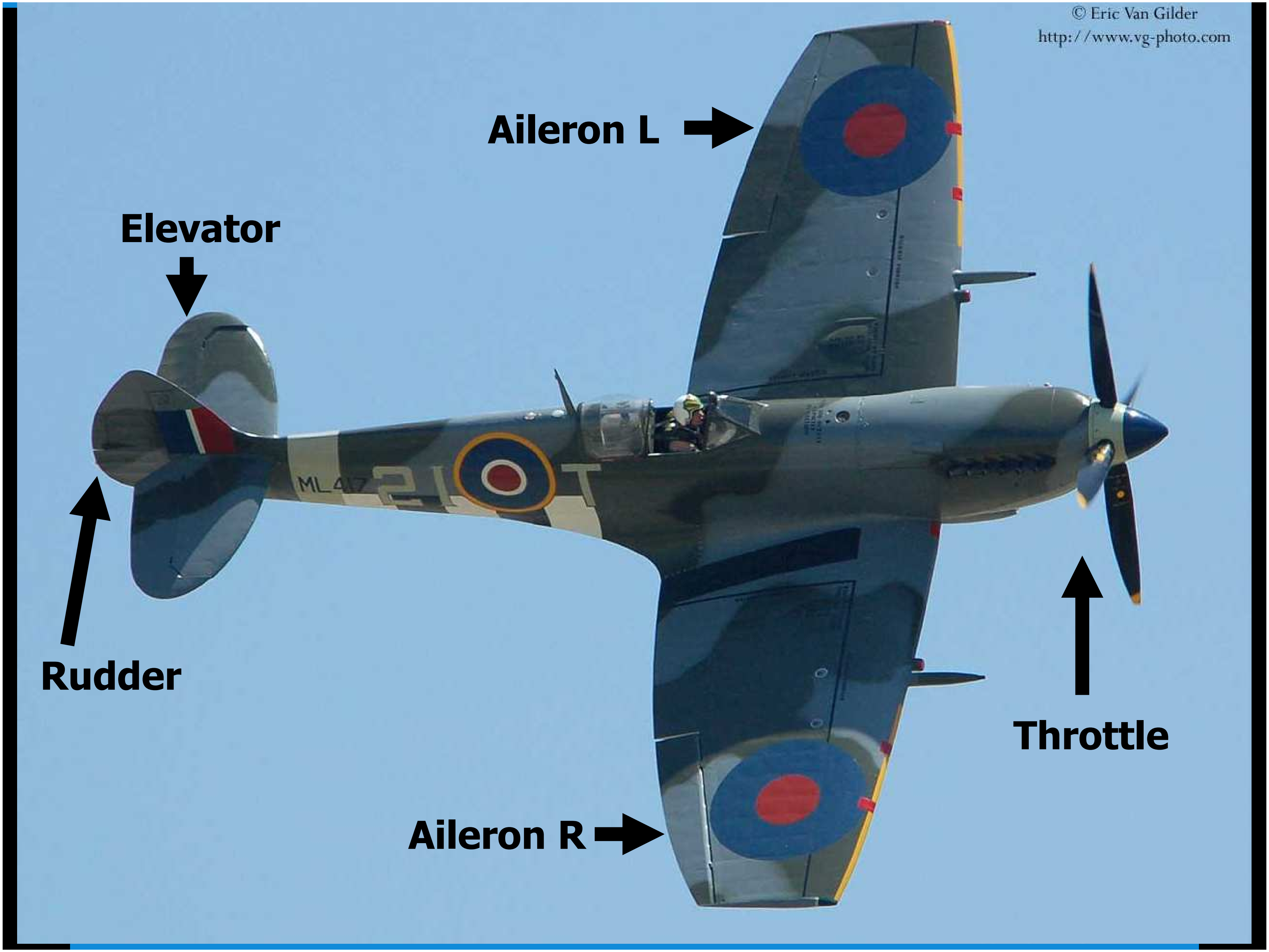
- Fokker Spin

First ailerons



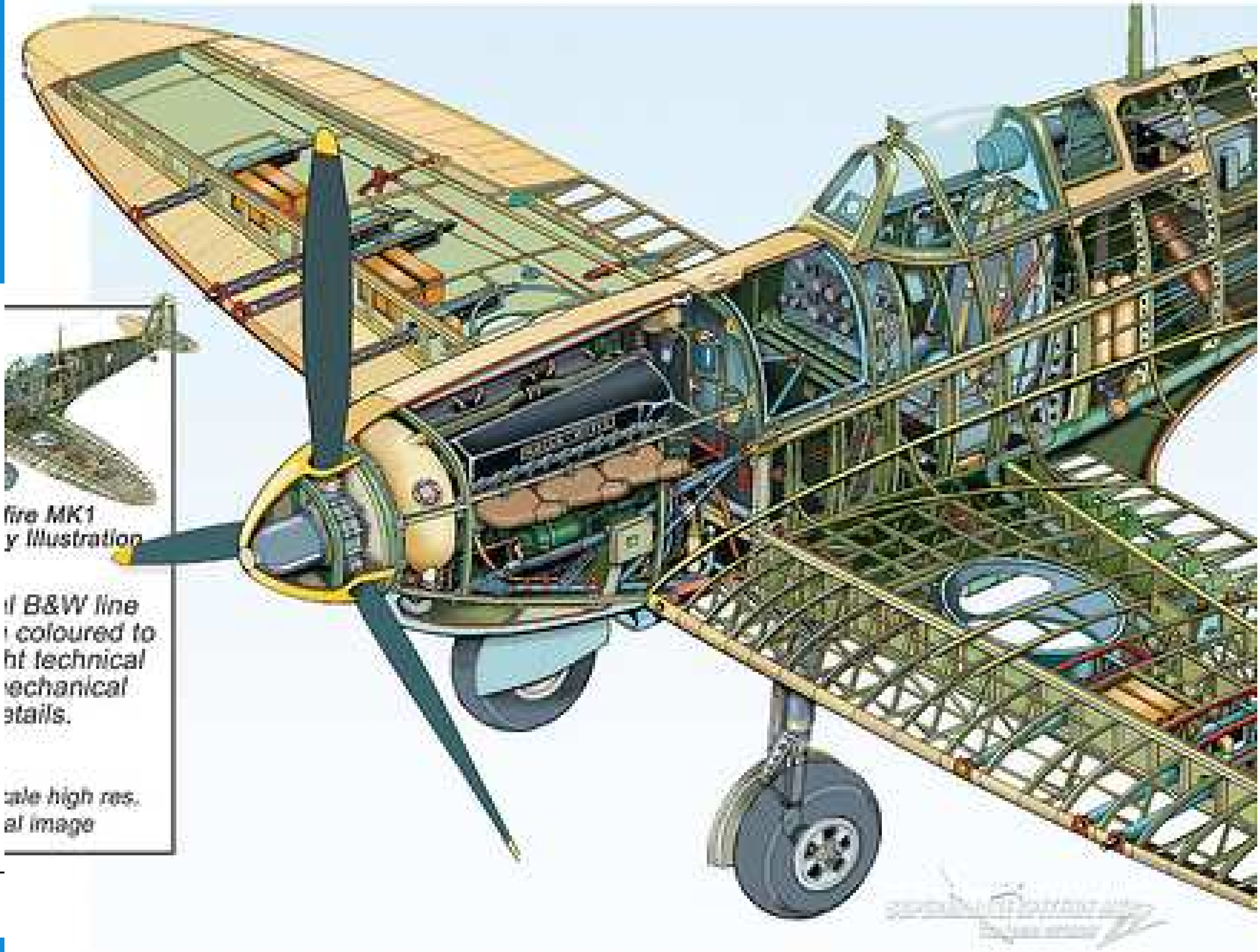
- **Monoplane**
- **Failed to cross channel on 19 July 1909**
- **World distance record: 154.6 km on 26 Augustus 1909 in 2 hr 17m**

Antoinette IV,1908 designed by Leon Lavasseur


© Eric Van Gilder
http://www.vg-photo.com
Aileron L
Elevator
ML417
21
T
Rudder
Throttle
Aileron R

"It is not immediately obvious how a pilot with <u>four</u> controls manages to control an aircraft with <u>six</u> degrees of freedom."

D. Stinton



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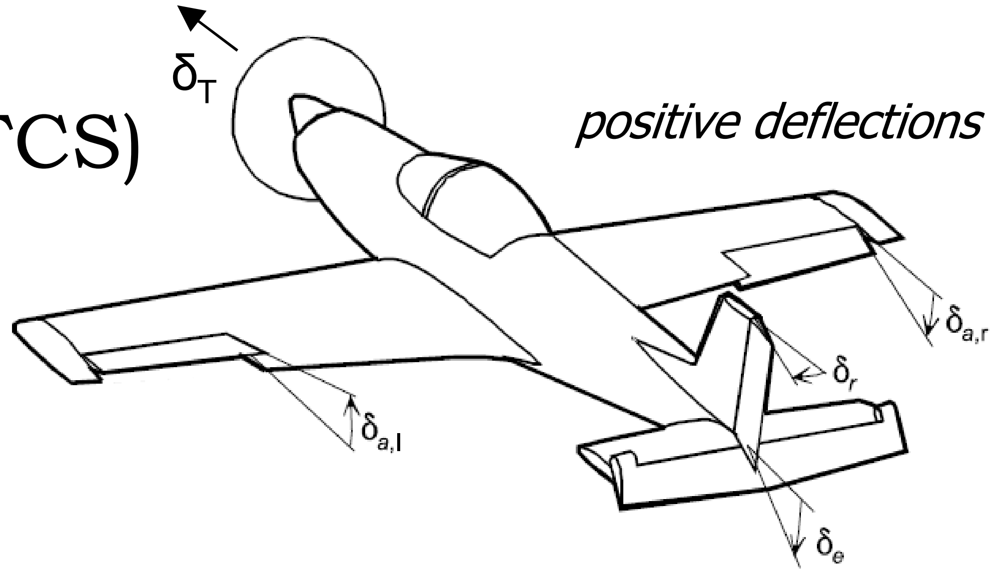
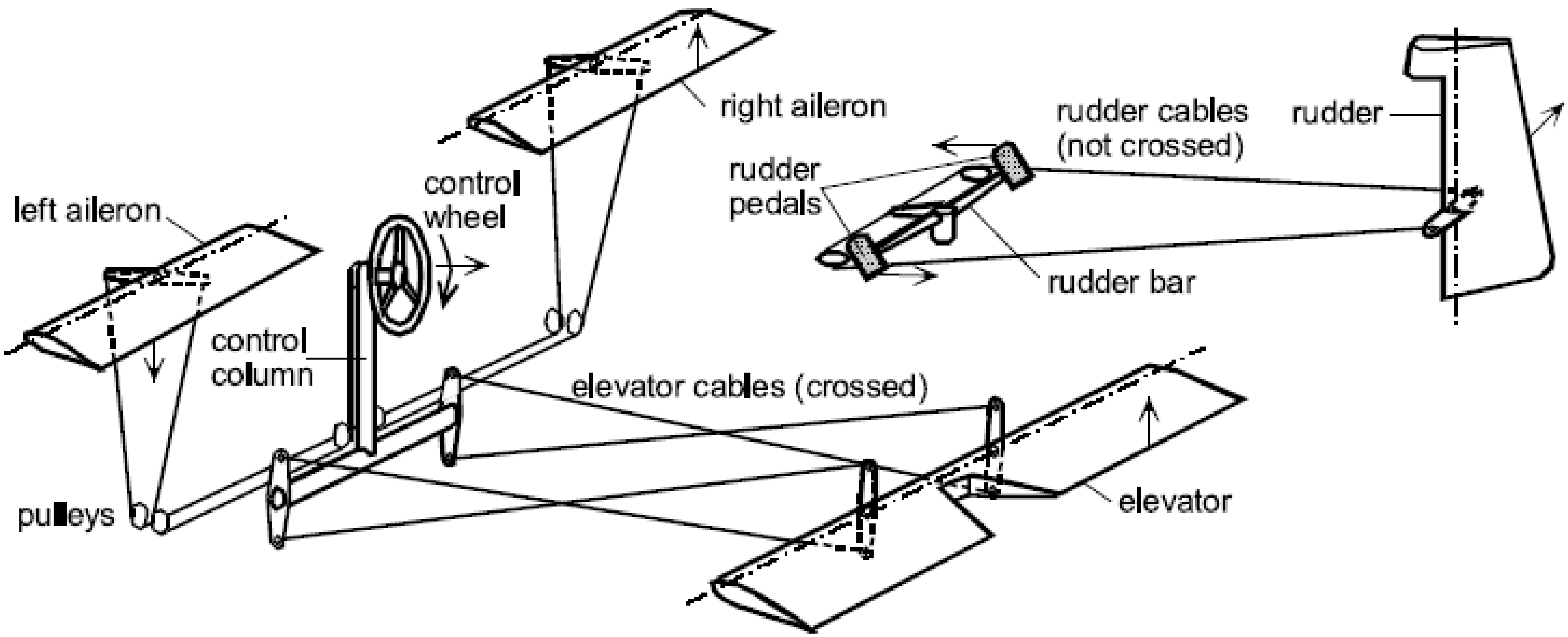
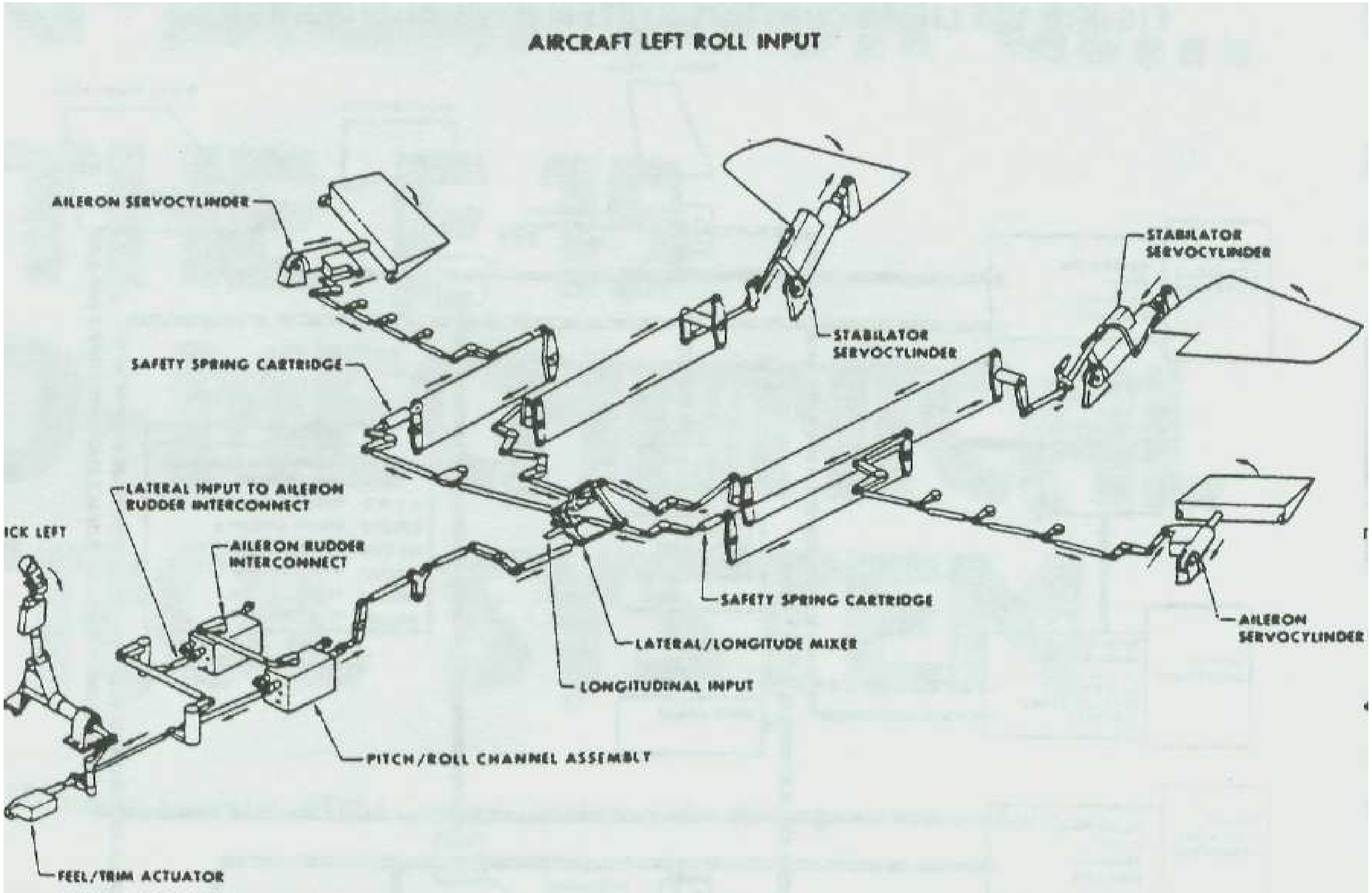
Classic Flight Control System (FCS)

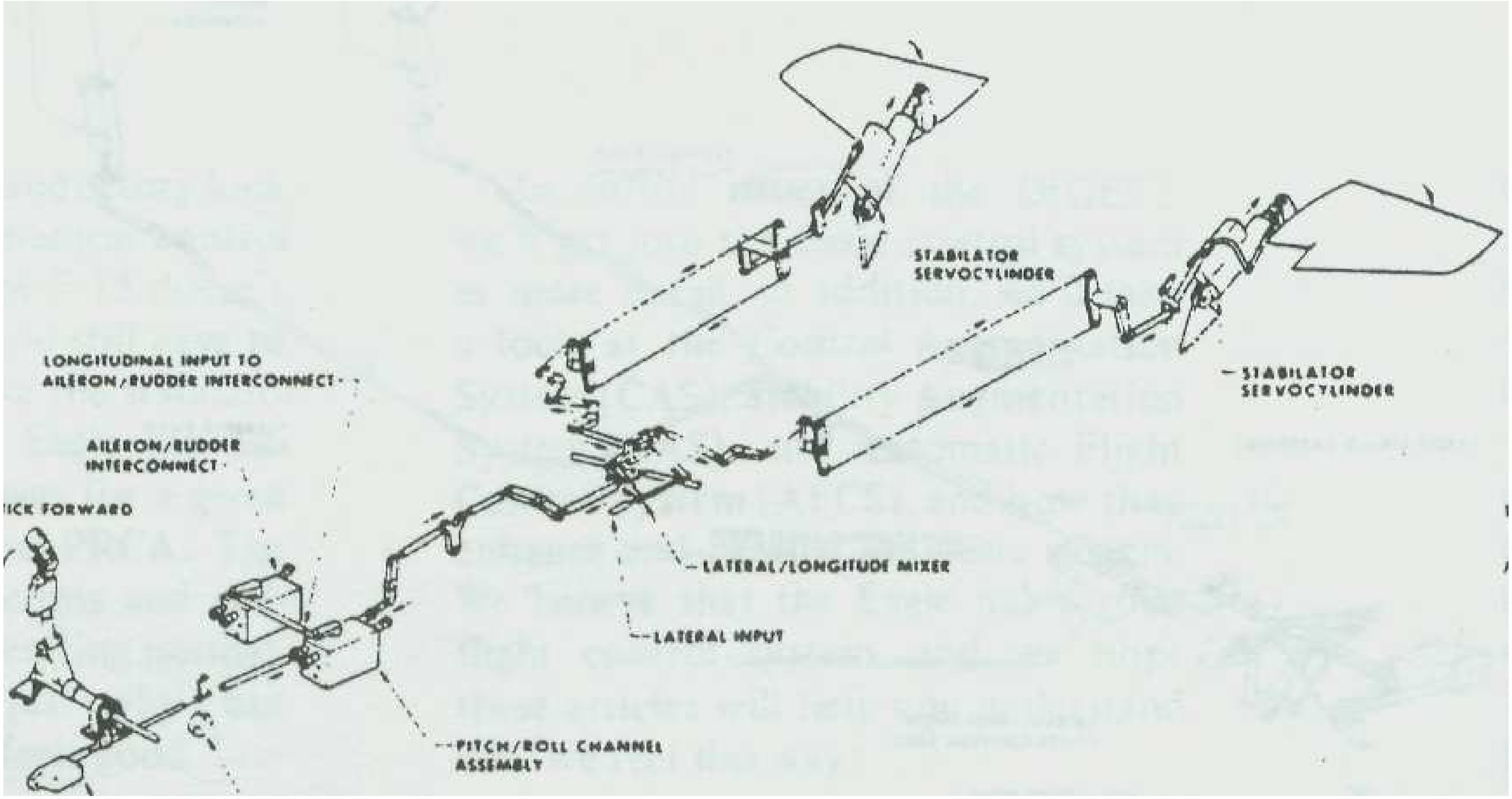




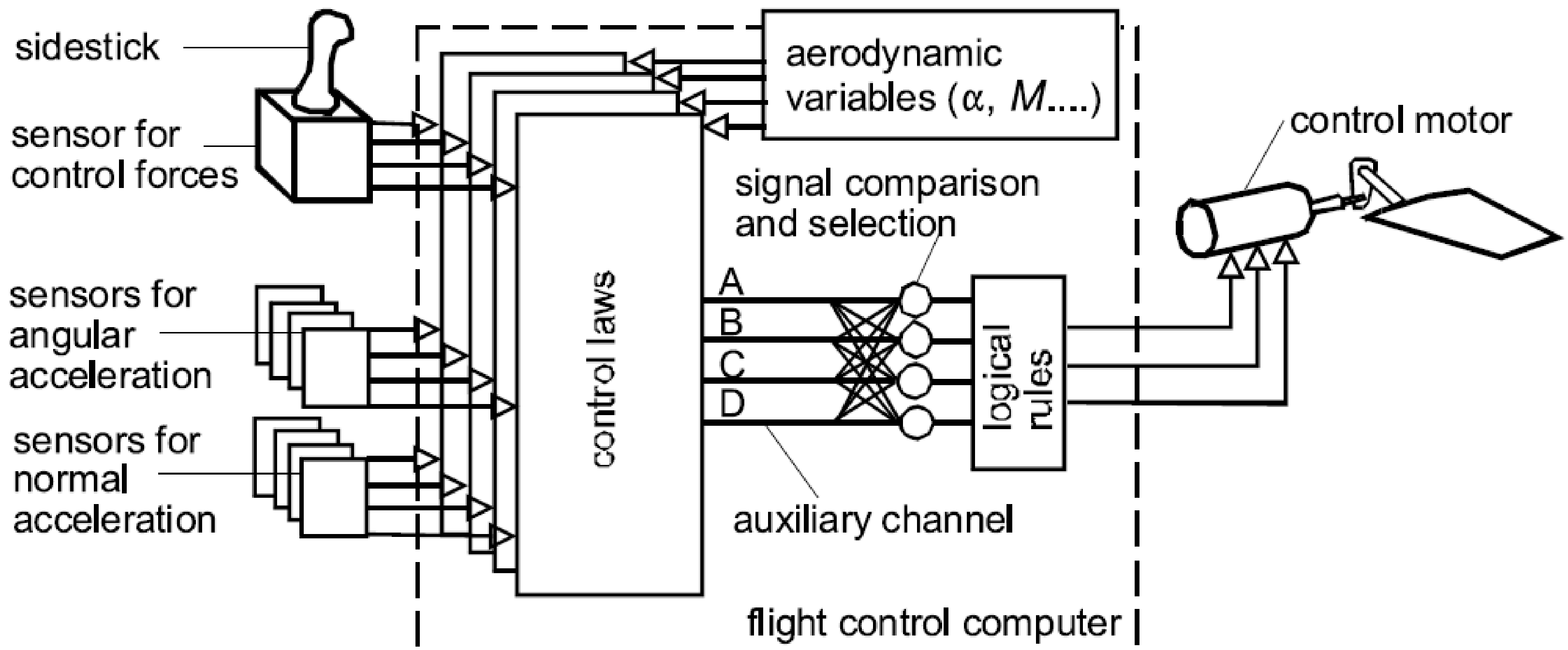
Classic FCS: F-15 Eagle



Classic FCS: F-15 fly by cable



Fly by wire FCS



First in military jets (agility) later in airliners (weight saving).

Demo

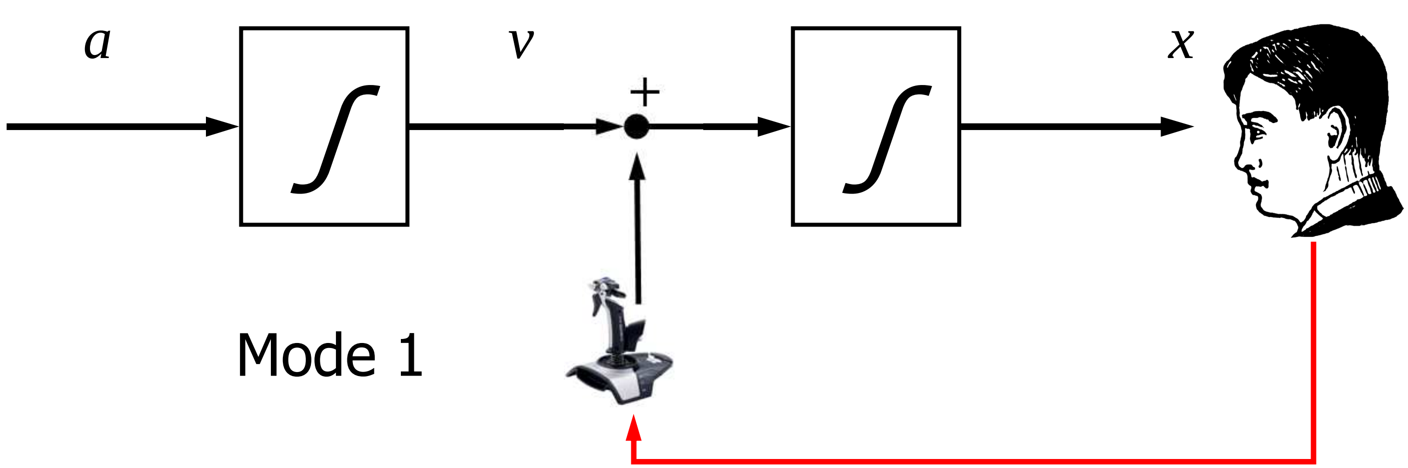
Stable Flight

- Mode 1: Controls vertical speed
- Mode 2: Controls vertical acceleration
- Mode 3: Control change of vertical acceleration

Integrators in control loop

speed $$v = \frac{\Delta x}{\Delta t}$$ $$x_{i+1} = x_i + v \cdot \Delta t$$

acceleration $$a = \frac{\Delta v}{\Delta t}$$ $$v_{i+1} = v_i + a \cdot \Delta t$$



Integrators in control loop

speed $$v = \frac{\Delta x}{\Delta t}$$ $$x_{i+1} = x_i + v \cdot \Delta t$$

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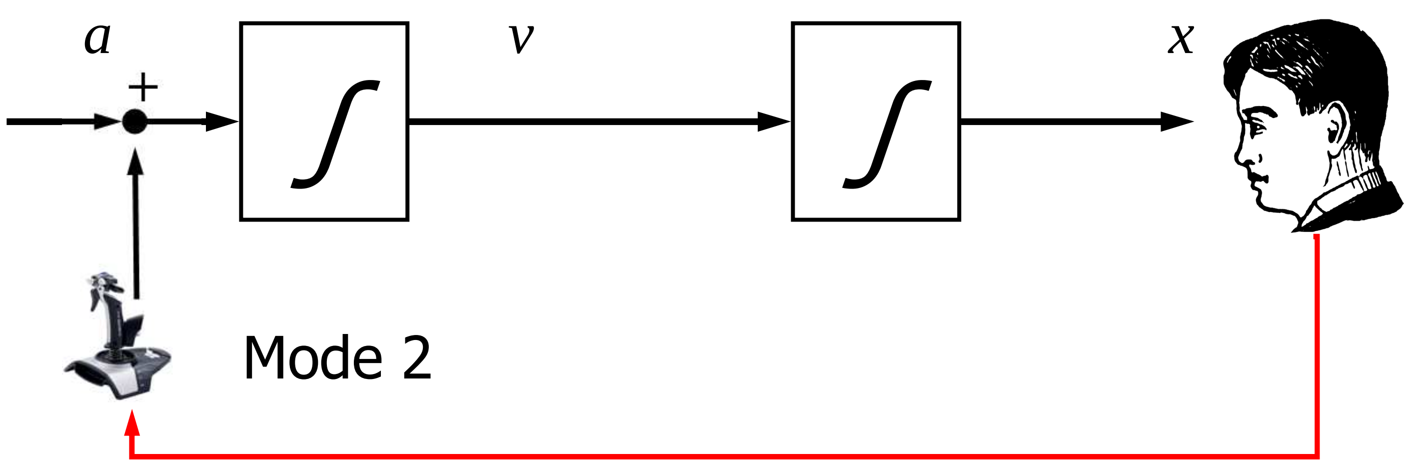


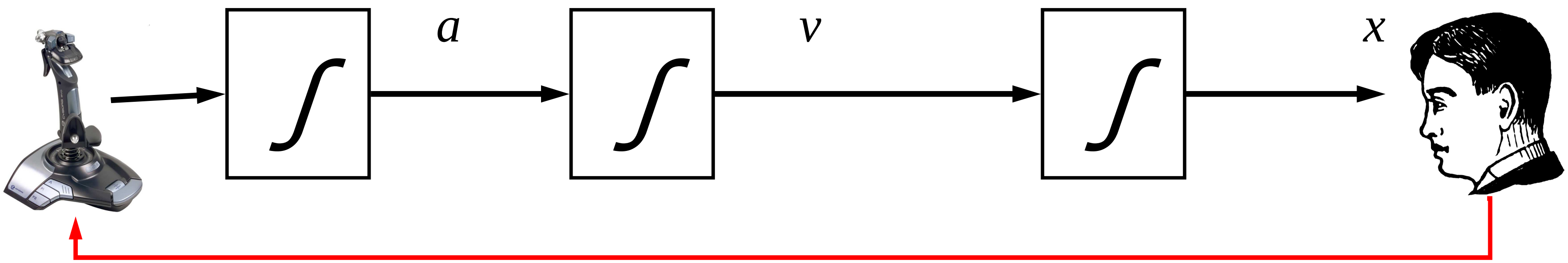
Integrators in control loop

speed $v = \frac{\Delta x}{\Delta t}$

acceleration $a = \frac{\Delta v}{\Delta t}$

x_{i+1}

v_{i+1}

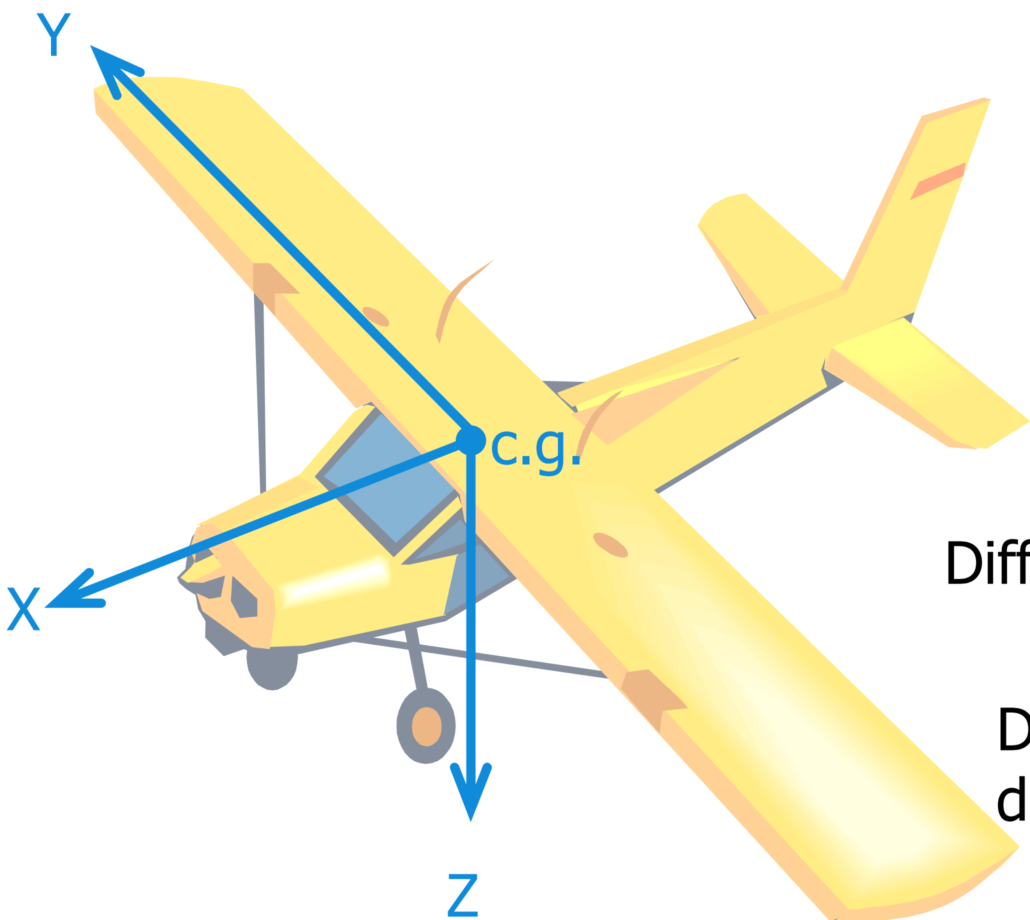


Mode 3

2.

Angles and axes

Body Axes

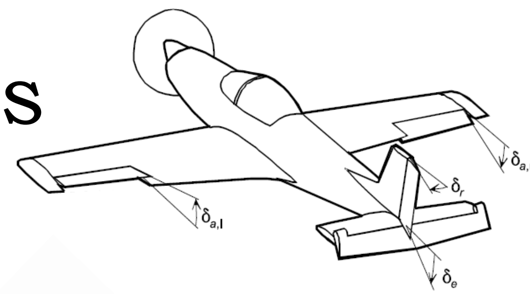


Forces in body axes

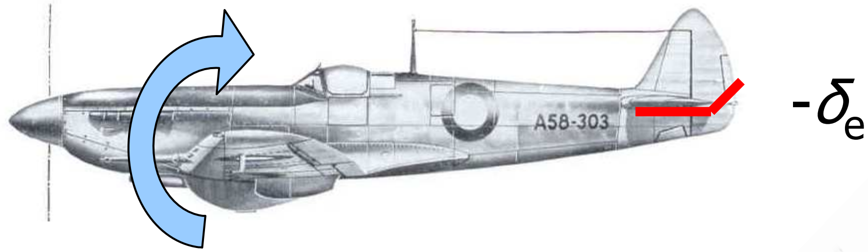
Difference with lift & drag?

Defined relative to direction of speed vector

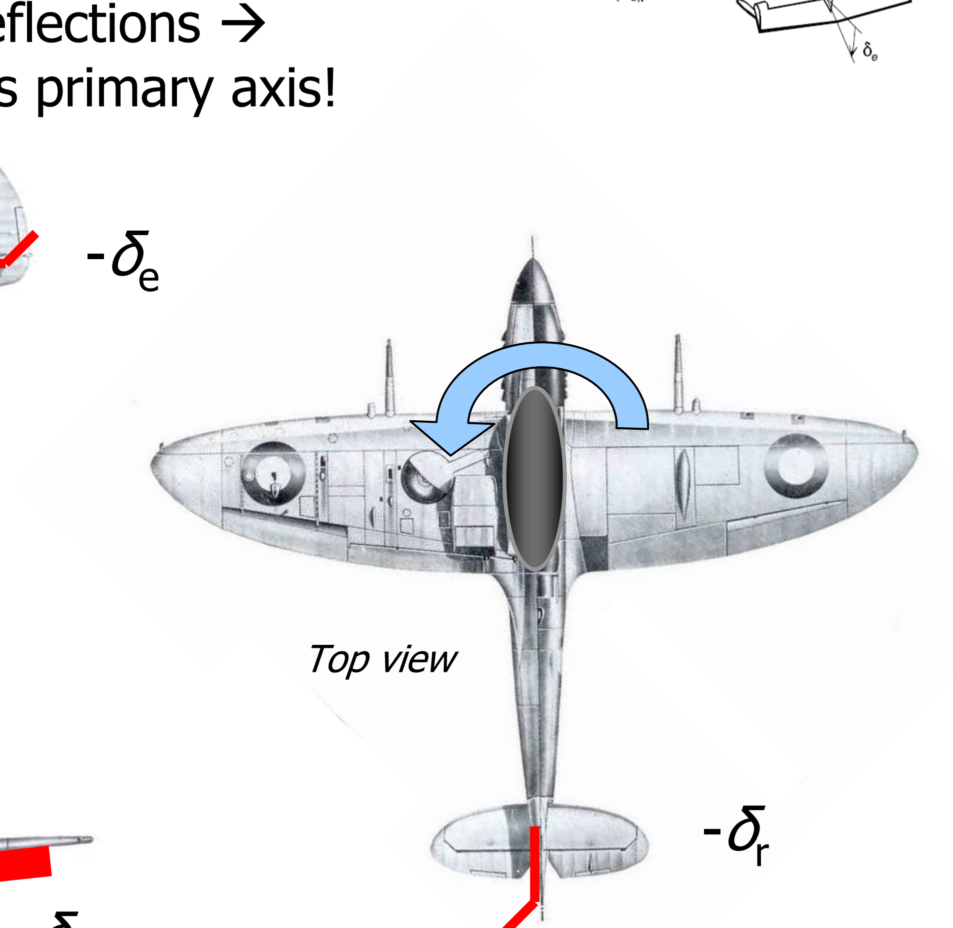
Control surfaces and rotations

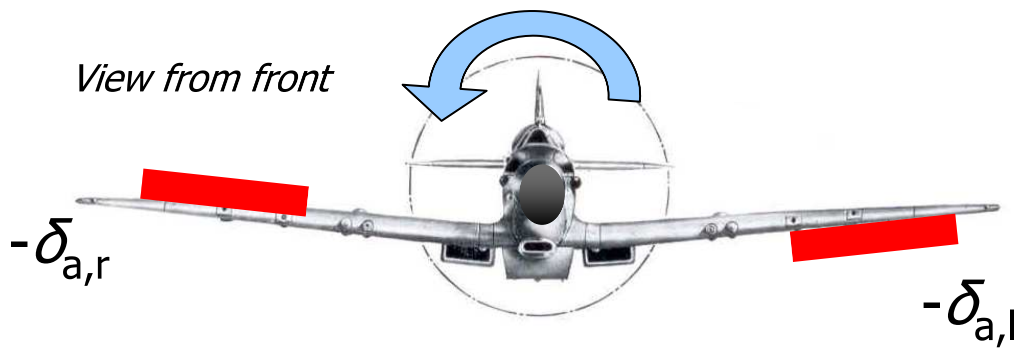


Sign convention: negative deflections → positive a/c response around its primary axis!



Elevator: pitch angle θ



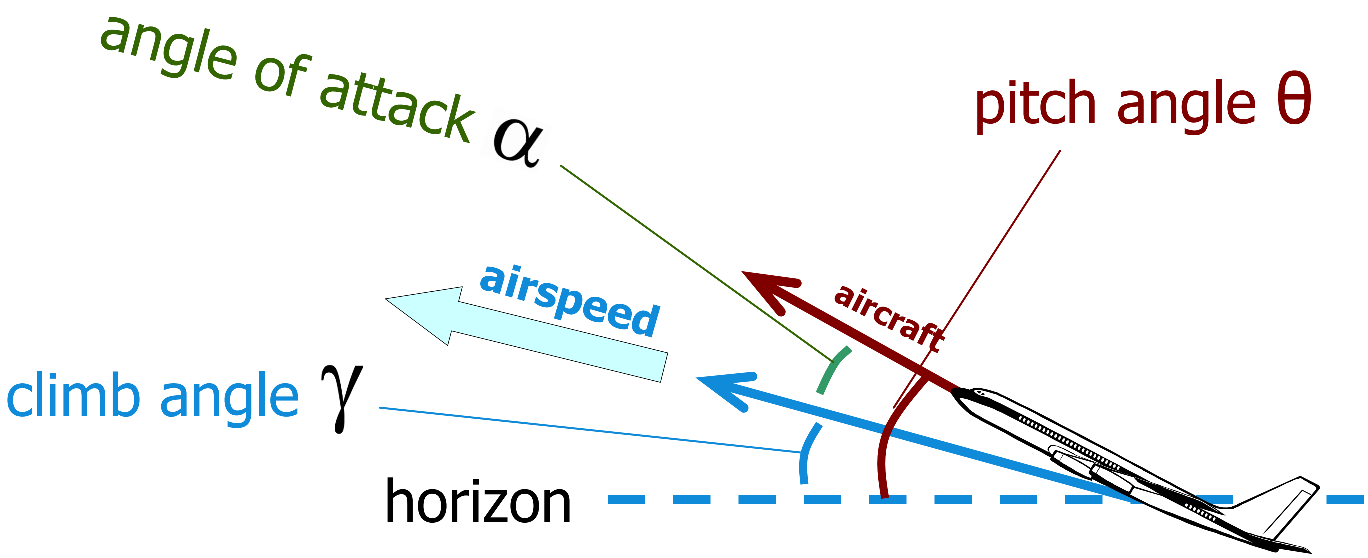


Ailerons: roll angle φ

Rudder: yaw angle ψ

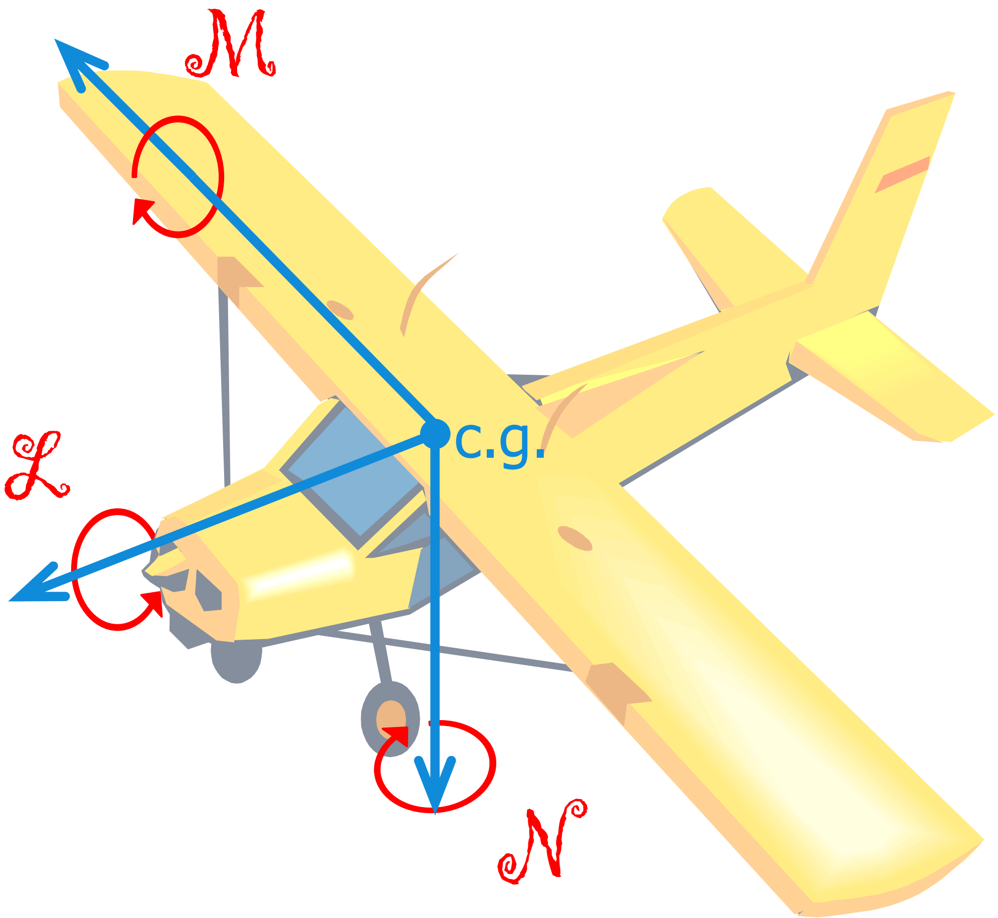
Stability axes and body axes

Stability: x_s-axis is attached to velocity
Body axes: x_b-axis is fixed to aircraft



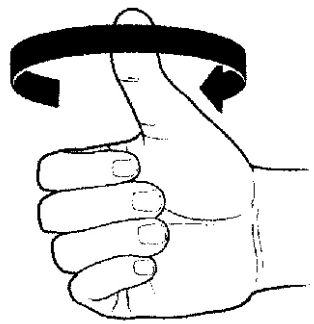
$$\alpha + \gamma = \theta$$

Moments



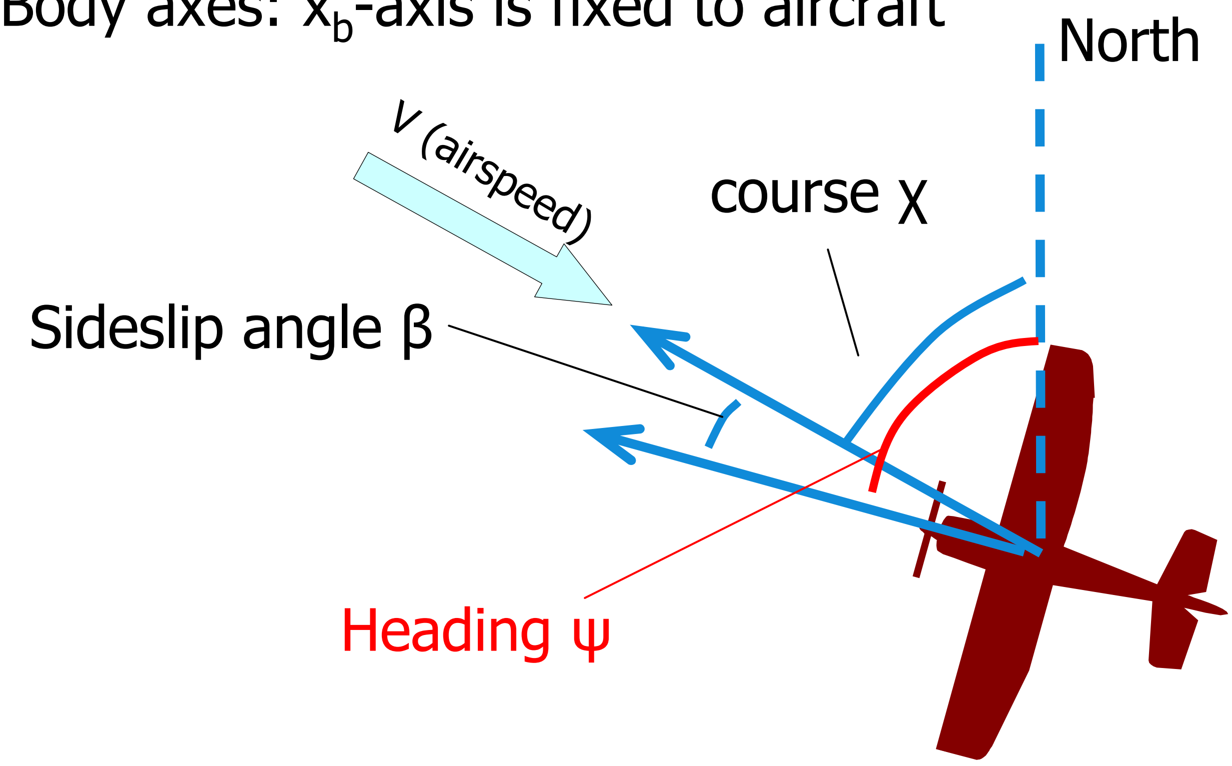
$\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$

Pitching moment $\mathcal{M}$
Nose up = positive



Stability axes and body axes

Stability: x_s-axis is attached to velocity
Body axes: x_b-axis is fixed to aircraft



Geodetic axes: x_g-axis is attached to North and horizon

Force & moment coefficients

- Forces dimensionless with $\frac{1}{2} \rho V^2 S$
- Moments dimensionless with:
  - Longitudinal $\mathcal{M}$: $\frac{1}{2} \rho V^2 S c$ $(c = chord)$
  - Lateral: $\mathcal{L}, \mathcal{N}$: $\frac{1}{2} \rho V^2 S b$ $(b = span)$
- C_X C_Y C_Z C_ℓ C_m C_n

For now: symmetrical movements in stability axes

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S}$$

For the wing+aircraft we use the surface area of the wing S!

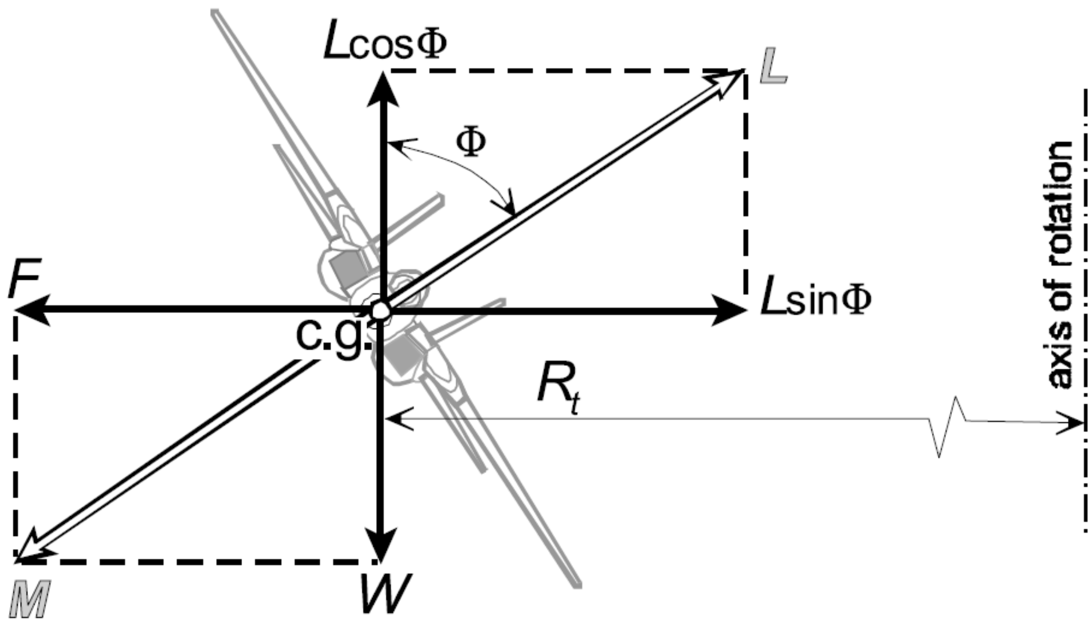
$$C_{L_H} = \frac{L_H}{\frac{1}{2}\rho V^2 S_H}$$

For the tail we use the surface of the tail: S_H !

$$C_m = \frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$$

pitching moment without dimensions
(so without influence of ρ, V and S)
it is a 'shape' parameter which
varies with the angle of attack.
Note the chord c in the denominator because of the unit Nm!

Bank angle: Horizontal steady turn



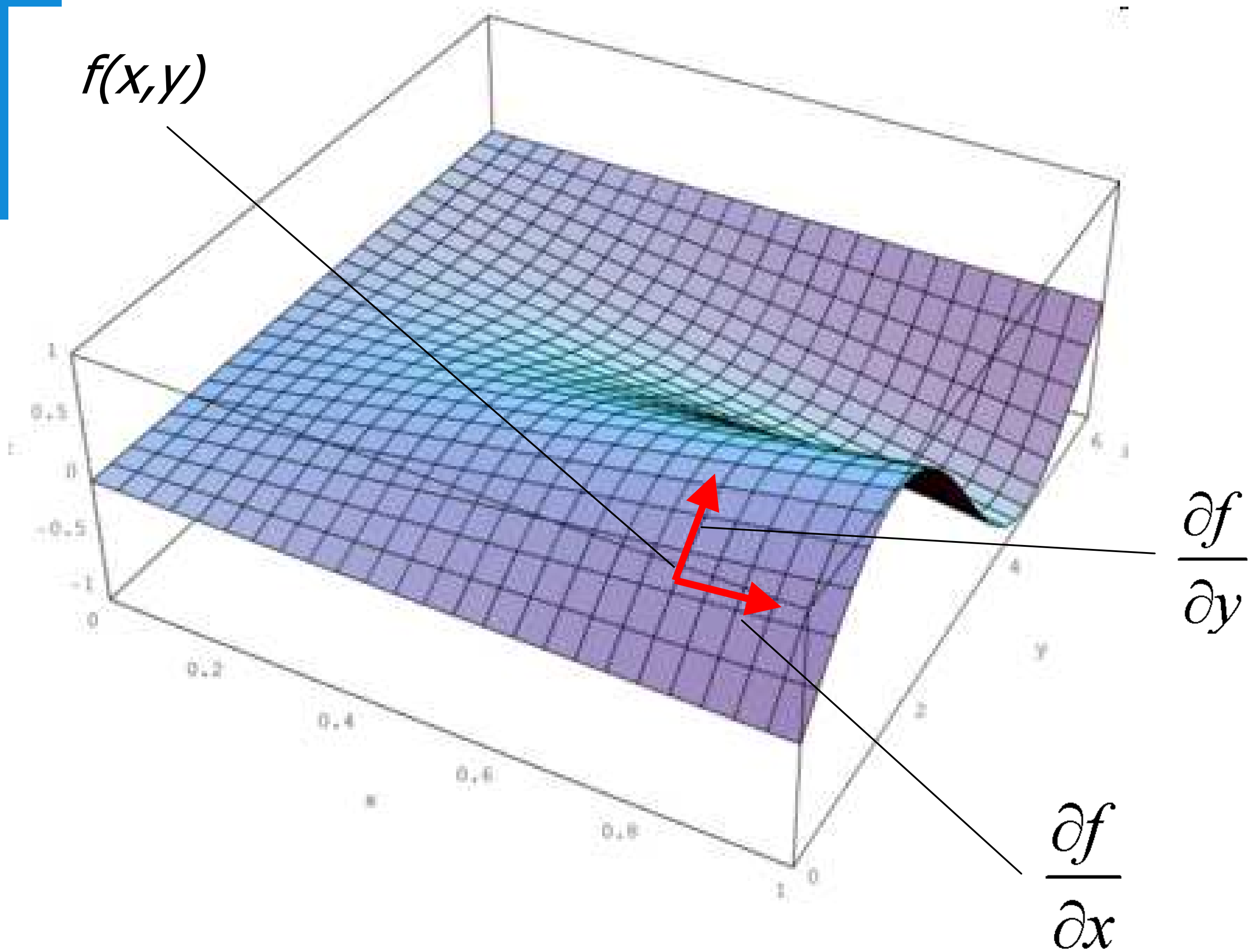
$$L \sin \Phi = F = \frac{W}{g} \frac{V^2}{R_t}$$

$$L \cos \Phi = W$$

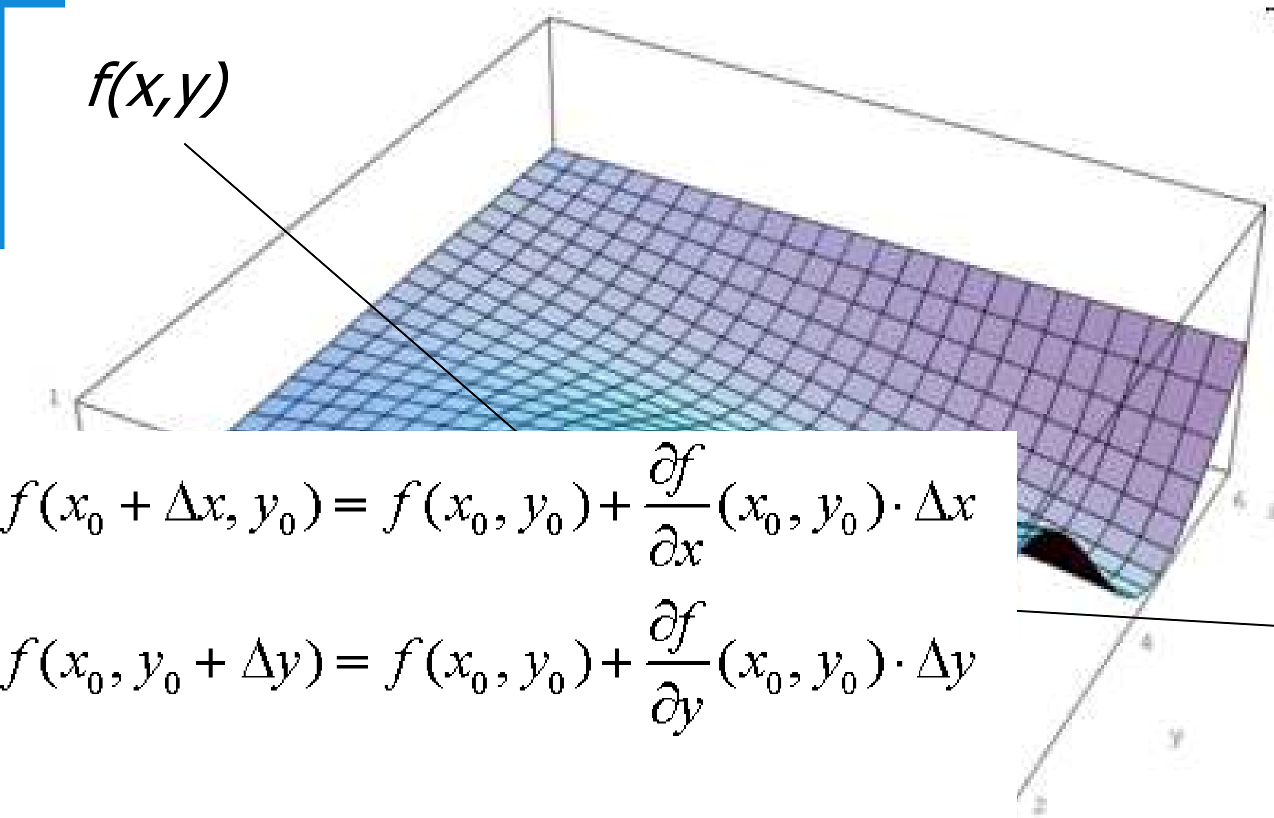
Load factor *n*:

$$n = 1/\cos \Phi$$

Partial derivatives:
use for small disturbances



Partial derivatives: use for small disturbances



$f(x,y)$

$$f(x_0+\Delta x, y_0) = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)\cdot \Delta x$$

$$f(x_0, y_0+\Delta y) = f(x_0, y_0) + \frac{\partial f}{\partial y}(x_0, y_0)\cdot \Delta y$$

$$\frac{\partial f}{\partial y} \qquad C_{f_y}$$

$$f(x_0+\Delta x, y_0) = f_0 + f_x \cdot \Delta x$$

$$f(x_0, y_0+\Delta y) = f_0 + f_y \cdot \Delta y$$

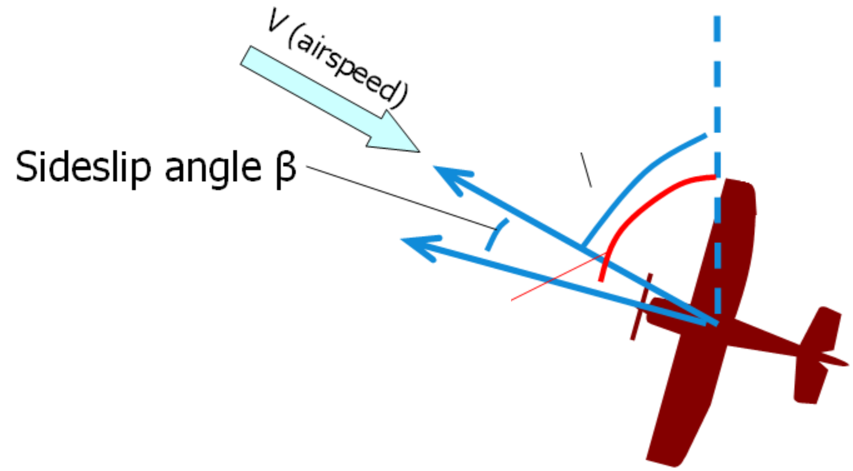
$$\frac{\partial f}{\partial x} \qquad C_{f_x}$$

Stability notation issue

C_{m_α} = change in pitch moment due to angle of attack

C_{n_β} = change in yawing moment due to sideslip angle

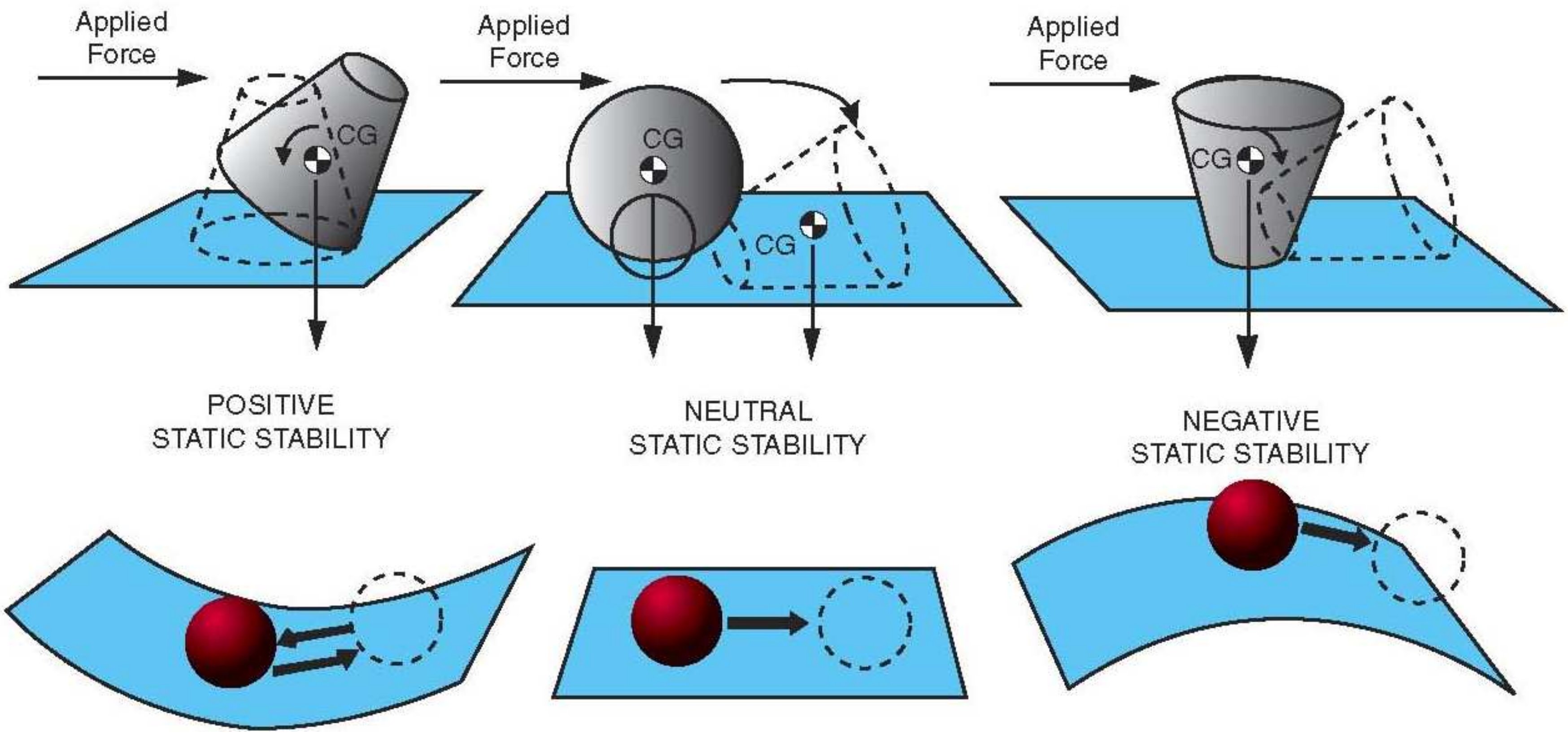
Etc. etc.



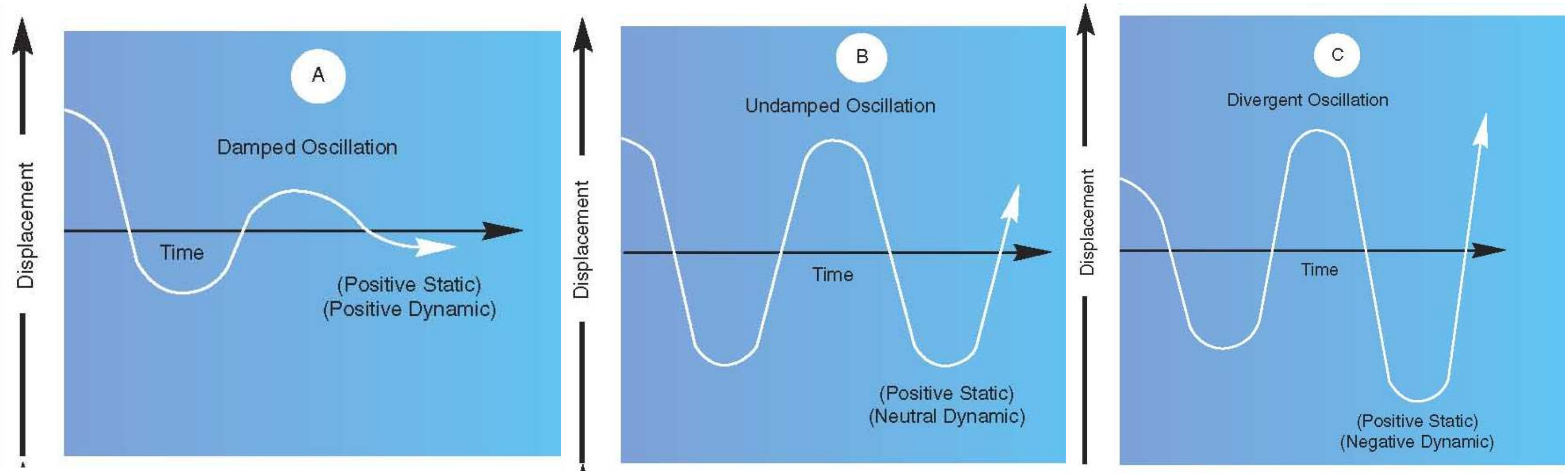
3.

Stability

Static stability



Dynamic stability



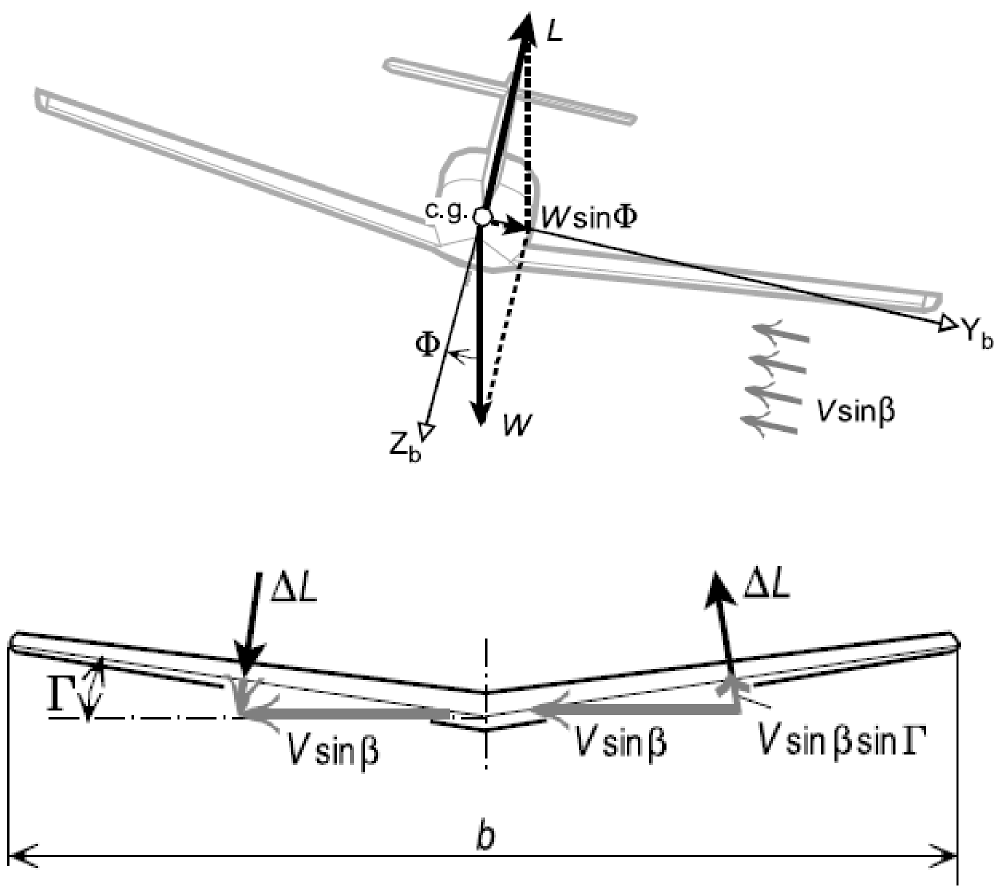
Harder to judge than static stability

4.

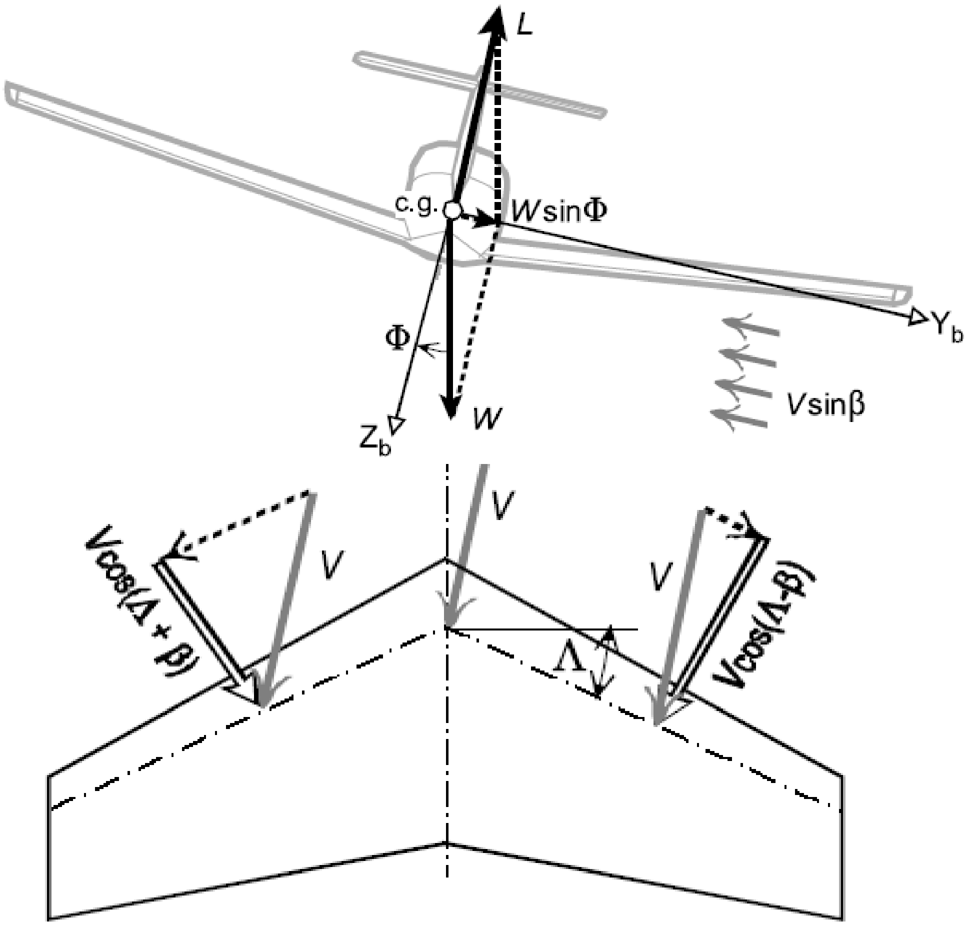
Static stability

- *Lateral examples*
- *Longitudinal*

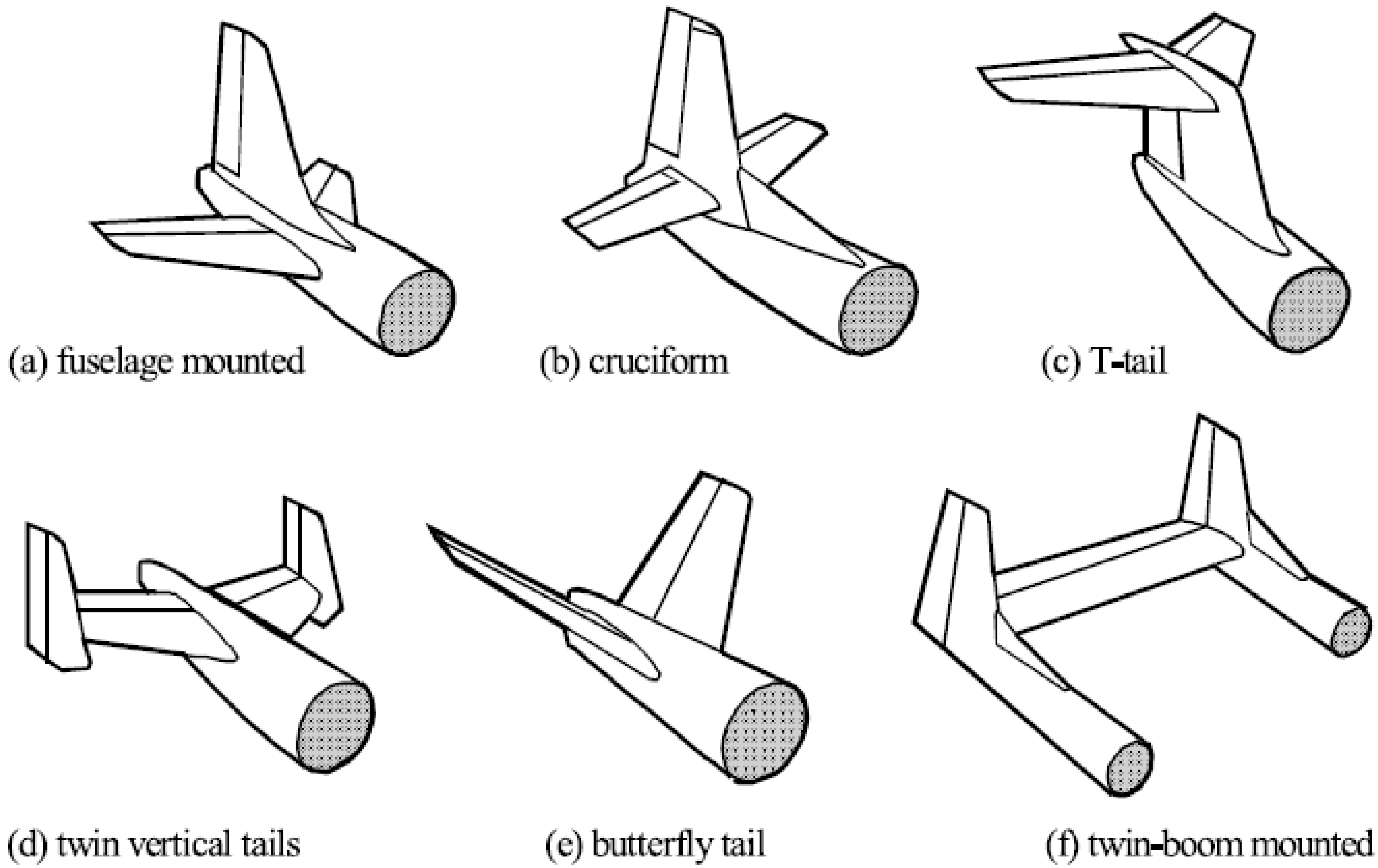
Lateral stability: dihedral



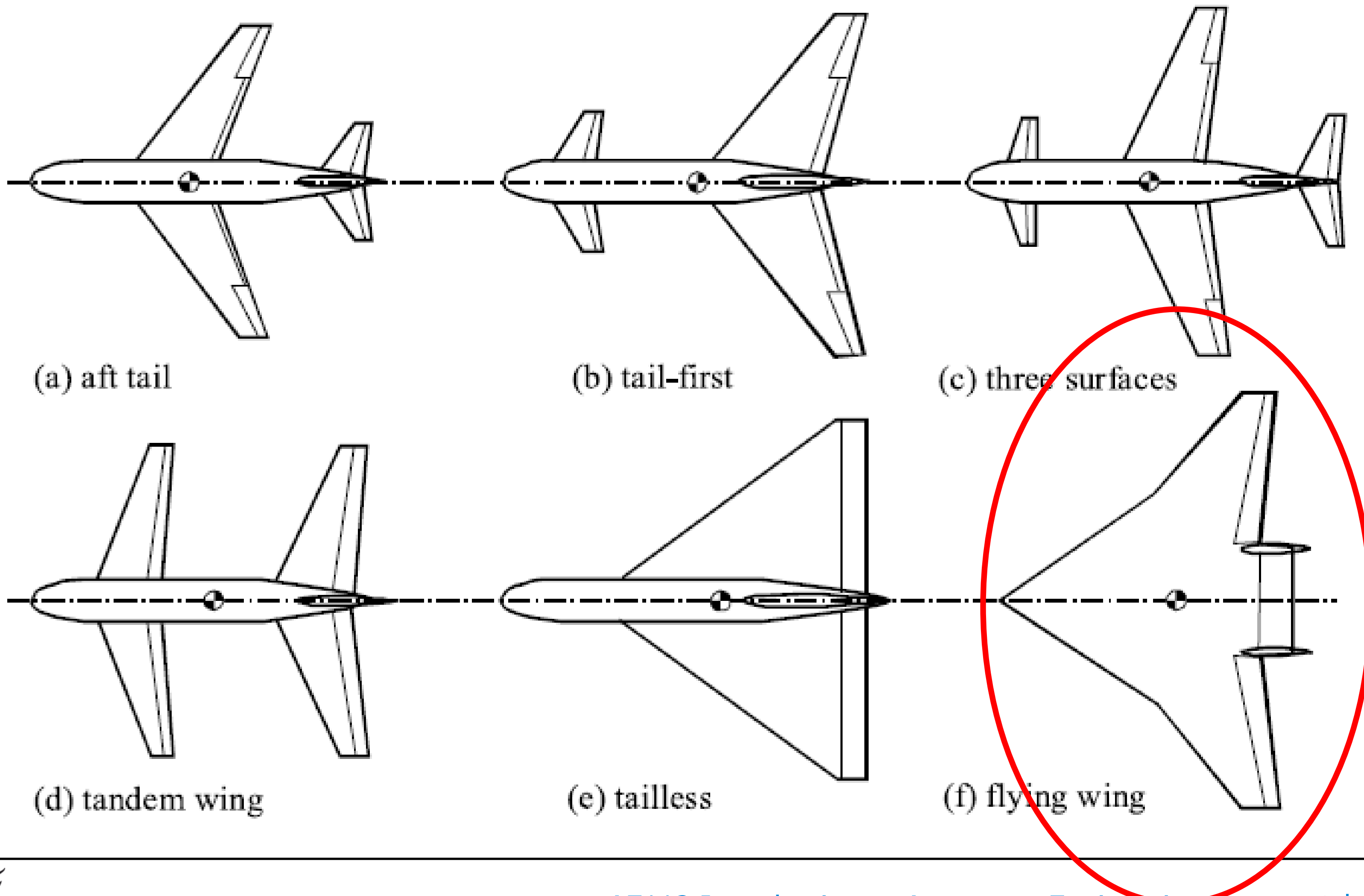
Lateral stability: wing sweep



Tail configurations …. or no tail?

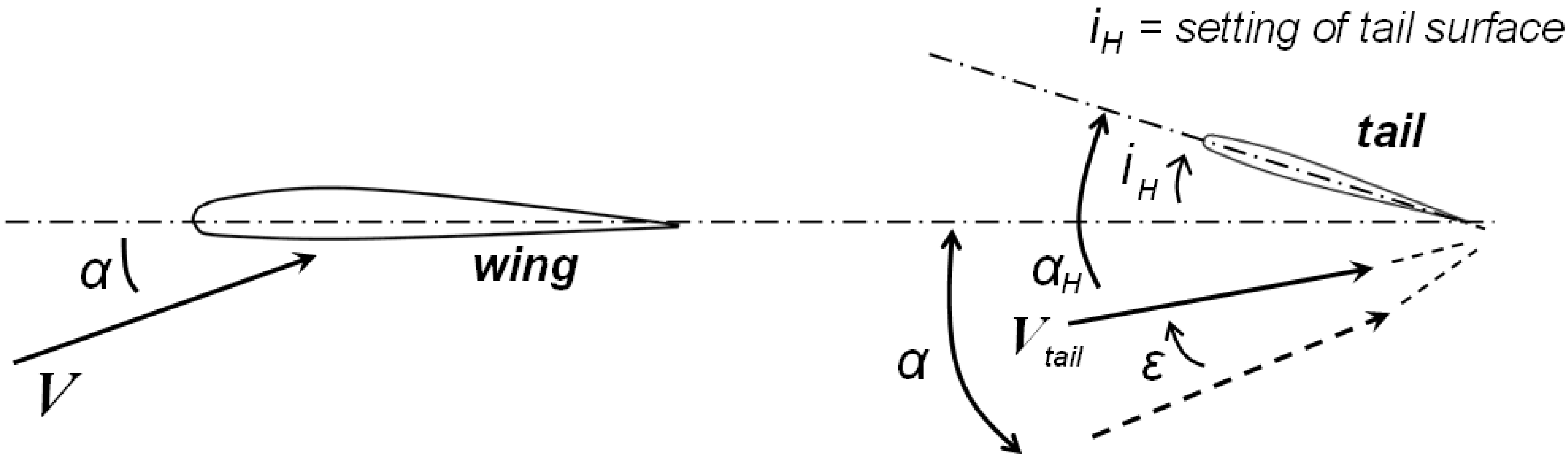


Tail-Wing Configurations



Longitudinal static stability

We have a situation at the tail...



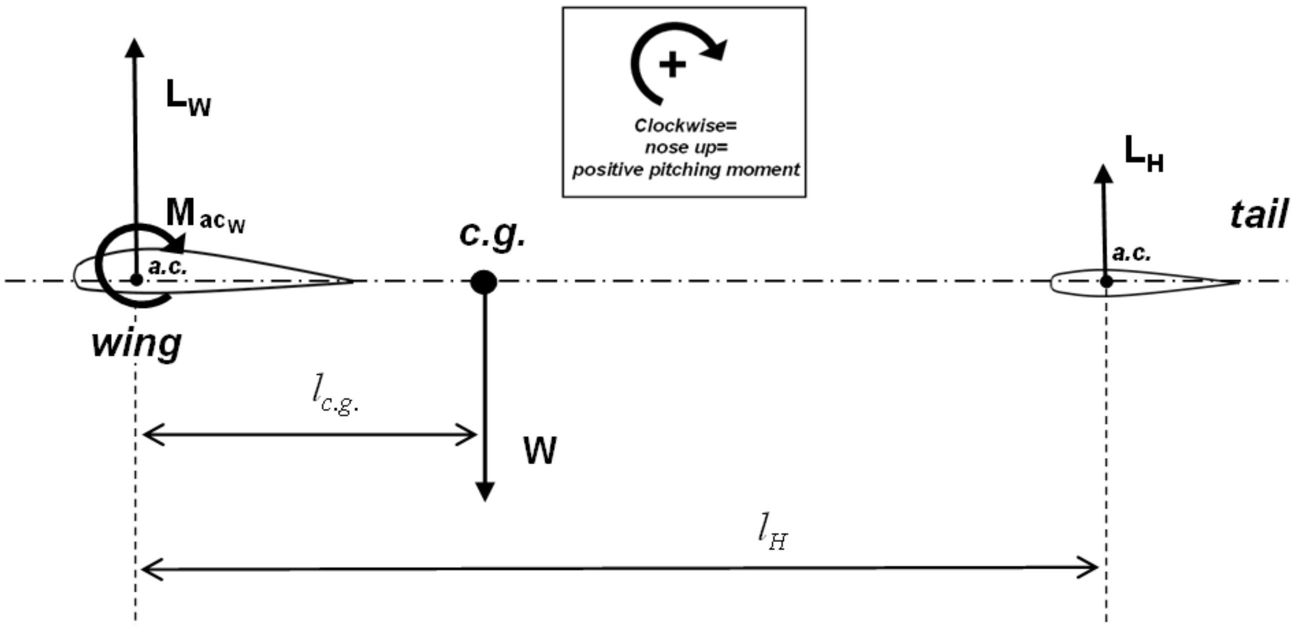
$$\alpha_H = \alpha - \varepsilon + i_H$$

ε = downwash angle

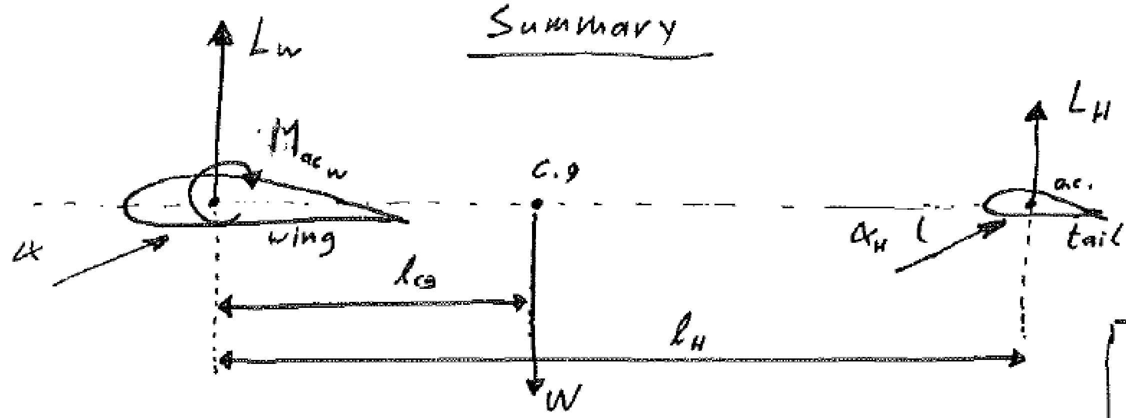
$$\frac{\Delta\alpha_H}{\Delta\alpha} = \frac{d\alpha_H}{d\alpha} = \frac{d}{d\alpha}(\alpha - \varepsilon + i_H) = 1 - \frac{d\varepsilon}{d\alpha}$$

Definition Aerodynamic center (subscript a.c.):

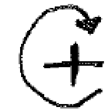
Point around which there is no change in moment due to a change in the angle of attack



Summary



$$L = L_w + L_H$$



$$\alpha_H = \alpha - \varepsilon + i_H$$

$$\frac{d\alpha_H}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}$$

Around c.g.:

$$M = M_{ac_w} + L_w \cdot l_{cg} - L_H\,(l_H - l_{cg})$$

$$= M_{ac_w} + (L_w + L_H)\,l_{cg} - L_H \cdot l_H$$

$$= M_{ac_w} + L \cdot l_{cg} - L_H \cdot l_H$$

Dimensionless:

Divide M by $\frac{1}{2}\rho V^2 S c$

$$\left(\frac{M}{\frac{1}{2}\rho V^2 S c} = \right) C_m = C_{m_{ac_w}} + \frac{C_L \frac{1}{2}\rho V^2 S\; l_{cg}}{\frac{1}{2}\rho V^2 S c} - \frac{C_{L_H} \frac{1}{2}\rho V^2 S_H \cdot l_H}{\frac{1}{2}\rho V^2 S c}$$

$$= C_{m_{ac_w}} + C_L \frac{l_{cg}}{c} - C_{L_H} \cdot \frac{S_H l_H}{S' c}$$

; Definition: $V_H = \frac{S_H l_H}{S \bar{c}}$ "tail volume"

$$= C_{m_{ac_w}} + C_L \frac{l_{cg}}{c} - V_H C_{L_H}$$

Differentiate to α:

$$\frac{dC_m}{d\alpha} = 0 + \frac{dC_L}{d\alpha} \frac{l_{cg}}{c} - V_H \frac{dC_{L_H}}{d\alpha}$$

$$= \frac{dC_L}{d\alpha} \cdot \frac{l_{cg}}{c} - V_H \cdot \frac{dC_{L_H}}{d\alpha_H} \cdot \frac{d\alpha_H}{d\alpha}$$

$$C_{m_\alpha} = C_{L_\alpha} \cdot \frac{l_{cg}}{c} - V_H \, C_{L_{\alpha_H}} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

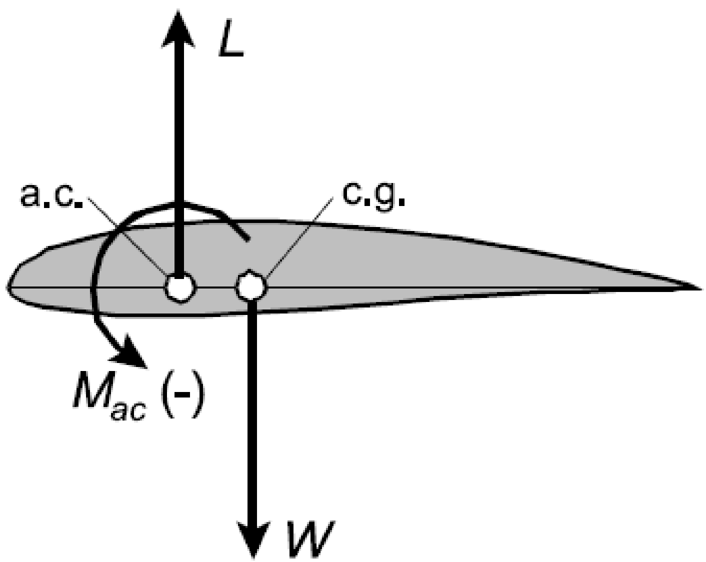
Stable: < 0 ↓

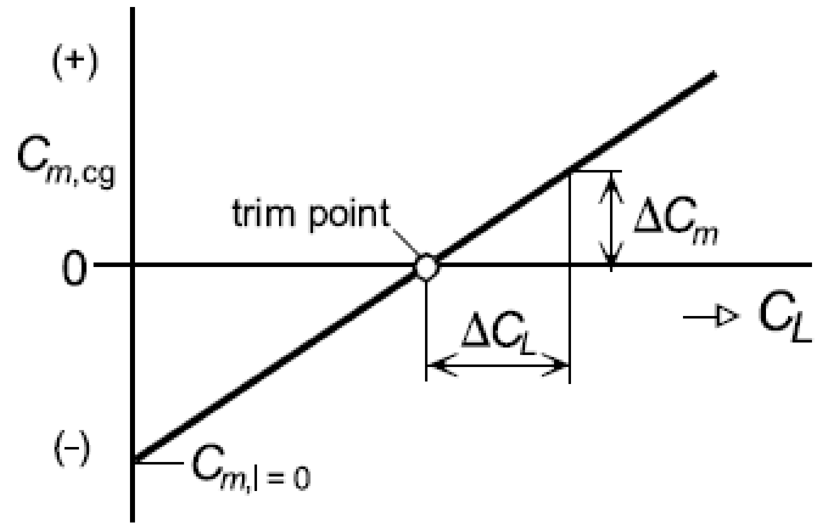
Neutral point: $C_{m_\alpha} = 0 \Rightarrow l_{cg} = l_{np}$

$$\Rightarrow \boxed{\frac{l_{np}}{c} = V_H \cdot \frac{C_{L_{\alpha_H}}}{C_{L_\alpha}} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}$$

→ Stable if $l_{cg} < l_{n.p.}$!

Wing alone is statically unstable





(b) statically unstable

Unfortunately wing with positive camber not stable!

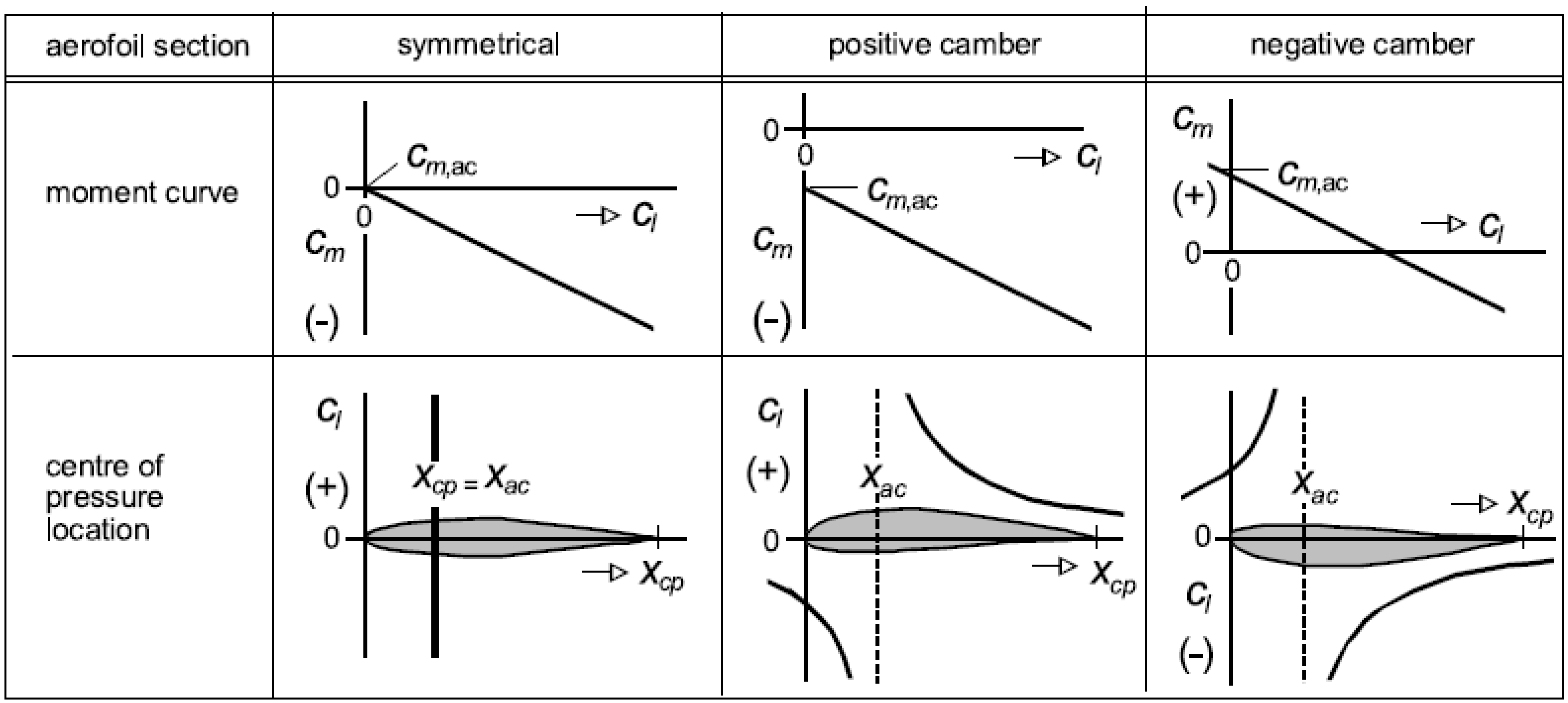
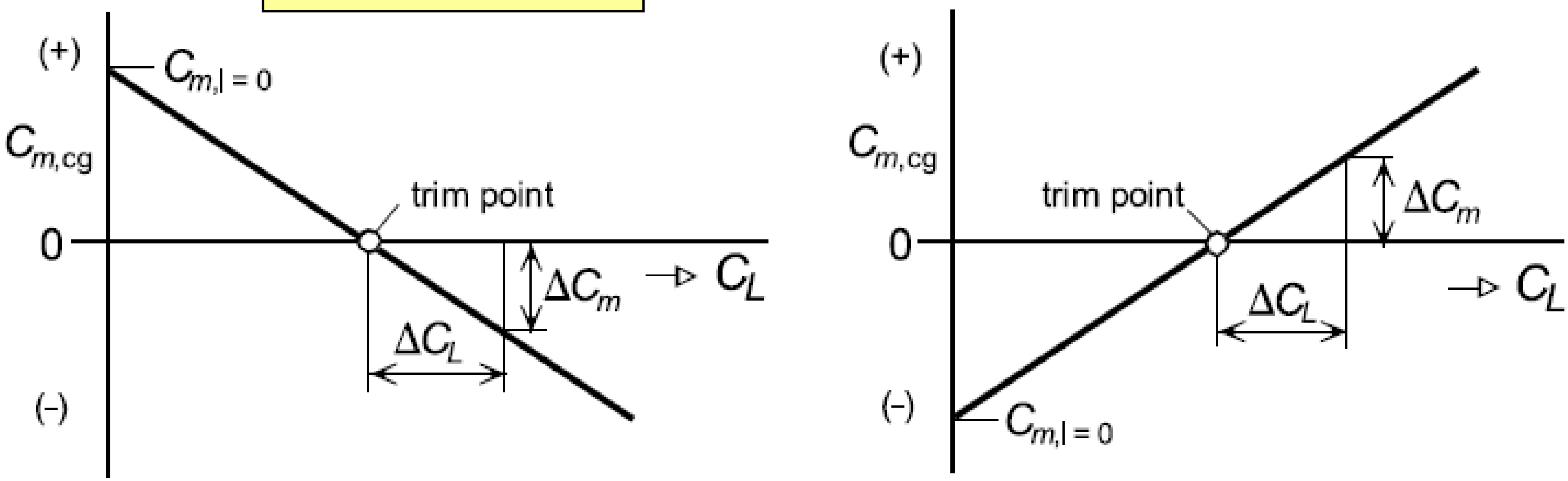


Figure 7.15: Moment curve and centre of pressure at small angles of attack for three classes of aerofoil. The reference point coincides with the nose point.

Longi atic stability

This is the situation we want
⇩



(a) statically stable (b) statically unstable

Stable when two conditions are both met:

1. $C_{m0} > 0$;sufficiently positive zero lift moment **AND**

2. $Cm_{\alpha} < 0$;negative change in moment due to angle of attack = same sign due to C_L

First condition:
positive zero lift moment

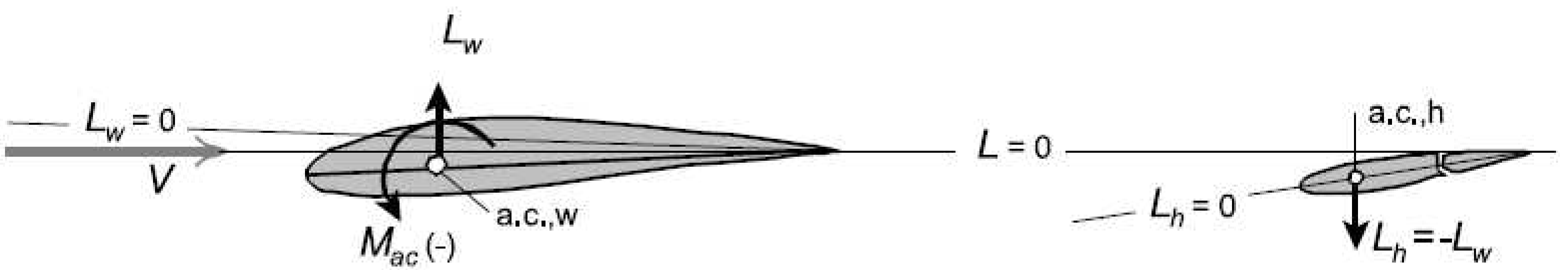
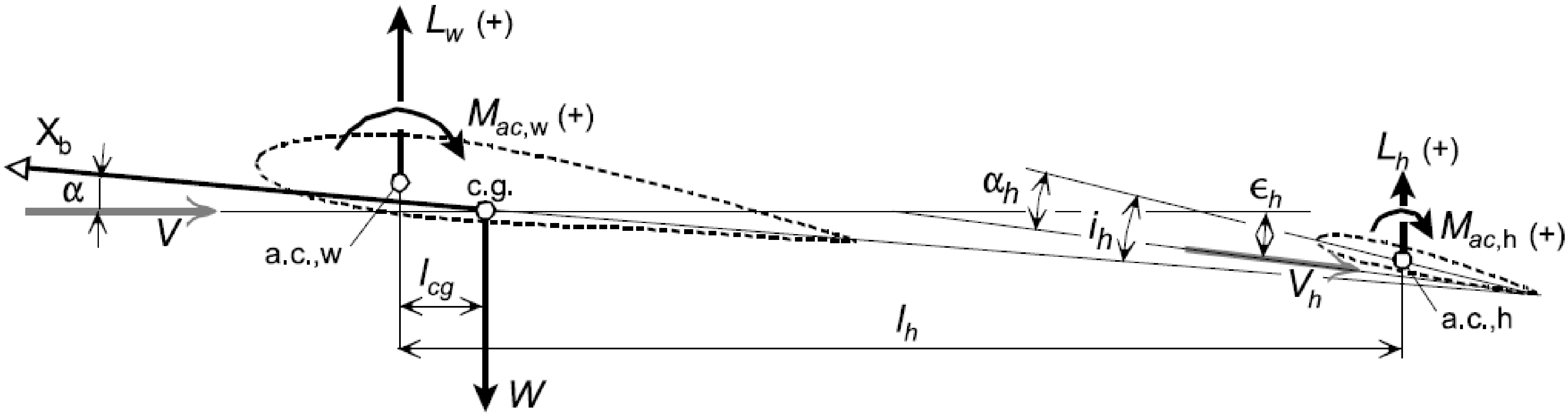


Figure 7.23: Position of the tailplane relative to a positive-cambered wing, resulting in a positive *zero-lift moment*.

Static longitudinal stability



$$L = L_w + L_h \,,$$

and the resulting moment about the c.g. is

$$M_{cg} = M_{ac_w} + M_{ac_h} + L_w l_{cg} - L_h(l_h - l_{cg})$$

$$L = L_w + L_h\ ,$$

and the resulting moment about the c.g. is

$$M_{\mathrm{cg}} = M_{\mathrm{ac_w}} + M_{\mathrm{ac_h}} + L_w l_{\mathrm{cg}} - L_h(l_h - l_{\mathrm{cg}})$$

$$M_{\mathrm{ac_h}} \approx 0$$

$$\Rightarrow \quad M_{\mathrm{cg}} = M_{\mathrm{ac}} + L l_{\mathrm{cg}} - L_h l_h.$$

$$C_{m_{\mathrm{cg}}} = \frac{M_{\mathrm{cg}}}{q_\infty S \bar{c}} = C_{m_{\mathrm{ac}}} + C_L \frac{l_{\mathrm{cg}}}{\bar{c}} - C_{L_{\mathrm{h}}} \frac{S_h l_h}{S \bar{c}} \,.$$

The product $S_h l_h$, known as the *horizontal tail volume*, is made dimensionless

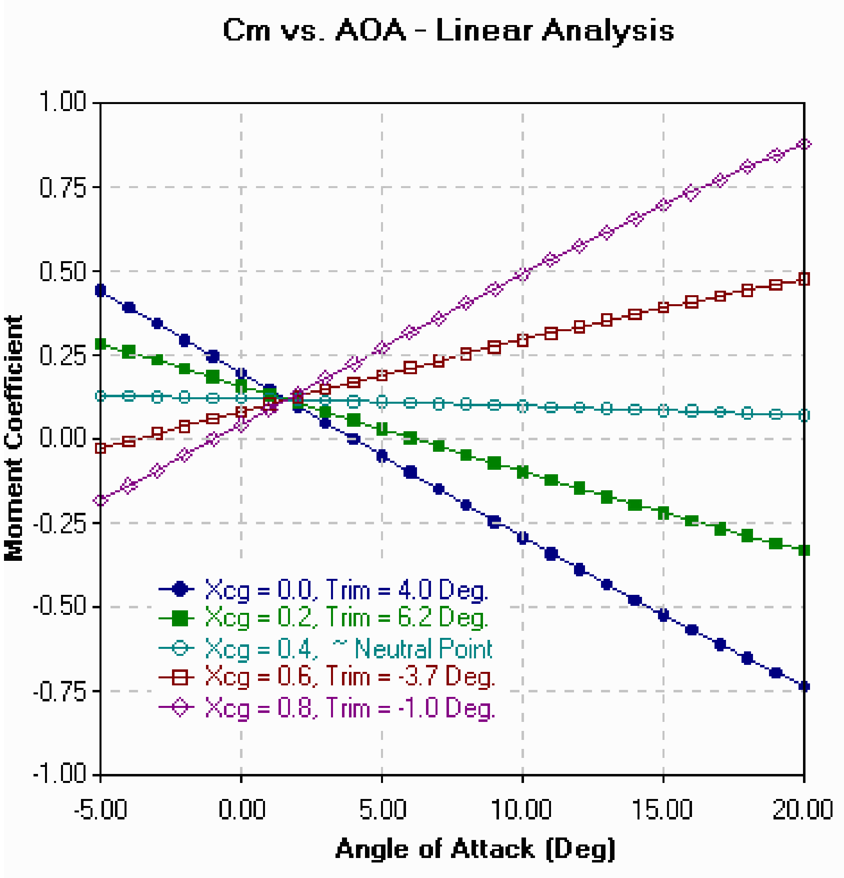
$$\bar{V}_h \triangleq \frac{S_h l_h}{S \bar{c}} \,.$$

$$\frac{\mathrm{d}C_{m_{\mathrm{cg}}}}{\mathrm{d}C_L} = \frac{l_{\mathrm{cg}}}{\bar{c}} - \frac{\mathrm{d}C_{L_{\mathrm{h}}}}{\mathrm{d}C_L} \bar{V}_h \,.$$

For static stability:

$$\frac{l_{\mathrm{cg}}}{\bar{c}} - \frac{\mathrm{d}C_{L_{\mathrm{h}}}}{\mathrm{d}C_L} \bar{V}_h < 0 \,.$$

Stability and C_{m_α}: neutral point

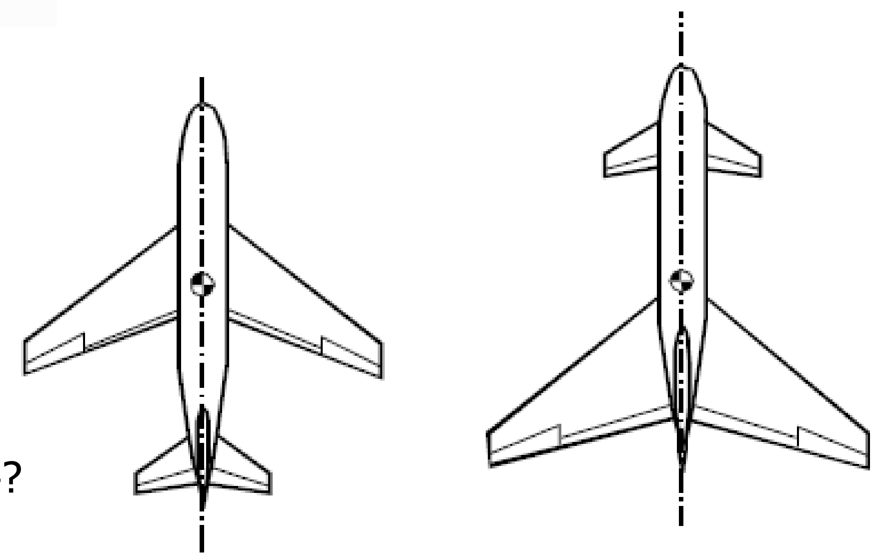


Estimate neutral point: more or less than 0.4?

Factors for pitch stability:

- Position of tail surface
- Position of center of gravity

Meaning of neutral point?



Neutral point

The tailplane angle of attack is equal to the aeroplane angle of attack increased by the tail *angle of incidence* i_h and reduced by the *downwash* angle due to wing lift (Figure 7.22),

$$\alpha_h = \alpha + i_h - \epsilon_h \,. \tag{7.38}$$

The tail incidence is invariable when the angle of attack is disturbed, hence

$$\mathrm{d}\alpha_h/\mathrm{d}\alpha = 1 - \mathrm{d}\epsilon_h/\mathrm{d}\alpha \,. \tag{7.39}$$

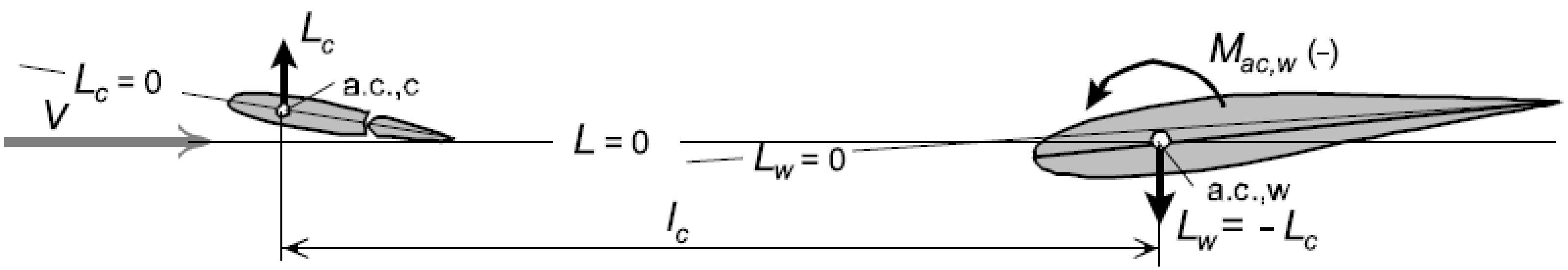
The location of the n.p. follows from the substitution of Equations (7.36), (7.34) and (7.39) into Equation (7.35):

$$\frac{l_{\mathrm{np}}}{l_h} = \frac{\mathrm{d}L_h/\mathrm{d}L_w}{1 + \mathrm{d}L_h/\mathrm{d}L_w} \quad \text{with} \quad \frac{\mathrm{d}L_h}{\mathrm{d}L_w} = \frac{(\mathrm{d}C_L/\mathrm{d}\alpha)_h}{(\mathrm{d}C_L/\mathrm{d}\alpha)_w}\left(1 - \frac{\mathrm{d}\epsilon_h}{\mathrm{d}\alpha}\right)\frac{S_h}{S} \,. \tag{7.40}$$

This expression shows, in a dimensionless form, the distance of the n.p. behind the a.c. of the wing as a result of the tailplane's stabilizing effect. For an aft-tail aeroplane, $\mathrm{d}L_h/\mathrm{d}L_w \approx 0.1$ and the following approximation can be made:

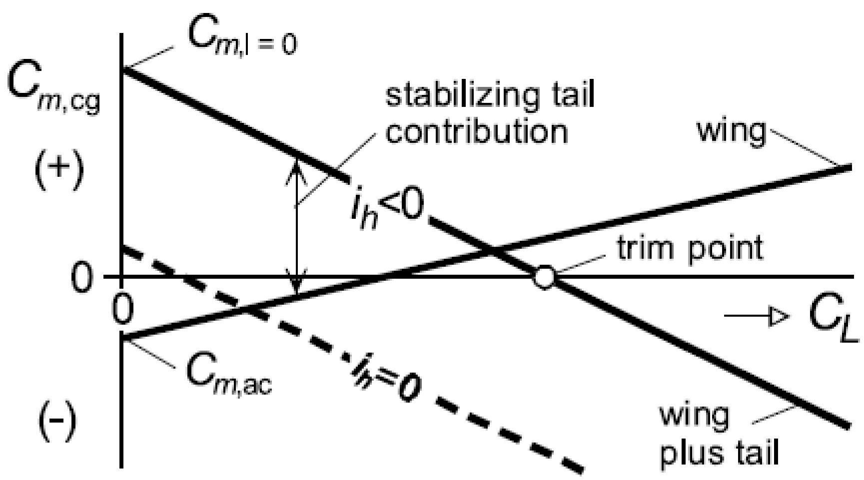
$$\frac{l_{\mathrm{np}}}{\bar{c}} = 0.9\frac{(\mathrm{d}C_L/\mathrm{d}\alpha)_h}{(\mathrm{d}C_L/\mathrm{d}\alpha)_w}\left(1 - \frac{\mathrm{d}\epsilon_h}{\mathrm{d}\alpha}\right)\bar{V}_h \,. \tag{7.41}$$

How about a canard?

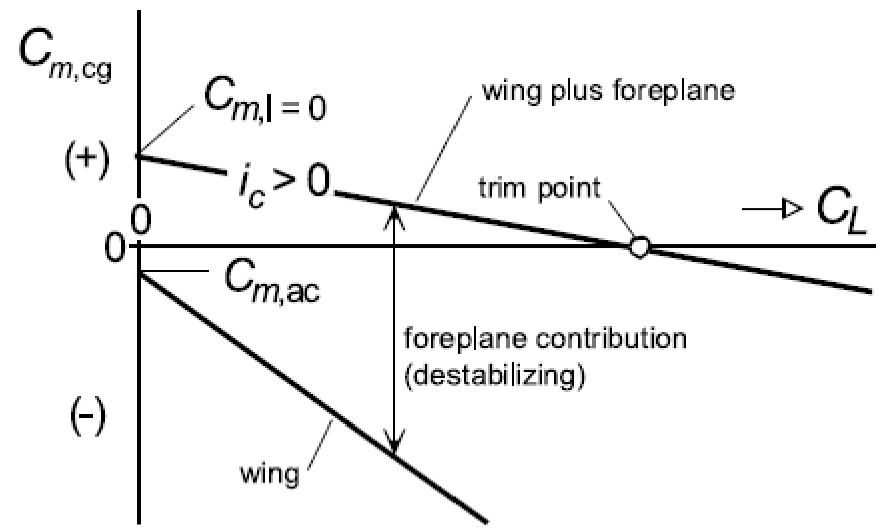


Zero lift situation

Tail vs. canard (foreplane)



Inherently stable tail config



Statically stable canard, by moving c.g. forward rel. to wing

Stability margin

A measure for the longitudinal stability can be determined for a given location of the *neutral point*. For this purpose, Equation (7.35) is expressed in a dimensionless form

$$\frac{l_{\mathrm{np}}}{\bar{c}} = \frac{\mathrm{d}C_{L_\mathrm{h}}}{\mathrm{d}C_L}\bar{V}_h \,. \tag{7.42}$$

If this expression is combined with Equation (7.32), the slope of the moment curve is found:

$$-\frac{\mathrm{d}C_m}{\mathrm{d}C_L} = \frac{l_{\mathrm{np}} - l_{\mathrm{cg}}}{\bar{c}} \,. \tag{7.43}$$

Piaggio P180 Avanti

Beechcraft Starship 2000

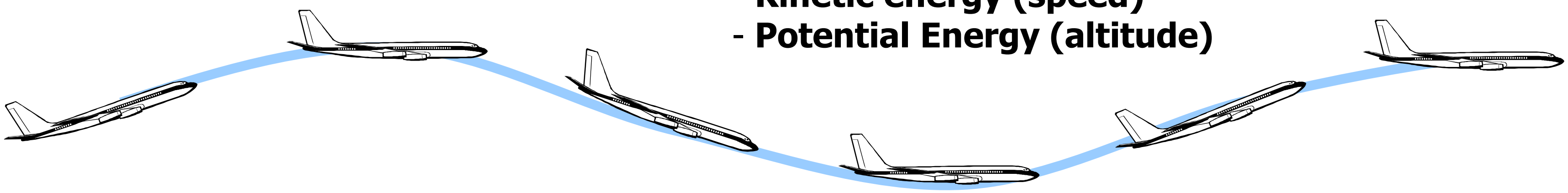
5.

Dynamic stability
- typical modes oscillations of conventional aircraft

Typical longitudinal oscillations

Period: 30 sec – several minutes

Exchanging:
- **Kinetic energy (speed)**
- **Potential Energy (altitude)**

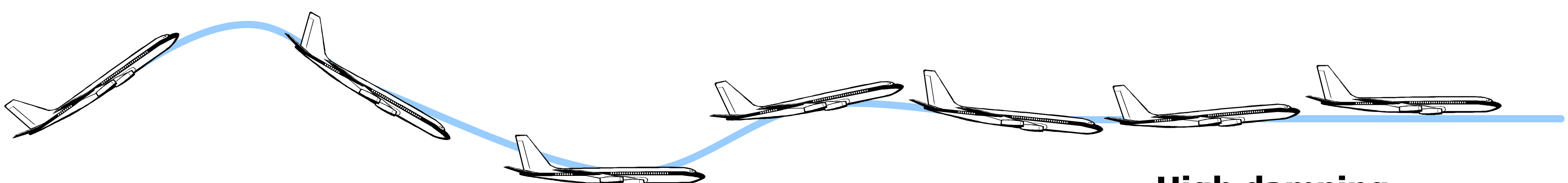


Langzame slingering (fugoïde)
Long period oscillation (phugoid)

Modern airliners:
Low drag, low damping
(sometimes noticeable as passenger)

Period: 2 - 5 seconds

Reaction on disturbance from balance

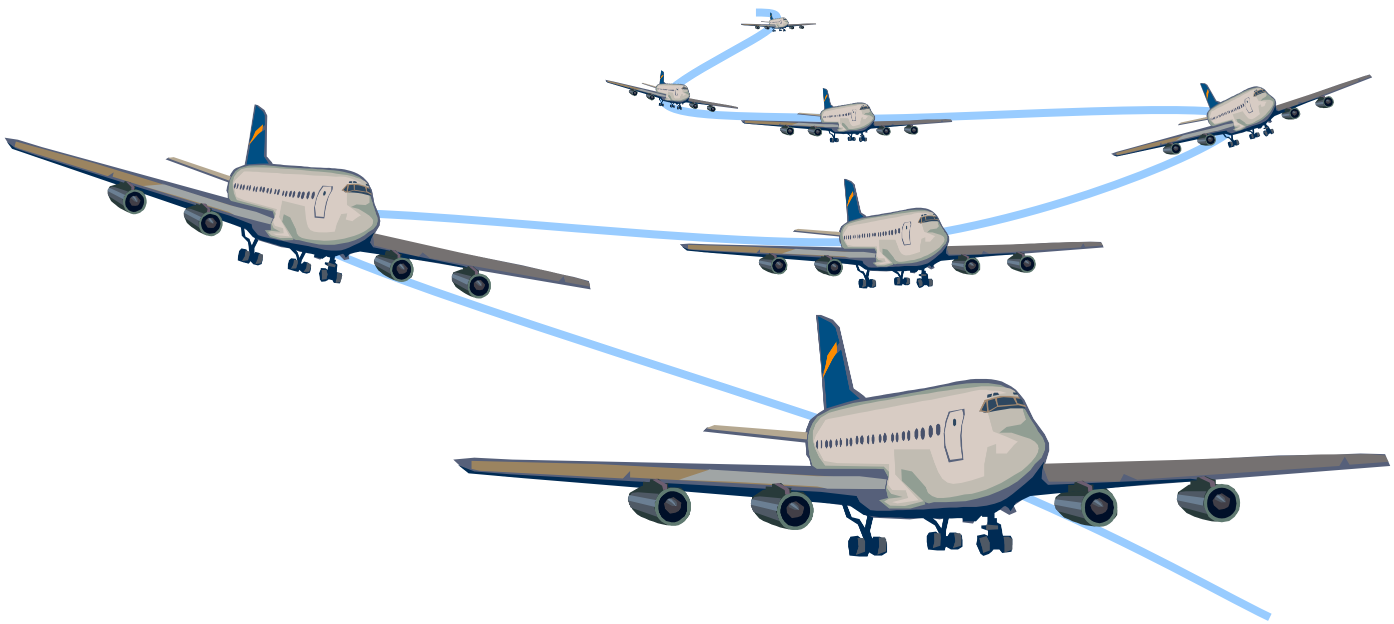


Snelle slingering
Short period pitching

High damping

Typical lateral oscillations

Zwierbeweging
Dutch roll

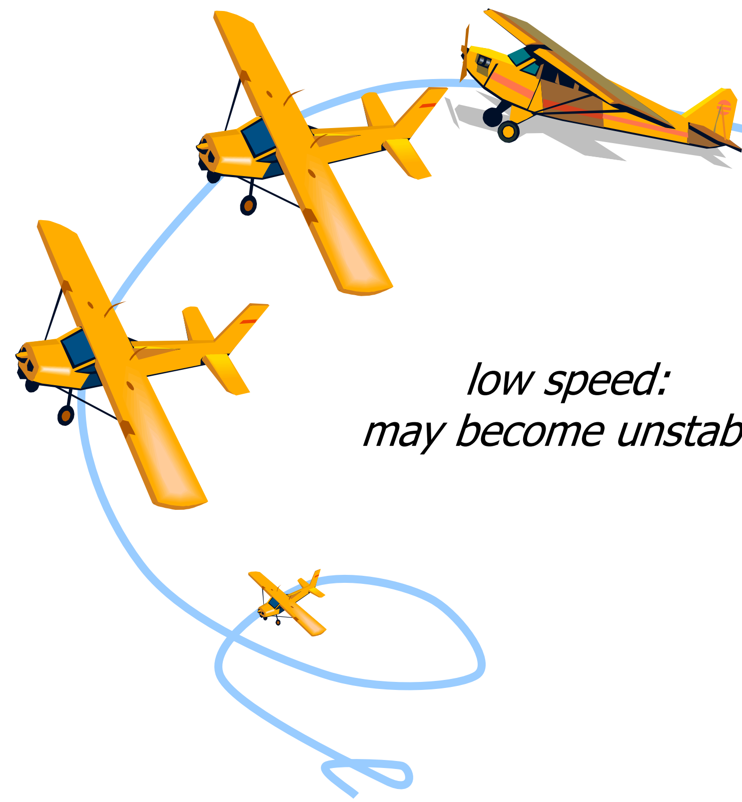


Typical lateral modes

Aperiodic rolling mode

high speed: stable

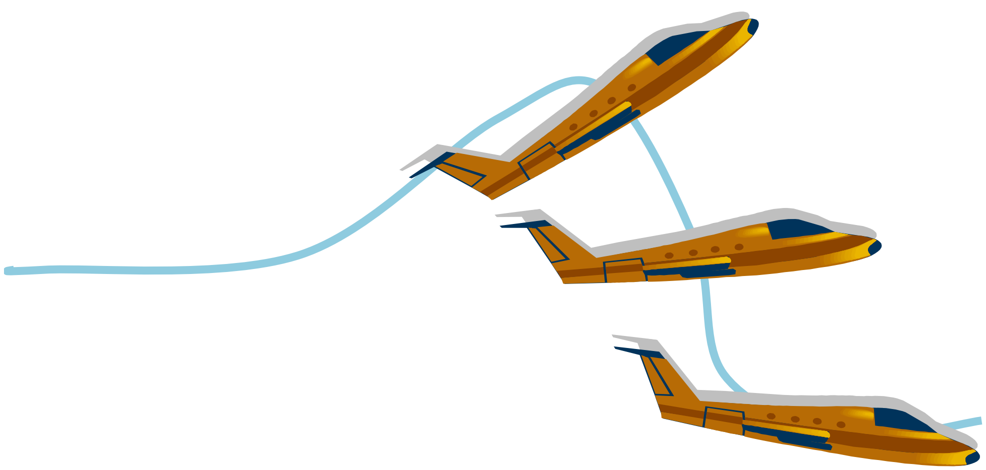
Spiral

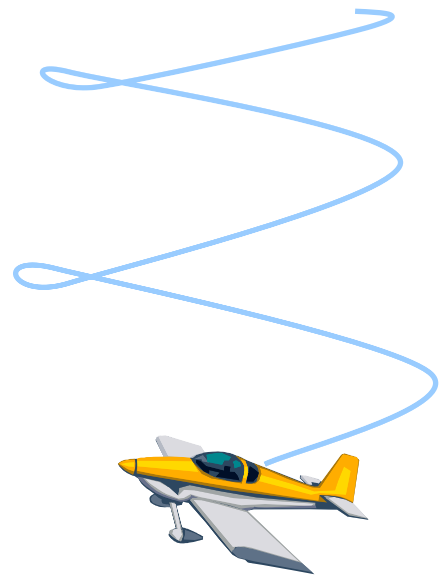


*low speed:
may become unstable*

Vrille, spin = stalled

Normal stall





Flat spin

(similar to steep spin)

Choose an aircraft…

- Estimate for your aircraft in which range the center of gravity would be from the planform

- For the following stability derivatives:
  - The sign of the derivative: negative, zero (negligible) or positive
  - Reason for the sign
    (contributing factors: change of lift of wing, position of surfaces etc)
  - Contribution to static stability (or reduction)

C_{l_r} C_{n_p} C_{l_β}

- Judge the configuration of your aircraft and the position of the control surfaces. Try to explain why this was chosen as it is from a static stability and/or control point of view.


EGYPT AIR

Example A300

$$\frac{l_{np}}{c} = \frac{a_t}{a} \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \text{with} \quad V_H = \frac{S_H \cdot l_H}{S \cdot c}$$

General data:

- Wing area S = 260 m^2
- Span b = 44.85 m
- Length 54.08 m
- Typical operating weight = 90,060 kg
- MTOW = 165,000 kg
- Distance wing ac to tail ac: l_H=25,0 m

Engineering data:

- CL-alpha wing, a_{wing} = 4.4 1/rad (=0.076 per degree)
- CL-alpha tail, a_{tail} = 2.7 1/rad (= 0.047 per degree)
- Downwash at tail 1.0 degree per 10.0 deg alpha
- When c.g. 3.55 m after a.c of wing, it should still be stable

Question:

- What is minimum horizontal tail area?

A300 stability.xls

Example A300

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- Downwash at tail 1.0 degree per 10.0 deg alpha
- When c.g. 3.55 m after a.c of wing, it should still be stable

Question:

- What is minimum horizontal tail area?
- S_H=67 m^2

Other potential questions: what is i_h?

Homework Stability & Control

- Anderson problems:

  7.1 - 7.6 & 7.9

- Notation is different: $h = 0.26$ means $x_{cg}/c = 0.26$

$$\frac{l_{np}}{c} = \frac{a_t}{a} \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \text{with} \quad V_H = \frac{S_H \cdot l_H}{S \cdot c}$$