Introduction to Aerospace Engineering

Exams

Problem 1

Antwoord is een beschrijving waar in ieder geval de gedachte aanwezig moet zijn dat het totale gewicht bestaat uit leeggewicht (vnl. constructie), betalende lading en brandstofgewicht. Lichtgewicht construeren is er dus op gericht om zoveel mogelijk payload + brandstof mee te nemen (welke verhouding optimaal is hangt af van de missie).

Problem 2

c. Structural properties depend on material properties and geometrical features

Problem 3

a)

$$dp = -\rho g_0 \, dh$$

$$\rho = \frac{p}{RT}$$

$$\Rightarrow \quad dp = -\frac{p}{RT} g_0 \, dh$$

$$\frac{dT}{dh} = a \Rightarrow \frac{dh}{dT} = \frac{1}{a}$$

$$\int \frac{dp}{p} = -\int \frac{1}{RT} g_0 \cdot \frac{dh}{dT} \cdot dT$$

$$\int_{p_1}^{p_2} \frac{1}{p} dp = \int_{T_1}^{T_2} -\frac{g_0}{aR} \cdot \frac{1}{T} dT$$

$$\ln p_2 - \ln p_1 = -\frac{g_0}{aR} \ln T_2 - \ln T_1$$

$$e^{\ln p_2 - \ln p_1} = e^{-\frac{g_0}{aR} \ln T_2 - \ln T_1}$$

$$\frac{p_2}{p_1} = \left(e^{\ln T_2 - \ln T_1}\right)^{-\frac{g_0}{aR}} = \left(\frac{T_2}{T_1}\right)^{-\frac{g_0}{aR}}$$

b) $$\boxed{L_{NET} = \rho_{atm} V g \left(1 - \frac{\rho_{gas}}{\rho_{atm}}\right) = \rho_{atm} V g \left(1 - \frac{M_{gas}}{M_{air}}\right)}$$

V = volume of gas

M = molar mass

ρ_{atm} = atmospheric air density

$$\Rightarrow \boxed{L_{NET} = 8.4544 \cdot \rho_{atm} \cdot V}$$

c) Given: M_{He}, M_{air},

$L_{NET} = W = 2{,}000$ kg

$h = 30.48$ km

at $h = 11$ km: $p = 22614$ Pa

$\rho = 0{,}3636$ kg/m3

$T = 216.65$ K

Needed: ρ_{min} for maximum volume!

First: $11 \to 20$ km isothermal: $T_0 = T_1 = 216.65$ K

Isothermal: $dp = -\rho g dh$ $\quad \rho = \frac{p}{RT}$

$$dp = -\frac{gp}{RT} dh$$

$$\frac{1}{p} dp = -\frac{g}{RT} dh$$

$$\ln p_2 - \ln p_1 = -\frac{g}{RT}(h_2 - h_1)$$

$$\frac{p_2}{p_1} = e^{-\frac{g}{RT}(h_2 - h_1)} = \frac{\rho_2 \cancel{RT}}{\rho_1 \cancel{RT}} = \frac{\rho_2}{\rho_1}$$

3 continued

$11 \rightarrow 20$ km: $\frac{p_1}{p_0} = e^{-\frac{g}{RT}(h_2 - h_1)}$

$$\frac{p_1}{22614} = e^{-\frac{9{,}81}{287 \cdot 216.65}(20000 - 11000)} = 0{,}241727 \cdot 22614$$

$$p_1 = 5466 \quad \text{Pa}$$

$$\rho = 0{,}241727 \cdot 0{,}3636 = 0{,}08789 \quad \text{kg/m}^3$$

$20 \rightarrow 30.418$ km: linear temperature variation, use earlier derived formulas

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{-\frac{g}{aR}} \Rightarrow \frac{\rho_2 \cdot T_2}{\rho_1 \cdot T_1} = \left(\frac{T_2}{T_1}\right)^{-\frac{g}{aR}}$$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{-\frac{g}{aR} - 1}$$

$$\Rightarrow T_2 = T_1 + a(h_2 - h_1)$$

$$= 216.65 + 1 \cdot 10{,}480$$

$$= 227.134$$

$$\frac{\rho_2}{0{,}08789} = \left(\frac{227{,}13}{216{,}65}\right)^{-\frac{9{,}81}{0{,}001 \cdot 287} - 1}$$

$$= \left(\frac{227.13}{216.65}\right)^{-35.18118}$$

$$\frac{\rho_2}{0{,}08789} = 0.18977$$

$$\rho_2 = 0{,}01668 \quad \text{kg/m}^3$$

So $\rho = 0{,}01668\ kg/m^3$ at $30.48\ km$

$$L_{NET} = W$$

$$\rho_{atm} \cdot V \cdot \cancel{g} \left(1 - \frac{M_{He}}{M_{air}}\right) = m \cdot \cancel{g}$$

$$0{,}01668 \cdot V \cdot \cancel{g} \left(1 - \frac{4{,}00}{28{,}97}\right) = 2{,}000$$

$$V = 139{,}1\ m^3$$

Problem 4

$*$ = throat

a) Energy equation:

$$C_p T_0 + \tfrac{1}{2} V_0^2 = C_p T^* + \tfrac{1}{2} {V^*}^2$$

$$V_0 = 0 \quad \Rightarrow \quad C_p T_0 = C_p T^* + \tfrac{1}{2} {V^*}^2$$

in the throat $M = 1$: $V^* = a = \sqrt{\gamma R T}$

so: $C_p T_0 = C_p T^* + \tfrac{1}{2} \gamma R T^* = T^*\left(C_p + \tfrac{1}{2}\gamma R\right)$

$$\Rightarrow T^* = \frac{C_p T_0}{C_p + \tfrac{1}{2}\gamma R}$$

$$R = C_p - C_v = C_p\left(1 - \frac{c_v}{c_p}\right) = C_p\left(1 - \frac{1}{\gamma}\right)$$

$R = 288$ J/kgK

filling in: $T^* = \dfrac{1008 \cdot 1200}{1008 + \frac{1}{2} 1.4 \cdot 288} = 1000\ K$

b) $V^* = a = \sqrt{\gamma R T^*} = \sqrt{1.4 \cdot 288 \cdot 1000} = 640$ m/s

c) Using the energy equation:

$$C_p T_0 = C_p T_e + \tfrac{1}{2} V_e^2$$

$$V_e = \sqrt{2 C_p (T_0 - T_e)} = \sqrt{2 \cdot 1008 (1200 - 310)} \approx 1339.5 \text{ m/s}$$

$$a_e = \sqrt{\gamma R T_e} = \sqrt{1.4 \cdot 288 \cdot 310} = 353.5 \text{ m/s}$$

$$M = \frac{V}{a} = \frac{1339.5}{353.5} = 3.79$$

d) $\dot{m} = \rho A V$

$\dot{m} = \rho^* A^* V^* \Rightarrow A^* = \dfrac{\dot{m}}{\rho^* V^*}$

$V^* = 640$ m/s.

Isentropic relation: $\dfrac{\rho_0}{\rho^*} = \left(\dfrac{T_0}{T^*}\right)^{1/\gamma-1}$

$\dfrac{P_0}{\rho_0} = RT_0 \Rightarrow \rho_0 = \dfrac{P_0}{RT_0} = \dfrac{9 * 1013250}{288 * 1200}$

$= 2.64$ kg/m^3

$\rho^* = \dfrac{\rho_0}{\left(\dfrac{T_0}{T^*}\right)^{1/\gamma-1}} = 1.674$ kg/m^3

$A^* = \dfrac{0.5}{1.674 * 640} = 4.67 * 10^{-4}$ m^2

$= 4.67$ cm^2

Problem 5

a) Tsjolkovski → M_p = 221,2 kg.

b) Momentum exchange → $(M_0 - M_p)V_1 = M_p V_e$ → M_p = 200,0 kg.

c) Instantaneous acceleration is most efficient for momentum exchange.

d) $a = \Delta V/10$ = 1 m/sec^2

e) $T = (a+g)M_0$ = 11 kN

Problem 6

1) The altitude goes down because of atmospheric drag.

2) The decay of Mir is high in the beginning and at the end of the plot, and less in the central part. This is caused by the solar activity: we have a Solar Maximum in the beginning and at the end -> large drag effect, and a Solar Minimum in the middle -> less drag and smaller loss of altitude

3) The decay of Mir and GFZ-1 is more-or-less the same, because they fly at about the same altitude and because their ballistic coefficient ($C_D S/m$, or, equally important the surface-to-mass ratio S/m) is more-or-less identical.

4) Mir is boosted back to its nominal altitude every now and then, whereas GFZ-1 is a purely passive satellite without any orbit correction possibilities.