Introduction to Aerospace Engineering

Exams

Solutions exam AE1101 Resit January 2011

Problem 1 (5 pts):
Answer **C**. The Wright Brothers managed to perform a powered and controlled flight for the first time

Problem 2 (10 pts):
In aerospace weight is a dominant factor. So options are always evaluated using "performance/weight" ratios. In case of mechanical properties of materials and structures, strength and stiffness are important properties. When these performances will be evaluated, we divide these with a function of the density, thereby obtaining so-called "specific properties".

Problem 3 (25 pts):

a) $\frac{dC_m}{d\alpha} < 0$, has to be negative for static stability

b) Moment equation:

$$\sum M = L_C \cdot (l_c + l_{cg}) + L_W \cdot l_{cg} + M_{ac_w} - L_H \cdot (l_H - l_{cg})$$

and since $L = L_C + L_W + L_H$, we write this as follows

$$\sum M = (L_C + L_W + L_H)\, l_{cg} + L_C\, l_c + M_{ac_w} - L_H \cdot l_H$$

$$M = L \cdot l_{cg} + L_C \cdot l_c + M_{ac_w} - L_H \cdot l_H$$

$$C_m = \frac{M}{\frac{1}{2}\rho V^2 S c}$$

$$C_m = \frac{L}{\frac{1}{2}\rho V^2 S} \frac{l_{cg}}{c} + \frac{L_C}{\frac{1}{2}\rho V^2 S} \frac{l_c}{c} + \frac{M_{ac_w}}{\frac{1}{2}\rho V^2 S \bar{c}} - \frac{L_H}{\frac{1}{2}\rho V^2 S} \frac{l_H}{c}$$

$$C_m = C_L \cdot \frac{l_{cg}}{c} + C_{L_C} \frac{S_C}{S} \cdot \frac{l_c}{\bar{c}} + C_{m_{ac_w}} - C_{L_H} \cdot \frac{S_H}{S} \frac{l_H}{c}$$

$$C_m = C_L \cdot \frac{l_{cg}}{\bar{c}} + C_{L_C} \cdot \bar{V}_C + C_{m_{ac_w}} - C_{L_H} \cdot \bar{V}_H$$

$$\frac{dC_m}{d\alpha} = C_{m_\alpha} = C_{L_\alpha} \cdot \frac{l_{cg}}{c} + C_{L_{\alpha_c}} \cdot \bar{V}_c + 0 - C_{L_{H_{\alpha_H}}} \cdot \frac{d\alpha_H}{d\alpha} V_H$$

$$\frac{d\alpha_H}{d\alpha} = \frac{d(\alpha + i_H - \varepsilon)}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}$$

$$a = C_{L_\alpha} \qquad a_c = C_{L_{\alpha_c}} \qquad a_t = C_{L_{\alpha_T}}$$

$$C_{m_\alpha} = a \frac{l_{cg}}{c} + a_c \cdot \bar{V}_c - a_t \left(1 - \frac{d\varepsilon}{d\alpha}\right) V_H$$

c) Prove unstable when $V_c = V_H$ dan $l_{cg} = 0$ and $a_c = a_t$ and $\frac{d\varepsilon}{d\alpha} = 0.05$

$$C_{m_\alpha} = a \frac{l_{cg}}{c} + a_c V_c - a_t \left(1 - \frac{d\varepsilon}{d\alpha}\right) V_H$$

$$= 0 + a_t \cdot V_H - a_t \cdot 0.95 \cdot V_H$$

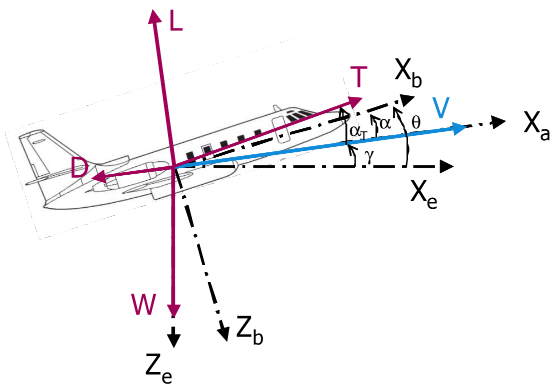
$$C_{m_\alpha} = 0.05\, a_t V_H > 0 \Rightarrow \text{unstable.}$$

d) C_{m_α} should become negative, so l_{cg} should be lower (hence < 0) to lower , because $a > 0$. This means the c.g. moves to in front of the wing (in direction of the nose).

e) The main advantage is more stability (especially by using a T-tail to minimize the downstream ε) and hence a larger c.g. range. The main disadvantage is the extra weight and/or extra drag

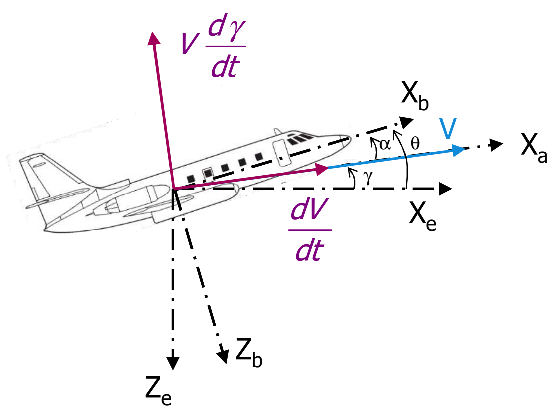
Problem 4: Solution to flight mechanics questions

A



B

C



$$\sum F_{//V}: \frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\sum F_{\perp V}: \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

$$\frac{P_a - P_r}{W} = V\sin\gamma + \frac{1}{2g}\frac{dV^2}{dt}$$

D

Steady symmetric horizontal flight:

$L = W, T = D$

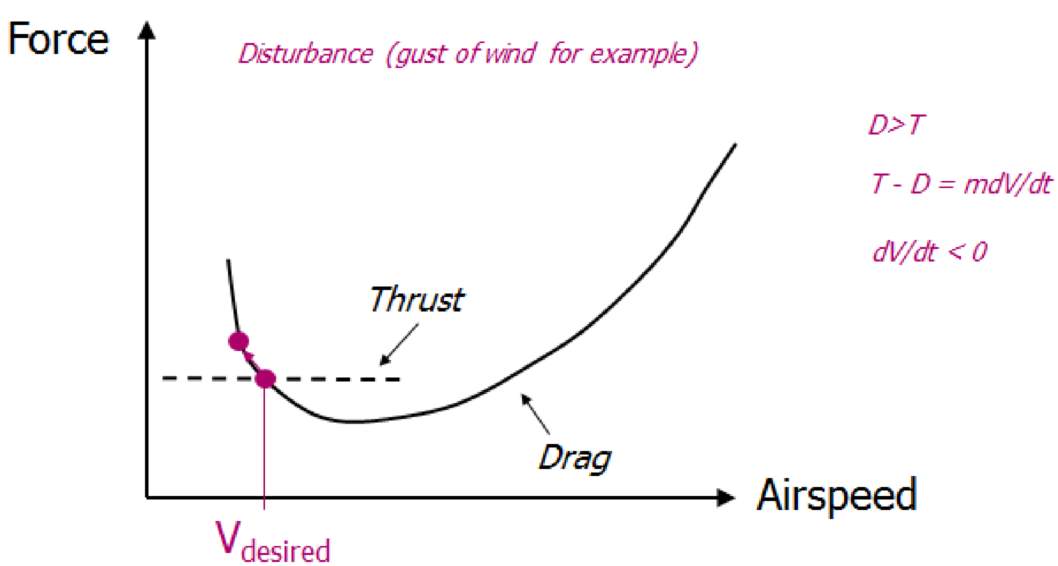
With

$L = C_L \frac{1}{2}\rho V^2 S$

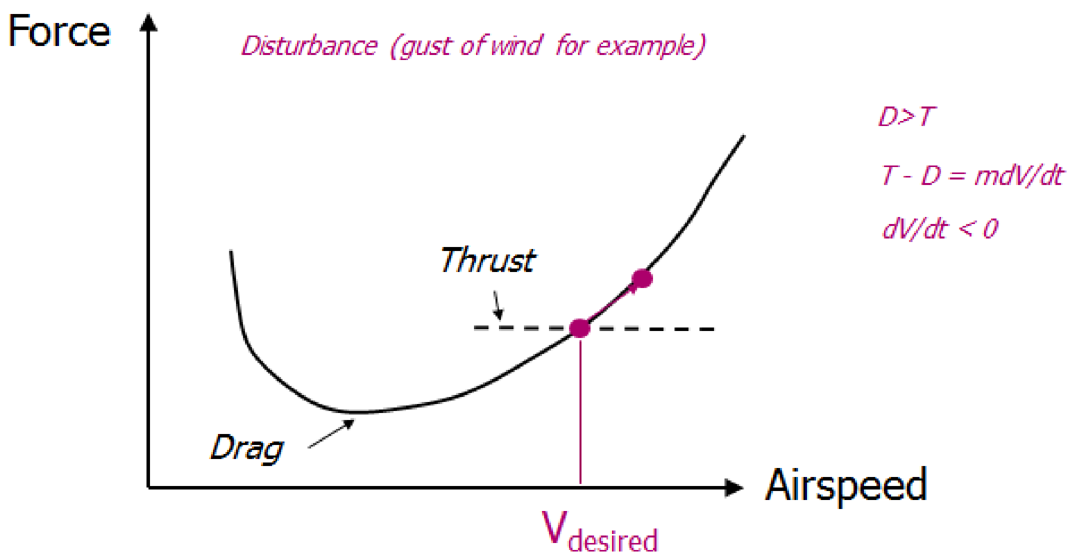
$D = C_D \frac{1}{2}\rho V^2 S$

Thrust must be at maximum to achieve V_{max}. Combining the equations yields:

$$\frac{C_L}{C_D} = \frac{W}{T} = \frac{4270}{950} = 4.49$$
$$C_D = 0.223 C_L$$
Lift drag polar:
$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} = 0.02 + \frac{C_L^2}{16.1}$$
Hence,
$$0.223 C_L = 0.02 + \frac{C_L^2}{16.1} \rightarrow C_L = 0.0923$$
$$L = W = C_L \frac{1}{2} \rho V^2 S$$
$$4270 = 0.0923 \cdot \frac{1}{2} \cdot 1.225 \cdot V^2 \cdot 3.51$$
$$V = 146.7 \; [m/s]$$
E.
Horizontal unsteady flight
$$\gamma = 0$$
$$\frac{W}{g}\frac{dV}{dt} = T - D$$
F.
At airspeeds below the minimum drag condition, the aircraft is unstable. If the aircraft decelerates (or accelerates) due to a disturbance, then the drag will increase (or decrease). The increased drag will make the aircraft decelerate (or accelerate) even more. Hence it is unstable.



At airspeeds above the minimum drag condition, the aircraft is stable. If the aircraft decelerates (or accelerates) due to a disturbance, then the drag will decrease (or increase). The decreased drag will make the aircraft accelerate (or decelerate). Hence it will return to the original airspeed and is stable.



Problem 5

a) Calculate p_o

Use the 2nd form of the isentropic relations:

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

P_e= 1 atm.=$1.01325*10^5$ N/m^2
M_e=3
γ=1.4

P_o=36.73 atm =$3.722*10^6$ N/m^2

b) Calculate T_o

Here too use the 2nd form of the isentropic relations, now for the temperature:

$$\frac{T_o}{T_e} = 1 + \frac{\gamma-1}{2} M_e^2$$

T_e=273.15 + 15=288.15 K
M_e=3
γ=1.4

T_o=806.82 K

c) Calculate V_e

$V_e = M_e * a_e$ $\qquad a_e = \sqrt{\gamma R T_e}$

R=287.15J/kgK
T_e=288.15 K

a_e=340.35 m/s

V_e=3*340.35=1021.05 m/s

d) Calculate V*

V*=M*.a*
M*=1 => V*=a* $\qquad a^* = \sqrt{\gamma R T^*}$
Now calculate T*

$$\frac{T_o}{T^*} = 1 + \frac{\gamma - 1}{2} M^{*2}$$

T*= T_o/1.2 = 672.35 K

V*=a*=$(1.4 \times 287.15 \times 672.35)^{0.5}$ = 519.90 m/s

e) Calculate A_e /A*

The continuity law applies: $\rho^* A^* V^* = \rho_e A_e V_e$ => $A_e / A^* = (\rho^* / \rho_e) \times (V^* / V_e)$

Both the velocities we have calculated. The exit density has the sea level value of 1.225, so now the only variable we need to determine is ρ*.

We have the 2nd form of the isentropic relations for the density:

$$\frac{\rho_o}{\rho^*} = \left(1 + \frac{\gamma - 1}{2} M^{*2} \right)^{\frac{1}{\gamma - 1}}$$

However, we do not yet have the value for the density in the reservoir. This can easily be calculated using the gas law for an ideal gas:

$$\frac{p_o}{\rho_o} = RT_o \qquad => \qquad \rho_o = \frac{p_o}{RT_o}$$

So ρ_o = 16.065 kg/m^3

This gives us ρ^*= 10.184 kg/m^3

Now all values are known we can derive the expansion ratio:

$A_e / A^* = (10.184/1.225) \times (519.9/1021.05)$ => $\underline{\mathbf{A_e / A^* = 4.233}}$

Points:

a) 4
b) 4
c) 5
d) 5
e) 7