# Introduction to Aerospace Engineering

## Exams

## Answers Resit January 27th, 2012

Problem 1.
Correct answer is D

The term snowball-effect describes the effect that if for one system (not only structural - B) the weight is reduced, this will have an impact on a number (not all - C) other systems as well (resulting in additional weight reductions). How the achieved weight reduction is used is not defined by the term (A).

Problem 2

a. For aluminum: Force = Allowable stress x width x thickness
   So, $t = F/(\sigma.w)$
   In direction 1: $t = 75000/(100x300) = 2,5$ mm
   In direction 2: $t = 120000/(100 \times 600) = 2$ mm
   The minimum required thickness is therefore 2,5 mm

   For Carbon composite we can use the same formula:

   In direction 1: $t = 75000/(150x300) = 1,667$ mm
   In direction 2: $t = 120000/(150x600) = 1,333$ mm
   Since the composite is UD, it is able to carry loads in ONE direction only.
   Therefore, we need to add the thicknesses.
   So, the minimum required thickness is this case is 3,0 mm

b. Weight = Width x Length x thickness x density = $w \times L \times t \times \rho$
   For aluminium: 0,3 x 0,6 x 0,0025 x 2800 = 1,26 kg
   For composite: 0,3 x 0,6 x 0,003 x 1800 = 0,972 kg

c. In this answer the choice is not important but the motivation!
   E.g. You can choose for aluminum with the argument that for small product series production costs are very important.
   You could also choose for composites with the argument that over the long life time of the product (30 years or more) the weight advantage of composite will result in much higher cost savings (less fuel required).

Problem 3

3a. [6 pts] Use standard atmosphere formulae for troposphere:
Calculate T with linear gradient a = -0.0065 K/m
T = 216.65 K, p = 22631.7 Pa, rho = 0.3639158 kg/m3

3b. [5 pts] L=W=2451662.5 N
$C_L$ = 0.432424350513
$C_D$ = 0.0340695414415
T = D = 193159.837195 N = 193 kN
Pa per eng = 11924390.3143 W or 11924 kW

3c. [6 pts] Standard configuration, so see Stability & Control hand-out for derivation, also derive downwash derivative deps/dalpha!

3d. [5 pts] From equation it follows: $l_{np}$ = 0.026607492312 $S_h$
Using c = 8.516667 m
$V_h$ = 0.544059
$S_h$ = 75.1668 $m^2$

3e [3 pts] Main advantage: - better longitudinal static stability, hence larger c.g. range
Two disadvantages:
- more weight
- more drag

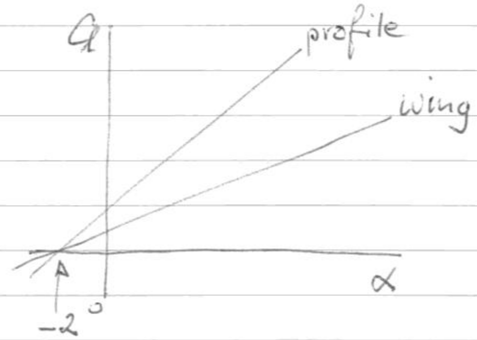
Problem 4

a) $M = ?$ We know that $a = \sqrt{\gamma R T}$
and $M = \frac{V}{a}$ by definition.

$V = \frac{810}{3.6} = 225\ m/s$ , $a = \sqrt{1.4 * 287.05 * 255.7}$
$= 320.56\ m/s$

$M = \frac{225}{320.56} = 0.702$

b)




The profile lift gradient is defined as $\frac{\Delta C_l}{\Delta \alpha}$

$a_0 = \frac{0.66 - 0}{4 - (-2)} = 0.11$ per degree

for the wing we can derive $a = \frac{a_0}{1 + \frac{a_0 * 57.3}{\pi A e}}$

$A = \frac{b^2}{S} = \frac{34.5^2}{149} = 7.99$

$e = 0.82$

hence $a = \frac{0.11}{1 + \left(\frac{57.3 * 0.11}{\pi * 7.99 * 0.82}\right)} = 0.0842$

This is however at the low Mach number.
The aircraft flies at $M = 0.7$

Using the Prandtl-Glauert correction for lift $\left( C_{L_M} = \frac{C_{L_0}}{\sqrt{1-M^2}} \right)$ we arrive at $a = \frac{0.0842}{\sqrt{1-0.7^2}}$
$= 0.1179$

The lift of the wing at $3^\circ$ and $M = 0.7$ is:

$$C_L = 5 * 0.1179 = 0.59$$

We also have $C_D = C_{d_p} + \frac{C_L^2}{\pi A e}$

hence $C_D = 0.0062 + \frac{0.59^2}{\pi * 7.99 * 0.82} = 0.0148$

and so we arrive at $\frac{C_L}{C_D} = \frac{0.59}{0.0148} = 39.86$

T in stagnation point is?

You can use the energy equation:

$$C_p T_1 + \tfrac{1}{2} V_1^2 = C_p T_0 + \tfrac{1}{2} V_0^2$$

or use $\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$, but note that in the latter case the velocity in station 0 is zero.

With the energy equation (0 is at $\infty$, 1 at the stagn. point)

$V_1 = 0$ : $T_1 = \frac{C_p T_0 + \frac{1}{2} V_0^2}{C_p} = 280.8\ K$

Using the second form of the isentropic relations:

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

In this case station 0 is the stagnation point (where $V_0 = 0$, which was the input to the derivation of this formula)

$$\frac{T_0}{T_1} = 1 + \frac{1.4-1}{2} * 0.7^2 = 1.098$$

$$T_0 = T_1 * 1.098 = 255.7 * 1.098 = 280.8 \text{ K}$$

By definition: $C_p = \frac{p - p_0}{q_0}$ or $\frac{p - p_\infty}{q_\infty}$

$$p = C_p q_0 + p_0$$

$$q_0 = \frac{1}{2} \rho V_0^2 = \frac{1}{2} * 0.736 \times 225^2 = 1.863 \times 10^4$$

$$p_0 = 5.41 * 10^4 \text{ N/m}^2$$

$$p = -1.29 * 1.863 * 10^4 + 5.41 * 10^4 = 3.01 \times 10^4 \text{ N/m}^2$$

**Answer to 5a:**
Steady: dV/dt = 0; straight: d$\gamma$/dt = 0; Assumption: small angle approximation → cos($\gamma$) ≅ 0
Thus, the equations of motion simplify to:

$$T - D - W\sin\gamma = 0$$
$$L = W$$

Multiply the first equation with airspeed in order to introduce rate of climb in the equation.

$$TV - DV - WV\sin\gamma = 0$$
$$\frac{P_a - P_r}{W} = V\sin\gamma = RC$$

Maximum rate of climb is achieved when the excess power ($P_a$ - $P_r$) is minimal since we assume a constant aircraft weight. Maximum power available is given to be independent of airspeed. Thus, maximum rate of climb occurs when power required is minimum.

$$P_r = DV$$
$$P_r = D\frac{L}{L}V = \frac{D}{L}WV = \frac{C_D}{C_L}WV$$
$$L = W \rightarrow V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$
$$P_r = \frac{C_D}{C_L}WV = \frac{C_D}{C_L}W\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{W^3}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

Aircraft weight and air density are given so the ratio $C_L^3$ / $C_D^2$ should be maximum. This is the case when the derivative of this ratio to $C_L$ is equal to 0.

$C_L = \sqrt{3C_{D_0}\pi Ae}$ (*this equation should be derived – the derivation can be found in the lecture sheets*)

Fill in the take-off configuration values:

$$C_L = \sqrt{3\cdot 0.03\cdot\pi\cdot 5} = 1.19$$

The drag coefficient, drag, power required and rate of climb can now easily be calculated

$$C_D = C_{D_0} + \frac{C_L^2}{\pi Ae} = 0.03 + \frac{1.19^2}{5\pi} = 0.12$$
$$P_r = \sqrt{\frac{W^3}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}} = \sqrt{\frac{8500^3}{9.84}\frac{2}{1.225}\frac{0.12^2}{1.19^3}} = 29.5\ \text{[kW]}$$
$$RC = \frac{P_a - P_r}{W} = \frac{115 - 29.5}{8.5} = 10.1\ \text{[m/s]}$$

The corresponding airspeed: $V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{8500}{9.84}\frac{2}{1.225}\frac{1}{1.19}} = 34.4\ \text{[m/s]}$

**Answer to 5b:**
Maximum horizontal distance
Steady: dV/dt = 0; straight: dγ/dt = 0; Assumption: small angle approximation → cos(γ) ≅ 0 and finally no engine thrust: T=0
Thus the equations of motion simplify to:

$$-D - W\sin\gamma = 0$$
$$L = W$$

We need to know the flight path angle to calculate the distance:

$$\sin\gamma = -\frac{D}{W}$$
$$\sin\gamma = -\frac{D}{L} = -\frac{C_D}{C_L}$$
$$\gamma = \arcsin\left(-\frac{C_D}{C_L}\right)$$

So the maximum distance is achieved when the ratio $C_L/C_D$ is maximum. This is the case when the derivative of the ratio to $C_L$ equals 0.

$C_L = \sqrt{C_{D_0}\pi Ae}$ (*this equation should be derived – the derivation can be found in the lecture sheets*)
Fill in the clean configuration values:

$$C_L = \sqrt{0.02\cdot\pi\cdot 5.5} = 0.59$$

The drag coefficient can now be calculated

$$C_D = C_{D_0} + \frac{C_L^2}{\pi Ae} = 0.02 + \frac{0.59^2}{5\pi} = 0.04$$

Hence,

$$\gamma = \arcsin\left(-\frac{0.04}{0.59}\right) = -3.9 \text{ [deg]}$$

The distance that can be flown follows from basic trigonometry (3000/tan(3.9)) and equals 44 km.

**Answer to 5c:**
Answer C is correct

**Answer to 5d:**
Answer B is correct