# Introduction to Aerospace Engineering

## Exams

TUDelft

# Answers exam AE1101 November 2010

English:

Question 1

Answers

a. Vertical force results in tension (A, D) and compression (B, C)
b. Horizontal force results in tension (A) and three Zero Force Elements (B,C, D)
c. A diagonal bar from point 2 to point 4 is possible; in case of the vertical force A remains in tension; D will be in compression and B and C will be ZFE. In case of the horizontal force, nothing will change
d. A web plate when air- or liquid tightness is required; when even shear distribution is favorable, etc.

Points: a = 4; b = 4; c = 4; d = 3

Dutch:

Vraag 1

Antwoorden

a. Vertikale kracht geeft trekkracht in A en D, en drukkracht in B en C
b. Horizontale kracht geeft trekkracht in A en geen krachten in B, C en D
c. Een diagonale staaf van punt 2 naar punt 4 is mogelijk; bij de vertikale kracht wordt A op trek belast, D op druk en B en C zijn onbelast; bij de horizontale kracht vindt er geen verandering plaats.
d. Een lijfplaat kan gebruikt worden als lucht- of vloeistofdichtheid een vereiste is, als een gelijkmatige schuifbelasting beter is, enz.

Punten: a = 4; b = 4; c= 4; d = 3

Opgave 2
Points: a=4 b=3 c=3 d=4 e=3 f=3

2a) ISA afleiding gradiënt uit sheets:

$$\frac{\rho_1}{\rho_0} = \left(\frac{T_1}{T_0}\right)^{-\frac{g}{aR}-1}$$

en $T_1 = T_0 + ah$

$$\rho_1 = \rho_0 \left(\frac{T_0 + ah}{T_0}\right)^{-\frac{g}{aR}-1} = \rho_0 \left(1 + \frac{a}{T_0} h\right)^{-\frac{g}{aR}-1}$$

$$\rho(h) = 1.225 \left(1 - \frac{h}{44331}\right)^{4.2577}$$ h in [m], $h < 11000$

b) $$C_{L_{max}} = \frac{m \cdot g}{\frac{1}{2}\rho V_{stall}^2 S'}$$ V EAS $\Rightarrow \rho = \rho_0$

no flaps: $C_{L_{max}} = 1.102 \approx 1.1$

full flaps: $C_{L_{max}} = 1.731 \approx 1.7$

c) $\rho(10668) = 0.3794$ kg/m3 (use 2a)

$$V_{TAS} = \sqrt{\frac{\rho_0}{\rho}} V_{EAS} = \sqrt{\frac{1.225}{0.3794}} \cdot 94 = 169 \text{ kts} = 87 \text{ m/s}$$

d) $$p = \rho R T \Rightarrow \rho = \frac{p}{RT} = \frac{103000}{287.05 \cdot 253.15} = 1.4174 \text{ kg/m}^3$$

$$V^2 = \frac{W}{C_{L_{max}} \cdot \frac{1}{2}\rho S'} \Rightarrow$$ $V_{stall}$ no flaps: 44.95 m/s = 87 kts

$V_{stall}$ full flaps = 35.86 m/s = 70 kts

e) Lift is proportional to dynamic pressure $\frac{1}{2}\rho V^2$, so when density decreases ($h\uparrow$) the speed needs to increase to create sufficient lift. When it is colder, the air is thicker, so the stall speed can be less.

or: via formula show $V_{stall} \sim \sqrt{\frac{1}{\rho}}$

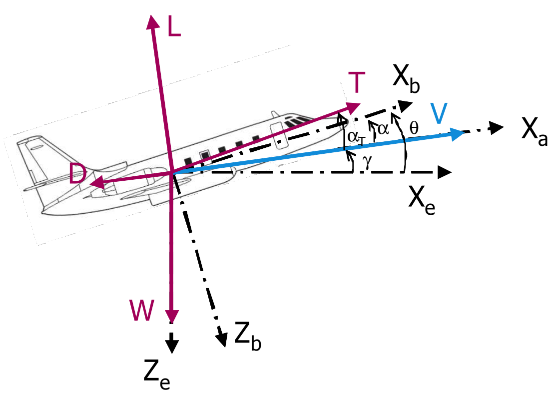
**2f**

- Altitude limit – pressurized cabin (maximum pressure differential on fuselage structure)
- Design diving speed $V_D$ / Maximum operating speed $V_{MO}$. The airplane is designed to withstand particular flight loads at this speed. (Positive and negative gusts of 25 ft/s should be considered $V_D$)
- Design diving Mach number $M_D$ / Maximum operating Mach number $M_{MO}$. The aircraft is designed to remain controllable up to this speed. (undesirable flying qualities, associated with buffeting effect can occur above this Mach number).

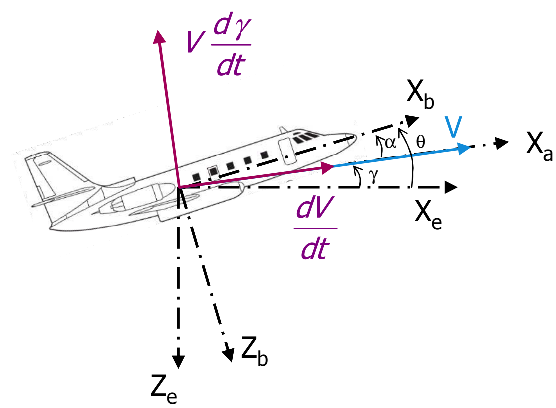
(Note: the difference between $V_D$ and $V_{MO}$, or $M_D$ and $M_{MO}$, is a safety margin. MO stands for maximum operating)

**3a**

Free body diagram




Kinetic diagram



3b

$$\sum F_{//V}: \frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\sum F_{\perp V}: \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

3c

Horizontal flight: The aircraft remains at a constant altitude (γ = 0; dγ/dt = 0)
Steady flight: Flight in which the forces and moments acting on the aircraft do not vary in time, neither in magnitude, nor in direction (dV/dt = 0)
Symmetric flight: flight in which both the angle of sideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth (β = 0 and the aircraft is not turning)

$$\frac{dV}{dt} = 0; \quad \gamma = 0; \quad \frac{d\gamma}{dt} = 0; \quad \alpha_T = 0$$

$$\sum F_{//V}: \frac{W}{g}\cdot 0 = T\cos 0 - D - W\sin 0$$

$$\sum F_{\perp V}: \frac{W}{g}V\cdot 0 = L - W\cos 0 + T\sin 0$$

$$L = W$$

$$T = D$$

3d

The ratio V/F represents airspeed [m/s] divided by fuel flow [N/s] or [kg/s]. Hence:

$$\left[\frac{V}{F}\right] = \left[\frac{m/s}{N/s}\right] = \left[\frac{m}{N}\right]$$

In other words, it is the distance that can be flown per unit of fuel. Clearly this must be maximized to obtain the maximum range.

3e

$$F \hat{=} c_P P_{br} \Leftrightarrow F = c_P \frac{P_a}{\eta}$$

$$T = D$$

$$P_a = P_r$$

$$F = c_P \frac{P_r}{\eta_j} = c_P \frac{DV}{\eta_j}$$

$$\frac{V}{F} = \frac{\eta_j}{c_P} \frac{1}{D}$$

$\eta_j$ and $c_P$ can be assumed to be constant as a function of airspeed (over the range of cruising speeds) for propeller aircraft

$$R_{\max} \Rightarrow \left(\frac{\mathrm{V}}{\mathrm{F}}\right)_{\max} \Rightarrow D_{\min} = \left(\frac{C_D}{C_L} W\right)_{\min} \Rightarrow \left(\frac{C_L}{C_D}\right)_{\max}$$

3f

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = 0$$

$$\frac{C_D \cdot 1 - C_L \cdot \frac{dC_D}{dC_L}}{C_D^2} = 0$$

$$\frac{dC_D}{dC_L} = \frac{C_D}{C_L}$$

$$C_D = C_{D_0} + kC_L^2$$

$$2kC_L = \frac{C_{D_0} + kC_L^2}{C_L}$$

$$C_L = \sqrt{\frac{C_{D_0}}{k}} = \sqrt{\frac{0.0275}{0.0456}} = 0.78$$

3g

$$L = W$$

$$V = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}} = \sqrt{\frac{45000}{28.8} \frac{2}{1.225} \frac{1}{0.78}} = 57.2 \text{ [m/s]}$$

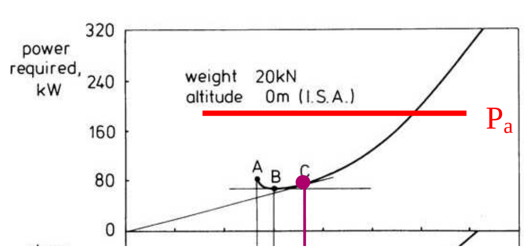
3h

$$C_D = 0.0275 + 0.0456 C_L^2 = 0.0275 + 0.0456 \cdot 0.78^2 = 0.055$$

$$\frac{V}{F} = \frac{\eta_j}{c_P} \frac{1}{D} = \underline{\frac{\eta_j}{c_P} \frac{C_L}{C_D} \frac{1}{W}} = \frac{0.35}{0.108 \cdot 10^{-6} \cdot 9.81} \frac{0.78}{0.055} \frac{1}{45000} = 104 \text{ [m/N]}$$

3i

(The numbers in the figure are not correct, it is only intended to show qualitatively the shape of the figure)



## Problem 4

The following data are given:

$T_0 := 3000 \cdot K$ $\qquad$ $A_t := 0.08 \cdot m^2$

$p_0 := 15 \cdot atm$ $\qquad$ $p_e := 1 \cdot atm$

$R := 378 \cdot \frac{J}{kg \cdot K}$ $\qquad$ $\gamma := 1.26$

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a )

Since the mach number at the exit is given by : $M_e = \frac{V_e}{a_e}$ the speed at the exit is known when $M_2$ and $a_2$ are available. With $a_e = \sqrt{\gamma \cdot R \cdot T_e}$ is becomes clear that we have to calculate the exit temperature first.

Apply the isentropic relations:

$$\frac{p_e}{p_0} = \left(\frac{T_e}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$

With this relation we find the temperature in the exit:

$$T_e := T_0 \cdot \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \quad => \quad T_e = 1715.7\ K$$

$$a_e := \sqrt{\gamma \cdot R \cdot T_e} \quad => \quad a_e = 903.96\ \frac{m}{s}$$

To find the mach number at the exit we apply the total pressure equation:

$$\frac{p_0}{p_e} = \left(1 + \frac{\gamma - 1}{2} \cdot M_e^{\ 2}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{or:} \quad M_e := \sqrt{\left[\left(\frac{p_0}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \cdot \frac{2}{\gamma - 1}} \quad => \quad M_e = 2.4$$

Hence the flow speed at exit becomes : $V_e := M_e \cdot a_e$ $\qquad$ $V_e = 2169.2\ \frac{m}{s}$

b )

The mass flow is given by: $m = \rho \cdot A \cdot V = \rho_t \cdot A_t \cdot V_t$ $\qquad$ (index t = throat)

With supersonic speed in the exit the mach number at the throat is : $M_t := 1$

The temparature in the throat is found by applying:

$$\frac{T_0}{T_t} = 1 + \frac{\gamma - 1}{2} \cdot M_t^2 \qquad \text{or :} \qquad T_t := \frac{T_0}{1 + \frac{\gamma - 1}{2} \cdot M_t^2} \qquad T_t = 2654.9 \text{ K}$$

The speed of sound in the throat becomes: $a_t := \sqrt{\gamma \cdot R \cdot T_t}$ => $a_t = 1124.5 \frac{m}{s}$

and the flow speed:

$$V_t := M_t \cdot a_t \qquad V_t = 1124.5 \frac{m}{s}$$

Finally the density in the throat is found from: $\frac{\rho_0}{\rho_t} = \left(1 + \frac{\gamma - 1}{2} \cdot M_t^2\right)^{\frac{1}{\gamma - 1}}$

Where $\rho_0$ is given by: $\rho_0 := \frac{p_0}{R \cdot T_0}$ => $\rho_0 = 1.340 \frac{kg}{m^3}$

Now : $\rho_t := \frac{\rho_0}{\left(1 + \frac{\gamma - 1}{2} \cdot M_t^2\right)^{\frac{1}{\gamma - 1}}}$ => $\rho_t = 0.838 \frac{kg}{m^3}$

And the massflow becomes: $\text{massflow} := \rho_t \cdot A_t \cdot V_t \qquad \text{massflow} = 75.4 \frac{kg}{s}$

Problem 4: continued

Alternatively we can calculate the velocity at the exit under a) via application of the energy equation:

$$C_p T_0 + \frac{1}{2} V_0^2 = C_p T_e + \frac{1}{2} V_e^2 \qquad 0 = \text{comb. chambe}$$

Since $V_0 = 0$ we find: $V_e^2 = 2 C_p T_0 - 2 C_p T_e$

or $$V_e = \sqrt{2 C_p (T_0 - T_e)} \qquad (1)$$

since only $T_0$ is given we have to find $C_p$ and $T_e$

How to calculate $T_e$ is already shown.

to calculate $C_p$ we write $R = C_p - C_v$ (2)

with $\gamma = \frac{C_p}{C_v}$ (2) transforms in $C_p = \frac{R\gamma}{\gamma - 1}$ (3)

with $R = 378$ J/kgK and $\gamma = 1.26$ we find

$$C_p = 1831.85 \text{ J/kg K}$$

with $T_e = 1715.7$ K we find

$$V_e = \sqrt{2 * 1831.85 (3000 - 1715.7)}$$

$$= 2169.2 \text{ m/s}$$

## Problem 5

a) Compressibility can be neglected when $M < 0.3$

$$M = \frac{V}{a} \qquad V = 245 \ km/hr = 68.06 \ m/s$$

$$a = 328.55 \ m/s$$

$$M = \frac{68.06}{328.55} = 0.207 < 0.3 \quad \text{Incompressible !}$$

b) Bernoulli along a stream line gives:

$$p_0 + \tfrac{1}{2}\rho V_0^2 = p_A + \tfrac{1}{2}\rho V_A^2 \qquad (1)$$

$p_0 = 70121 \ N/m^2$
$\rho_0 = 0.90926 \ kg/m^3$
$V_A = 85 \ m/s$

$$p_A = p_0 + \tfrac{1}{2}\rho\left(V_0^2 - V_A^2\right) \qquad (2)$$

$$= 70121 + 0.5 * 0.90926\left(68.06^2 - 85^2\right)$$

$$= 68942.22 \ N/m^2$$

c) $C_{p_A} = \dfrac{P_A - P_0}{\frac{1}{2}\rho V_0^2} = \dfrac{68942.22 - 70121}{0.5 * 0.90926 * 68.06^2}$

$= -0.560$

d) $\left.\begin{array}{l} C_r = 3 \text{ m} \\ C_t = 2 \text{ m} \end{array}\right\} S = 50 \text{ m}^2$

$\left.\begin{array}{l} b = 20 \text{ m} \\ A = b^2/S \end{array}\right] \longrightarrow A = \dfrac{400}{50} = 8$

Since this is a wing we write

$C_D = C_d + \dfrac{C_L^2}{\pi A e}$ in which $C_d$ is the profile drag coefficient

the drag force comes from

$D = C_d \cdot \frac{1}{2}\rho V^2 S$

First we have to calculate the wing lift coefficient $C_L$ with the information we have from the profile

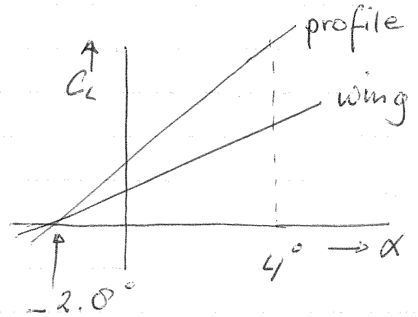
The profile lift gradient is given by:

$a_0 = \dfrac{(0.97 - 0)}{(6 - (-2.8))} = \dfrac{0.97}{0.08} = 0.11 /\text{degr.}$

For a wing we know that the lift-gradient is given by:

$$a = \frac{a_0}{1 + \frac{57.3 * a_0}{\pi A e}}$$

It follows that $a = \frac{0.11}{1 + \frac{57.3 * 0.11}{\pi * 8 * 0.9}} = 0.0855$




The lift coefficient of the wing follows from:

$$C_L(4\,degr) = C_L(-2.8\,degr) + a*(4-(-2.8))$$

$$C_L(4) = 0 + 0.0855 * 6.8 = 0.5814$$

Then: $C_{D_{4\,degr}} = 0.0065 + \frac{0.5814^2}{\pi * 0.8 * 0.9}$

$= 0.0065 + .01494$

$= 0.02144$

The drag force at 4 degrees follows from:

$$D = 0.02144 * \frac{1}{2} * 0.90926 * 68.06^2 * 50$$

$$= 2258.0 N$$

e) The lift-drag ratio of the wing $L/D$ can also be written as $C_L/C_D$

$C_L$ at 4 degr is 0.5814
$C_D$ at 4 degr is 0.02144

$$C_L/C_D = \frac{0.5814}{0.02144} = 27.12$$